Money and Banking

Interest Rates 1/2

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Interest Rates Directly Affect Our Everyday Lives

Why Are Interest Rates Important To Economy?

- The interest rate is the fee for using money, expressed in annual percentage terms.
- Interest rates are one of the most important numbers in the economy because they influence how likely people are to borrow money.
- Low Interest rates $\longrightarrow$ People borrow more money $\longrightarrow$ invest in big things like a business $\longrightarrow$ boost job growth or even wages.

Roadmap

- Presnet value (PV) and discounting the future
- Four Types of Credit Market Intruments
- Yield to Maturity
- Distinction Between Real and Nominal Interest Rates

Present Value

Definition

A dollar paid to you one year from now is less valuable than a dollar paid to you today.

Example

If you make \$100 loan, at the end of the year you will have \$110, which can be written as

$$\$100 \times (1 + 0.1) = \$110 \tag{1}$$

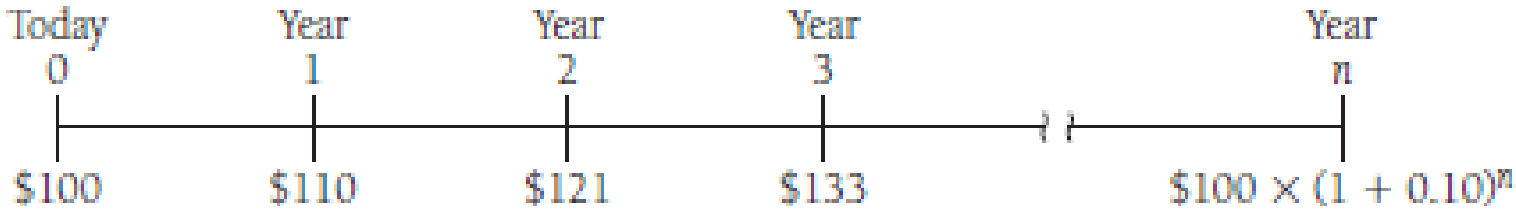
Example

If you then lent out the \$110 at the same interest rate, at the end of the second year you would have

$$\$110 \times (1 + 0.1) = \$121 \tag{2}$$

$$\$100 \times (1 + 0.1) \times (1 + 0.1) = \$121 \tag{3}$$

Present Value



- You are as happy having \$100 today as $100\times(1+0.1)^n$ dollars n years from now.
- $\$133 = 100\times(1+0.1)^3$ three years from now is worth \$100 today.

Discounting the Future

Definition

The process of calculating today's value of dollars received in the future.

$$PV = \frac{CF}{(1+i)^n} \tag{5}$$

PV : present value

CF : cash flow (future payment)

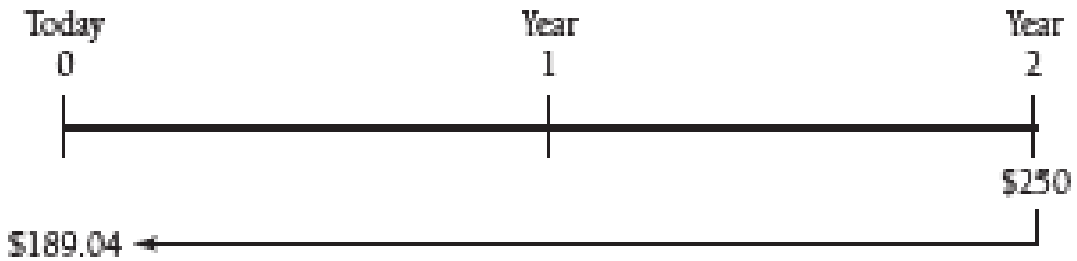
Application 1: Simple Present Value

Problem

What is the present value of $250 to be paid in two years if t he interest rate is 15%?

Solution

$$PV = \frac{\$250}{(1+0.15)^2} = \$189.04 \tag{6}$$



Application 2: Evaluate your friend's offer

Your best friend has just won the scratch-off lottery that promises to pay him \$30,000. Your friend has been told that he must take three yearly payments of \$10,000 each. He knows that you have a huge savings account and says to you, "Here. I'll give you my lottery ticket and you give me \$30,000. It's the same thing."

Should you take the lottery ticket if the interest rate is 10% or 5%?

Application 2: Evaluate your friend's offer

Evaluate your friend's offer.

- If the interest rate is 10%, calculate the present value of receiving \$10,000 per year for three years, compounded annually. [\$24,868.52]
- If the interest rate is 5%, calculate the present value of receiving \$10,000 per year for three years, compounded annually. [\$27,232.48]
- If the interest rate is high, say 20%, will the present value of a payment in the future be large or small? [SMALL]

Four Types of Credit Market Instruments

In terms of the timing of their cash flow payments, there are four basic types of credit market instruments.

- Simple loan. For example, short-term commercial loans to business
- Fixed-payment loan.

Example

If you borrow \$1000, a fixed payment loan might require you to pay \$126 every year for 25 years.

- Coupon bond.

Example

A coupon bond with \$1000 face value, pay you a coupon payment of \$100 per year for ten years, and then repay you the face value amount of \$1000 at the maturity date. The coupon rate is then 10%.

Four Types of Credit Market Instruments

- Discount bond. Bought at a price below its face value (at a discount), and the face value is repaid at the maturity time.

Example

Chris bought a one-year discount bond with a face value of \$1000 for \$900 from Austin, in a year's time, Austin , the owner, would be reapid the face value of \$1000.

YTM on a Simple Loan

Problem

If Austin borrows \$100 from his sister and next year she wants \$110 back from him, what is the yield to maturity on this loan?

Solution

$$100 = \frac{110}{1+i} \tag{7}$$

$$i = 10\% \tag{8}$$

Fact

For simple loans,

$$i = YTM \tag{9}$$

YTM on a Fixed-Payment Loan

Problem

The loan is \$1000 and the yearly payment is \$126 for the next 25 years.

Solution

$$1000 = \frac{126}{1+i} + \frac{126}{(1+i)^2} + ... + \frac{126}{(1+i)^{25}} \tag{10}$$

More generally, for any fixed-payment loan,

$$\begin{aligned} LV &= \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + ... + \frac{FP}{(1+i)^n} \\ LV &= \text{loan value; } FP = \text{fixed yearly payment} \\ n &= \text{number of years until maturity} \end{aligned}$$

Fact

For fixed-payment loan,

$$i = YTM \tag{11}$$

YTM on a Coupon Bond

Problem

Find the price of a 10% coupon bond with a face value of \$1000, a 12.25% yield to maturity, and 8 years to maturity.

Solution

$$PV = C \times \frac{1}{YTM} \times (1 - \frac{1}{(1+YTM)^n}) + \frac{FV}{(1+YTM)^n} \tag{12}$$

FV: face value; PV: present value; t: How many years it takes the security to reach maturity; C: coupon payment

$$C = \frac{\text{coupon rate} \times FV}{\text{number of coupons per year}} \tag{13}$$

$$PV = 100 * \frac{1}{0.1225} * (1 - \frac{1}{(1+0.1225)^8}) + \frac{1000}{(1+0.1225)^8} = 889.24 \tag{14}$$

YTM on a Discount Bond

Problem

What is the yield to maturity on a one-year, \$1000 Treasury bill with a current price of \$900?

Solution

$$PV = \frac{CF}{(1+YTM)^n} \tag{15}$$

$$900 = \frac{1000}{1+YTM} \rightarrow YTM = 11.1\% \tag{16}$$

YTM on a Discount Bond

For any one-year discount bond, the yield to maturity can be written as

$$YTM = \frac{F - P}{P} \tag{17}$$

F : face value of the discount bond

P : current price of the discount bond

YTM on a Discount Bond

Lemma

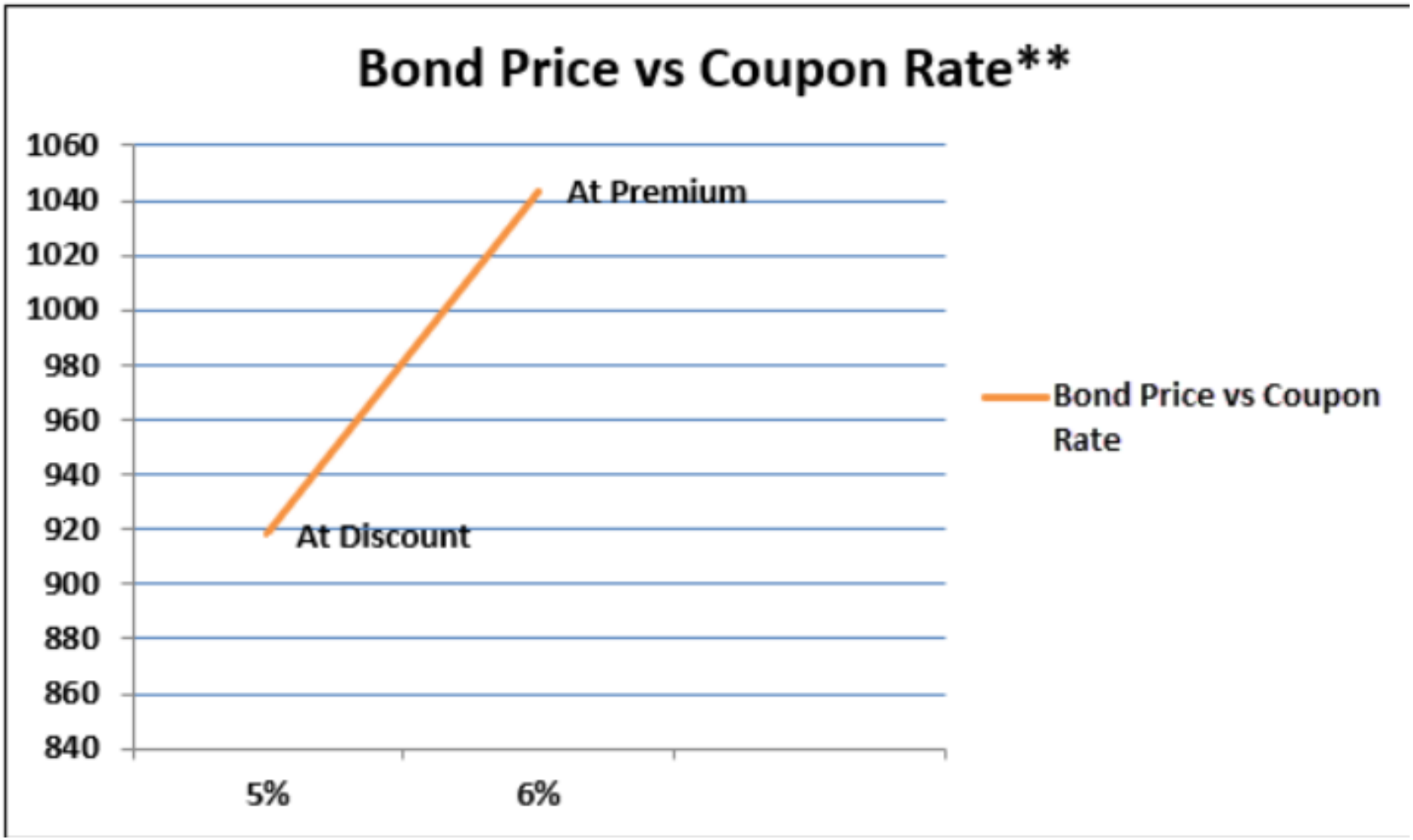
current bond prices and interest rates are negatively related: When the interest rate rises, the price of the bond falls, and vice versa.

YTM vs CR(r)

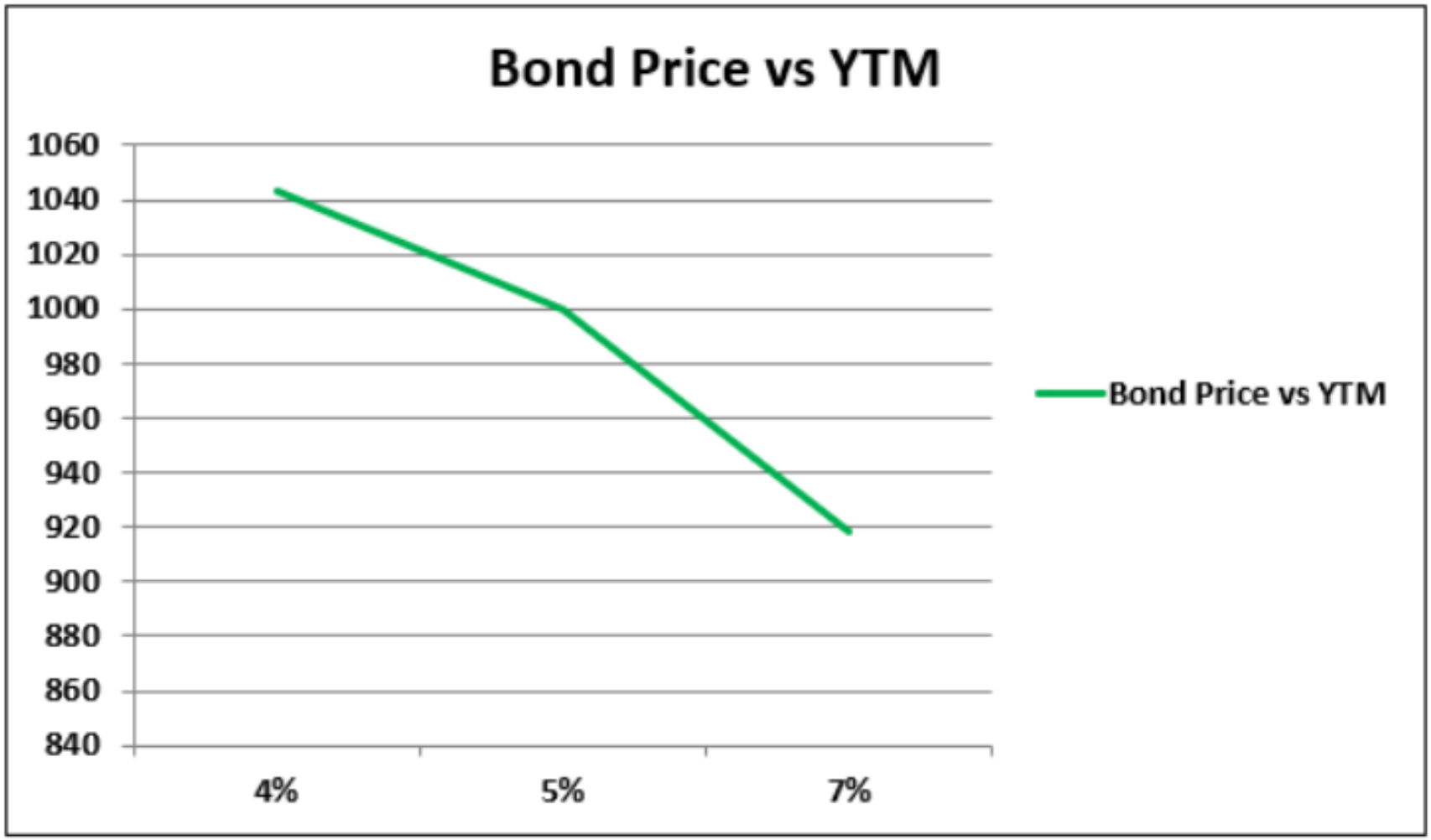
- If r<YTM, the bond price will be less than face value.
- if r=YTM, the bond price will be equal to its face value. This also suggests an inverse relationship between YTM and bond prices.

Face Value	Coupon Rate	YTM	Bond Price	Trading At
1000	6%	4%	1043.3	Premium
1000	5%	7%	918	Discount
1000	5%	5%	1000	Par

Bond Prive vs Coupon Rate



Bond Price vs YTM



Return on a Bond

The return on a bond held from time t to time t+1 can be written as

$$R = \frac{C + P_{t+1} - P_t}{P_t} = \underset{(i_c)}{\frac{C}{P_t}} + \underset{(g)}{\frac{P_{t+1} - P_t}{P_t}} \Rightarrow R = i_c + g \qquad (18)$$

R : return from holding the bond from time t to time t+1

C : coupon payment

P_t : price of the bond at time t

P_{t+1} : price of the bond at time t+1

i_c : current yield

g : rate of capital gain

Return on a Bond

Example

\$1000 face value bond, with coupon rate of 10%, that is bought for \$1000, held for a year, and then sold for \$1200. Then the one-year holding-period return for this bond:

$$\frac{100+200}{1000} = 30\% \tag{19}$$

Example

What is the YTM?

$$1200 = 100 \times \frac{1}{YTM} \times (1 - \frac{1}{1+YTM}) + \frac{1000}{1+YTM} \Rightarrow YTM = ? \tag{20}$$

Return on a Bond

$$YTM \neq R \tag{21}$$

- the return on bond will not necessarily equal the yield to maturity on that bond.

Concept

- A real (effective) interest rate is an interest rate that has been adjusted to remove the effects of inflation
- A nominal interest rate refers to the interest rate before taking inflation into account.
- When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.

Application: Calculating Real Interest Rates

Example

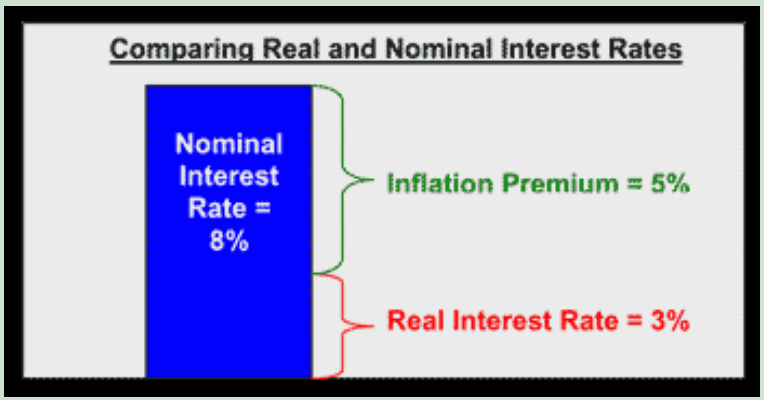
What is the real interest rate if the nominal interest rate is 8% and the expected inflation rate is 5% over the course of a real?

Solution

$$r = i - \pi^e = 8\% - 5\% = 3\% \tag{22}$$

i : *nominal interest rate*

π^e : *expected inflation rate*



Application: Real vs Effective

Problem

Suppose your savings account pay 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have at the end of the year?

$$r = (1 + \frac{i}{n})^n - 1 \tag{23}$$

- interest rate per quarter:

$$i = \frac{9\%}{4} = 2.25\% \tag{24}$$

- Annual real interest rate:

$$r = (1 + \frac{9\%}{4})^4 - 1 = 9.31\% \tag{25}$$

Application: Real vs Effective

- Balance at the end of one year (after 4 quarters)

$$F = \$10,000 * (1 + 9.31\%) = \$10,931 \tag{26}$$