

# Two-Tank Mixing System: Analytical and Numerical Analysis

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## **Abstract**

This project examines the mixing dynamics of salt and pure water in interconnected tanks, studying how external inflows, outflows, and exchanges between the tanks affect salt concentrations over time. A system of first-order differential equations was created and solved both analytically and numerically. In the course of the analytical evaluation, an equation was derived by which the relationship between the concentrations in both tanks can be predicted using only the values of the system parameters. This expression was used to select values for two different cases for which the system was solved.

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# 1 List of Symbols

$t$	Time (minutes).
$S_A(t)$	Salt mass in Tank A at time $t$ (kg).
$S_B(t)$	Salt mass in Tank B at time $t$ (kg).
$\mathbf{S}(t)$	Salt mass vector as time $t$ : $[S_A(t), S_B(t)]^T$ .
$S_A, c_A$	Salt mass and concentration in Tank A respectively at steady-state.
$S_B, c_B$	Salt mass and concentration in Tank B respectively at steady-state.
$\mathbf{S}_{\text{steady}}$	Steady-state salt mass vector $[S_A, S_B]^T$ .
$V_{A+}$	Inflow rate to Tank A (L/min).
$V_{B+}$	Inflow rate to Tank B (L/min).
$V_{A-}$	Outflow rate from Tank A (L/min).
$V_{B-}$	Outflow rate from Tank B (L/min).
$V_{AB}$	Flow rate from Tank A to Tank B (L/min).
$V_{BA}$	Flow rate from Tank B to Tank A (L/min).
$K_A$	Salt concentration in inflow to Tank A (kg/L).
$K_B$	Salt concentration in inflow to Tank B (kg/L).
$A$	Coefficient matrix of the steady-state solution.
$M$	Coefficient matrix of the system.
$\mathbf{b}$	External input vector.
$\lambda_1, \lambda_2$	Eigenvalues of matrix $M$ .
$\mathbf{v}_1, \mathbf{v}_2$	Eigenvectors of matrix $M$ .
$A_\alpha$	Pipe delivering inflow to Tank A.
$B_\alpha$	Pipe delivering inflow to Tank B.
$A_\beta$	Pipe connecting Tank A to Tank B (mixing/exchange).
$B_\beta$	Pipe connecting Tank B to Tank A (mixing/exchange).
$A_\gamma$	Pipe for outflow from Tank A.
$B_\gamma$	Pipe for outflow from Tank B.

## 2 Introduction

We propose a solution to a classic two-tank mixing liquids problem. The liquids in this case are saline and pure water. The system consists of two 100 L interconnected tanks which are referred to as Tank A and Tank B. [Figure 1](#) illustrates the setup of the tanks and connections between them and an external system that feeds and drains saltwater from both tanks

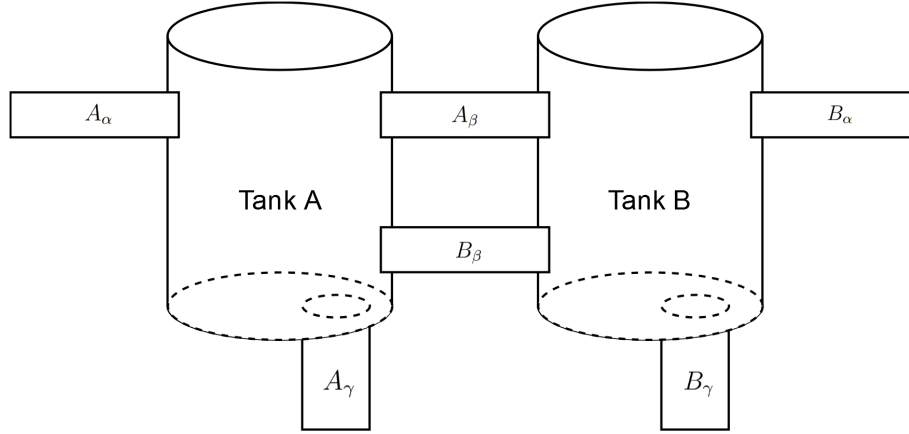


Figure 1: An illustration of the two-tank system

Each tank has four pipes connected to it. For Tank A, the pipe labelled as  $A_\alpha$  delivers water with a salt concentration of  $K_A$  at a rate of  $V_{A+}$ . The two pipes labelled  $A_\beta$  and  $A_\gamma$  drain water from Tank A into Tank B at a rate of  $V_{AB}$  and into the external environment at  $V_{A-}$  respectively. Finally, the pipe  $B_\beta$  delivers salt water from Tank B to Tank A at a rate of  $V_{BA}$ . The setup is the same for Tank B only that the symbols  $A$  and  $B$  are interchanged in the parameters of Tank A to get those of Tank B. [Table 1](#) summarizes inflow and outflow parameters for both tanks.

Table 1: Flow parameters and initial salt contents of both tanks.

	Inflow	Outflow	Initial salt mass
Tank A	$V_{A+} (K_A \text{ conc.}), V_{BA}$	$V_{A-}, V_{AB}$	0 kg
Tank B	$V_{B+} (K_B \text{ conc.}), V_{AB}$	$V_{B-}, V_{BA}$	1 kg

At time  $t = 0$ , the initial salt concentrations in each tank are known, and the system evolves dynamically according to the flow rates and concentrations. The objective is to determine the steady-state salt concentrations in each tank, as well as to analyze how the system evolves under various parameter choices.

### 3 Model

We base our model chiefly on the principle of conservation of mass. That is, the rate of change of salt in each tank is equal to the difference between the rate at which salt enters the tank and the rate at which it leaves.

Let  $S_A(t)$  and  $S_B(t)$  represent the salt concentrations in Tanks A and B respectively at time  $t$ . The total mass of salt in each tank at any time is given by the product of the concentration and the tank volume. This follows from the constraint that the volumes in the tanks remain constant, which can be ensured by choosing the right values for the flow rate parameters. Also, we know from dimensional analysis that the rate at which salt enters or leaves a tank is the product of the flow rate and the concentration.

#### 3.1 Tank A Equation

Each of the four pipes connected to Tank A contribute a term to its differential equation. We designate inflows as positive terms and outflows as negative.  $A_\alpha$  and  $B_\beta$  are inflow pipes while  $A_\beta$  and  $A_\gamma$  are outflow pipes. If we use the names of the pipes to represent their salt throughputs, the rate of change of salt in Tank A is

$$\frac{dS_A(t)}{dt} = A_\alpha + B_\beta - (A_\beta + A_\gamma). \quad (3.1)$$

*Equation 3.1* can be written in terms of the flow rates and concentrations of the liquids in the pipes as

$$\frac{dS_A(t)}{dt} = V_{A+}K_A + V_{BA}\frac{S_B(t)}{100\text{ L}} - \left( V_{AB}\frac{S_A(t)}{100\text{ L}} + V_{A-}\frac{S_A(t)}{100\text{ L}} \right). \quad (3.2)$$

This gives the first half of a system of differential equations whose solution can be used to predict the salt concentrations of both tanks over time.

#### 3.2 Tank B Equation

We perform the same analysis for Tank B as for Tank A, yielding the following differential equation in terms of the salt throughputs of the pipes connected to it:

$$\frac{dS_B(t)}{dt} = B_\alpha + A_\beta - (B_\beta + B_\gamma). \quad (3.3)$$

Hence, the differential equation for Tank B is

$$\frac{dS_B(t)}{dt} = V_{B+}K_B + V_{AB}\frac{S_A(t)}{100\text{ L}} - \left( V_{BA}\frac{S_B(t)}{100\text{ L}} + V_{B-}\frac{S_B(t)}{100\text{ L}} \right). \quad (3.4)$$

*Equation 3.4* is the other half of the system of ODEs that model the given problem.

### 3.3 Assumptions

The equations derived in the preceding sections rely on simplifying assumptions about the system which are outlined below.

1. The salt concentration in a tank at any moment in time is the same throughout its volume.
2. The volumes in each tank remain constant at 100 L. We achieve this by making the sum of inflow rates for each tank equal the sum of outflow rates, which is expressed as

$$V_{A+} + V_{BA} = V_{A-} + V_{AB}, \text{ and} \quad (3.5)$$

$$V_{B+} + V_{AB} = V_{B-} + V_{BA}, \quad (3.6)$$

which simply reduce to

$$V_{A+} + V_{B+} = V_{A-} + V_{B-}. \quad (3.7)$$

These assumptions simplify the system's dynamics, removing the need to account for spatial variations or changing volumes.

## 4 Solutions

Since our model is a system of two linear ordinary differential equations, an analytical solution is feasible. We begin by solving the system analytically for choice values of the parameters. Next, we provide a numerical solution for the same set of parameters.

### 4.1 Analytical solution

We start by considering the steady state situation. Let  $t_s$  be the time at which the system reaches its equilibrium. Also, let

$$\begin{aligned}
S_A &= S_A(t_s) & c_A &= \frac{S_A}{100} \\
S_B &= S_B(t_s) & c_B &= \frac{S_B}{100}
\end{aligned}$$

be the masses and concentrations of salt in Tank A and Tank B respectively when  $t = t_s$ . At equilibrium, the rates of change of salt in both tanks are zero. That is, for Tank A,

$$\frac{dS_A(t)}{dt} = 0 \quad (4.1)$$

$$\Rightarrow V_{A+}K_A + V_{BA}\frac{S_B}{100\text{ L}} - \left( V_{AB}\frac{S_A}{100\text{ L}} + V_{A-}\frac{S_A}{100\text{ L}} \right) = 0 \quad (4.2)$$

$$\Rightarrow (100\text{ L})V_{A+}K_A = (V_{AB} + V_{A-})S_A - V_{BA}S_B. \quad (4.3)$$

The steady-state equation for Tank B can be derived in a similar manner as

$$(100\text{ L})V_{B+}K_B = -V_{AB}S_A + (V_{BA} + V_{B-})S_B. \quad (4.4)$$

[Equation 4.3](#) and [Equation 4.4](#) form a system of two linear equations which can be solved for  $S_A$  and  $S_B$  simultaneously. We employ the use of matrices to simplify the problem by setting

$$\begin{aligned}
A &= \begin{bmatrix} V_{AB} + V_{A-} & -V_{BA} \\ -V_{AB} & V_{BA} + V_{B-} \end{bmatrix}, \\
\mathbf{S}_{\text{steady}} &= \begin{bmatrix} S_A \\ S_B \end{bmatrix}, \text{ and} \\
\mathbf{b} &= \begin{bmatrix} V_{A+}K_A \\ V_{B+}K_B \end{bmatrix}.
\end{aligned}$$

It follows that

$$A\mathbf{s}_{\text{steady}} = (100\text{ L})\mathbf{b} \quad (4.5)$$

$$\Rightarrow \mathbf{s}_{\text{steady}} = (100\text{ L})A^{-1}\mathbf{b} \quad (4.6)$$

$$\Rightarrow \begin{bmatrix} S_A \\ S_B \end{bmatrix} = \frac{(100\text{ L})}{V_{A-}V_{BA} + V_{AB}V_{B-} + V_{A-}V_{B-}} \begin{bmatrix} K_A V_{A+}(V_{BA} + V_{B-}) + K_B V_{BA}V_{B+} \\ K_B V_{B+}(V_{AB} + V_{A-}) + K_A V_{AB}V_{A+} \end{bmatrix} \quad (4.7)$$

$$\Rightarrow \begin{bmatrix} c_A \\ c_B \end{bmatrix} = \frac{1}{V_{A-}V_{BA} + V_{AB}V_{B-} + V_{A-}V_{B-}} \begin{bmatrix} K_A V_{A+}(V_{BA} + V_{B-}) + K_B V_{BA}V_{B+} \\ K_B V_{B+}(V_{AB} + V_{A-}) + K_A V_{AB}V_{A+} \end{bmatrix}. \quad (4.8)$$

$$(4.9)$$

We consider two possible cases at equilibrium:

1. The two tanks contain the same quantity of salt. That is

$$c_A = c_B \quad (4.10)$$

$$\Rightarrow K_A V_{A+}(V_{BA} + V_{B-}) + K_B V_{BA}V_{B+} = K_B V_{B+}(V_{AB} + V_{A-}) + K_A V_{AB}V_{A+} \quad (4.11)$$

$$\Rightarrow K_A V_{A+}(V_{BA} + V_{B-}) - K_A V_{AB}V_{A+} = K_B V_{B+}(V_{AB} + V_{A-}) - K_B V_{BA}V_{B+} \quad (4.12)$$

$$\Rightarrow K_A V_{A+}(V_{BA} + V_{B-} - V_{AB}) = K_B V_{B+}(V_{AB} + V_{A-} - V_{BA}) \quad (4.13)$$

$$\Rightarrow \frac{K_A V_{A+}}{K_B V_{B+}} = \frac{V_{AB} + V_{A-} - V_{BA}}{V_{BA} + V_{B-} - V_{AB}}. \quad (4.14)$$

2. Tank A contains more salt than Tank B. From [Equation 4.14](#), this implies

$$\frac{K_A V_{A+}}{K_B V_{B+}} > \frac{V_{AB} + V_{A-} - V_{BA}}{V_{BA} + V_{B-} - V_{AB}}. \quad (4.15)$$

We find parameters that satisfy each relation defined above as well as [Equation 3.7](#) by creating one comparator for each relation and calling the `find_params()` function defined in [8.1](#). The code listing below shows how this is done.

```
gt_comp = @(a,b) a > b;
eq_comp = @(a,b) a == b;

gt_params = find_params(gt_comp);
eq_params = find_params(eq_comp);
```

[Table 2](#) shows the result of running the above code block.



Table 2: Parameter values that satisfy the two cases.

	=	>
$V_{A+}$ (L/min)	2.00	2.00
$V_{A-}$ (L/min)	2.10	2.05
$V_{B+}$ (L/min)	2.20	2.05
$V_{B-}$ (L/min)	2.10	2.00
$V_{AB}$ (L/min)	2.80	2.00
$V_{BA}$ (L/min)	2.10	2.25
$K_A$ (kg/L)	0.22	0.10
$K_B$ (kg/L)	0.10	0.12

Let  $M = -\frac{1}{100}A$  and

$$\mathbf{S}(t) = \begin{bmatrix} S_A(t) \\ S_B(t) \end{bmatrix}. \quad (4.16)$$

Consequently,

$$\frac{d}{dt}\mathbf{S}(t) = M\mathbf{S}(t) + \frac{1}{100}\mathbf{b}, \quad (4.17)$$

which is solved by (see [1])

$$\mathbf{S}(t) = c_1 e^{\lambda_1 t} \mathbf{v}_1 + c_2 e^{\lambda_2 t} \mathbf{v}_2 + \mathbf{S}_{\text{steady}}. \quad (4.18)$$

$c_1$  and  $c_2$  are constants that can be determined from the initial conditions of the system, while  $\lambda_1$  and  $\lambda_2$  are the eigenvalues of  $M$ , with corresponding eigenvectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  respectively. Lastly,  $\mathbf{S}_{\text{steady}}$  is the vector of steady-state salt quantities from earlier.

#### 4.1.1 A solution with equal concentrations

From the choice of parameters in [Table 2](#), the coefficient matrix when  $c_A = c_B$  is

$$M = \frac{1}{100} \begin{bmatrix} -4.90 & 2.10 \\ 2.80 & -4.20 \end{bmatrix}, \quad (4.19)$$

having the following eigenvalues and eigenvectors.

$$\begin{aligned}\lambda_1 &= -0.070 & \mathbf{v}_1 &= \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \lambda_2 &= -0.021 & \mathbf{v}_2 &= \begin{bmatrix} -0.60 \\ -0.80 \end{bmatrix}\end{aligned}$$

The steady-state solution for these parameters is

$$\mathbf{S}_{\text{steady}} = \begin{bmatrix} S_A \\ S_B \end{bmatrix} = \frac{110}{7} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (4.20)$$

Hence, by [Equation 4.18](#),

$$\mathbf{S}(t) = \frac{1}{\sqrt{2}} c_1 e^{-0.070t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{-0.021t} \begin{bmatrix} -0.60 \\ -0.80 \end{bmatrix} + \frac{110}{7} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4.21)$$

$$\Rightarrow \mathbf{S}(0) = \frac{1}{\sqrt{2}} c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -0.60 \\ -0.80 \end{bmatrix} + \frac{110}{7} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4.22)$$

$$\Rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -0.60 \\ -0.80 \end{bmatrix} + \frac{110}{7} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4.23)$$

$$\Rightarrow \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} 110 \\ 103 \end{bmatrix} \quad (4.24)$$

$$\Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}^{-1} \begin{bmatrix} 110 \\ 103 \end{bmatrix} \quad (4.25)$$

$$= \frac{1}{49} \begin{bmatrix} 131\sqrt{2} \\ 1065 \end{bmatrix} \quad (4.26)$$

$$\Rightarrow \mathbf{S}(t) = \frac{131}{49} e^{-0.070t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{1065}{49} e^{-0.021t} \begin{bmatrix} -0.60 \\ -0.80 \end{bmatrix} + \frac{110}{7} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (4.27)$$

[Equation 4.27](#) gives the quantity of salt in both tanks for every time  $t > 0$  in minutes. Note also that as  $t \rightarrow \infty$ , the mass of salt in both tanks are equal and approach  $\frac{110}{7}$  kg—the steady-state result.

#### 4.1.2 A solution with a higher concentration of salt in Tank A

Similar to the previous section, we define the coefficient matrix for when  $c_A > c_B$  using the parameters in the  $>$  column of [Table 2](#):

$$M = \frac{1}{100} \begin{bmatrix} -4.05 & 2.25 \\ 2.00 & -4.25 \end{bmatrix}, \quad (4.28)$$

with eigenvalues and eigenvectors (rounded to five decimal places) and values of

$$\begin{aligned} \lambda_1 &= -0.02026 & \mathbf{v}_1 &= \begin{bmatrix} 0.74351 \\ 0.66872 \end{bmatrix} \\ \lambda_2 &= -0.06274 & \mathbf{v}_2 &= \begin{bmatrix} -0.71126 \\ -0.70293 \end{bmatrix}. \end{aligned}$$

At steady-state, the quantities of salt in both tanks are

$$\mathbf{S}_{\text{steady}} = \begin{bmatrix} S_A \\ S_B \end{bmatrix} = \begin{bmatrix} 11.04031 \\ 10.98368 \end{bmatrix}. \quad (4.29)$$

[Equation 4.29](#) shows that, as predicted, Tank A has a greater salt content than Tank B. We determine the transient solution, for all time  $t > 0$ , in a similar manner as in [4.1.1](#) to be

$$\mathbf{S}(t) = -14.88728e^{-0.02026t} \begin{bmatrix} 0.74351 \\ 0.66872 \end{bmatrix} - 0.04014e^{-0.06274t} \begin{bmatrix} -0.71126 \\ -0.70293 \end{bmatrix} + \begin{bmatrix} 11.04031 \\ 10.98368 \end{bmatrix}. \quad (4.30)$$

Unlike [Equation 4.27](#), [Equation 4.30](#) uses approximate values for the different quantities which could pose a problem during comparative analysis with the numerical method.

## 4.2 Numerical solution

We solve the system numerically by employing the use of MATLAB's `ode45` solver as documented in [2]. The function `numerical_solution()`, which is defined in [8.2](#) takes a vector of the parameters of the system as an argument and returns the result of calling `ode45` on a model defined with those parameters. In the code listing below, `numerical_solution()` is called twice—once for the set of parameters of each case, then the results are plotted in two separate figures—[Figure 2](#) and [Figure 3](#).

```

[t_eq, c_eq] = numerical_solution(eq_params);
[t_gt, c_gt] = numerical_solution(gt_params);

% Plot results
figure;
plot(t_eq, c_eq);
legend('Concentration in Tank A', 'Concentration in Tank B');
xlabel('Time (minutes)');
ylabel('Salt Concentration (kg/L)');
title('Salt Dynamics for Case 1: Equal Concentrations');
grid on;

figure;
plot(t_gt, c_gt);
legend('Concentration in Tank A', 'Concentration in Tank B');
xlabel('Time (minutes)');
ylabel('Salt Concentration (kg/L)');
title('Salt Dynamics for Case 2: Tank A > Tank B');
grid on;

```

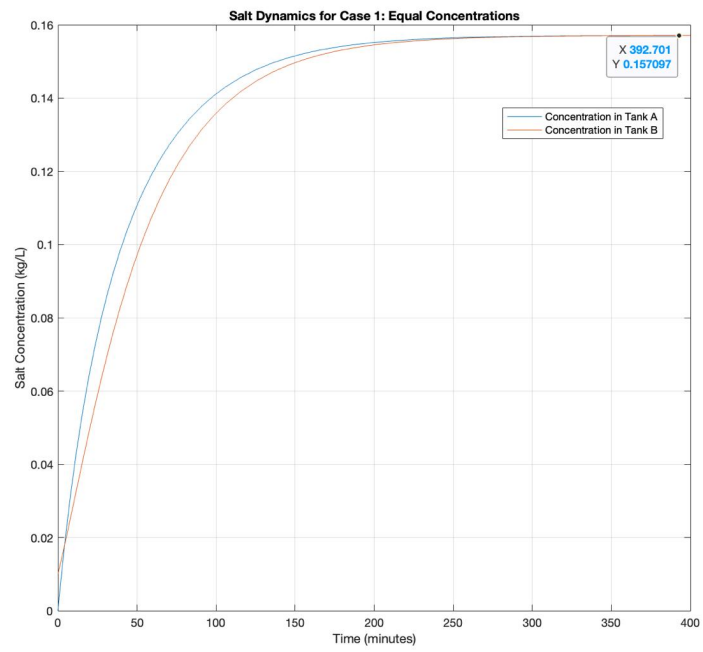


Figure 2: Evolution of salt concentration in Tanks A and B for Case 1.

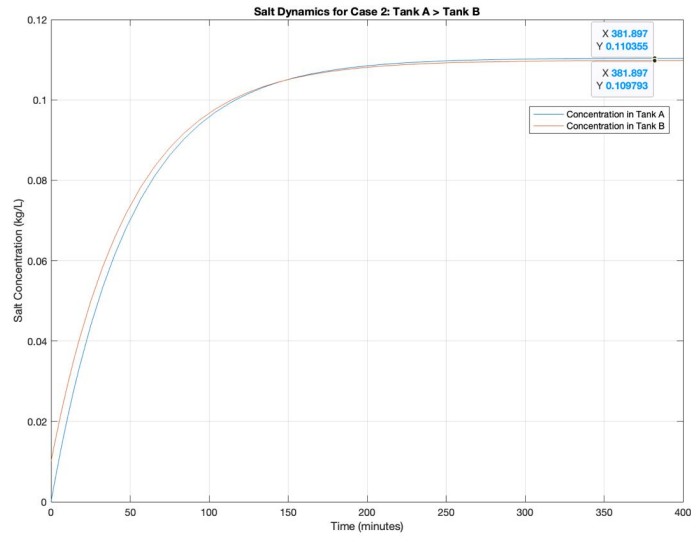


Figure 3: Evolution of salt concentration in Tanks A and B for Case 2.

A cursory glance at both figures would show that in both cases, the salt concentrations converge to about the same values that the analytical solution predicted. This, as well as the time evolution of both models, will be investigated in the next chapter.

## 5 Methods

To see how well the analytical and numerical solutions match, we plot the time evolution of the concentration of salt in Tank A predicted by both methods on the same figure. *Figure 4* shows this plot for Case 1, while *Figure 5* shows that for Case 2.

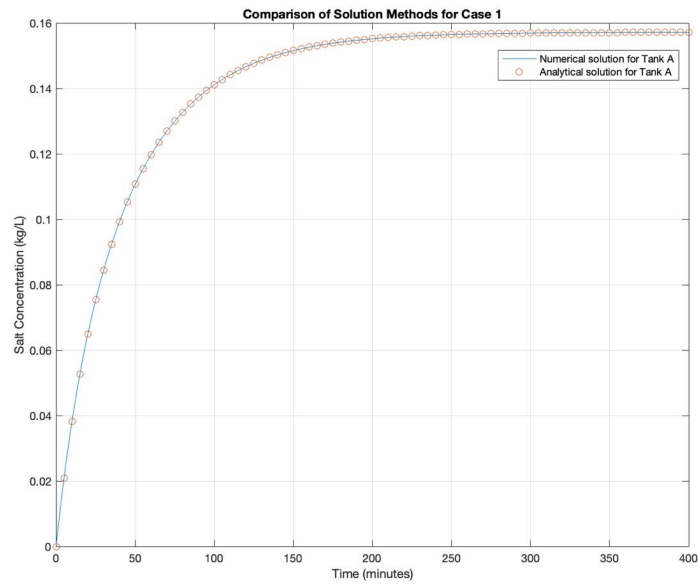


Figure 4: Salt concentration in Tank A over time for Case 1 as predicted by both solution methods.

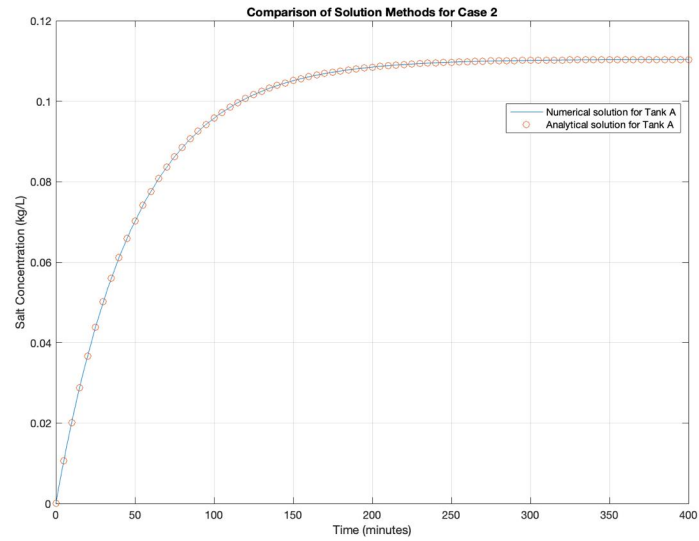


Figure 5: Salt concentration in Tank A over time for Case 2 as predicted by both solution methods.

As can be seen, the numerical solution matches closely with the analytical solution and vice versa even in the second case when non-exact values of the equation constants were used.

The analytical solution offers a precise representation of the system's behaviour in terms of eigenvalues, eigenvectors, and steady-state values, which allows for a deeper understanding of the system's dynamics. Deriving the analytical solution, however, is mathematically intensive. Additionally, analytical methods are often limited to linear systems with constant coefficients, making them intractable for more complex or non-linear problems. In contrast, numerical methods are well-suited for solving a broader range of problems, including non-linear systems and those with time-varying parameters.

## 6 Results

From the solutions to the system presented in 4, both tanks approach a constant concentration of approximately 0.1571 kg/L in the first case. In Case 2, the liquid in Tank A approaches a constant concentration of 0.1104 kg/L, while the concentration of that of Tank B approaches 0.1098 kg/L. Also, neither tank approach its steady-state faster than the other in each case.

Basing our model on the principle of conservation of mass provides a solid foundation for a correct analysis of the actual, real-world behaviour of the system. Though simple, the solutions proposed remain valid for a variety of system parameters, as long as they do not break the starting assumptions.

These assumptions, however, limit the model's practicability. In real-world situations, the assumptions of perfect mixing and constant liquid volume may be very difficult to ensure, leading to significant disparities between model predictions and actual measurements. Moreover, the model's exclusion on non-linearities is sure to hamper its performance in actual liquid-mixing problems.

## 7 Summary

Throughout the project, we explored the mixing dynamics of a two-tank saltwater system, with an in-depth usage of analytical and numerical methods to model and predict the system's evolution and steady-state behavior. Using mass conservation principles, a model of first-order differential equations was derived to illustrate the salt concentrations in the two tanks. Regarding the analytical solution, it provided insights into steady-state conditions and transient dynamics, with cases examined for both equal and unequal steady-state concentrations. While with the numerical method, its simulations with MATLAB's ode45 confirmed the analytical results, ensuring the consistency despite approximated values.

To sum up, the study showed that the system reliably reaches steady-state concentrations under given assumptions, such as perfect mixing or constant tank volumes. However, despite their simplicity, these assumptions may limit real-world applicability. Hence for more complex scenarios, we can pivot by, for instance, extending the model to account for non-linearities and variable volume. Combining both analytical and numerical precision and versatility, the project has



provided an in-depth, complete approach to coupled mixing systems studies and designs within various engineering and environmental contexts.

## 8 Appendices

### 8.1 Parameter search function

```
function params = find_params(comparator)
ks = 0.1:0.02:0.5;
vs = 2:0.05:3;

N = numel(vs) * 4;
i = 1;
rates = zeros(N,4);
stop = 0;
for v_ap = vs
    for v_bp = vs
        for v_am = vs
            for v_bm = vs
                if (v_ap + v_bp) == (v_am + v_bm)
                    rates(i,:) = [v_ap, v_am, v_bp, v_bm];
                    i = i + 1;
                    if i > N
                        stop = 1;
                        break;
                    end
                end
            end
        end
    end
    if stop == 1
        break;
    end
end
if stop == 1
    break;
end
end
if stop == 1
    break
end
```

```

    end
end

for i = 1:N
    v_ap = rates(i, 1);
    v_am = rates(i, 2);
    v_bp = rates(i, 3);
    v_bm = rates(i, 4);

    for ka = ks
        for kb = ks
            for v_ab = vs
                for v_ba = vs
                    lhs = (ka*v_ap) / (kb*v_bp);
                    rhs = (v_ab + v_am - v_ba) / (v_ba + v_bm - v_ab);
                    is_interesting = v_ap ~= v_am && v_ab ~= v_ba && ka ~= kb;

                    if comparator(lhs,rhs) && is_interesting
                        params = [v_ap, v_am, v_bp, v_bm, ka, kb, v_ab, v_ba]';
                        return
                    end
                end
            end
        end
    end
end
end
end
end
end
end
end

```

## 8.2 Numerical solution functions

```

function [t, c] = numerical_solution(params)
    V_Ap = params(1); V_Am = params(2); V_Bp = params(3); V_Bm = params(4);
    K_A = params(5); K_B = params(6); V_AB = params(7); V_BA = params(8);

    % Time span
    tspan = [0, 400];

    % Initial conditions

```

```

x0 = [0; 1];

% Solve ODE
[t, x] = ode45(@(t, x) salt_dynamics(t, x, V_Ap, V_Am, V_Bp, ...
    V_Bm, V_AB, V_BA, K_A, K_B), tspan, x0);

% Convert salt amounts to concentrations
c = S / 100;
end

function dxdt = salt_dynamics(~, x, V_Ap, V_Am, V_Bp, V_Bm, V_AB, V_BA, K_A, K_B)
    x_A = x(1); % Salt in Tank A
    x_B = x(2); % Salt in Tank B

    % Tank A
    dx_A = V_Ap * K_A - V_Am * x_A / 100 - V_AB * x_A / 100 + V_BA * x_B / 100;

    % Tank B
    dx_B = V_Bp * K_B - V_Bm * x_B / 100 - V_BA * x_B / 100 + V_AB * x_A / 100;

    dxdt = [dx_A; dx_B];
end

```

## References

- 1 Boyce, W.E. and DiPrima, R.C. (2017) Elementary Differential Equations and Boundary Value Problems. 11th ed., Wiley, Hoboken, NJ, USA.
- 2 MathWorks. (2024) Solve Nonstiff Differential Equations — MATLAB Ode45. <https://www.mathworks.com/help/matlab/ref/ode45.html>.