

①

a)  $f$  - center frequency

$c$  - speed of sound

$\lambda$  - wavelength

$$\underline{\underline{\lambda = c/f}}$$

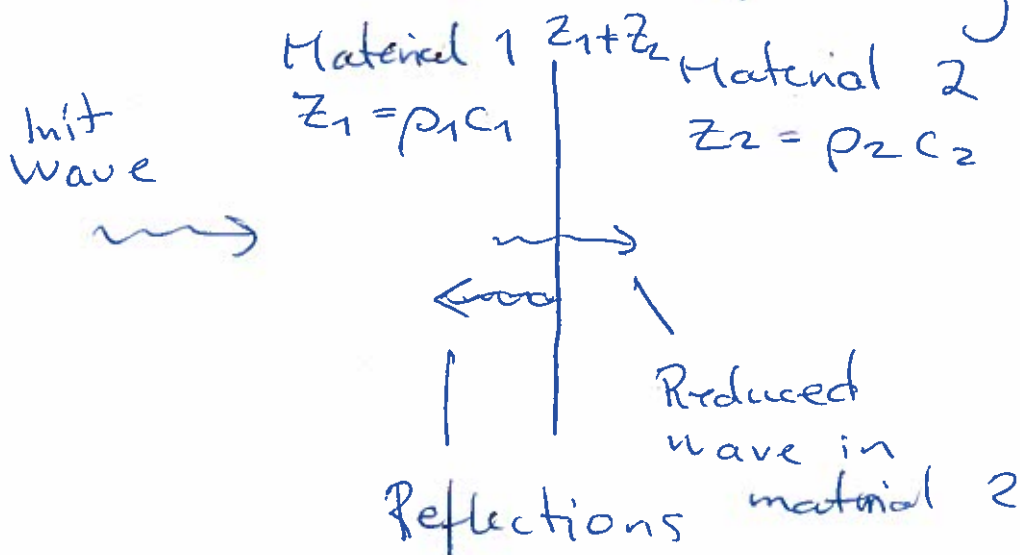
b)

Acoustic impedance gives the relationship between acoustic pressure and volume flow:

$$Z = P/U \Rightarrow \text{Impedance} = \frac{\text{Pressure}}{\text{Flow}}$$

The specific impedance for a material is dependent on the material  $z = \rho c$ .

Variations in specific acoustic impedance between two neighbouring materials will lead to reflections of incoming waves



- 1 c) The speed of sound,  $c$ , varies dependent of materials mass density,  $\rho$ , and compressibility,  $\chi$ .  
The relation is given as

$$c = \frac{1}{\sqrt{\rho \chi}}$$

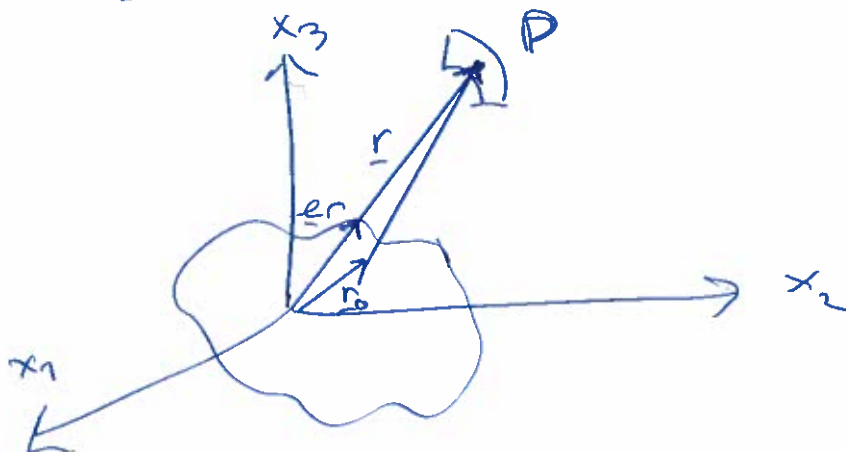
- d) The Fraunhofer approximation defines simplifications/assumptions we can use when the distance from the transducer is large enough.

The Fraunhofer approximation gives that the time/phase lag can be approximated by:

$$\frac{|\underline{r} - \underline{r}_0|}{c} \approx \frac{r - \underline{e}_r \cdot \underline{r}_0}{c}$$

And for amplitude:

$$\frac{1}{4\pi |\underline{r} - \underline{r}_0|} \approx \frac{1}{4\pi r}$$



1e) The Fraunhofer approximation holds for distance larger than  $D^2/2\lambda$  where  $D$  is diameter and  $\lambda$  wavelength.

f) For soft biological materials three major mechanisms for attenuation are:

- Absorption
- Reverberation
- Phase front aberration

g) In ultrasound imaging the blood produces backscattering in the form of Rayleigh scattering. Rayleigh scattering have much less dimensions than ultrasound wavelength.

h) When we send out a ultrasound wave we can in the receiving sweep from far distance to very close, this is called dynamic focusing. In the receive we can adjust for the time delays in the generated pulses, and easier make the dynamic focusing.

1i) Harmonic imaging means to filter the received image by the second harmonic of the sent frequency. This will give an image which is less influenced by low frequency noise elements, and the filtered image will potentially contain less noise.

j) For imaging one would use a probe with phased linear arrays. For heart imaging a center frequency of 2.5 MHz is typically used.

Using phased array gives the opportunity to obtain an image between the ribs, which potentially would lead to no image with linear arrays.

2.5 MHz gives the opportunity to obtain a detailed image depicting the motion of heart valves and blood flow. To get precise motion of valves higher frequency would be useful, but would give ~~more noisy image~~ less depth due to attenuation.

1k) The point spread function defines the spatial resolution of the image. The resolution along the beam is determined by the length of the transmitted pulse. The resolution transverse to the beam is determined by the width of the beam. The width of the beam depends on the aperture and the length of the transmitted pulse depends on the frequency.

l) Mechanical Index (MI) estimates the potential for mechanical bioeffects.

Transfer of momentum from propagating waves can release energy due to collapse and formation of gas bubbles in liquid.

Thermal Index (TI) estimates the potential for bioeffects due to change in temperature in the imaged area.

The concern is based on the knowledge that cellular activity can change as a consequence of temperature by ultrasound waves.



m)

- Using high MI can lead to cavitation, meaning formation and collapse of gas bubbles in liquid where the pressure falls below the vapor pressure.

n)

- Adjusting the gain level and average the received signal may enhance SNR.

Adjusting gain will give a signal with more intensity, but will potentially also gain up the noise.

- Averaging the signal will remove random noise contribution, and only leave real noise in the received signal and also enhance the SNR greatly.

- Reverberations are caused by multiple reflections of the transmitted pulse. The reflected pulse will potentially be reflected by other interfaces. ~~and~~ The reflected pulse may be partly reflected and transmitted multiple times.

- Reverberations emerged ~~for~~ from inhomogeneous areas with multiple materials with different mass density and compressibility.

②  $f_0 = 2.5 \text{ MHz}$ ,  $B = 2 \text{ MHz}$ , Dimension =  $2 \times 15 [\text{m}]$   
 $C = 1540 \text{ m/s}$ ,  $T = 2 \text{ cycles}$

a) PRF must follow:

$$\frac{1}{\text{PRF}} > 2 \cdot \frac{\text{depth}}{C}$$

$$\text{depth} = 0.12 \text{ m}$$

$$\frac{1}{\text{PRF}} > 2 \cdot \frac{0.12 \text{ m}}{1540 \text{ m/s}} = \frac{0.24}{1540} \text{ s}$$

$$\frac{1}{\text{PRF}} > 1.56 \cdot 10^{-4} \text{ s}$$

$$\text{PRF} < 6416.7 \text{ Hz}$$

b)

Sampling frequency needs to be twice the highest frequency content to avoid loss of information.

$$f_s \geq 2 \cdot \left( f_0 + \frac{B}{2} \right)$$

highest frequency content

$$f_s \geq 7.5 \text{ MHz}$$

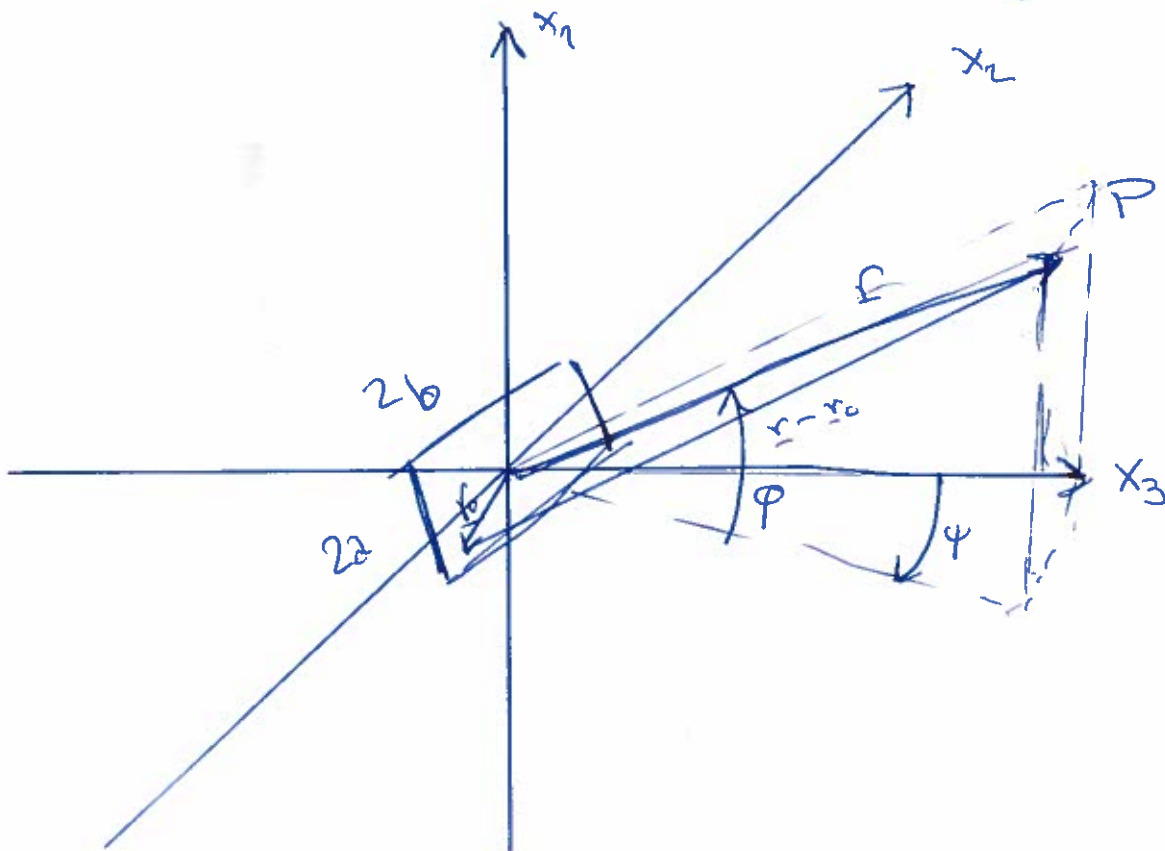
2c)

If the distance between two neighbouring elements exceeds  $\lambda/2$  there will potentially be generated grating lobes.

$d_w$  = distance between

$$d_w < \lambda/2 = \frac{c}{2f} = \frac{1540 \text{ m/s}}{2.5 \cdot 10^6 \frac{1}{s}} = \underline{\underline{0.62 \text{ mm}}}$$

d) Rectangular aperture in 3D-space:



Looking at the variation in pressure in 1D we get a sinc-function dependent on the azimuth angle  $\varphi$ , the wavelength  $\lambda$  and the aperture azimuth size  $2a$ .

$$p(r) = \text{sinc}\left(\frac{2a}{\lambda} \sin \varphi\right) = \text{sinc}\left(\frac{2df_0}{c} \sin \varphi\right)$$



e)  $p(r)$  can be written depend on some azimuth direction  $x$ , relative to the distance straight ahead  $R$ .

$$\sin \phi = \frac{x}{R}$$

$\Downarrow$

$$\begin{aligned} p(r) &= \text{sinc}\left(\frac{2a}{\lambda} \sin \phi\right) = \text{sinc}\left(2a \cdot \frac{f_0}{c} \cdot \frac{x}{R}\right) \\ &= \underline{\underline{\text{sinc}\left(\frac{2axf_0}{Rc}\right)}} \end{aligned}$$

f) Using Rayleigh condition the beam width,  $w$  can be calculated as:

$$w = f_{\#} \lambda$$

$\uparrow$   
 F-number

$$f_{\#} = \frac{R}{2a}, \quad \lambda = \frac{c}{f_0}$$

Given  $R = 10 \text{ cm}$ :

$$w = \frac{R \cdot c}{2a f_0} = \frac{0.1 \text{ m} \cdot 1540 \text{ m/s}}{2 \cdot 0.02 \text{ m} \cdot 2.5 \cdot 10^6 \frac{1}{\text{s}}} = 3.08 \cdot 10^{-3} \text{ m}$$

$= 3.1 \text{ mm}$

2f)

For a sinc function FWHM is approximated to:

$$\text{FWHM} \approx 1.2 f_{\#} \lambda$$

$$= 1.2 \cdot w = 1.2 \cdot 3.1 \text{ mm}$$

$$= \underline{\underline{3.7 \text{ mm}}}$$

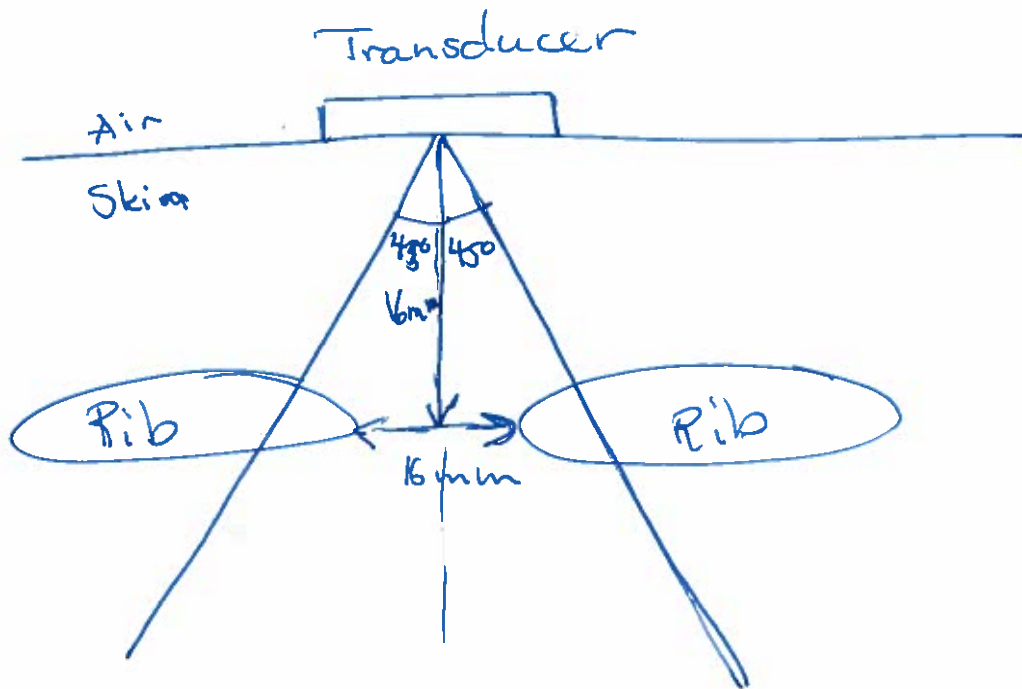
g) x

3

$$f_0 = 3 \text{ MHz}$$

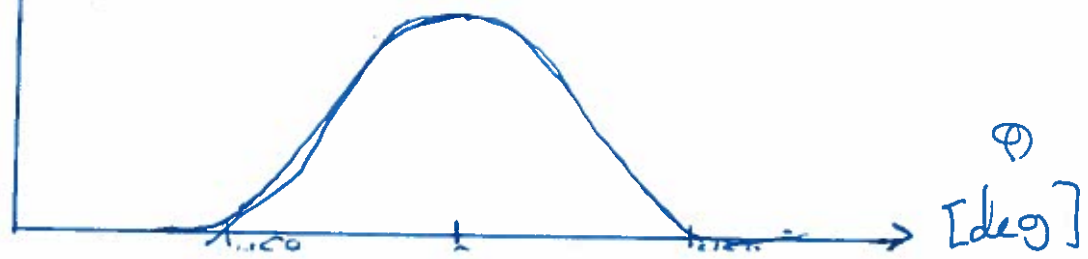
x - Azimuth aperture

$$x = 16 \text{ mm}$$



- d) The ribs have quite different acoustic impedance than the skin, and will lead to total reflection in these areas of the image. The reflection will lead to noise in the rest of the image as well.

b) Effective Aperture size



$$3c) p(r) = \text{sinc}\left(\frac{2f_0}{c} \sin \phi\right)$$

$$p(r) = \text{sinc}\left(\frac{x}{f_{\#} \lambda}\right)$$

Beam width:

$$w = f_{\#} \lambda$$

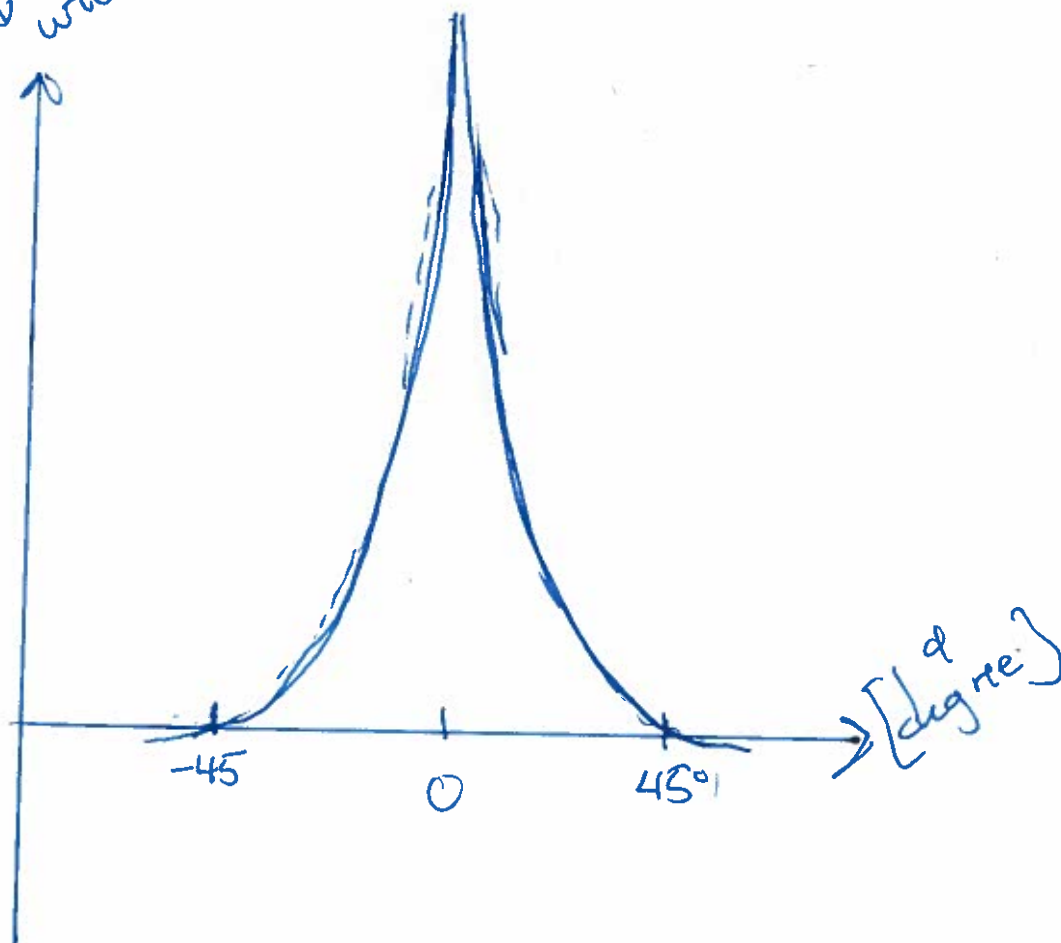
$$\frac{2f_0}{c} \sin \phi = \frac{x}{f_{\#} \lambda}$$

$$f_{\#} = \frac{x}{2 \sin \phi}$$

$$w(\phi) = \frac{x \lambda}{2 \sin \phi} = \frac{10 \text{ cm}}{16 \text{ cm}} \cdot \frac{0.51 \text{ nm}}{\sin \phi}$$

$$= \frac{0.32}{\sin \phi} \text{ mm}$$

Beam width



3d) By reducing the distance to 8mm, by applying pressure on the chest of the patient we can reduce the beam width at the height of the ribs. This can reduce the effect of the ribs in the image and potentially lead to a better image of the heart inside the ribs.

e) Using a micro convex aperture may possibly help image the heart by being able to image in between the ribs.