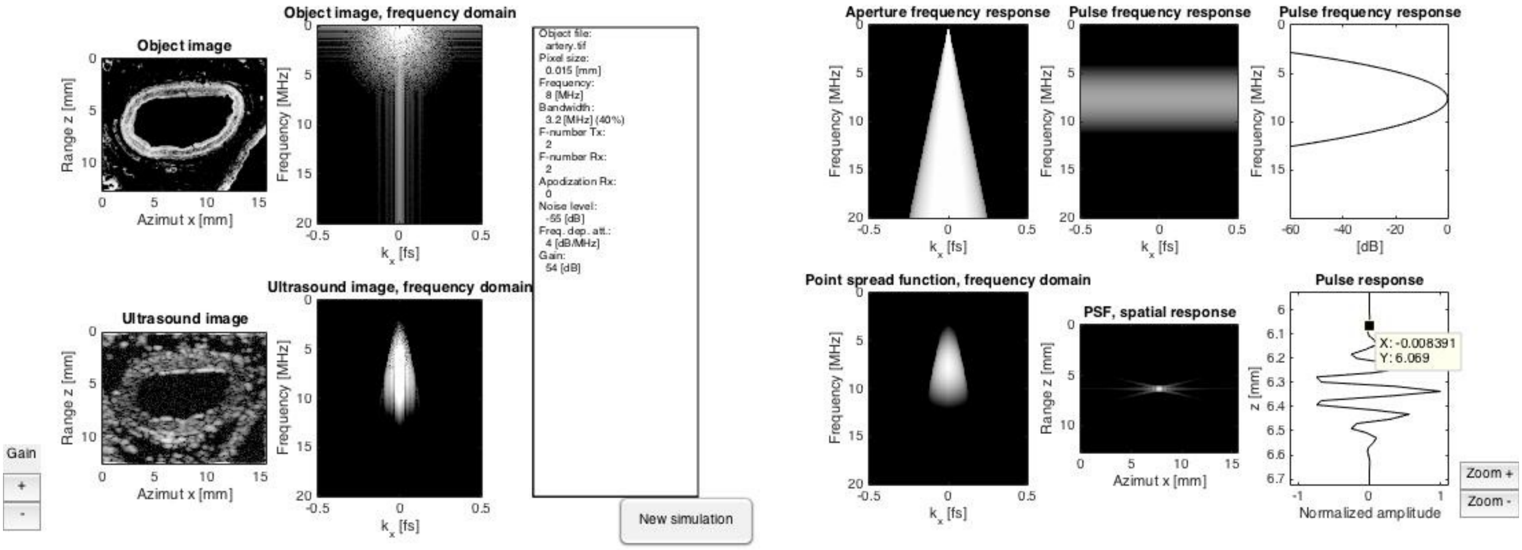
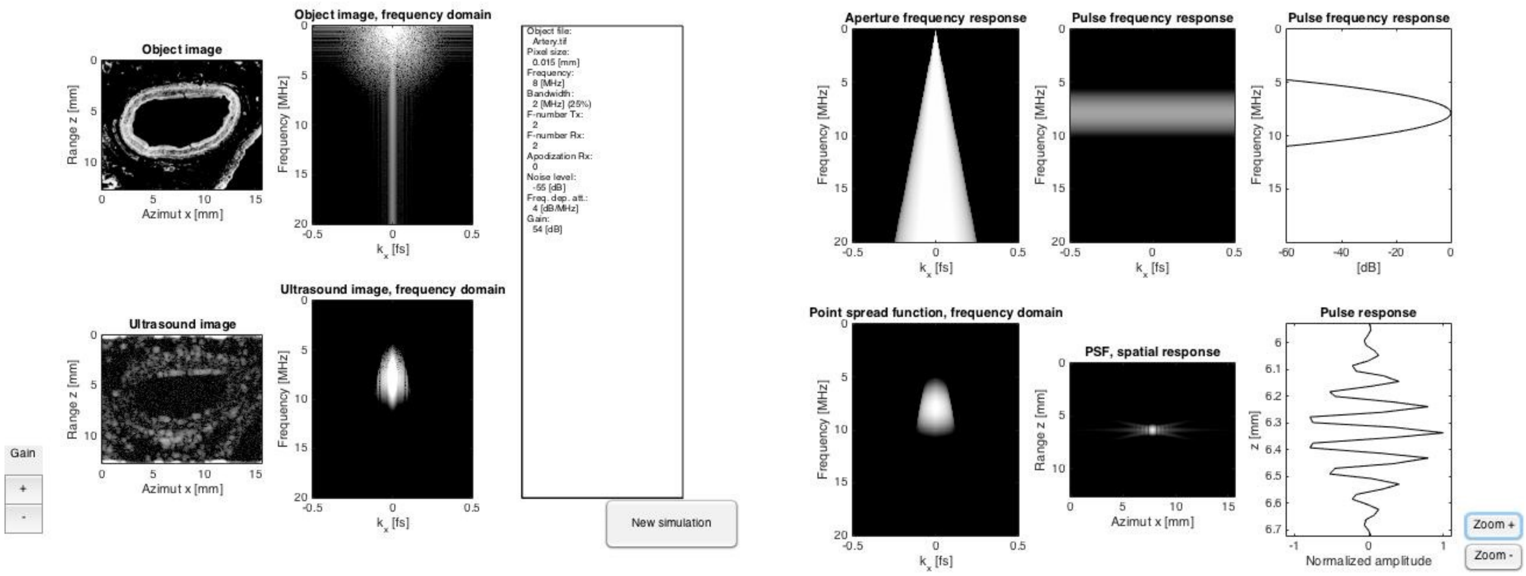
Exercise 7 - K-space ultrasound simulation

Part 1 - Vary the bandwidth



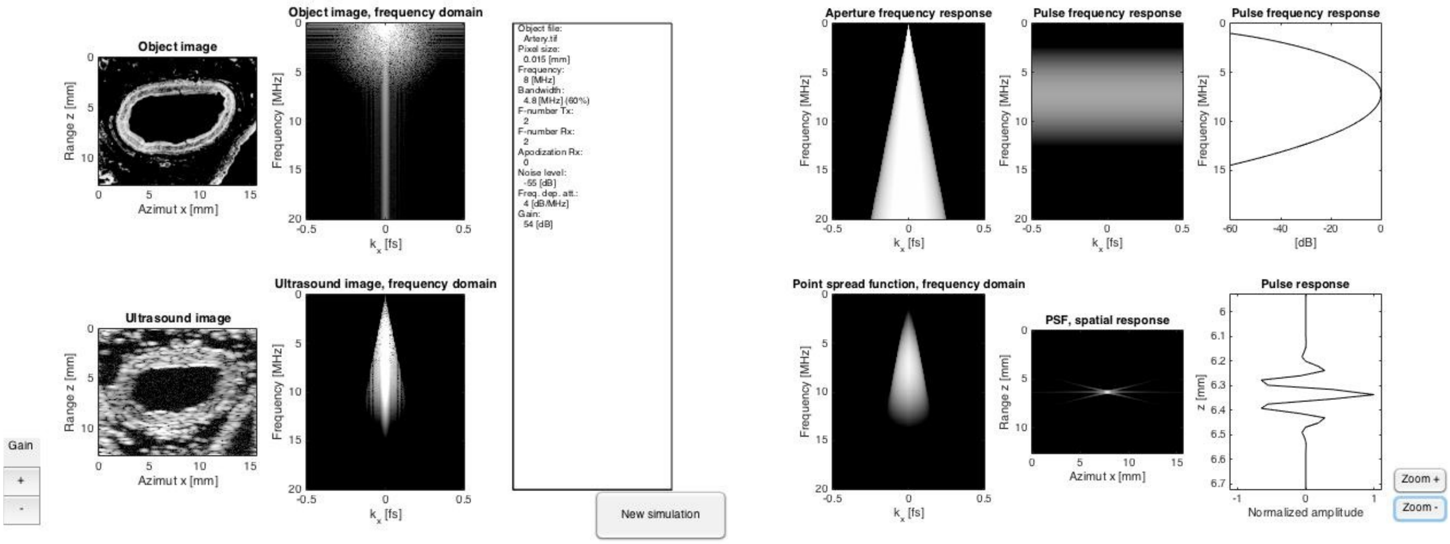
Using a bandwidth of 3.2 MHz gives a beam width of 0.537 mm.

Decrease to 25%



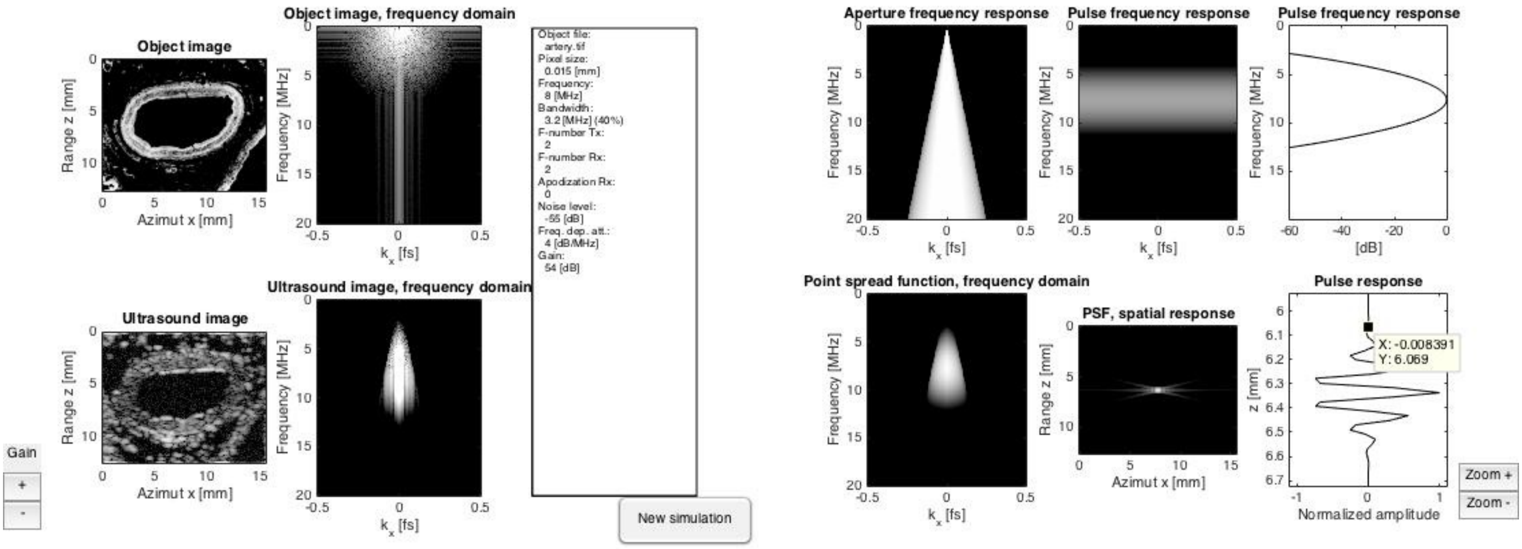
Using a short bandwidth of 2 MHz the beam pulse becomes long with a measured beam width of 0.9 mm. The resulting point spread function in frequency will have variations for areas in the area of 6 MHz to 10 MHz with a relative little magnitude in the k- dimension (shown in lower left corner of the right figure).

Increase to 60%:



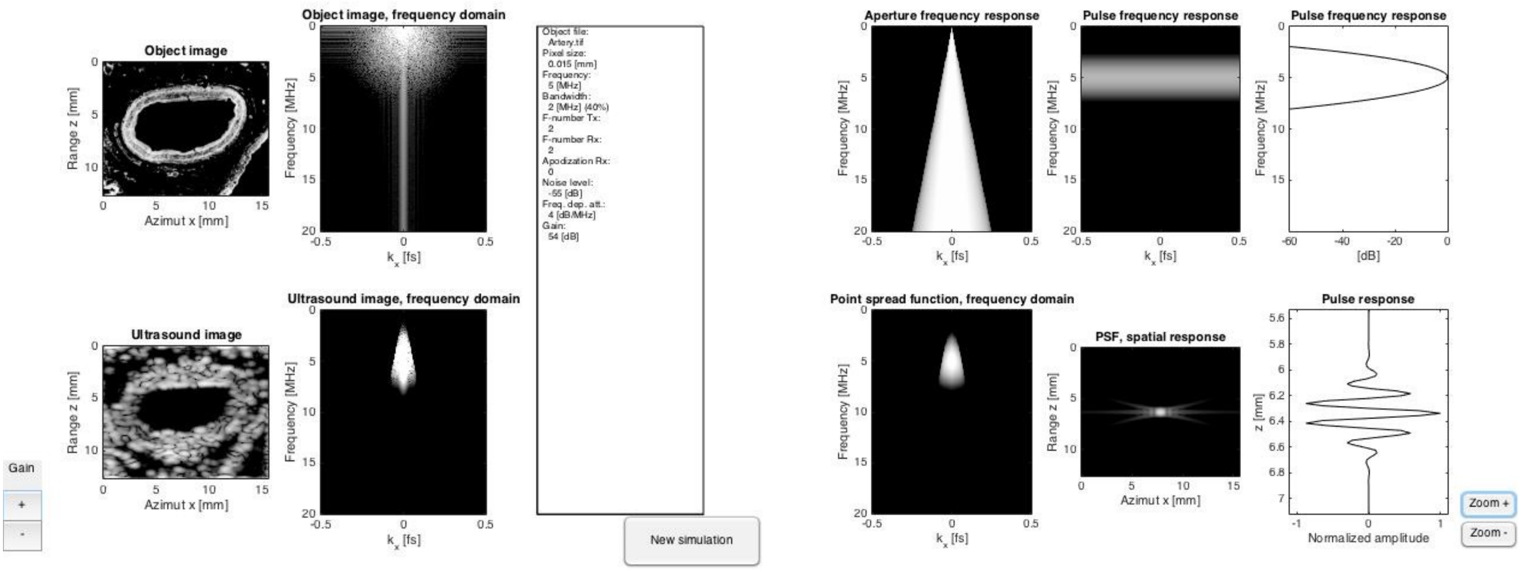
Using a larger bandwidth of 4.5 MHz the beam pulse becomes shorter with a measured beam width of 0.385 mm. The resulting point spread function in frequency will have cover larger areas in both the frequency range and k-space. As the pulse length becomes shorter, the psf in frequency will cover frequencies from about 3.5 MHz to 12.5 MHz. In the k-space the magnitude will be a bit larger then for 25 % for areas with higher frequency (shown in lower left corner of the right figure).

Part 2 - Vary the pulse frequency



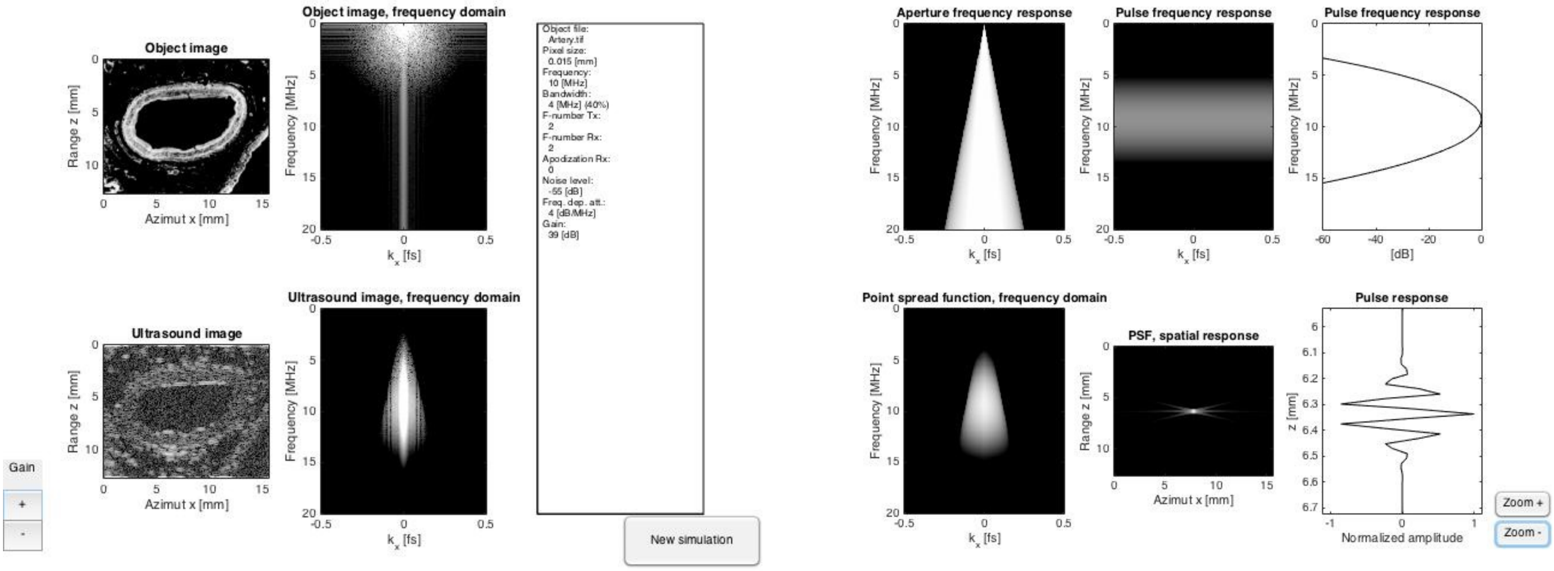
For a frequency of 8 MHz the noise level is -55 dB and signal gain 54 dB. This gives a signal-to-noise ratio in dB of -1 dB.

Reduce the frequency to 5 MHz



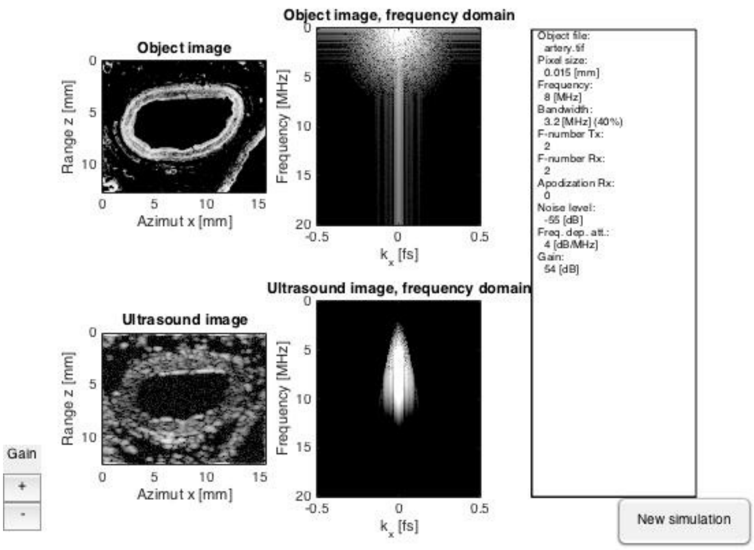
Reducing the frequency to 5 MHz gives a noise level of -55 dB and signal gain of 54 dB, which gives a signal-to-noise ratio in dB of -1 dB. This show that reducing the frequency from 8 MHz to 5 MHz does not change the signal-to-noise ratio.

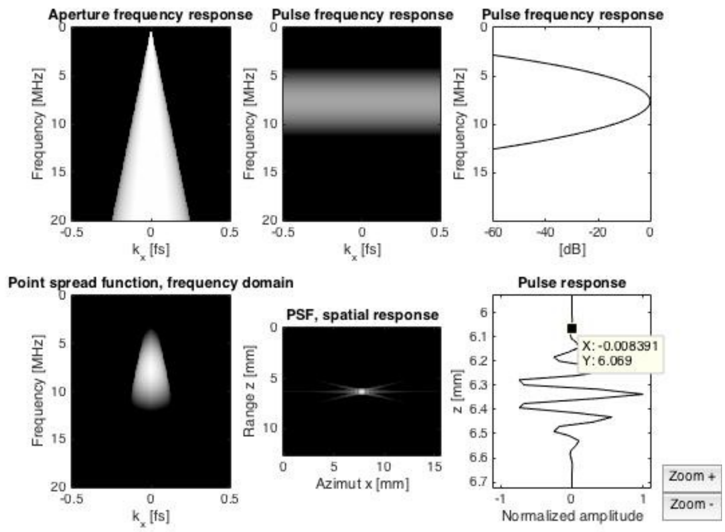
Increase the frequency to 10 MHz



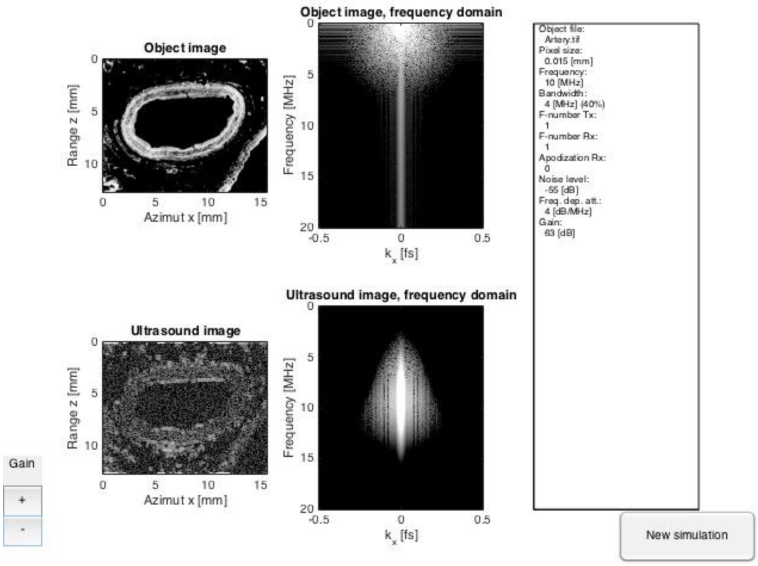
Increasing the frequency has the same noise levels as for decreased frequency, -55 dB. The increasing of frequency leads to lose in signal gain, as we need to compensate for increased absorption due to frequency dependent absorption. The signal gain for 10 MHz is 39 dB, and gives a signal-to-noise ratio of -16 dB, a loss of 15 dB compared to frequency of 5 MHz and 8 MHz.

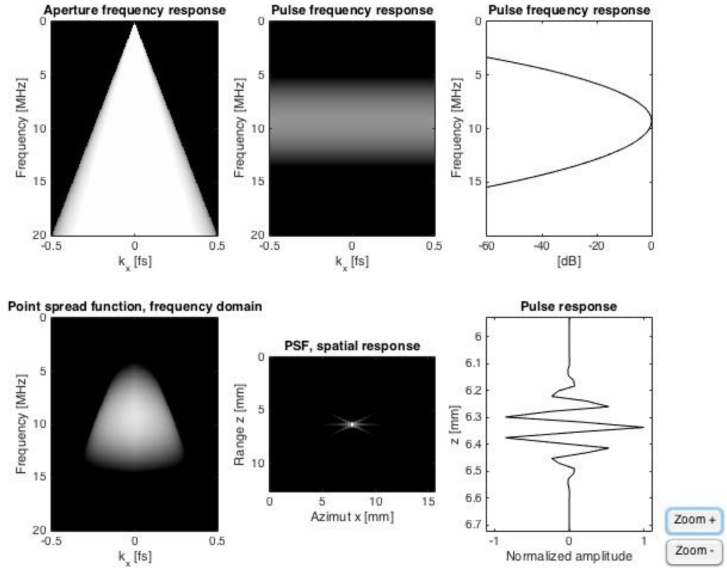
Part 3 - Vary the F-number





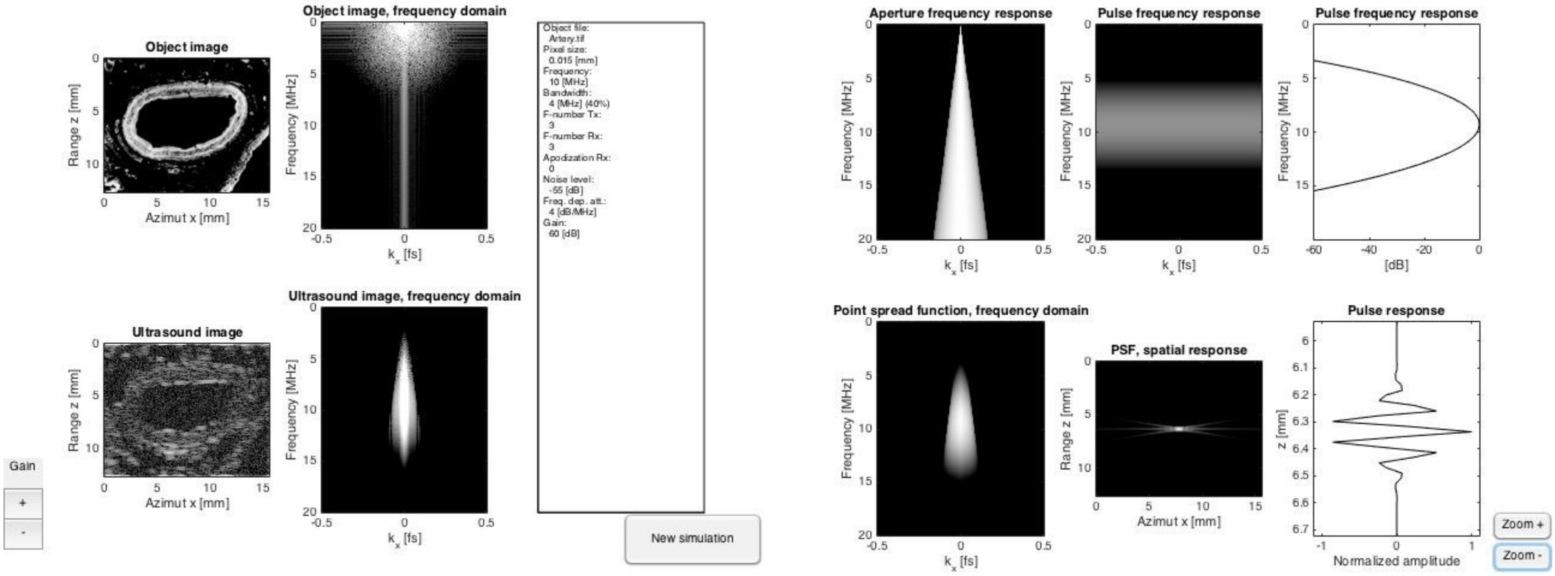
Reduce F-number to 1 i.e increase aperture





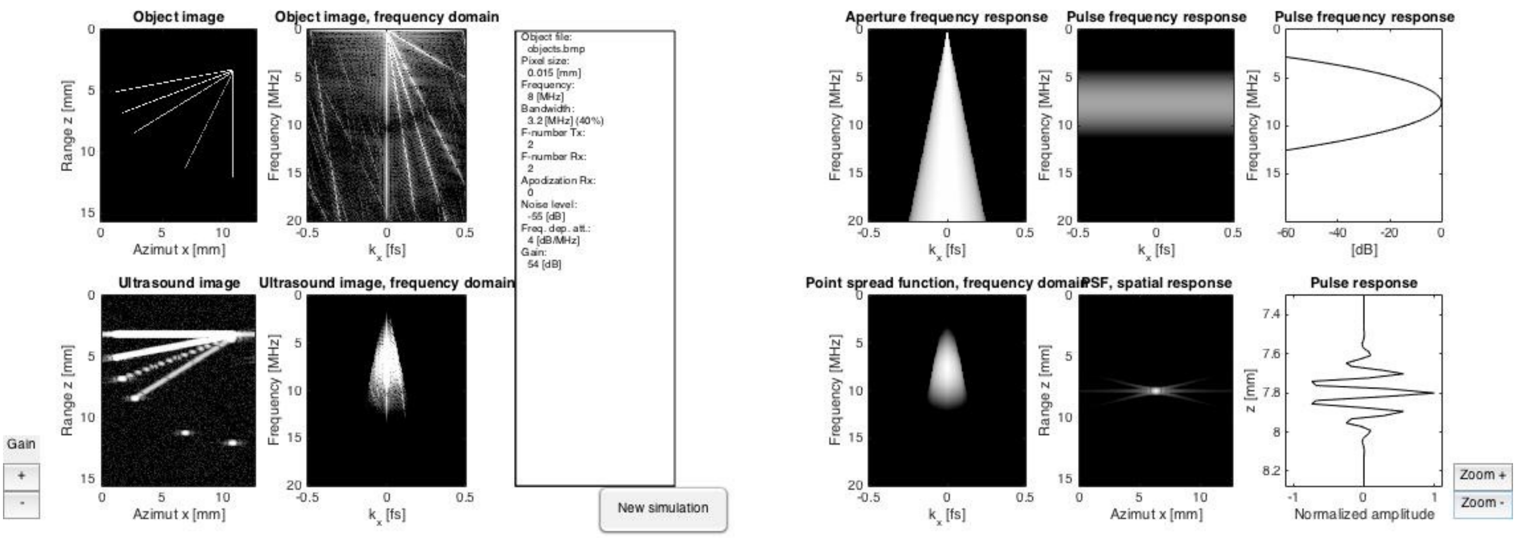
Reducing the F-number from 2 to 1 results in wider opening angle of the frequency response of the aperture(top right plot in the right figure). This gives a wider frequency response in the k-direction. Increasing the aperture results in a narrower beam, as seen for the spatial response of the PSF (lower middle plot in the right figure). A narrower beam will give improved lateral resolution.

Increase F-number to 3, i.e reduce the aperture



For F-number equal to 3 the opening angle of of the frequency response of the aperture is significantly narrower. The spatial response of the point spread function has also become wider, which shows that the beam will be wider. A wider beam will lead to loss in lateral resolution.

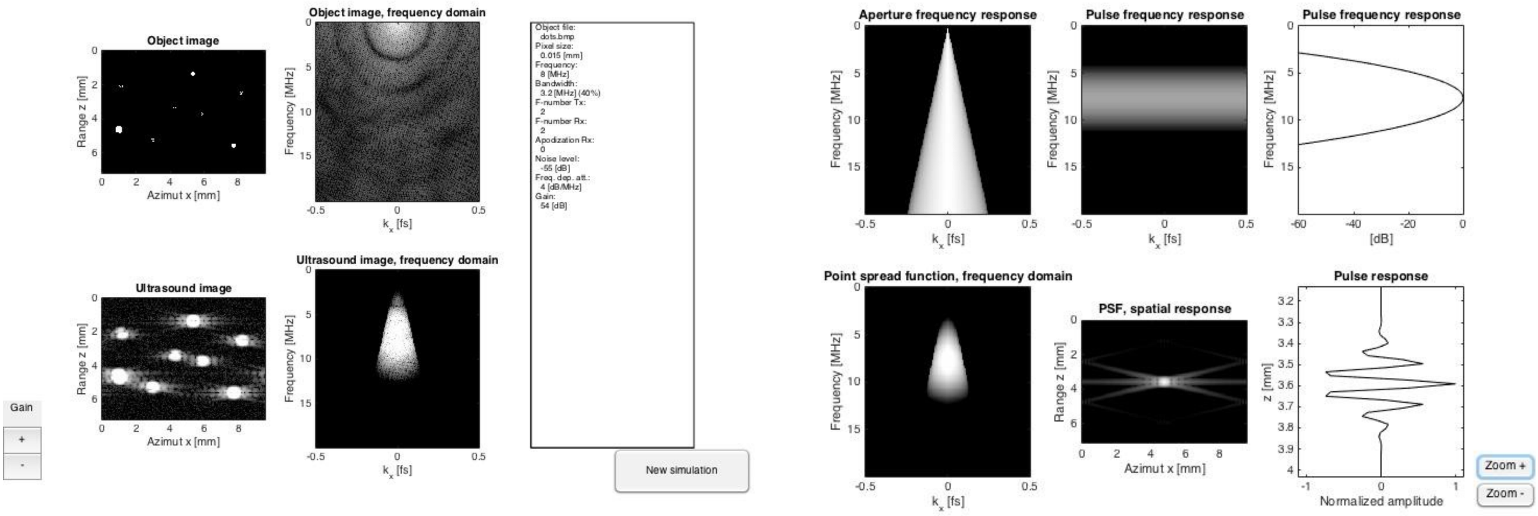
Part 4 - Simulate image with multiple different lines



The frequency domain corresponding to the object image has been mirrored respectively to the spatial domain. A vertical line in the spatial domain will be horizontal in the frequency domain, and the rotated lines will be rotated 90 degrees. The object frequency domain also contain scattering due to lack of pixels leading to the lines not being perfectly straight, and mismatch will lead to different less intensive lines in the frequency image.

The ultrasound image will contain content in the object image which passes through the point spread function. The vertical lines and rotated lines which are close to vertical will be removed, as they do not pass through the point spread function.

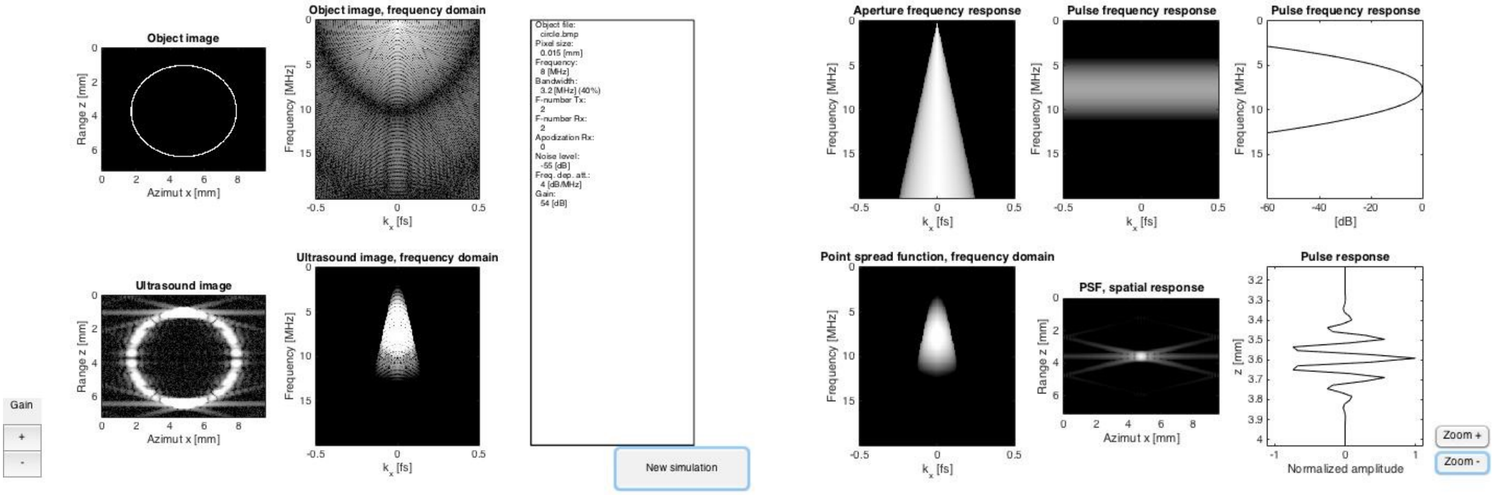
Part 5 - Simulate image with multiple different dots



The Fourier domain for the object image is noisy and the dots have been smeared over the whole spectrum. It is not possible to differ between the different dots in the frequency domain for the object image.

The ultrasound image of the object image shows the dots clearly. The point spread function have made the dots wider and with som smearing around the dots. The point spread function also leads to the dots being of roughly the same size, even do they vary a bit in the object image.

Part 6 - Simulate image with a circle



The frequency domain of the object image has ringing effects in from the top of the image in direction of all angles. The effects is circular symmetric around the top middle of the image.

The ultrasound image has almost a full circle, but the circle is somewhat weaker in the vertical parts of the circle. There are also multiple shaded lines in the image produced by the lack of pixels giving that the circle is not perfect. The lines are both rotated and in the horizontal direction

Part 7 - Rayleigh criterion

The diffraction pattern of a one-way focused rectangular aperture is proportional to

$$p(x) = \mathrm{sinc}\left(\frac{x}{f_{\#}\lambda}\right) \quad \text{where } \mathrm{sinc}(u) = \frac{\sin(\pi u)}{\pi u}$$

The lateral resolution of this system is described by the beam width the aperture can produce.
For a sinc function this corresponds to the distance to the first zero point

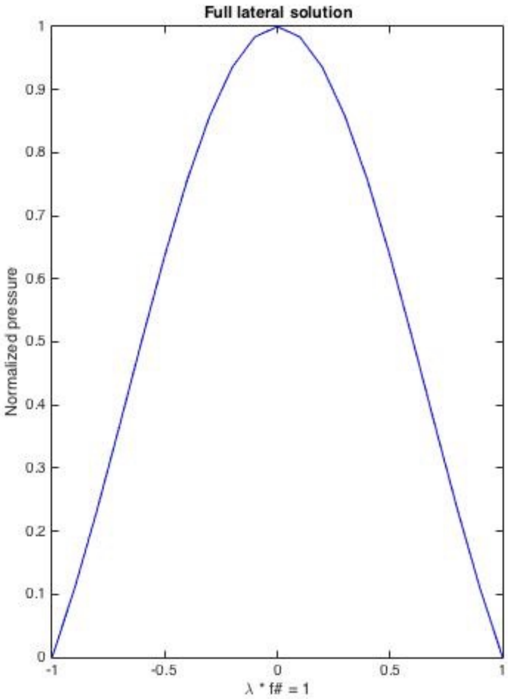
$$\mathrm{sinc}\left(\frac{w}{f_{\#}\lambda}\right) = 0 \quad , w - \text{beam width}$$

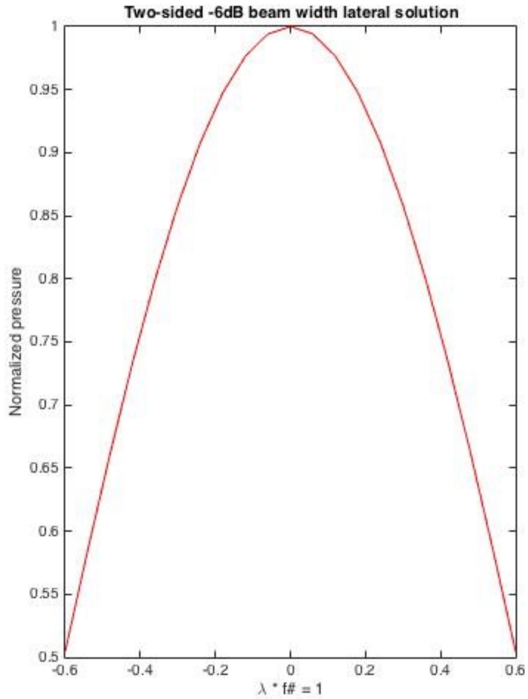
$$\Downarrow$$

$$\frac{\sin\left(\pi \frac{w}{f_{\#}\lambda}\right)}{\pi \cdot \frac{w}{f_{\#}\lambda}} = 0$$

$$\frac{w}{f_{\#}\lambda} = 1$$

$$\underline{\underline{w = f_{\#}\lambda}}$$





When the system is focused on both transmit and receive using the same aperture, one will have loss in lateral resolution. Have much loss one will have in transmit and receive depends on how the aperture is shared between the two, but both will experience a loss in maximum available beam width which leads to loss in lateral resolution.

Part 8 - Gaussian pulse

$$H_p(f) = e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2}$$

If we set dB-level to $H_p(f)$ equal to -6 we can find B_{RMS}:

$$20\log_{10}(H_p(f)) = -6$$

$$20\log_{10}\left(e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2}\right) = -6$$

$$\log_{10}\left(e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2}\right) = \frac{-6}{20}$$

$$e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2} = 10^{-6/20} = 0.5012 \simeq 1/2$$

$$-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2 = -\ln(2)$$

$$\left(\frac{f-f_0}{B_{RMS}}\right)^2 = 2\ln 2$$

$$f-f_0 = B_{RMS}\sqrt{2\cdot\ln 2}$$

$$B = 2\cdot(f-f_0) = 2\cdot B_{RMS}\sqrt{2\cdot\ln 2}$$

$$B_{RMS} = \frac{1}{2\sqrt{2\ln 2}}B = 0.43\cdot B$$

Part 9 - Pulse length

$$H_p(f) = e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2}$$

Fourier Transform → Gaussian (Wolfram MathWorld mathworld.wolfram.com):

$$\mathcal{F}\{e^{-af^2}\} = \sqrt{\frac{\pi}{a}}\, e^{-\pi^2 t^2/a}$$

$f_{diff} = f - f_0$

$$H_p(f_{diff}) = e^{-\frac{1}{2}\left(\frac{f_{diff}}{B_{RMS}}\right)^2} = e^{-\frac{1}{2B_{RMS}^2} f_{diff}^2}$$

$$a = \frac{1}{2B_{RMS}^2}$$

$$\mathcal{F}^{-1}\{H_p(f_{diff})\} = \sqrt{\frac{\pi}{\frac{1}{2B_{RMS}^2}}}\, e^{-\pi^2 t^2 \cdot (2B_{RMS}^2)}$$

$$= \underline{\sqrt{2\pi}\, B_{RMS}\, e^{-2(\pi B_{RMS} t)^2}}$$

$$p(t) = \mathcal{F}^{-1}\{H_p(f)\} = \mathcal{F}^{-1}\{H_p(f_{diff})\} e^{j2\pi f_0 t}$$

$$= \frac{1}{2}\cos(2\pi f_0 t) \cdot \sqrt{2\pi}\, B_{RMS}\, e^{-2(\pi B_{RMS} t)^2}$$

$$= \underline{\underline{\sqrt{\frac{\pi}{2}}\, B_{RMS}\, e^{-2(\pi B_{RMS} t)^2} \cos(2\pi f_0 t)}}$$

Envelope is given by the exponential $e^{-2(\pi B_{RMS} t)^2}$. Need to find when that goes below 6 dB.

$$20\log_{10}(\text{envelope}) = -6$$

$$20\log_{10}\left(e^{-2(\pi B_{RMS} t_{6dB})^2}\right) = -6$$

$$\log_{10}\left(e^{-2(\pi B_{RMS} t_{6dB})^2}\right) = -6/20$$

$$e^{-2(\pi B_{RMS} t_{6dB})^2} = 10^{-6/20}$$

$$\cong 1.2$$

$$\underbrace{\ln(e)}_{1} \cdot \left(-2(\pi B_{RMS} t_{6dB})^2\right) = -\ln(2)$$

$$2\pi^2 B_{RMS}^2 t_{6dB}^2 = \ln(2)$$

$$t_{6dB}^2 = \frac{1}{2(\pi B_{RMS})^2} \ln(2) \quad \left(B_{RMS} = \frac{B}{2\sqrt{2\ln 2}}\right)$$

$$t_{6dB}^2 = \frac{1}{2\pi^2 \cdot \frac{B^2}{4 \cdot 2 \cdot \ln 2}} \ln 2$$

$$t_{6dB}^2 = \frac{4 \cdot \cancel{2} \ln 2}{\cancel{2}\pi^2 \cdot B^2} \ln 2 = \left(\frac{2 \cdot \ln 2}{\pi B}\right)^2$$

$$t_{6dB} = \underline{\frac{2\ln(2)}{\pi B}}$$

$$T_{6dB} = 2 \cdot t_{6dB} = 2 \cdot \frac{2\ln(2)}{\pi B} = \underline{\underline{\frac{4\ln 2}{\pi B}}}$$