Exercise: K-space ultrasound simulation

Aims

Understanding how different pulse and focus parameters affect the ultrasound image.
Interpretation of 2D-Fourierplane (k-space, frequency space).

Background

In this exercise we will use a matlab tool that simulates ultrasound imaging in the Fourier plane. By changing focus parameters, pulse frequency and pulse length for the imaging system, we will se how this affects the image.

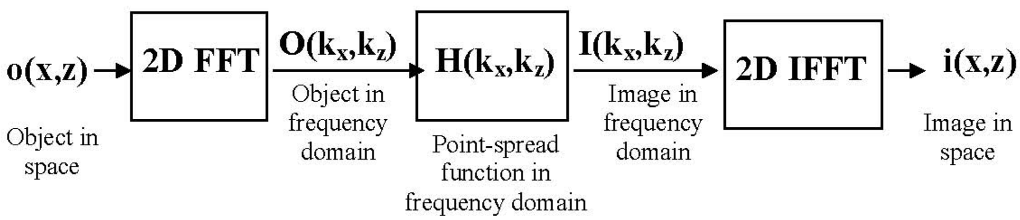

Figure 1: Simulation of the ultrasound system in the Fourier domain. The object that is being imaged is Fourier transformed in 2 dimensions. The object in the Fourier domain is then multiplied with a point spread function in the frequency domain before the image is inverse-transformed back to spatial coordinates.

The transmitted pulse has a finite length that limits the resolution along the beam. The resolution in the azimuth direction is given by the transducer aperture size relative to the wavelength. This makes the ultrasound image a smeared version of the original object.

Under certain assumptions the point spread function in the frequency domain, $H(k_x,k_z)$, can be approximated by two components, one part describing the frequency response of the transmitted pulse, and one part that describes aperture and focusing at transmission and reception. The point-spread function is the product of the aperture- and pulse response.

According to the Fraunhofer approximation, the lateral variation of the continuous wave (CW) ultrasound field at the focal depth can be approximated as the Fourier transform of the aperture. The Fourier transform of this ultrasound field is thus a scaled version of the aperture. Extending this principle from CW to pulsed wave (PW) acquisition, containing more frequency components, the Frequency domain (K-space) representation of the ultrasound focal field is a lowpass-process in the lateral and elevation dimension and a bandpass process radially. This is illustrated in Fig. 2.

Radially, the region of support is centered on $2k_0=2f_0/c$, and bounded by the pulse bandwidth. In the k_x and k_y directions, the region of support is bounded by the aperture function, where the lateral width increases linearly with increasing radial frequency, meaning it is not a separable process.

Upon reception, the lateral K-space response is convolved with the reception aperture, and the radial resolution is approximately doubled due to the pulse compression. With equal transmit and receive apertures of size a focused at depth R, the critical angle as shown in Fig. 2 is phi = $\tan^{-1}(1/2f_{\#})$, where f#=R/a.

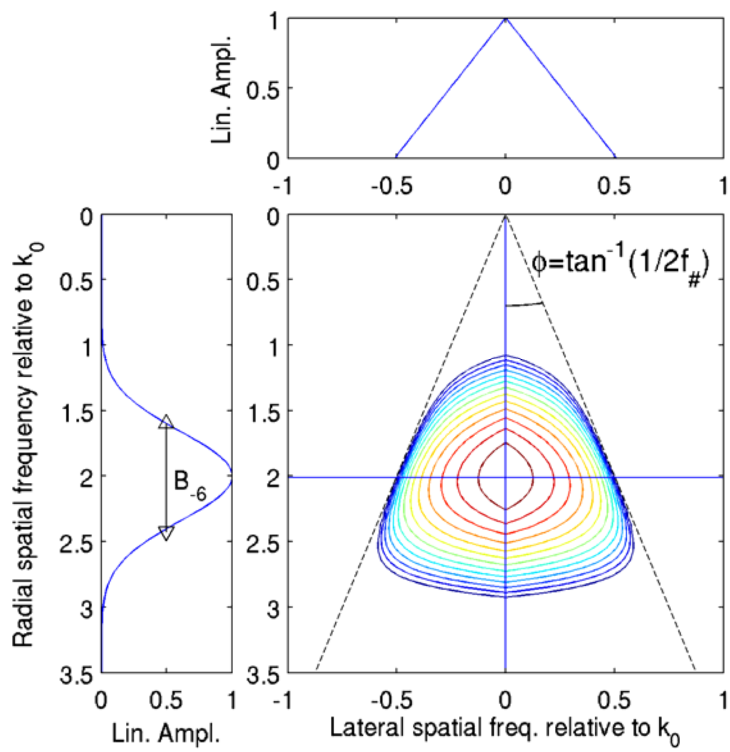

Figure 2
The bottom right pane shows a contour plot of a K-space point spread function (only showing positive radial frequencies); 3 dB separates each contour, with a total dynamic range of 40 dB for this figure. The top pane shows the aperture response at $k_z=2k_0$ in a linear scale, indicated by the horizontal line of the image. Likewise the left pane shows the pulse response as indicated by the vertical line. The transmit- and receive apertures are equal, which results in a triangular aperture response. The maximum lateral frequency content for each radial frequency is governed by the opening angle of the region of support. For this case of equal transmit- and reception apertures phi = tan^{-1}(1/2f_{\#}).

Focusing parameters.

F-number: The F-number (f#) of a focused transducer gives the ratio between the focal distance and the aperture. A large F-number gives a long, wide focus, while a small F-number gives a short, narrow focus. The F-number can be different for transmission and reception.

Pulse parameters:

Pulse length: A short pulse provides the best resolution along the beam, which means it has a wide bandwidth in the frequency domain. Normally probes are limited to a relative bandwidth of about 60-70%, which means that the bandwidth of the pulse is 60-70% of the center frequency. This gives the lower limit for the transmitted pulse length.

Pulse frequency: A high frequency provides a better resolution both radially and laterally. Radially because the pulse length decreases when the frequency increases while holding the number of wavelengths constant. The lateral resolution increases because the aperture size is larger relative to the wavelength for higher frequencies.

Exercises:
Download the zip-file with the matlab-code and example pictures from the course homepage. Unzip the files and start matlab. Go to the directory with the extracted files in matlab, and start the simulation program by typing

» echo2dsim

in the command window. The simulation program will store the previous parameters used. To start the program with the default parameters type:

» echo2dsim(1)

A dialog box will appear (Figure 3). You will be asked to specify the parameters for the simulation. All the fields have default settings, so the simulation will start when you press OK. Later in the exercise we will change the parameters.

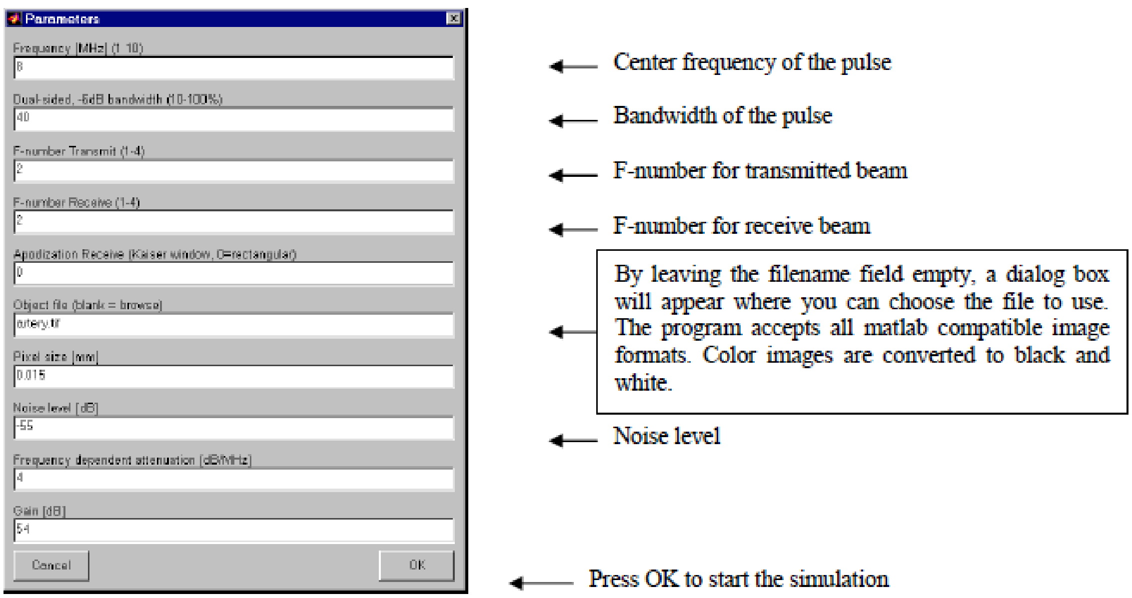

Figure 3: Dialog box for specifying the simulation parameters.

In figure window 1 (Figure 4) the object image (upper) and the simulated ultrasound image (lower) will appear. To the left in spatial coordinates and in the middle in the frequency domain. If the resulting image is too bright or too dark use the buttons marked "+" and "-" to adjust the gain. The simulation parameters used are listed to the right in the figure window.

In figure window 2 (Figure 5) the response of the aperture (upper left corner) and the pulse in the frequency domain (upper middle right) will be shown. The point-spread function in the frequency domain (lower left) is the product of the aperture response and the pulse response. The point spread function in spatial coordinates is found by taking the inverse Fourier transform of the point spread function in the frequency domain. The lower right window shows the transmitted pulse. Use the buttons marked "Zoom+" and "Zoom-" to enlarge the pulse, for example to measure the pulse-length.

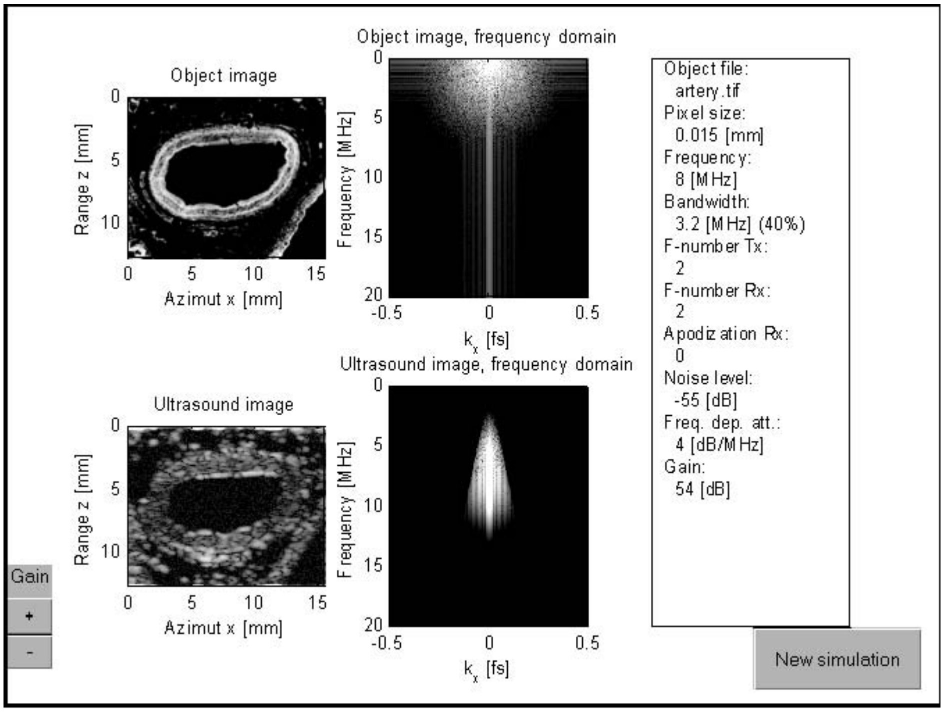

Figure 4: Figure window 1. Object in space (upper left), object in frequency plane (upper middle), simulated ultrasound image (lower left) and the simulated ultrasound image in the frequency domain (lower middle). The simulation parameters are listed to the right.

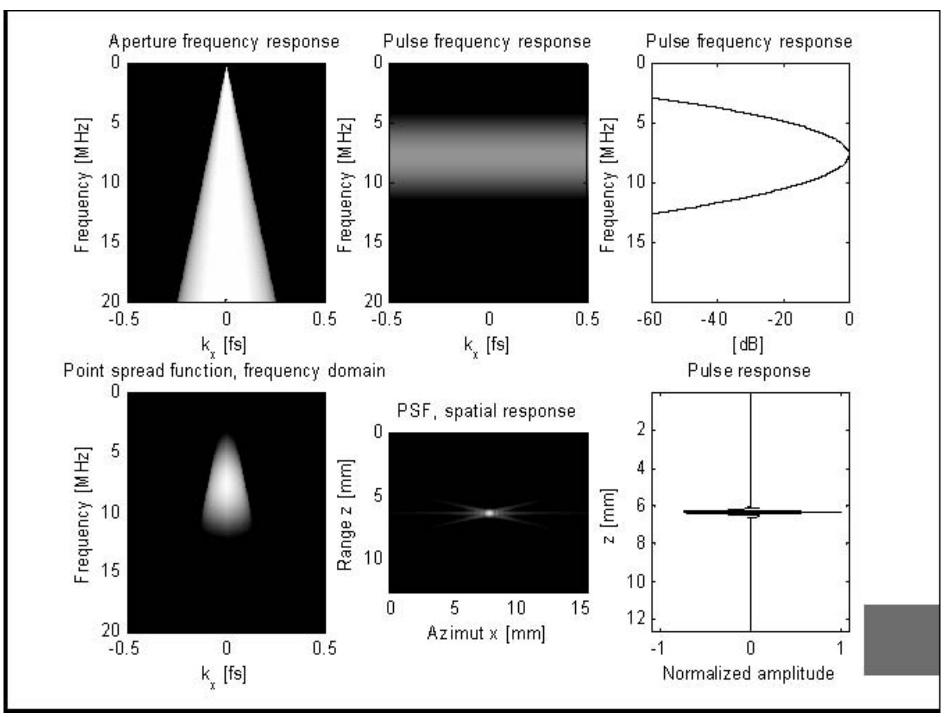

Figure 5: Figure window 2 shows the frequency response of the aperture (upper left), the frequency response of the pulse (upper middle and right). To the lower left corner the combined point spread function is shown in the frequency domain.

Part 1. Vary the pulse bandwidth. Comment on what happens with the resolution in the image when you:
i) Reduce the bandwidth to 25%
ii) Increase the bandwidth to 60%

Measure the length of the pulse in the two cases. What happens to the point spread function in the frequency domain?

Part 2. Set the bandwidth back to 40%, and vary the pulse frequency. Comment on what happens to the PSF in the frequency domain when you:
i) Reduce the frequency to 5 MHz.
ii) Increase the frequency to 10 MHz.

Use the buttons marked "+" and "-" to adjust the gain in the image.
Frequency dependent attenuation is included in the simulations. What effect will this have on the signal to noise ratio when you increase the frequency?

Part 3. Keep the frequency at 10 MHz and the bandwidth at 40%. What happens to the frequency response of the aperture when you:
i) Reduce the F-number for transmission and reception to 1, i.e. increase the aperture.
ii) Increase the F-number for transmission and reception to 3, i.e. reduce the aperture.

How does this affect the resolution in the image?

Part 4. Make an image with one horizontal, one vertical and 3-5 rotated lines at different angles. Use the included picture objects.bmp, and edit it in paint. Leave the background black, and the lines in different nuances of gray and white. Use wide lines. Run the simulation on the image and comment on how the simulated images look in space and in the frequency domain.

Part 5. Make another image with point noise, i.e. place white or gray dots on a black background. Use for example the spray can tool in Windows Paint. How does it look in the Fourier domain after imaging?

Part 6. Make another object with a circle, and comment on Fourier transform of it as well as the simulated image of the circle.

Part 7. For a one-way focused imaging system the resolving power was first given by Lord Rayleigh as the distance from the peak to the first zero crossing in the diffraction pattern. This is called the Rayleigh criterion. The diffraction pattern of a one-way focused rectangular aperture is proportional to $p(x)=\mathrm{sinc}(x / f\#\lambda)$, where $\mathrm{sinc}(x)=\sin(\pi x)/(\pi x)$.

What is the lateral resolution of this system as given by the Rayleigh criterion?
How does this compare to the lateral resolution given by the two-sided -6dB beam width? (Numerical solution is OK, for instance by plotting in matlab.)
What happens to the lateral resolution when the system is focused on both transmit and receive, using the same aperture?

Part 8. In the computer simulation above we have used a Gaussian pulse given by the center frequency f_0 and the two-sided -6dB bandwidth B_{-6}. See Fig. 6. Derive the conversion factor from this -6dB bandwidth to the RMS bandwidth B used in the standard Gaussian equation.

$$H_p(f) = e^{-0.5\left(\frac{f-f_0}{B_{RMS}}\right)^2}$$

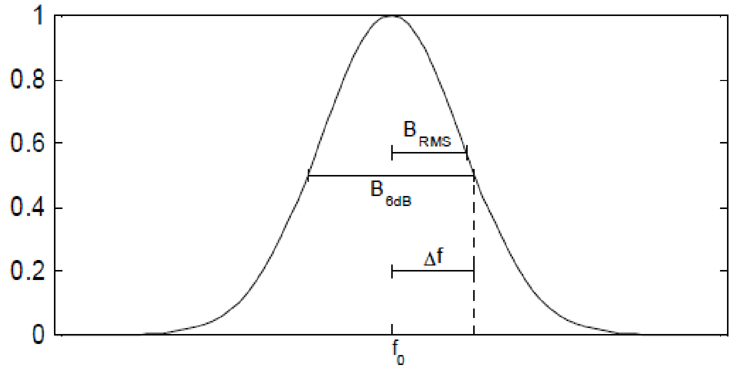

Figure 6. Transmit pulse.

Part 9. The radial resolution is given by half the transmitted pulse length. Calculate the inverse Fourier transform of H_p to find the time-domain pulse, and then calculate the -6dB radial resolution.