Spatial Colour Gamut Mapping by Orthogonal Projection of Gradients onto Constant Hue Lines

Ali Alsam and Ivar Farup

Sør-Trndelag University College, Trondheim, Norway
Gjøvik University College, Gjøvik, Norway

Abstract. We present a computationally efficient, artifact-free, spatial gamut mapping algorithm. The proposed algorithm offers a compromise between the colorimetrically optimal gamut clipping and an ideal spatial gamut mapping. This is achieved by the iterative nature of the method: At iteration level zero, the result is identical to gamut clipping. The more we iterate the more we approach an optimal spatial gamut mapping result. Our results show that a low number of iterations, 20-30, is sufficient to produce an output that is as good or better than that achieved in previous, computationally more expensive, methods. More importantly, we introduce a new method to calculate the gradients of a vector valued image by means of a projection operator which guarantees that the hue of the gamut mapped colour vector is identical to the original. Furthermore, the algorithm results in no visible halos in the gamut mapped image a problem which is common in previous spatial methods. Finally, the proposed algorithm is fast- Computational complexity is $O(N)$, N being the number of pixels. Results based on a challenging small destination gamut supports our claims that it is indeed efficient.

1 Introduction

To accurately define a colour three independent variables need to be fixed. In a given three dimensional colour space the colour gamut is the volume enclosing all the colour values that can be reproduced by the reproduction device or present in the image. Colour gamut mapping is the problem of representing the colour values of an image within the gamut of a reproduction device, typically a printer or a monitor. Furthermore, in the general case, when an image gamut is larger than the destination gamut some visual image information will be lost. We therefore redefine gamut mapping as the problem of representing the colour values of an image within the gamut of a reproduction device with minimum loss of visual information, i.e., as visually close as possible.

Unlike single colours, images are represented in a higher dimensional space than three, i.e. knowledge of the exact colour values is not, on its own, sufficient to reproduce an unknown image. In order to fully define an image, the spatial context of each colour pixel needs to be fixed. Based on this, we define two categories of gamut mapping algorithms. In the first, colours are mapped independent of their spatial context [1]. In the second, the mapping is influenced

G. Bebis et al. (Eds.): ISVC 2012, Part I, LNCS 7431, pp. 556–565, 2012.

by the local context of each colour value [2–5]. The latter category is referred to as spatial colour gamut mapping.

Eschbach [6] stated that although the accuracy of mapping of a single colour is well defined, the reproduction accuracy of images isn't. To elucidate this claim, with which we agree, we consider a single colour that is defined by its hue, saturation and lightness. Assuming that such a colour is outside the target gamut, we can modify its components independently. That is to say, if the colour is lighter or more saturated than what can be achieved inside the reproduction gamut, we shift its lightness and saturation to the nearest feasible values. Further, in most cases it is possible to reproduce colours without shifting their hue.

Taking the spatial context of colours into account presents us with the challenge of defining the spatial components of a colour pixel and incorporating this information into the gamut mapping algorithm. Generally speaking, we need to define rules that would result in mapping two colours with identical hue, saturation and lightness to two different magnitudes depending on their context in the image. The main challenge is, thus, defining the spatial context of an image pixel in a manner that results in an improved gamut mapping. By improved we mean that the appearance of the resultant in-gamut image is closer to the original as judged by a human observer. Further, from a practical point of view, the new definition needs to result in an algorithm that is fast and does not result in image artifacts.

It is well understood that the human visual system is more sensitive to spatial ratios than to absolute luminance values [7]. This knowledge is at the heart of all spatial gamut mapping algorithms. A rephrasing of spatial gamut mapping is then the problem of representing the colour values of an image within the gamut of a reproduction device while preserving the spatial ratios between different colour pixels. In an image, spatial ratios are the difference, given some metric, between a pixel and its surround. This can be the difference between one pixel and its adjacent neighbors or pixels far away from it. Thus, we face the problem that spatial ratios are defined in different scales and dependent on the chosen difference metric.

McCann suggested to preserve the spatial gradients at all scales while applying gamut mapping [8]. Meyer and Barth [9] suggested to compress the lightness of the image using a low-pass filter in the Fourier domain. As a second step the high-pass image information is added back to the gamut compressed image. Many spatial gamut mapping algorithms have been based upon this basic idea [2, 10–12, 4].

A completely different approach was taken by Nakauchi et al. [13]. They defined gamut mapping as an optimization problem of finding the image that is perceptually closest to the original and has all pixels inside the gamut. The perceptual difference was calculated by applying band-pass filters to Fourier-transformed CIELAB images and then weighing them according to the human contrast sensitivity function. Thus, the best gamut mapped image is the image having contrast (according to their definition) as close as possible to the original. Kimmel et al. [3] presented a variational approach to spatial gamut mapping where it was shown that the gamut mapping problem leads to a quadratic programming formulation, which is guaranteed to have a unique solution if the gamut of the target device is convex. However, they did not apply their method to colour images.

Finding an adequate description of the surface of the gamut, commonly denoted a gamut boundary descriptors (GBDs) is an important step in any colour gamut mapping algorithm. One of the main challenges is the fact that gamut surfaces are most often concave. Many methods for finding the GBD have been proposed over the years. Recently, Bakke et al. [14] presented an evaluation of the most common method, showing that the modified convex hull algorithm by Balasubramanian and Dalal [15] is generally the most reliable one.

The algorithm presented in this paper adheres to our previously stated definition of spatial gamut mapping in that we aim to preserve the spatial ratios between pixels in the image while preserving hue. The task of preserving three dimensional gradients while maintaining the original hue values is challenging–the correction of the image gradients in a three dimensional colour space results in unavoidable change in the hue. This is true because adding a three-dimensional gradient vector to the colour triplet results in a modified vector that isn't necessarily parallel to the original.

The first contribution of this paper is, thus, to derive a new n-dimensional gradient operator that is, mathematically, guaranteed to results in gradient vectors that are parallel to the original colour. In the literature, there are a number of gradient operators that provide estimations of the three dimensional colour-image-gradients. As an example the first and second eigenvalues of the tensor matrix are used to define gradients. From a spatial gamut mapping point of view, these operators share two drawbacks: The first is that the gradient vector is not in the direction of the original colour while the second is that operators result in the absolute value of the gradient without its orientation, i.e. the gradient is defined along a line not a vector. The operator derived in this paper remedies both these problems. We define the difference between two colour vectors as the norm of the first minus the norm of orthogonally projected component of the second onto the first. In so doing we arrive at an oriented gradient that is in the direction of the first vector.

The second contribution of this paper is the use of a new computationally efficient approach to restrict the gamut of the spatially mapped image to be within the destination gamut. This is achieved by observing that the resultant image pixels are a convex combination of the colour values that are obtained by clipping the gamut to the gamut boundaries and the neutral gray. Thus we start by calculating the gradients of the original image in the CIELab colour space. The image is then gamut mapped by projecting the colour values to the nearest, in gamut, point along hue-constant lines. The difference between the gradients of the gamut mapped image and that of the original is then iteratively minimized with the constraint that the norm of resultant colour is no greater than that of the gamut clipped vector. The scale at which the gradient is preserved is related to the number of iterations and the extent to which we can fit the original gradients into the destination gamut.

The third contribution relates to halos which are a main drawback in previous spatial gamut mappings techniques. We observe that halos are visible in the resultant gamut mapped images at strong lightness or chromatic edges. Furthermore, those edges are generally visible in the gamut clipped image. That is to say that halos are the result of over enhancing visible edges. We avoid this problem by using anisotropic diffusion [16] where the gradients of the gamut mapped image are improved based on their strength. In other words, diffusion is encouraged within regions and prohibited across strong edges thus avoiding the introduction of halos.

Finally, our results show that as few as ten to thirty iterations are sufficient to produce an output that is similar or better than previous methods. Being able to improve upon previous results using such low number of iterations allows us to state that the proposed algorithm is fast.

2 Spatial Gamut Mapping: A Mathematical Definition

Let's say we have an original image with pixel values $\mathbf{p}(x, y)$ (bold face to indicate vector) in CIELab or any similarly structured colour space. A gamut clipped image can be obtained by leaving in-gamut colours untouched, and moving out-of-gamut colours along straight lines towards $\mathbf{g}$, the center of the gamut, G, on the L axis until they hit the gamut surface. Let's denote the gamut clipped image $\mathbf{p}_c(x, y)$. In a previous papers [17], we showed that spatial gamut mapping can be achieved by minimising

$$\min \int ||\nabla \mathbf{p}_s - \nabla \mathbf{p}||^2 \, dA \quad \text{subject to} \quad \mathbf{p}_s \in G. \tag{1}$$

where $\mathbf{p}_s$ is the spatially gamut mapped image that we are solving for. The numerical solution to this problem was found by solving the corresponding Euler–Lagrange equation,

$$\nabla^2(\mathbf{p}_s - \mathbf{p}) = 0 \tag{2}$$

using a finite difference method with Jacobi iteration, subject to the constraint that the resultant colour vectors are inside the gamut boundaries defined by the gamut clipper image.

One of the problems with this approach was that there was a tendency towards the creation of halos near strong edges. In Reference [18], we therefore proposed to exchange the simple diffusion equation with the anisotropic diffusion equation proposed by Perona and Malik [16]:

$$\nabla \cdot (D\nabla(\mathbf{p}_s - \mathbf{p})) = 0. \tag{3}$$

The diffusion constant was chosen in accordance with Perona and Malik: $D = 1/(1 + |\nabla \mathbf{p}/\kappa|^2)$, κ being a regularisation parameter, which resulted in the following equation:

$$\nabla \cdot \left(\frac{\nabla(\mathbf{p}_s - \mathbf{p})}{1 + |\nabla \mathbf{p}/\kappa|^2} \right) = 0. \tag{4}$$

In order to simplify the problem further, we solved this equation for the grayscale versions of the original, p, and gamut mapped images, p_s only. The final colour gamut mapped image was assumed to be a convex linear combination of the original image and the neutral gray color at any pixel position. The main sacrifice by this approach was that we were not able to recover details that were lost in the conversion between the colour gamut mapped image and its grayscale version.

Here, we propose a new way to deal with Equation (4). Instead of working only on the grayscale images, as in Reference [18], we perform the gamut mapping directly in the full three-dimensional colour space. However, in order to ensure hue constancy during the mapping process, we force the changes to occur on lines of constant hue by projecting the gradients of the original image onto the vectors $\mathbf{p}-\mathbf{g}$, i.e. instead of using the gradient of the original image directly, we substitute it with

$$(\nabla \mathbf{p})_{||} = \mathbf{e}(\mathbf{p})(\mathbf{e}(\mathbf{p}) \cdot \nabla \mathbf{p}), \tag{5}$$

where $\mathbf{e}(\mathbf{p}) = (\mathbf{p}-\mathbf{g})/|\mathbf{p}-\mathbf{g}|$ is a unit vector in a direction of constant hue. Thus, the equation we are seeking to solve for $\mathbf{p}_s$ is

$$\nabla \cdot \left(\frac{\nabla \mathbf{p}_s}{1 + |(\nabla \mathbf{p})_{||}/\kappa|^2} \right) = \nabla \cdot \left(\frac{(\nabla \mathbf{p})_{||}}{1 + |(\nabla \mathbf{p})_{||}/\kappa|^2} \right), \tag{6}$$

subject to $\mathbf{p}_s \in G$. This equation is discretised using the finite difference method with homogeneous boundary conditions, and iterated using the steepest decent method. In order to avoid loss of saturation, we imposed a constraint on how much the out-of-gamut colours can be compressed. This constraint is described by the parameter s (suggesting "saturation"). A value of, e.g., $s = 0.7$ constrains the resulting colour of an out-of-gamut pixel to be outside the inner 70% of the target gamut as measured along the line from $\mathbf{g}$ to $\mathbf{p}_c$.

3 Results

Figure 1 shows two original colour images and the results of pure gamut clipping. Clearly, many details are rendered invisible in the clipped images. For the first image, this loss of detail is evident in the left side of the face, the hair and the face paint.

Figures 2 and 3 show the results of running our algorithm for various number of iterations and for different values of the saturation parameter for the two images, respectively. The saturation parameters are $s = 0.65, 0.75, s = 0.85$ are in the three columns from left to right column, and the number of iterations are $N = 5, 10, 20, 50, 100, 500$ from top to bottom. We observe that small details and edges are corrected to match the original better. With more iterations, the local changes are propagated to larger regions in order to maintain the spatial ratios, however, already at ten iterations, the result resemble that presented in [4], which is, according to Dugay et al. [19] a state-of-the-art algorithm. For many of the images tried, an optimum seems to be found around ten to twenty iterations. Thus, the algorithm is very fast, the complexity of each iteration being $O(N)$

Fig. 1. The original colour images (left) and the gamut clipped images (right)

for an image with N pixels. At the bottom of Figures 2 and 3, the result with as many as 400 iterations are shown. Here, we notice that the details are preserved in a fashion that indicates the power of spatial gamut mapping were we observe that the details are as good as those in the original image.

We further notice that improving the details have not resulted in halos around strong edges. This is due to the use of anisotropic diffusion where we limit diffusion over strong edges. Finally, we notice that, with the introduction of the s parameter, the de-saturation of some colours resulting from previous versions of the algorithm [17, 18] is much more controllable.

As part of this work, we have experimented with 20 images which we mapped to a small destination gamut. Our results shows that the proposed algorithm results in improvement in the visualisation of all the images.

4 Conclusion

In this paper, we presented a spatial gamut mapping algorithm that is derived to minimise the difference, in local contrast, between an original image and its in-gamut counterpart. The first contribution of this paper, is the introduction of a gradient operator that results in three dimensional gradients that are in the direction of the original colour. The motivation behind the use of this operator is to maintain the hue of the original colour (subject to the linearity of the hue lines in the colour space). By employing anisotropic diffusion and a constraint

Fig. 2. The first image from Figure 1 gamut mapped by the proposed algorithm for the three values $s = 0.65, 0.75, s = 0.85$ in the three columns from left to right, and for the number of iterations $N = 5, 10, 20, 50, 100, 500$ from top to bottom

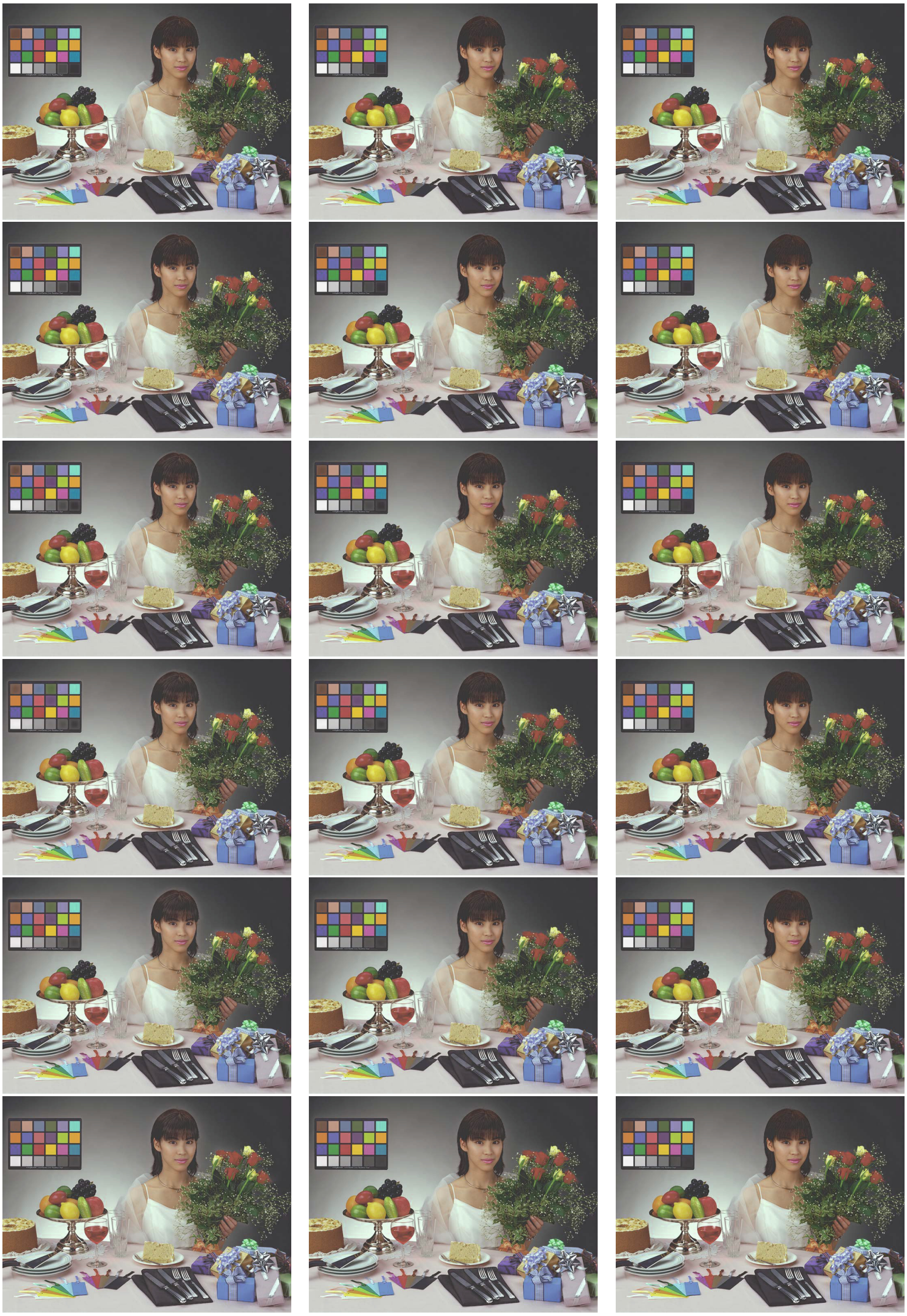

Fig. 3. The second image from Figure 1 gamut mapped by the proposed algorithm for the three values $s = 0.65, 0.75, s = 0.85$ in the three columns from left to right, and for the number of iterations $N = 5, 10, 20, 50, 100, 500$ from top to bottom

on the extent to which gradient correction is allowed to result in de-saturation, we were able to achieve results that improve on the state of art algorithm- our results are free from halos, without loss of saturation. Finally, this improvement is achieved to a small computational cost which makes this algorithm suited for practical implementation as part of a colour reproduction pipeline.

References

1. Morovič, J., Luo, M.R.: The fundamentals of gamut mapping: A survey. Journal of Imaging Science and Technology 45, 283–290 (2001)
2. Bala, R., de Queiroz, R., Eschbach, R., Wu, W.: Gamut mapping to preserve spatial luminance variations. Journal of Imaging Science and Technology 45, 436–443 (2001)
3. Kimmel, R., Shaked, D., Elad, M., Sobel, I.: Space-dependent color gamut mapping: A variational approach. IEEE Transactions on Image Processing 14, 796–803 (2005)
4. Farup, I., Gatta, C., Rizzi, A.: A multiscale framework for spatial gamut mapping. IEEE Transactions on Image Processing 16 (2007), doi:10.1109/TIP.2007.904946
5. Giesen, J., Schubert, E., Simon, K., Zolliker, P.: Image-dependent gamut mapping as optimization problem. IEEE Transactions on Image Processing 16, 2401–2410 (2007)
6. Eschbach, R.: Image reproduction: An oxymoron? Colour: Design & Creativity 3, 1–6 (2008)
7. Land, E.H., McCann, J.J.: Lightness and retinex theory. Journal of the Optical Society of America 61, 1–11 (1971)
8. McCann, J.J.: A spatial colour gamut calculation to optimise colour appearance. In: MacDonald, L.W., Luo, M.R. (eds.) Colour Image Science, pp. 213–233. John Wiley & Sons Ltd. (2002)
9. Meyer, J., Barth, B.: Color gamut matching for hard copy. SID Digest, 86–89 (1989)
10. Morovič, J., Wang, Y.: A multi-resolution, full-colour spatial gamut mapping algorithm. In: Proceedings of IS&T and SID's 11th Color Imaging Conference: Color Science and Engineering: Systems, Technologies, Applications, Scottsdale, Arizona, pp. 282–287 (2003)
11. Eschbach, R., Bala, R., de Queiroz, R.: Simple spatial processing for color mappings. Journal of Electronic Imaging 13, 120–125 (2004)
12. Zolliker, P., Simon, K.: Retaining local image information in gamut mapping algorithms. IEEE Transactions on Image Processing 16, 664–672 (2007)
13. Nakauchi, S., Hatanaka, S., Usui, S.: Color gamut mapping based on a perceptual image difference measure. Color Research and Application 24, 280–291 (1999)
14. Bakke, A.M., Farup, I., Hardeberg, J.Y.: Evaluation of algorithms for the determination of color gamut boundaries. Journal of Imaging Science and Technology 54, 050502–050511 (2010)
15. Balasubramanian, R., Dalal, E.: A method for quantifying the color gamut of an output device. In: Proc. SPIE. Color Imaging: Device-Independent Color, Color Hard Copy, and Graphic Arts II, San Jose, CA, vol. 3018 (1997)

16. Perona, P., Malik, J.: Scale-space and edge detection using anisotropic diffusion. IEEE Transactions on Pattern Analysis and Machine Intelligence 12, 629–639 (1990)
17. Alsam, A., Farup, I.: Colour Gamut Mapping as a Constrained Variational Problem. In: Salberg, A.-B., Hardeberg, J.Y., Jenssen, R. (eds.) SCIA 2009. LNCS, vol. 5575, pp. 109–118. Springer, Heidelberg (2009)
18. Alsam, A., Farup, I.: Spatial Colour Gamut Mapping by Means of Anisotropic Diffusion. In: Schettini, R., Tominaga, S., Trémeau, A. (eds.) CCIW 2011. LNCS, vol. 6626, pp. 113–124. Springer, Heidelberg (2011)
19. Dugay, F., Farup, I., Hardeberg, J.Y.: Perceptual evaluation of color gamut mapping algorithms. Color Research and Application 33, 470–476 (2008)