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December 3, 2017

1

1.1

:

$$z = f(x, y) \tag{1}$$

$$F(x, y, z) = 0 \tag{2}$$

z (1) (2) x, y.
(1), x y (, .3,). (2), (1), , .
(2) , . 3, ,

$$x^2 + y^2 + (z - c)^2 = R^2.$$

(1), : , z=c,

$$z = c - \sqrt{R^2 - (x^2 + y^2)}, \tag{3}$$

$z = c$

$$z = c + \sqrt{R^2 - (x^2 + y^2)}; \tag{4}$$

, $z = c$, (3), (4).
, (1) .

1.2 .

ka [1] (. . 6):

$$k_a = \lim_{l \to 0} \frac{2h}{l^2} = \lim_{l \to 0} \frac{2f(x, y)}{l^2},$$

, $x \to 0$ $y \to 0$ ϵ , :

$$k_a = \lim_{l \to 0} \frac{f_{xx}(0,0)x^2 + 2f_{xy}(0,0)xy + f_{yy}(0,0)y^2 + 2\epsilon l^2}{l^2} = \\ = f_{xx}(0,0)\cos^2\alpha + f_{xx}(0,0)\sin^2\alpha + 2f_{xy}(0,0)\sin\alpha\cos\alpha \tag{5}$$

,

$$\frac{x}{l} = \cos\alpha; \frac{y}{l} = \sin\alpha.$$

,

$$k_x = f_{xx}(0,0); k_y = f_{yy}(0,0). \tag{6}$$

, A, k_x k_y, $f_{xy}(0,0)$, .

1.3

:

$$\left.\begin{array}{c} a_{x_1'x_1'} = a_{x_1x_1}l_1^2 + a_{x_2x_2}m_1^2 + 2a_{x_1x_2}l_1m_1 = a_{x_1x_1}\cos^2\alpha + a_{x_2x_2}\sin^2\alpha + \\ +2a_{x_1x_2}\cos\alpha\sin\alpha \\ a_{x_2'x_2'} = a_{x_1x_1}l_2^2 + a_{x_2x_3}m_2^2 + 2a_{x_1x_2}l_2m_2 = a_{x_1x_1}\sin^2\alpha + a_{x_2x_2}\cos^2\alpha - \\ -2a_{x_1x_2}\cos\alpha\sin\alpha \\ a_{x_1'x_3'} = a_{x_3x_1}l^2 = a_{x_2x_2}l_1l_2 + a_{x_2x_2}m_1m_2 + a_{x_1x_1}(l_1m_2 + l_2m_1) = \\ = -a_{x_1x_1}\cos\alpha\sin\alpha + a_{x_2x_2}\sin\alpha\cos\alpha + a_{x_1x_1}(\cos^2\alpha - \sin^2\alpha) = \\ = \frac{1}{2}(a_{x_2x_2} - a_{x_1x_1})\sin 2\alpha + a_{x_1x_1}\cos 2\alpha. \end{array}\right\}$$

, (10) . $f_{x_1y_1}(0,0)$:

$$f_{x_1y_1} = \frac{\partial^2 f}{\partial x_1 \partial y_1} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial y_1}\right) = \frac{]\partial}{\partial x_1}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y_1} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y_1}\right) = \\ = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y_1}\right)\frac{\partial x}{\partial x_1} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y_1}\right)\frac{\partial y}{\partial x_1} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y_1}\right)\frac{\partial y}{\partial x_1} + \\ + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y_1}\right)\frac{\partial y}{\partial x_1} = \frac{\partial^2 f}{\partial x^2}\frac{\partial x}{\partial y_1}\frac{\partial x}{\partial x_1} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial x}{\partial y_1}\frac{\partial x}{\partial x_1} + \\ + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial x}{\partial y_1}\frac{\partial x}{\partial x_1} + \frac{\partial^2 f}{\partial y^2}\frac{\partial x}{\partial y_1}\frac{\partial x}{\partial x_1}. \tag{7}$$

$f_{x_1y_1}$ f $x = 0$, $y = 0$.

2

2.1

α_1 α_2, - - r , (α_1, α_2). , r α_1 α_2:

$$\mathrm{r} = \mathrm{r}(\alpha_1, \alpha_2) \tag{8}$$

(8) :

$$x = x(\alpha_1, \alpha_2); y = y(\alpha_1, \alpha_2); z = z(\alpha_1, \alpha_2).$$