ELEMENTARY COMBINATORICS & PROBABILITY

Lecture 8

Conditional probability, Bayes' rule

LAST TIME

- Probability of an event
- Frequentist interpretation of probability (Python)

TODAY

- Conditional probability
- The law of total probability
- Bayes' rule

WARM-UP

TWO COIN FLIPS

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$$P(E) = \frac{C(4,2)}{2^4} = \frac{4!}{2^6} = \frac{3}{8}$$

NUMBERED BALLS

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$$P(E) = \frac{2 \cdot 2 + 3 \cdot 3}{5 \cdot 5} = \frac{13}{25}$$

MORE BALLS

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$$P(E) = \frac{C(4,2) \cdot 2^2 \cdot 3^2}{5^4} = \frac{6^3}{5^4} = 0.3456$$

CONDITIONAL PROBABILITY

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- 14% of the population aged 65+.
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Note: $P(M \,\&65+) = P(M|65+) \cdot P(65+)$

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$$P(X_1 + X_2 = 7 | multiple\ spots) = \frac{4}{5 \cdot 5} = \frac{4}{25} = 0.16$$

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BOWLS AND BALLS

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$$P(X_1 + X_2 = 5) = \frac{4}{36} = \frac{1}{9}$$

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THE LAW OF TOTAL PROBABILITY

https://youtu.be/U3_783xznQI

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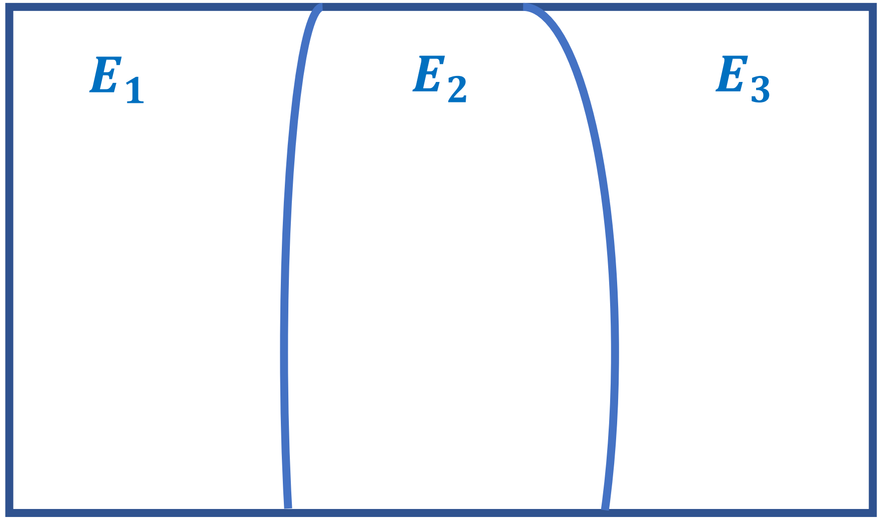
$$S = E_1 \cup E_2 \cup \cdots \cup E_n,$$

$$E_1 \cap E_2 \cap \cdots \cap E_n = \emptyset$$



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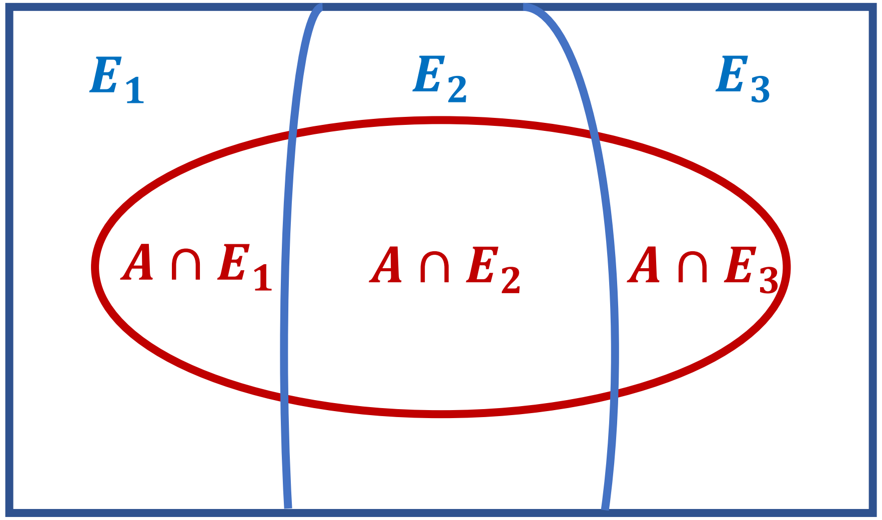

Then $\mathrm{P}(A)$ can be computed as follows:

$$P(A) = P(A, E_1) + P(A, E_2) + \cdots + P(A, E_n) =$$
$$= P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) +$$
$$+ \cdots + P(A|E_n) \cdot P(E_n)$$

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$$= (1 - 0.6) \cdot 0.4 + (1 - 0.1) \cdot 0.6 = 0.7$$

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TRY IT!

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HEALTHY $\approx 100\ \%$	**FALSE POSITIVES** 10% of 99% healthy people ~~= 9.9% of the population~~	**TRUE NEGATIVES** 90% of 99% healthy people $\boxed{= 89.1\% \text{ of the population}}$
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SAME THING, BUT IN TERMS OF PROBABILITY

$P(D) = 0.01, \qquad P(+ \mid D) = P(- \mid no\ D) = 0.9, \qquad P(D \mid +) = ?$

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$$\boldsymbol{P(A|B)} = \frac{P(AB)}{P(B)} = \frac{\boldsymbol{P(B|A)}}{\boldsymbol{P(B)}} \cdot \boldsymbol{P(A)}$$

BAYES’ RULE: AN OVERVIEW

https://youtu.be/BcvLAw-JRss

BALLS IN TWO BOWLS

- There are two bowls: bowl A contains 2 blue and 3 red balls, and bowl B contains 1 blue and 4 red balls. A blue ball was randomly picked from one of the bowls. What's the probability that it was bowl A?

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$$= \frac{0.2}{0.3} = \frac{1}{3}$$

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LET’S PRACTICE!

https://docs.google.com/document/d/1hI1IP9_YRMh4RRoL6RjvTwh6ynGd1YLs8-jVR0wp49M/edit?usp=sharing