ELEMENTARY COMBINATORICS & PROBABILITY

Lecture 10

Random variables

TODAY

- Review interim exam 2
- Random variables

REVIEW INTERIM EXAM 2

DRUG STUDY

- In a drug study of a group of 500 patients, 150 patients responded positively to drug A, 200 patients responded positively to drug B and 90 patients responded positively to both drug A and drug B. What is the probability that a patient responds positively to drug A given that this patient responded positively to drug B?

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- 200 patients responded positively to drug B $\quad P(B) = \frac{200}{500} = 0.4$

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$$P(A \text{ } and \text{ } B) = \frac{90}{500} = 0.18$$

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$$P(A|B) =$$

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$$P(A|B) = \frac{P(A \cap \ B)}{P(B)} =$$

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$$P(A|B) = \frac{P(A \cap \ B)}{P(B)} = \frac{90/500}{200/500} = \frac{90}{200} = \frac{9}{20}$$

INDEPENDENCE

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$$E = \{one\ or\ three\ 1s\}, \qquad P(E) =$$

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$$P(E\ and\ F) = P(100 \text{ or } 111) =$$

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$$P(E\ and\ F) = P(100 \text{ or } 111) = \frac{1+1}{2^3} = \frac{1}{4}$$

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$$P(E\ and\ F) = P(100 \text{ or } 111) = \frac{1+1}{2^3} = \frac{1}{4}$$

$$\frac{1}{4} = P(E\ and\ F)$$

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$\frac{1}{4} = P(E\ and\ F) \quad P(E) \cdot P(F) = \frac{1}{2} \cdot \frac{1}{2}$

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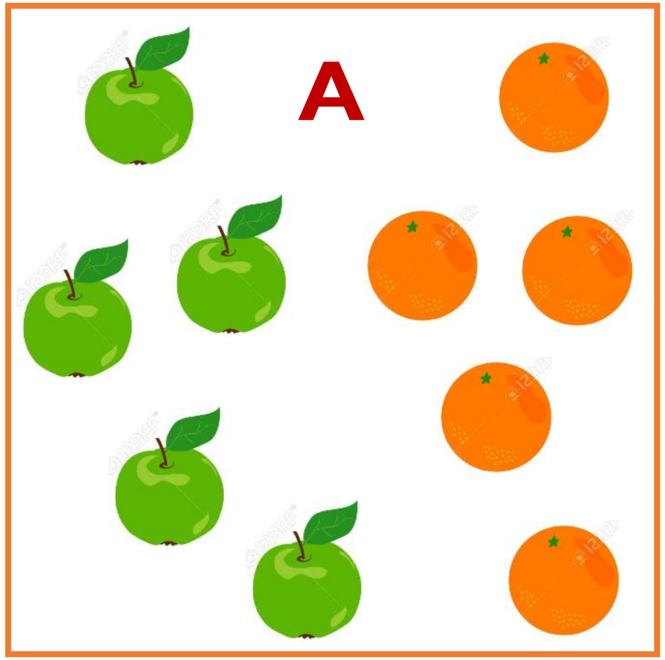
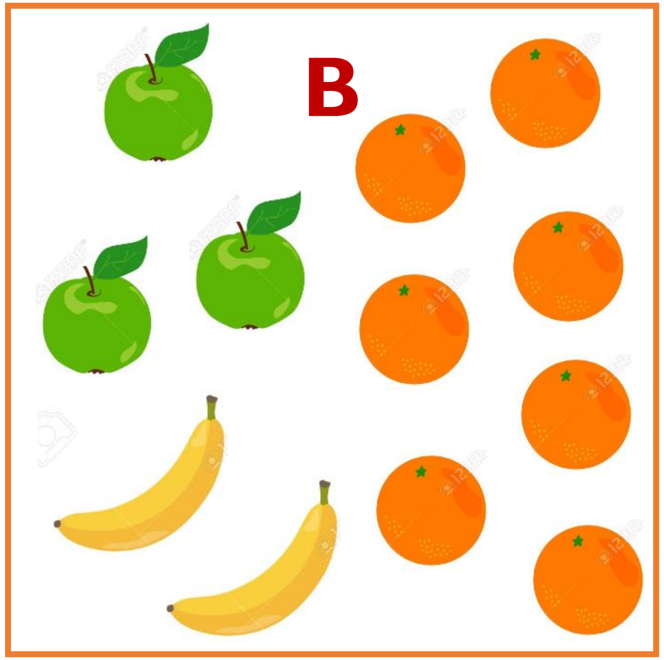
$\frac{1}{4} = P(E\ and\ F) = P(E) \cdot P(F) = \frac{1}{2} \cdot \frac{1}{2} \rightarrow E\ and\ F$ are independent

FRUITS





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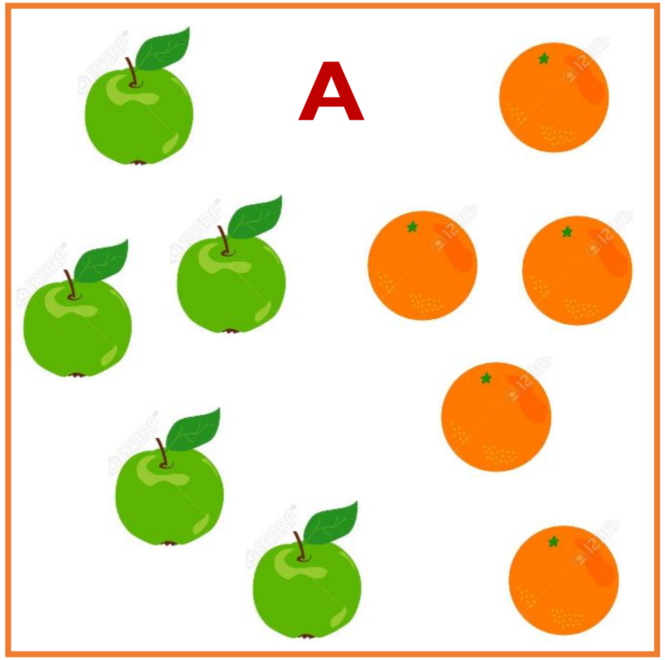

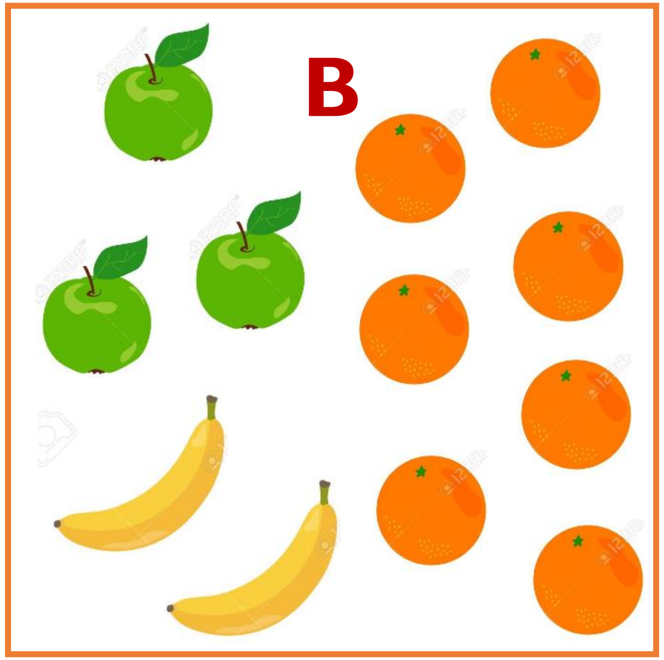

- What is the probability to pick an <u>orange</u> given that you've chosen <u>box A</u>?

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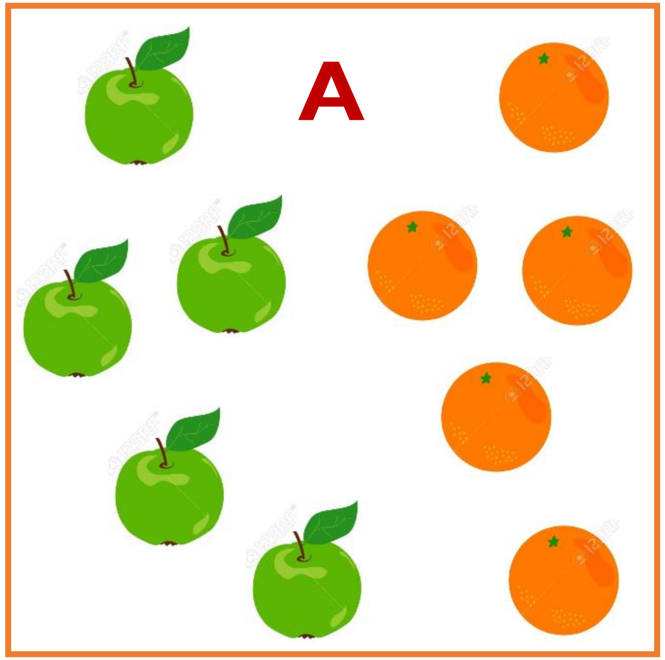

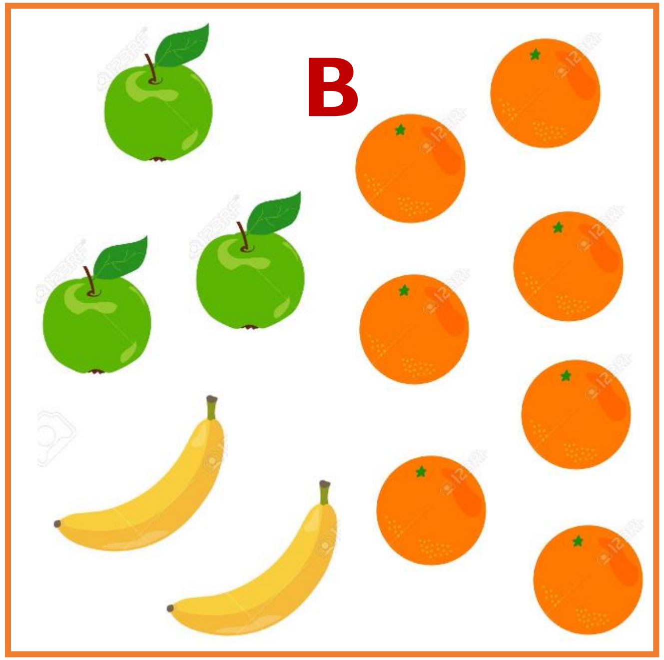

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$$P(orange|A) =$$

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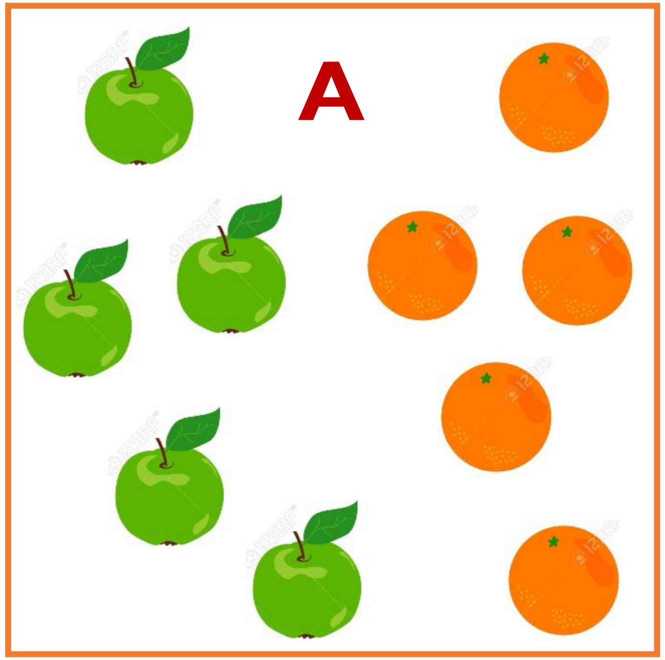

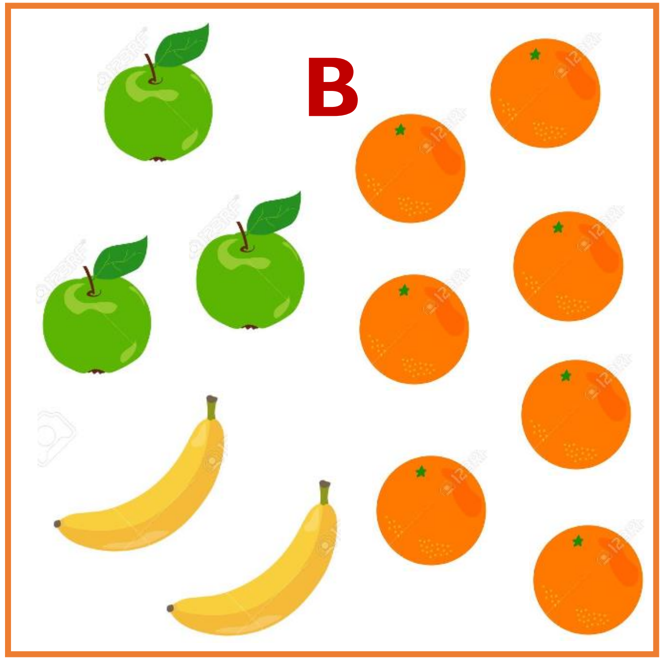

- What is the probability to pick an orange given that you've chosen box A?

$$P(orange|A) = \frac{5}{10} = 0.5$$

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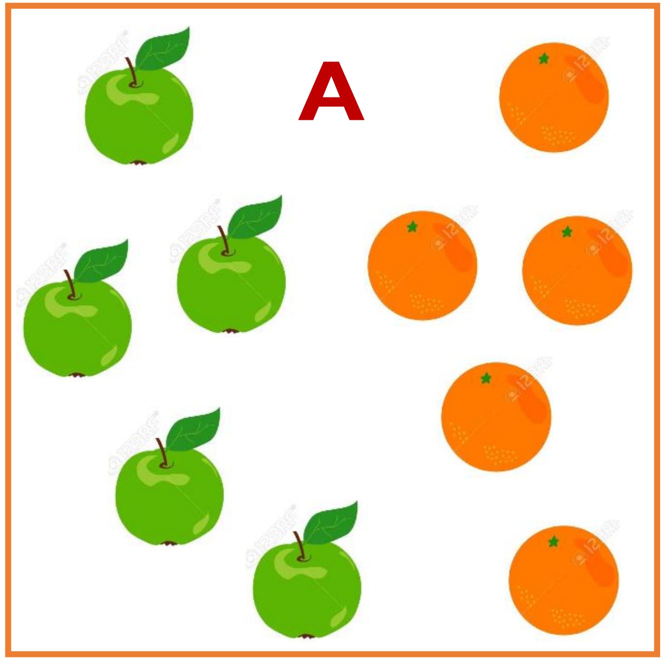

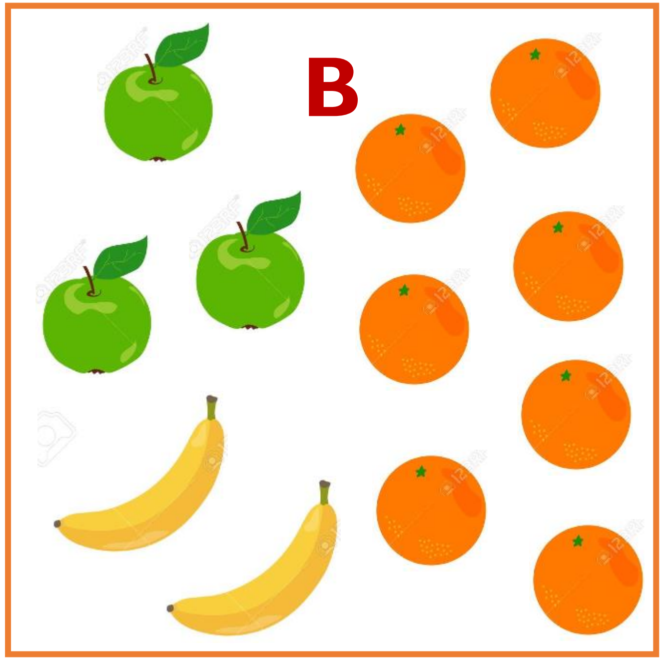

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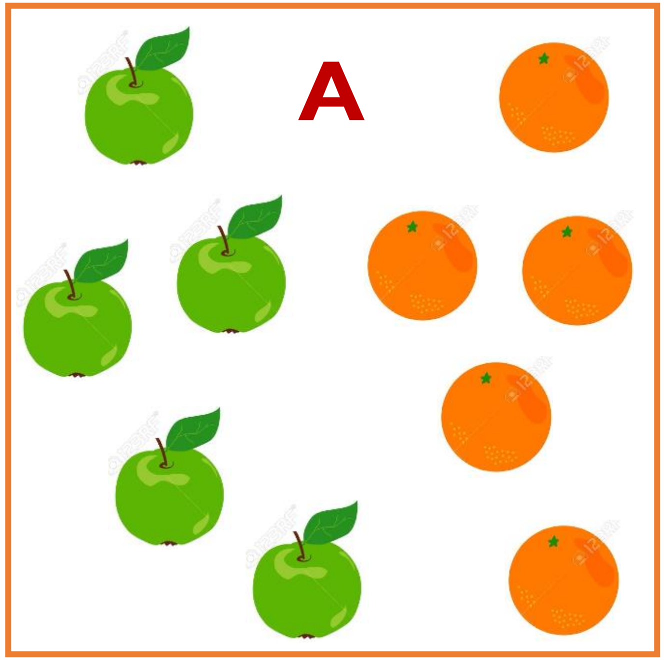

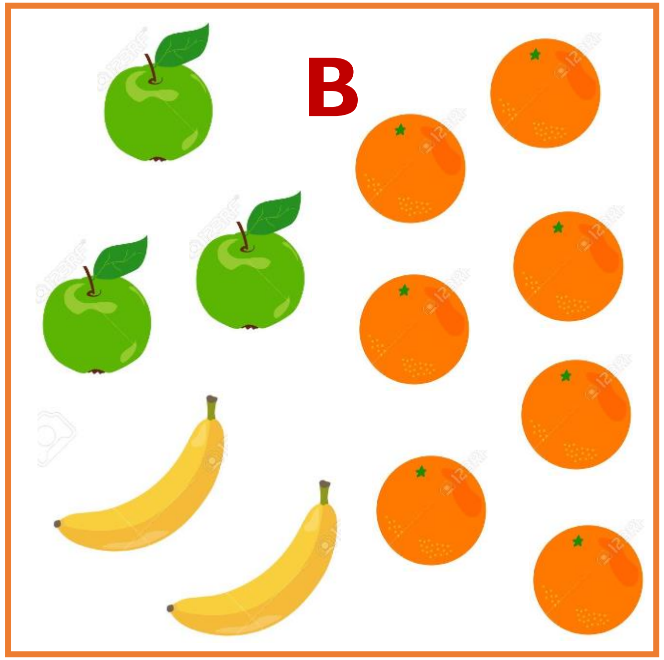

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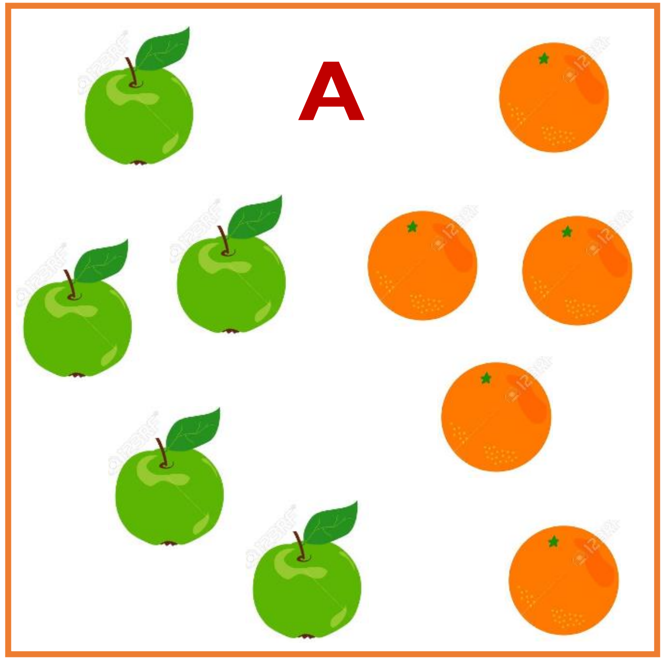

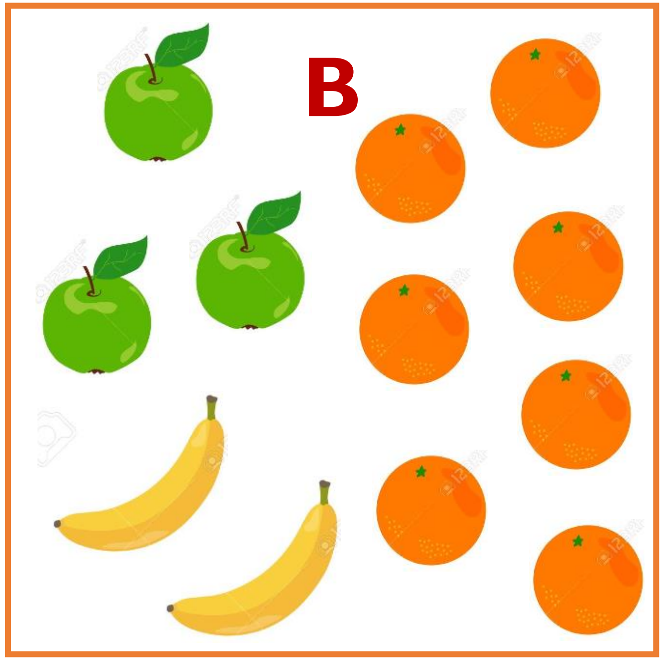

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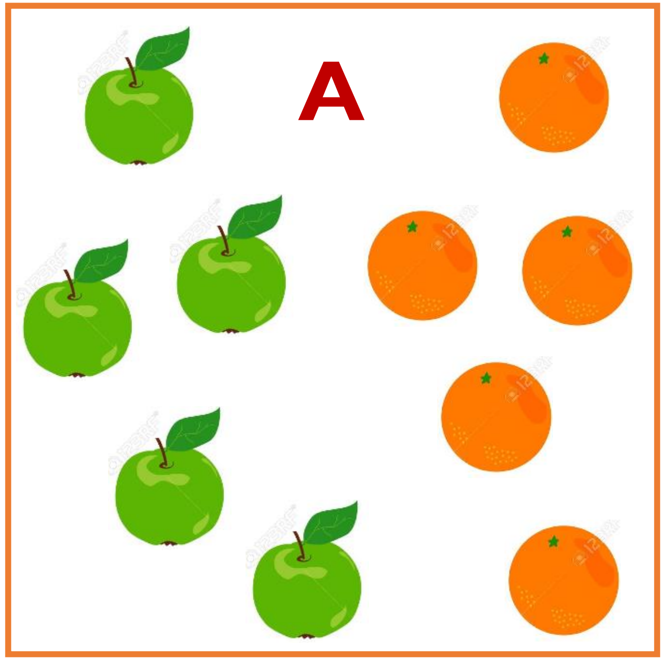

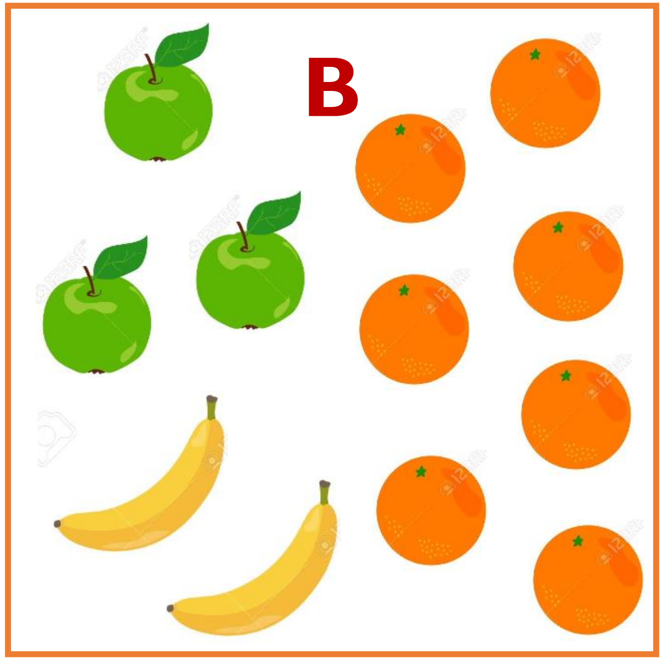

- What is the probability to pick an <u>apple</u> given that you've chosen <u>box B</u>?

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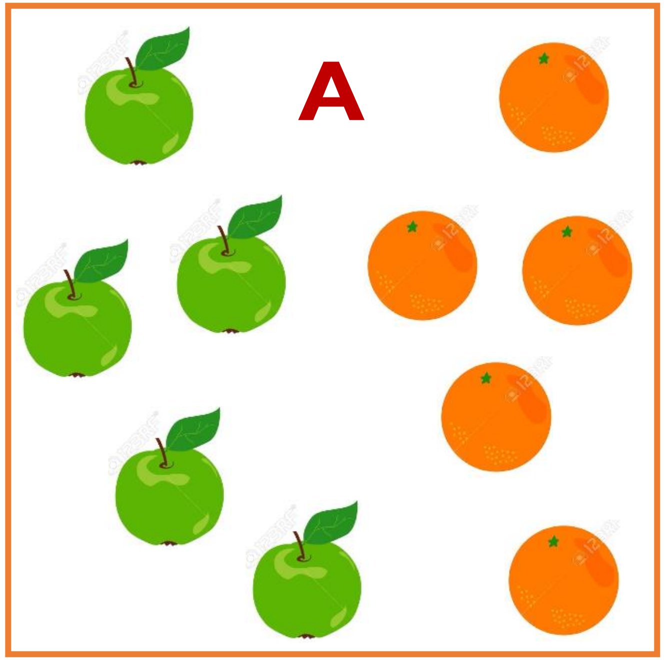

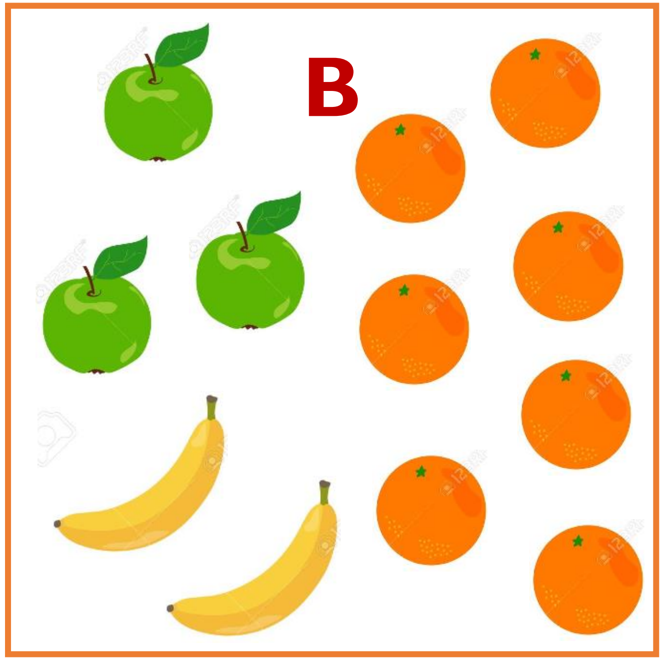

- What is the probability to pick an apple given that you've chosen box B?

$$P(apple|B) = \frac{3}{12} = 0.25$$

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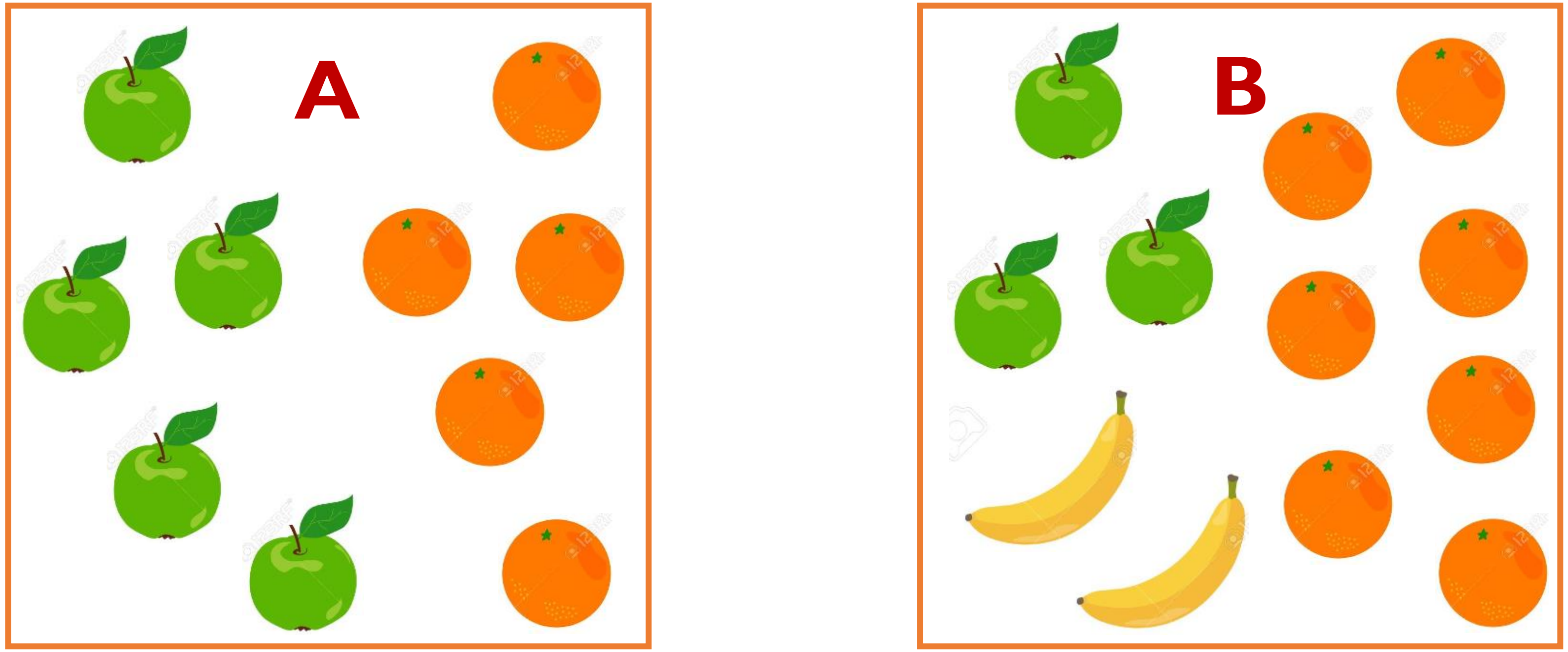

- What is the probability to pick an apple?

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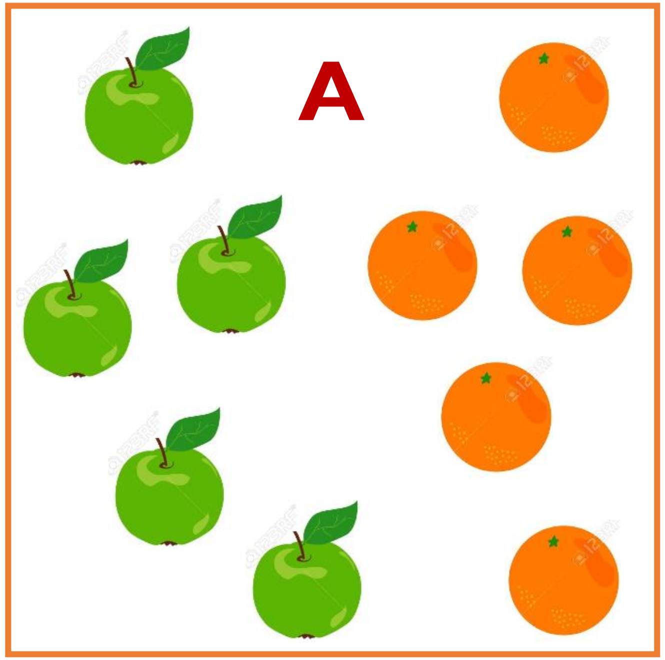

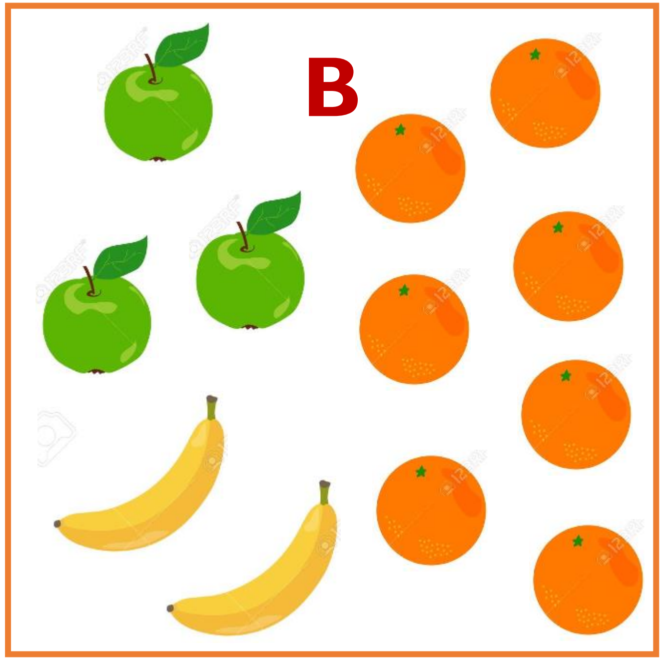

- What is the probability to pick an apple?

$$P(apple) =$$

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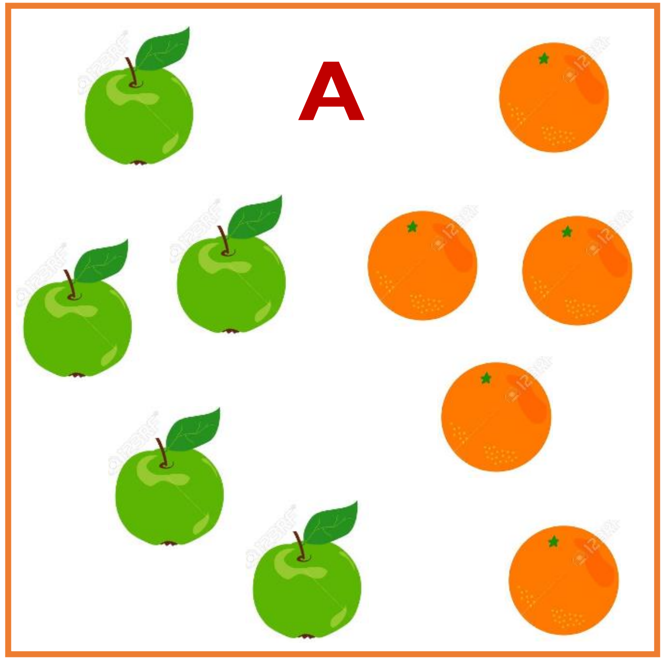

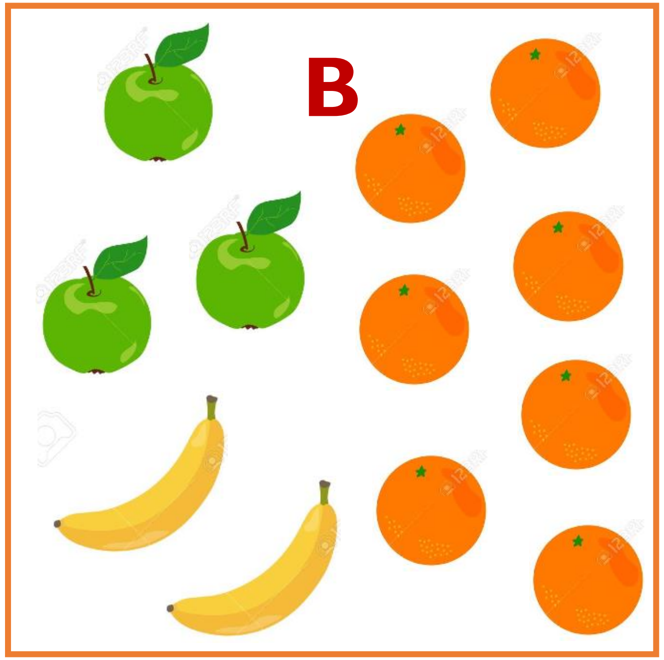

- What is the probability to pick <u>an apple</u>?

$$P(apple) = P(apple|A) \cdot P(A) + P(apple|B) \cdot (B) =$$

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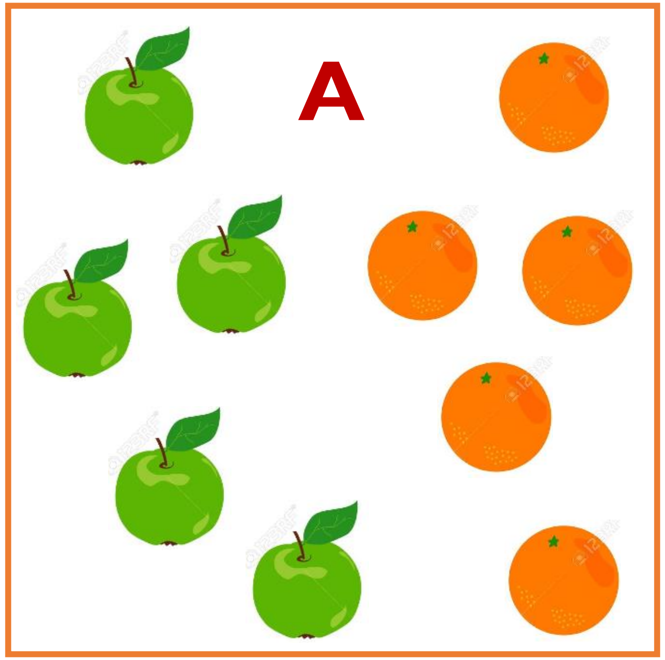

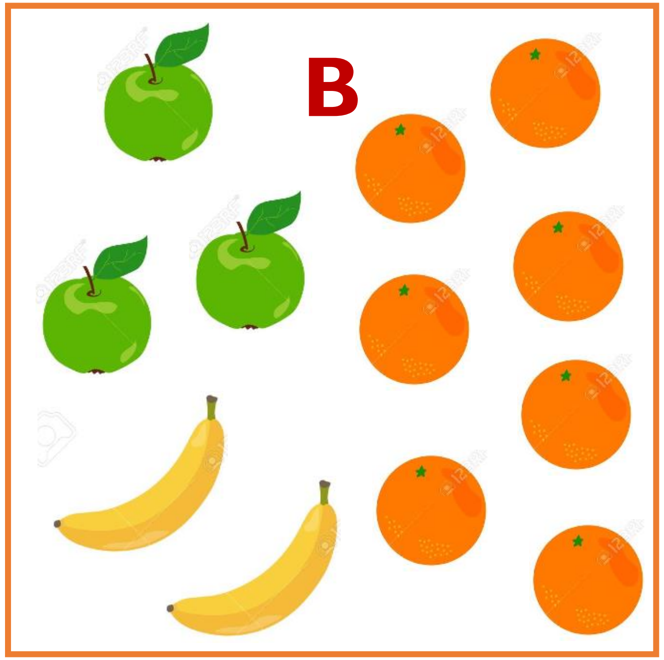

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$$P(apple) = P(apple|A) \cdot P(A) + P(apple|B) \cdot (B) =$$

$$= 0.5 \cdot 0.5 + 0.25 \cdot 0.5 = 0.375$$

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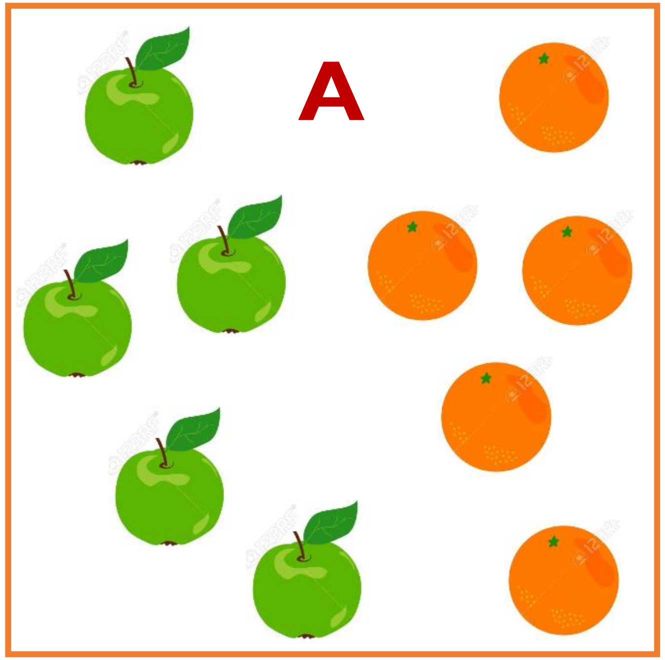

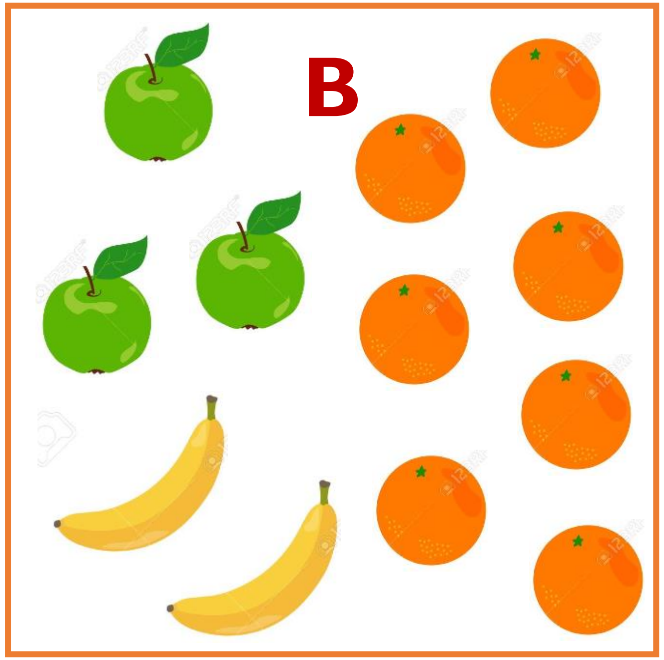

- What is the probability to pick <u>an orange</u>?

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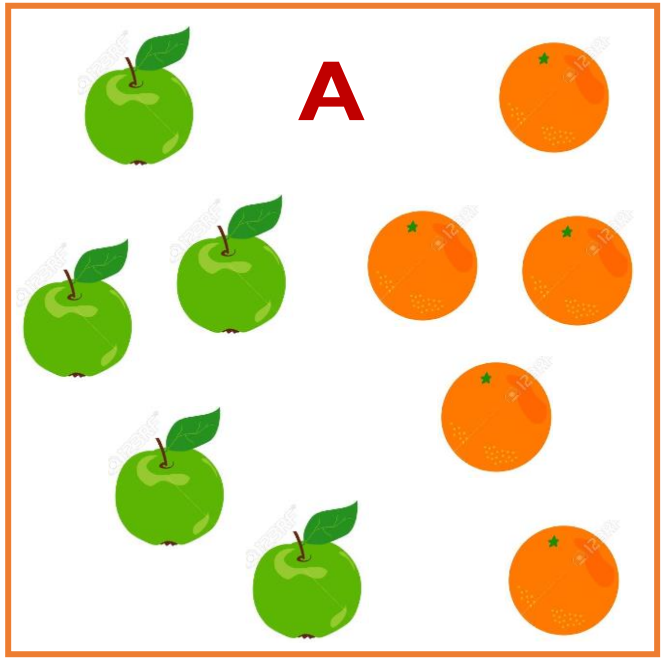

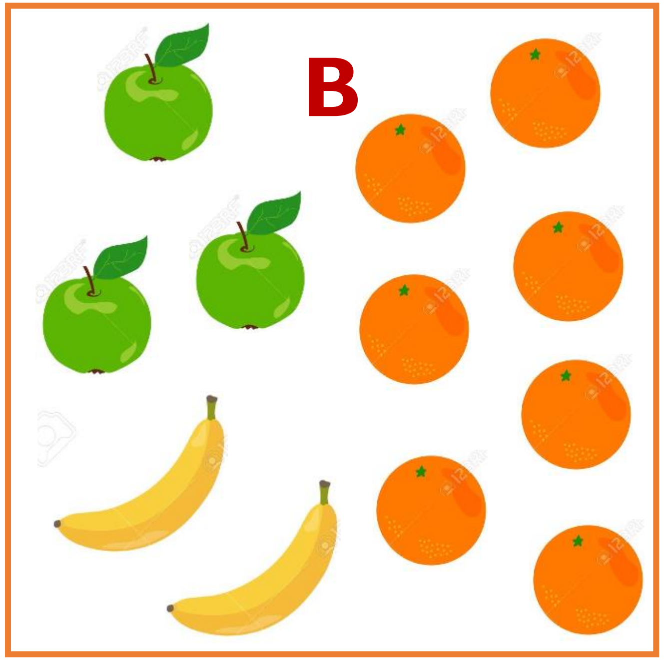

- What is the probability to pick <u>an orange</u>?

$$P(orange) =$$

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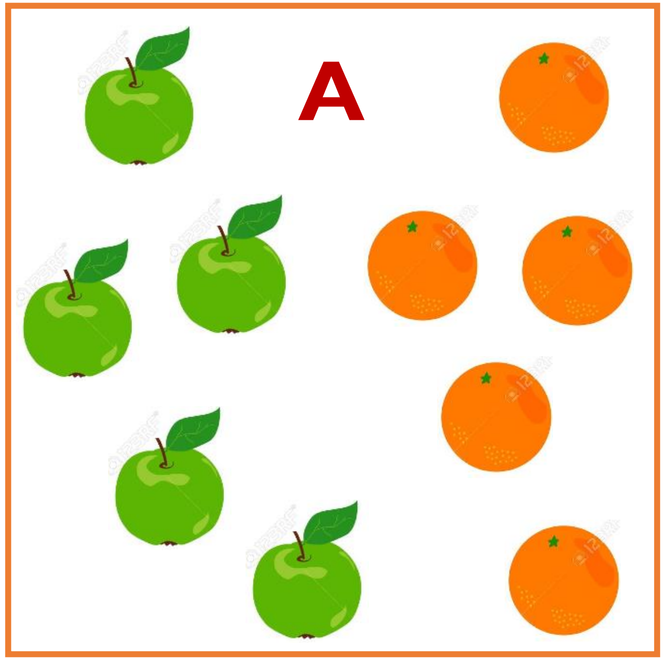

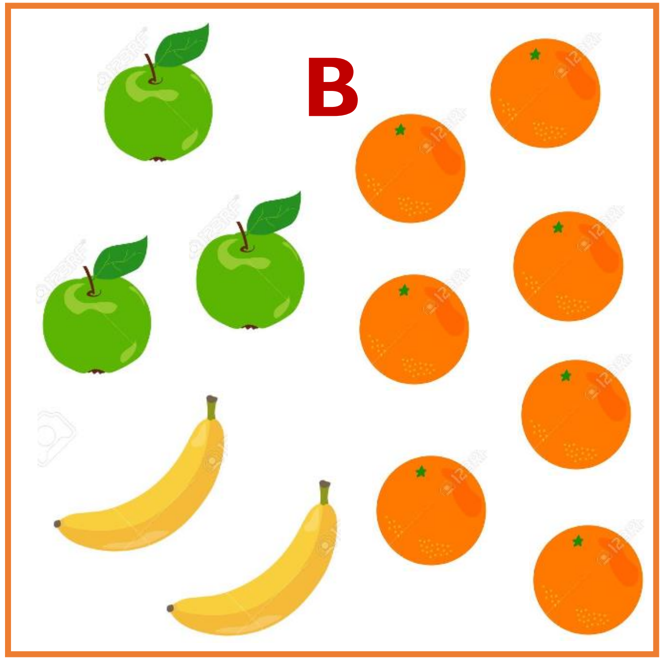

- What is the probability to pick <u>an orange</u>?

$$P(orange) = P(orange|A) \cdot P(A) + P(orange|B) \cdot P(B) =$$

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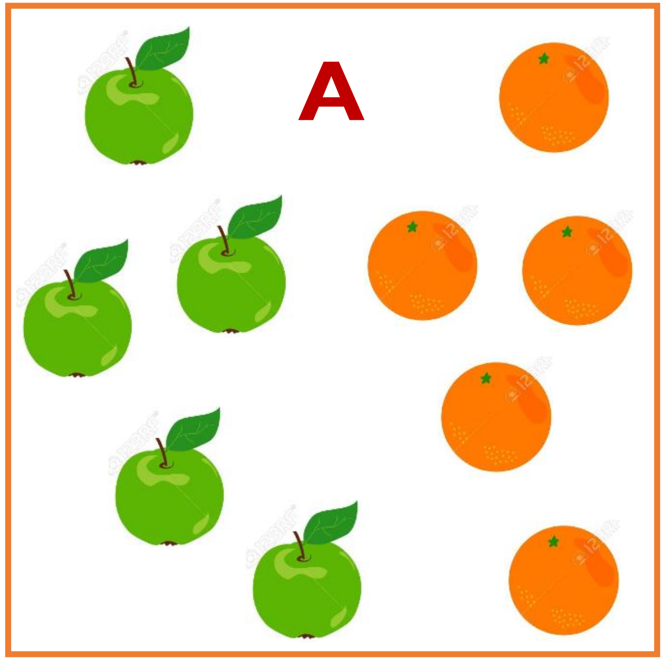

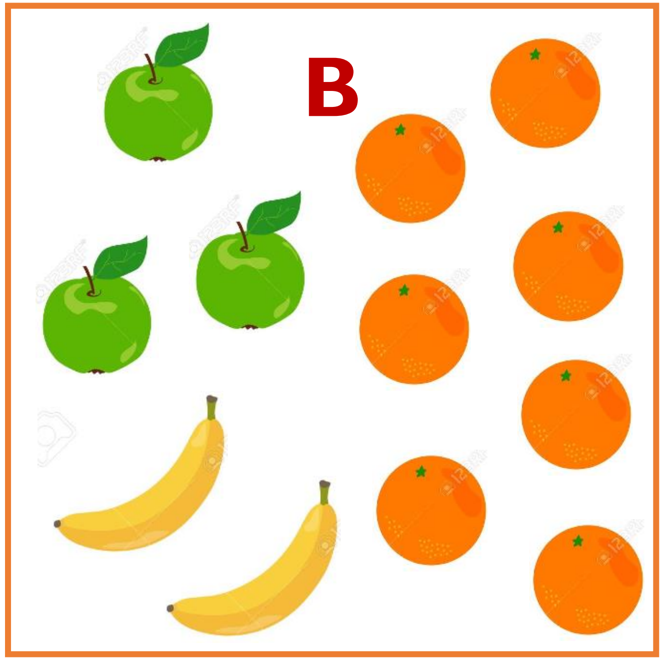

- What is the probability to pick <u>an orange</u>?

$$P(orange) = P(orange|A) \cdot P(A) + P(orange|B) \cdot P(B) =$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{7}{12} \cdot \frac{1}{2} = \frac{1}{4} + \frac{7}{24} = \frac{13}{24}$$

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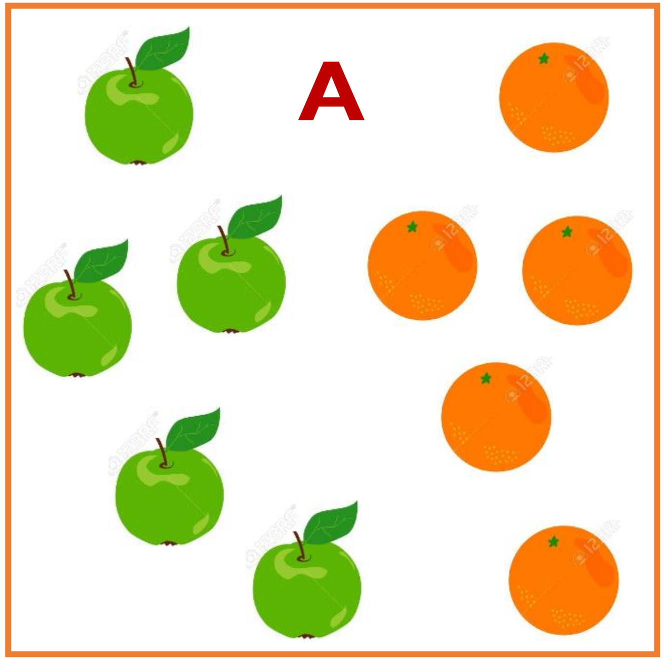

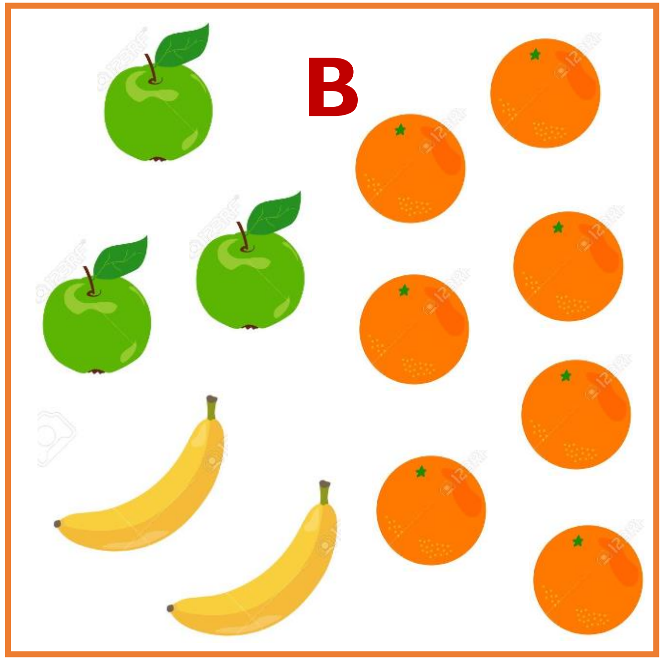

- What is the probability to pick <u>a banana</u>?

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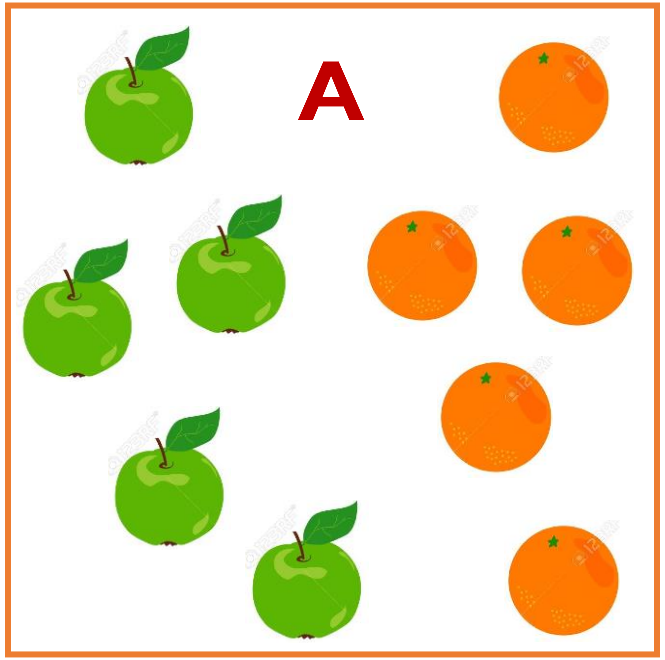

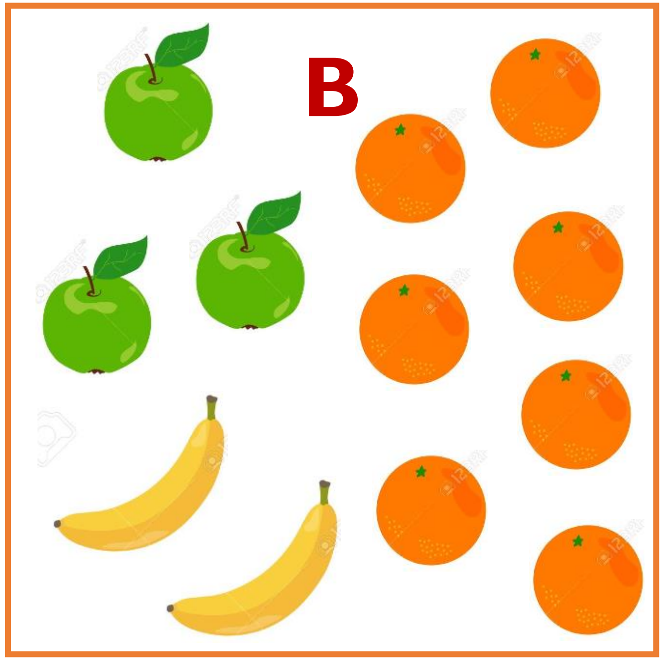

- What is the probability to pick a banana?

$$P(banana) =$$

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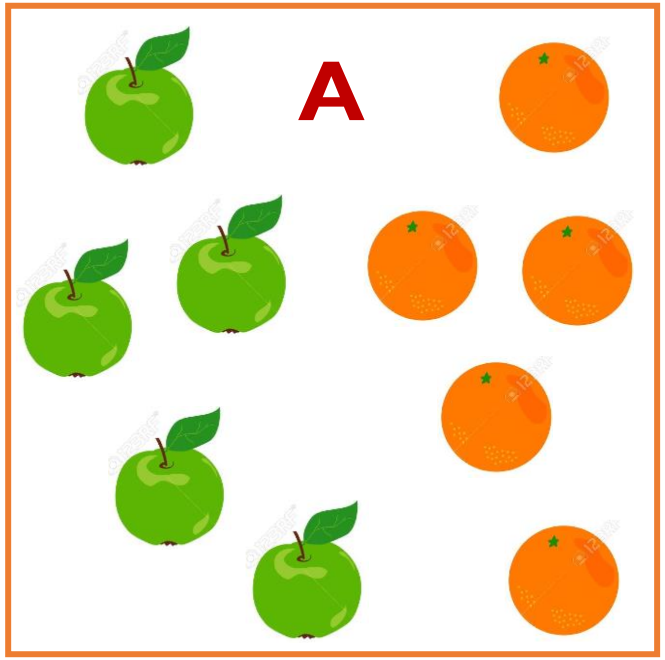
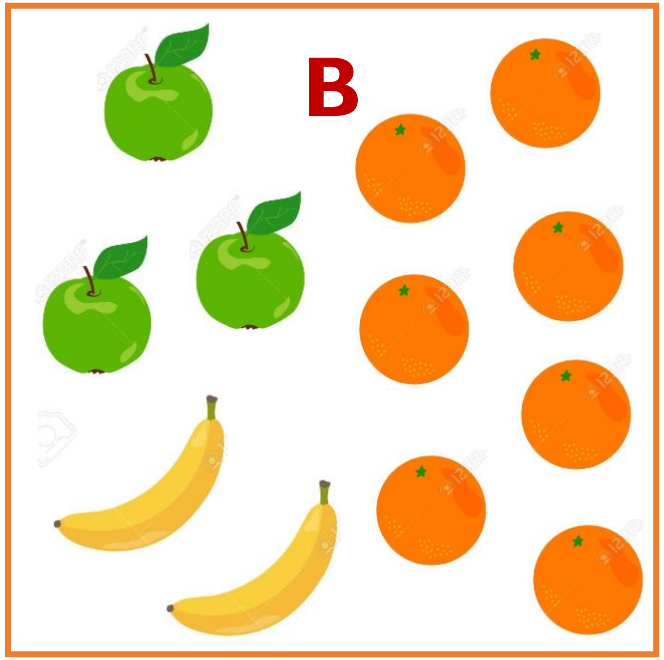
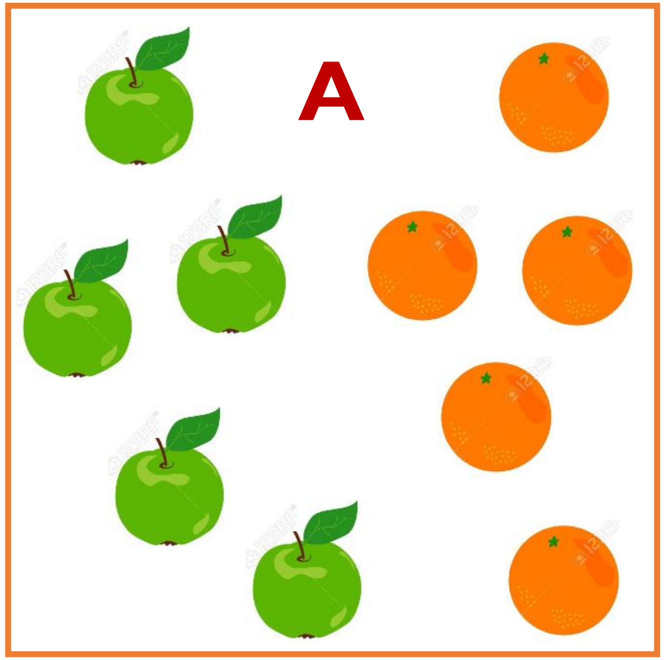


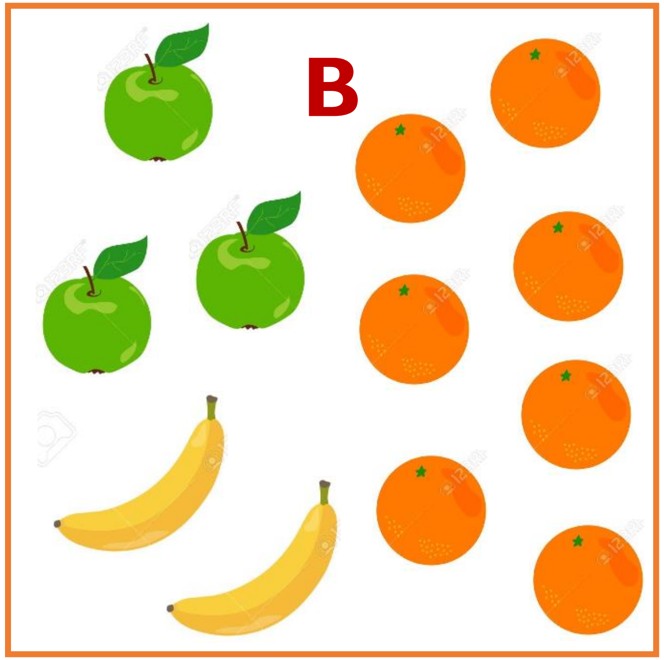


- What is the probability to pick <u>a banana</u>?

$$P(banana) = P(banana|A) \cdot P(A) + P(banana|B) \cdot P(B) =$$

FRUITS



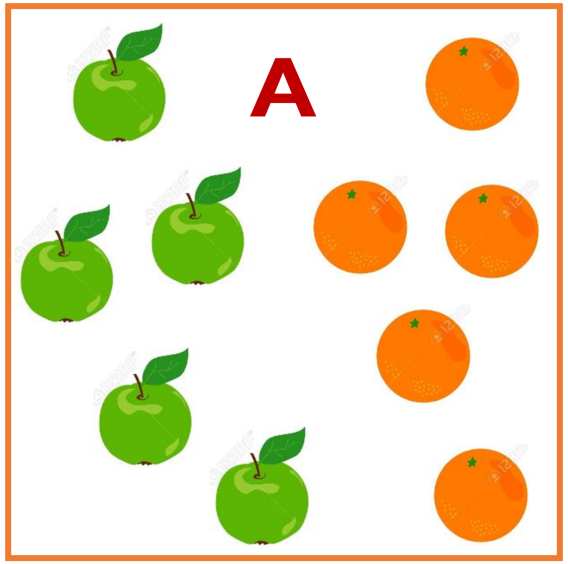


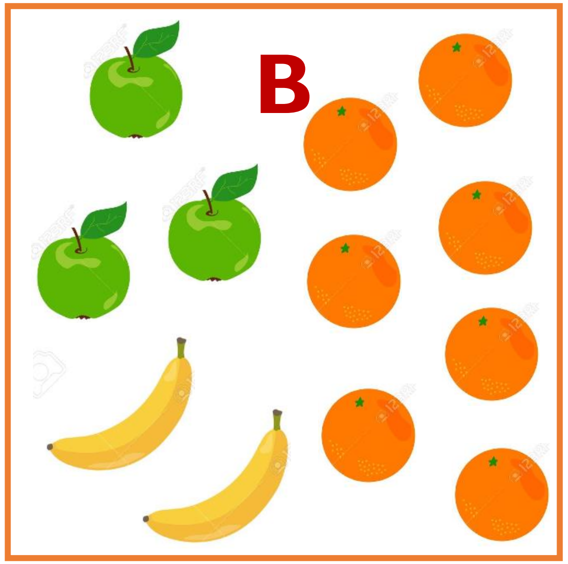
- What is the probability to pick a banana?

$$P(banana) = P(banana|A) \cdot P(A) + P(banana|B) \cdot P(B) =$$

$$= 0 \cdot \frac{1}{2} + \frac{2}{12} \cdot \frac{1}{2} = \frac{1}{12}$$

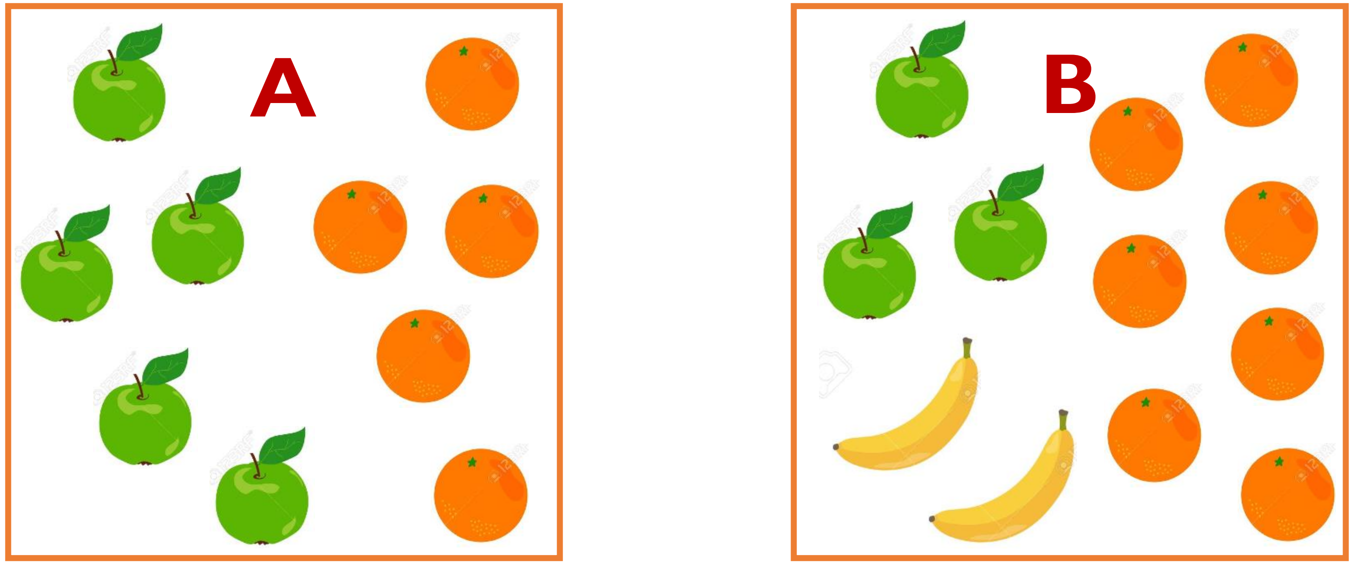
FRUITS





- What is the probability that you chose box B given that you picked a banana?

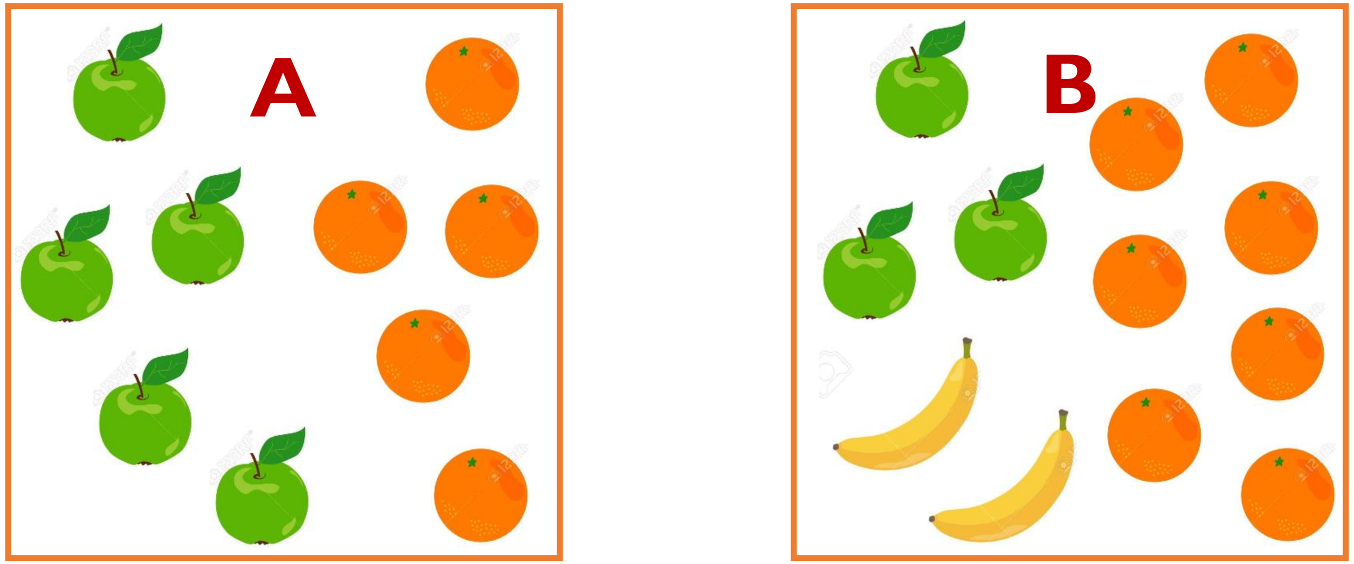
FRUITS



- What is the probability that you chose <u>box B</u> given that you picked <u>a banana</u>?

$$P(B|banana) =$$

FRUITS



- What is the probability that you chose box B given that you picked a banana?

$$P(B|banana) = \frac{P(banana|B) \cdot P(B)}{P(banana)} =$$

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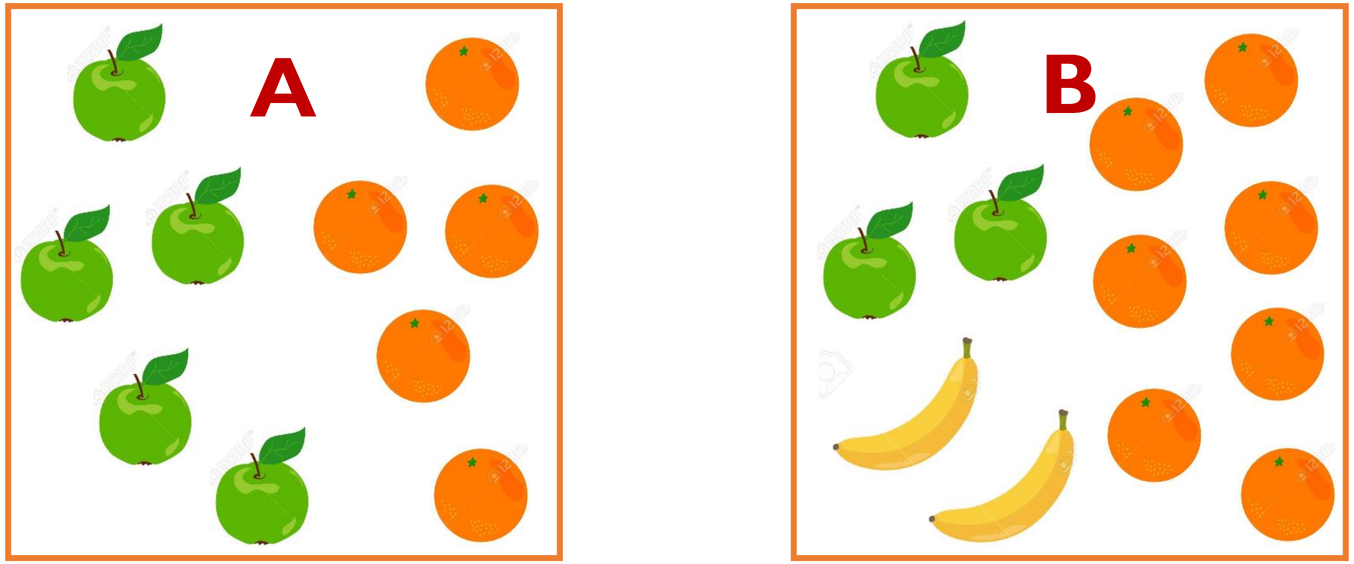

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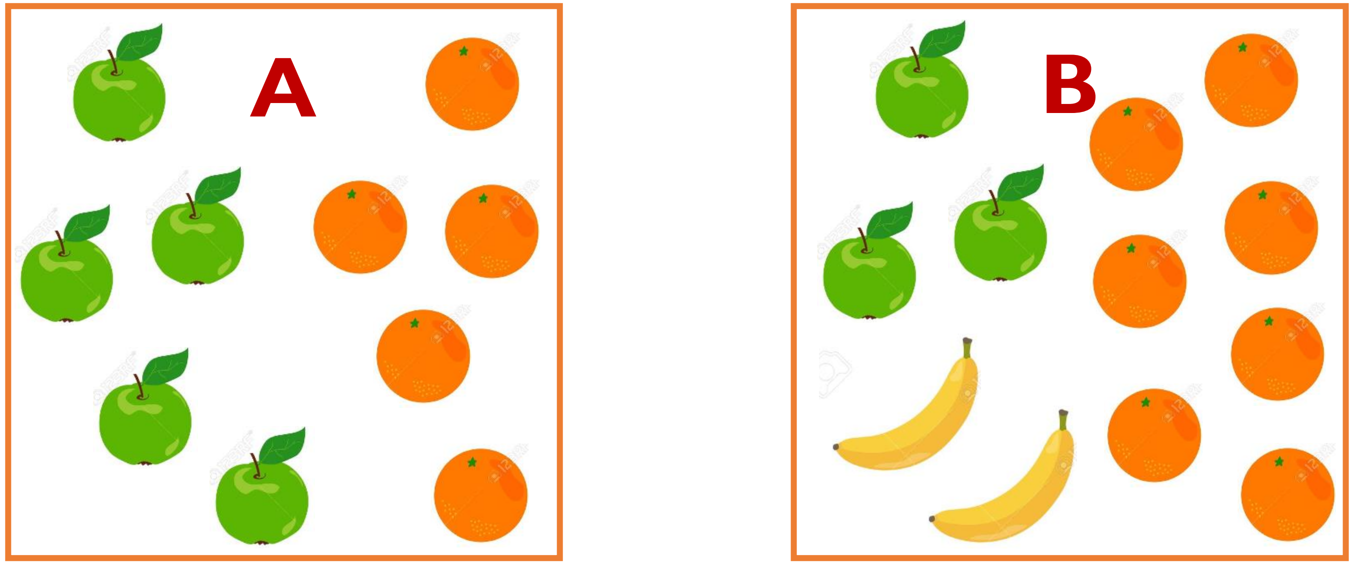

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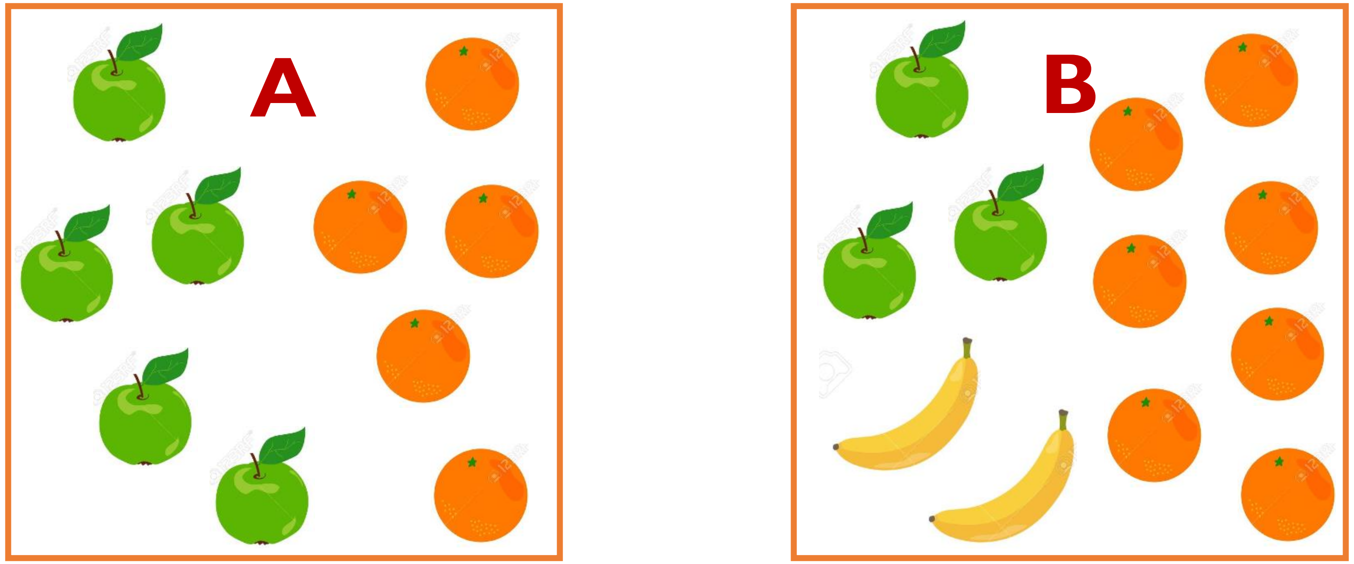

- What is the probability that you chose <u>box A</u> given that you picked <u>an apple</u>?

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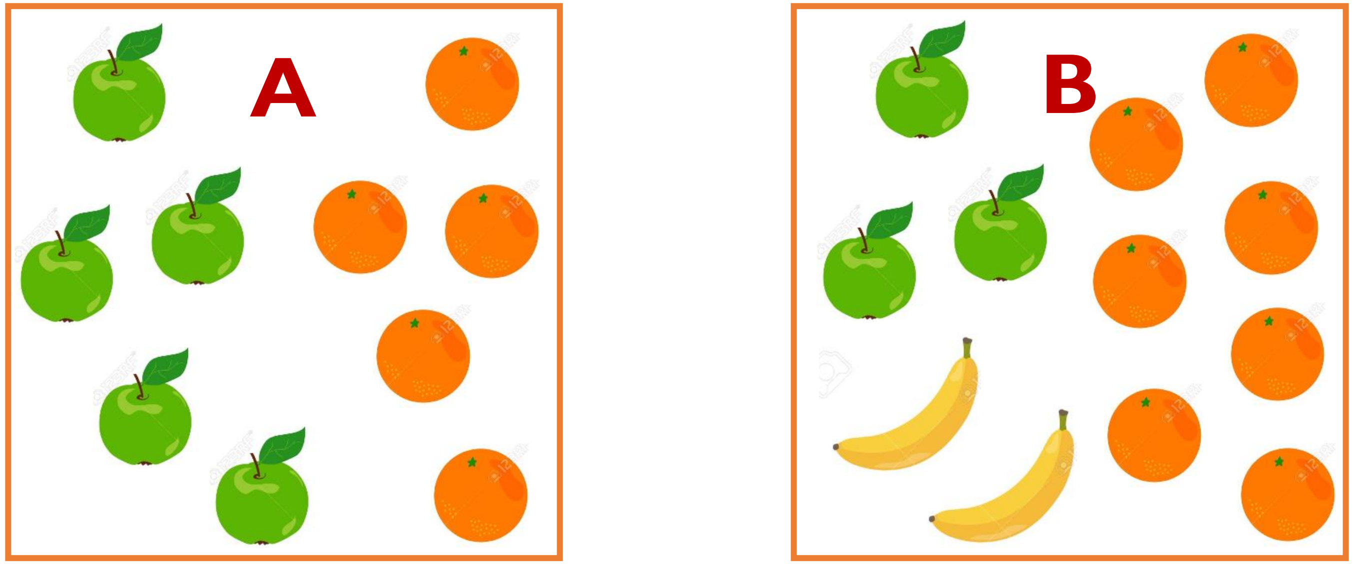

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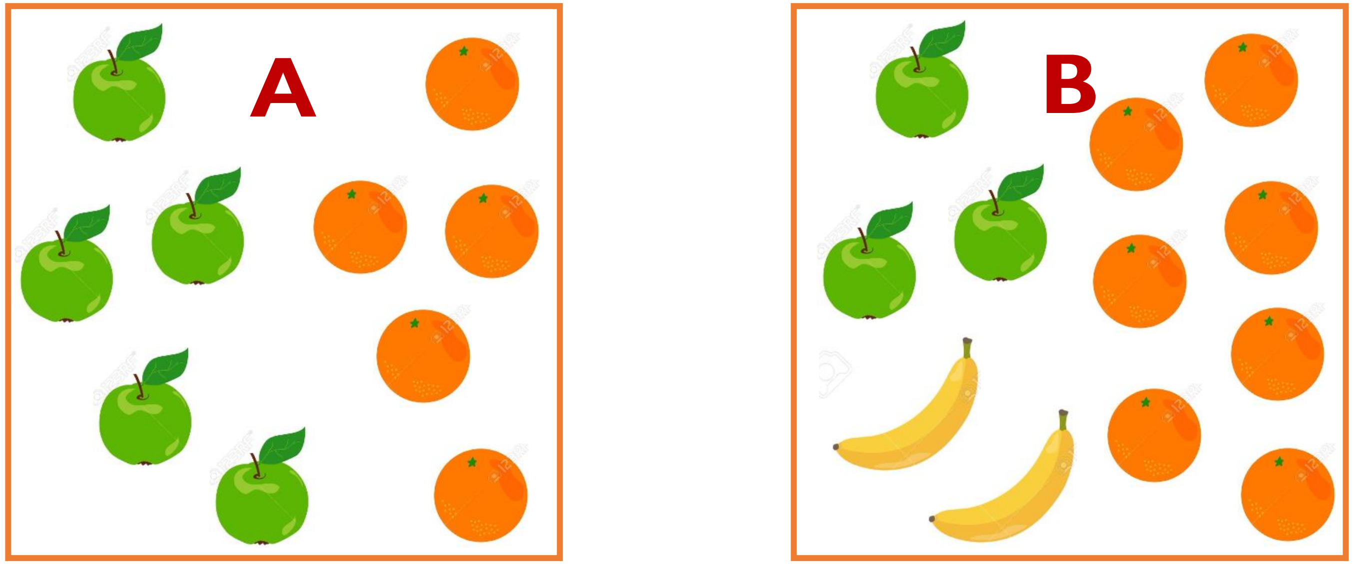

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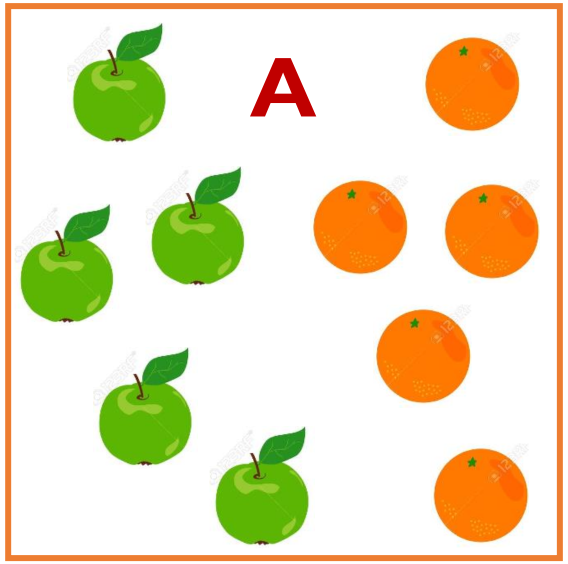

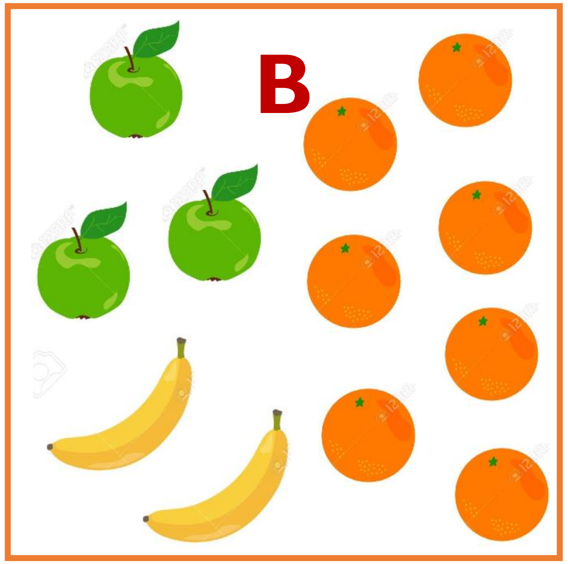

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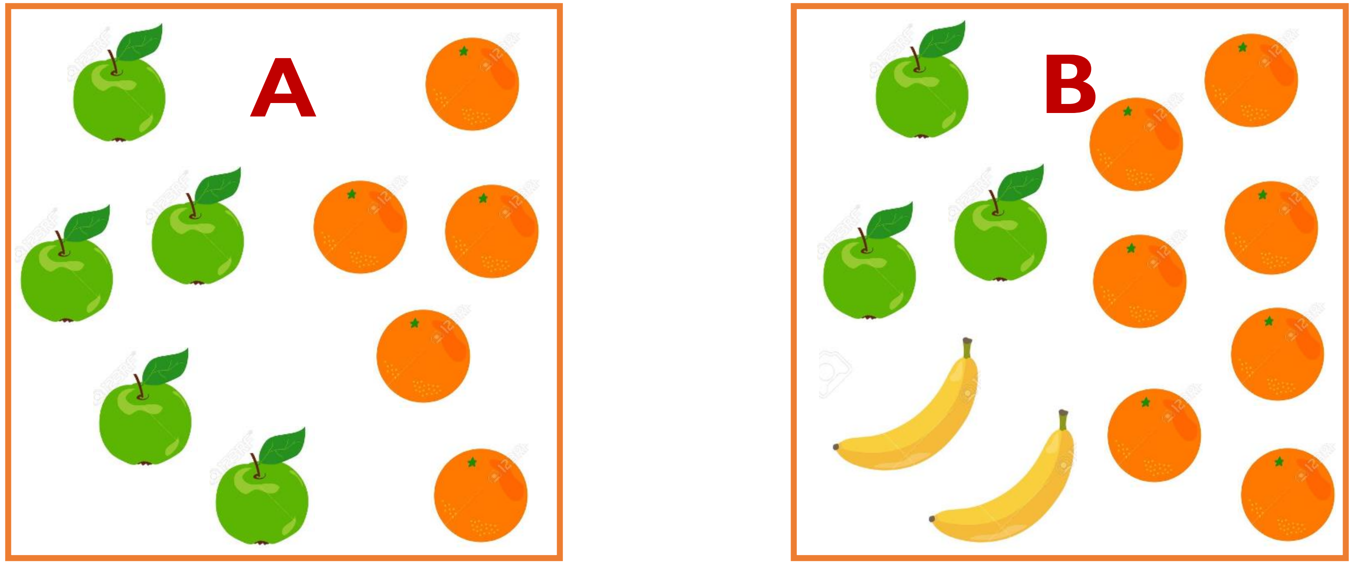

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JAR OF BEANS

- You have three jars of jellybeans. Jar A contains 25 red beans, 50 blue beans and 25 green beans. Jar B contains 50 beans of each of the three colors. Jar C contains 10 red beans and 90 blue beans. You are randomly choosing one of the three jars and then randomly picking one bean from it. What's the probability that it's green?

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$$= \frac{25}{100} \cdot \frac{1}{3} + \frac{50}{150} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}$$

ALARM

- A fire alarm system consists of two smoke detectors. The alarm will go on if at least one of the detectors detects smoke. Smoke detector A detects smoke with probability 0.85, while smoke detector B detects it with probability 0.9. Both systems detect smoke with probability 0.8. What is the probability that the fire alarm does **not** go on if there is a fire?

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DRUG TEST

- 0.5% of the population are using drugs. If a person taking the test has used drugs, the test will show a positive result with a 98% chance, and if they haven't used drugs, the test will be negative with a 98% probability. A random person tests positive. What is the probability that this person has actually used drugs?

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$$= \frac{0.98 \cdot 0.005}{0.98 \cdot 0.005 + (1 - 0.98) \cdot (1 - 0.005)} \approx 0.2$$

RANDOM VARIABLES

https://youtu.be/S_obHZJZ5EM

MOTIVATION

- We usually focus on some numerical aspects of the experiment
  - number of heads in 100 coin tosses;
  - number of boys among 4 kids;
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- These are *random variables*
  - A real-valued variable whose value is determined by an underlying random experiment.

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- A random variable is a *function* from the sample space to the real numbers:

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- A set of all possible values a random variable can take is called **range**.

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(UN)COUNTABLE SETS

- You know that sets can be finite and infinite.

$$A = \{1, 2, 3\}, \qquad B = \{2, 4, 6, \dots\}, \qquad C = [1; 2], \qquad D = \mathbb{R}$$

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- **Probability distribution** of X:

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PROBABILITY MASS FUNCTION

- For a discrete random variable X with range $R_X = \{x_1, x_2, ...\}$ **probability mass function (PMF)** is defined as

$$P_X(x) = \begin{cases} P(X = x), & x \in R_X \\ 0, & x \notin R_X \end{cases}$$

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PROPERTIES OF PMF

- Let X be a random variable with range R_X and a PMF $P_X(x)$.
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1. For all x, $0 \leq P_X(x) \leq 1$

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3. For any set $A \subseteq R_X$, $P(X \in A) = \sum_{x_k \in A} P_X(x_k)$
   $A = \{0, 1\}$, $\qquad P(X = 0 \; or \; X = 1) = 0.25 + 0.5 = 0.75$

UNFAIR COIN

- You are tossing an unfair coin with the probability of heads 0.6 twice. Let random variable X denote the total number of heads.

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$$P_X(x) = \begin{cases} 0.4^2, & x = 0 \\ & x = 1 \\ & x = 2 \\ & otherwise \end{cases}$$

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$$P(X = 10) = P_X(10)$$

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$$P(X = 10) = P_X(10) = 0,$$
$$P(X \leq 1) =$$

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ANOTHER EXAMPLE

- Random experiment: tossing a fair coin until heads appear.
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$$P_X(k) = P(X = k) =$$

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- PMF: $P_X(k) = ?$

$$P_X(k) = P(X = k) = \begin{cases} 0.5^k, & k \geq 1 \\ 0, & otherwise \end{cases}$$

CUMULATIVE DISTRIBUTION FUNCTION

- Another way to represent distribution.
- **Cumulative distribution function (CDF)** of a random variable X is defined as follows:

$$F_X(x) = P(X \leq x) \quad \forall x \in \mathbb{R}$$

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PMF AND CDF: EXAMPLE

- You are rolling a fair die. Y – the outcome.
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