ELEMENTARY COMBINATORICS & PROBABILITY

Review week 2

LAST WEEK

- Random experiments
  - outcome;
  - sample space;
  - events.
- Probability of an event
- Conditional probability
- Independent events
- The law of total probability
- Bayes' rule

RANDOM EXPERIMENT

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- The set of all possible outcomes is called the **sample space** (denoted by S).

EVENT

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  $E_1 = \{1\} -$ we've got number 1.

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  Example: rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

  $E_1 = \{1\} -$ we've got number 1.

  $E_2 = \{2, 4, 6\} -$ we've got an even number.

EVENT

- Each subset of a sampling space is called an **event** (denoted by E).

  Example: rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

  $E_1 = \{1\}$ – we've got number 1.

  $E_2 = \{2, 4, 6\}$ – we've got an even number.

  $E_3 = \{3, 5\}$ – we've got 3 or 5.

INTERSECTION OF TWO EVENTS

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$$F_1 = \{a\ male\ prof.\ is\ selected\}, \qquad F_2 = \{a\ female\ prof.\ is\ selected\}$$

$$F_1 \cap F_2 = \emptyset, \qquad |F_1 \cap F_2| = 0$$

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$$F = \{a\ female\ student\ is\ selected\},$$

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COMPLEMENTARY EVENTS

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$$E^C = \{a\ student\ is\ selected\ \}$$

$$F = \{a\ female\ student\ is\ selected\},$$
$$F^C = \{a\ male\ student\ or\ a\ professor\ is\ selected\}$$

COMPUTING PROBABILITY

- If a sample space S is finite, and each outcome is equally likely, than probability of an event E can be computed as

$$P(E) = \frac{\#\text{ ways } E \text{ can occur}}{\#\text{ possible outcomes}} = \frac{|E|}{|S|}$$

PROBABILITY

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PROBABILITY

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$$P(student) = \frac{10}{15+10} = \frac{10}{25} = \frac{2}{5}$$

$$P(professor) + P(student) = 1$$

CONDITIONAL PROBABILITY

- Let A and B be events in a sample space S with $P(B) > 0$.

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(AB)}{P(B)} = \frac{P(A \,\&\, B)}{P(B)}$$

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$$P(woman|student) =$$

CONDITIONAL PROBABILITY

- Faculty board, consisting of 15 professors (10 men and 5 women) and 10 students (5 men and 5 women), is randomly choosing a candidate to represent the faculty at the visiting day.

$$P(man|professor) = \frac{10}{15} = \frac{2}{3}$$

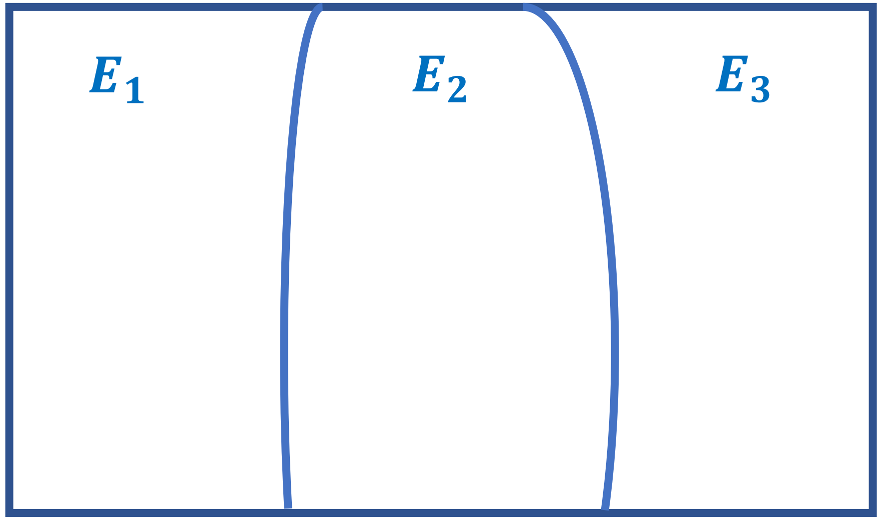
$$P(woman|student) = \frac{5}{10} = \frac{1}{2}$$

THE LAW OF TOTAL PROBABILITY

Suppose that the sample space S is split into n disjoint events:

$$S = E_1 \cup E_2 \cup \cdots \cup E_n,$$

$$E_1 \cap E_2 \cap \cdots \cap E_n = \emptyset$$



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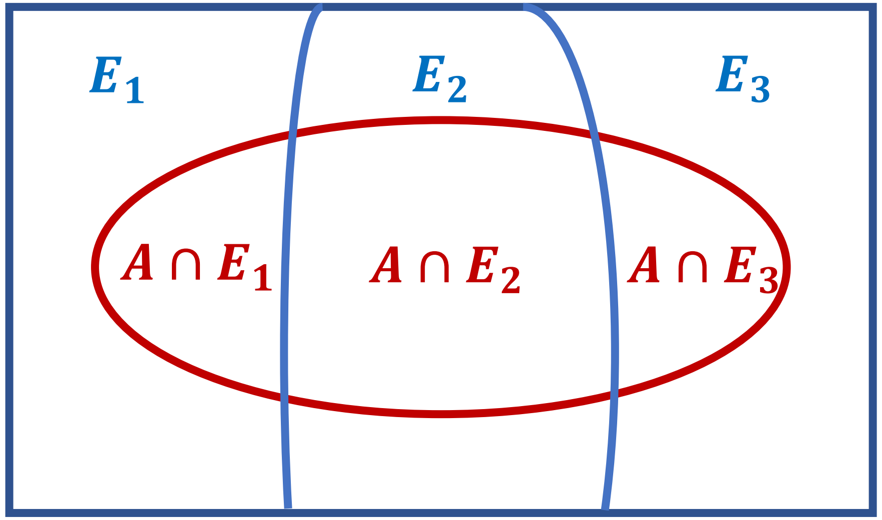

Then $\mathrm{P}(A)$ can be computed as follows:

$$P(A) = P(A, E_1) + P(A, E_2) + \cdots + P(A, E_n) =$$
$$= P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) +$$
$$+ \cdots + P(A|E_n) \cdot P(E_n)$$

TOTAL PROBABILITY

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$$P(man) =$$
$$= P(man|student) \cdot P(student) + P(man|professor) \cdot P(professor) =$$
$$=$$

TOTAL PROBABILITY

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$$= P(man|student) \cdot P(student) + P(man|professor) \cdot P(professor) =$$
$$= \frac{5}{10} \cdot \frac{10}{25} + \frac{10}{15} \cdot \frac{15}{25} = \frac{5}{25} + \frac{10}{25} = \frac{15}{25} = \frac{3}{5}$$

BAYES' RULE

$$\boldsymbol{P(A|B)} = \frac{P(AB)}{P(B)} = \frac{\boldsymbol{P(B|A)}}{\boldsymbol{P(B)}} \cdot \boldsymbol{P(A)}$$

BAYES' RULE

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$$P(professor|man) =$$

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$$P(professor|man) = \frac{P(man|professor) \cdot P(professor)}{P(man)} =$$

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$$= \frac{\frac{10}{15} \cdot \frac{15}{25}}{\frac{10}{15} \cdot \frac{15}{25} + \frac{5}{10} \cdot \frac{10}{25}} =$$

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$$= \frac{\frac{10}{15} \cdot \frac{15}{25}}{\frac{10}{15} \cdot \frac{15}{25} + \frac{5}{10} \cdot \frac{10}{25}} = \frac{2}{5} \cdot \frac{5}{3} = \frac{2}{3}$$

INDEPENDENT EVENTS

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$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A) \cdot P(B)}{P(A)} = P(B)$$

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$$E_1 = \{man\}, \qquad E_2 = \{professor\}$$

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$P(E_1) =$

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$$P(E_1 \cap E_2) \qquad P(E_1) \cdot P(E_2)$$

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$$P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2)$$

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$$P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2) \rightarrow$$

the events are not independent

WEATHER FORECAST

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$$P(Sat \,\&\, Sun) = P(Sat) \cdot P(Sun) = \frac{1}{4}$$

BOXES

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$$= \frac{1}{2} \cdot 1 + \frac{14}{29} \cdot \frac{1}{2} = \frac{1}{2} + \frac{7}{29} = \frac{43}{58}$$

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$$P(B|white) = \frac{P(white|B) \cdot \mathrm{P}(B)}{P(white)} = \frac{\frac{14}{29} \cdot \frac{1}{2}}{\frac{14}{29} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}} = \frac{14}{43}$$

QUEUE

- Six people form a queue, including A and B. Assume that all 6! orderings are equiprobable. What is the probability that B is before A given that A is not the first?

$$P(B < A|A \neq 1) =$$

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$$= \frac{1 \cdot 4! + 2 \cdot 4! + 3 \cdot 4! + 4 \cdot 4! + 5 \cdot 4!}{5 \cdot 5!} =$$

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$$= \frac{1 \cdot 4! + 2 \cdot 4! + 3 \cdot 4! + 4 \cdot 4! + 5 \cdot 4!}{5 \cdot 5!} = \frac{15 \cdot 4!}{5 \cdot 5!} = \frac{15}{25} = \frac{3}{5}$$

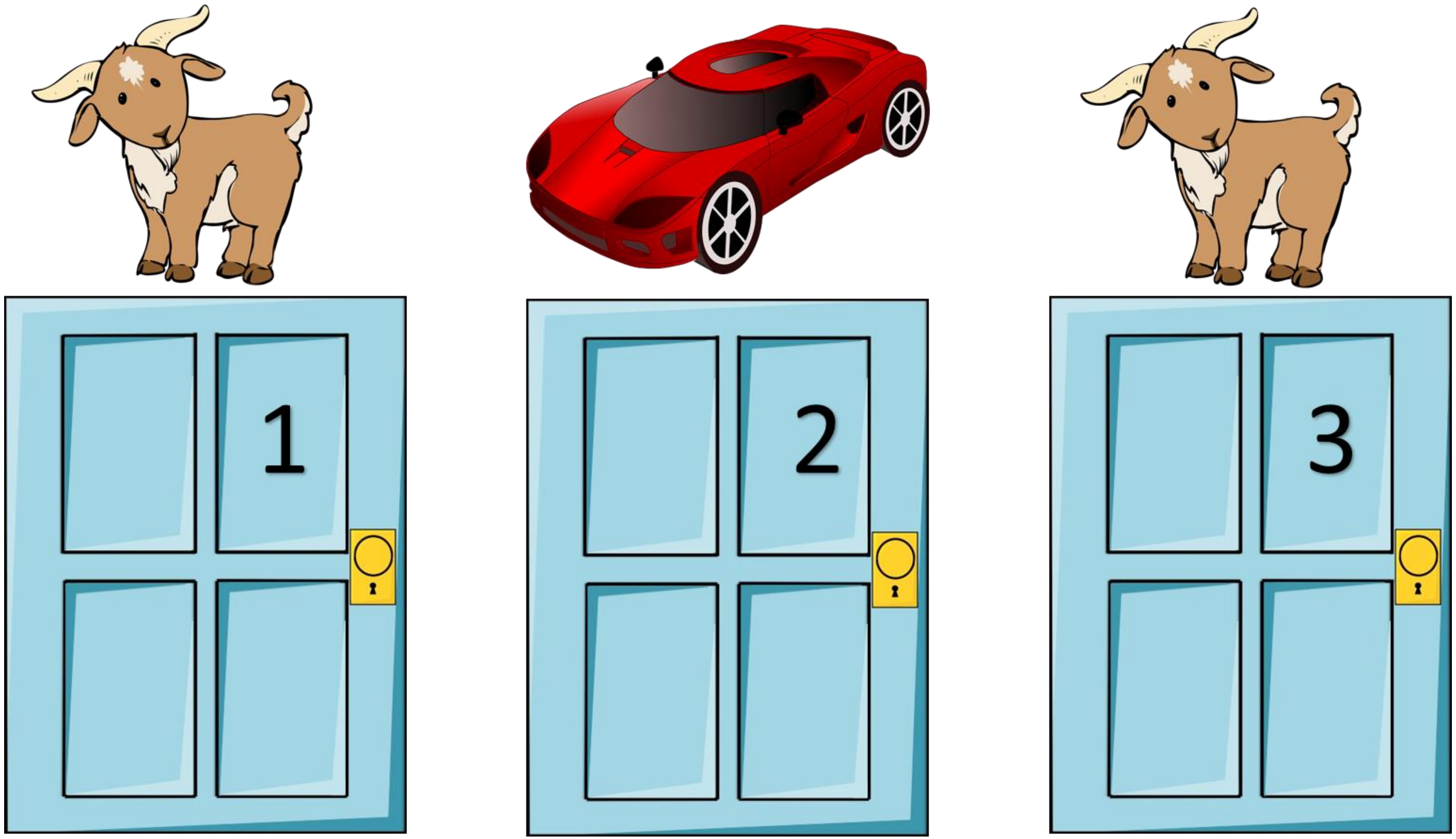
MONTY HALL

INTERIM EXAM

- Exam available on Google classroom as of 10:20
- Open book
- 15 questions
  - 5 one-point questions;
  - 10 two-point questions.
- Solutions must be written in the file
  - typed or pictures;
  - should contain explanations.
- **Deadline: 12:30 Barcelona time**

GOOD LUCK!

MONTY HALL



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