ELEMENTARY COMBINATORICS & PROBABILITY

Probability of an event

TODAY

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MOTIVATION

- Some things in the world are random
  - flipping a coin;
  - roll of a die;
  - picking a random card from a shuffled deck of cards.

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- Started to evolve in the 17th century
  - gambling games.

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- *(1933)* Kolmogorov developed the first rigorous approach to probability.

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- The set of all possible outcomes is called the **sample space** (denoted by S).

RANDOM EXPERIMENTS: EXAMPLES

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  Sample space:
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- Random experiment: rolling a die
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  Outcome: 5

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  $E_3 = \{3, 5\}$ – we've got 3 or 5.

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$$E_1 = \{1, 3, 5\}, \qquad E_2 = \{2, 4, 6\}, \qquad E_1 \cap E_2 = \emptyset$$

WHAT IS PROBABILITY

RELATIVE FREQUENCY

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- Example:
  If we flip a fair coin infinitely many times, it will come up heads half of the times.

COMPUTING PROBABILITY

- If a sample space S is finite, and each outcome is equally likely, than probability of an event E can be computed as

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- Another interpretation of probability: quantification of our degree of **subjective personal belief** that something will happen.

- Good news: interpretation doesn't matter, same principles hold.

MORE EXAMPLES

FLIPPING A COIN

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- What's the probability of getting two heads?

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$$P(E) = \frac{3}{6} = \frac{1}{2}$$

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Therefore,

$$P(E) = \frac{6}{20} = 0.3$$

PLAYING A DICE GAME

https://youtu.be/Kgudt4PXs28

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   Example: getting a baby

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   E: ‘get boy or girl’ $P(E) = 1$

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3. If $E_1, E_2, E_3, \ldots$ are mutually exclusive events, then

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$$P(E_A \cup E_B) = P(E_A) + P(E_B) = 0.2 + 0.4 = 0.6$$

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DRAWING NAMES

A total of 20 students participate in a drawing in which one name is chosen at random. There are 6 seniors (2 men and 4 women), 5 juniors (3 men and 2 women) and 9 freshmen (5 men and 4 women). What is the probability that the name that is drawn belongs to **either a woman or to a junior**?

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$$= \frac{50}{100} + \frac{20}{100} - \frac{10}{100} = 0.6$$

LET'S PRACTICE!

https://docs.google.com/document/d/1RaSYs_32U0RHtzooHtjs7wJOrRGjCQBDLjR2kpvxg-s/edit?usp=sharing