ELEMENTARY COMBINATORICS & PROBABILITY

Lecture 11

Random variables

LAST TIME

- Random variables
- Probability mass function
- Cumulative distribution function

TODAY

- Review PMFs and CDFs
- Special distributions

PMF AND CDF

- For a discrete random variable X with range $R_X = \{x_1, x_2, \dots\}$ **probability mass function (PMF)** is defined as

$$P_X(x) = \begin{cases} P(X = x), & x \in R_X \\ 0, & x \notin R_X \end{cases}$$

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- **Cumulative distribution function (CDF)** of a random variable X is defined as follows:

$$F_X(x) = P(X \leq x) \quad \forall x \in \mathbb{R}$$

PMF AND CDF: EXAMPLE

- X – total number of heads after two tosses of a coin.

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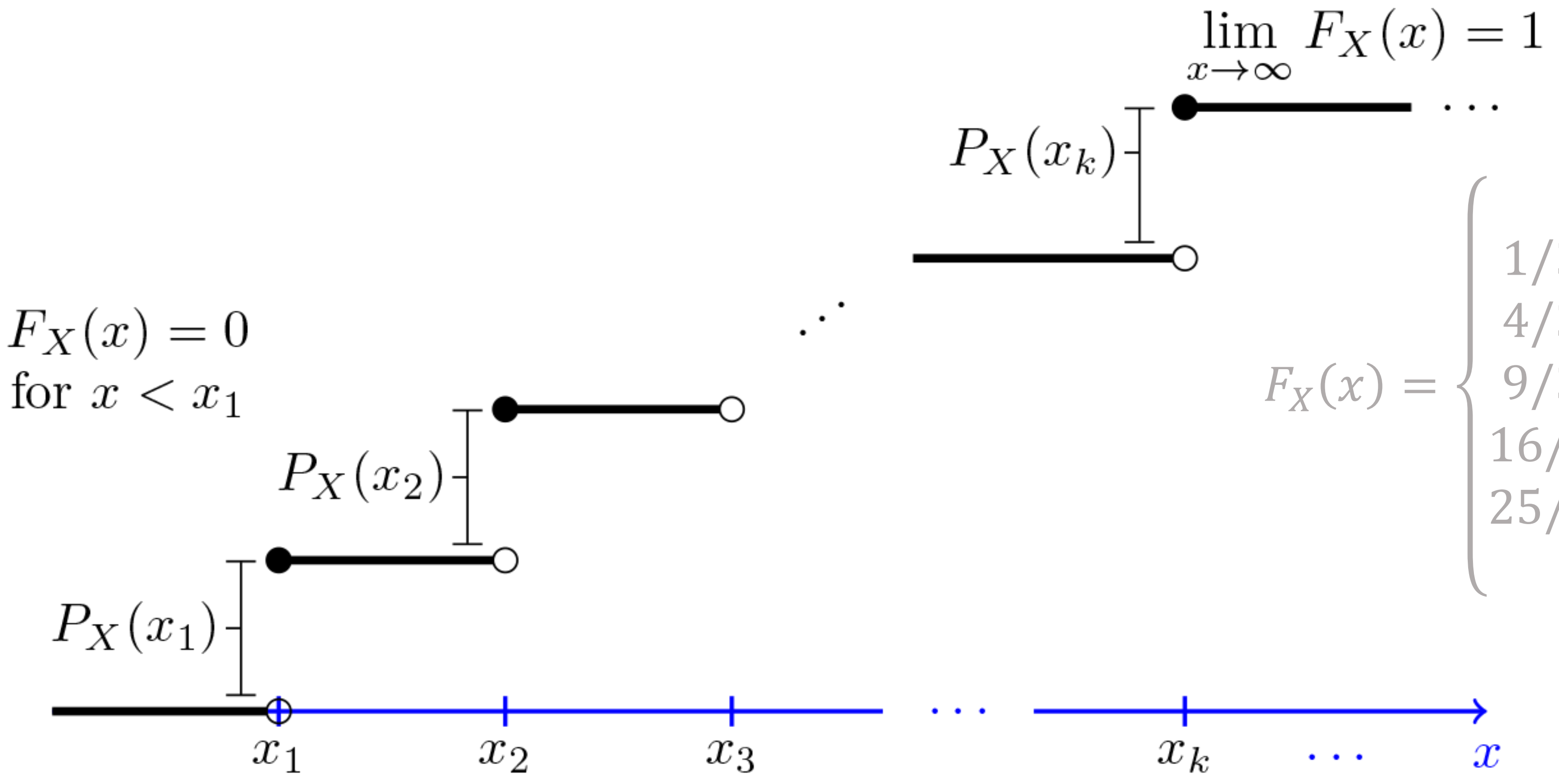
CDF OF A DISCRETE RANDOM VARIABLE



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PMF AND CDF: BASIC PROPERTIES

- Which basic properties does a PMF function have?
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- Which basic properties does a CDF function have?
  - $0 \leq F(x) \leq 1$
  - F(x) is non-decreasing

ROLLING DICE: PMF AND CDF

Google Classroom -> Programming exercise

SPECIAL DISTRIBUTIONS

BERNOULLI DISTRIBUTION

- Consider a random experiment with two possible outcomes: “success” (with probability p) or “failure” (with probability $1 - p$)
  - tossing a coin: H or T;
  - a new child: a boy r a girl;
  - you take an exam: pass or fail.

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- Consider a random variable X

x	0	1
$P(X = x)$		

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  - you take an exam: pass or fail.

- Consider a random variable $X \sim \boldsymbol{Bernoulli(p)}$

x	0	1
$P(X = x)$	$1 - p$	p

BERNOULLI DISTRIBUTION

- A random variable X is said to be a Bernoulli random variable with parameter p if its PMF is given by

$$P_X(x) = P(X = x) = \begin{cases} p, & x = 1 \\ 1 - p, & x = 0 \\ 0, & \text{otherwise} \end{cases}$$

EXAMPLE: PASSING AN EXAM

- You need to answer a multiple-choice question
  (4 options, only 1 is correct).
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$$P_X(x) = P(X = x) =$$

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$$P_X(x) = P(X = x) = \begin{cases} 0.75, & x = 0 \\ 0.25, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

BINOMIAL DISTRIBUTION

- You toss a coin once. Random variable X denotes if you get heads:

$$R_X = \{0,1\}, \qquad P(X = 0) = P(T) = 1 - p, \qquad P(X = 1) = P(H) = p$$

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- Represents the number of successes in a series of n independent Bernoulli trials, each of which results in success with probability p.

EXAMPLE: EXAM

- You are randomly guessing the answers to 5 multiple choice questions (4 options, 1 correct in each).

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