ELEMENTARY COMBINATORICS & PROBABILITY

Lecture 9

Conditional probability, Bayes' rule

LAST TIME

- Conditional probability
- The law of total probability
- Bayes' rule
- (Python) Testing for a rare disease

TODAY

- More examples
- Independent events
- (Maybe) modelling in Python

WARM-UP

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. A single ball is selected at random.
- **What is the probability that it's red?**

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$$P(R) =$$

BALLS IN A BOWL

- There’re 2 red balls and 3 blue balls in a box. A single ball is selected at random.
- **What is the probability that it’s red?**

$$P(R) = \frac{2}{2+3} = \frac{2}{5} = 0.4$$

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
- **… they're both red?**
-

- **… they're of different colors?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
- **… they're both red?**
  You can consider it both ordered or unordered
- **… they're of different colors?**

BALLS IN A BOWL

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- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
- **… they're both red?**
  You can consider it both ordered or unordered

$$P(RR) = \frac{2 \cdot 1}{5 \cdot 4} =$$

- **… they're of different colors?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
- **… they're both red?**
  You can consider it both ordered or unordered
-

$$P(RR) = \frac{2 \cdot 1}{5 \cdot 4} = \frac{C(2,2)}{C(5,2)} =$$

- **… they're of different colors?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
- **… they're both red?**
  You can consider it both ordered or unordered
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$$P(RR) = \frac{2 \cdot 1}{5 \cdot 4} = \frac{C(2,2)}{C(5,2)} = \frac{1}{10}$$

- **… they're of different colors?**

BALLS IN A BOWL

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- **… they're both red?**
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- **… they're of different colors?**

$$P(E) =$$

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- **… they're of different colors?**

$$P(E) = \frac{C(2,1) \cdot 2 \cdot 3}{5 \cdot 4} =$$

BALLS IN A BOWL

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- **… they're of different colors?**

$$P(E) = \frac{C(2,1) \cdot 2 \cdot 3}{5 \cdot 4} = \frac{C(2,1) \cdot C(3,1)}{C(5,2)} =$$

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected at random. What is the probability that…
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- **… they're of different colors?**

$$P(E) = \frac{C(2,1) \cdot 2 \cdot 3}{5 \cdot 4} = \frac{C(2,1) \cdot C(3,1)}{C(5,2)} = \frac{3}{5}$$

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected one by one. The first ball is returned to the box before the second one is selected. What is the probability that…
- **… they're both red?**
- **… they're of different colors?**

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$$P(RR) = \frac{2 \cdot 2}{5 \cdot 5} =$$

- **… they're of different colors?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected one by one. The first ball is returned to the box before the second one is selected. What is the probability that…
- **… they're both red?**

$$P(RR) = \frac{2 \cdot 2}{5 \cdot 5} = \frac{4}{25}$$

- **… they're of different colors?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Two balls are selected one by one. The first ball is returned to the box before the second one is selected. What is the probability that…
- **… they're both red?**

$$P(RR) = \frac{2 \cdot 2}{5 \cdot 5} = \frac{4}{25}$$

- **… they're of different colors?**

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BALLS IN A BOWL

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- **… they're of different colors?**

$$P(E) = \frac{C(2,1) \cdot 2 \cdot 3}{5 \cdot 5} =$$

BALLS IN A BOWL

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- **… they're both red?**

$$P(RR) = \frac{2 \cdot 2}{5 \cdot 5} = \frac{4}{25}$$

- **… they're of different colors?**

$$P(E) = \frac{C(2,1) \cdot 2 \cdot 3}{5 \cdot 5} = \frac{12}{25}$$

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Three balls are selected one by one. Each ball is returned to the box before the next one is selected. What is the probability that…
- **… all of them are red?**
- **… 2 of them are red and the other one is blue?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Three balls are selected one by one. Each ball is returned to the box before the next one is selected. What is the probability that…
- **… all of them are red?**

$$P(RRR) =$$

- **… 2 of them are red and the other one is blue?**

BALLS IN A BOWL

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- **… all of them are red?**

$$P(RRR) = \frac{2 \cdot 2 \cdot 2}{5^3} =$$

- **… 2 of them are red and the other one is blue?**

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Three balls are selected one by one. Each ball is returned to the box before the next one is selected. What is the probability that…
- **… all of them are red?**

$$P(RRR) = \frac{2 \cdot 2 \cdot 2}{5^3} = \frac{8}{125}$$

- **… 2 of them are red and the other one is blue?**

BALLS IN A BOWL

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- **… 2 of them are red and the other one is blue?**

$$P(E) = \frac{C(3,1) \cdot 3 \cdot 2^2}{5^3} =$$

BALLS IN A BOWL

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- **… all of them are red?**

$$P(RRR) = \frac{2 \cdot 2 \cdot 2}{5^3} = \frac{8}{125}$$

- **… 2 of them are red and the other one is blue?**

$$P(E) = \frac{C(3,1) \cdot 3 \cdot 2^2}{5^3} = \frac{36}{125}$$

BALLS IN A BOWL

- There're 2 red balls and 3 blue balls in a box. Four balls are selected one by one. Each ball is returned to the box before the next one is selected. What is the probability that…
- **… 1 ball is red and 3 balls are blue?**
- **… 2 balls are red and 2 balls are blue?**

BALLS IN A BOWL

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- **… 1 ball is red and 3 balls are blue?**

$$P(E) = \frac{C(4,1) \cdot 2 \cdot 3^3}{5^4}$$

- **… 2 balls are red and 2 balls are blue?**

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$$P(E) = \frac{C(4,1) \cdot 2 \cdot 3^3}{5^4}$$

- **… 2 balls are red and 2 balls are blue?**

$$P(E) = \frac{C(4,2) \cdot 2^2 \cdot 3^2}{5^4}$$

PROBLEM SET 7

Conditional probability, the law of total probability

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first flip came up heads?

$$P(3H|H **) =$$

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$$P(3H|H **) = \frac{P(3H \text{ and } H **)}{P(H **)} =$$

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$$P(3H|H**) = \frac{P(3H \text{ and } H**)}{P(H**)} =$$

$$P(3H \text{ and } H**) = \quad , \qquad P(H**) =$$

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first flip came up heads?

$$P(3H|H\ **) = \frac{P(3H \text{ and } H\ **)}{P(H\ **)} =$$

$$P(3H \text{ and } H\ **) = \frac{1}{2^3}, \qquad P(H\ **) =$$

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$$P(3H|H **) = \frac{P(3H \text{ and } H **)}{P(H **)} =$$

$$P(3H \text{ and } H **) = \frac{1}{2^3}, \qquad P(H **) = \frac{2^2}{2^3} =$$

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$$P(3H|H **) = \frac{P(3H \text{ and } H **)}{P(H **)} =$$

$$P(3H \text{ and } H **) = \frac{1}{2^3}, \qquad P(H **) = \frac{2^2}{2^3} = \frac{1}{2}$$

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first flip came up heads?

$$P(3H|H **) = \frac{P(3H \text{ and } H **)}{P(H **)} = \frac{2}{2^3} = \frac{1}{4}$$

$$P(3H \text{ and } H **) = \frac{1}{2^3}, \qquad P(H **) = \frac{2^2}{2^3} = \frac{1}{2}$$

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first two flips resulted in two heads?

$$P(3H|HH\,*) =$$

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COINS

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$$P(3H|HH*) = \frac{P(3H \text{ and } HH*)}{P(HH*)} =$$

$$P(3H \text{ and } HH*) = \qquad , \qquad P(HH*) =$$

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COINS

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$$P(3H|HH\,*) = \frac{P(3H \text{ and } HH\,*)}{P(HH\,*)} =$$

$$P(3H \text{ and } HH\,*) = \frac{1}{2^3}, \qquad P(HH\,*) = \frac{2}{2^3} =$$

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first two flips resulted in two heads?

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$$P(3H \text{ and } HH*) = \frac{1}{2^3}, \qquad P(HH*) = \frac{2}{2^3} = \frac{1}{4}$$

COINS

- A coin is flipped three times. What is the probability of getting three heads given that the first two flips resulted in two heads?

$$P(3H|HH*) = \frac{P(3H \text{ and } HH*)}{P(HH*)} = \frac{4}{2^3} = \frac{1}{2}$$

$$P(3H \text{ and } HH*) = \frac{1}{2^3}, \qquad P(HH*) = \frac{2}{2^3} = \frac{1}{4}$$

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$$P(2H|T***) = \frac{P(2H \text{ and } T***)}{P(T***)} =$$

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COINS

- A coin is flipped 4 times. What is the probability that heads occurs twice given that the first flip is tails?

$$P(2H|T***) = \frac{P(2H \text{ and } T***)}{P(T***)} =$$

$$P(2H \text{ and } T***) = \frac{3}{2^4}, \qquad P(T***) =$$

COINS

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$$P(2H \text{ and } T***) = \frac{3}{2^4}, \qquad P(T***) = \frac{2^3}{2^4} =$$

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- A coin is flipped 4 times. What is the probability that heads occurs twice given that the first flip is tails?

$$P(2H|T\ ***) = \frac{P(2H \text{ and } T\ ***)}{P(T\ ***)} =$$

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COINS

- A coin is flipped 4 times. What is the probability that heads occurs twice given that the first flip is tails?

$$P(2H|T***) = \frac{P(2H \text{ and } T***)}{P(T***)} = \frac{3}{8}$$

$$P(2H \text{ and } T***) = \frac{3}{2^4}, \qquad P(T***) = \frac{2^3}{2^4} = \frac{1}{2}$$

THE LAW OF TOTAL PROBABILITY

MARKETING

- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

$$P(P) =$$

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$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$

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$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$
$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$

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$$P(M \cup T) =$$

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$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) =$$

MARKETING

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$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$$

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$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$
$$= \frac{1}{3} \cdot 0.21 + 0.1 \cdot (1 - 0.21) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$$

MARKETING

- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$
$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$
$$= \frac{1}{3} \cdot 0.21 + 0.1 \cdot (1 - 0.21) = 0.07 + 0.079 = 0.149$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$$

BAYES' RULE

A TEST FOR DISEASE

- Imagine that a rare disease affects 1% of the population

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$$P(D) = 0.01$$

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$$P(+|D) = 0.9, \qquad P(-|no\ D) = 0.9$$

A TEST FOR DISEASE

- Imagine that a rare disease affects 1% of the population

$$P(D) = 0.01$$

- A 90% - accurate test has been developed to detect the disease

$$P(+|D) = 0.9, \qquad P(-|no\ D) = 0.9$$

- You test positive. How much should you be worried?

A TEST FOR DISEASE

- Imagine that a rare disease affects 1% of the population

$$P(D) = 0.01$$

- A 90% - accurate test has been developed to detect the disease

$$P(+|D) = 0.9, \qquad P(-|no\ D) = 0.9$$

- You test positive. How much should you be worried?

$$P(D \mid +) = ?$$

HOW RELIABLE THE TEST IS?

$$P(D) = 0.01, \qquad P(+\mid D) = P(-\mid no\ D) = 0.9$$

REALITY / TEST RESULTS	POSITIVE	NEGATIVE
HEALTHY $\approx 92\%$	**FALSE POSITIVES** 10% of 99% healthy people $\boxed{= 9.9\% \text{ of the population}}$	**TRUE NEGATIVES** 90% of 99% healthy people = 89.1% of the population
ILL $\approx 8\%$	**TRUE POSITIVES** 90% of 1% ill people $\boxed{= 0.9\% \text{ of the population}}$	**FALSE NEGATIVES** 10% of 1% ill people = 0.1% of the population

SAME THING, BUT IN TERMS OF PROBABILITY

$$P(D) = 0.01, \qquad P(+ \mid D) = P(- \mid no\ D) = 0.9, \qquad P(D \mid +) = ?$$

$$P(D \mid +) = \frac{P(+ \text{ and } D)}{P(+)} =$$

SAME THING, BUT IN TERMS OF PROBABILITY

$$P(D) = 0.01, \qquad P(+ \mid D) = P(- \mid no\ D) = 0.9, \qquad P(D \mid +) = ?$$

$$P(D \mid +) = \frac{P(+ \text{ and } D)}{P(+)} =$$

$$P(+ \mid D) = \frac{P(+ \text{ and } D)}{P(D)} \ \rightarrow \ P(+ \text{ and } D) =$$

SAME THING, BUT IN TERMS OF PROBABILITY

$$P(D) = 0.01, \qquad P(+ \mid D) = P(- \mid no\ D) = 0.9, \qquad P(D \mid +) = ?$$

$$P(D \mid +) = \frac{P(+ \text{ and } D)}{P(+)} =$$

$$P(+ \mid D) = \frac{P(+ \text{ and } D)}{P(D)} \;\rightarrow\; P(+ \text{ and } D) = P(+ \mid D) \cdot P(D) =$$

SAME THING, BUT IN TERMS OF PROBABILITY

$$P(D) = 0.01, \qquad P(+\mid D) = P(-\mid no\ D) = 0.9, \qquad P(D\mid +) = ?$$

$$P(D\mid +) = \frac{P(+\text{ and }D)}{P(+)} =$$

$$P(+\mid D) = \frac{P(+\text{ and }D)}{P(D)} \;\rightarrow\; P(+\text{ and }D) = P(+\mid D)\cdot P(D) = 0.009$$

SAME THING, BUT IN TERMS OF PROBABILITY

$$P(D) = 0.01, \qquad P(+\mid D) = P(-\mid no\ D) = 0.9, \qquad P(D\mid +) = ?$$

$$P(D\mid +) = \frac{P(+\text{ and }D)}{P(+)} =$$

$$P(+\mid D) = \frac{P(+\text{ and }D)}{P(D)} \;\rightarrow\; P(+\text{ and }D) = P(+\mid D)\cdot P(D) = 0.009$$

$$P(+) =$$

SAME THING, BUT IN TERMS OF PROBABILITY

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$$= 0.9\cdot 0.01 + 0.1\cdot 0.99 = 0.108$$

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$$P(D \mid +) = \frac{P(+ \text{ and } D)}{P(+)} = \frac{\mathbf{0.009}}{\mathbf{0.108}} \approx \mathbf{0.083}$$

$$P(+ \mid D) = \frac{P(+ \text{ and } D)}{P(D)} \ \rightarrow\ P(+ \text{ and } D) = P(+ \mid D) \cdot P(D) = 0.009$$

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$$= 0.9 \cdot 0.01 + 0.1 \cdot 0.99 = 0.108$$

HOW RELIABLE THE TEST IS?

$$P(D) = 0.01, \qquad P(+\mid D) = P(-\mid no\ D) = 0.9$$

REALITY / TEST RESULTS	POSITIVE	NEGATIVE
HEALTHY $\approx 92\%$	**FALSE POSITIVES** 10% of 99% healthy people = 9.9% of the population	**TRUE NEGATIVES** 90% of 99% healthy people = 89.1% of the population
ILL $\approx 8\%$	**TRUE POSITIVES** 90% of 1% ill people = 0.9% of the population	**FALSE NEGATIVES** 10% of 1% ill people = 0.1% of the population

BAYES' RULE

$$\boldsymbol{P(A|B)} = \frac{P(AB)}{P(B)} = \frac{\boldsymbol{P(B|A)}}{\boldsymbol{P(B)}} \cdot \boldsymbol{P(A)}$$

BALLS IN TWO BOWLS

- There are two bowls: bowl A contains 2 blue and 3 red balls, and bowl B contains 1 blue and 4 red balls. A blue ball was randomly picked from one of the bowls. What's the probability that it was bowl A?

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$$= \frac{P(blue|A) \cdot P(A)}{P(blue|A) \cdot P(A) + P(blue|B) \cdot P(B)} = \frac{0.4 \cdot 0.5}{0.4 \cdot 0.5 + 0.2 \cdot 0.5} =$$

$$= \frac{0.2}{0.3} = \frac{2}{3}$$

COINS

- A box contains 3 coins with a head on each side, 4 coins with a tail on each side, and 2 fair coins. One of these nine coins is selected at random and tossed once. Suppose that the toss does result in a head. What is the probability that the coin used for tossing was fair?

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E_H – a coin with 2 H is selected $\qquad P(E_H) =$

E_T – a coin with 2 T is selected $\qquad P(E_T) =$

E_F – a fair coin is selected $\qquad P(E_F) =$

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$E_H -$ a coin with 2 H is selected $\quad P(E_H) = \frac{3}{3+4+2} = \frac{1}{3}$

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$$P(H) = \frac{4}{9}, \qquad P(E_F|H) = \frac{P(H|E_F) \cdot P(E_F)}{P(H)} = \frac{\frac{1}{2} \cdot \frac{2}{9}}{\frac{4}{9}} = \frac{1}{4}$$

EXAM

- Exam is divided into two parts. The probability of getting B or better on Part 1 is 0.8, the probability of getting B or better on Part 2 is 0.6, and the probability of getting B or better on both parts is 0.5. What is the probability of getting B or better on Part 2 if you got B or better on Part 1?

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$$P(E_2|E_1) =$$

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$$P(E_2|E_1) = \frac{P(E_1 \& E_2)}{P(E_1)} = \frac{0.5}{0.8} = \frac{5}{8}$$

A MAGICIAN

- A magician has five coins in his pocket. Four of the coins are fair but the other is a biased coin and has two heads. He pulls a coin at random from his pocket and flips it five times. It comes up heads all five times. What is the probability that he selected a fair coin from his pocket?

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$$= \frac{0.5^5 \cdot 0.8}{0.5^5 \cdot 0.8 + 1 \cdot 0.2}$$

TEACHING

- Methods A and B are available for teaching a skill. The failure rate is 20% for A and 10% for B. Method B is used only 30% of the time, and A is used the other 70%. A worker was taught the skill by one of the methods but failed to learn it. What is the probability that she was taught by method A?

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$$P(A) = 0.7, \qquad P(B) = \quad , \qquad P(F|A) = \quad , \qquad P(F|B) =$$

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$$= \frac{0.2 \cdot 0.7}{0.2 \cdot 0.7 + 0.1 + 0.3} =$$

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$$= \frac{0.2 \cdot 0.7}{0.2 \cdot 0.7 + 0.1 + 0.3} = \frac{14}{17} \approx 0.82$$

$$P(A) = 0.7, \qquad P(B) = 0.3, \qquad P(F|A) = 0.2, \qquad P(F|B = 0.1)$$

INDEPENDENT EVENTS

COINS

- When a coin is flipped four times, what is the probability that …
- **… heads comes up exactly twice?**

$$P(2H) =$$

- **… heads comes up exactly twice given that heads comes up on the first flip?**

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$$P(2H|H***) = \frac{P(2H\ and\ H***)}{P(H***)} = \frac{3/2^4}{2^3/2^4} = \frac{3}{8}$$

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INDEPENDENT EVENTS

- Two events A and B from the same sample space S are independent if

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$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A) \cdot P(B)}{P(A)} = P(B)$$

ROLLING DICE

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$P(EF) = P(E) \cdot P(F) = \frac{1}{36} \rightarrow E\&F$ are independent!

FLIPPING A COIN

- A coin is flipped 6 times. Let E be the event that heads and tails come up an equal number of times. Let F be the event that heads and tails come up once each during the first two flips of the coin. Are E and F independent events?

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$$P(E) =$$

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$$P(E) = \frac{C(6,3)}{2^6} = \frac{5}{2^4}, \qquad P(F) =$$

$$P(EF) =$$

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$$P(E) = \frac{C(6,3)}{2^6} = \frac{5}{2^4}, \qquad P(F) = \frac{2 \cdot 2^4}{2^6} = \frac{1}{2},$$

$$P(EF) =$$

FLIPPING A COIN

- A coin is flipped 6 times. Let E be the event that heads and tails come up an equal number of times. Let F be the event that heads and tails come up once each during the first two flips of the coin. Are E and F independent events?

$$P(E) = \frac{C(6,3)}{2^6} = \frac{5}{2^4}, \qquad P(F) = \frac{2 \cdot 2^4}{2^6} = \frac{1}{2},$$

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$\rightarrow E$ & F aren't independent

FLIPPING A COIN 2

- Let E be the event that when a coin is flipped three times, we don't get all heads or all tails. Let F be the event that when a coin is flipped three times, heads comes up at most once. Are E and F independent events?

$$P(E) =$$

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FLIPPING A COIN 2

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$$P(E) = 1 - 2 \cdot \frac{1}{2^3} = \frac{3}{4}$$

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$\rightarrow E$ & F are independent!

WEATHER FORECAST

- There is a 50% chance of rain on Saturday and a 50%chance of rain on Sunday. If having rain on Saturday and having rain on Sunday are independent events, then what is the probability of having no rain over the weekend?

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