ELEMENTARY COMBINATORICS & PROBABILITY

Lecture 2

Principle of Inclusion-Exclusion

LAST TIME

- The basics of set theory
- Basic counting principles
  - Rule of sum
  - Rule of product
  - Other useful tricks
- Applying those to solve problems

TODAY

- Basic counting principles: review
- Finish Problem Set 1
- Principle of exclusion-inclusion
  - *How to apply rule of sum when the sets are not disjoint?*
- Problem Set 2.

PROBLEM SET 1

COMPLEMENT OF A COMPLEMENT

- $(A^C)^C = ?$

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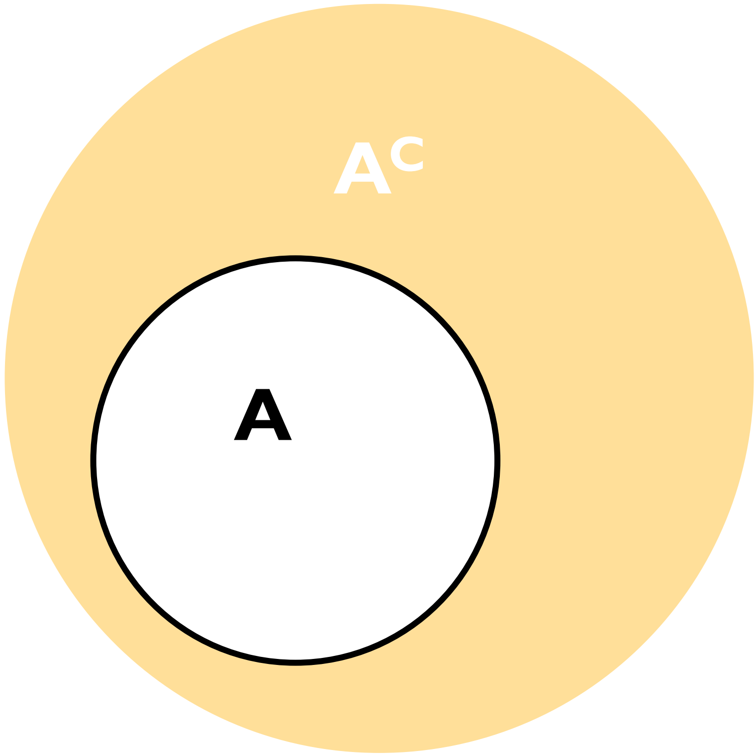
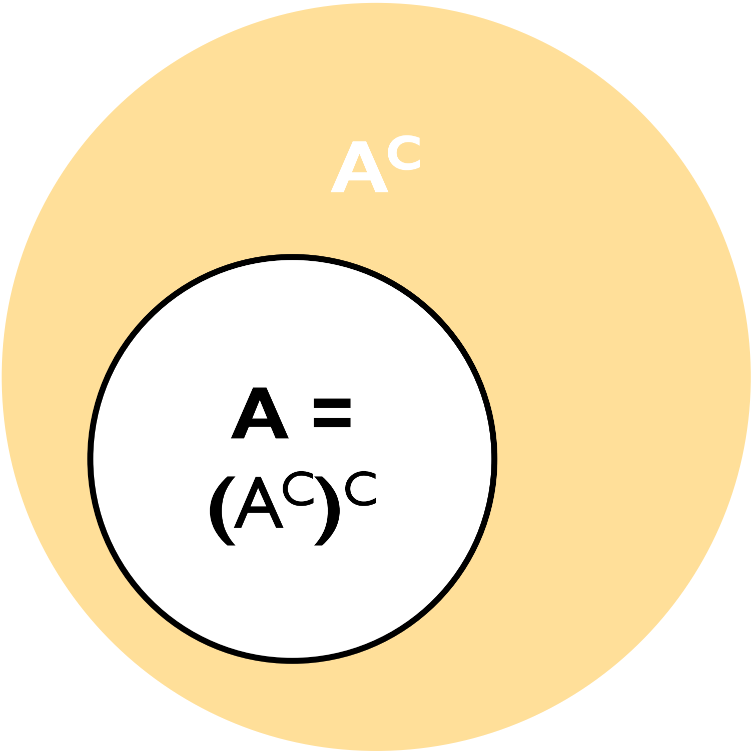


COMPLEMENT OF A COMPLEMENT

- $(A^C)^C = A$

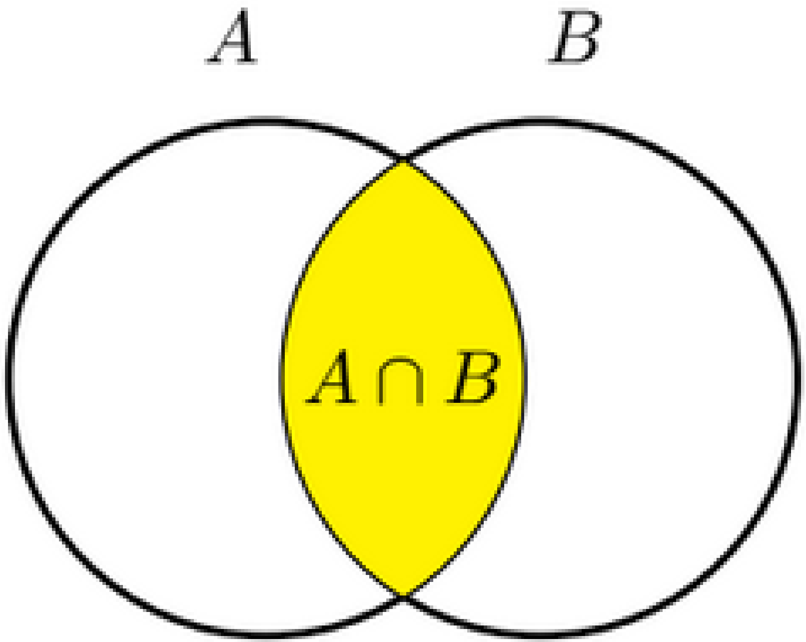


ABSORPTION LAW 2

- $A \cup (A \cap B) = ?$

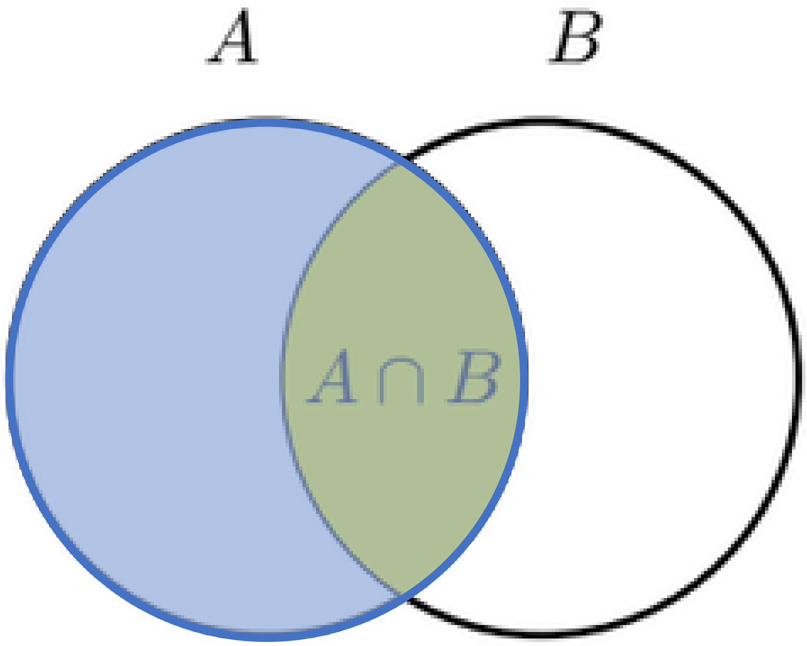
ABSORPTION LAW 2

- A ∪ (A ∩ B) = ?



ABSORPTION LAW 2

- **A** ∪ (A ∩ B) = A



SHOPPING

- A woman has decided to shop at one store today, either in the north part of town or the south part of town.
    - North: shop at either a mall, a furniture store, or a jewelry store.
    - South: shop at either a clothing store or a shoe store.
- How many stores can the woman choose from?

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- How many stores can the woman choose from?

Sum rule:

3 options in the north part **+ 2** options in the south part =
= 2 + 3 = 5 different stores to visit

WORKOUTS

- You want to design a 30-minute workout.
  - First 15 minutes: running, rowing, kickboxing or skipping.
  - Second 15 minutes: squats, pull-ups or core routine.
- How many such workouts are possible?

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  - First 15 minutes: running, rowing, kickboxing or skipping.
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Product rule:

4 options for the first part x 3 options for the second part =
= 4 x 3 = 12 options for the

NUMBERS

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1 – 9:

10 – 99:

100 – 999:

NUMBERS

- How many integer numbers from 1 to 999 **don't** contain digit 7?

1 – 9:

8 numbers (everything but 7)

10 – 99:

100 – 999:

NUMBERS

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8 x 9 numbers (1st digit: anything but 0 or 7, 2nd digit: anything but 7)

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8 x 9 x 9 (1st digit: anything but 0 or 7, 2nd and 3rd digits: anything but 7)

Sum rule:

8 + 8 x 9 + 8 x 9 x 9 numbers from 1 to 999 **don't** contain digit 7

NUMBERS

- How many integer numbers from 1 to 999 **don't** contain digit 7?

Other solution:

NUMBERS

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Other solution:

Number between 1 and 999:

$$* \; * \; *$$

(6 is represented as 006, 45 is represented as 045)

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Every digit * can be anything but 7, therefore:

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Other solution:

Number between 1 and 999:

* * *

(6 is represented as 006, 45 is represented as 045)

Every digit * can be anything but 7, therefore:

9 x 9 x 9

NUMBERS

- How many integer numbers from 1 to 999 **don't** contain digit 7?

Other solution:

Number between 1 and 999:

* * *

(6 is represented as 006, 45 is represented as 045)

Every digit * can be anything but 7, therefore:

$9 \times 9 \times 9 - 1$

(000 should not be considered,
because we're interested numbers from 1 to 999)

NUMBERS

- How many integer numbers from 1 to 999 contain digit 7?

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- How many integer numbers from 1 to 999 contain digit 7?

There are 999 numbers between 1 and 999.

728 don’t contain digit 7 (previous step).

NUMBERS

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Therefore, 999 – 728 – 271 numbers contain digit 7.

NUMBERS

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Numbers from 1 to 999: * * *

NUMBERS

- How many integer numbers from 1 to 999 contain digit 7 **exactly** once?

Numbers from 1 to 999: $* * *$

Contain 7 exactly once $\Leftrightarrow$

NUMBERS

- How many integer numbers from 1 to 999 contain digit 7 **exactly** once?

Numbers from 1 to 999: * * *

Contain 7 exactly once ⇔ three possibility:

7 * *

* 7 *

* * 7

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Numbers from 1 to 999: * * *

Contain 7 exactly once ⇔ three possibility:

7 * * 9 x 9 options

* 7 * 9 x 9 options

* * 7 9 x 9 options

Sum rule:

9 x 9 + 9 x 9 + 9 x 9 = 243

numbers from 1 to 999 contain digit 7 **exactly** once

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Other solution:

1 – 9:

1 option (number 7)

10 – 99:

100 – 999:

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Other solution:

<u>1 – 9:</u>

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<u>10 – 99:</u>

7* *7

<u>100 – 999:</u>

NUMBERS

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Other solution:

<u>1 – 9:</u>

1 option (number 7)

<u>10 – 99:</u>

7* (9 options) *7

<u>100 – 999:</u>

NUMBERS

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Other solution:

<u>1 – 9:</u>

1 option (number 7)

<u>10 – 99:</u>

7* (9 options) *7 (8 options)

<u>100 – 999:</u>

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7* (9 options) *7 (8 options)

<u>100 – 999:</u>

7** *7* **7

NUMBERS

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Other solution:

<u>1 – 9:</u>

1 option (number 7)

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7* (9 options) *7 (8 options)

<u>100 – 999:</u>

7** (9 x 9 options) *7* (8 x 9 options) **7 (8 x 9 options)

NUMBERS

- How many integer numbers from 1 to 999 contain digit 7 **exactly** once?

Other solution:

<u>*1 – 9:*</u>

1 option (number 7)

<u>*10 – 99:*</u>

7* (9 options) *7 (8 options)

<u>*100 – 999:*</u>

7** (9 x 9 options) *7* (8 x 9 options) **7 (8 x 9 options)

Sum rule: 1 + 9 + 8 + 9 x 9 + 8 x 9 + 8 x 9 = 243 numbers from 1 to 999 contain digit 7 **exactly** once

NUMBERS

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Therefore,

NUMBERS

- How many integer numbers from 1 to 999 contain digit 7 **more than** once?

There are 999 numbers from 1 to 999.

728 don't contain digit 7 (previous steps).

243 of them contain digit 7 exactly once (previous step).

Therefore,

$$999 - 728 - 243 = 28$$

numbers from 1 to 999 contain digit 7 **more than** once

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

How many numbers between 0 and 9999 contain digit 3 three times?

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How many numbers between 0 and 9999 contain digit 3 three times?

333

333*

33*3

3*33

*333

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

How many numbers between 0 and 9999 contain digit 3 three times?

333	1
333*	9
33*3	9
3*33	9
*333	8

=> 1 + 9 + 9 + 9 + 8 = 36 numbers

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

How many numbers between 0 and 9999 contain digit 3 four times?

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

How many numbers between 0 and 9999 contain digit 3 four times?

Just one: 3333

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

NUMBERS 2

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There are 10000 numbers between 0 and 9999.

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There are 10000 numbers between 0 and 9999.

1 contains digit 3 four times (previous step).

36 contain digit 3 three times (previous step).

NUMBERS 2

- How many numbers between 0 and 9999 contain digit 3 less than 3 times?

There are 10000 numbers between 0 and 9999.

1 contains digit 3 four times (previous step).

36 contain digit 3 three times (previous step).

Therefore, 10000 – 1 – 36 = 9963
numbers between 0 and 9999 contain digit 3 less that three times.

PASSWORDS

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$$36 \times 36 \times 36 \times 36 \times 36 \times 36 = 36^6$$

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6-character passwords with *no* digits

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6-character passwords with *any* number of digits

$26 \times 26 \times 26 \times 26 \times 26 \times 26 = 26^6$

6-character passwords with *no* digits

$36^6 - 26^6$ passwords with at least one digit

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- How many numbers between 0 and 999999 don’t have two similar digits next to each other?

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… and so on (do for 4-, 5- and 6-digit numbers, then sum up)

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10 numbers (anything)

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SET OPERATIONS: REVISION

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- Let C, M and I be the sets of people who like **C**hinese, **M**exican and **I**talian food respectively.

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- People who like Chinese or Mexican cuisine: $C \cup M$

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THE PRINCIPLE OF INCLUSION-EXCLUSION

COUNT NUMBERS DIVISIBLE BY 3

- How many numbers between 1 and 100 are divisible by 3?

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  Every 3rd number is divisible by 3: 3, 6, 9, …, 93, 96, 99.

COUNT NUMBERS DIVISIBLE BY 3

- How many numbers between 1 and 100 are divisible by 3?

  Every 3^{rd} number is divisible by 3: 3, 6, 9, …, 93, 96, 99.

  Thus, there are [100 / 3] = 33 of them.

COUNT NUMBERS DIVISIBLE BY 5

- How many numbers between 1 and 100 are divisible by 5?

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- How many numbers between 1 and 100 are divisible by 5?

  Every 5^{th} number is divisible by 5: 5, 10, 15, …, 95, 95

COUNT NUMBERS DIVISIBLE BY 5

- How many numbers between 1 and 100 are divisible by 5?

  Every 5th number is divisible by 5: 5, 10, 15, …, 95, 95, 100.

  Thus, there are [100 / 5] = 20 of them.

COUNT NUMBERS DIVISIBLE BY 3 & 5

- How many numbers between 1 and 100 are divisible by 3?
  33
- How many numbers between 1 and 100 are divisible by 5?
  20
- How many numbers between 1 and 100 are divisible by 3 or 5?

COUNT NUMBERS DIVISIBLE BY 3 & 5

- How many numbers between 1 and 100 are divisible by 3?

  33

- How many numbers between 1 and 100 are divisible by 5?

  20

- How many numbers between 1 and 100 are divisible by 3 or 5?

  Sum rule:

  33 numbers divisible by 3 + 20 numbers divisible by 5 =
  = 53 numbers divisible by 3 or 5.

COUNT NUMBERS DIVISIBLE BY 3 & 5

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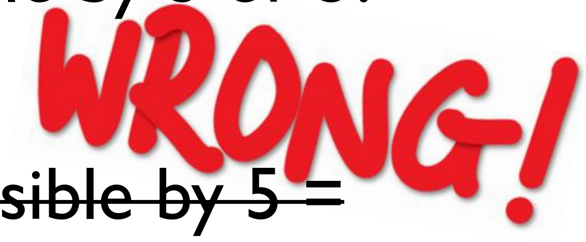
  ~~*Sum rule:*~~



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ADDITION PRINCIPLE

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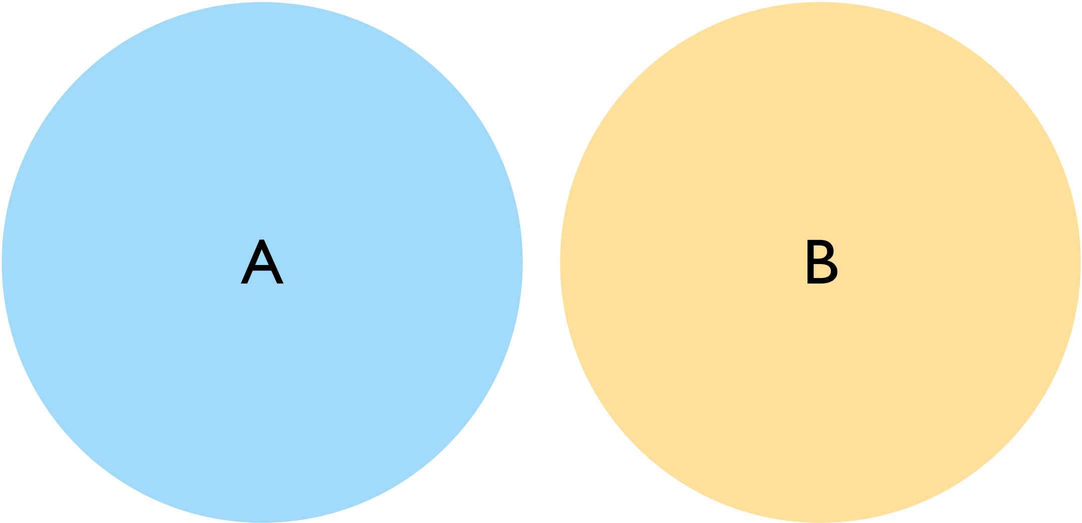
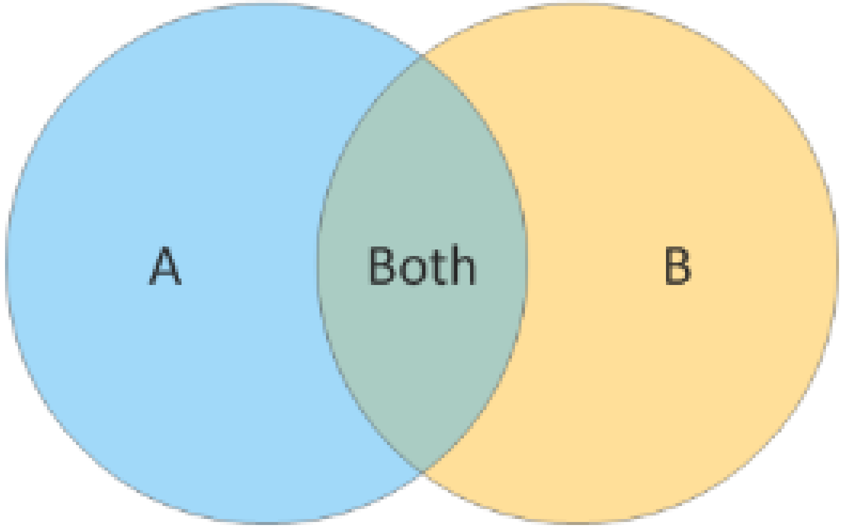


INCLUSION-EXCLUSION (2 SETS)

- If A and B are not disjoint ($A \cap B = \emptyset$), then

$$|A \cup B| =$$



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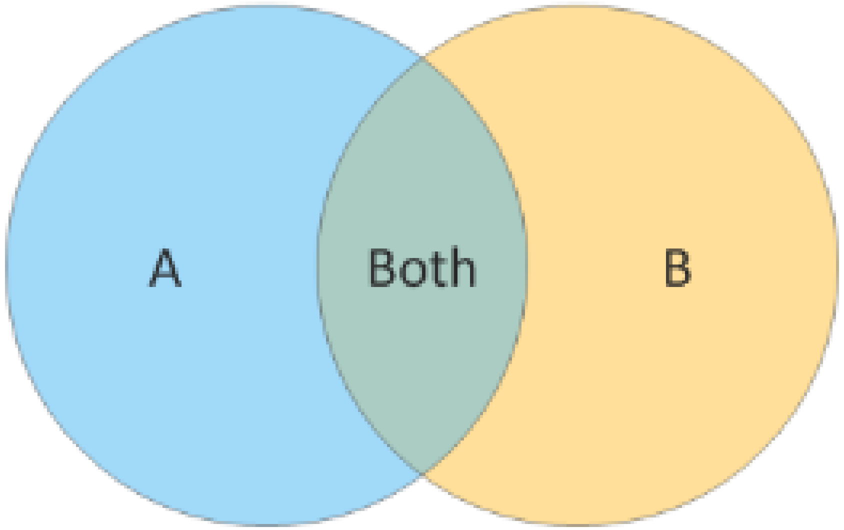

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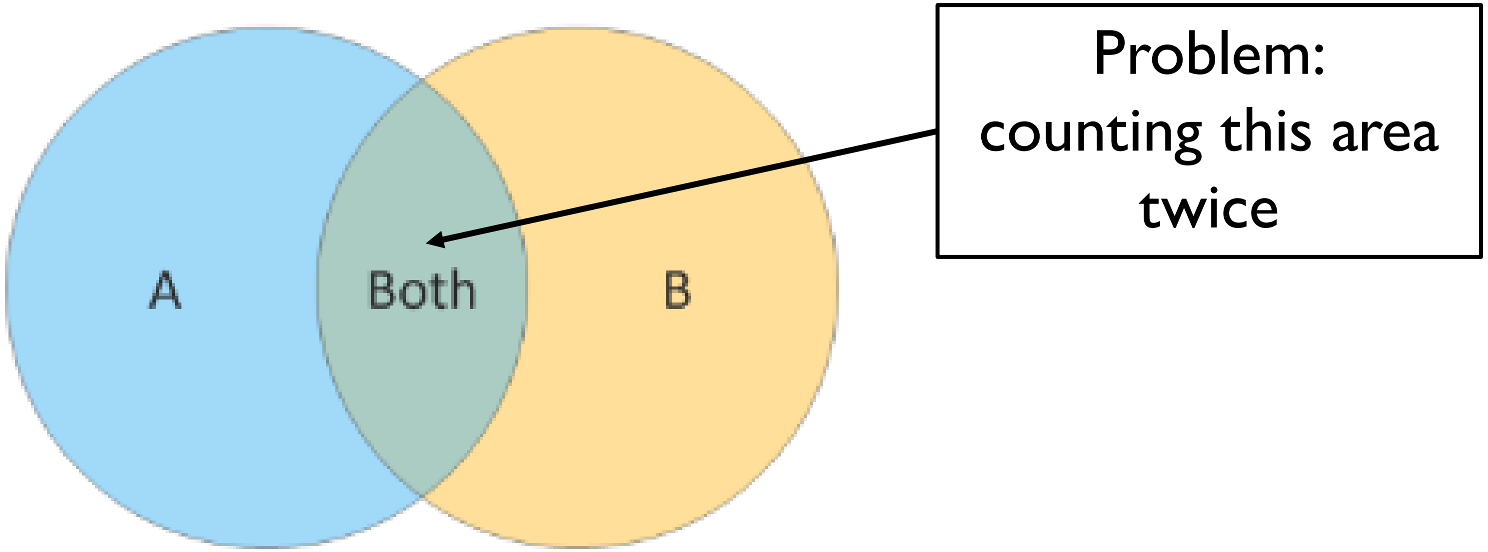
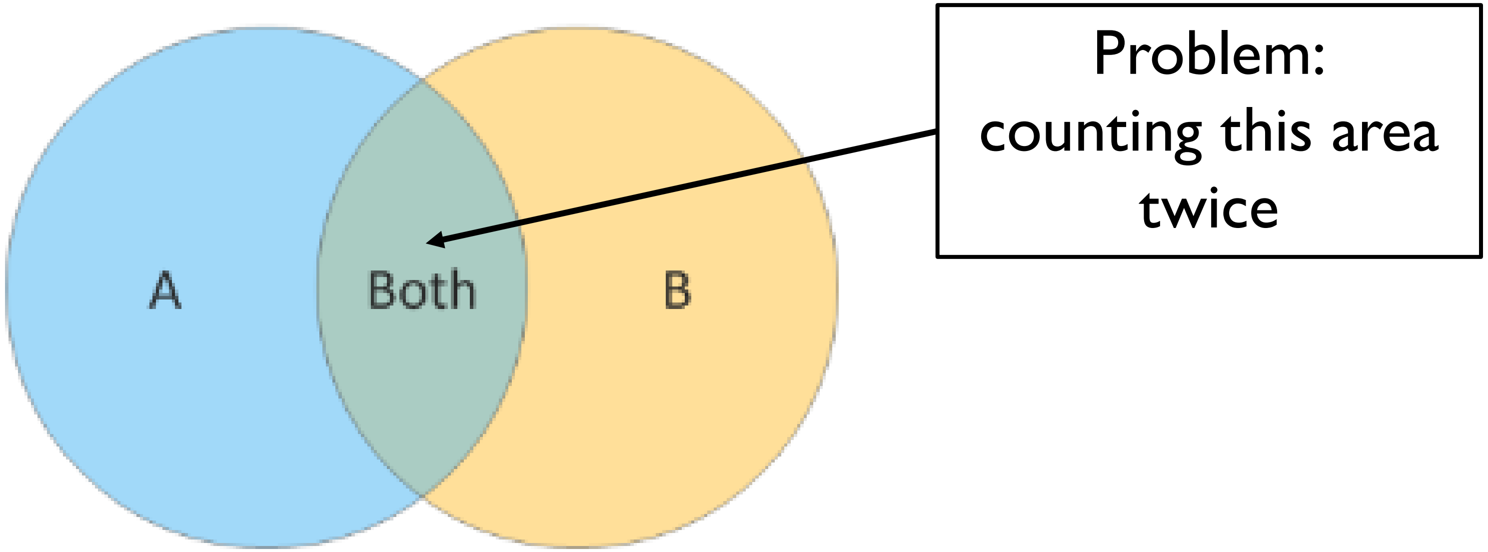


INCLUSION-EXCLUSION (2 SETS)

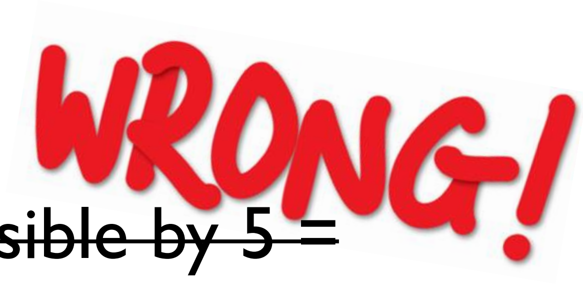
- If A and B are not disjoint ($A \cap B = \emptyset$), then

$$|A \cup B| = |A| + |B| - |A \cap B|$$



BACK TO OUR EXAMPLE

- ~~*Sum rule:*~~



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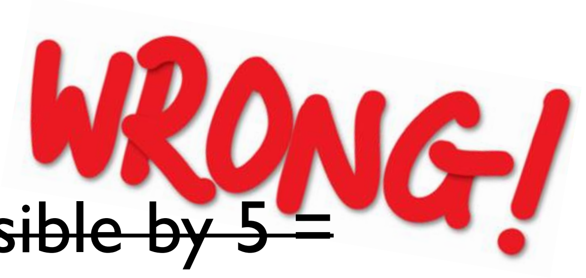
~~= 53 numbers divisible by 3 or 5.~~

- Problem:

BACK TO OUR EXAMPLE



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- Problem: there are numbers which are divisible both by 3 and 5 (e.g., 15, 30, 45, …).

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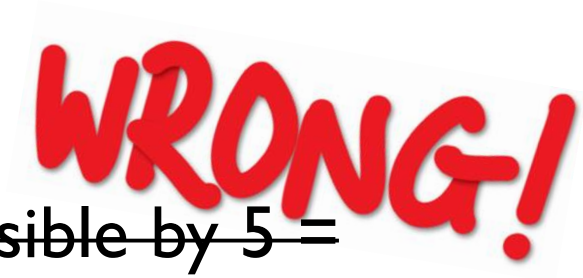

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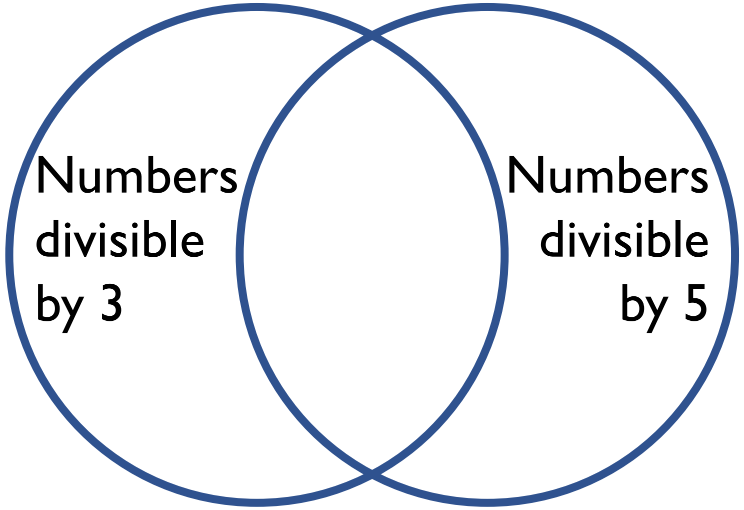
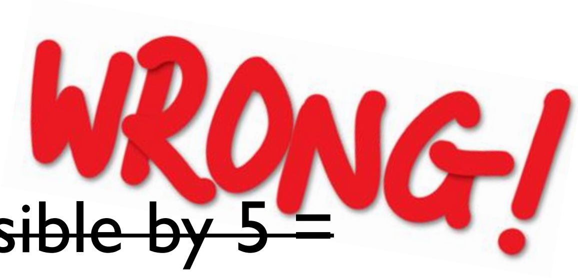


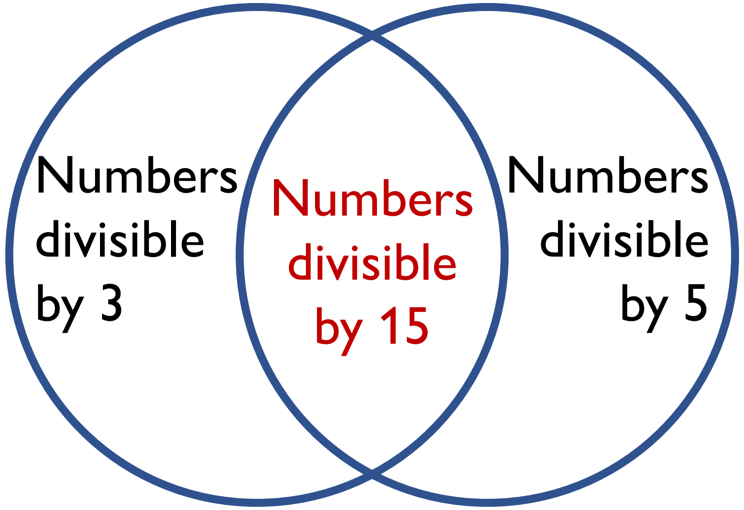
BACK TO OUR EXAMPLE

- ~~*Sum rule:*~~



~~33 numbers divisible by 3 + 20 numbers divisible by 5 =~~
~~= 53 numbers divisible by 3 or 5.~~

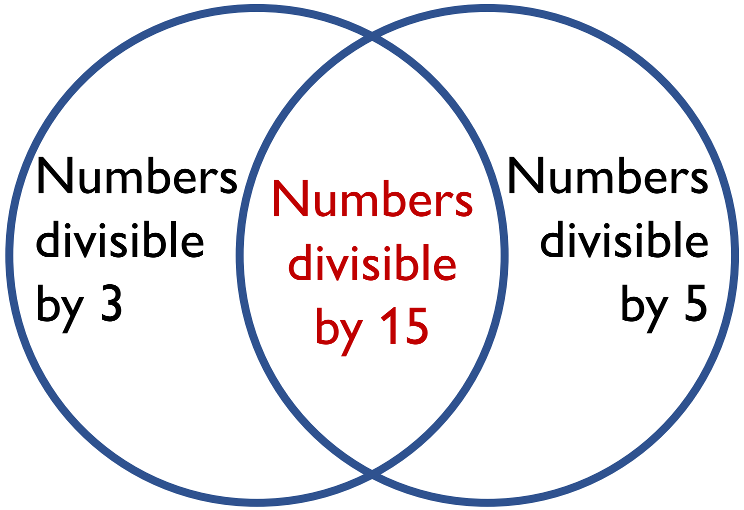
- Problem: there are numbers which are divisible both by 3 and 5 (e.g., 15, 30, 45, …).



BACK TO OUR EXAMPLE

- How many numbers between 1 and 100 are divisible by 3 or 5?

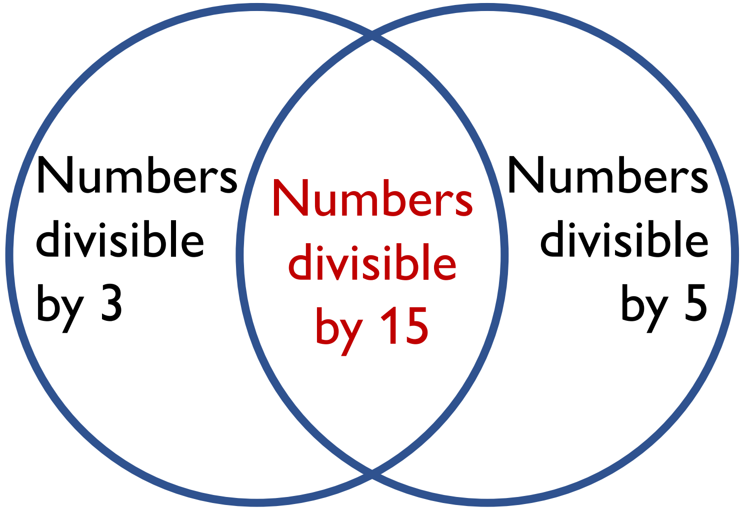
$$|D_3 \cup D_5| =$$



BACK TO OUR EXAMPLE

- How many numbers between 1 and 100 are divisible by 3 or 5?

$$|D_3 \cup D_5| = |D_3| + |D_5| - |D_{15}| =$$

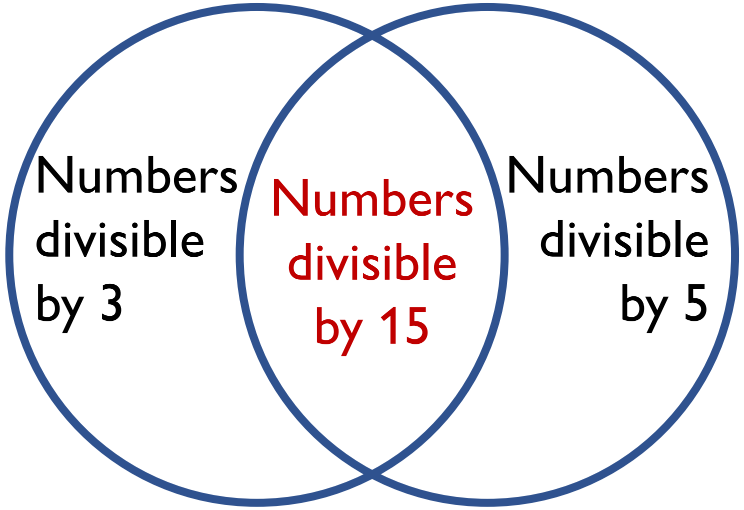


BACK TO OUR EXAMPLE

- How many numbers between 1 and 100 are divisible by 3 or 5?

$$|D_3 \cup D_5| = |D_3| + |D_5| - |D_{15}| =$$

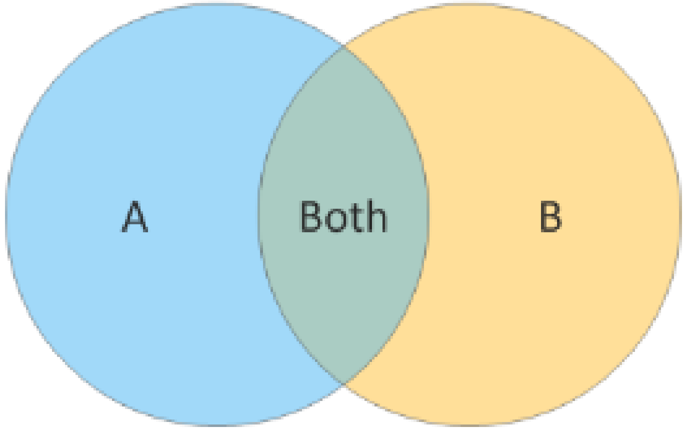
$$= 33 + 20 - 6 = 47.$$



PRINCIPLE OF INCLUSION-EXCLUSION FOR TWO SETS

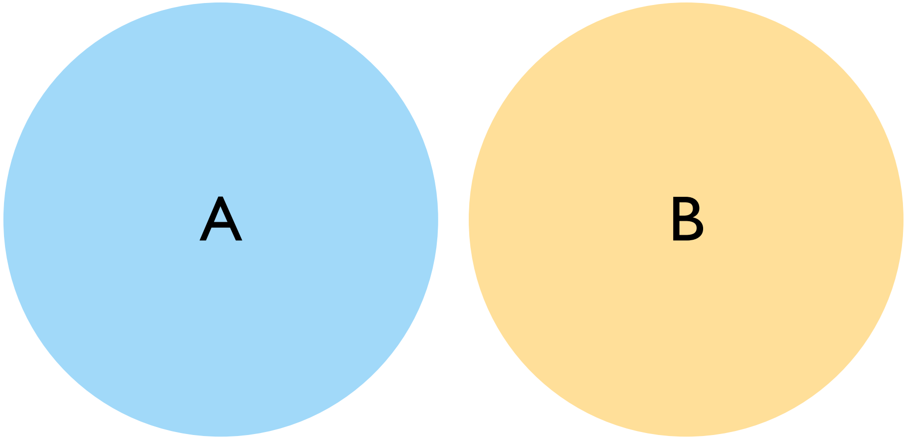
- For every two finite sets A and B

$$|A \cup B| = |A| + |B| - |A \cap B|.$$



- In particular, if A and B are disjoint, then $|A \cap B| = \emptyset$ and

$$|A \cup B| = |A| + |B|.$$



EXAMPLE: STUDENTS

- In a discrete mathematics course, there are
  - 17 students majoring in computer science
  - 11 students majoring in mathematics
  - 5 students majoring in both.

How many students major in computer science or mathematics?

EXAMPLE: STUDENTS

- In a discrete mathematics course, there are
  - 17 students majoring in computer science   set C
  - 11 students majoring in mathematics   set M
  - 5 students majoring in both.

How many students major in computer science or mathematics?

EXAMPLE: STUDENTS

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Students majoring in CS or Math:

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How many students major in computer science or mathematics?

Students majoring in CS or Math: $C \cup M$

$$|C \cup M| = |C| + |M| - |C \cap M| =$$

$= 17 + 11 - 5 = 23$ students majoring in CS of Math

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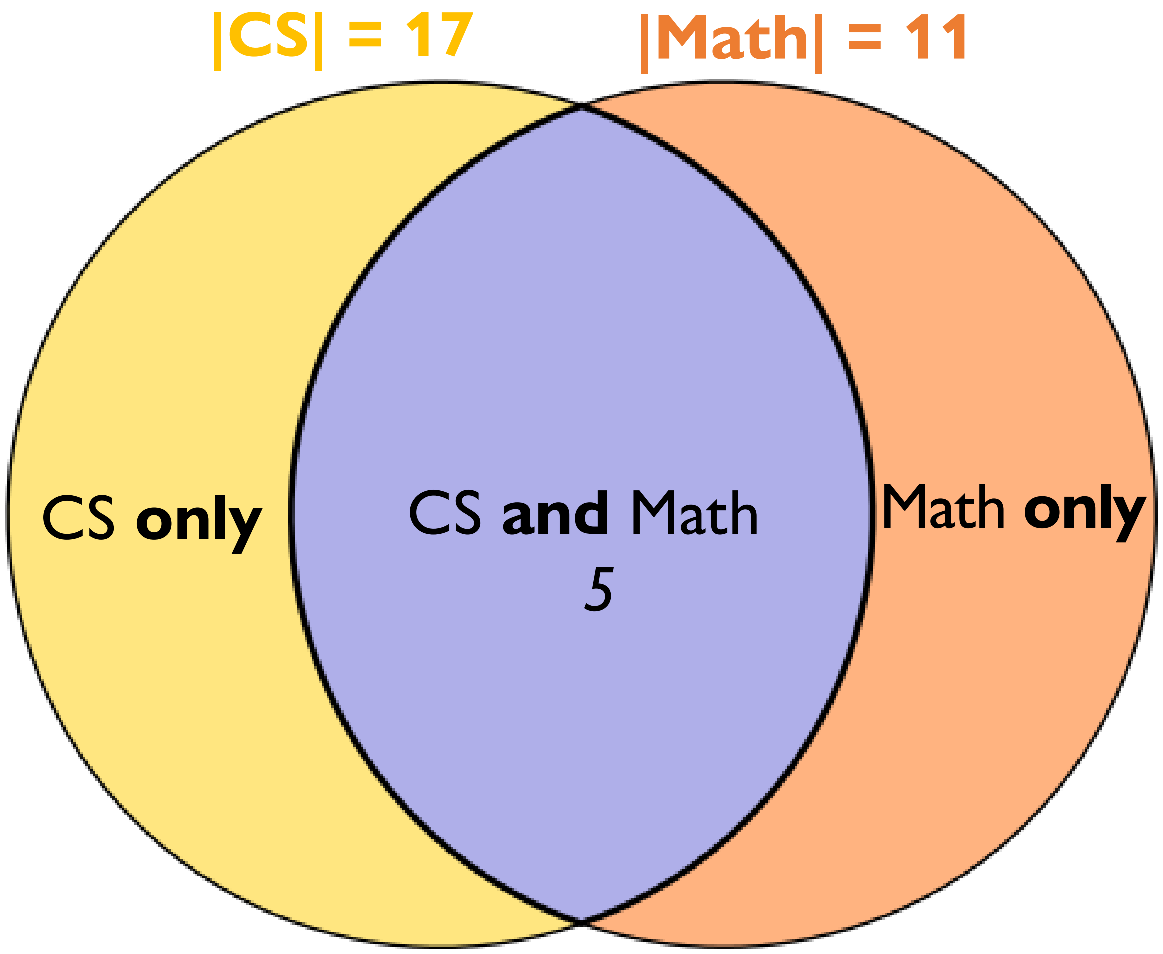

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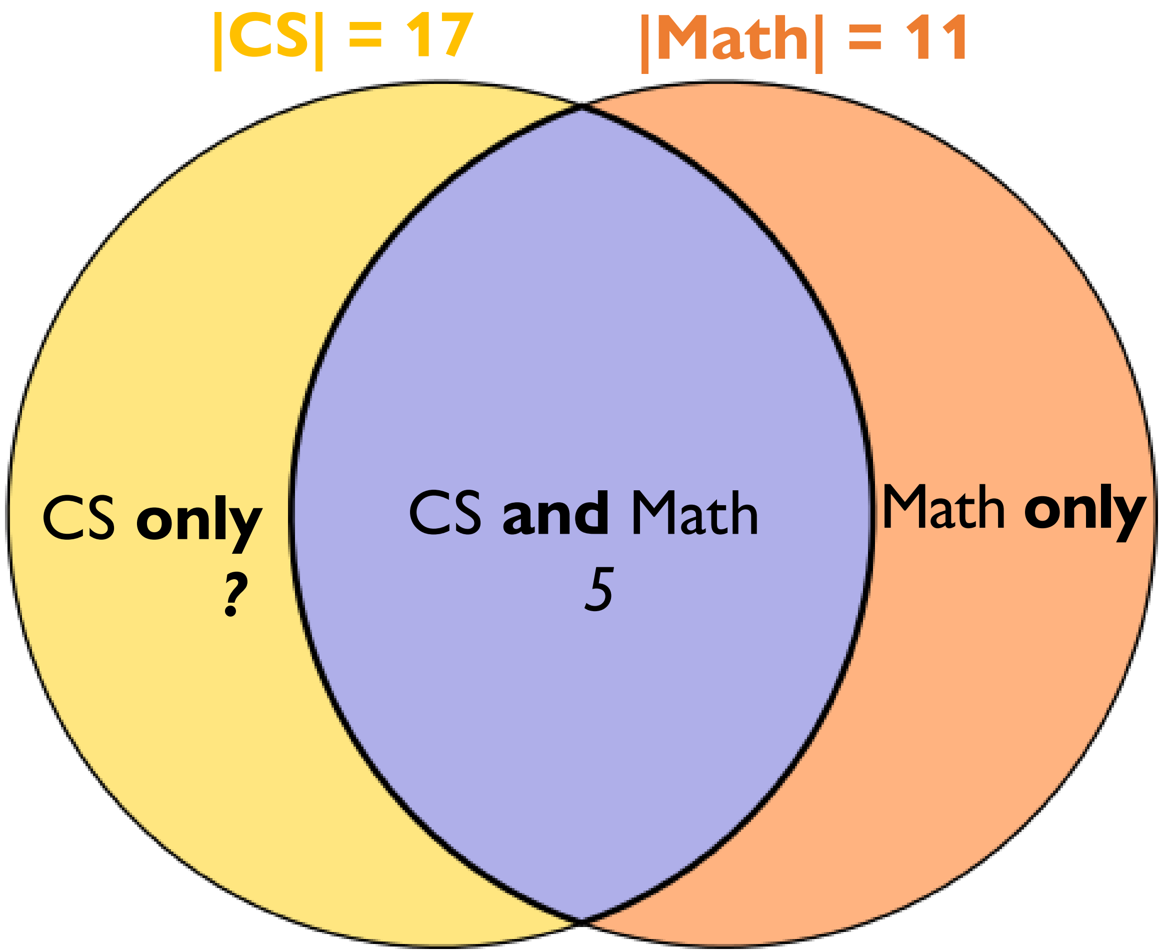

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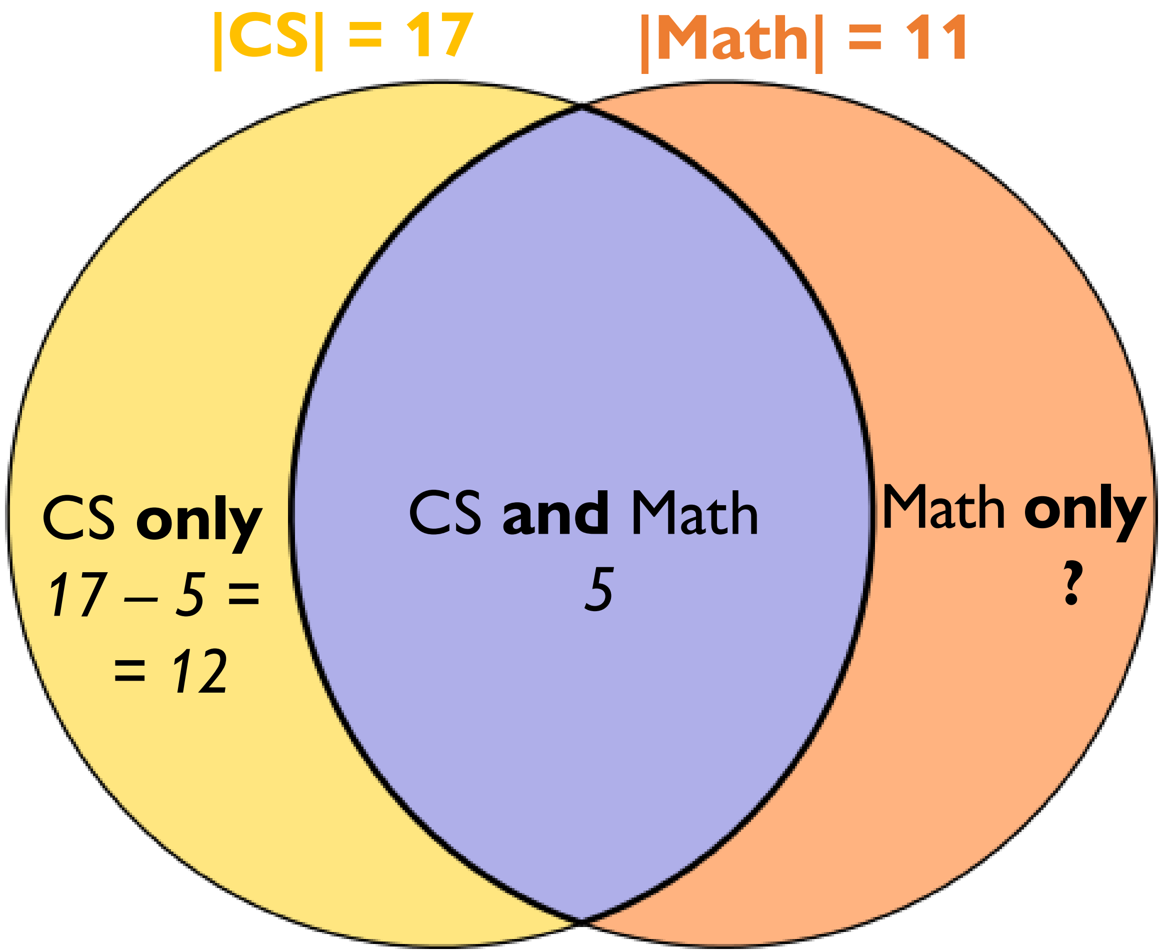

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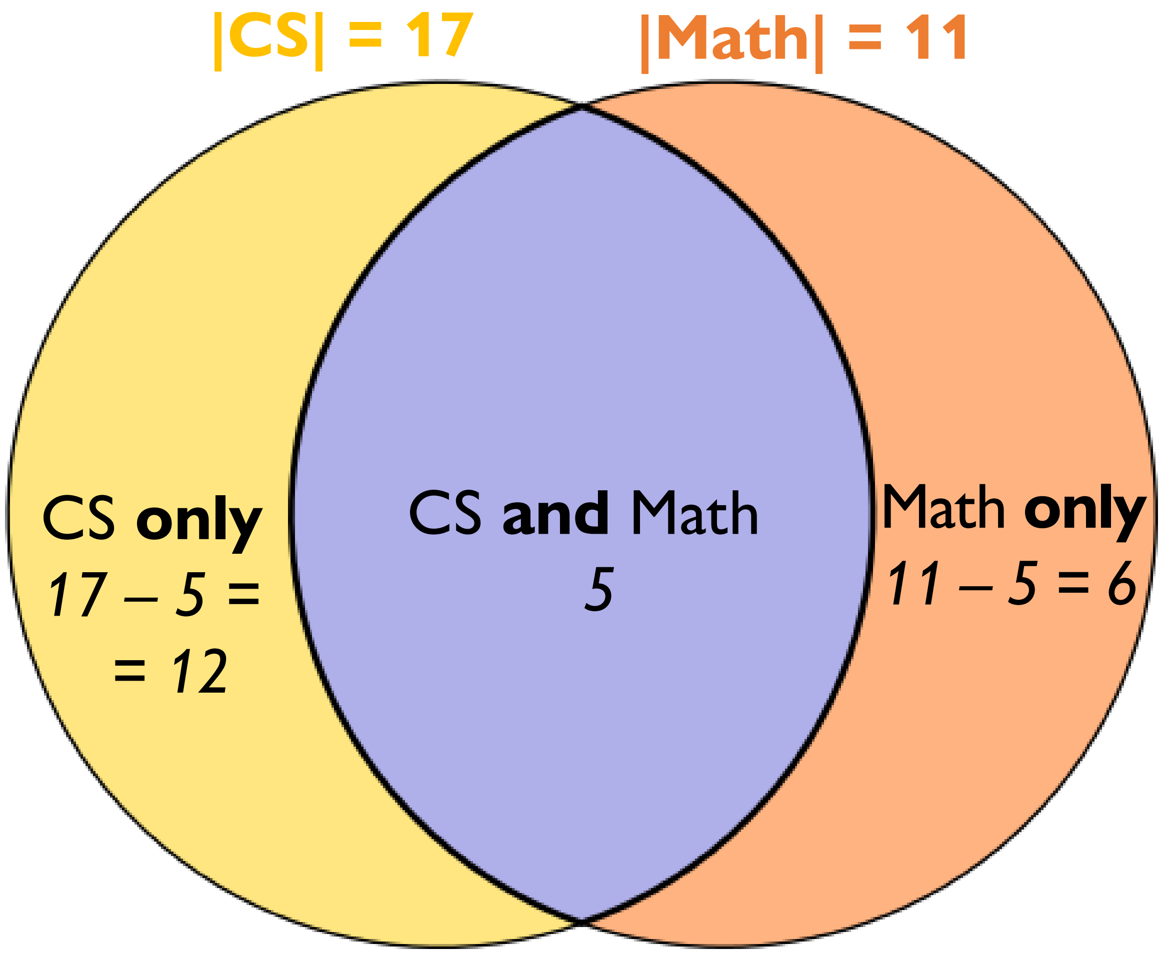

EXAMPLE: 8-BIT STRINGS

- How many 8-bit sequences begin with 110 or end with 1100?

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$$N = N_{\text{begin with 110}} + N_{\text{end with 1100}} - N_{\text{begin with 110 and end with 1100}}$$

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  # begin with 110:

  # end with 1100:

  # begin with 110 **and** end with 1100:

EXAMPLE: 8-BIT STRINGS

- How many 8-bit sequences begin with 110 or end with 1100?

  # begin with 110:

1	**1**	**0**	*	*	*	*	*

  # end with 1100:

  # begin with 110 **and** end with 1100:

EXAMPLE: 8-BIT STRINGS

- How many 8-bit sequences begin with 110 or end with 1100?

\# begin with 110: $N_{110} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

1	**1**	**0**	*	*	*	*	*

\# end with 1100:

\# begin with 110 **and** end with 1100:

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end with 1100:

*	*	*	*	**1**	**1**	**0**	**0**

begin with 110 **and** end with 1100:

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1	**1**	**0**	*	*	*	*	*

\# end with 1100: $N_{1100} = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$

*	*	*	*	**1**	**1**	**0**	**0**

\# begin with 110 **and** end with 1100:

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*	*	*	*	**1**	**1**	**0**	**0**

\# begin with 110 **and** end with 1100: $N_{110,1100} = 2$

1	**1**	**0**	*	**1**	**1**	**0**	**0**

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1	**1**	**0**	*	**1**	**1**	**0**	**0**

$N =$

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$$N = N_{110} + N_{1100} - N_{110,1100} =$$

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*	*	*	*	**1**	**1**	**0**	**0**

\# begin with 110 **and** end with 1100: $N_{110,1100} = 2$

1	**1**	**0**	*	**1**	**1**	**0**	**0**

$$N = N_{110} + N_{1100} - N_{110,1100} = 32 + 16 - 2 = 46$$

AN INEQUALITY APPLICATION

- In a group of 50 people, 30 own a dog and 25 own a cat. What can we say about the number of people who own both a dog and a cat?

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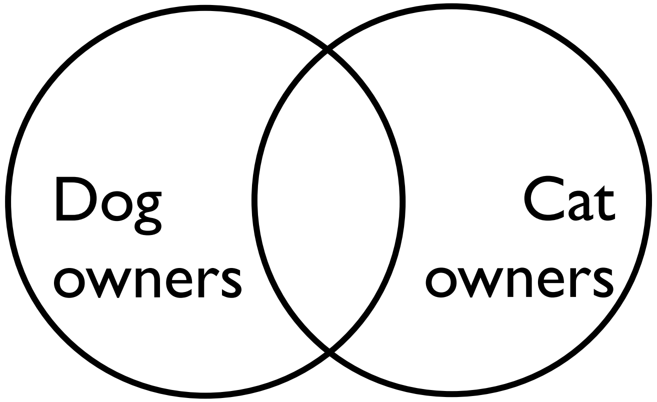

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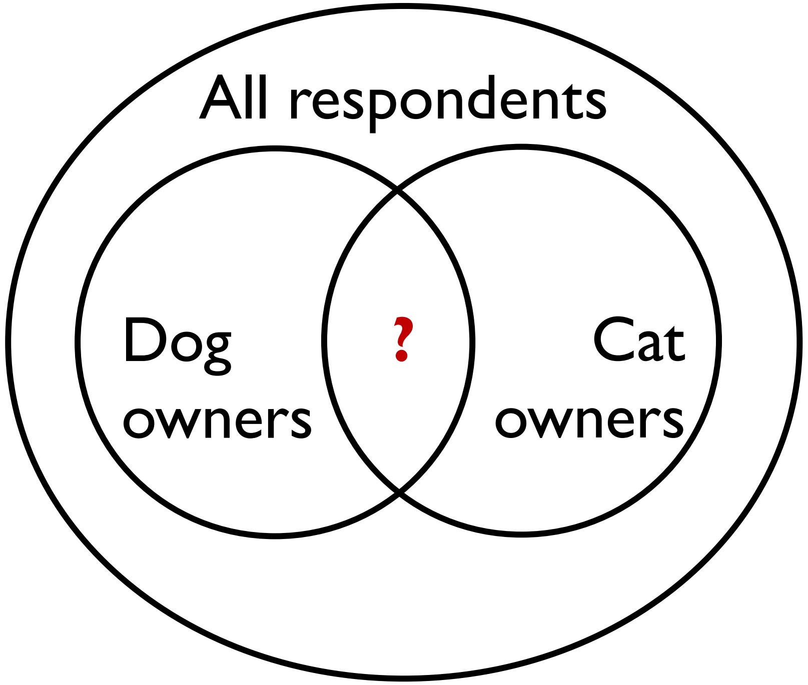

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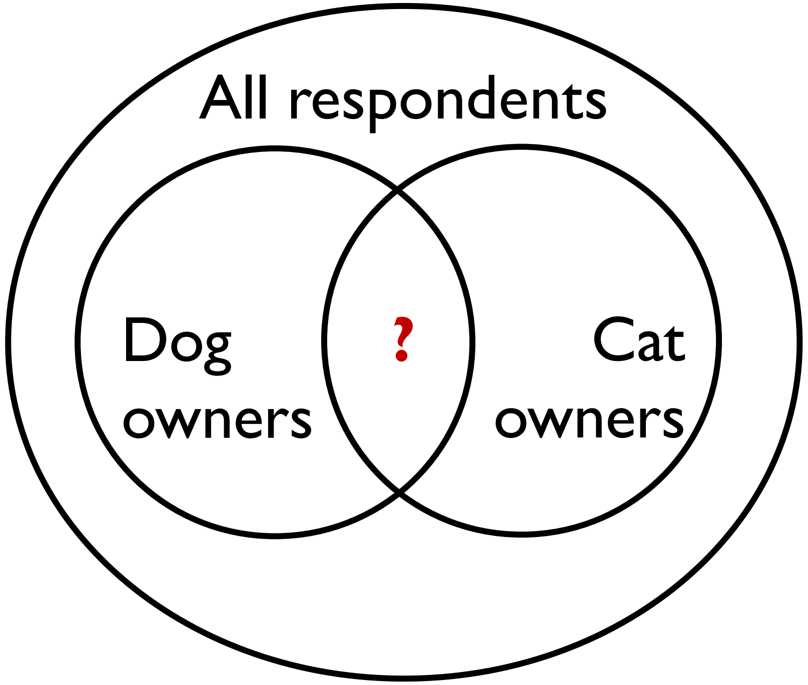

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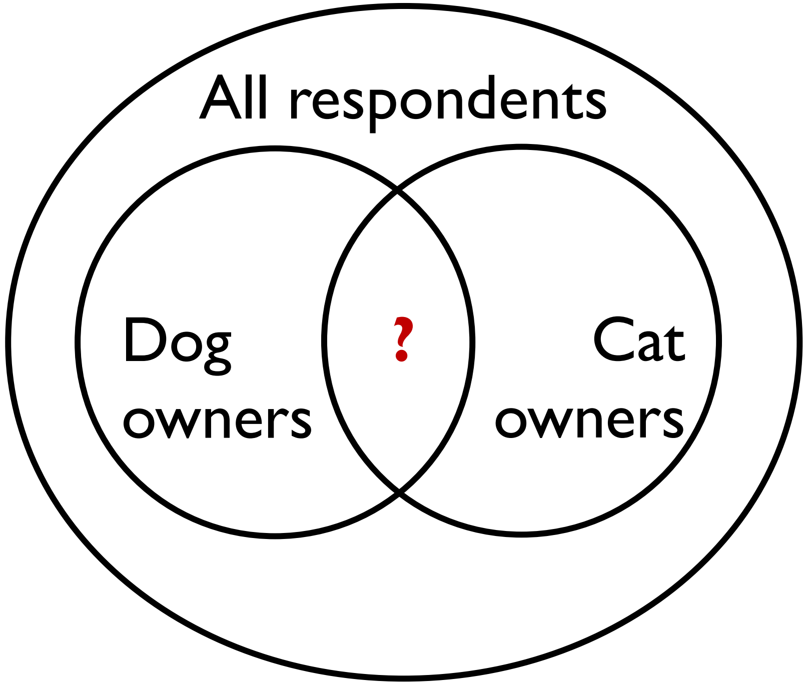

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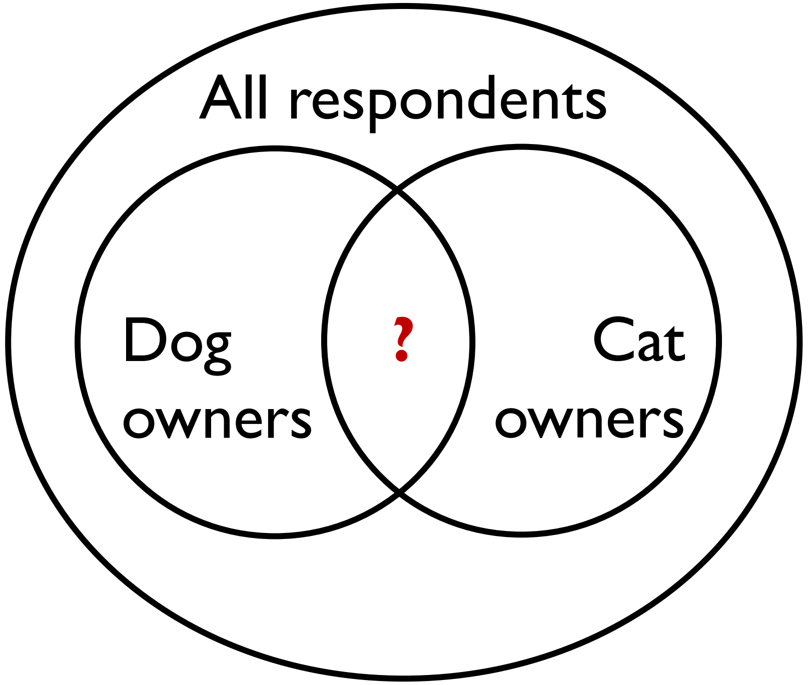

$$|D \cup C| = |D| + |C| - |\boldsymbol{D} \cap \boldsymbol{C}|$$

$$|D| + |C| = 30 + 25 = 55$$

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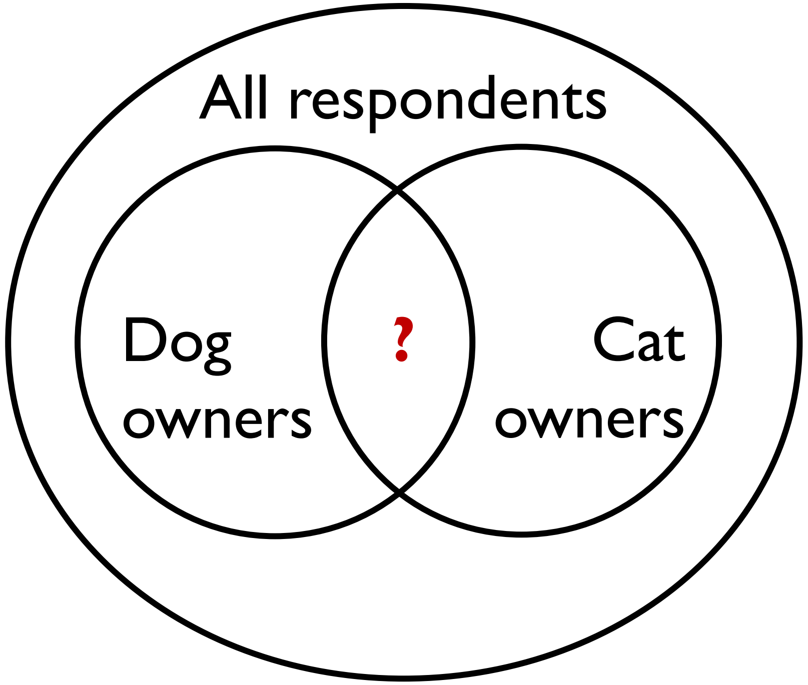

$$|D \cup C| = |D| + |C| - |\boldsymbol{D} \cap \boldsymbol{C}|$$

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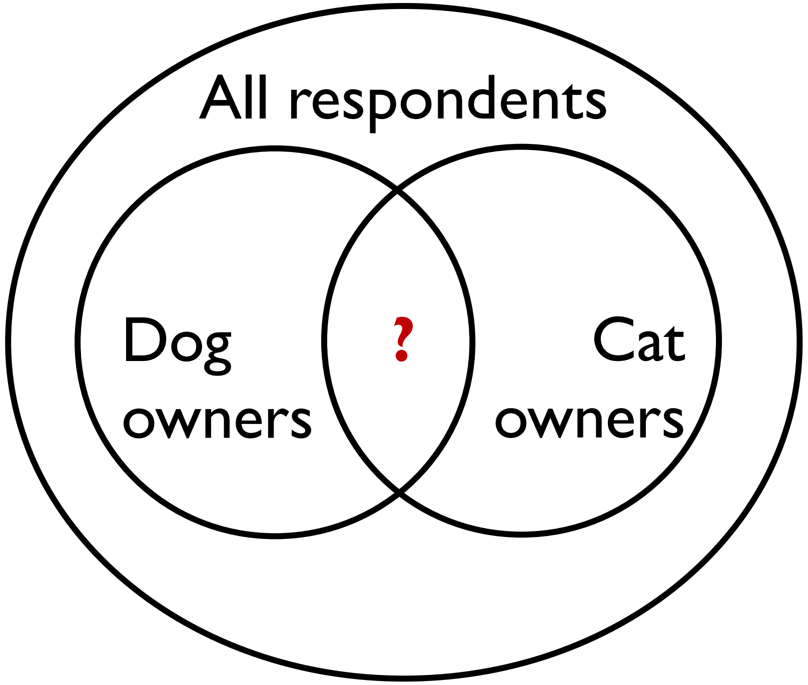

$$|D \cup C| = |D| + |C| - \boldsymbol{|D \cap C|}$$

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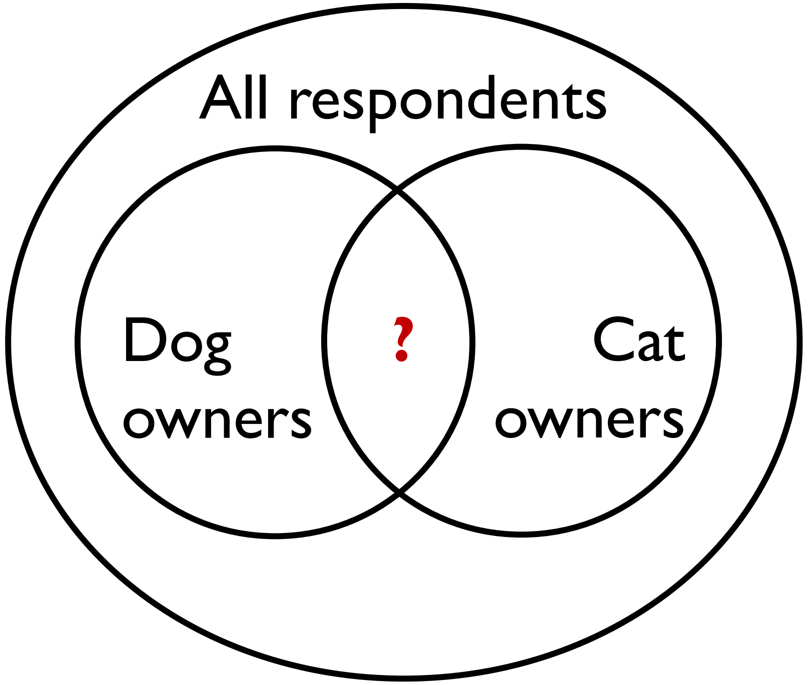

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$$|D \cup C| \leq 50$$

$$50 \geq 55 - |D \cap C|$$

$$|D \cap C| \geq 5$$

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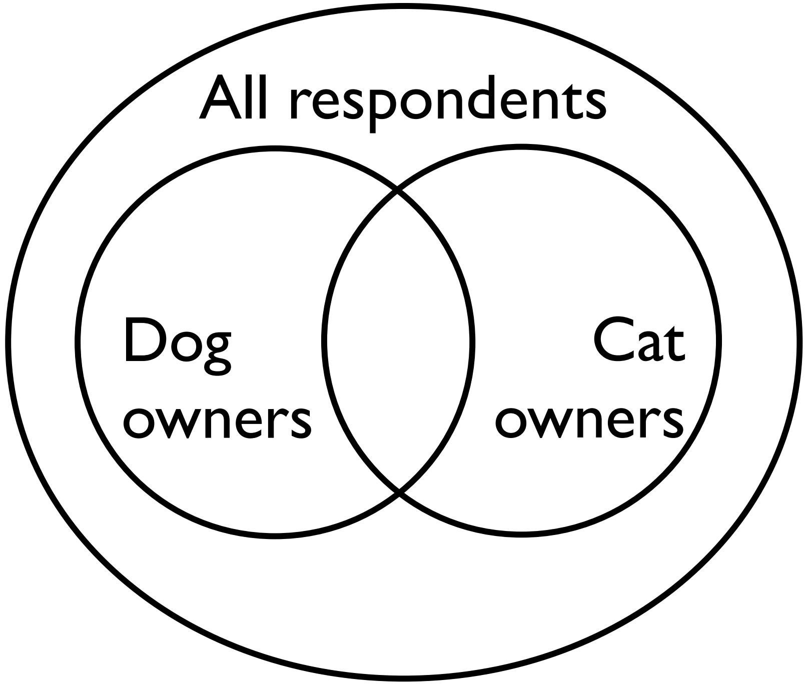

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$$|D| + |C| = 30 + 25 = 55$$

$$|D \cup C| \leq 50$$

$$50 \geq 55 - |D \cap C|$$

$$|D \cap C| \geq 5$$

At least 5 people must own both a dog and a cat.

LET’S PRACTICE!

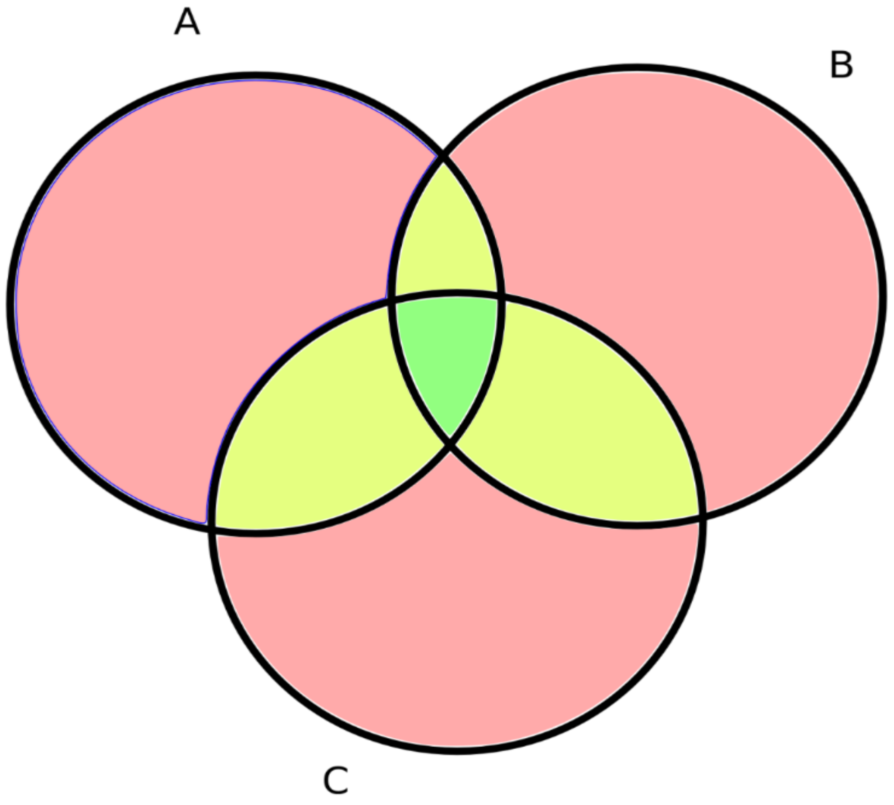
https://docs.google.com/document/d/1VIDmtgY9qrhqisifcF6R_E37QSOH7-g0Ib25bX4vxiw/edit?usp=sharing

PART I

PRINCIPLE OF INCLUSION-EXCLUSION FOR THREE SETS

- For any finite sets A, B and C

$$|A \cup B \cup C| =$$



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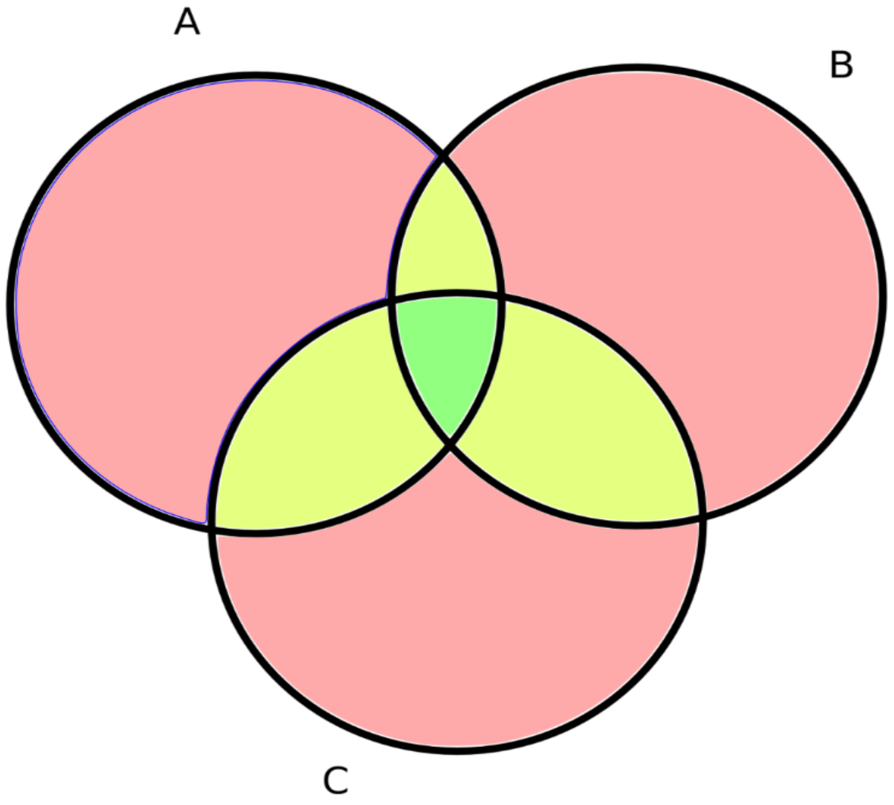

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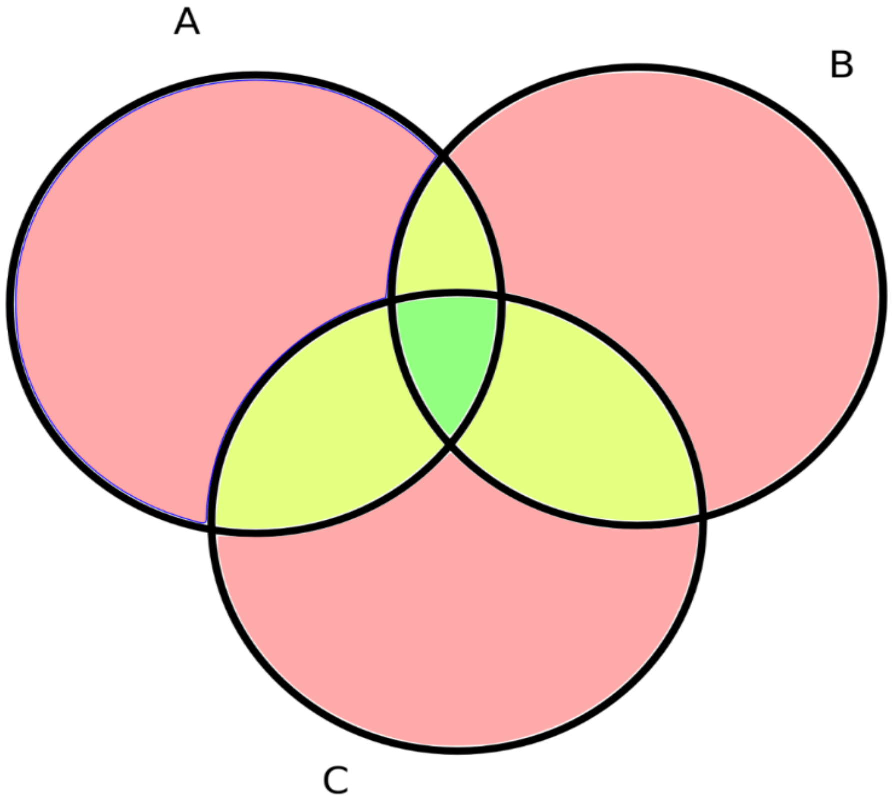

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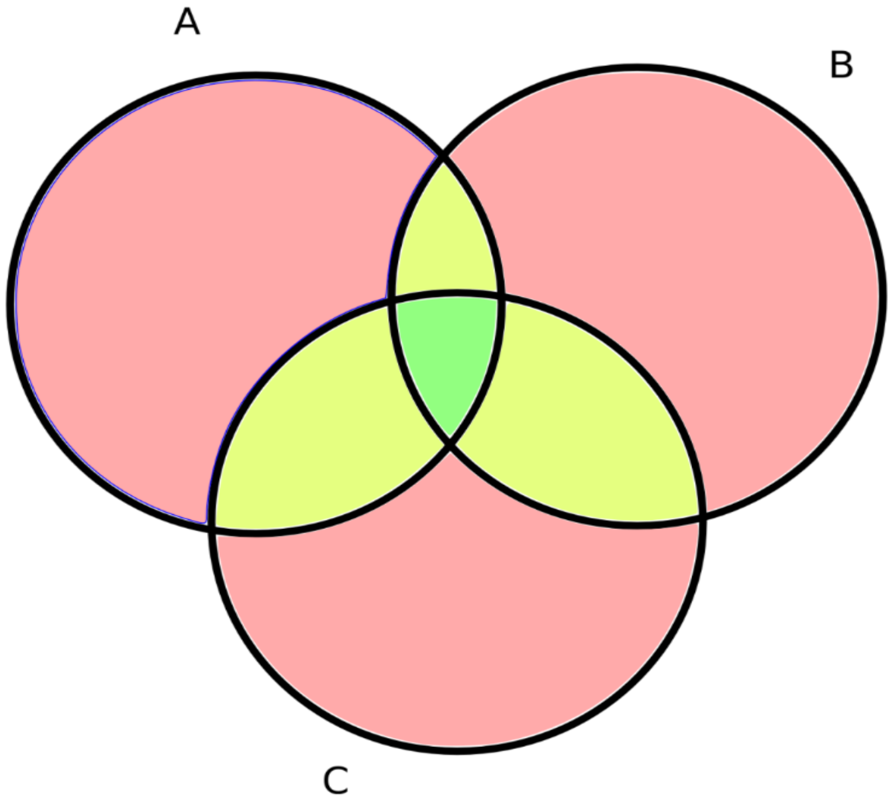
$$- |A \cap B| - |A \cap C| - |B \cap C| +$$

$$+ |A \cap B \cap C|$$



TRAVELLERS

Travelers are surveyed about the city they want to visit:

- Las Vegas: 26
- NY: 31
- Orlando: 36
- Las Vegas and NY: 12
- Las Vegas and Orlando: 11
- NY and Orlando: 13
- all three: 5

How many of these travelers want to visit *at least one* of the cities?

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$$|L \cup N \cup O| =$$

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$$|L \cup N \cup O| =$$

$$= |L| + |N| + |O| -$$

$$-|L \cap N| - |L \cap O| - |N \cap O| +$$

$$+|L \cap N \cap O| =$$

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$$+|L \cap N \cap O| =$$

$$= 26 + 31 + 36 -$$

$$-12 - 11 - 13 +$$

$$+ 5 = 62$$

TRAVELLERS 2

Travelers are surveyed about the city they want to visit:

- Las Vegas: 26
- NY: 31
- Orlando: 36
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How many of these travelers want to visit *at least* **two** of the cities?

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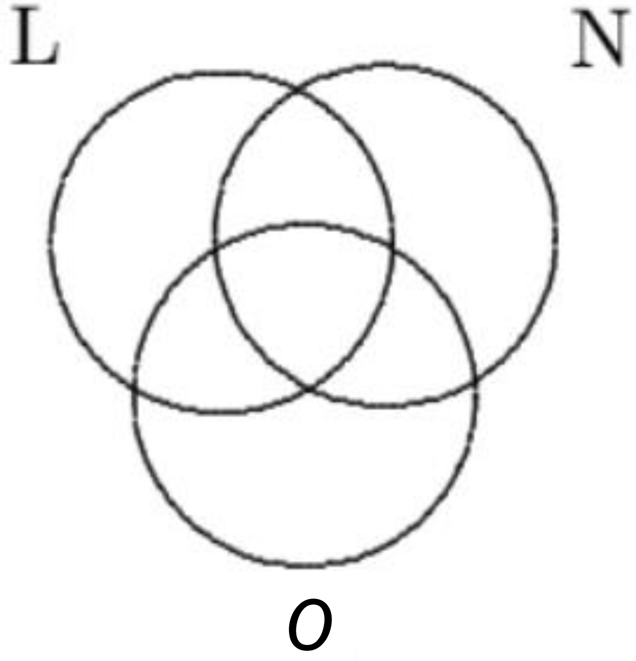

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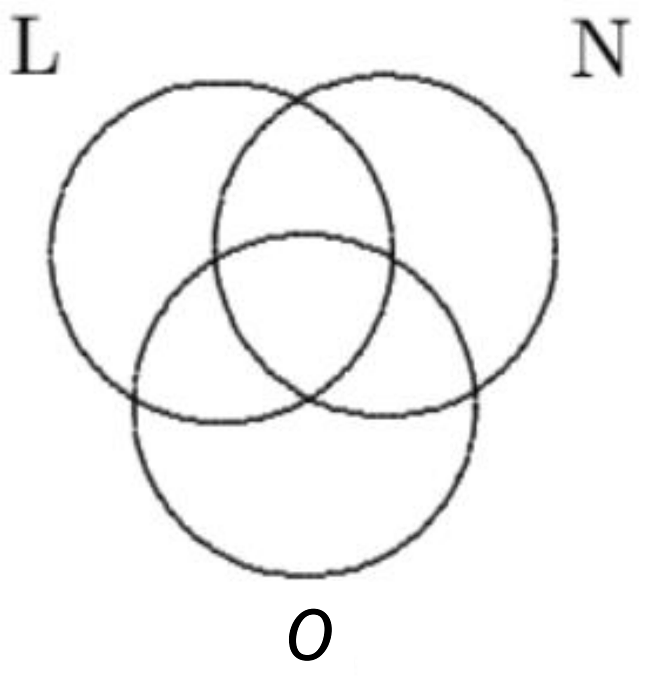
"at least two cities" =

all three cities

NY and L but not O

NY and O but not L

L and O but not NY



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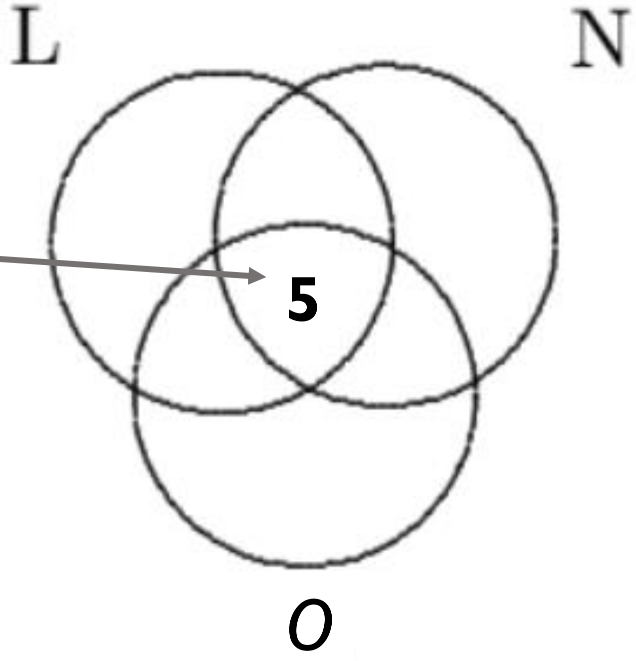

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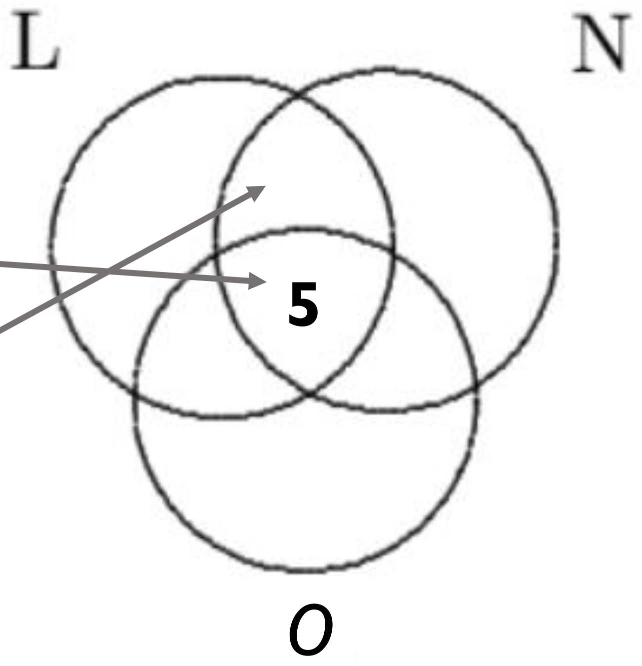

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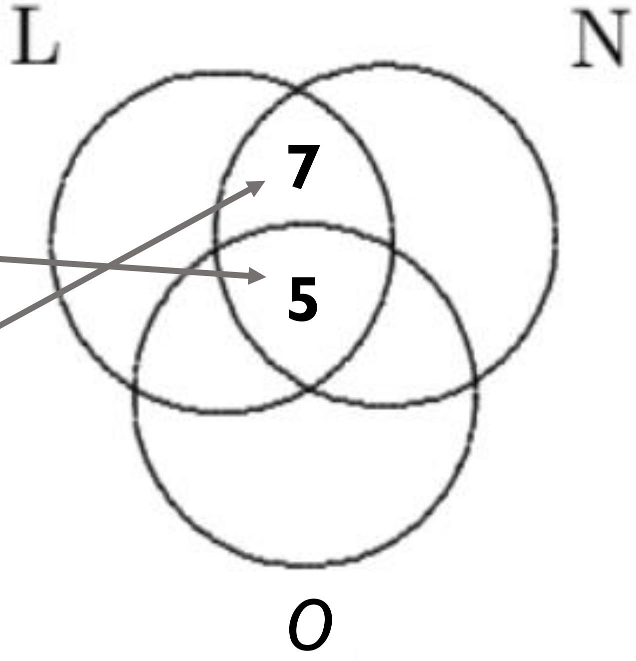
L and O but not NY



How many of these travelers want to visit *at least* **two** of the cities?

TRAVELLERS 2

Travelers are surveyed about the city they want to visit:

- Las Vegas: 26
- NY: 31
- Orlando: 36
- Las Vegas and NY: 12
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- all three: 5

"at least two cities" =

all three cities

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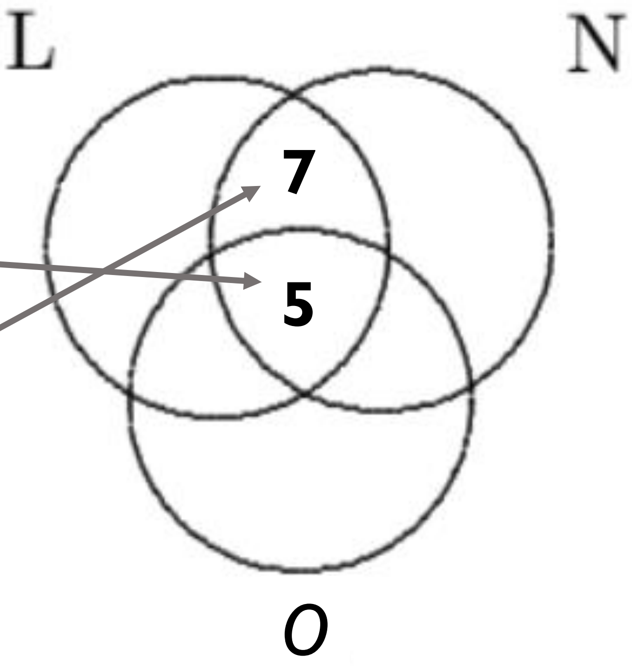

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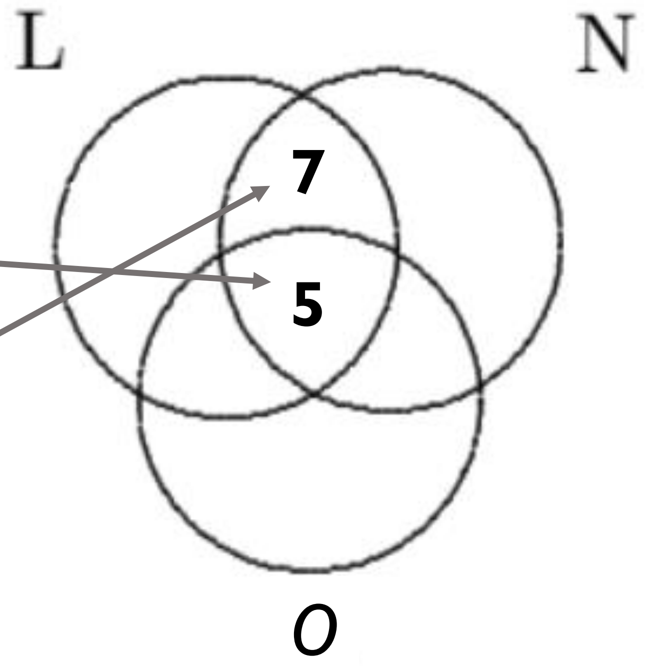

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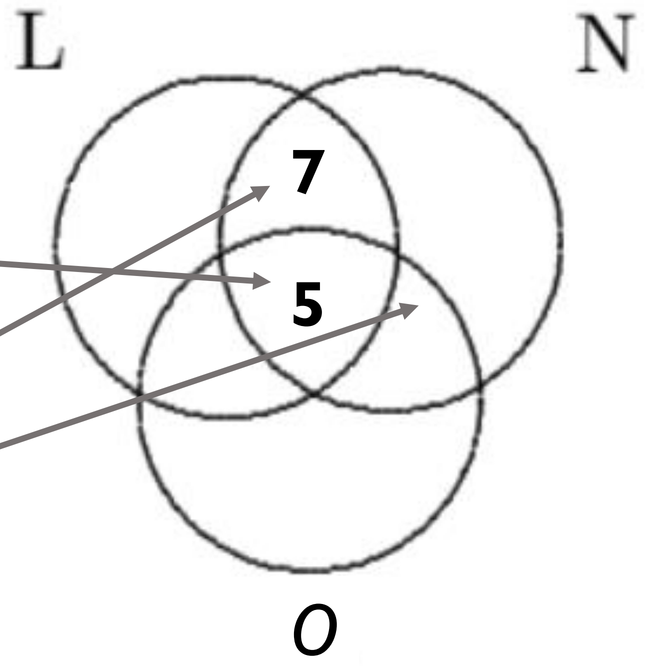

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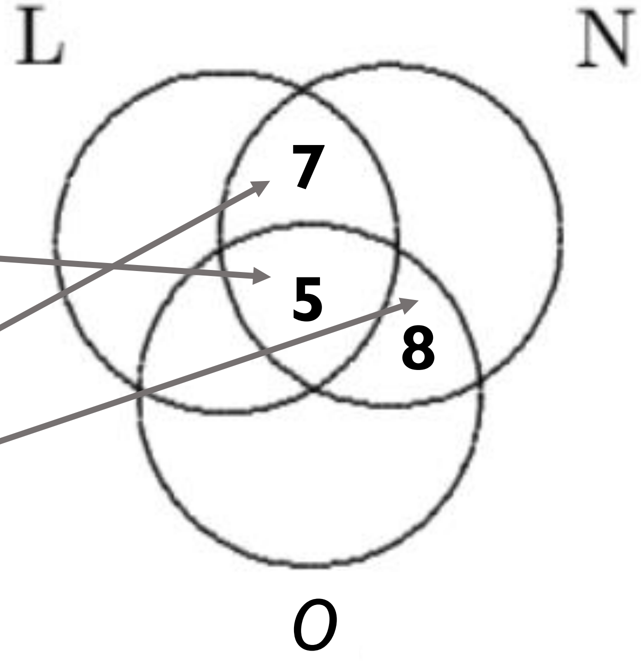

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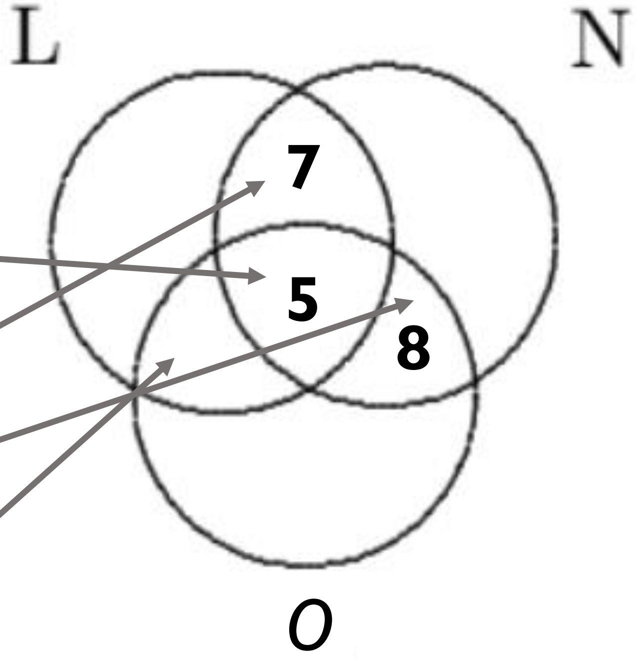

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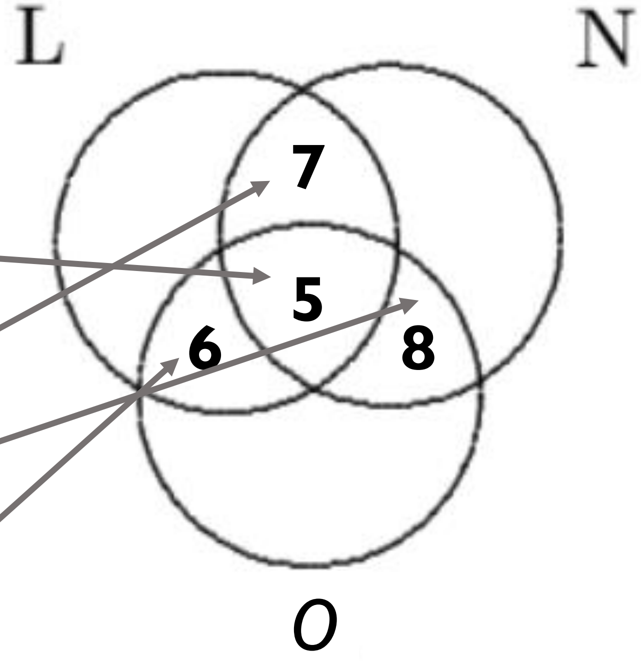

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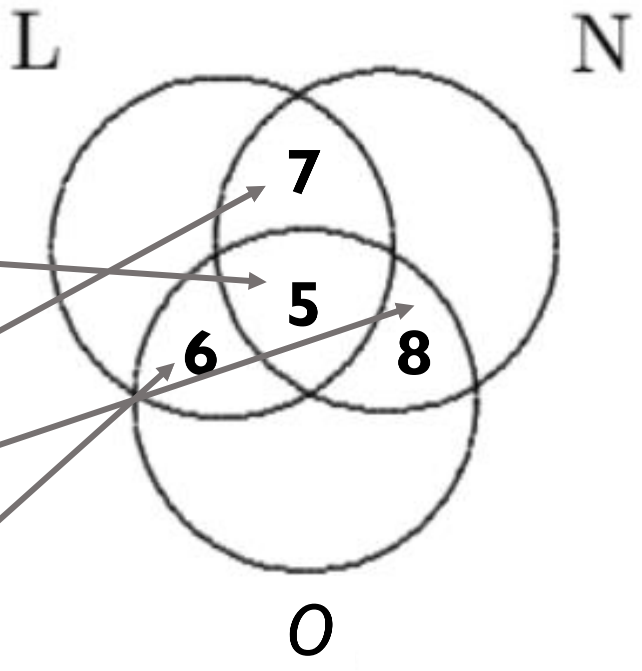

How many of these travelers want to visit *at least* **two** of the cities?

5 + 6 + 8 + 7 = 26 people

EXAMPLE: STUDENT CANTEEN

- 60 students:
  - 31 students like **b**eef;
  - 32 students like **c**hicken;
  - 31 students like **f**ish;
  - 15 students like beef and chicken;
  - 12 students like beef and fish;
  - 19 students like chicken and fish;
  - 8 students like all three.
- How many like none of these?

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Students who like b, c or f: $B \cup C \cup F$

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$$|B \cup C \cup F| =$$
$$= |B| + |C| + |F| -$$
$$-|B \cap C| - |B \cap F| - |C \cap F|$$
$$+|B \cap C \cap F| =$$

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Students who don't like either:

$$(B \cup C \cup F\ )^c$$

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Students who don't like either:

$$(B \cup C \cup F\ )^c$$

$$|(B \cup C \cup F\ )^c| = |U| - |B \cup C \cup F|$$

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Students who don't like either:

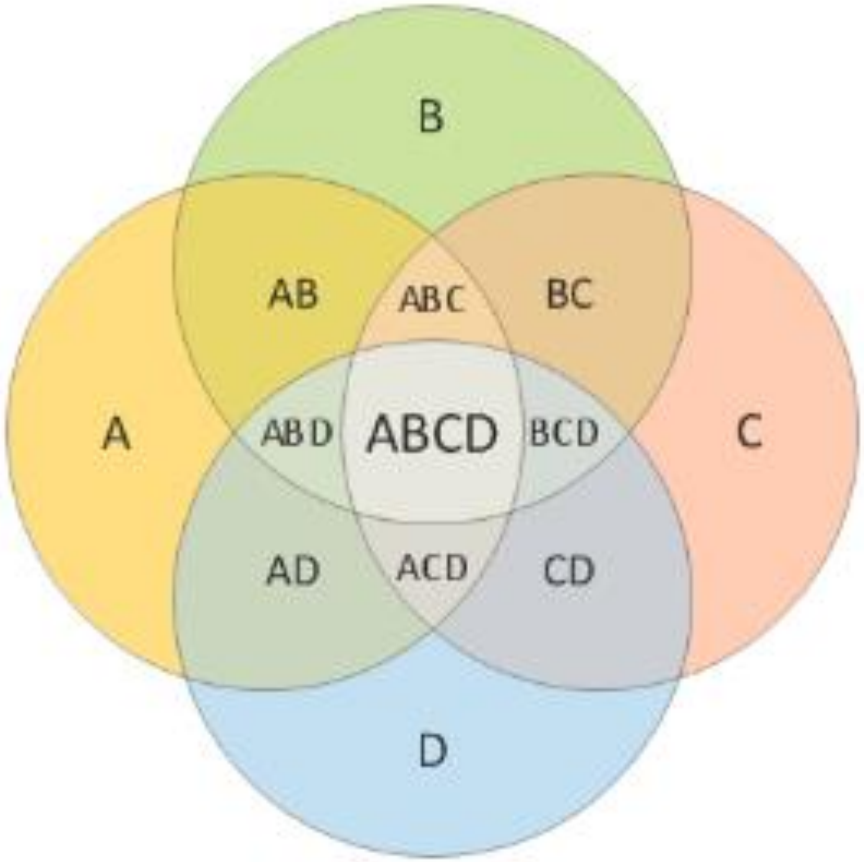
$$(B \cup C \cup F\ )^c$$

$$|(B \cup C \cup F\ )^c| = |U| - |B \cup C \cup F|$$
$$= 60 - 56 = 4$$

PRINCIPLE OF INCLUSION-EXCLUSION FOR FOUR SETS

For any finite sets A, B, C and D

$$|A \cup B \cup C \cup D| =$$



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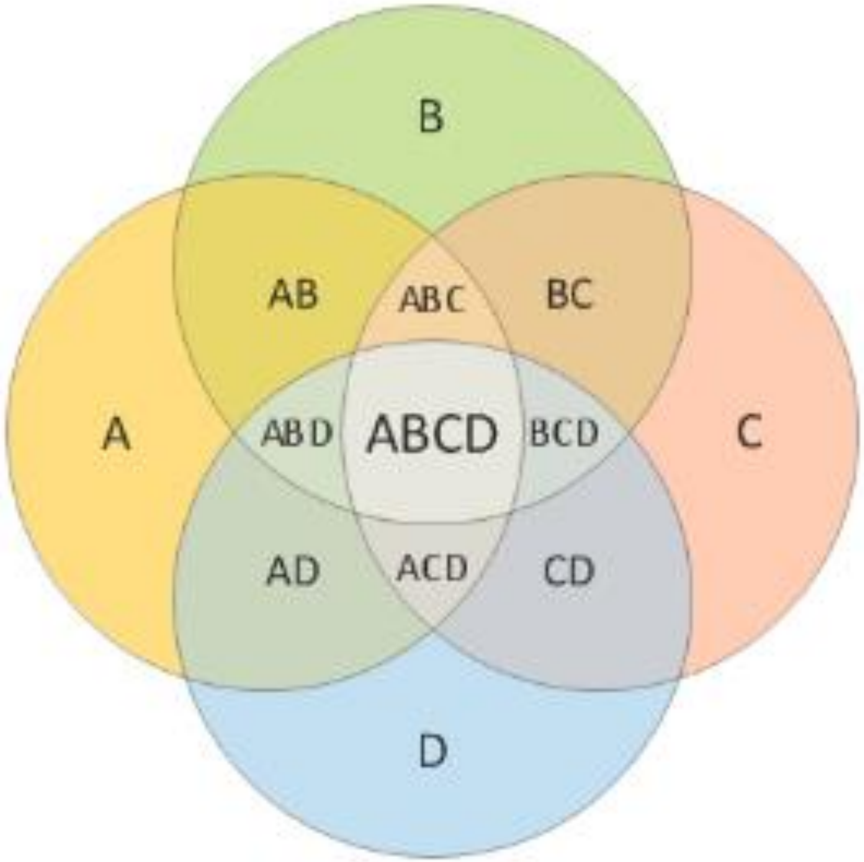

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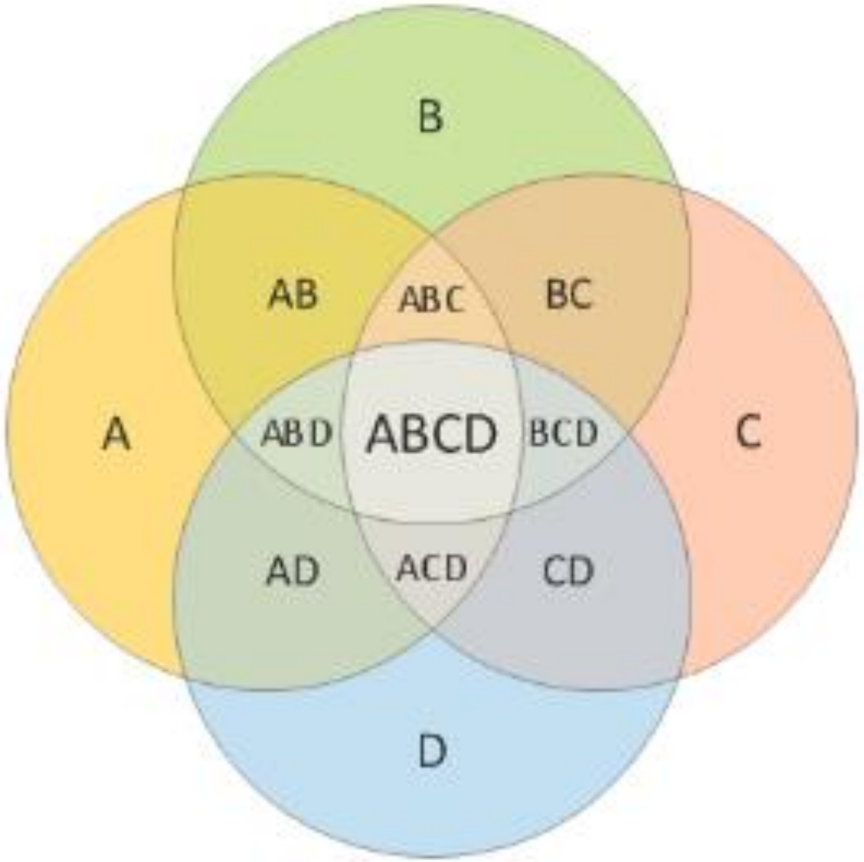

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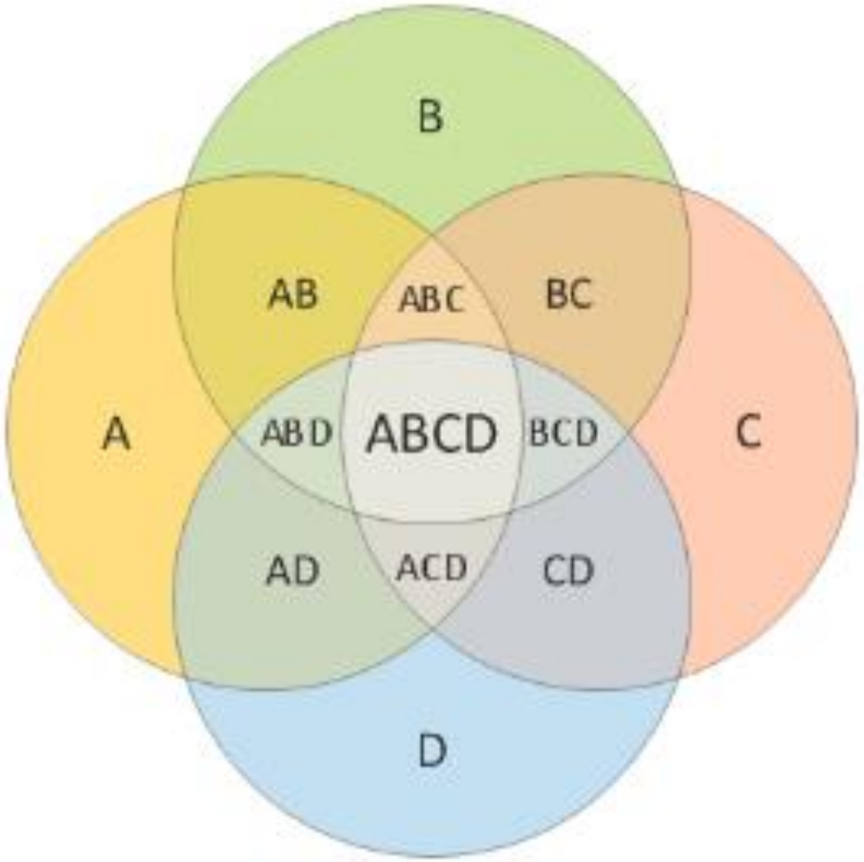

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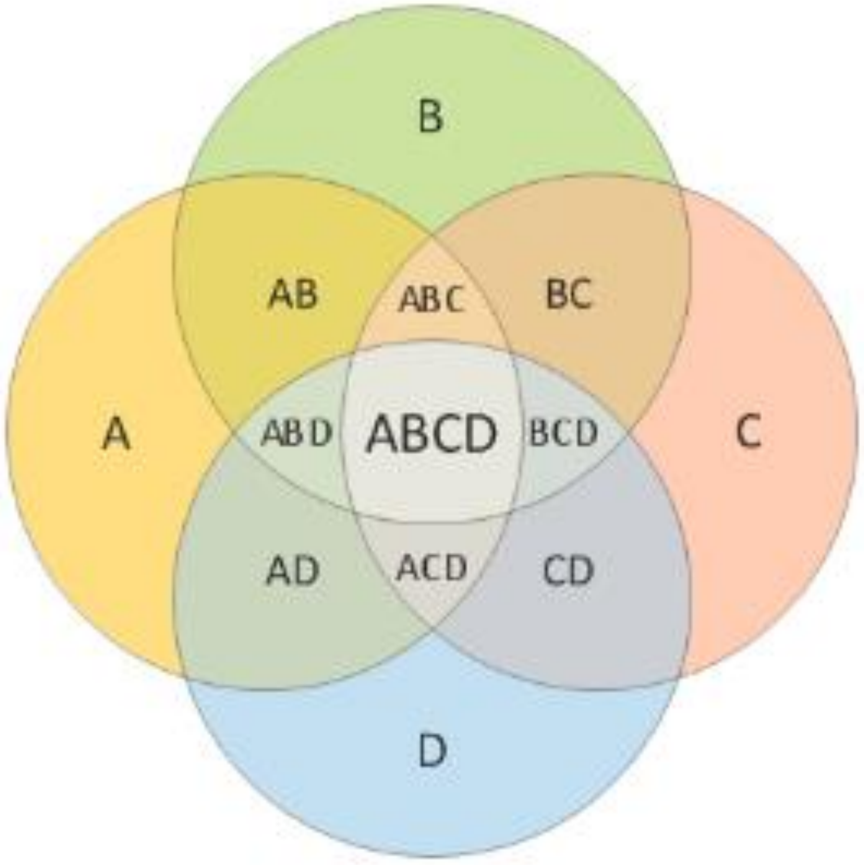
$$+|A \cap B \cap C| + |B \cap C \cap D| +$$
$$+|A \cap B \cap D| + |A \cap C \cap D| -$$
$$-|A \cap B \cap C \cap D|.$$



LET'S PRACTICE!

https://docs.google.com/document/d/1VIDmtgY9qrhqisifcF6R_E37QSOH7-g0Ib25bX4vxiw/edit?usp=sharing

PART II