ELEMENTARY COMBINATORICS & PROBABILITY

Probability of an event

LAST TIME

- Random experiments
- Sample space
- Events
- Probability of an event

TODAY

- Problem set 6
- More pen-and-paper exercises
- Python exercises

WARM-UP

PROBABILITY OF AN EVENT

- Consider a random experiment where all outcomes are equally likely.
- Probability $P(E)$ of an event E is such an experiment is

$$P(E) = \frac{\#\, ways\ E\ can\ happen}{\#\, possible\ outcomes}$$

- Let E^C be the complementary event. Then

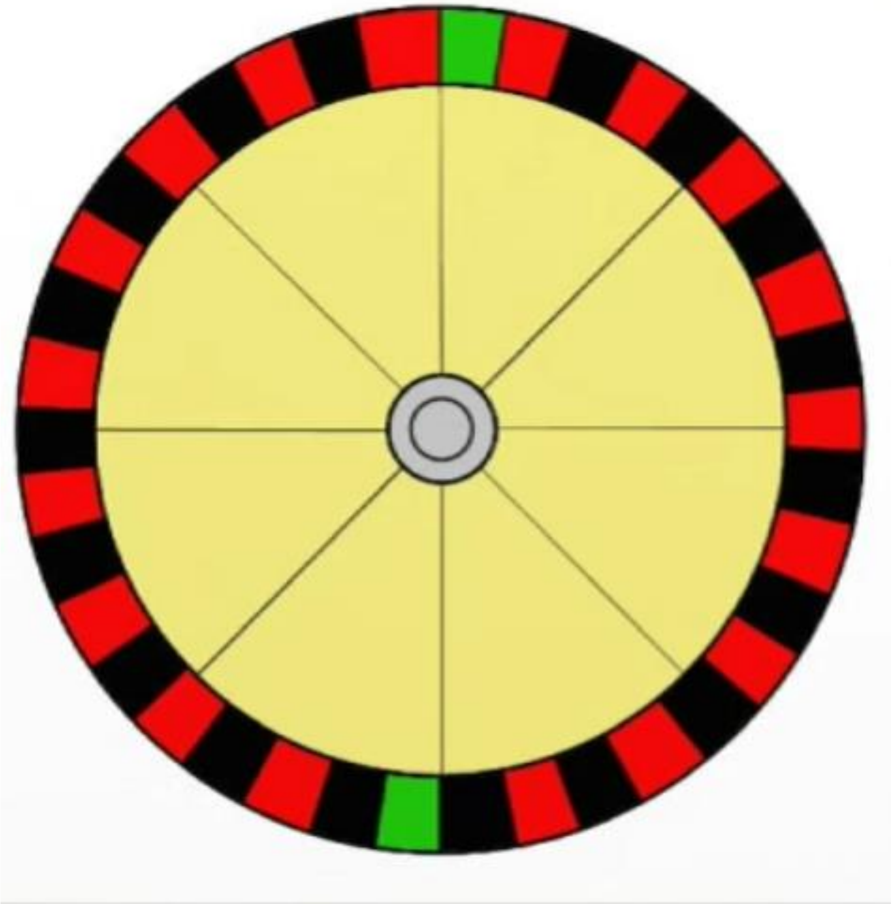
$$P(E^C) = 1 - P(E)$$

- Let F be some other event. Then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

ROULETTE

- American roulette: 38 sectors
  - 18 red
  - 18 black
  - 2 green (0 and 00)



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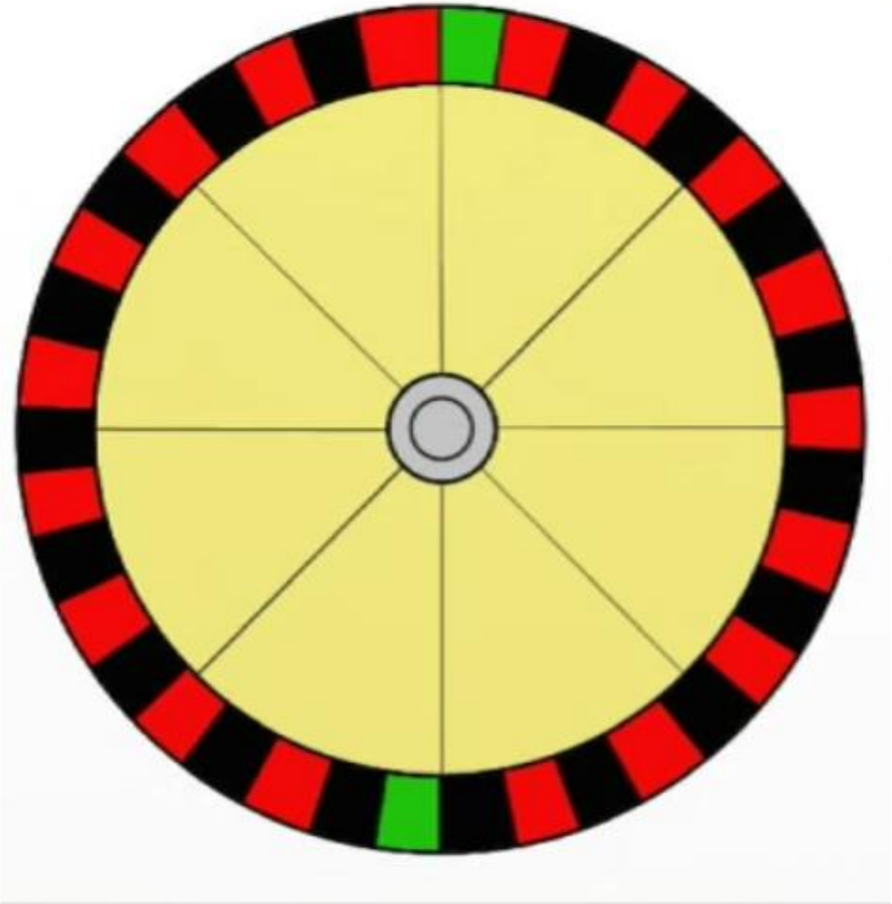

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    $$P(win) = 18/38 \approx 0.474$$
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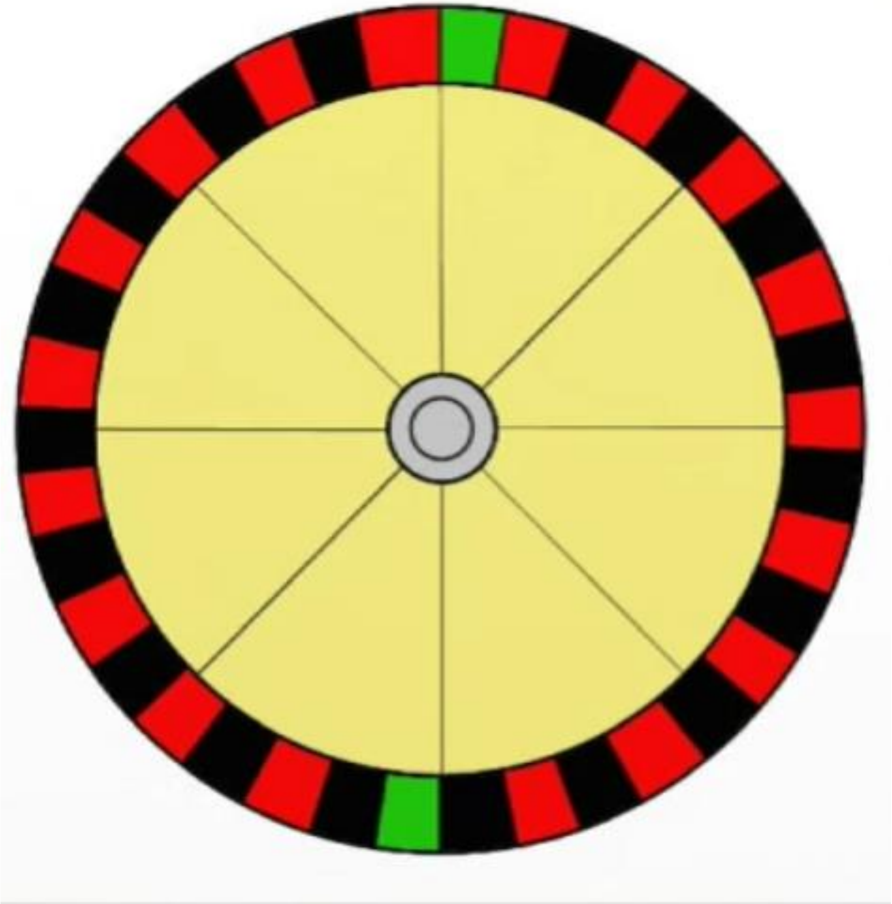

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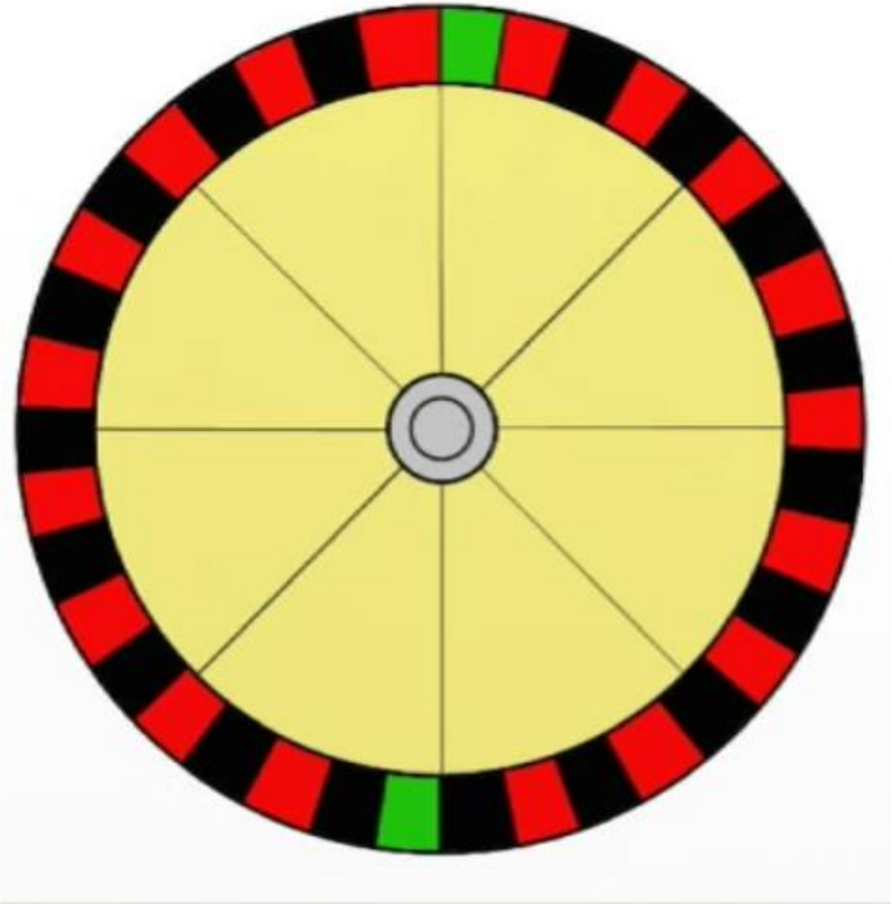

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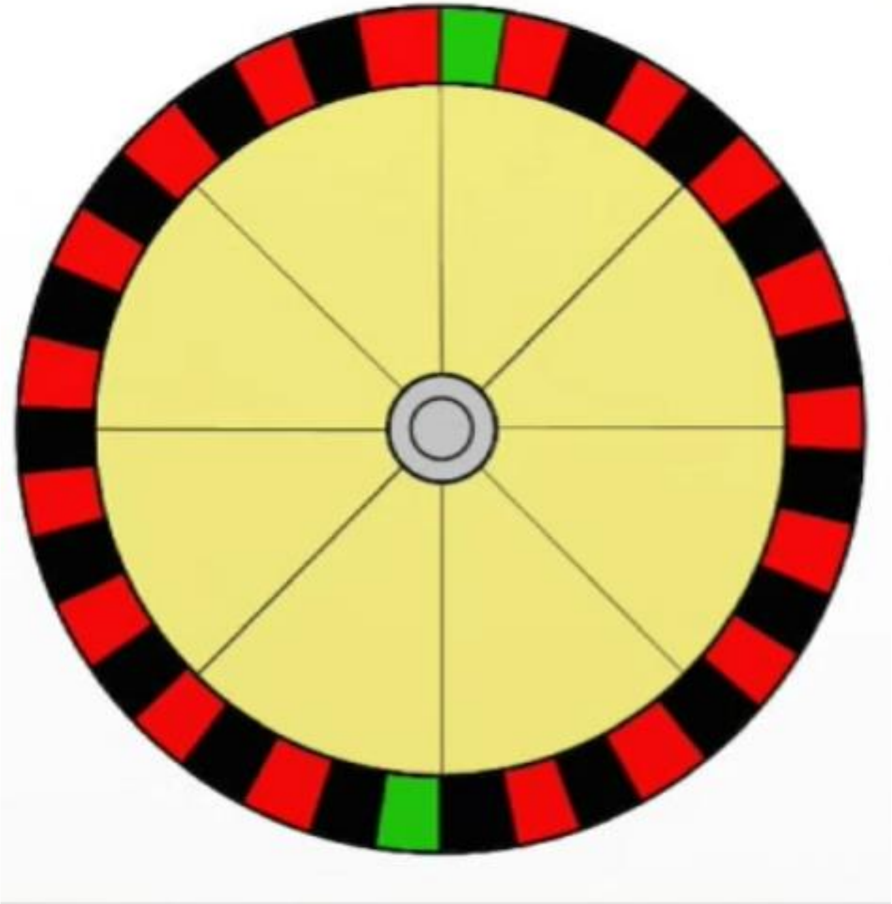
    $P(win) = 2/38 \approx 0.05$



PROBLEM SET 6

Selected problems

BALLS FROM A BOWL

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  ways to pick two same-color balls one by one

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$$P(E) = 62/210 = 31/105$$

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  Compute the probability that the sum of the faces is 3.

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$$P(E) = \frac{2}{36} = \frac{1}{18} \approx 0.056$$

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- In a lottery, 5 balls are selected at random from a collection of 53 numbered white balls, and one ball is selected at random from a collection of 42 numbered red balls. A lottery ticket lists the numbers of 5 white balls (in any order) as well as the number of the red ball.
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  Number of winning combinations: 1

  Probability of winning: $1/[C(53, 5) \cdot 42] \approx 0.0000000083$

BOYS AND GIRLS

- Assuming equal outcomes, calculate the probability that …
- **… in a 3-child family, there are 2 boys and 1 girl**

  $|S| =$

- **… in a 4-child family, there are 2 boys and 2 girls**

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  $P(E) = 3/8$

RANDOM NUMBER

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- **… it doesn't contain digit 7?**
- **… it contains unique digits (e.g., 123 but 111)?**

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$$= \frac{33}{100} + \frac{14}{100} - \frac{4}{100} = \frac{43}{100} = 0.43$$

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  $$P(E) = \frac{C(14,6)}{C(21,6)} = \frac{3003}{54264} = \ 0.055$$

PROBABILITY AS LIMITING FREQUENCY

MORE EXAMPLES

PLAYING CARDS

- 52 cards in a deck
- 4 suits (types):
  - clubs ♣
  - diamonds ♦
  - hearts ♥
  - spades ♠
- 13 cards of each type

	2	3	4	5	6	7	8	9	10	J	Q	K	A
$\heartsuit$													
$\diamondsuit$		$3\diamondsuit$											
$\clubsuit$										$J\clubsuit$			
$\spadesuit$													

POKER

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$$C(52, 5) = \frac{52!}{5! \cdot 47!} = 2{,}598{,}960$$

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- What is the probability of being dealt a poker hand that consists of the 3 ♥ , 5 ♦ , 6 ♠ , 10 ♣ and Q ♦ ?

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$$P(E) = \frac{1}{2{,}598{,}960} \approx 0.000000385$$

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$$P(E) = \frac{123552}{2{,}598{,}960} \approx 0.048$$