INTRODUCTION TO STATISTICS

LECTURE 5

LAST TIME

- Understanding probability density functions (PDFs)
- PDF and CDF
- Uniform distribution
- Exponential distribution

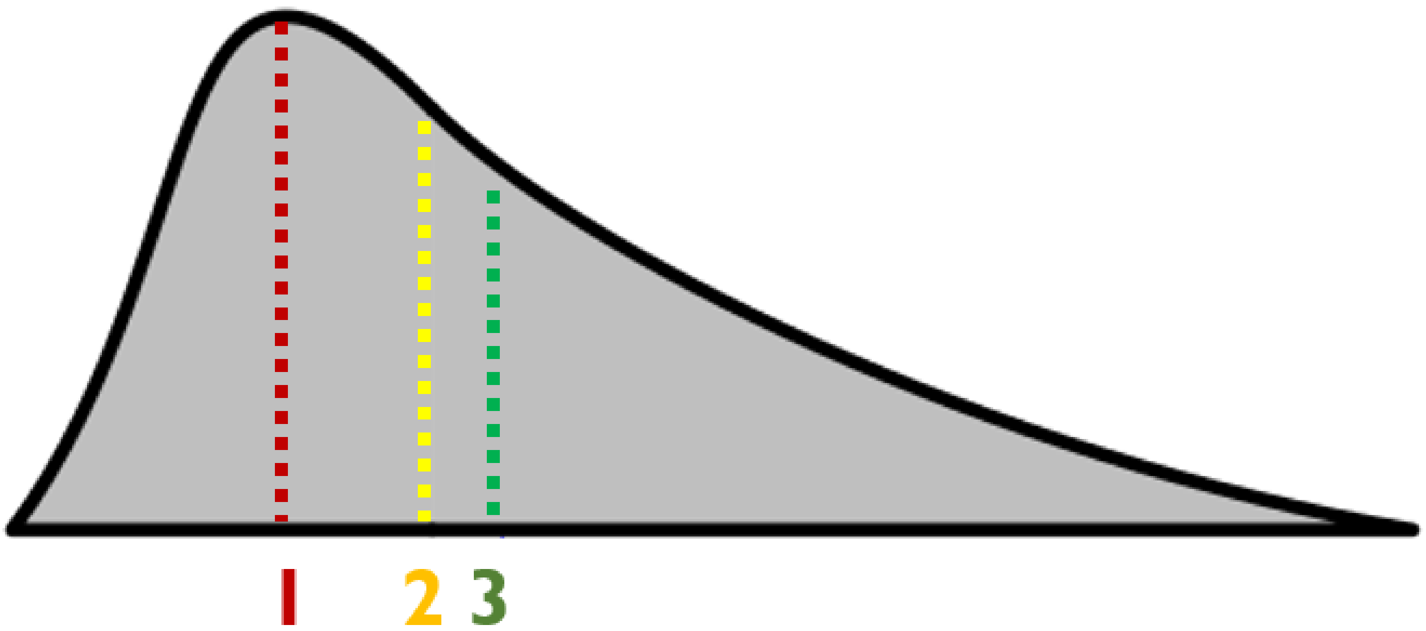
TODAY

- Finish the uniform distribution exercise
- Review density
- Maximum Likelihood for continuous distributions
- Normal distribution
- “Strange” distributions

MEAN, MEDIAN AND MODE

ON PDFs

MEAN, MEDIAN AND MODE



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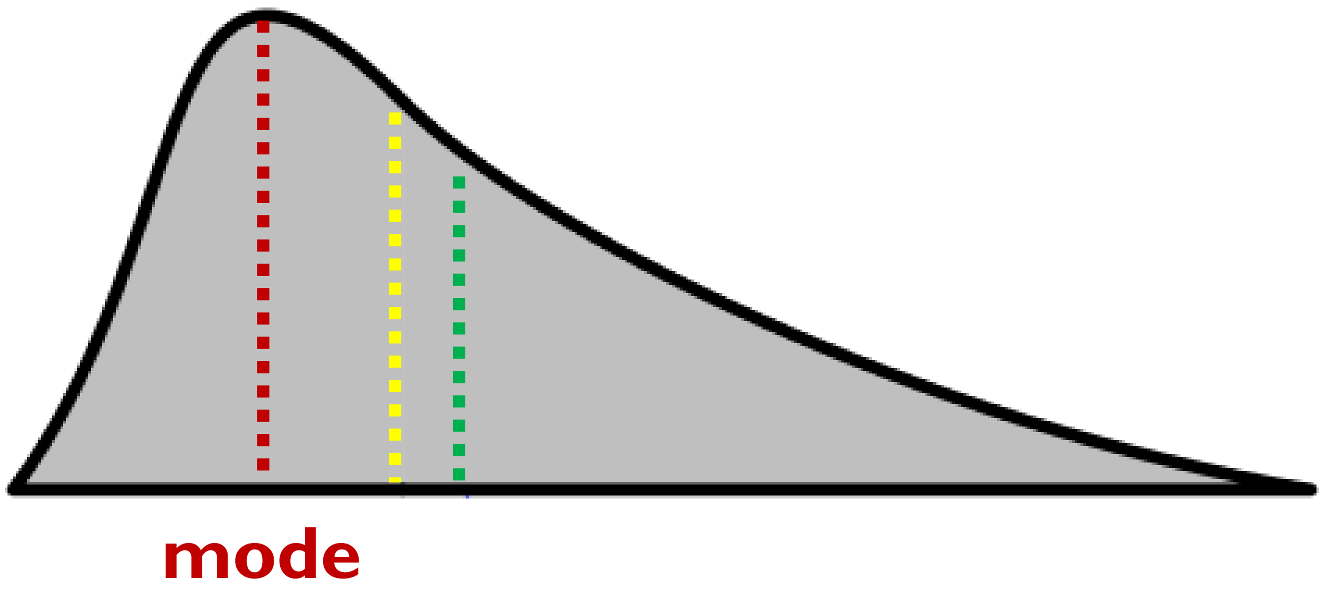

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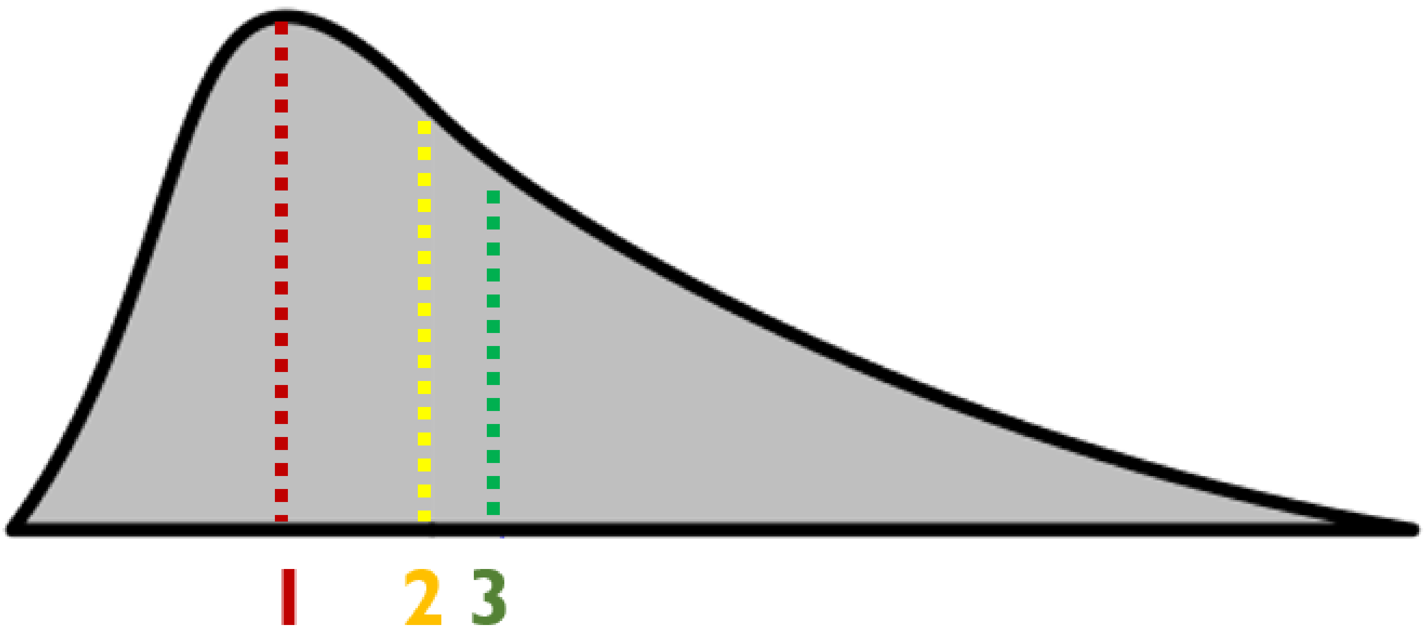

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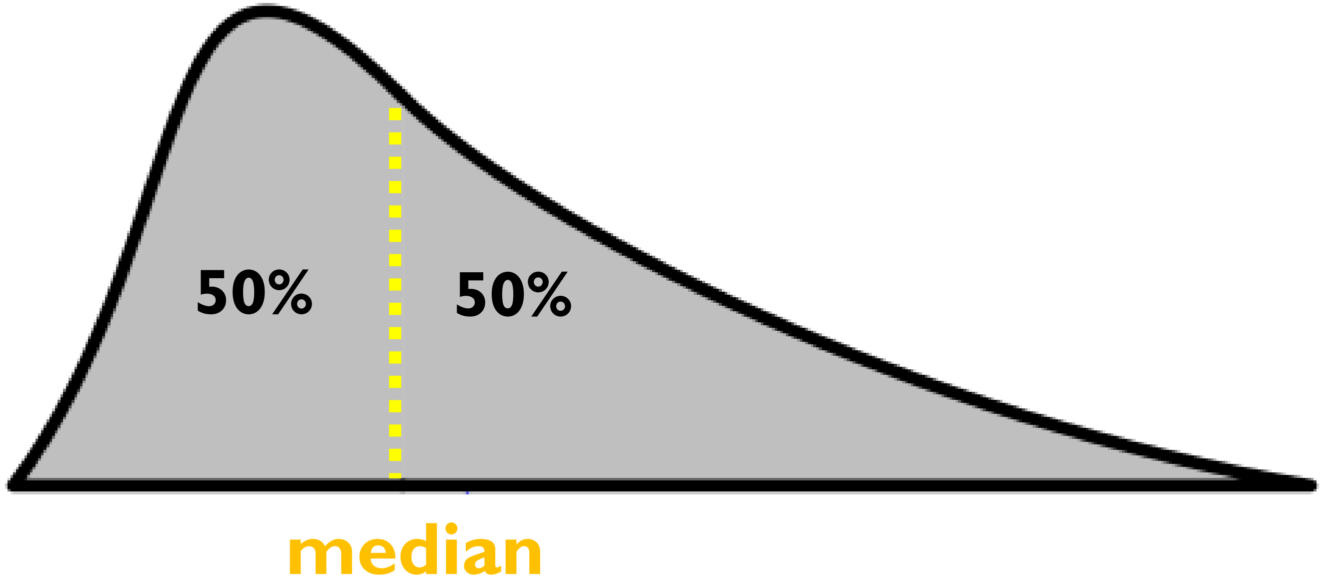

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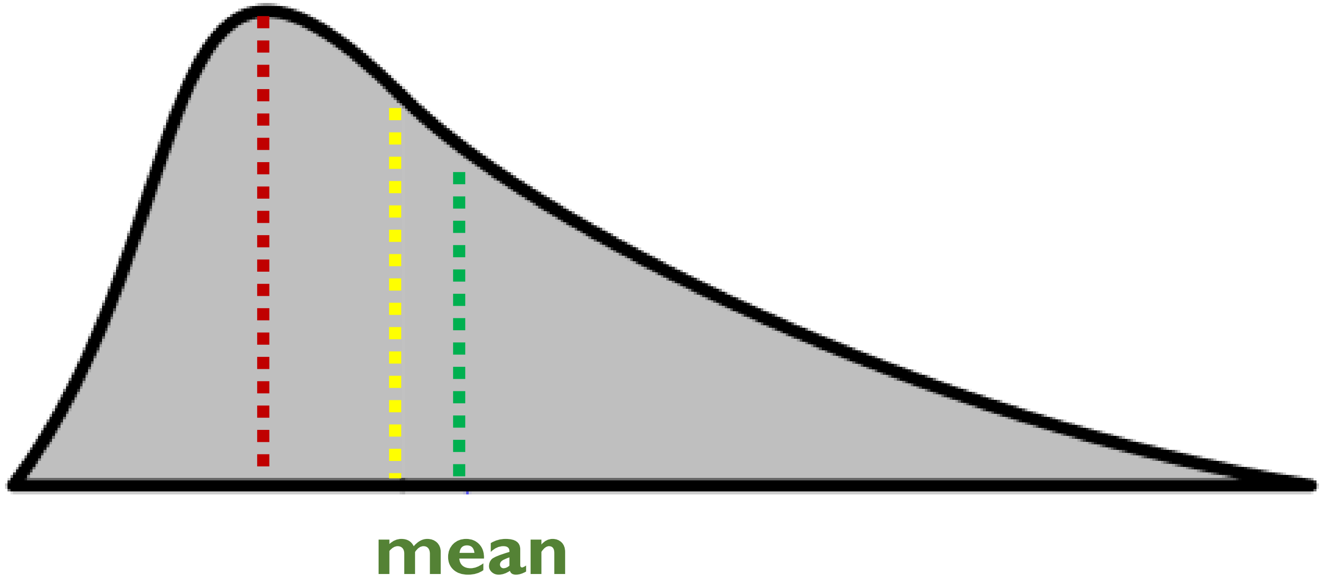

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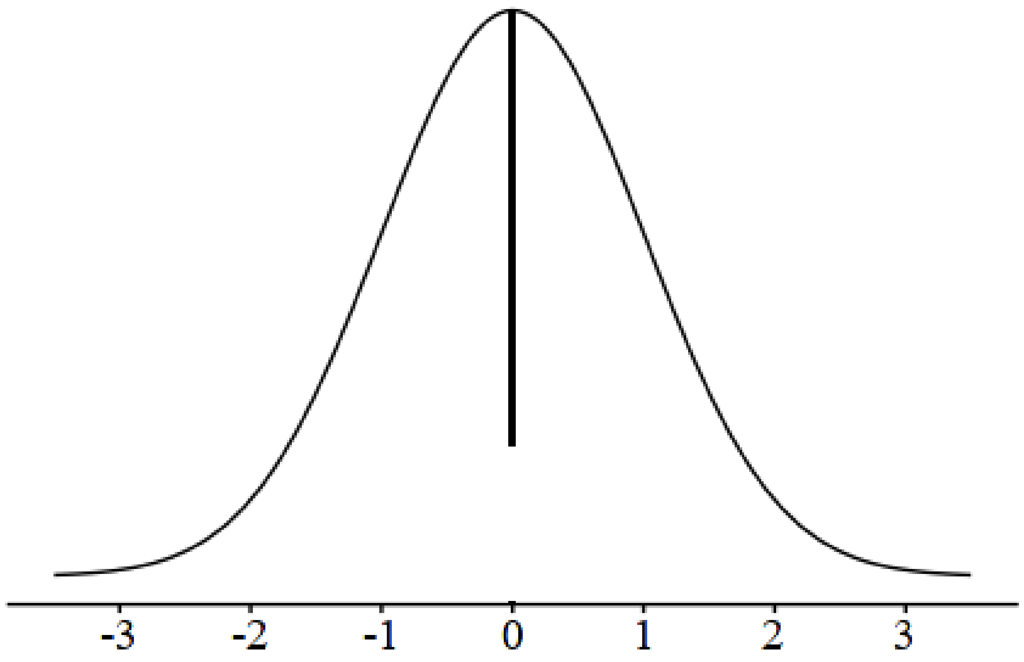

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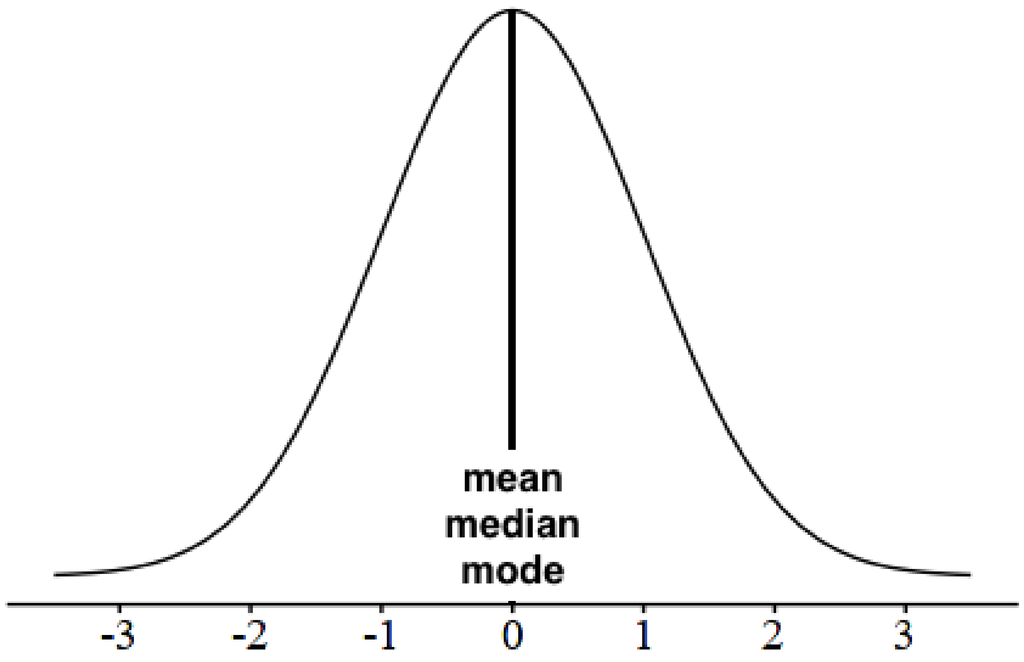

REMINDER: EXPONENTIAL DISTRIBUTION

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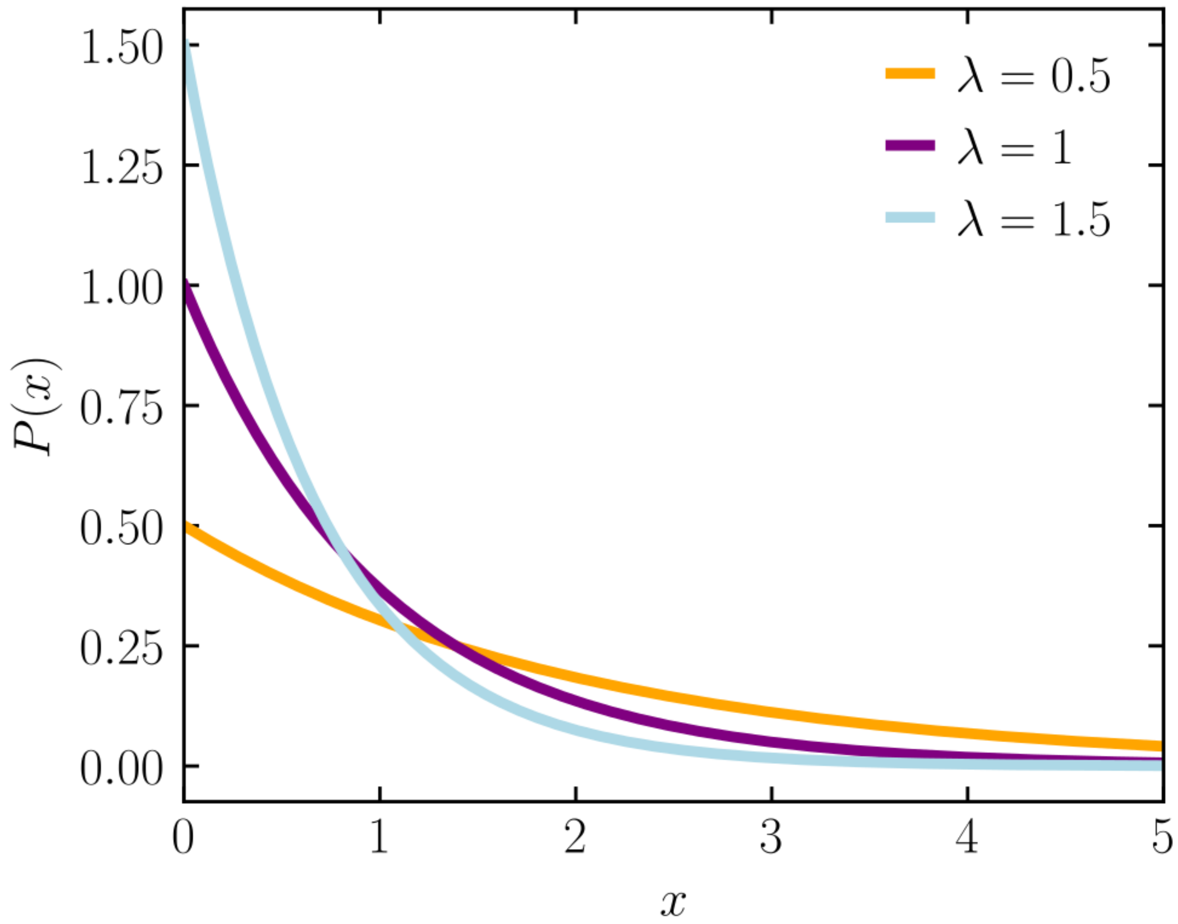

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- How to estimate λ?

MAXIMUM LIKELIHOOD ESTIMATE

FOR PARAMETERS OF CONTINUOUS DISTRIBUTIONS

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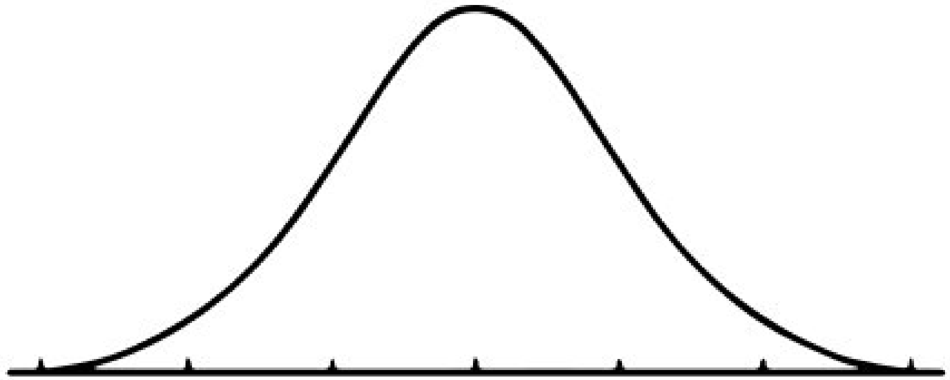
BREAK

NORMAL DISTRIBUTION



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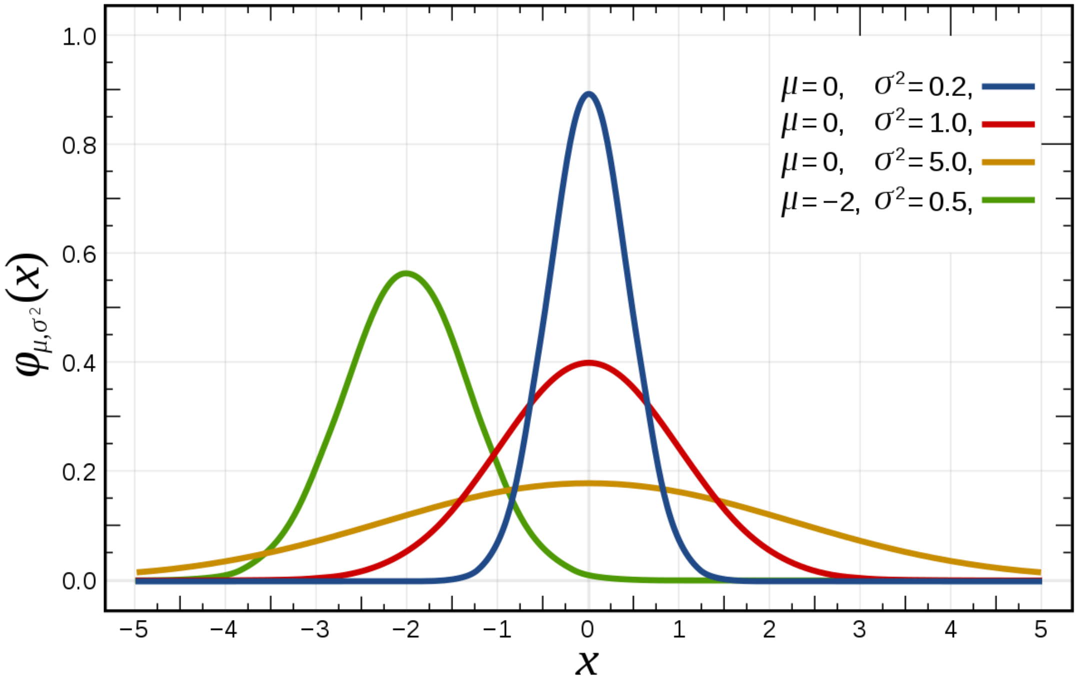

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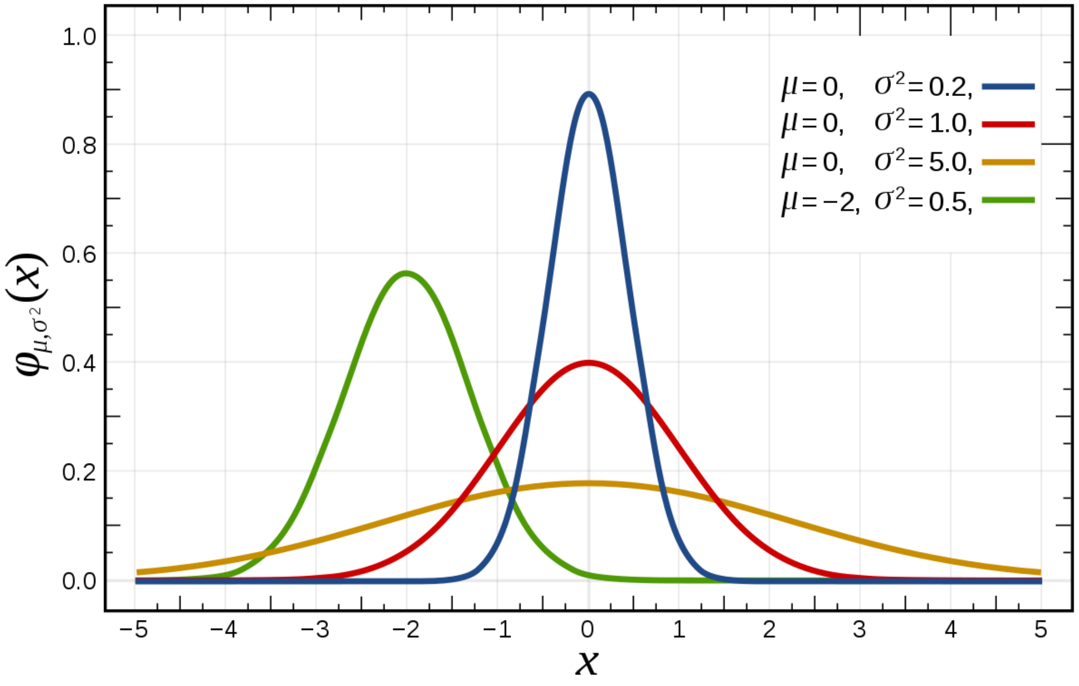

STANDARD NORMAL DISTRIBUTION

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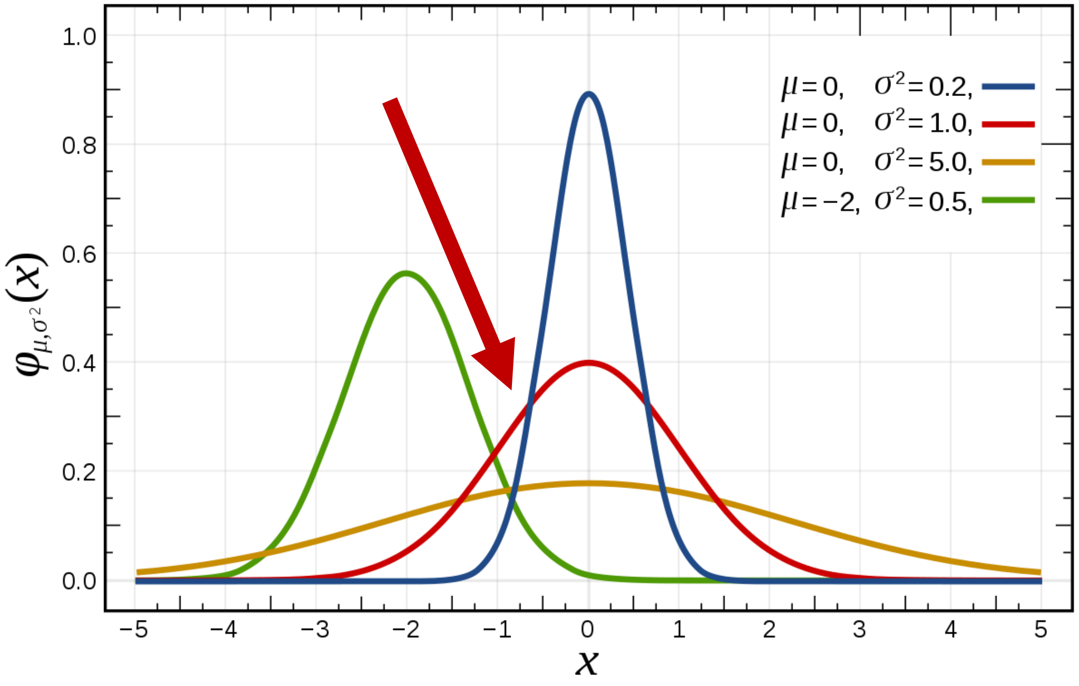

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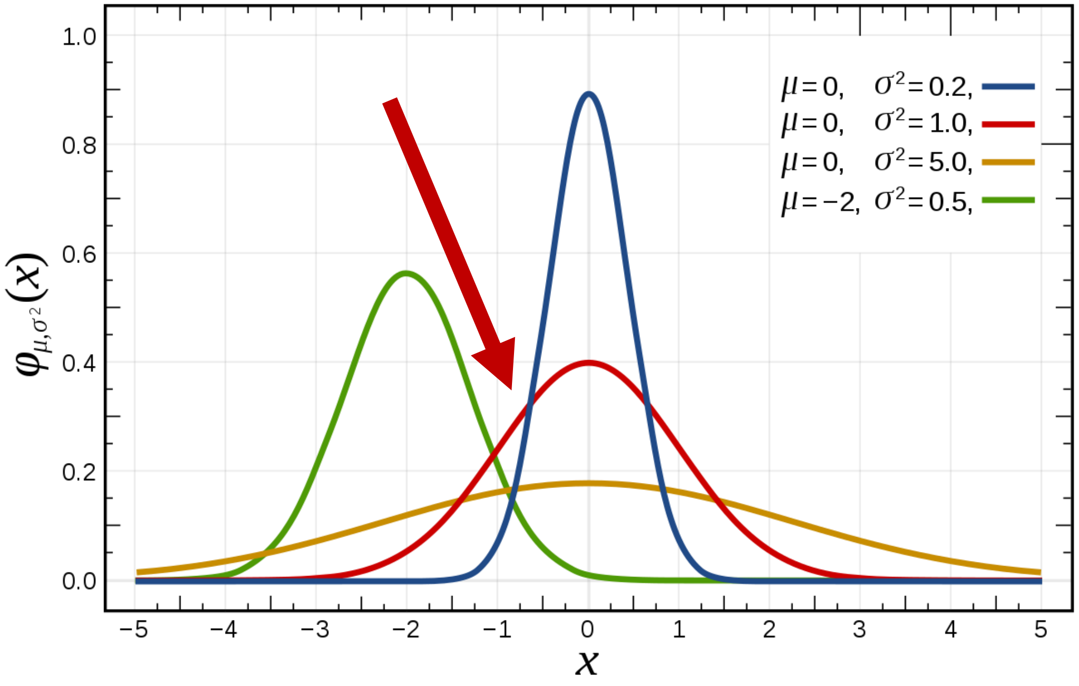

LET'S GET TO KNOW THE NORMAL DISTRIBUTION BETTER!

Google Classroom -> Lecture 5 -> Normal Distribution Basics

PARAMETER ESTIMATION FOR THE NORMAL DISTRIBUTION

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- What is the MLE $\hat{\mu} =$?

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$$L(\mu) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X_i-\mu}{\sigma}\right)^2}$$

$$\log L(\mu) =$$

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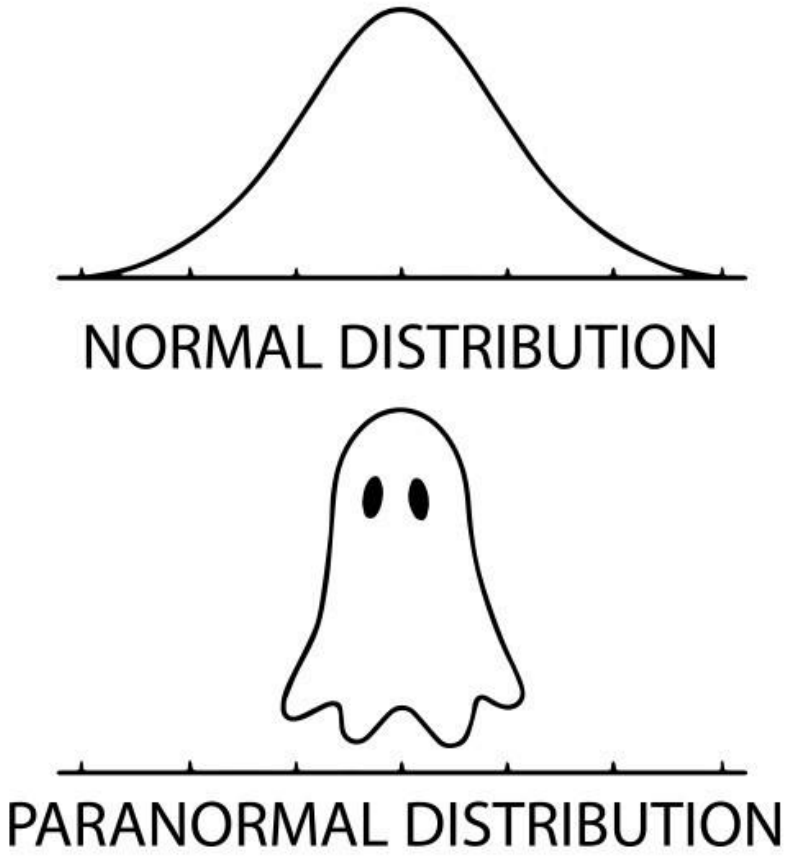
BREAK



LONG-TAIL DISTRIBUTIONS

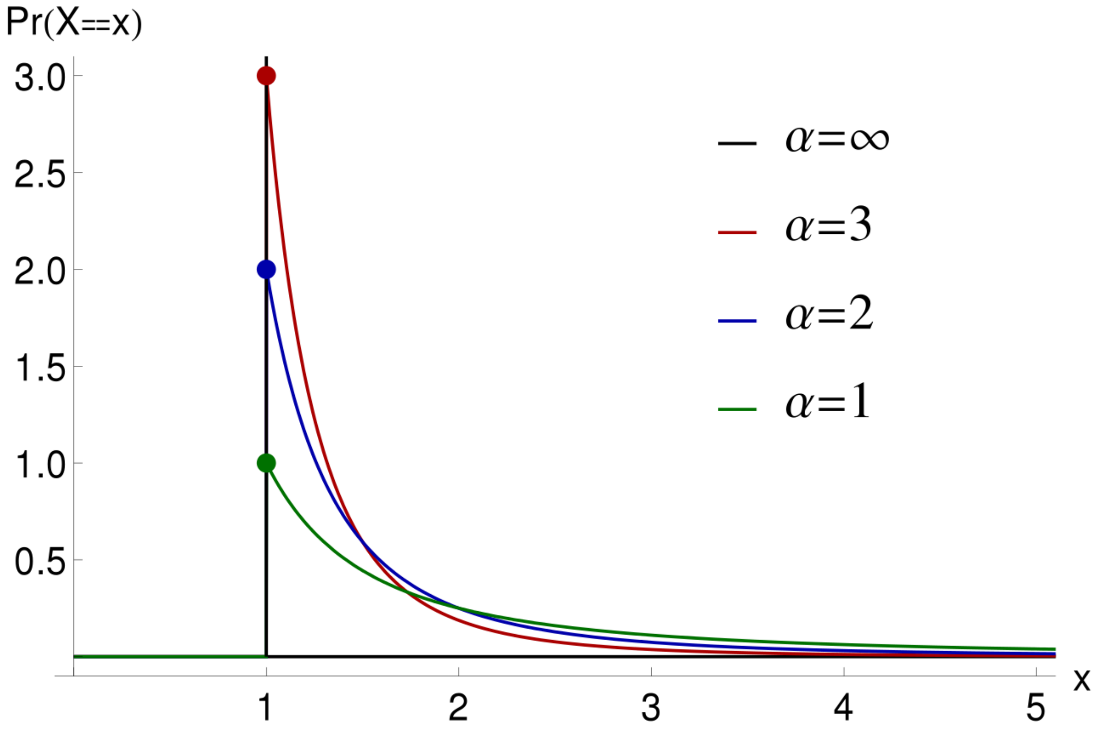
EXPECTED VALUE

$$E(X) = \int_{-\infty}^{+\infty} x \cdot p(x) dx$$

PARETO DISTRIBUTION

$$p\ \ x\ \ = \begin{cases} \dfrac{\alpha}{x^{\alpha+1}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$\alpha > 0$$



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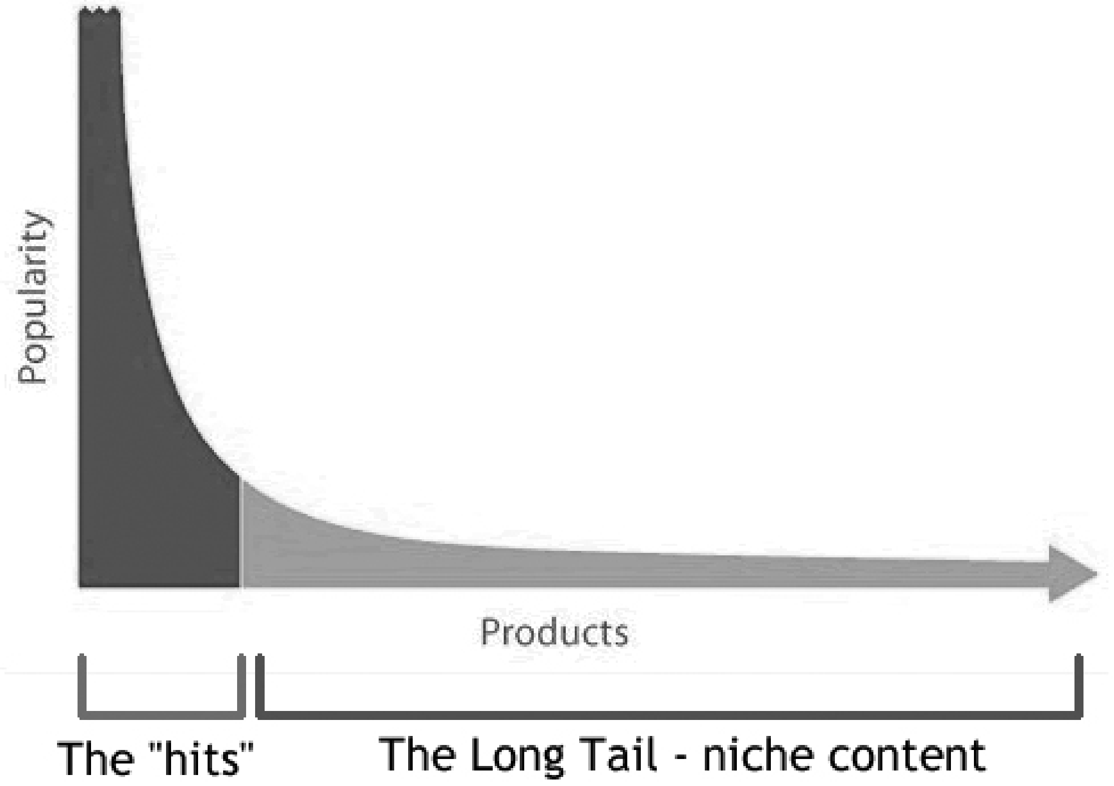

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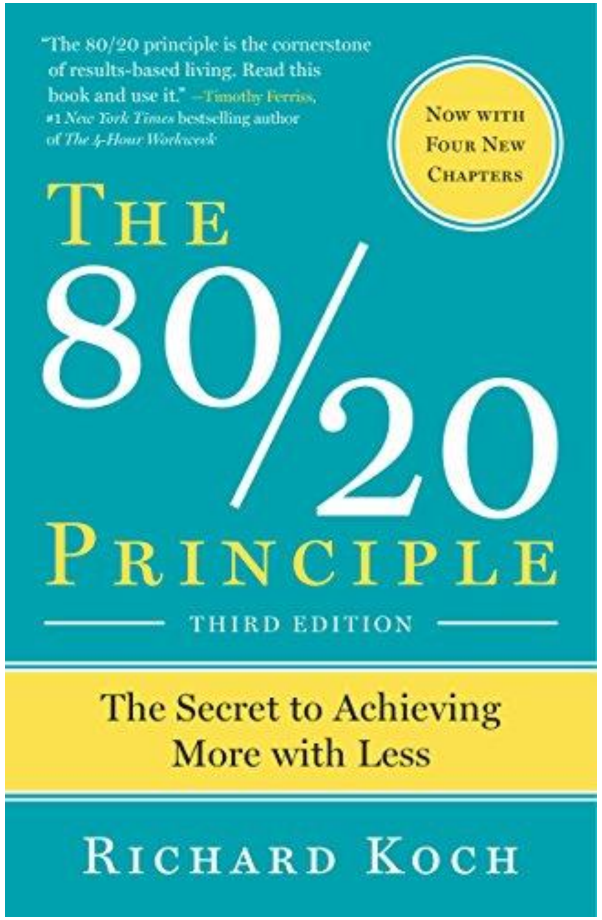
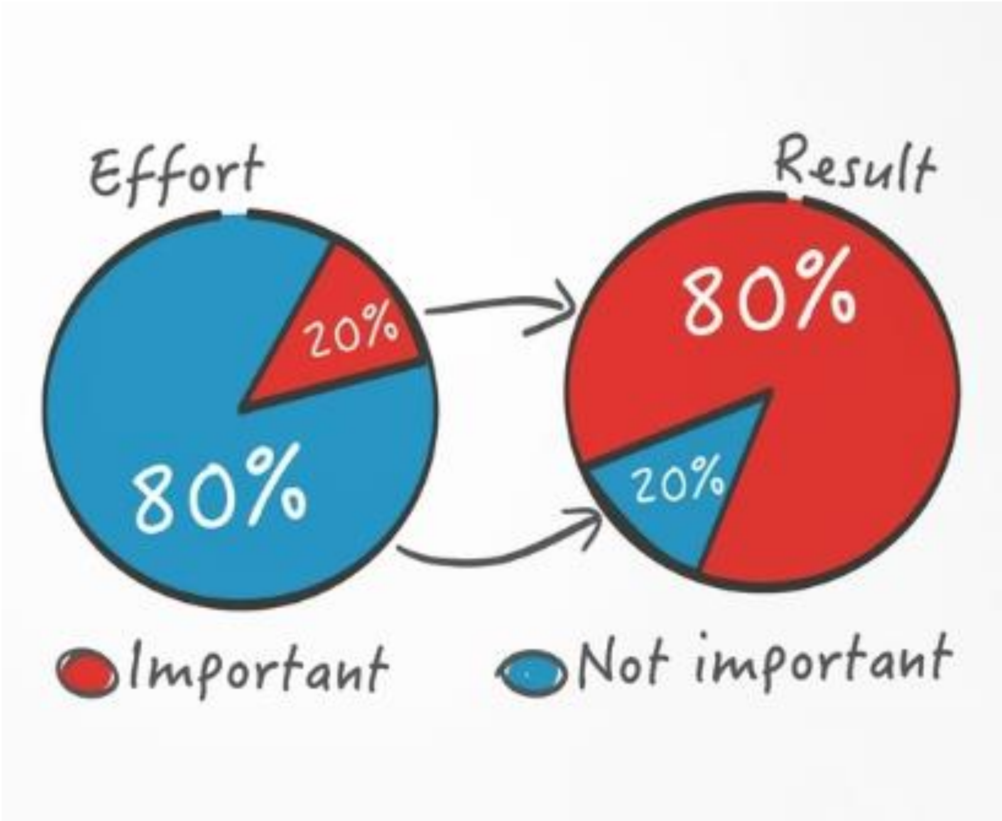
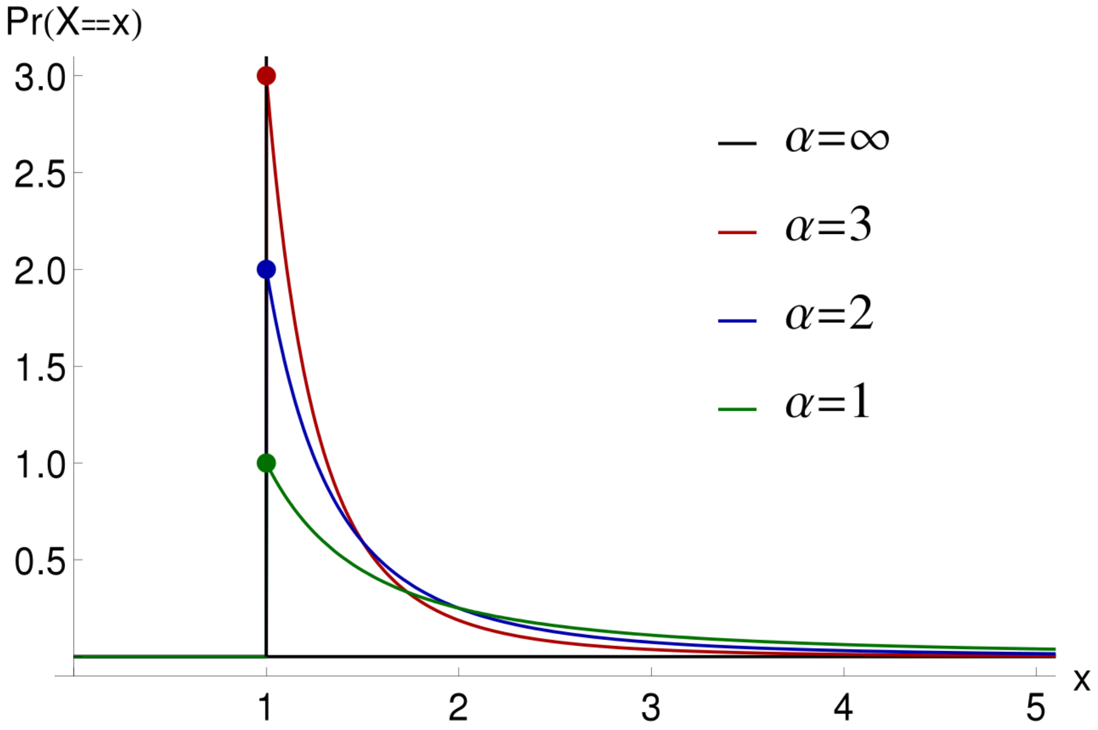




PARETO DISTRIBUTION

$$\boldsymbol{\alpha = 1}$$

$$p(x) = \begin{cases} \dfrac{1}{x^2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



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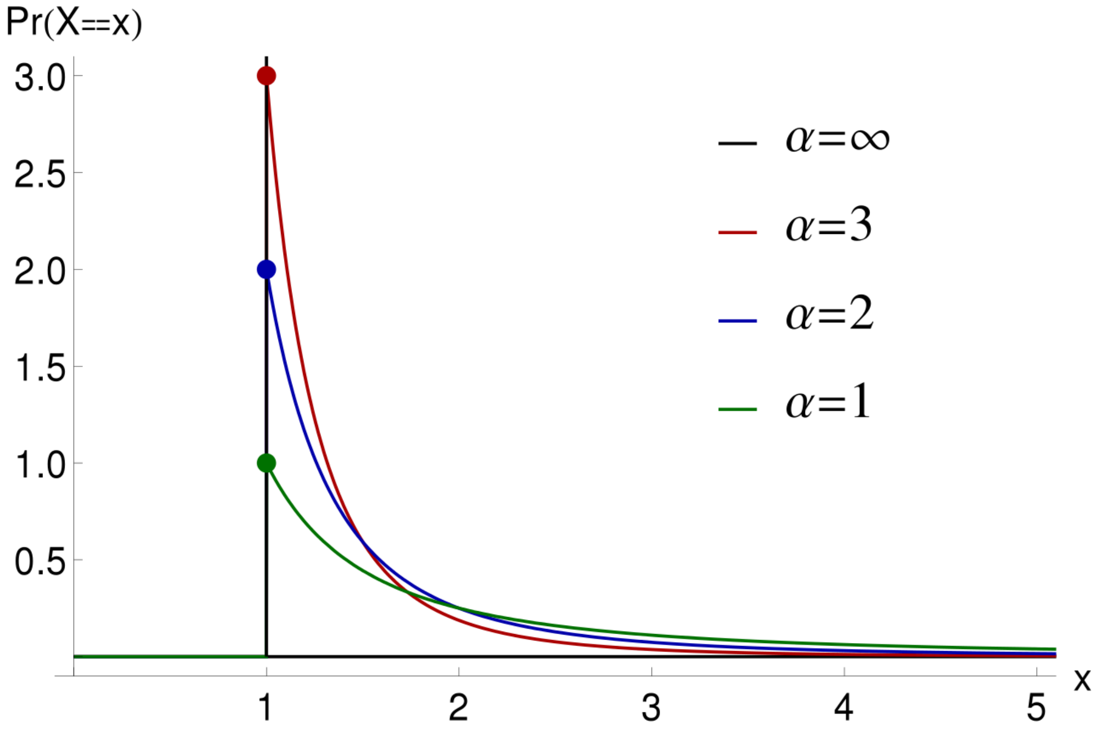

EXPECTED VALUE FOR PARETO

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EXPECTED VALUE FOR PARETO

$$\int_{0}^{+\infty} \frac{x}{x^2}\,dx = \int_{0}^{1} \frac{1}{x}\,dx + \int_{1}^{+\infty} \frac{1}{x}\,dx =$$

$$= \log(x)\Big|_{0}^{1} + \log(x)\Big|_{1}^{+\infty} =$$

EXPECTED VALUE FOR PARETO

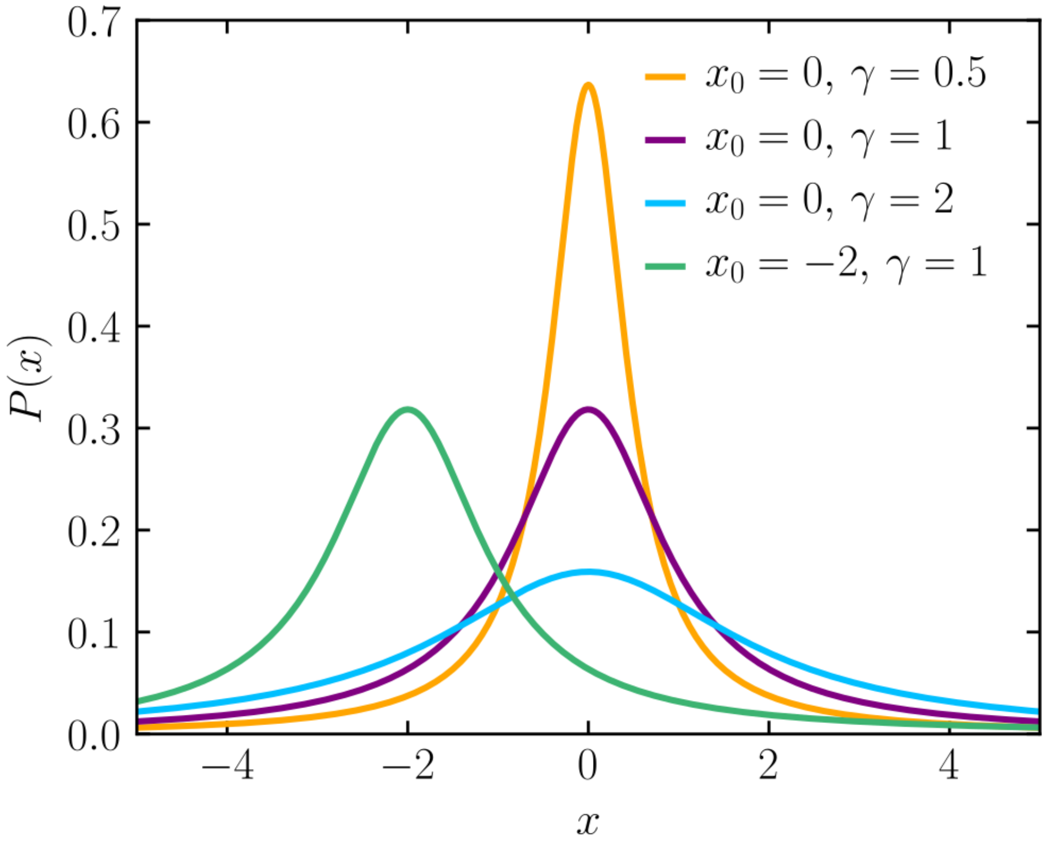
$$\int_0^{+\infty} \frac{x}{x^2}\,dx = \int_0^1 \frac{1}{x}\,dx + \int_1^{+\infty} \frac{1}{x}\,dx =$$

$$= \log(x)\Big|_0^1 + \log(x)\Big|_1^{+\infty} =$$

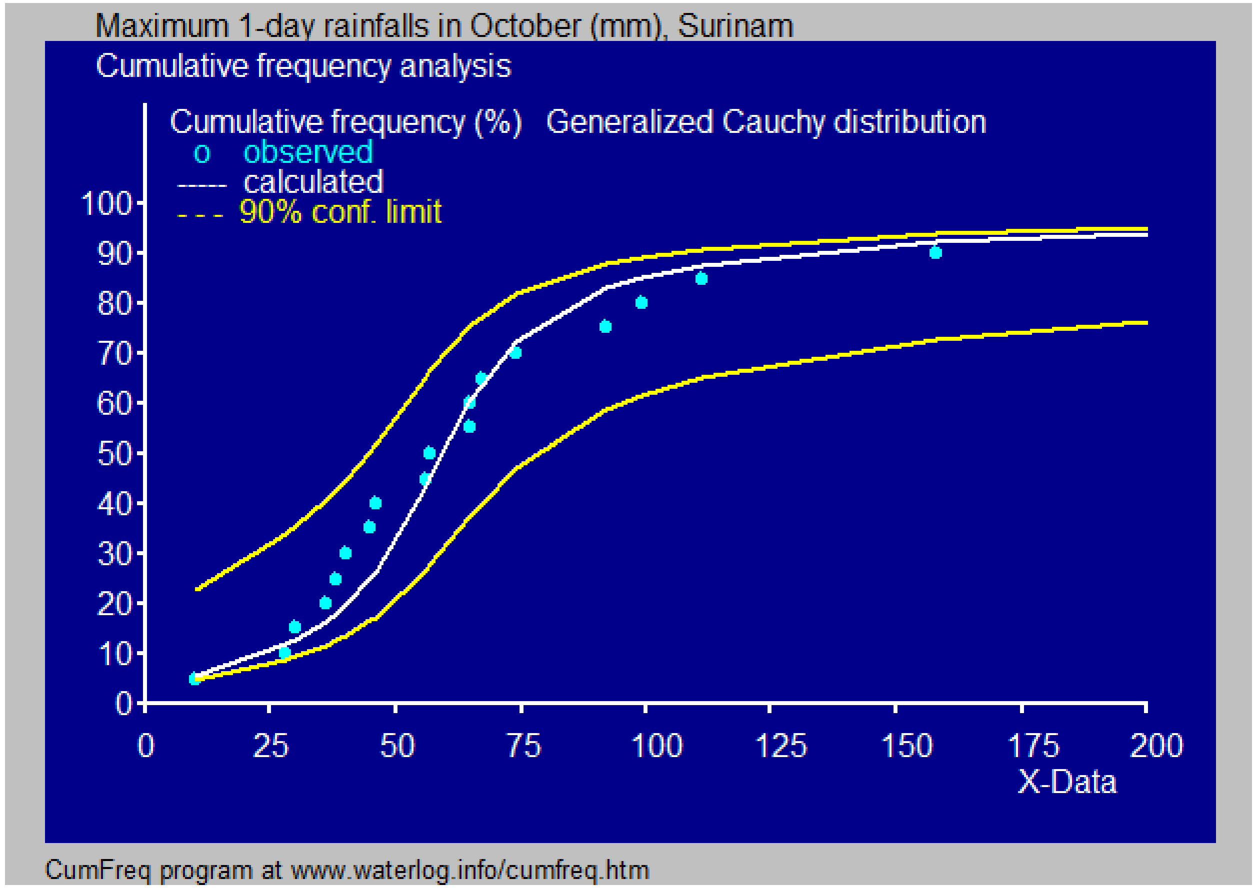
$$= \infty + \infty$$

CAUCHY DISTRIBUTION

$$p(x) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x - x_o}{\gamma}\right)^2\right]}$$



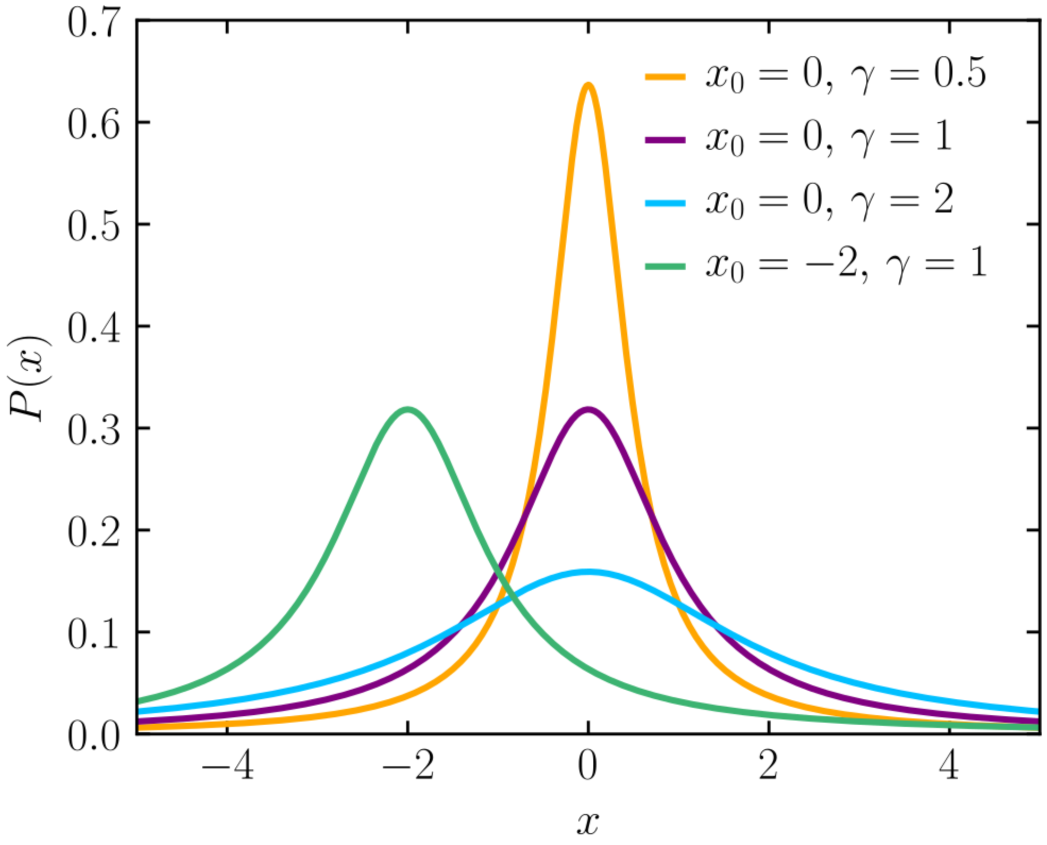
CAUCHY DISTRIBUTION



CAUCHY DISTRIBUTION

$$\boldsymbol{x_0 = 0, \qquad \gamma = 1}$$

$$p(x) = \frac{1}{\pi(1 + x^2)}$$

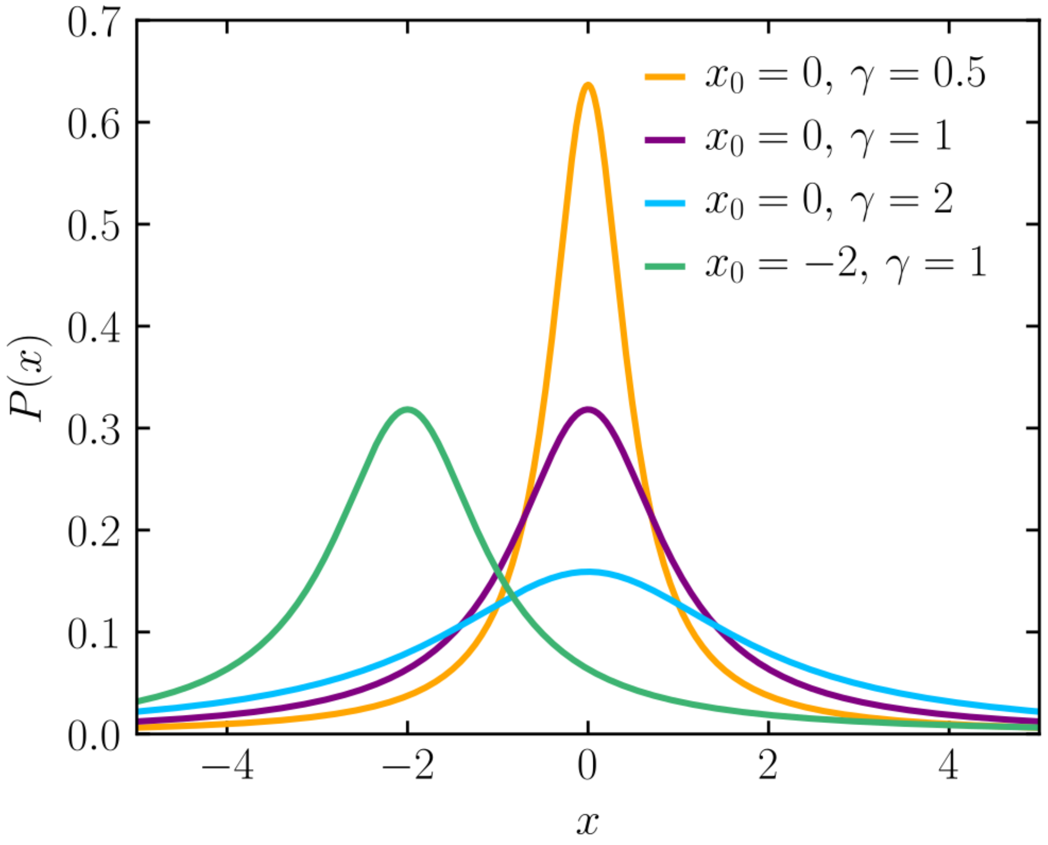


CAUCHY DISTRIBUTION

$$\boldsymbol{x_0 = 0}, \qquad \boldsymbol{\gamma = 1}$$

$$p(x) = \frac{1}{\pi(1 + x^2)}$$

$$E(X) = ?$$



EXPECTED VALUES FOR CAUCHY

$$\int_{-\infty}^{+\infty} \frac{x}{\pi(1+x^2)}\,dx =$$

EXPECTED VALUES FOR CAUCHY

$$\int_{-\infty}^{+\infty} \frac{x}{\pi(1+x^2)}\,dx = \int_{-\infty}^{0} \frac{x}{\pi(1+x^2)}\,dx + \int_{0}^{+\infty} \frac{x}{\pi(1+x^2)}\,dx =$$

EXPECTED VALUES FOR CAUCHY

$$\int_{-\infty}^{+\infty} \frac{x}{\pi(1+x^2)}\,dx = \int_{-\infty}^{0} \frac{x}{\pi(1+x^2)}\,dx + \int_{0}^{+\infty} \frac{x}{\pi(1+x^2)}\,dx =$$

$$= \int_{-\infty}^{0} \frac{dx^2}{2\pi(1+x^2)} + \int_{0}^{+\infty} \frac{dx^2}{2\pi(1+x^2)} =$$

EXPECTED VALUES FOR CAUCHY

$$\int_{-\infty}^{+\infty} \frac{x}{\pi(1+x^2)}dx = \int_{-\infty}^{0} \frac{x}{\pi(1+x^2)}dx + \int_{0}^{+\infty} \frac{x}{\pi(1+x^2)}dx =$$

$$= \int_{-\infty}^{0} \frac{dx^2}{2\pi(1+x^2)} + \int_{0}^{+\infty} \frac{dx^2}{2\pi(1+x^2)} =$$

$$= \frac{1}{2\pi}\log(1+x^2)\Big|_{-\infty}^{0} + \frac{1}{2\pi}\log(1+x^2)\Big|_{0}^{+\infty} =$$

EXPECTED VALUES FOR CAUCHY

$$\int_{-\infty}^{+\infty} \frac{x}{\pi(1+x^2)} dx = \int_{-\infty}^{0} \frac{x}{\pi(1+x^2)} dx + \int_{0}^{+\infty} \frac{x}{\pi(1+x^2)} dx =$$

$$= \int_{-\infty}^{0} \frac{dx^2}{2\pi(1+x^2)} + \int_{0}^{+\infty} \frac{dx^2}{2\pi(1+x^2)} =$$

$$= \frac{1}{2\pi} \log(1+x^2)\Big|_{-\infty}^{0} + \frac{1}{2\pi} \log(1+x^2)\Big|_{0}^{+\infty} = \infty - \infty$$

BUT WHAT DOES IT MEAN?

Google Classroom -> Lecture 5 -> Long-tailed distributions

DETECTING HEAVY TAILS: LOG-LOG SCALE

POWER LAW

$$y = x^a$$

$$\log y = a \log x$$

LINEAR in a log-log plot

EXPONENTIAL

$$y = e^{ax}$$

$$\log y = ax = e^{\log ax}$$

EXPONENTIAL in a log-log plot

TO SUM UP EVERYTHING…

https://youtu.be/iA0xRUNvLV8

SUMMARY OF WEEK 1

- Descriptive statistics
  - Summary statistics, tables and plots

- Continuous random variables:
  - CDFs and PDFs
  - Standard distributions
    (uniform, exponential, normal)
  - Long-tailed distributions
    (Pareto, Cauchy)

- Parameter estimation:
  - Maximum Likelihood;
  - discrete and continuous variants.

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- You can still submit Assignment 1 (-10% of the score each late day)
- **Graded assignment 2 due Monday evening.**
- New assignment on Monday.