INTRODUCTION TO STATISTICS

LECTURE 8

TODAY

- Confidence intervals based on normal data
  - for μ when σ is known;
  - for μ when σ is unknown;
  - for σ.

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  - for μ when σ is known;
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- Large sample CI

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- Confidence intervals based on normal data
  - for μ when σ is known;
  - for μ when σ is unknown;
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- Large sample CI

- Bernoulli data and polling

A QUICK REMINDER

What we saw last time

CI: DEFINITION

A $1-\alpha$ confidence interval for a parameter θ is an interval $C_n = (T_1, T_2)$ such that $\mathrm{T}_1 = t_1(X_1, \dots, X_n)$, $T_2 = t_2(X_1, \dots, X_n)$ and

$$P(T_1 < \theta < T_2) \geq 1-\alpha$$

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- **Random** intervals: T_1 and T_2 are functions of random samples.

- θ is unknown, but fixed
  T_1 and T_2 are random

CI: INTERPRETATION

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- **Not** a probability statement about θ since it's fixed.
- Common interpretation:

CI: INTERPRETATION

$$C_n = (T_1, T_2): \;\; P(T_1 < \theta < T_2) \geq 1 - \alpha$$

- **Not** a probability statement about θ since it's fixed.
- Common interpretation:

If I repeat the experiment many times, the interval will contain the true value of θ 95% of the time (α=0.05).

CI FOR NORMAL DATA

CI for μ, *known* σ

CI FOR μ, KNOWN σ

$X_1, X_2, \ldots, X_n$ – samples from $N(\mu, \sigma^2)$, σ is known.
How to construct a CI for μ?

CI FOR μ, KNOWN σ

$X_1, X_2, \dots, X_n$ – samples from $N(\mu, \sigma^2)$, σ is known.
How to construct a CI for μ? Consider $\bar{X}$:

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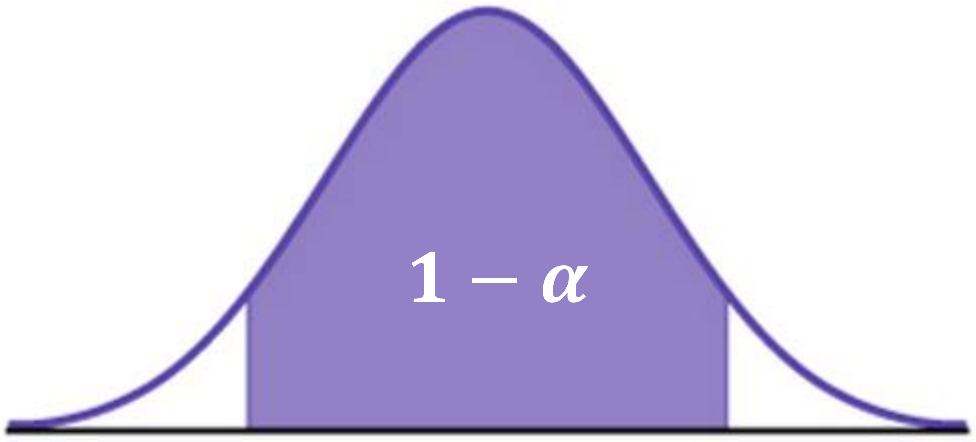
CI FOR μ, KNOWN σ



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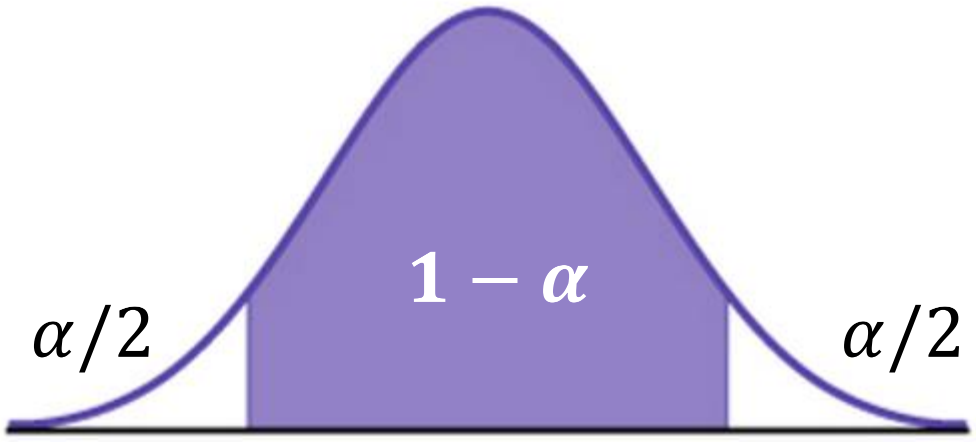

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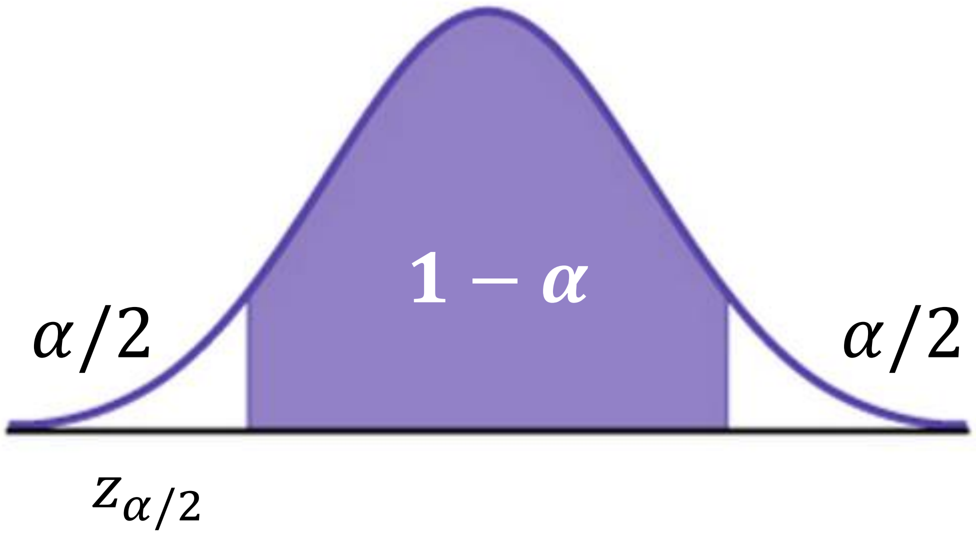

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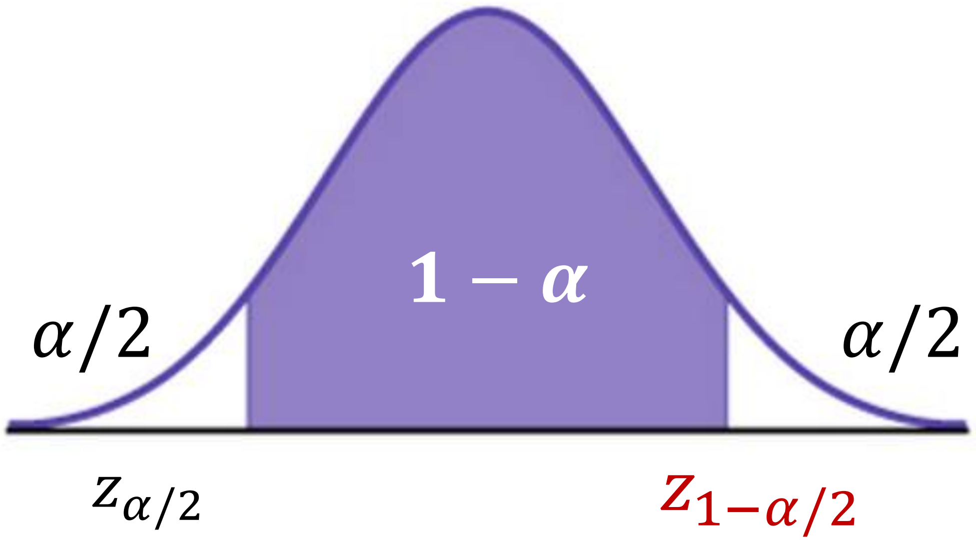

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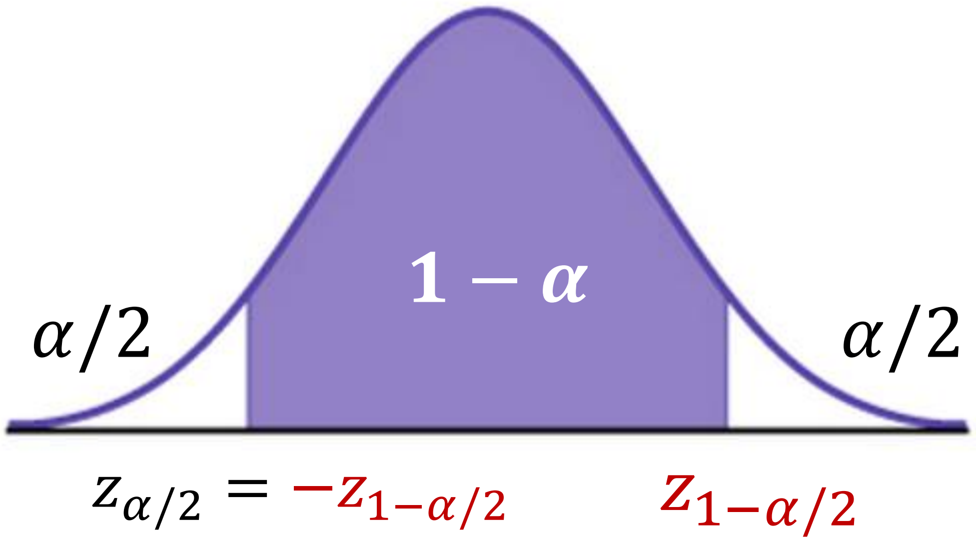

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CI FOR μ, KNOWN σ: EXAMPLE 1

- $X_1, X_2, \dots, X_{100}$ – samples from $N(\mu, 3^2)$, $\bar{X} = 12.$
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Quantile (p)	Φ^{-1}(p, 0, 1)
0.995	2.58
0.99	2.33
0.975	1.96
0.95	1.64
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PRACTICE!

Google Classroom -> Lecture 8 -> z-intervals

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STUDENT DISTRIBUTION

William Sealy Gosset

STUDENT DISTRIBUTION

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STUDENT DISTRIBUTION

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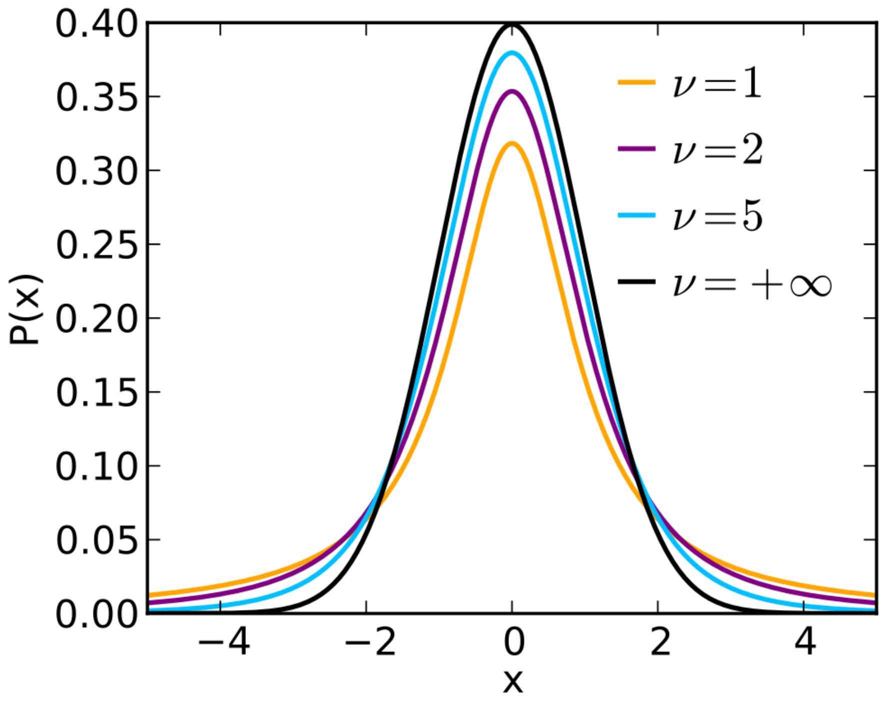

William Sealy Gosset

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Now:

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CI FOR μ, UNKNOWN σ: EXAMPLE

- $X_1, X_2, \dots, X_{20}$ − samples from $N(\mu, \sigma^2)$, σ is unknown.
- $\bar{X} = 42, \quad s^2 = 36$

- Give the 95%-CI for μ.

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$$\mu = 42 \pm \frac{6}{\sqrt{20}} \cdot 2.093$$

PRACTICE!

Google Classroom -> Lecture 8 -> t-intervals

CI FOR NORMAL DATA

CI for σ, unknown μ, σ

CI FOR σ, UNKNOWN μ, σ

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$$\frac{(n-1)s^2}{\sigma^2}$$

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$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1) -$$

Chi-squared distribution

CHI-SQUARED DISTRIBUTION

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$$\Rightarrow$$

$$Q = Z_1^2 + \cdots + Z_n^2 \sim \chi^2(n-1)$$

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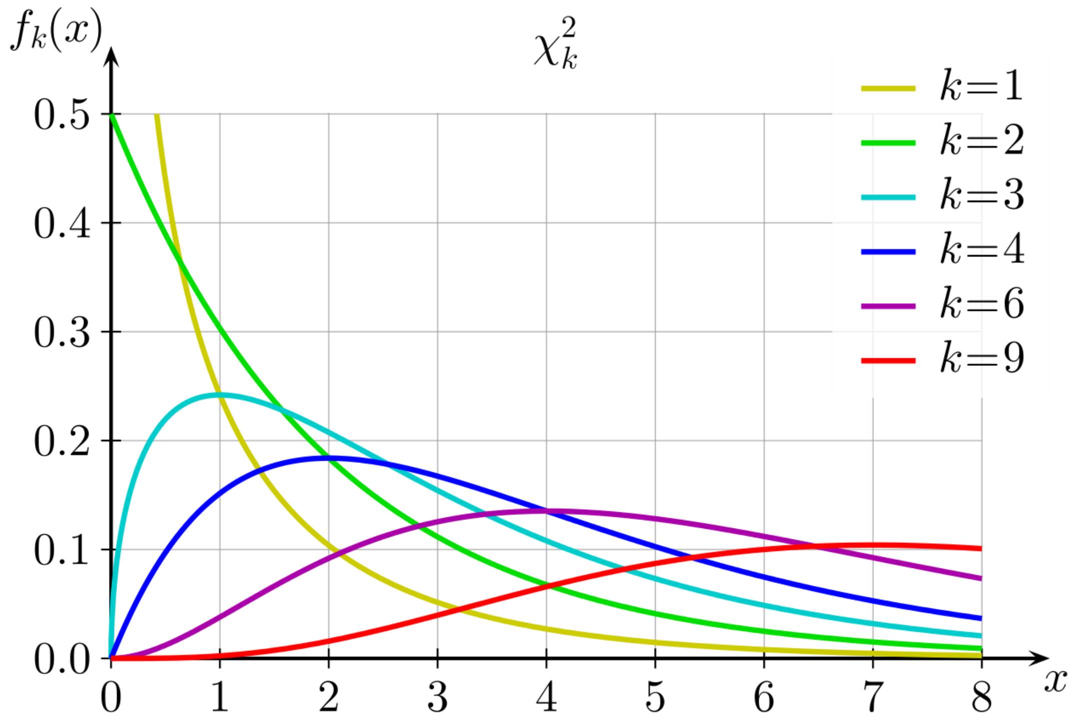

CI FOR $\boldsymbol{\sigma}$, UNKNOWN $\boldsymbol{\mu}, \boldsymbol{\sigma}$

- $X_1, X_2, \dots, X_{100}$ – samples from $N(\mu, \sigma^2)$, σ is unknown.
- How to construct a CI for σ? Consider S^2:

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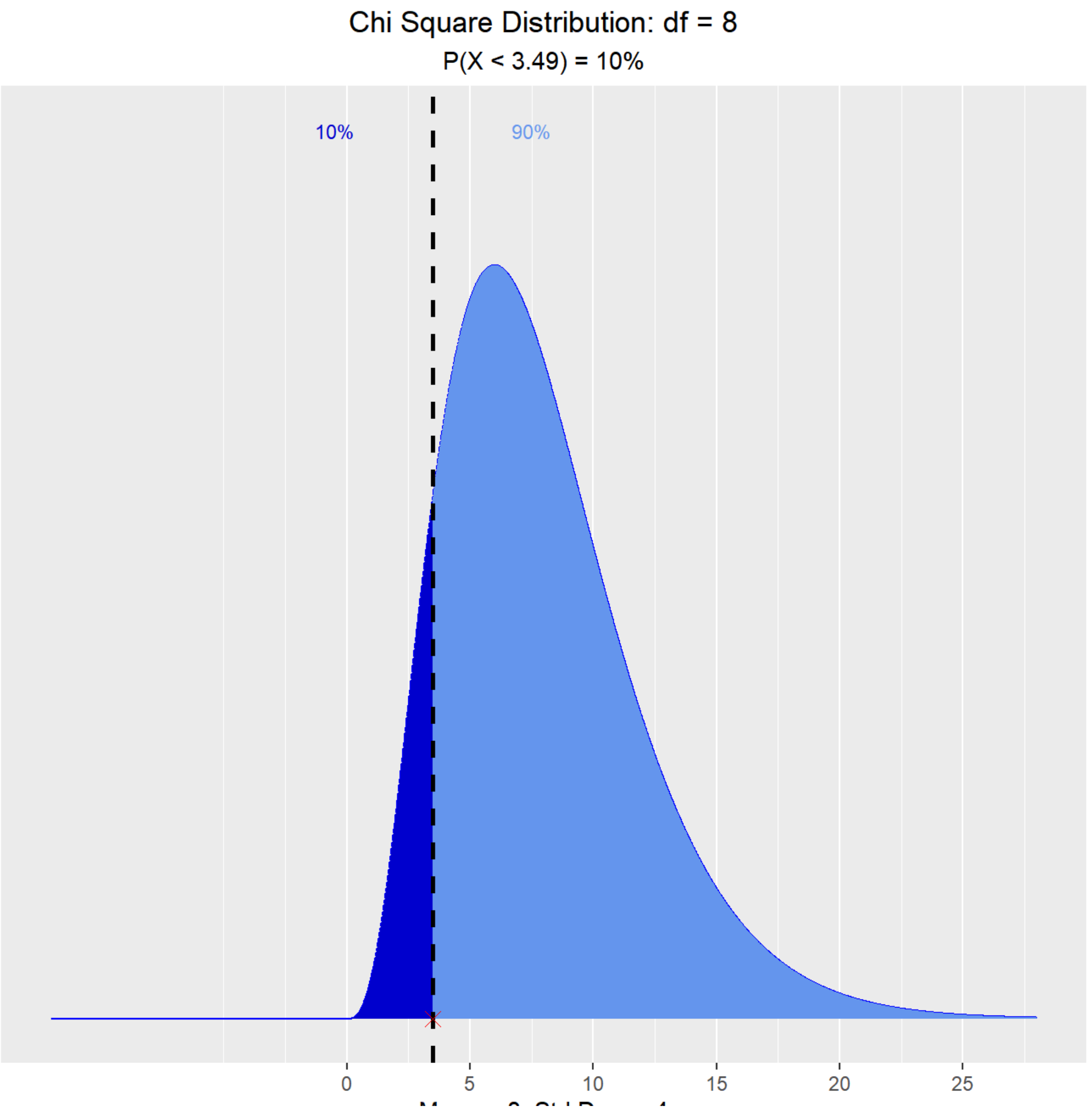

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CI FOR σ, UNKNOWN μ, σ: EXAMPLE

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$$\left[\frac{19 \cdot 36}{32.85}; \frac{19 \cdot 36}{8.91}\right]$$

LARGE SAMPLES

Using Central Limit Theorem

CI FOR LARGE SAMPLES

- Typical task: estimating the mean of a distribution.
- Suppose $X_1, \dots X_n$ is drawn from an unknown distribution.
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- CLT:

If $\mu, \sigma^2 < \infty$ and if n is sufficiently large, then:

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PRACTICE!

Google Classroom -> Lecture 8 -> Large sample CI