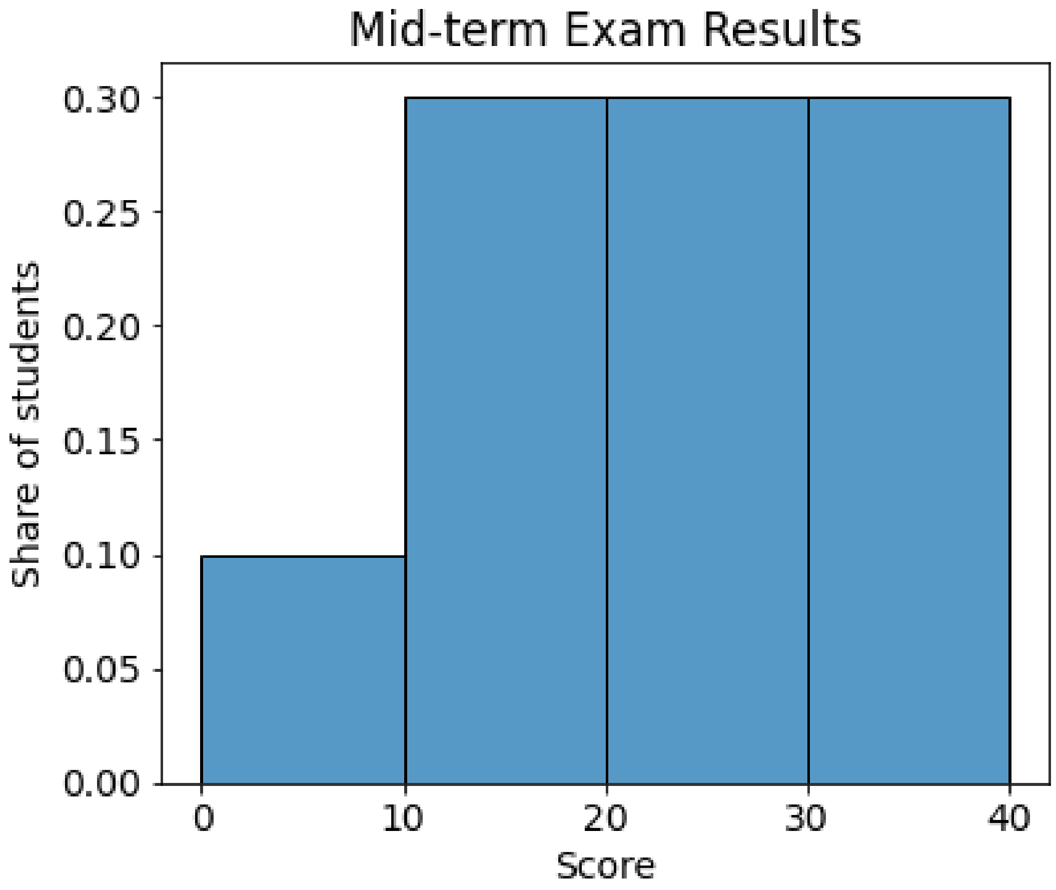
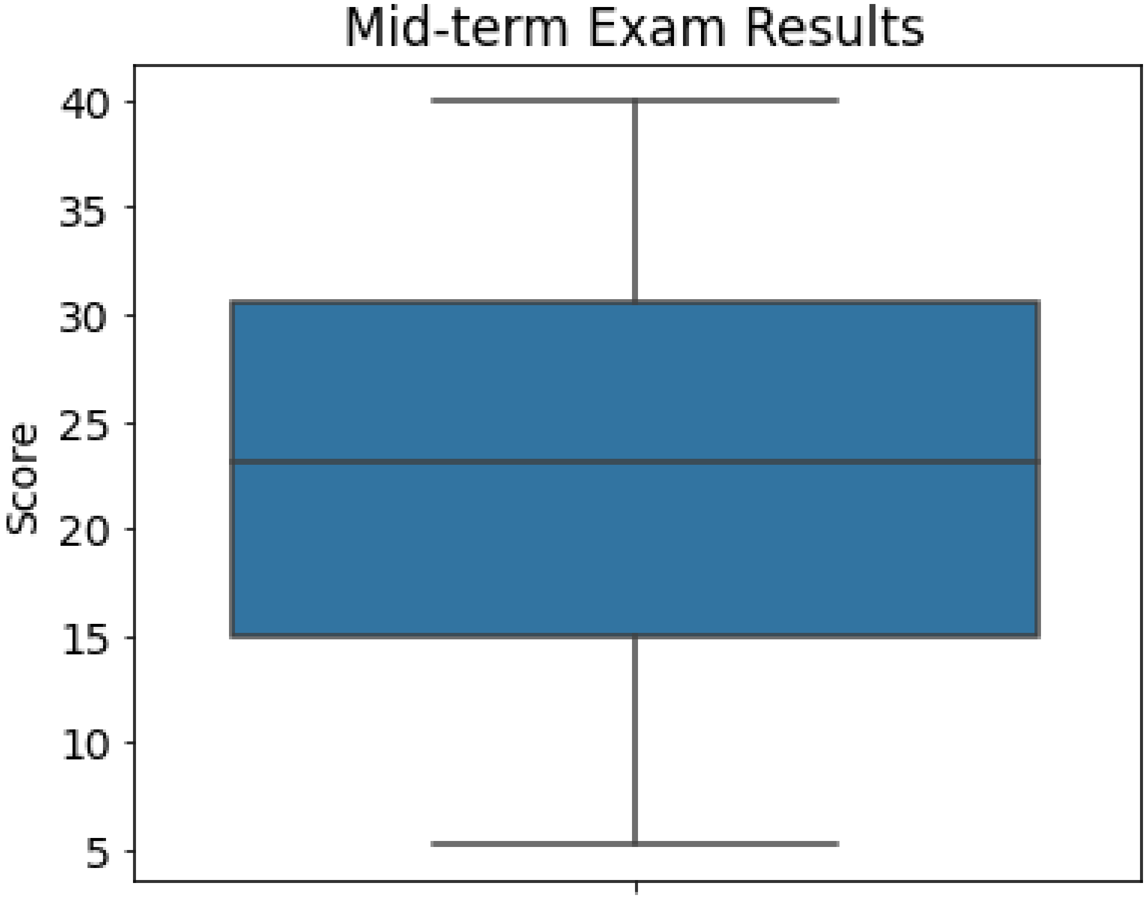
INTRODUCTION TO STATISTICS

LECTURE 10

MID-TERM RESULTS: AN OVERVIEW



MID-TERM RESULTS: AN OVERVIEW



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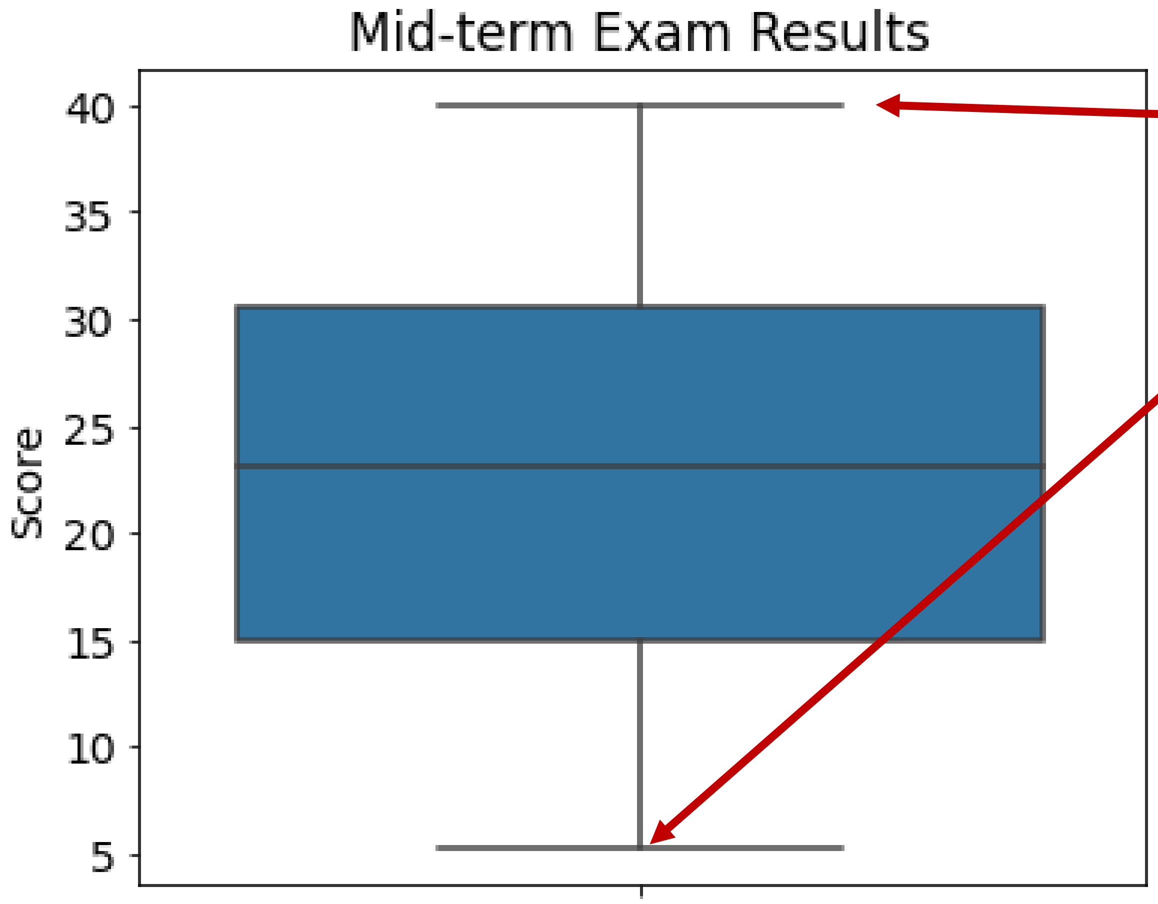
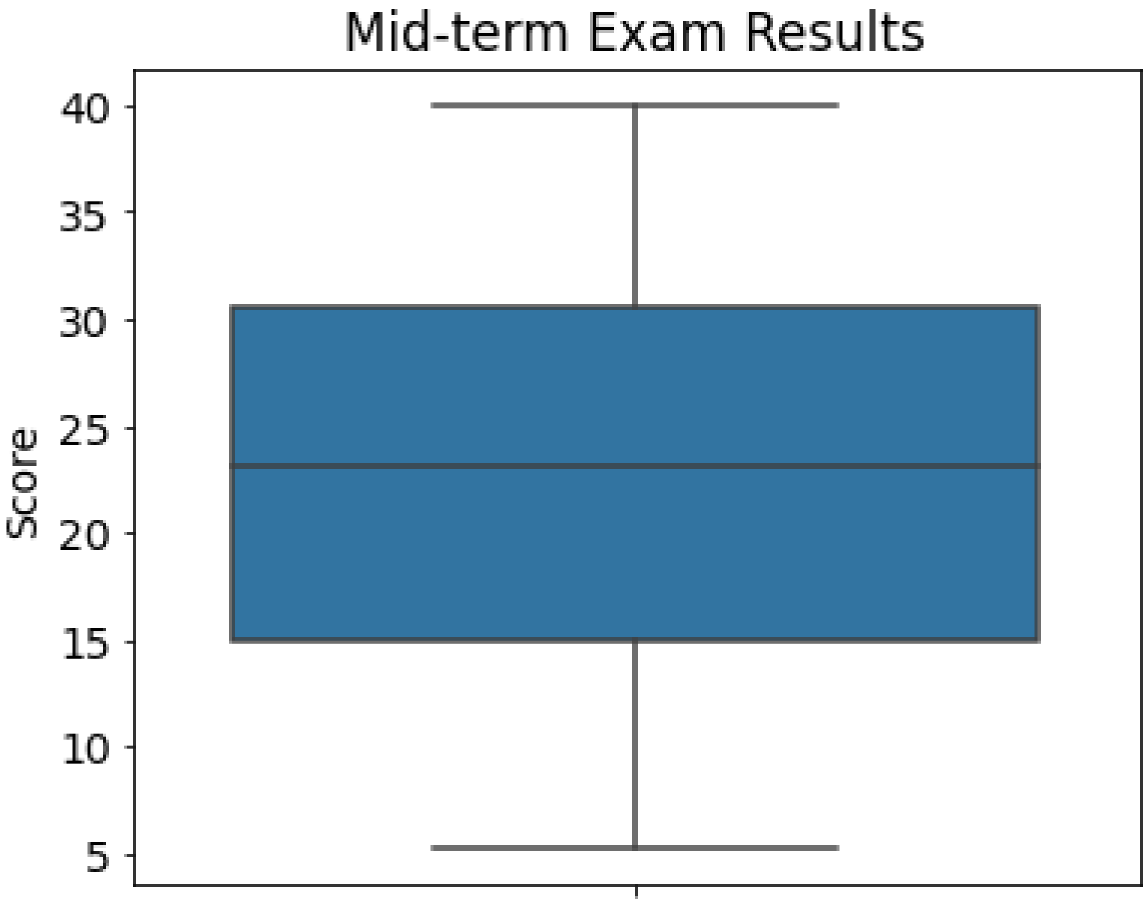


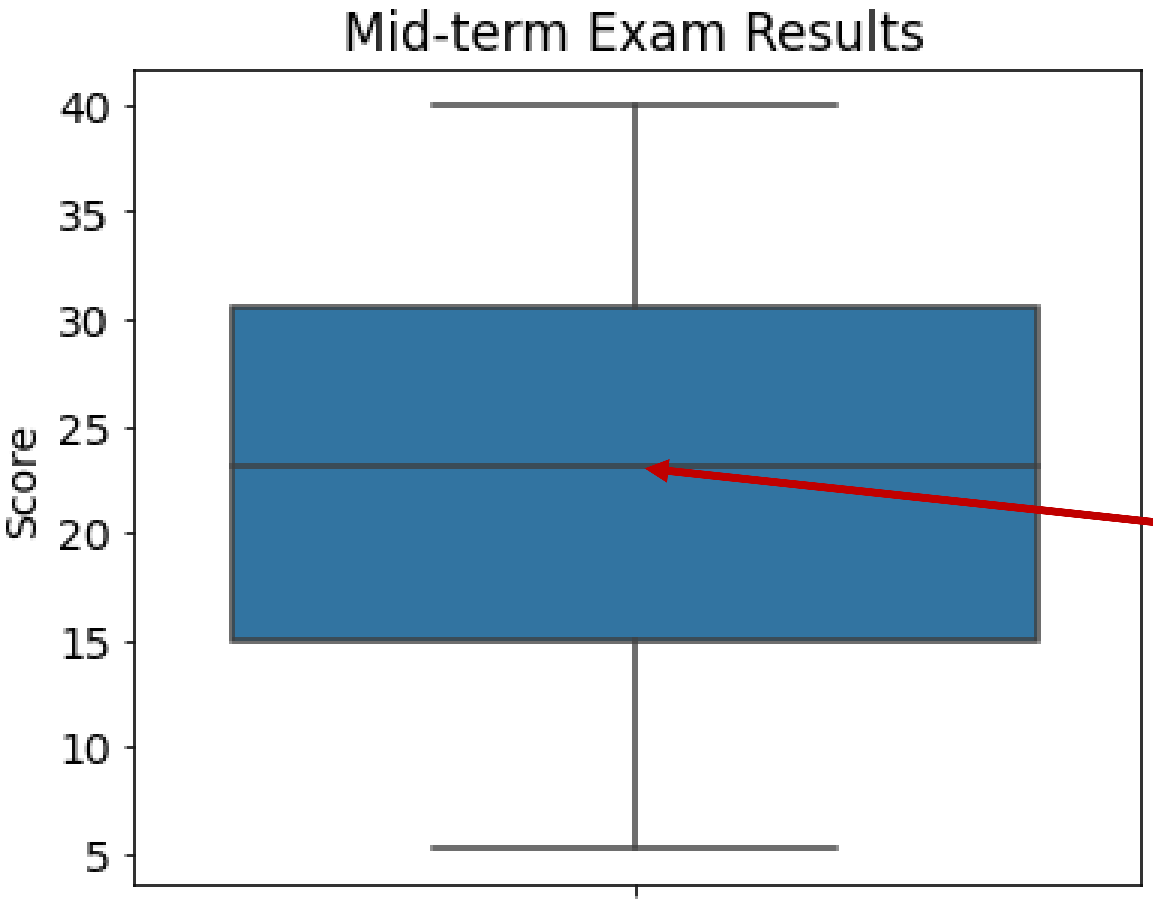
- MIN and MAX scores:
  - 5.25 and 40

MID-TERM RESULTS: AN OVERVIEW



- MIN and MAX scores:
  - 5.25 and 40
- Mean score:
  - 23 points

MID-TERM RESULTS: AN OVERVIEW



- MIN and MAX scores:
  - 5.25 and 40
- Mean score:
  - 23 points
- Median score:
  - 23.1 points

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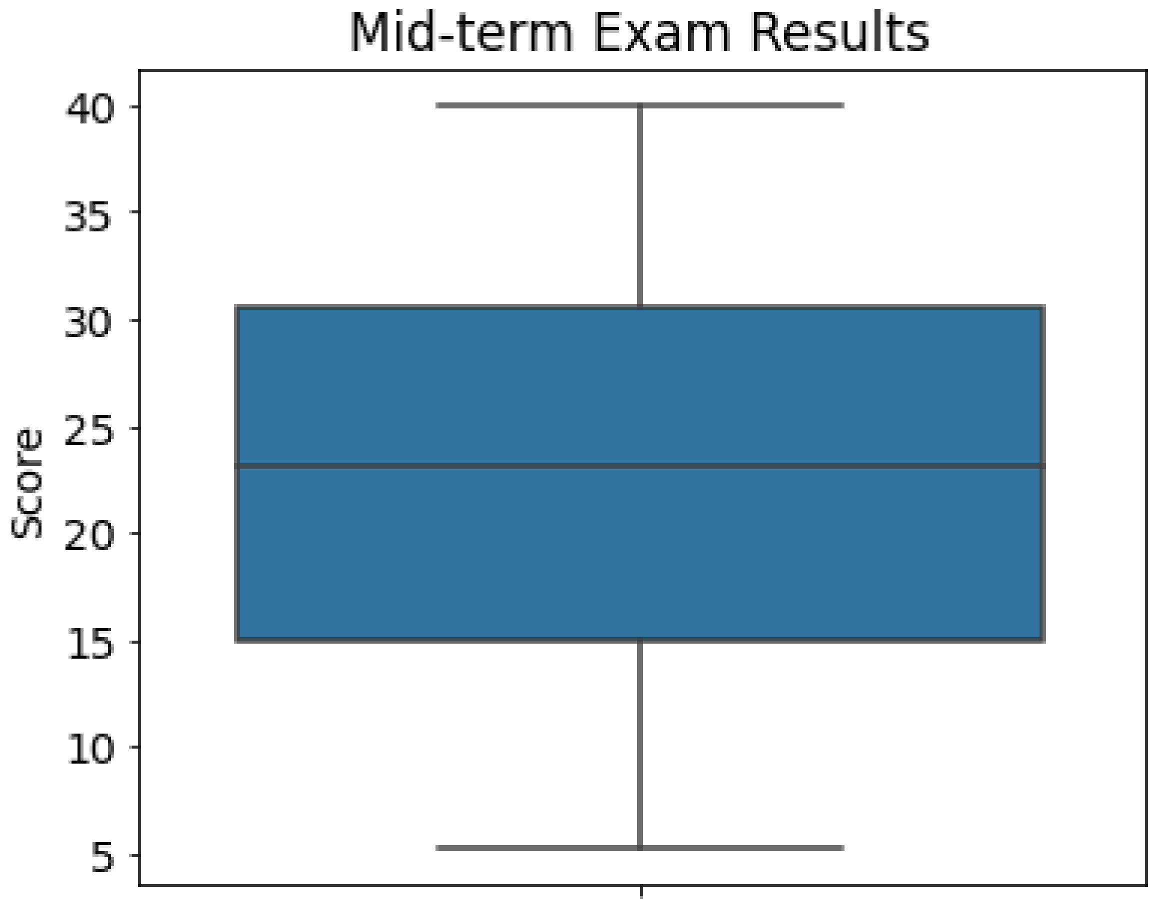


- MIN and MAX scores:
  - 5.25 and 40
- Mean score:
  - 23 points
- Median score:
  - 23.1 points
- Standard deviation:
  - 10.5 points

PLANS FOR THE WEEK

- MONDAY
  - CI recap
  - Hypothesis testing I
- TUESDAY
  - Hypothesis testing II
- WEDNESDAY
  - Covariance and correlation
  - Linear regression
- THURSDAY
  - Big recap
- FRIDAY
  - Final exam

PLANS FOR THE WEEK

- MONDAY
  - CI recap
  - Hypothesis testing I

- Assignment 4, part I is due 23:59
- Assignment 4, part II is out

- TUESDAY
  - Hypothesis testing II

- WEDNESDAY
  - Covariance and correlation
  - Linear regression

- Assignment 4, part II is due 23:59
- Assignment 5 is out

- THURSDAY
  - Big recap

- FRIDAY
  - Final exam

- Assignment 5 is due 23:59

LAST TIME

- Confidence intervals:
  - z-interval;
  - t-interval;
  - χ^2-interval;
  - some examples.

- Hypothesis testing
  - *The Lady Tasting Tea experiment;*
  - *an introduction to hypothesis testing.*

LAST TIME

- Confidence intervals:
  - z-interval;
  - t-interval;
  - χ^2-interval;
  - some examples.

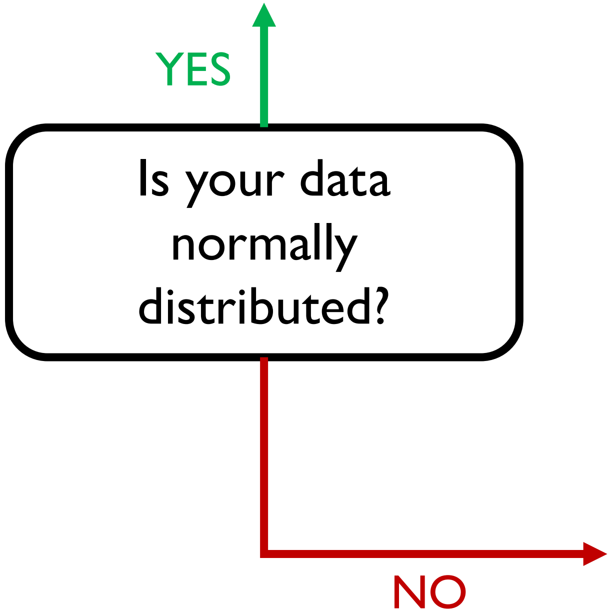
CONFIDENCE INTERVALS

A BRIEF RECAP

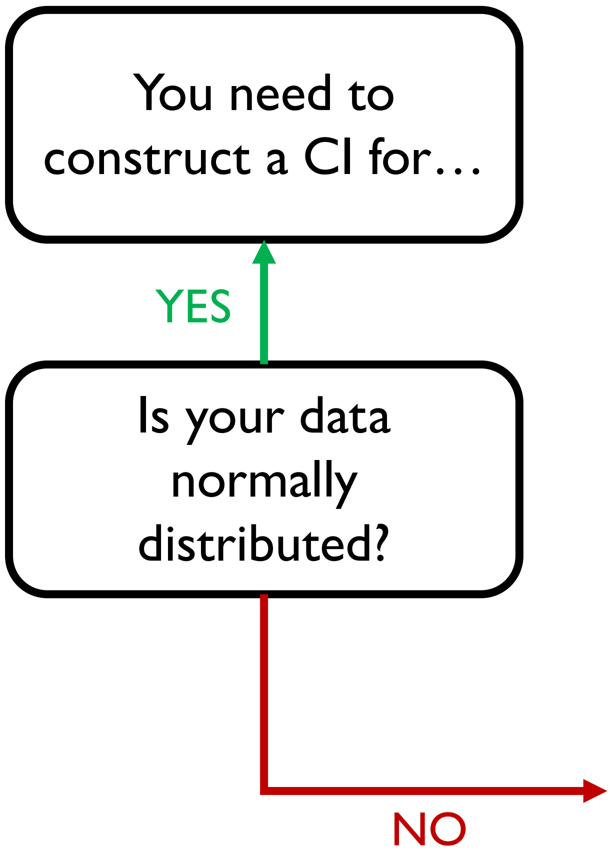
CI: A BRIEF RECAP

Is your data
normally
distributed?

CI: A BRIEF RECAP



CI: A BRIEF RECAP



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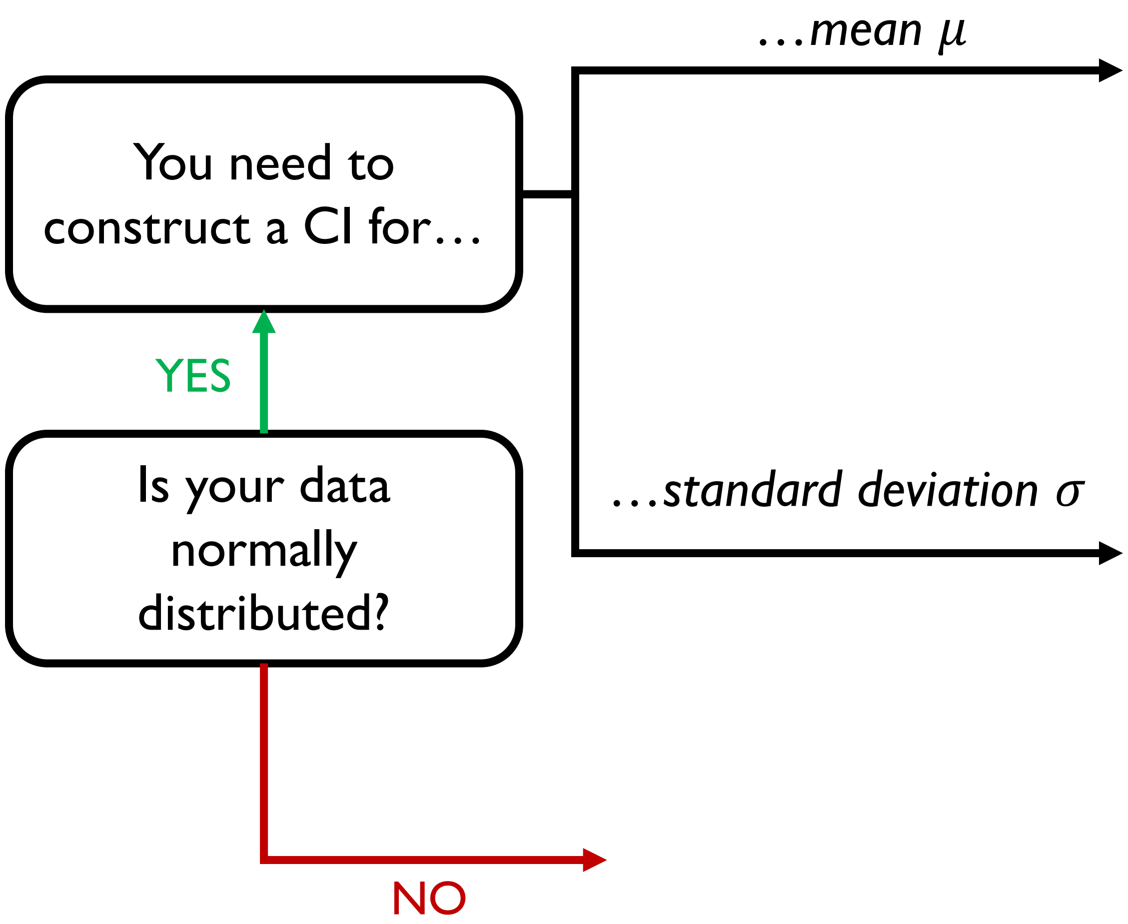

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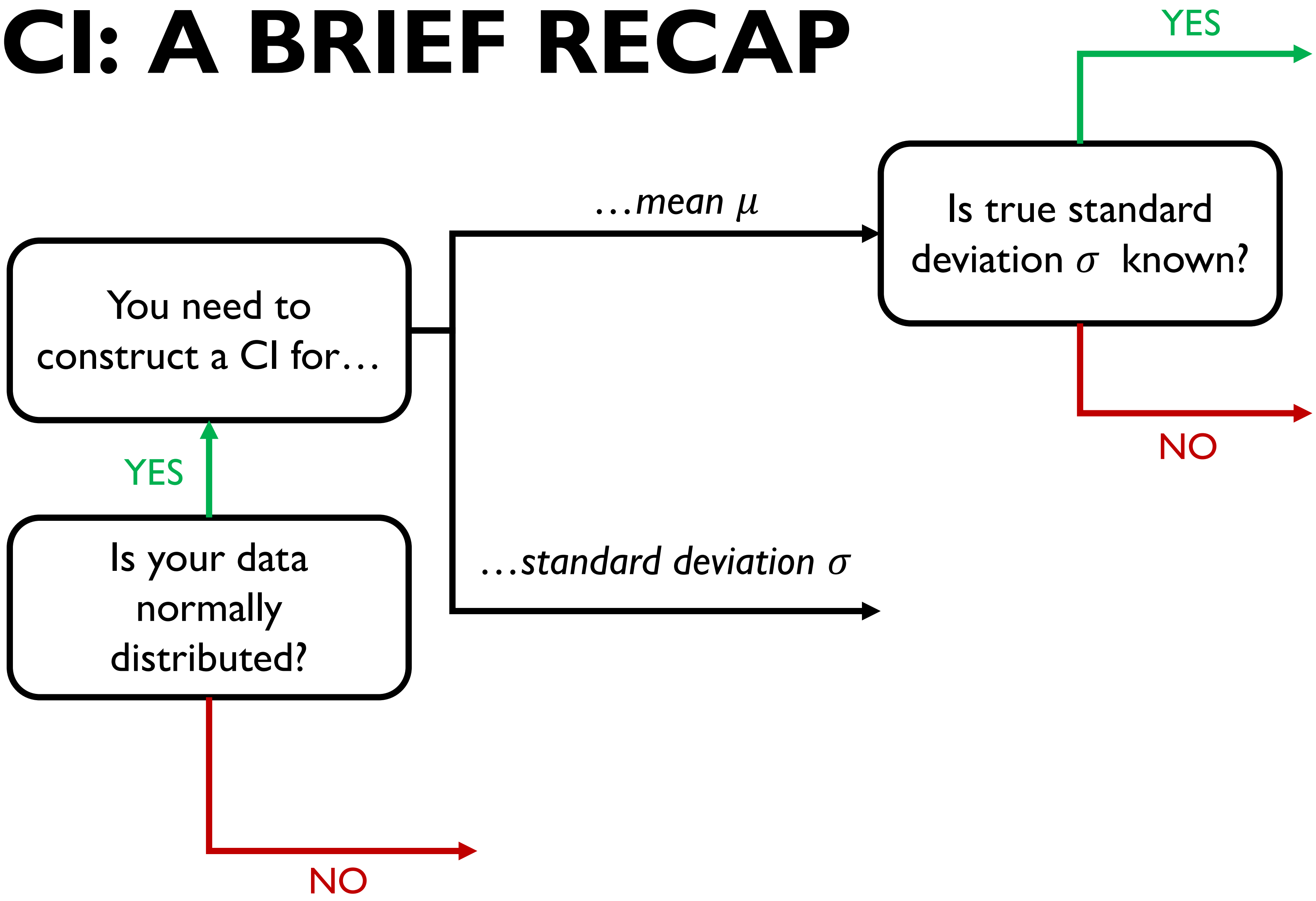

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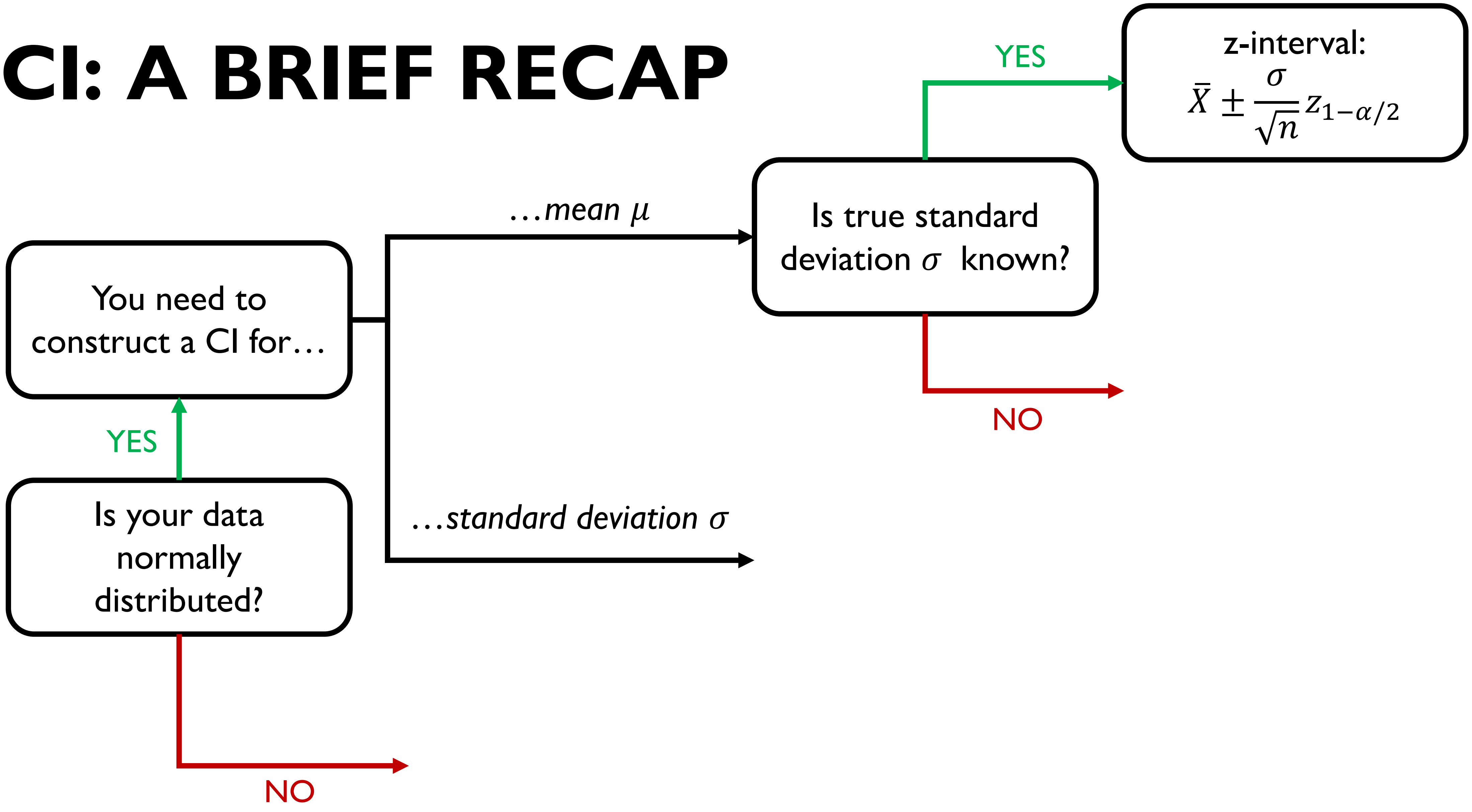

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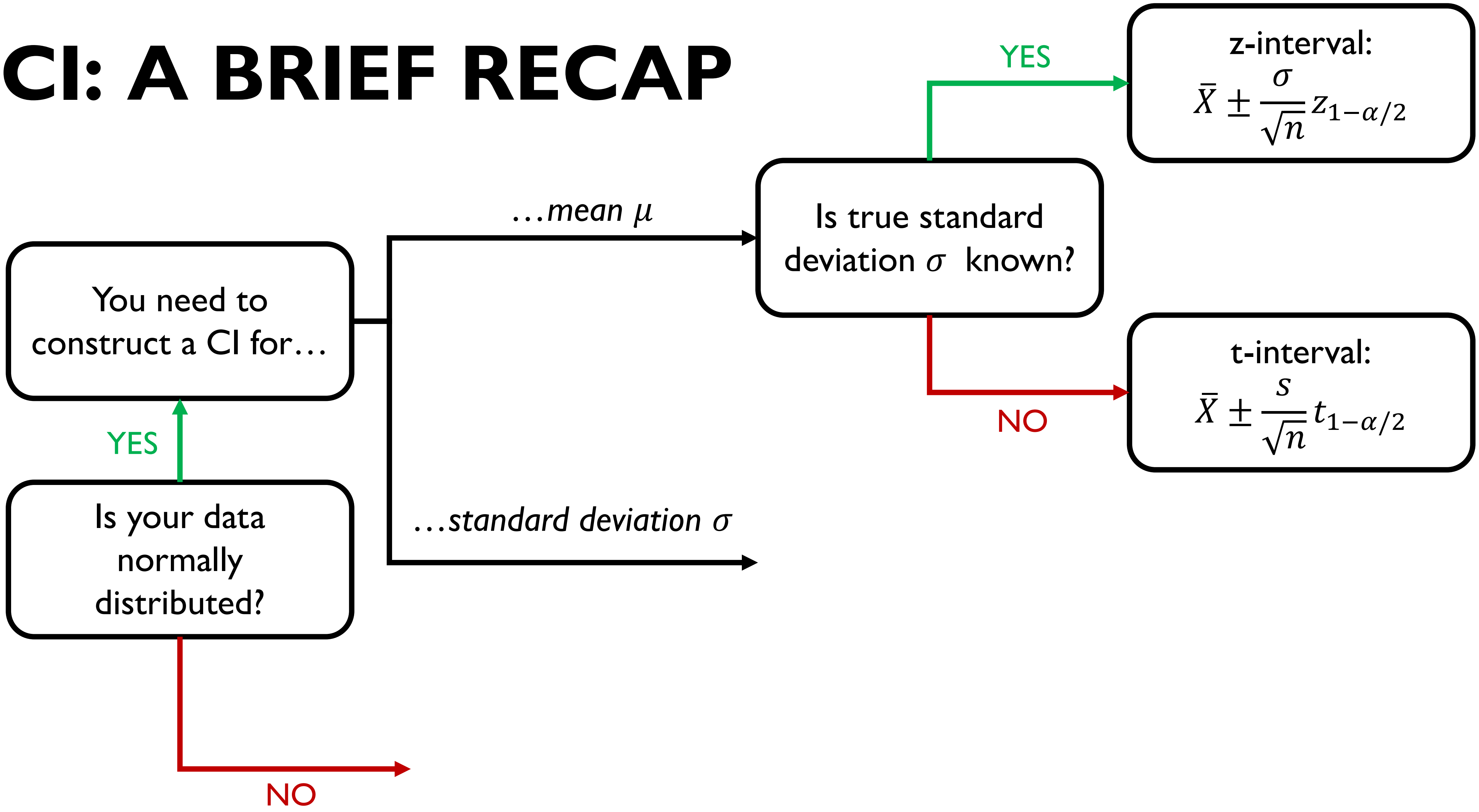

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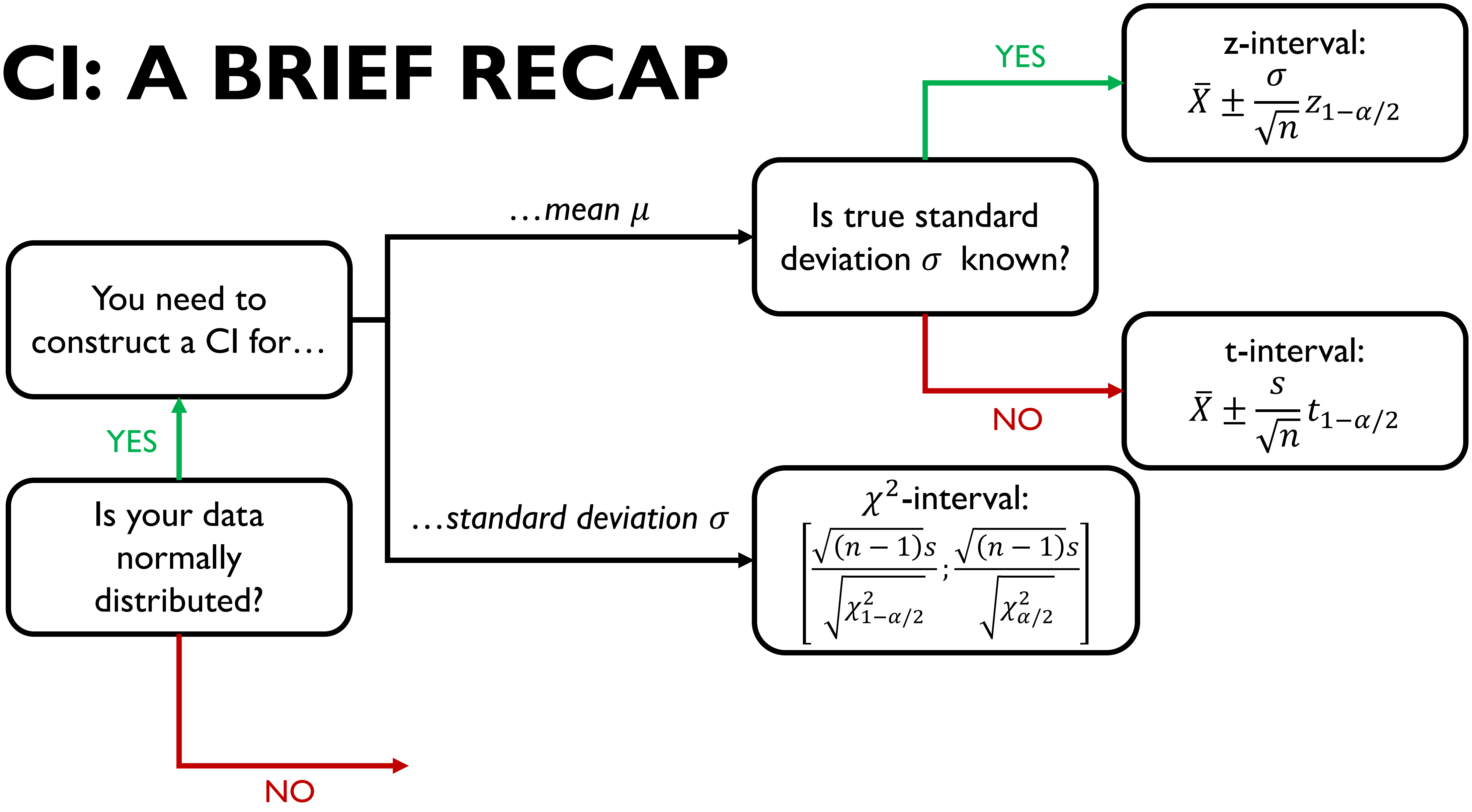

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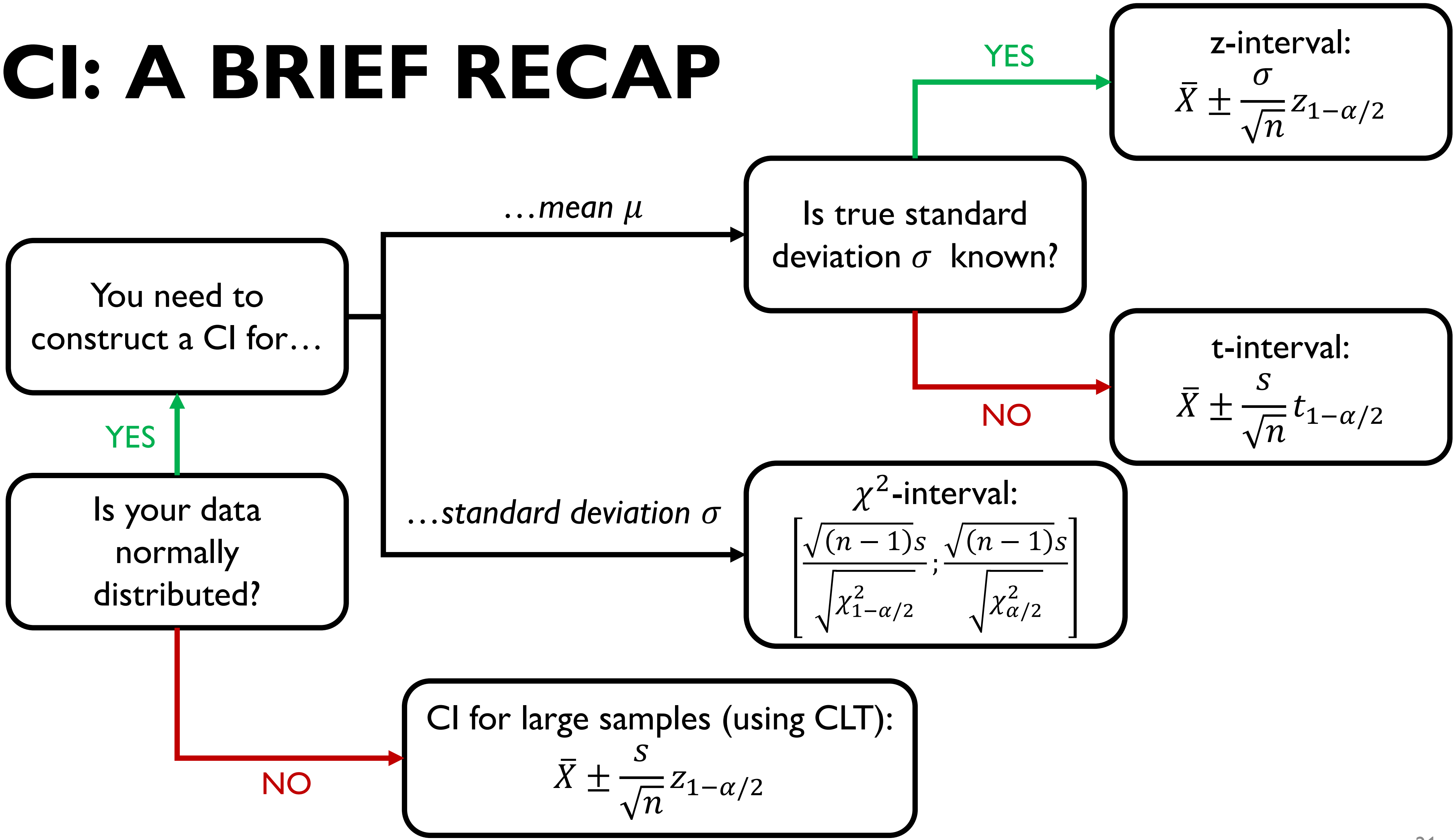

QUICK QUIZ

CI: A BRIEF RECAP

A machine fills in wine bottles with a random amount of wine that follows normal distribution with unknown mean μ and unknown standard deviation σ.

You check the last 100 bottles and see that on average they were filled with $\bar{X} = 705$ ml of wine, with sample std $s = 3$ ml.

Which interval would you use to construct a 95%-CI for μ?

CI: A BRIEF RECAP

In a study on cholesterol levels a sample of 1000 patients was chosen.

The average plasma cholesterol levels subjects was $\bar{X} = 6$ mmol/L, with sample std $s = 0.4$ mmol/L.

Which interval would you use to construct a 95%-CI for the true mean?

CI: A BRIEF RECAP

Height of a female student is a random variable following normal distribution with unknown mean μ and standard deviation $\sigma = 5$ cm.

The average height of 100 female students $\bar{X} = 165$ cm, and you sample std $s = 4$ cm

Which interval would you use to construct a 95%-CI for μ?

CI: A BRIEF RECAP

On a candy factory, a machine fills packs with random number of candies which follows normal distribution with unknown mean μ and unknown standard deviation σ.

In the last 50 packs, there were on average $\bar{X} = 90$ g of sweets, with sample std $s = 7$ g.

Which interval would you use to construct a 95%-CI for σ?

HYPOTHESIS TESTING

HYPOTHESIS TESTING

- Test whether a default assumption about the data is plausible.

- Key idea: you can't *prove* something, but you can disprove it.

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HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

HYPOTHESIS TESTING

ASSUMPTION:
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EVIDENCE:

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:
NOT CONVINCING…

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:
NOW IT's CONVINCING!

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:

HYPOTHESIS TESTING

ASSUMPTION:
THERE’RE NO ALIENS ON THE MOON

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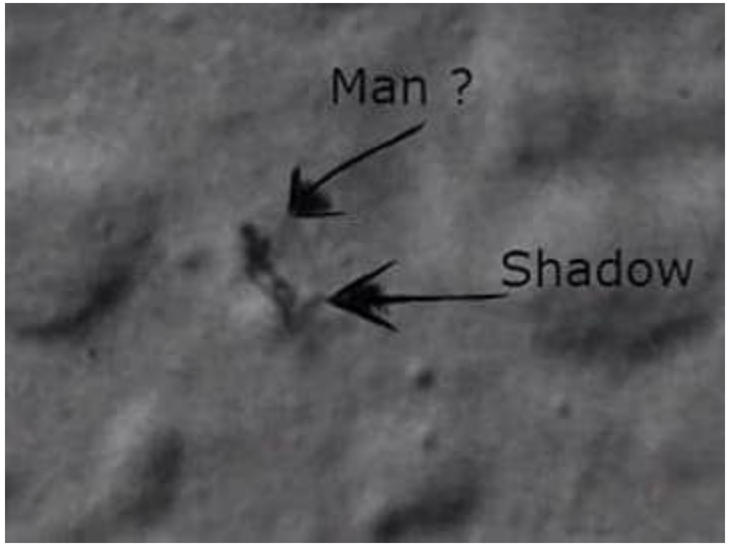
EVIDENCE:

STILL NO REASON TO BELIEVE THERE ARE ALIENS



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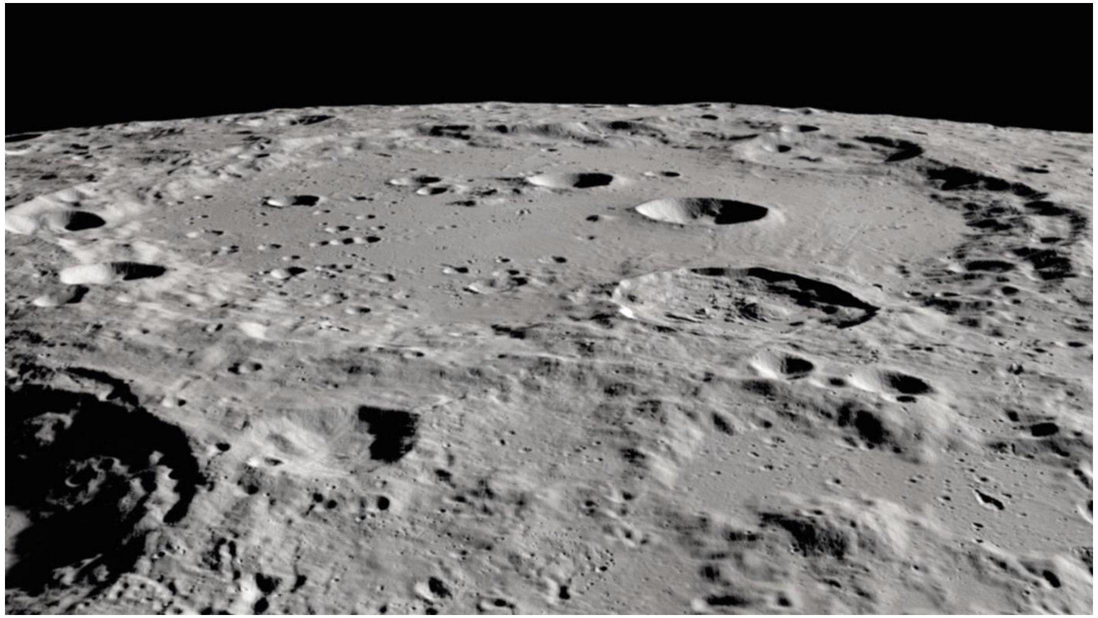

HYPOTHESIS TESTING

ASSUMPTION:

THERE'RE NO ALIENS ON THE MOON

EVIDENCE:

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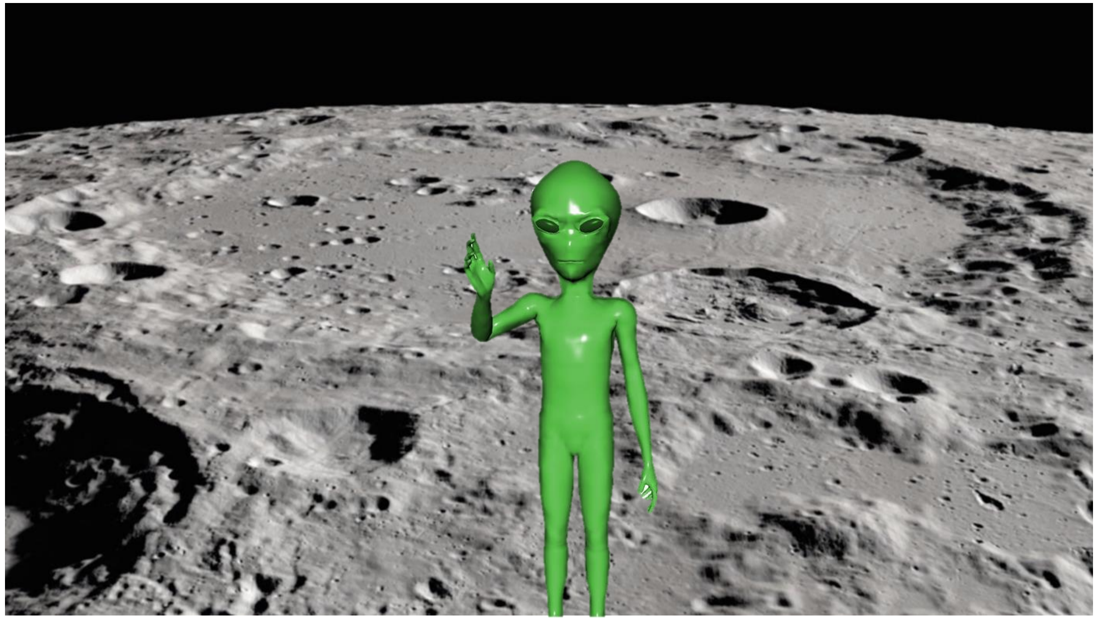

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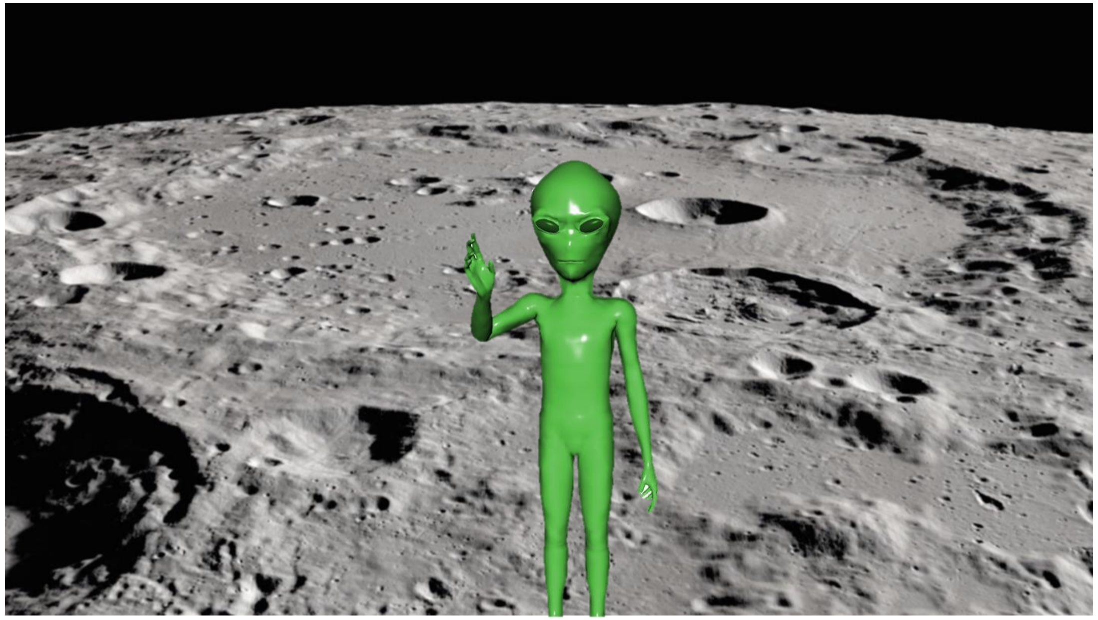

HYPOTHESIS TESTING

- Note:
  - Default assumption = "status quo".

HYPOTHESIS TESTING

- Note:
    - Default assumption = “status quo”.
    - Alternative: something we can find evidence for.

HOW TO TEST A HYPOTHESIS

BASIC INGREDIENTS

RUNNING EXAMPLE

How to check if a coin is a fair one?

RUNNING EXAMPLE

We flip a coin 10 times to test if it’s a fair one.

INGREDIENTS

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$$H_1\colon p > 0.5$$

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$$X \sim$$

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$$X \sim Binomial(\quad\quad)$$

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X											
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X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
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- **Rejection region R**: if X is in the rejection region, we reject H_0 in favor of H_1.

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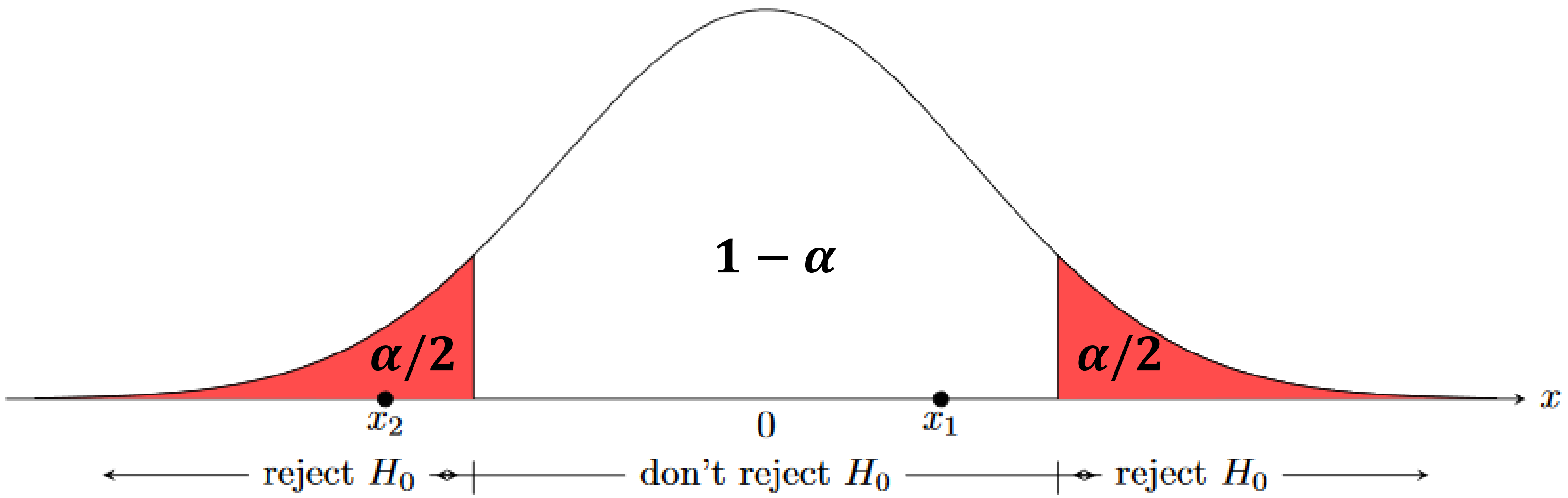
- **Rejection region R**: if X is in the rejection region, we reject H_0 in favor of H_1.
- **Significance level** α **:** $P(X \in R|H_0) \leq \alpha$
  - Typically chosen in advance (common values are 0.1, 0.05, 0.01.)

INGREDIENTS

- Distribution of the test statistic under H_0, two-sided alternative:



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$$\boldsymbol{\alpha = 0.05}$$

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$$\boldsymbol{\alpha = 0.2}$$

X	0	1	2	3	4	5	6	7	8	9	10
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  - we won’t claim the coin is unfair unless we have compelling evidence.

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- **Rejection region**: data that is *extreme* <u>under the null hypothesis</u> = outcomes in the tail(s) of the null distribution.
  - depends on the significance level α of the test.

REMARKS

H_0: $p = 0.5$ ‘the coin is fair’, H_1: $p \neq 0.5$ ‘the coin is not fair’ $\alpha = \mathbf{0.05}$

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- If we get *0, 1, 9, 10* H
  - Reject H_0

REMARKS

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- Based on the result, do you reject the null hypothesis?

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$$P(Reject\ H_0|H_0) = P(X \in R|H_0) = \alpha -$$

significance level

ONE-SIDED ALTERNATIVES

INGREDIENTS

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Z-TEST

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z-test

- IQ follows $N(\mu_0, \sigma^2)$ where σ is known.

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- Can we reject H_0?

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- Test statistic:

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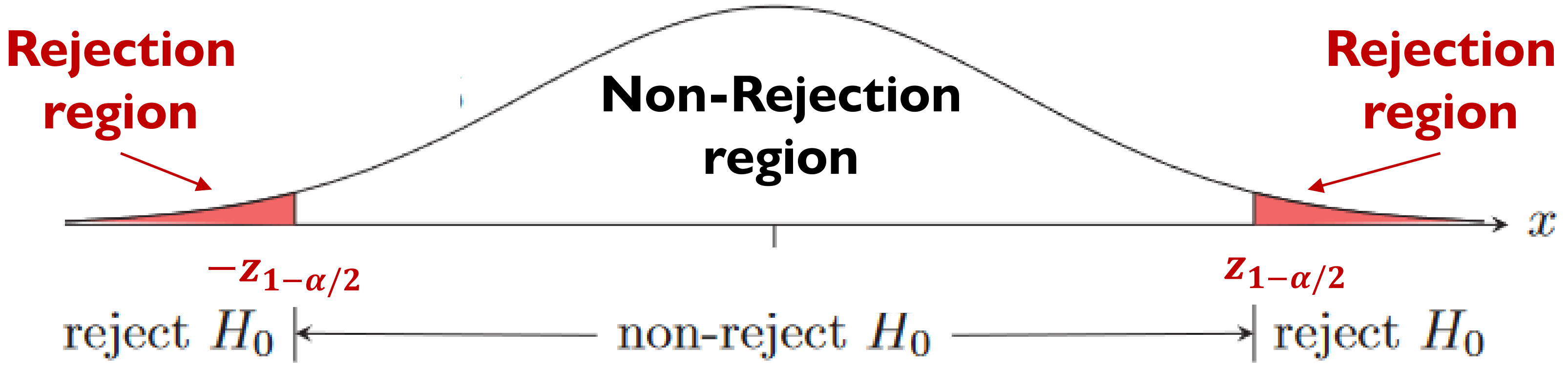

ONE-SIDED ALTERNATIVES

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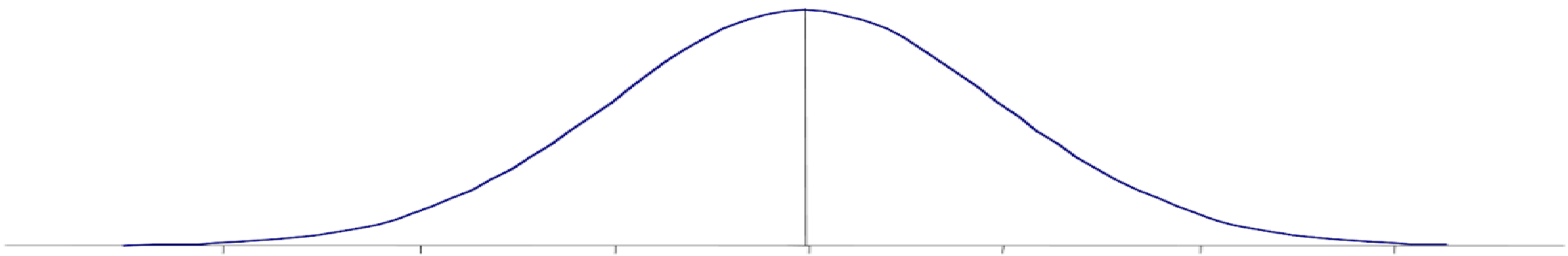

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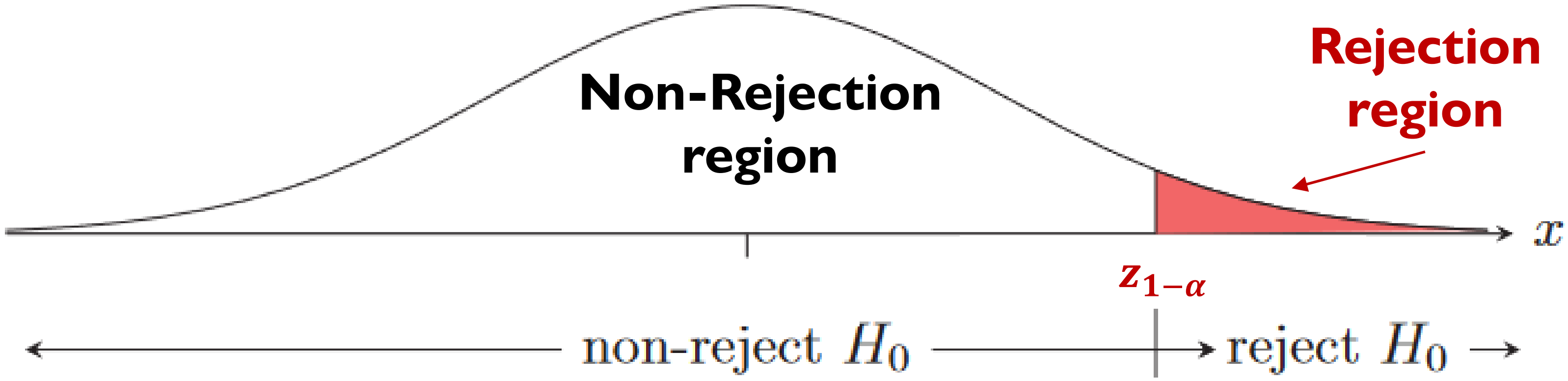

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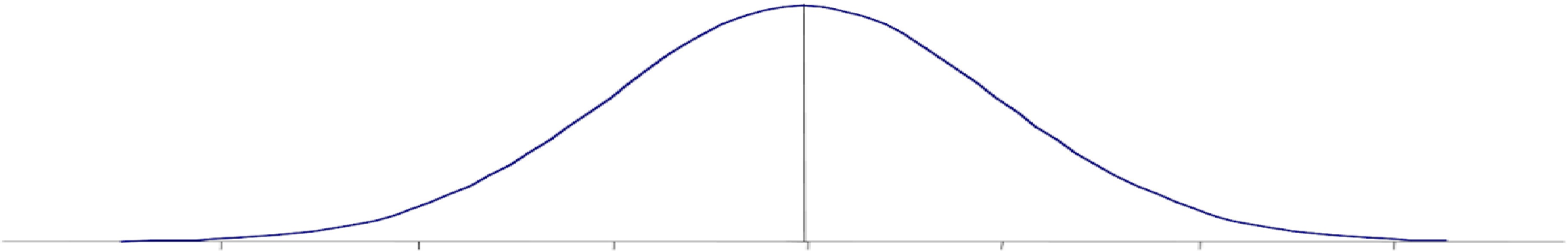

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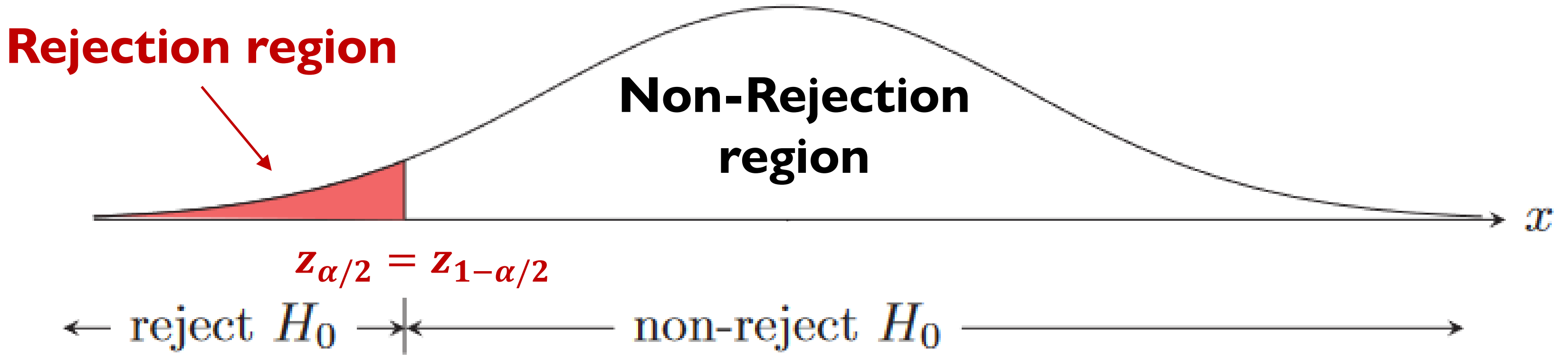

Z-TEST: EXAMPLE 1

- Weight loss is described by $X \sim N(5, 7^2)$.
- $n = 30$ patients followed an experimental diet.
- Average weight loss $\bar{X} = 6.1$ kg.

Does the diet make any difference?

Z-TEST: EXAMPLE 2

- A bakery supplies loaves of bread to supermarkets. Not every loaf weighs the same: loaf weight $X \sim N(\mu_0, 0.1^2)$, supermarket expects $\mu_0 = 2$ kg.
- A supermarket draws a sample of $n = 20$ loaves: average weights is $\bar{X} = 1.97$ kg.
- The supermarket wants to be sure that the weights are, on average, not lower than 2 kg.

Is there evidence against this?

T-TEST

When the data is coming from normal distribution
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T-TEST: EXAMPLE 2

- Sales at your company: $X \sim N(\mu_0, \sigma^2)$, $\mu_0 = 100$ dollars per transaction.

- You hire new employees. Based of a sample of $\mathrm{n} = 25$ salesmen, average sale is $\bar{X} = 130$ dollars, with sample standard deviation $\mathrm{s} = 15$.

Are the new employees better than the old ones?
Test your hypothesis at $\boldsymbol{\alpha = 0.05}$.

TO SUM UP

- Hypothesis testing ingredients
  - H0 and H1
  - Test statistic
  - Null distribution
  - Rejection region
  - Two- and one-sided alternatives

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- One-sample tests:
  - Z-test: normal distribution, variance known
  - T-test: normal distribution, variance unknown