INTRODUCTION TO STATISTICS

LECTURE 9

CONFIDENCE INTERVALS: RECAP

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- Often constructed as follows:

$$\textit{point estimate} \pm \textit{quantile} \cdot \textit{variance of point estimate}$$

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  - CI for μ, σ is known: **t-interval**

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  - CI for σ, μ is unknown: $\boldsymbol{\chi^2}$**-interval**

$$\left(\sqrt{\frac{(n-1)s^2}{q_{1-\alpha/2}}}; \sqrt{\frac{(n-1)s^2}{q_{\alpha/2}}}\right), q - \text{ quantiles from } \chi^2 \text{ distribution } (n-1) \text{ d.f.}$$

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$$\mu = \bar{X} \pm \frac{s}{\sqrt{n}} z_{1-\alpha/2}$$

CI FOR BERNOULLI DISTRIBUTION

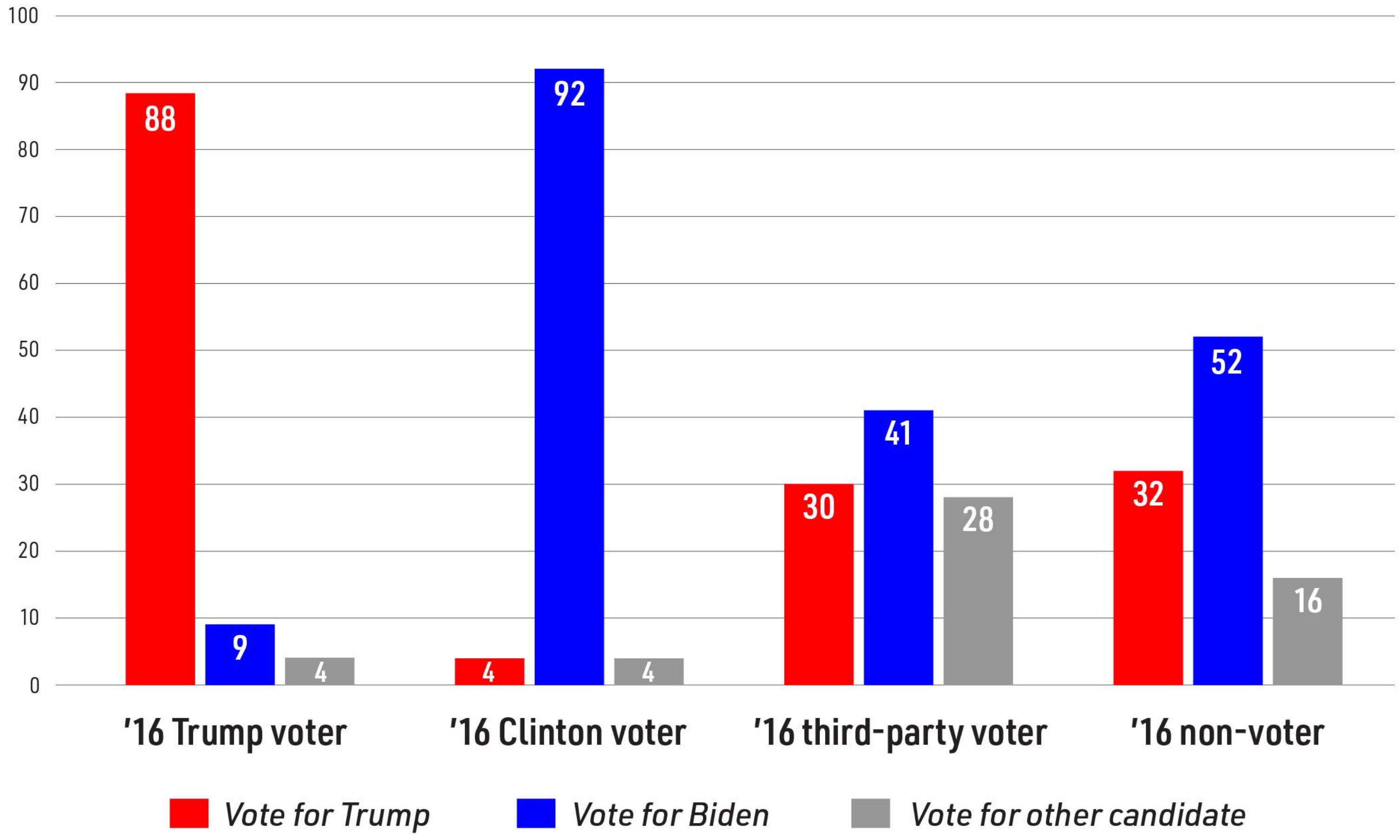
POLITICAL POLLS

- Political polls are often reported as a value with a margin-of-error.

52% favor candidate A with a margin-of-error of ±5%.

Presidential Election Preview:

How Clinton, Trump Voters from 2016 Plan to Vote in 2020



A total of 1,510 eligible voters, who are adult members of the USC Dornsife Center for Economic and Social Research's Understanding America Study internet panel, participated from August 11 – 16, 2020. Margin of sampling error for this preliminary sample is +/-3 percentage points. Tracking graphs will be updated every day. For full question text, methodology, and other information, visit https://uasdata.usc.edu/index.php

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and the 95% - CI is $52\% \pm 5\%$

CI FOR p (BERNOULLI)

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$$p \in (0.504; 0.696)$$

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    polls 40 random voters and finds 22 support A.
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SAMPLE SIZE

How large should my sample be?

SAMPLE SIZE DETERMINATION

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$$n \geq \frac{\sigma^2 z_{1-\alpha/2}^2}{\epsilon^2}$$

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$$n^* = 385$$

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Let $p_0 = 0.5$ – the worst possible variance

$$1.96 \sqrt{\frac{0.25}{n}} \leq 0.03 \iff$$

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$$1.96 \sqrt{\frac{0.25}{n}} \leq 0.03 \iff n \geq 1067.1$$

HYPOTHESIS TESTING

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- After tossing a coin 100 times, we observed H only 30 times. Is it a fair coin?

- Did extra tuition help students?

- Is drug A more efficient than drug B?

HOW IT ALL STARTED

https://youtu.be/lgs7d5saFFc

LADY TASTING TEA

Ronald Fisher, 1913

LADY TASTING TEA

- 8 cups of tea
  - 4 cups: milk first
  - 4 cups: tea first

- The lady must select 4 cups prepared by one method.

- *How to check her ability to distinguish the teas?*

Ronald Fisher, 1913

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Tea-Tasting Distribution Assuming H_0		
Success count	Combinations of selection	Number of Combinations
0	oooo	1 × 1 = 1
1	ooox, ooxo, oxoo, xooo	4 × 4 = 16
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	6 × 6 = 36
3	oxxx, xoxx, xxox, xxxo	4 × 4 = 16
4	xxxx	1 × 1 = 1
Total		70

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- How unlikely would it be to randomly guess all 4?

$$1/70 \approx 0.014$$

Tea-Tasting Distribution Assuming H_0		
Success count	Combinations of selection	Number of Combinations
0	oooo	$1 \times 1 = 1$
1	ooox, ooxo, oxoo, xooo	$4 \times 4 = 16$
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	$6 \times 6 = 36$
3	oxxx, xoxx, xxox, xxxo	$4 \times 4 = 16$
4	xxxx	$1 \times 1 = 1$
Total		70

LADY TASTING TEA

- The default assumption:

H_0: the lady can’t distinguish the teas

- How unlikely would it be to randomly guess all 4?

$$1/70 \approx 0.014$$

That’s *surprising enough* to reject H_0

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