

Discrete random variables: Practice Problems

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1. Consider two independent discrete random variables X and Y . Their distributions are given by the tables below.

x	-1	0
$P(X = x)$	0.2	0.8

y	1	0
$P(Y = y)$	0.4	0.6

- (a) Find distribution, expected value and variance of $Z = X + Y$.
- (b) Find expected value and variance of $W = 10X - Y$.
2. 5 people enter the elevator on the ground floor of a 10 story building. Each of them exits on one of the 10 floors with equal probability, independently of the others. What is the probability that ...
- (a) ... at least one person exits on the 6th floor?
- (b) ... everyone exits on the 6th floor or above?
- (c) ... no one exits on the 6th floor or above?
3. A customer support center uses the Poisson distribution to model the number of requests they are likely to receive so that they know how many call center reps to keep on staff. Suppose that they receive, on average, 10 requests per hour. What is the probability that in the next hour they will ...
- (a) ... receive between 8 and 12 calls?
- (b) ... they receive less than 10 calls?
- (c) ... they will receive at least one call?
4. A student is given a multiple-choice exam with 10 questions, where each question has five possible answers. She guesses randomly for each question.
- (a) What is the expected number and variance of the correctly guessed answers?

- (b) What is the probability that no answer will be correct?
 - (c) In order to pass the exam, the student needs to answer at least 60% of the questions correctly. What is the probability that the student will pass?
 - (d) Imagine that the student needs to pass 5 such multiple choice exams in order to graduate. What are the chances to graduate answering the exam questions in each exam randomly?
5. Occasional disruptions are possible in the normal functioning of a certain device. The number of such disruptions per day follows the Poisson distribution and is independent from that on any other day. On average, there are 3 disruptions a day. What is the probability that ...
- (a) ... at least one disruption will happen on a random day?
 - (b) ... no disruptions will happen on two consecutive days?