PROBABILITY & STATISTICS

Lecture 8 – More on Normal distribution

HOW TO COMPUTE PROBABILITIES?

Let's start with the *standard* normal distribution.

PROBABILITIES FROM THE NORMAL DISTRIBUTION

$$X \sim N(10, 2^2)$$

$$P(2 < X \leq 6) =$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
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1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
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2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

PROBABILITIES FROM THE NORMAL DISTRIBUTION

$$X \sim N(10, 2^2)$$

$$P(2 < X \leq 6) =$$

$$= P(2 - 10 < X - 10 \leq 6 - 10) =$$

$$= P\left(\frac{2-10}{2} < \frac{X-10}{2} \leq \frac{6-10}{2}\right) =$$

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2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
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$$= \Phi(-2) - \Phi(-4) =$$

$$= 1 - \Phi(0.2) - 1 + \Phi(0.4) =$$

$$1 - 0.9772 - 1 + 1$$

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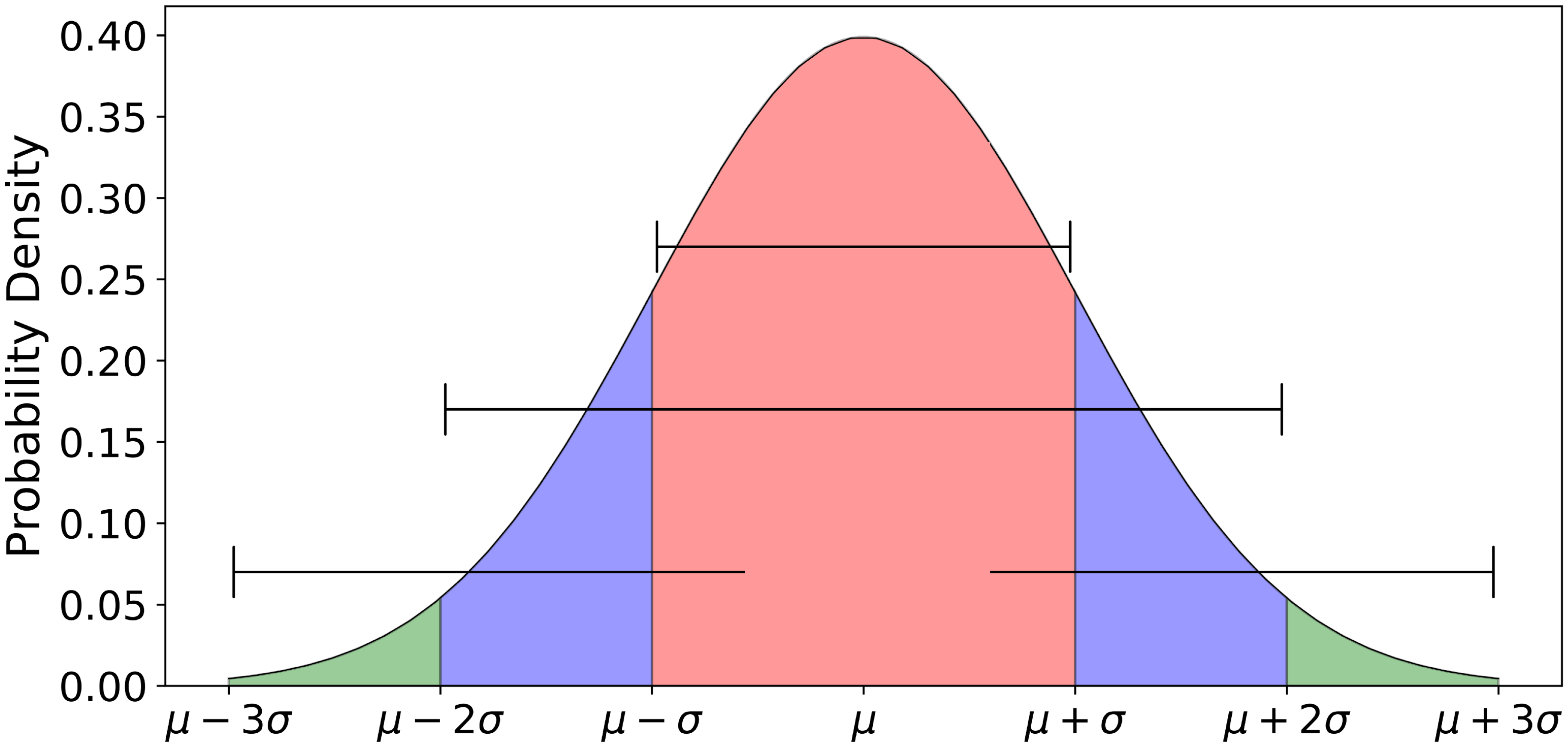
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

WHAT % IS WITHIN 1σ, 2σ OR 3σ FROM THE MEAN?



WHAT % IS WITHIN 1σ FROM THE MEAN?

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$$P(\mu - \sigma < X \leq \mu + \sigma) =$$

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$$= P(-\sigma < X - \mu \leq \sigma) =$$

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$$= P\left(-1 < \frac{X - \mu}{\sigma} \leq 1\right) \cong$$

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$$P(\mu - \sigma < X \leq \mu + \sigma) =$$

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~N(0,1)

$$= P\left(-1 < \boxed{\frac{X - \mu}{\sigma}} \leq 1\right) \cong$$

WHAT % IS WITHIN 1σ FROM THE MEAN?

$$X \sim N(\mu, \sigma)$$

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$$= P(-\sigma < X - \mu \leq \sigma) =$$

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$$\cong 2\Phi(1) - 1 =$$

WHAT % IS WITHIN 1σ FROM THE MEAN?

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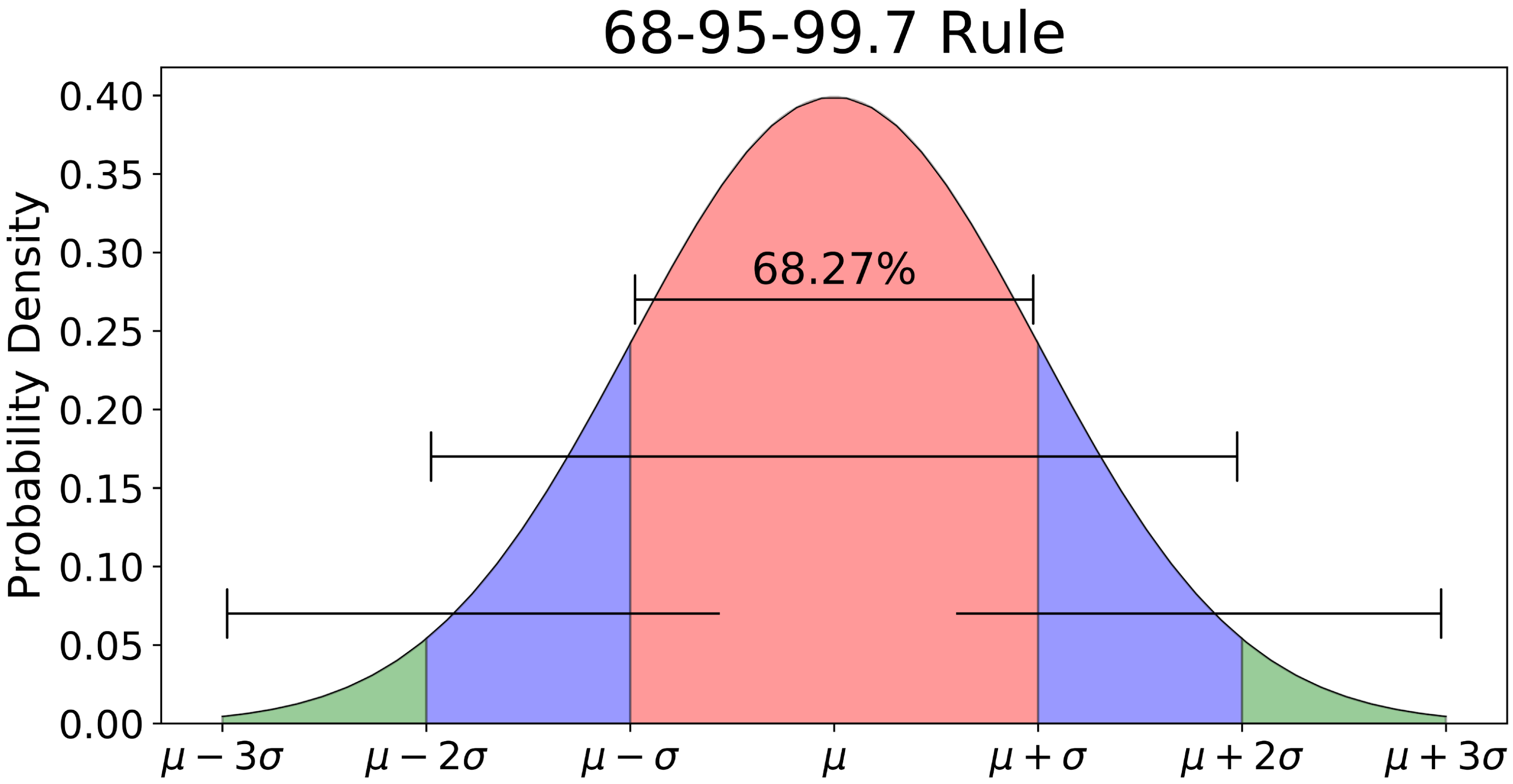
$$= P(-\sigma < X - \mu \leq \sigma) =$$

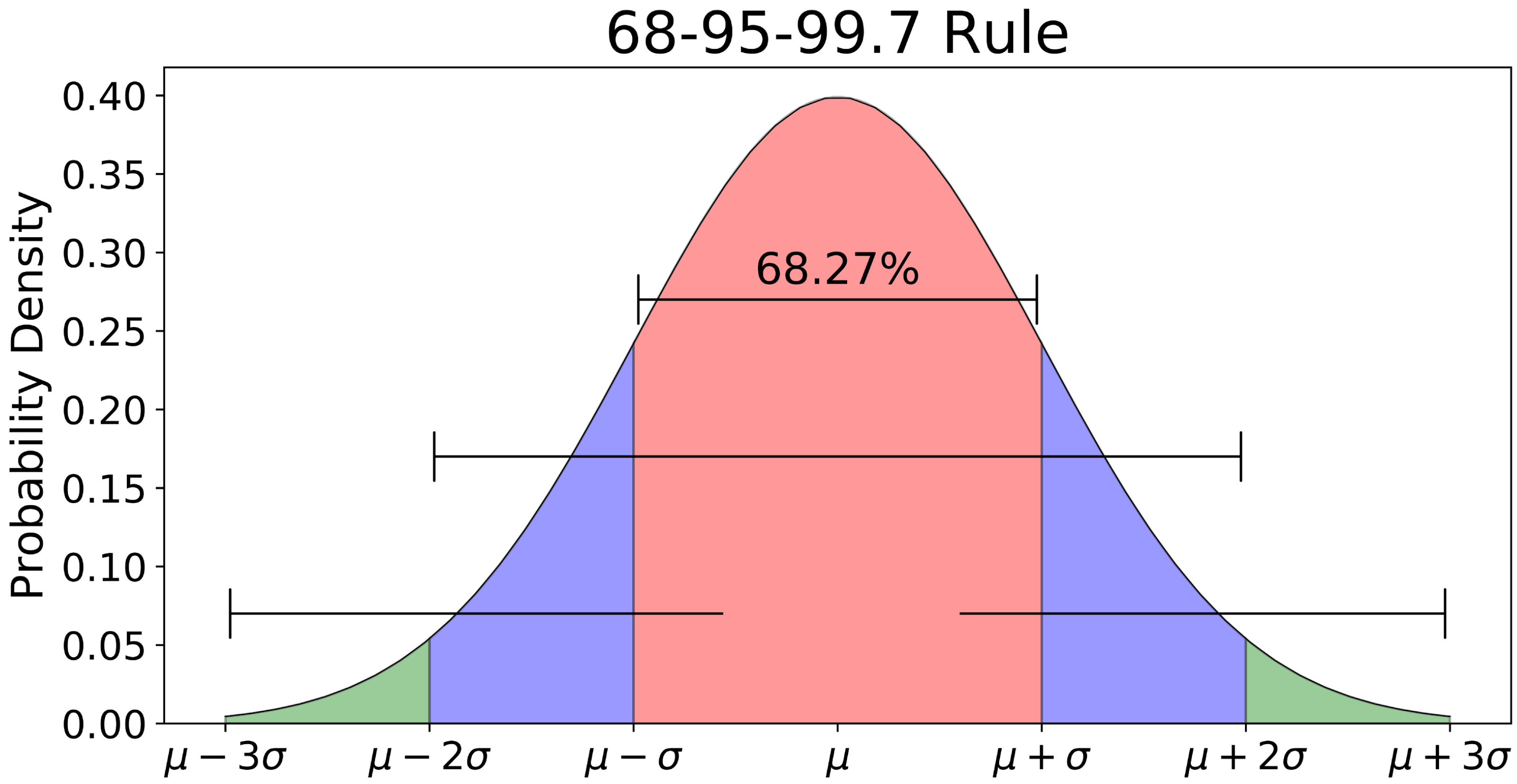
$$= P\left(-1 < \frac{X - \mu}{\sigma} \leq 1\right) \cong$$

$$\cong 2 * 0.84134 - 1 \cong 0.6827$$

WHAT % IS WITHIN 1σ FROM THE MEAN?



WHAT % IS WITHIN 2σ FROM THE MEAN?



WHAT % IS WITHIN 2σ FROM THE MEAN?

$$X \sim N(\mu, \sigma)$$

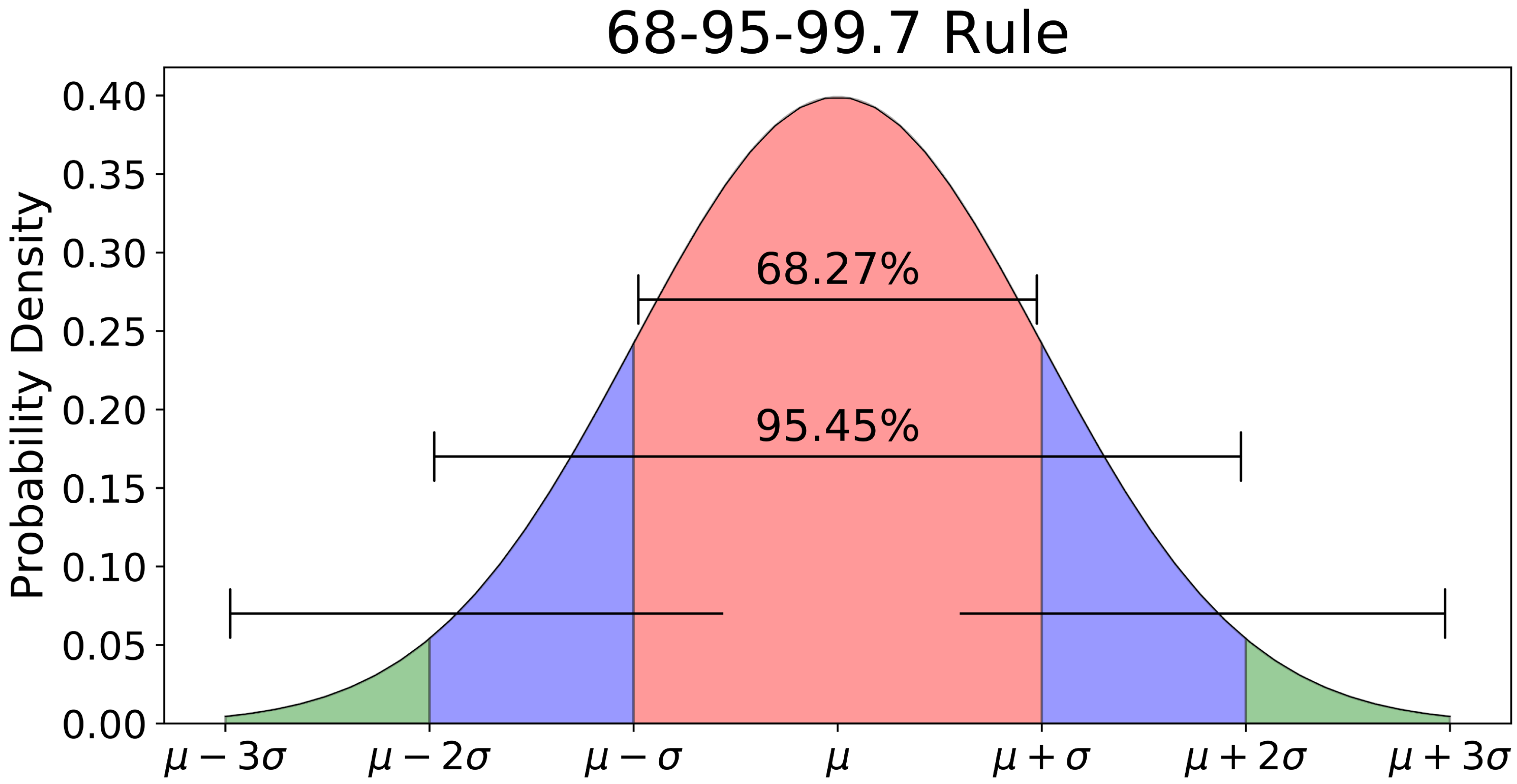
$$P(\mu - 2\sigma < X \leq \mu + 2\sigma) =$$

$$= P(-2\sigma < X - \mu \leq 2\sigma) =$$

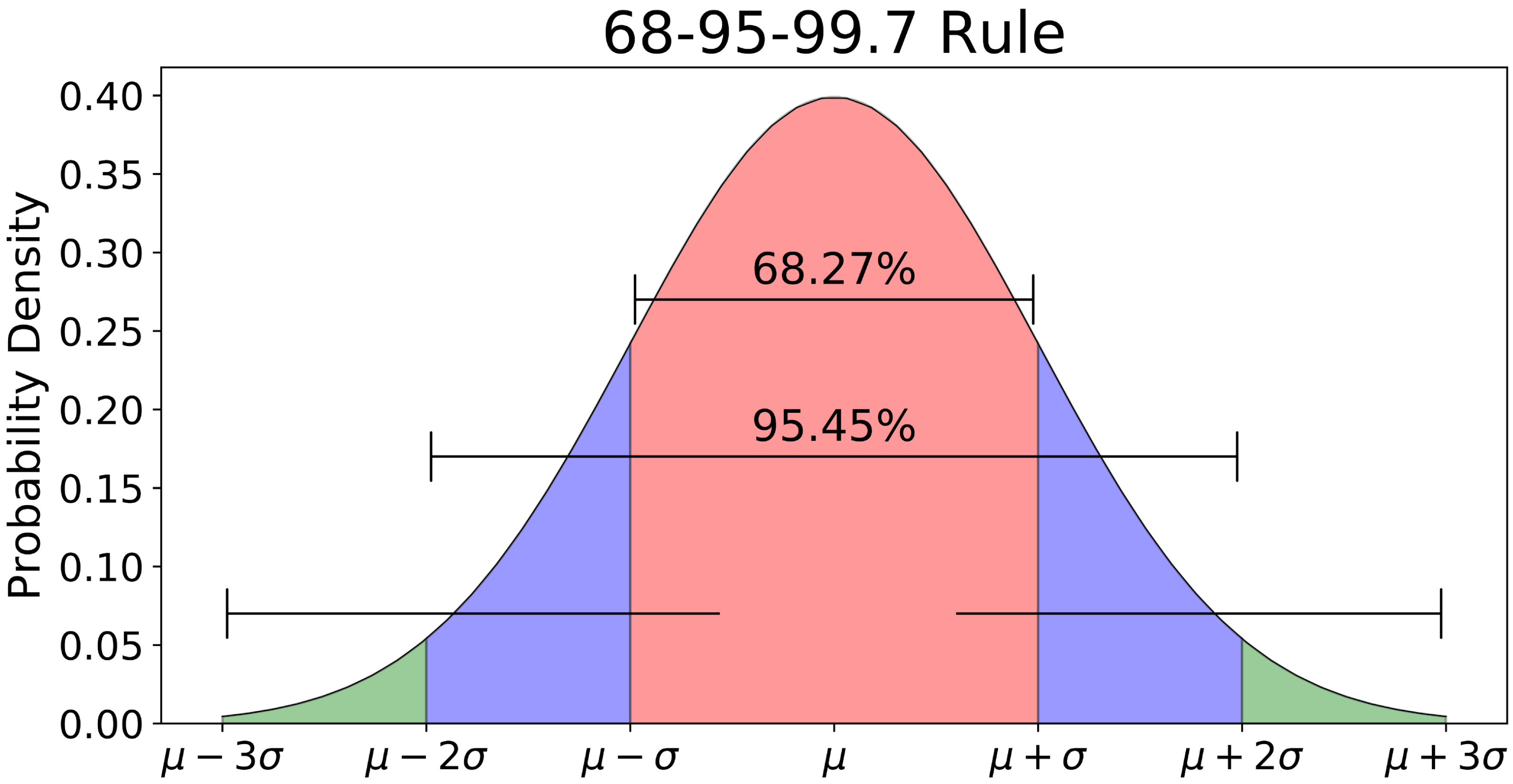
$$= P\left(-2 < \frac{X - \mu}{\sigma} \leq 2\right) \cong$$

$$\cong 2\Phi(2) - 1 \cong 0.9545$$

WHAT % IS WITHIN 2σ FROM THE MEAN?



WHAT % IS WITHIN 3σ FROM THE MEAN?



WHAT % IS WITHIN 3σ FROM THE MEAN?

$$X \sim N(\mu, \sigma)$$

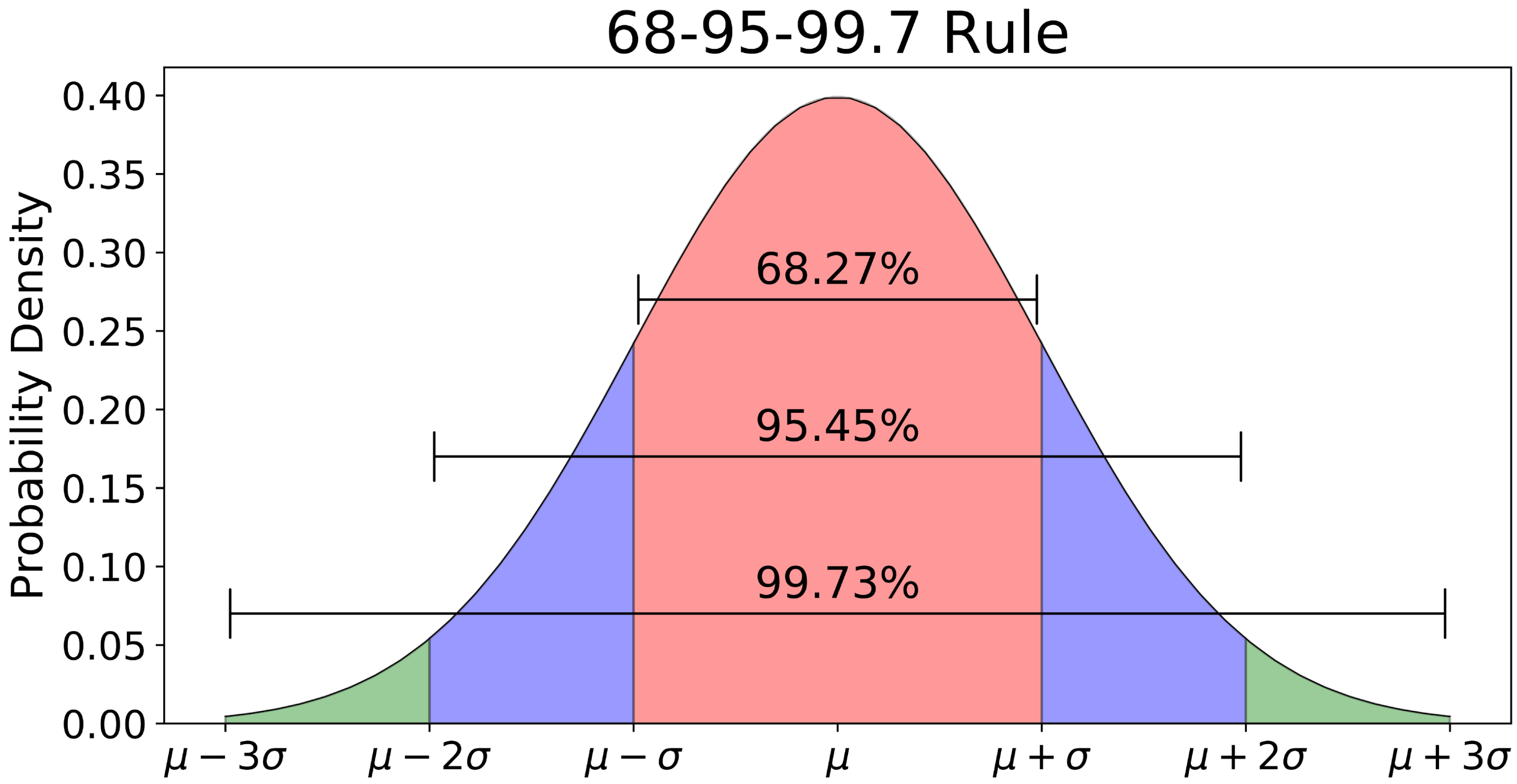
$$P(\mu - 3\sigma < X \leq \mu + 3\sigma) =$$

$$= P(-3\sigma < X - \mu \leq 3\sigma) =$$

$$= P\left(-3 < \frac{X - \mu}{\sigma} \leq 3\right) \cong$$

$$\cong 2\Phi(3) - 1 \cong 0.9973$$

WHAT % IS WITHIN 1σ, 2σ OR 3σ FROM THE MEAN?



QUANTILES

GMAT

- Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112.

  What is the probability of an individual scoring above 500 on the GMAT?

$$X \sim N(527, 112^2)$$

$$P(X > 500) = 1 - P(X \leq 500) =$$

GMAT

- Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112.

  What is the probability of an individual scoring above 500 on the GMAT?

$$X \sim N(527, 112^2)$$

$$P(X > 500) = 1 - P(X \leq 500) = 1 - P\left(\frac{X - 527}{112} \leq \frac{500 - 527}{112}\right) =$$

$$= 1 - \Phi(-0.241) = 1 - 1 + \Phi(0.241) = 0.4948$$

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QUANTILE

- Given $0 < q < 1$, answer the question:

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What is the value x such that $P(X \leq x) = q$?

QUANTILE

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What is the value x such that $P(X \leq x) = q$?

x – q-quantile.

QUANTILE

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  What is the value x such that $P(X \leq x) = 0.95$?

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$$x\colon\ \Phi(x) = 0.95 \iff$$

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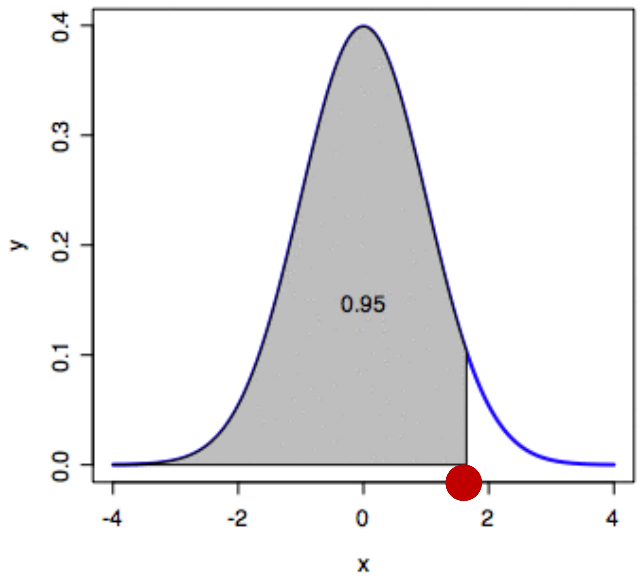
$$x\colon\ \Phi(x) = 0.95 \iff x = \Phi^{-1}(0.95)$$



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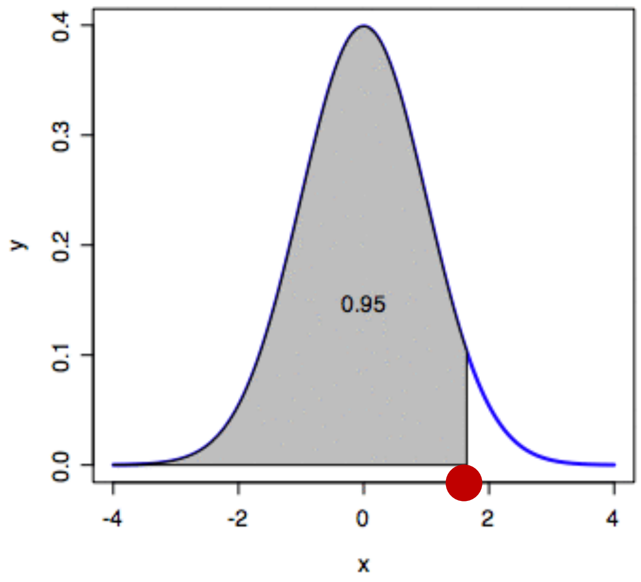

How to compute this? There are tables!

QUANTILE

- $X \sim N(0,1)$.
  What is the value x such that
  $P(X \leq x) = 0.95$?

$x =$

α	0.9	0.95	0.975	0.99	0.995	0.999
z_α	1.282	1.645	1.960	2.326	2.576	3.090

QUANTILE

- $X \sim N(0,1)$.
  What is the value x such that
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$$x = \Phi^{-1}(0.95) = 1.645$$

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QUANTILE

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$$P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = 0.95$$

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$$\frac{x-\mu}{\sigma} = \Phi^{-1}(0.95)$$

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$$\frac{x-\mu}{\sigma} = \Phi^{-1}(0.95)$$

$$x = \sigma\Phi^{-1}(0.95) + \mu$$

α	0.9	0.95	0.975	0.99	0.995	0.999
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QUANTILE

- $X \sim N(10,6)$.
  What is the value x such that
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$$P(X \leq x) = 0.95$$

α	0.9	$\boxed{0.95}$	0.975	0.99	0.995	0.999
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α	0.9	$\boxed{0.95}$	0.975	0.99	0.995	0.999
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$$\frac{x-10}{6} = \Phi^{-1}(0.95)$$

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$$\frac{x-10}{6} = \Phi^{-1}(0.95)$$

$$x = 6\Phi^{-1}(0.95) + 10 = 19.84$$

α	0.9	0.95	0.975	0.99	0.995	0.999
z_α	1.282	1.645	1.960	2.326	2.576	3.090

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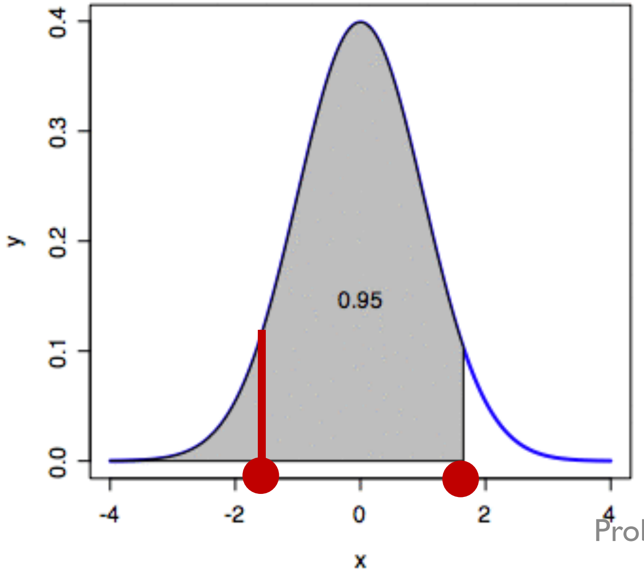
$$x^* = 112 \cdot 1.645 + 527 = 711.24$$

QUANTILE

- $X \sim N(0,1)$.
  What is the value x such that $P(X \leq x) = 0.05$?

  $x = -$

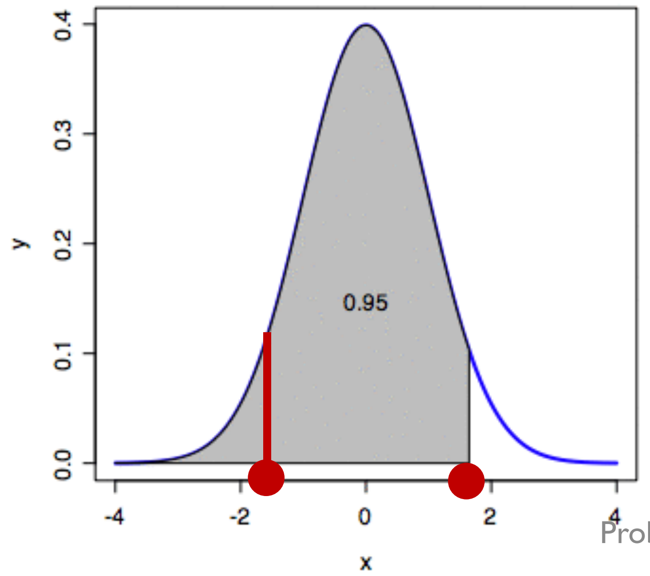


Quantile (p)	$\Phi^{-1}(p, 0, 1)$
0.995	2.58
0.99	2.33
0.975	1.96
0.95	1.64
0.9	1.28

QUANTILE

- $X \sim N(0,1)$.
  What is the value x such that $P(X \leq x) = 0.05$?

$$x = -\Phi^{-1}(0.95) = -1.64$$



Quantile (p)	$\Phi^{-1}(p, 0, 1)$
0.995	2.58
0.99	2.33
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0.9	1.28

PRACTICAL EXERCISE

Google Classroom -> Day 8 ->
Probabilities from the normal distribution

LINEAR COMBINATION
OF NORMALLY DISTRIBUTED VARIABLES

LINEAR COMBINATION OF NORMALLY DISTRIBUTED VARIABLES

- A linear combination of independent random variables having a normal distribution also has a normal distribution:

$$X_1, X_2, \dots, X_n \text{ - independent}$$

$$X_i \sim N\left(\mu_i, \sigma_i^2\right)$$

$$Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n \Rightarrow$$

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$$Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n \Rightarrow$$

$$Y \sim N(\mu_Y, \sigma_Y^2)$$

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$$Y \sim N(\mu_Y, \sigma_Y^2)$$

$$\mu_Y = \qquad\qquad , \qquad \sigma_Y^2 =$$

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$$Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n \Rightarrow$$

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$$\mu_Y = a_1 \mu_1 + a_2 \mu_2 + \cdots + a_n \mu_n, \qquad \sigma_Y^2 =$$

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- A linear combination of independent random variables having a normal distribution also has a normal distribution:

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$$Y \sim N(\mu_Y, \sigma_Y^2)$$

$$\mu_Y = a_1 \mu_1 + a_2 \mu_2 + \cdots + a_n \mu_n, \qquad \sigma_Y^2 = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + \cdots + a_n^2 \sigma_n^2$$

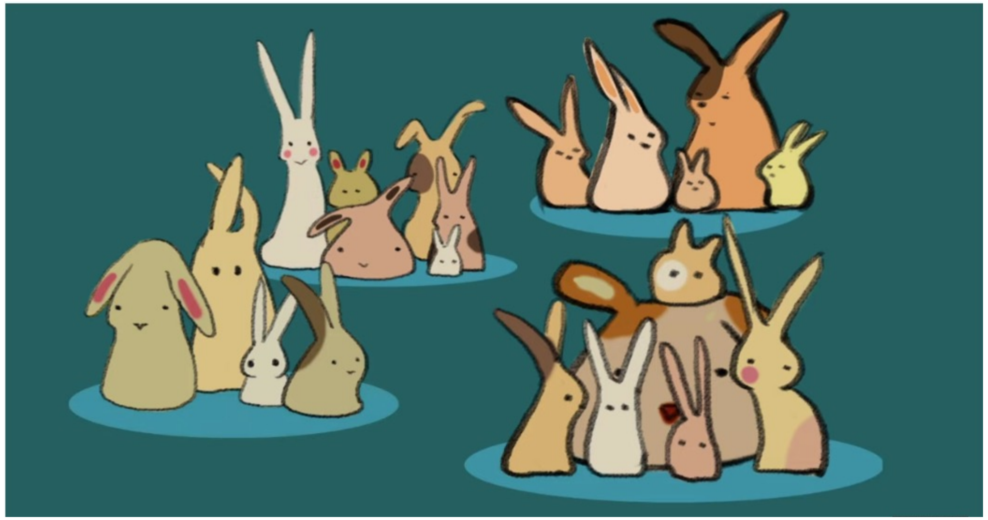
CENTRAL LIMIT THEOREM

Why normal distribution is so important

CENTRAL LIMIT THEOREM

In the video:

- Instead of measuring every single rabbit, weigh samples of size N and compute sample means.

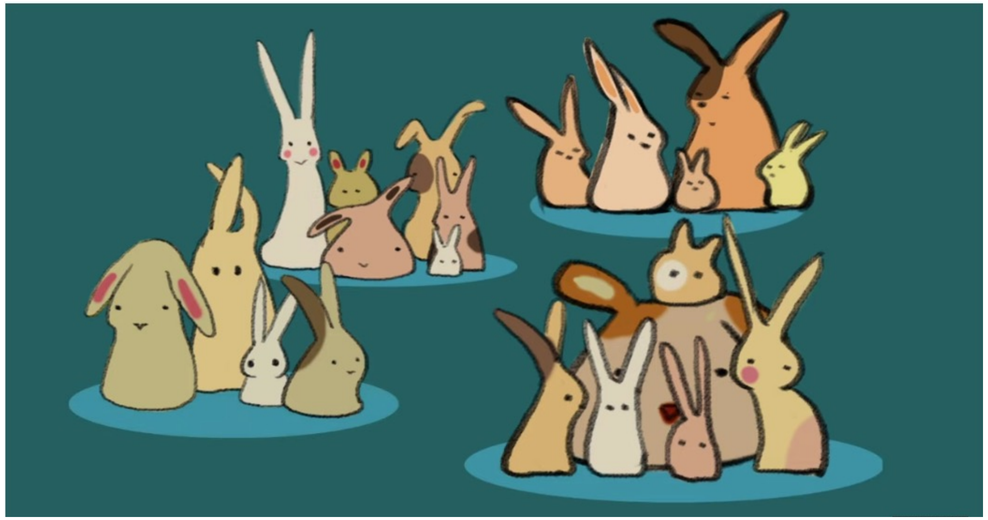


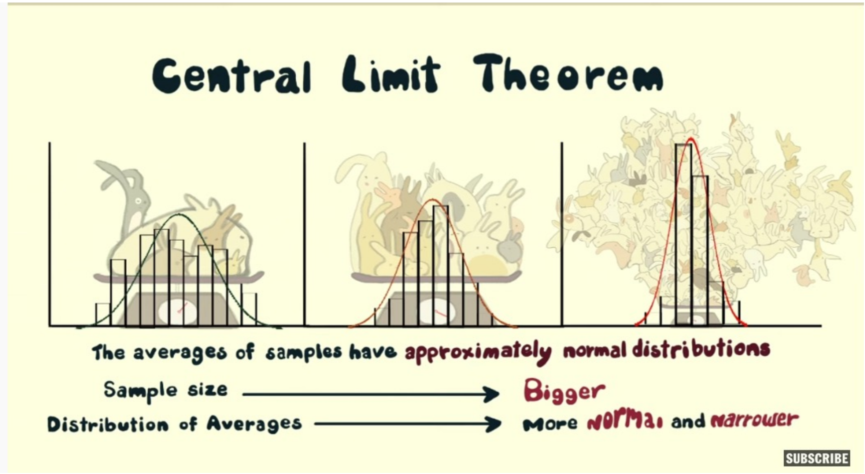
CENTRAL LIMIT THEOREM

In the video:

- Instead of measuring every single rabbit, weigh samples of size N and compute sample means.

- ***Central limit theorem (informally):*** the larger the N, the more "normal" the distribution of the sample averages is.





CENTRAL LIMIT THEOREM

Samples $X_1, X_2, \dots, X_n$:

- i.i.d.
- a *finite* mean μ and *finite* variance σ^2

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$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

CENTRAL LIMIT THEOREM

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Then

$$\bar{X}_n \approx N\left(\quad , \quad \right)$$

CENTRAL LIMIT THEOREM

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- i.i.d.
- a *finite* mean μ and *finite* variance σ^2

Let

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Then

$$\bar{X}_n \approx N\left(\mu, \frac{\sigma^2}{n}\right)$$

CENTRAL LIMIT THEOREM

- $X \sim Po(5)$ - number of errors per computer program
  - $E(X) = Var(X) = 5$

- $X_1, X_2, \ldots, X_{125}$ - number of errors in the programs.

$$P(\bar{X}_n \leq 5.5) =$$

CENTRAL LIMIT THEOREM

- $X \sim Po(5)$ - number of errors per computer program
  - $E(X) = Var(X) = 5$

- $X_1, X_2, \ldots, X_{125}$ - number of errors in the programs.

$$P(\bar{X}_n \leq 5.5) =$$

$$= P\left(\frac{\sqrt{125}(\bar{X}_n - 5)}{\sqrt{5}} \leq \qquad\qquad \right) \approx$$

CENTRAL LIMIT THEOREM

- $X \sim Po(5)$ - number of errors per computer program
  - $E(X) = Var(X) = 5$

- $X_1, X_2, \dots, X_{125}$ - number of errors in the programs.

$$P(\bar{X}_n \le 5.5) =$$

$$= P\left(\frac{\sqrt{125}(\bar{X}_n - 5)}{\sqrt{5}} \le \frac{\sqrt{125}(5.5 - 5)}{\sqrt{5}}\right) \approx$$

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$$\approx P(Z \leq 2.5) = 0.9938$$

CLT IN ACTION

Google Classroom -> Lecture 7 -> Mean of means