PROBABILITY AND STATISTICS

Lecture 13 – Hypothesis testing

HYPOTHESIS TESTING

HYPOTHESIS TESTING

- After tossing a coin 100 times, we observed H only 30 times. Is it a fair coin?
- Did extra tuition help students?
- Is drug A more efficient than drug B?

HOW IT ALL STARTED

https://youtu.be/lgs7d5saFFc

LADY TASTING TEA

Ronald Fisher, 1913

LADY TASTING TEA

- 8 cups of tea
  - 4 cups: milk first
  - 4 cups: tea first

- The lady must select 4 cups prepared by one method.

- *How to check her ability to distinguish the teas?*

Ronald Fisher, 1913

LADY TASTING TEA

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H_0: the lady can't distinguish the teas

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- How unlikely would it be to randomly guess all 4?

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Tea-Tasting Distribution Assuming H_0		
Success count	Combinations of selection	Number of Combinations
0	oooo	1 × 1 = 1
1	ooox, ooxo, oxoo, xooo	4 × 4 = 16
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	6 × 6 = 36
3	oxxx, xoxx, xxox, xxxo	4 × 4 = 16
4	xxxx	1 × 1 = 1
Total		70

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- How unlikely would it be to randomly guess all 4?

$$1/70 \approx 0.014$$

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- How unlikely would it be to randomly guess all 4?

$$1/70 \approx 0.014$$

That's *surprising enough* to reject H_0

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Total		70

HYPOTHESIS TESTING

- Test whether a default assumption about the data is plausible.

- Key idea: you can't *prove* something, but you can disprove it.

HYPOTHESIS TESTING

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HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

HYPOTHESIS TESTING

ASSUMPTION:
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EVIDENCE:

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:
NOT CONVINCING…

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:
NOW IT's CONVINCING!

HYPOTHESIS TESTING

ASSUMPTION:
ALL THE SWANS ARE WHITE

EVIDENCE:

HYPOTHESIS TESTING

ASSUMPTION:
THERE'RE NO ALIENS ON THE MOON

HYPOTHESIS TESTING

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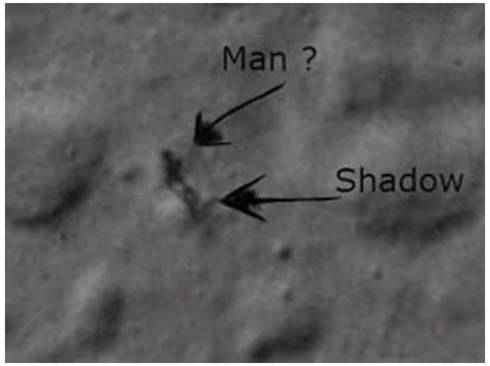
HYPOTHESIS TESTING

ASSUMPTION:
THERE'RE NO ALIENS ON THE MOON

EVIDENCE:

STILL NO REASON TO BELIEVE THERE ARE ALIENS



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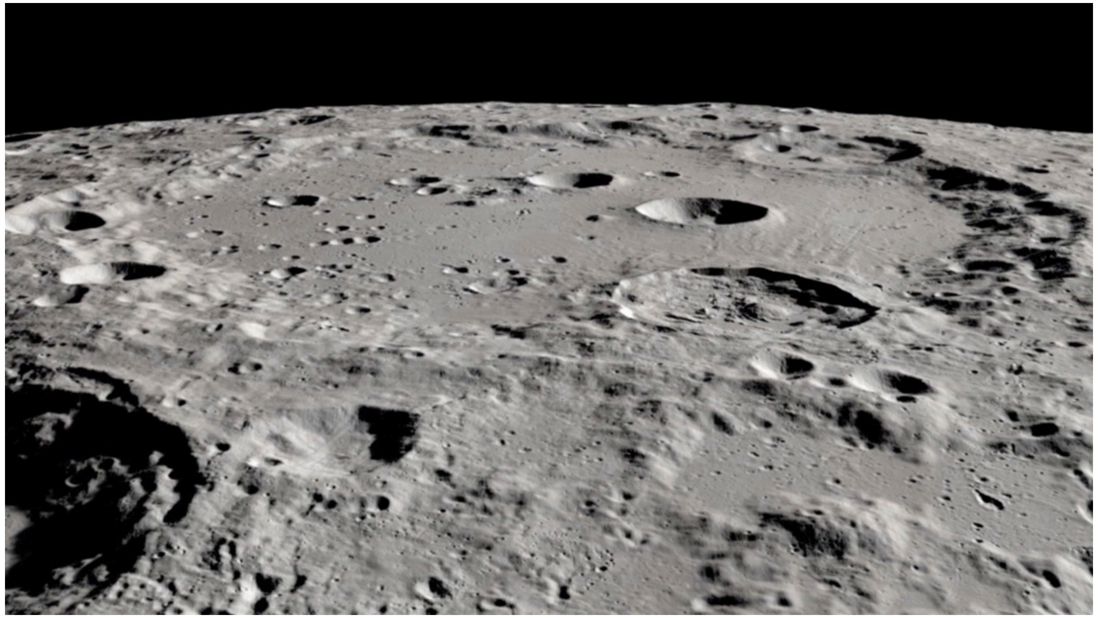

HYPOTHESIS TESTING

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EVIDENCE:

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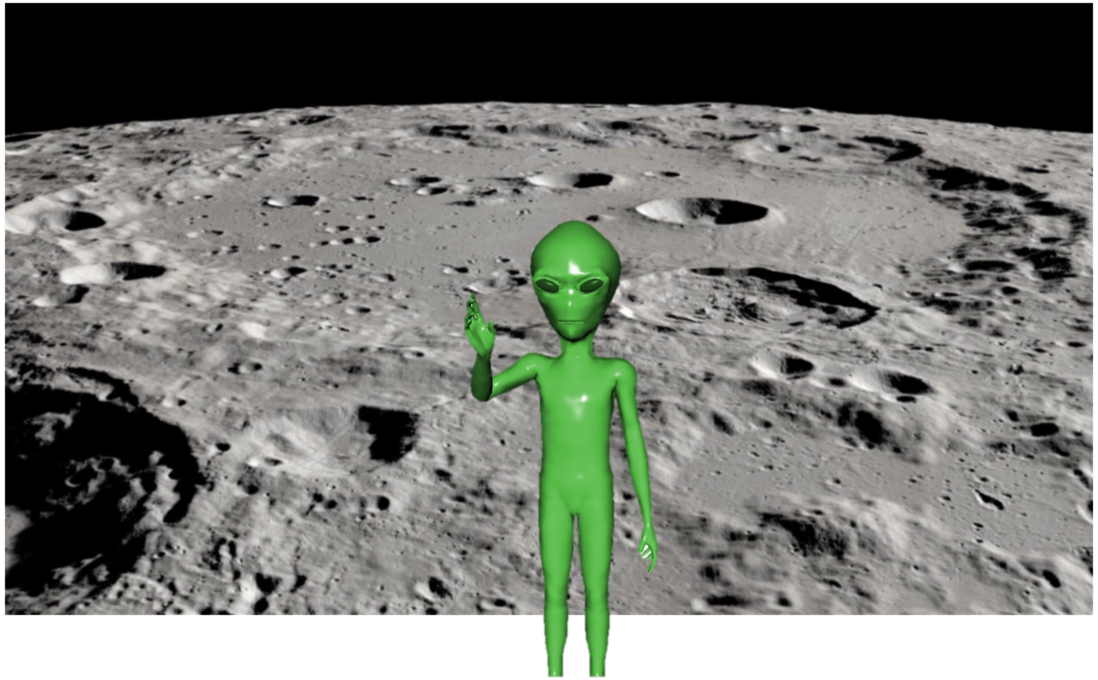

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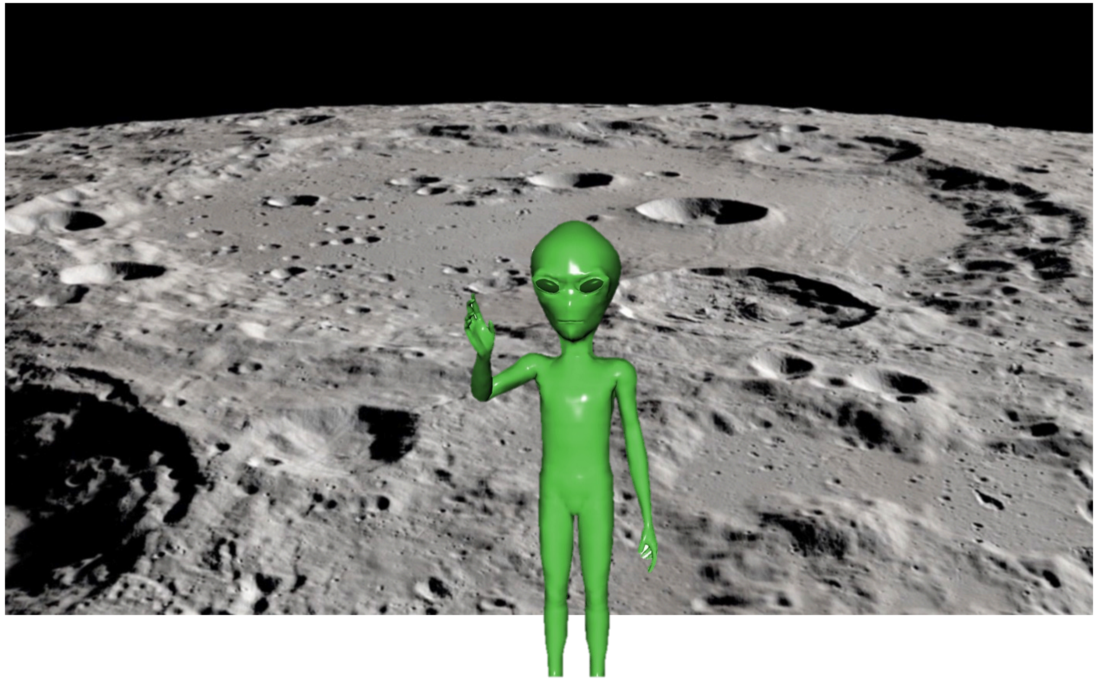

HYPOTHESIS TESTING

- Note:
  - Default assumption = “status quo”.

HYPOTHESIS TESTING

- Note:
  - Default assumption = “status quo”.
  - Alternative: something we can find evidence for.

HOW TO TEST A HYPOTHESIS

BASIC INGREDIENTS

RUNNING EXAMPLE

How to check if a coin is a fair one?

RUNNING EXAMPLE

We flip a coin 10 times to test if it's a fair one.

INGREDIENTS

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$$H_1\colon p > 0.5$$

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One-sided: $H_1 \colon p > 0.5$ or $H_1 \colon p < 0.5$

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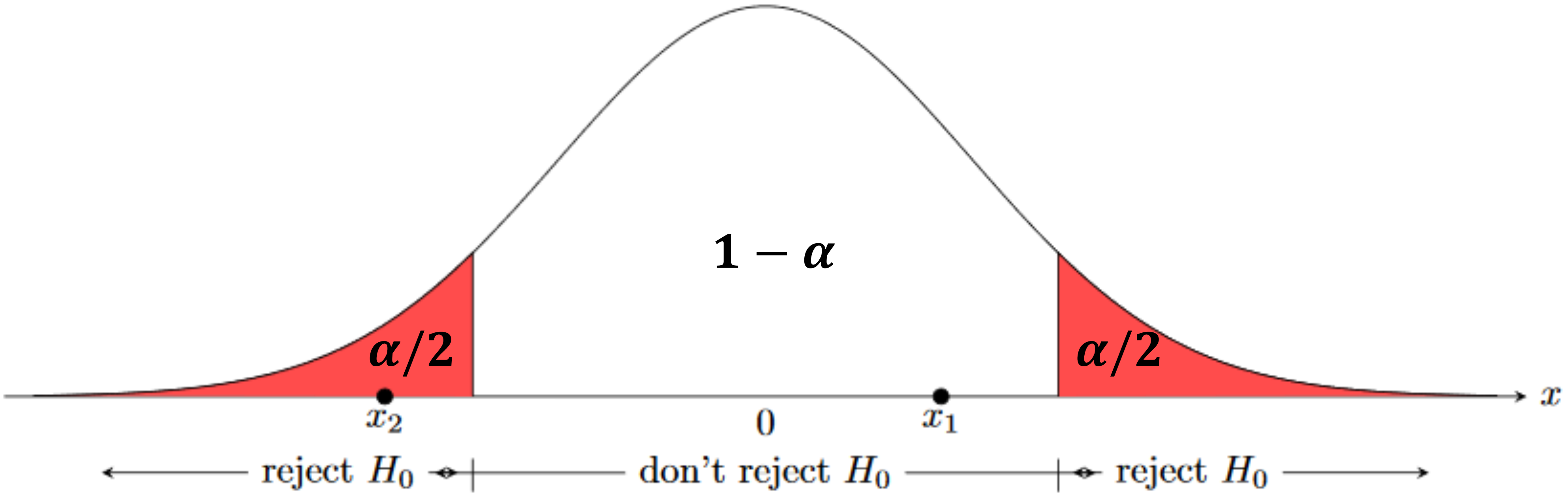
- **Rejection region R**: if X is in the rejection region, we reject H_0 in favor of H_1.
- **Significance level** α **:** $P(X \in R|H_0) \leq \alpha$
  - Typically chosen in advance (common values are 0.1, 0.05, 0.01.)

INGREDIENTS

- Distribution of the test statistic under H_0, two-sided alternative:



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REMARKS

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  - we won’t claim the coin is unfair unless we have compelling evidence.

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- **Rejection region**: data that is *extreme* under the null hypothesis = outcomes in the tail(s) of the null distribution.
  - depends on the significance level α of the test.

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- If we get *0, 1, 9, 10* H
  - Reject H_0

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- If we get 2, *3, 4, 5, 6, 7, 8* H, then the test statistic is in the non-rejection region.
  - Interpretation: the data 'does not support rejecting the null hypothesis'.
  - **Never claim that the data proves the null hypothesis is true.**

EXPERIMENT

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- Toss a coin 10 times, record the number of heads you got.

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- Toss a coin 10 times, record the number of heads you got.

- Based on the result, do you reject the null hypothesis?

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What is the probability to incorrectly reject H_0?

$$P(Reject\ H_0|H_0) = P(X \in R|H_0) = \alpha -$$

significance level

ONE-SIDED ALTERNATIVES

INGREDIENTS

We flip a coin 10 times to test if it's a fair one.

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H_0: $p = 0.5$, $\boldsymbol{H_1}$: $\boldsymbol{p < 0.5}$ 'the biased towards T'

$\boldsymbol{\alpha = 0.05}$

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$P(X\|H_0)$	0.001	0.01	0.044	0.117	0.205	0.246	0.205	0.117	0.044	0.01	0.001

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Z-TEST

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'HarbourSpace students' IQs are the same as those of the general population'

$$H_1: X \sim N(\mu_{HS}, \sigma^2), \qquad \mu_{HS} > \mu_0$$

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- Can we reject H_0?

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- $H_0: X \sim N(\mu_{HS}, \sigma^2),\ \mu = \mu_0$ $\quad$ $H_1: X \sim N(\mu_{HS}, \sigma^2),\ \mu \neq \mu_0$
- Test statistic:

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Assuming H_0, $X_i \sim N(\mu_0, \sigma^2)$

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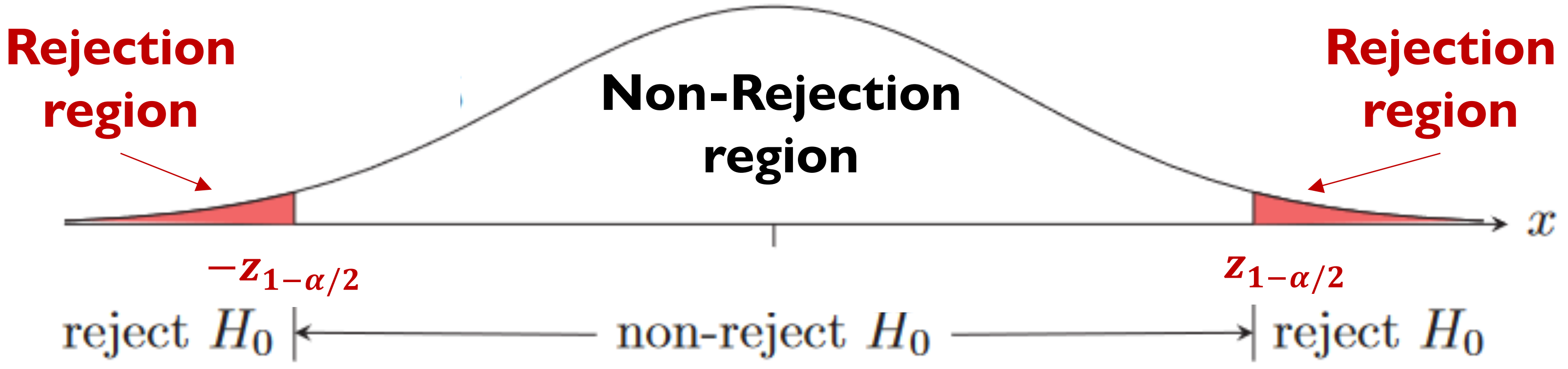

ONE-SIDED ALTERNATIVES

Z-TEST: ONE-SIDED

- $H_0: X \sim N(\mu_{HS}, \sigma^2),\ \mu = \mu_0$ $\quad H_1: X \sim N(\mu_{HS}, \sigma^2),\ \mu > \mu_0$
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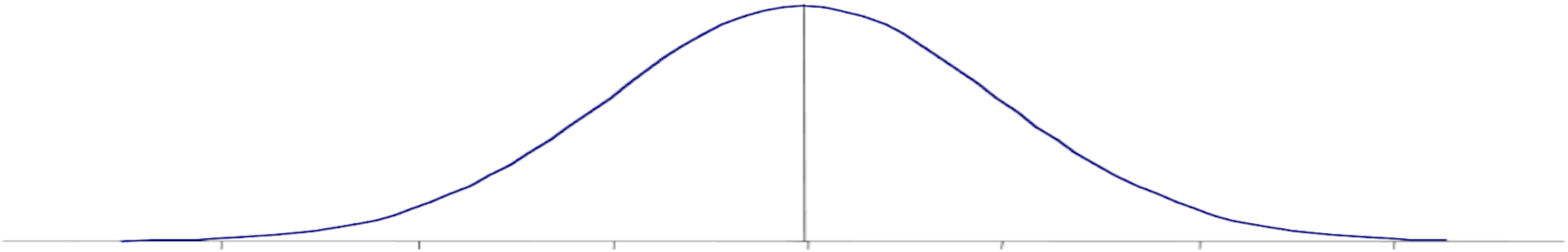

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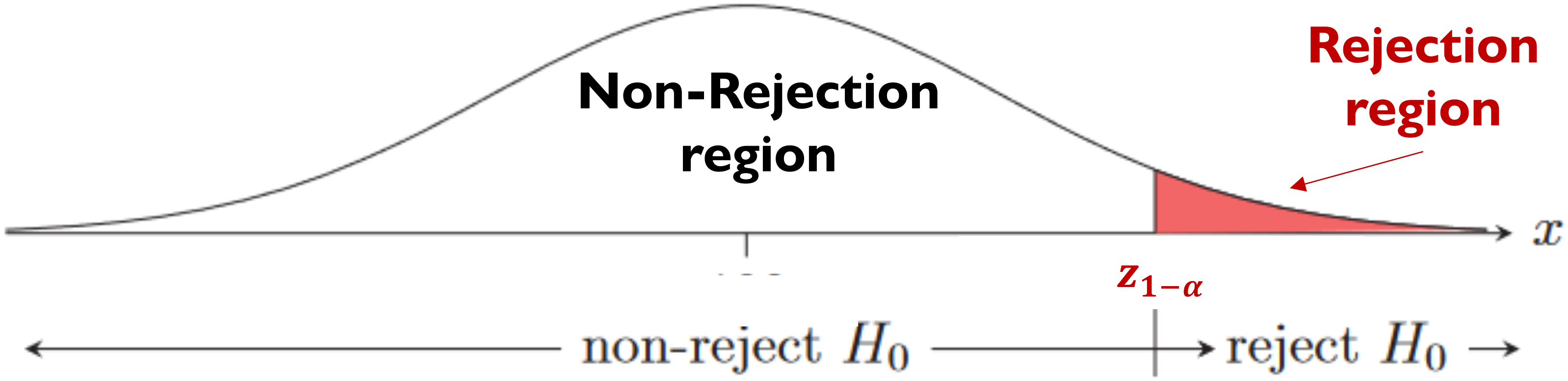

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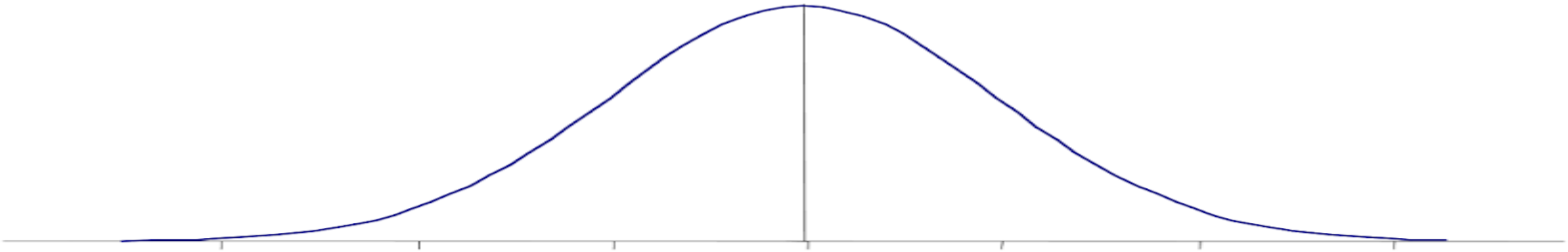

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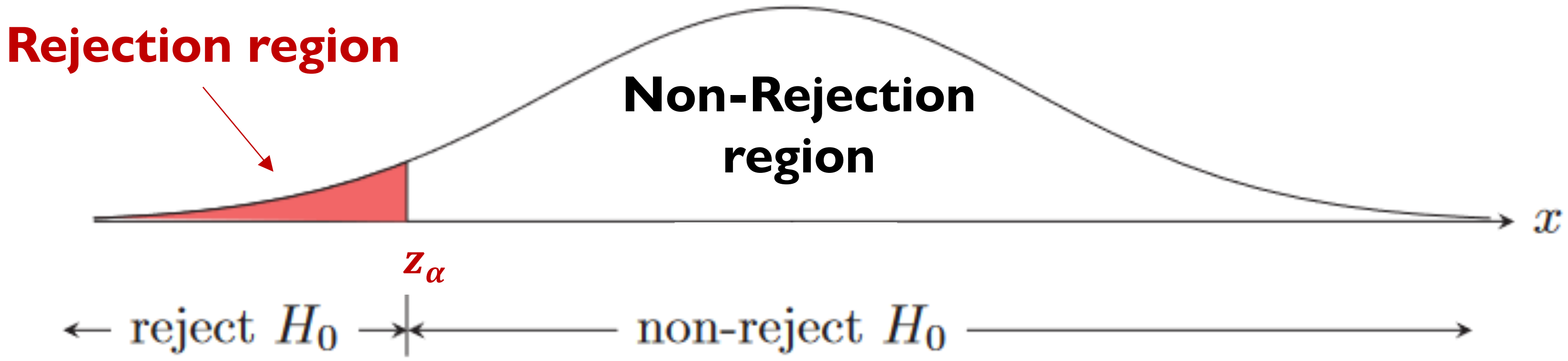

Z-TEST: EXAMPLE 1

- Weight loss is described by $X \sim N(5, 7^2)$.
- $n = 30$ patients followed an experimental diet.
- Average weight loss $\bar{X} = 6.1$ kg.

Does the diet make any difference?

Z-TEST: EXAMPLE

- Assume that weight loss follows $N(\mu, 7^2)$. 30 patients followed an experimental diet. Average weight loss $\bar{X} = 6.1$ kg. Is there evidence that $\mu \neq 5$? Test at $\alpha = 0.05$.

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$$T(X) = \frac{(6.1-5)\sqrt{30}}{7} = 0.86, \quad |T(X)| < 1.96 \implies \text{don't reject } H_0$$

Z-TEST: EXAMPLE 2

- A bakery supplies loaves of bread to supermarkets. Not every loaf weighs the same: loaf weight $X \sim N(\mu_0, 0.1^2)$, supermarket expects $\mu_0 = 2$ kg.
- A supermarket draws a sample of $n = 20$ loaves: average weights is $\bar{X} = 1.97$ kg.
- The supermarket wants to be sure that the weights are, on average, not lower than 2 kg.

Is there evidence against this?

T-TEST

When the data is coming from normal distribution
and variance is unknown

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T-TEST: EXAMPLE 2

- Sales at your company: $X \sim N(\mu_0, \sigma^2)$, $\mu_0 = 100$ dollars per transaction.

- You hire new employees. Based of a sample of $\mathrm{n} = 25$ salesmen, average sale is $\bar{X} = 130$ dollars, with sample standard deviation $\mathrm{s} = 15$.

Are the new employees better than the old ones?
Test your hypothesis at $\alpha = 0.05$.

T-TEST: EXAMPLE

- Based of a sample of $\mathrm{n} = 25$ salesmen, average sale is $\bar{X} = 130$ dollars, with sample standard deviation $\mathrm{s} = 15$. Are the sales larger than \$100 on average? Test at $\alpha = 0.05$. Assume that sales follow $N(\mu, \sigma^2)$.

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$$H_0 : \mu = 100, \qquad H_1 : \mu > 100$$

$$T(X) = \frac{(\bar{X} - \mu_0)\sqrt{n}}{s} \sim t(24), \qquad t_{1-0.05}(24) = 1.711$$

$$T(X) = \frac{(130-100)\sqrt{25}}{15} = 10 > 1.711 \ \Longrightarrow \text{reject } H_0$$

TYPES OF ERRORS

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		True state of nature	
		H_0	H_A
Our decision	Reject H_0		
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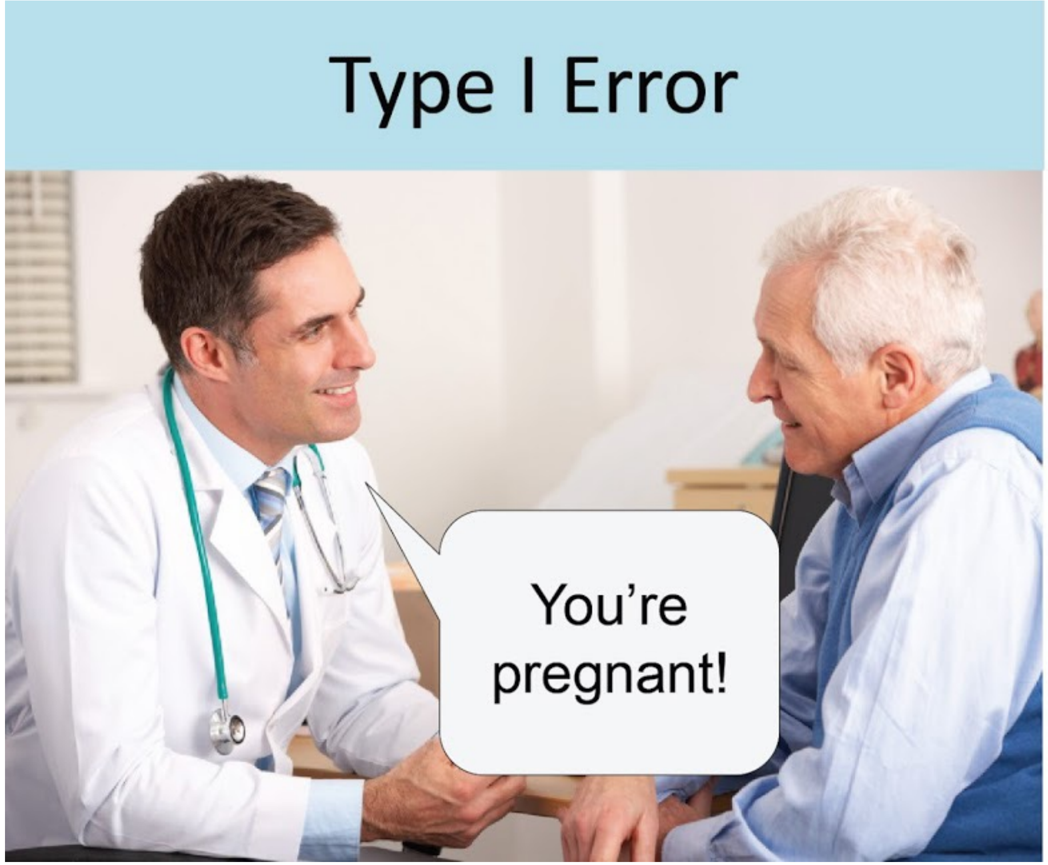

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$$\alpha = P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0) = P(T \in R \mid H_0)$$

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We want power near and significance near

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We want power near 1 and significance near 0

P-VALUES

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- Up till now:
  - Rejecting H_0 based on the value of the test statistic:

$$T \in R \implies \text{reject } H_0, \qquad T \notin R \implies \text{do not reject } H_0$$

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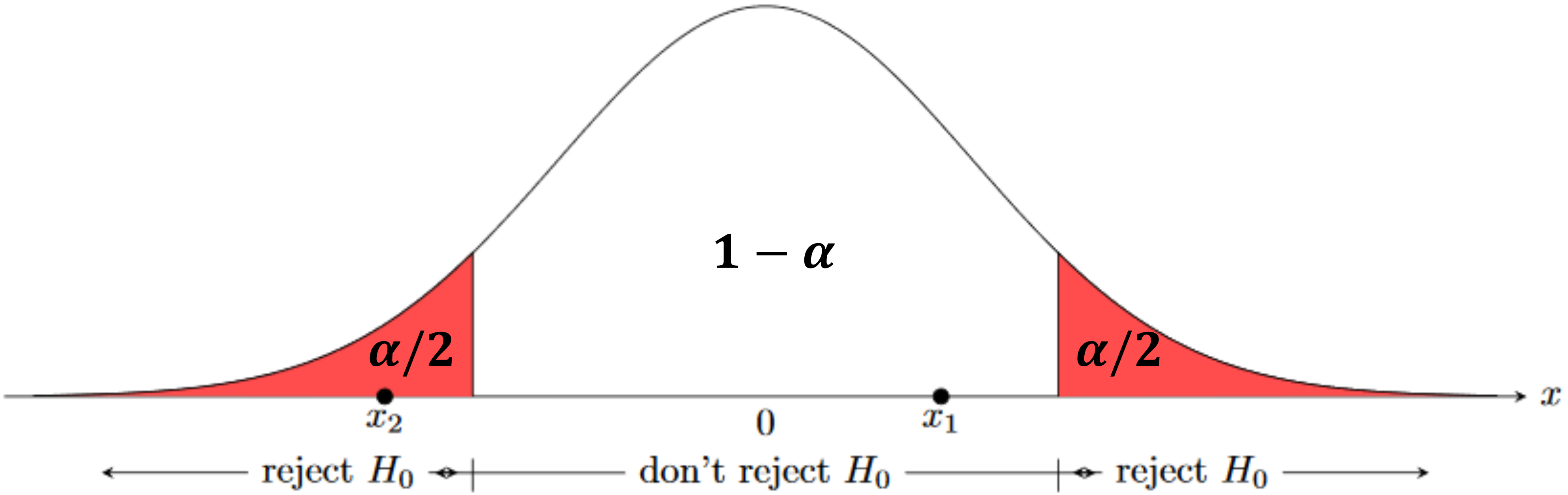
- A more unified approach: report **p-value**.

P-VALUES



$1-\alpha$
$\alpha/2$
$\alpha/2$
x
x_2
0
x_1
reject H_0
don't reject H_0
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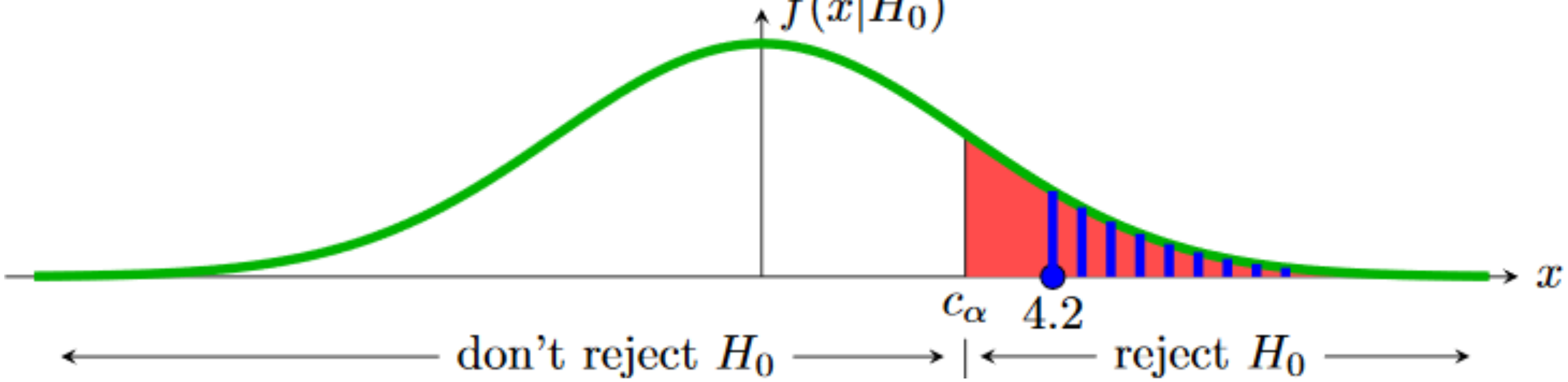
P-VALUES

- Example:
  - Suppose we have the right-sided rejection region. We see data with test statistic t = 4.2. Should we reject H_0?



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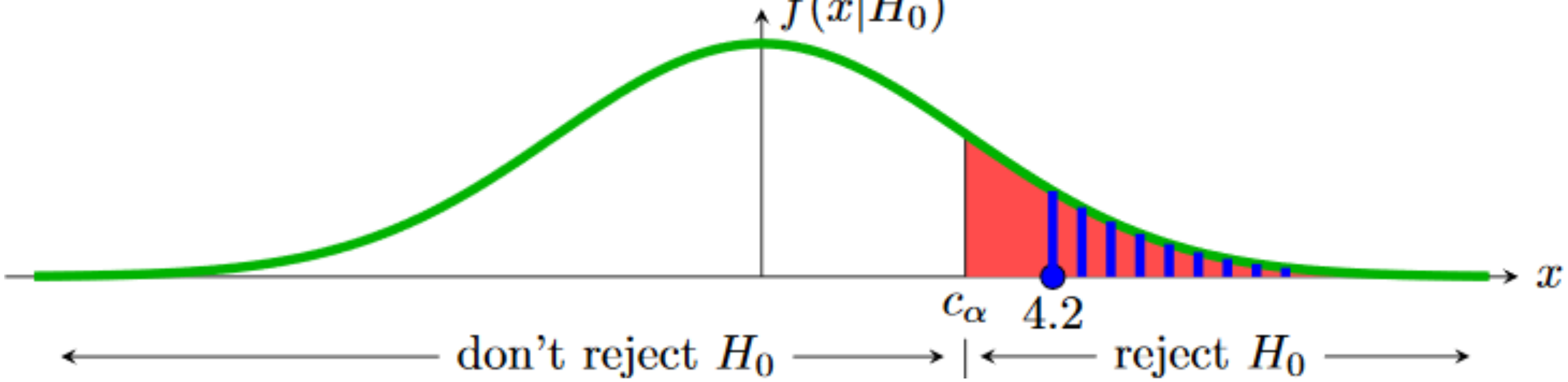

- Answer:
  - Yes, if $T = 4.2$ is in the rejection region.

P-VALUES

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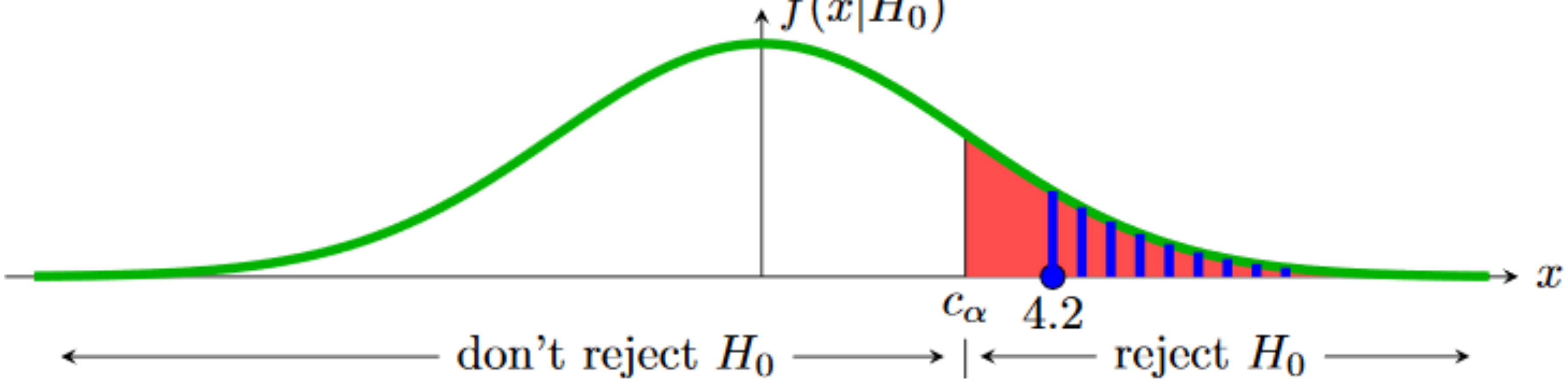

- Answer:
  - Yes, if $T = 4.2$ is in the rejection region.
- Alternatively:
  - Yes, if **blue area** < **red area**

P-VALUES

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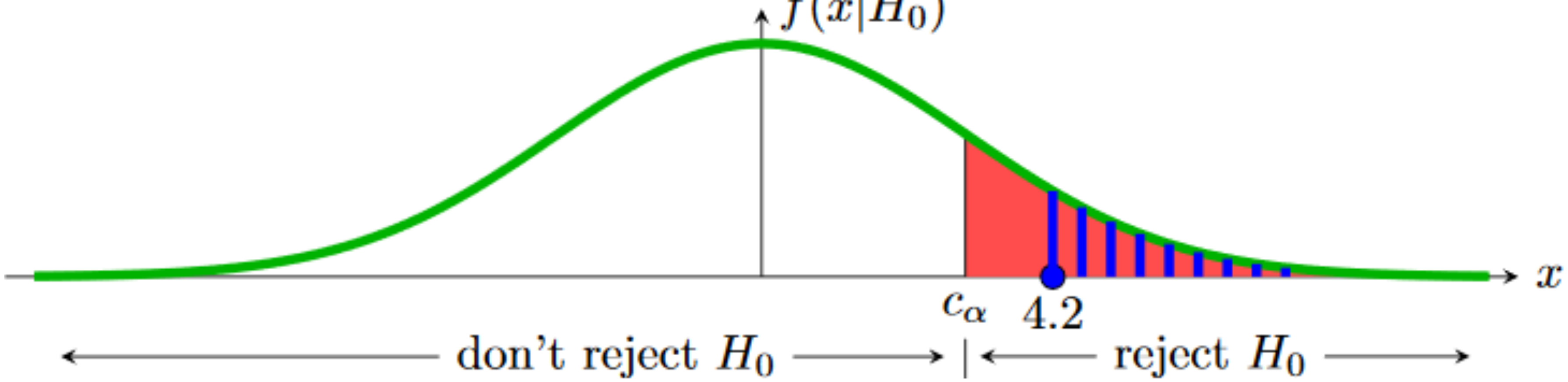

- Significance: $\alpha = P(T \in R|H_0)$ = red area.
- p-value: $p = P(\text{data at least as extreme as T}|H_0)$ = blue area

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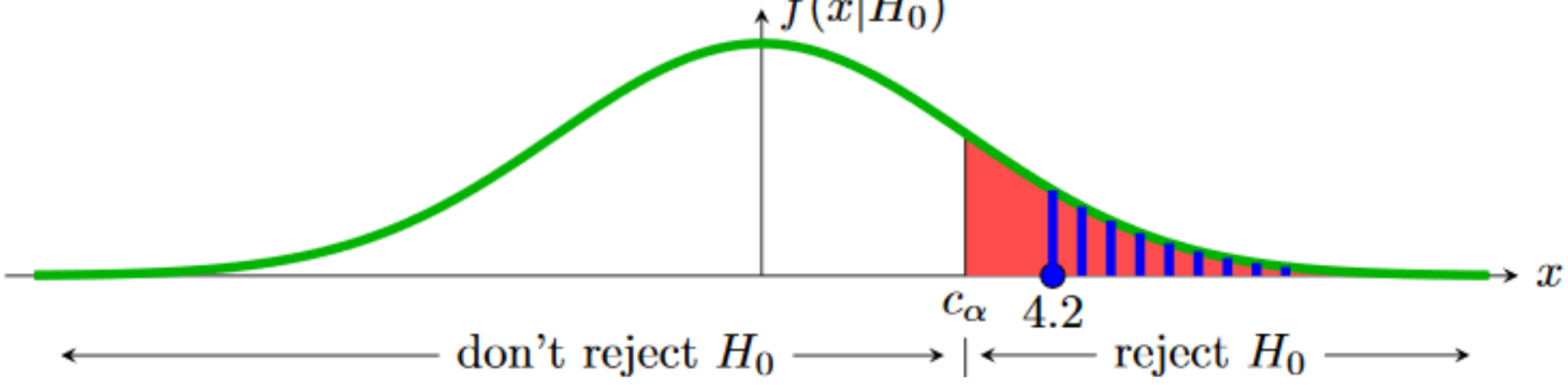

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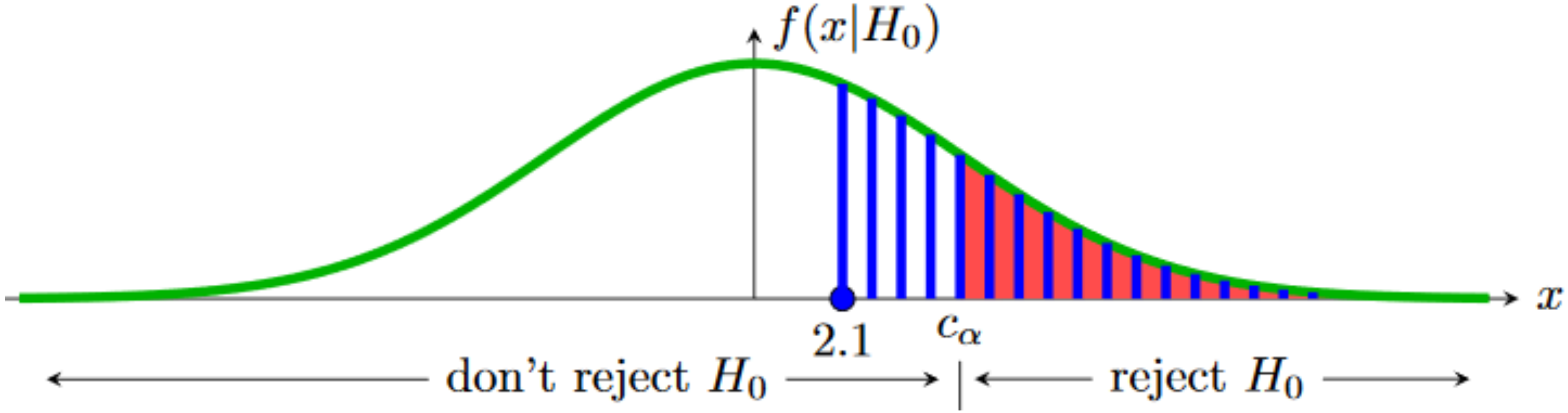
P-VALUES

- Example:
  - Suppose we have the right-sided rejection region. We see data with test statistic t $= 2.1$. Should we reject H_0?



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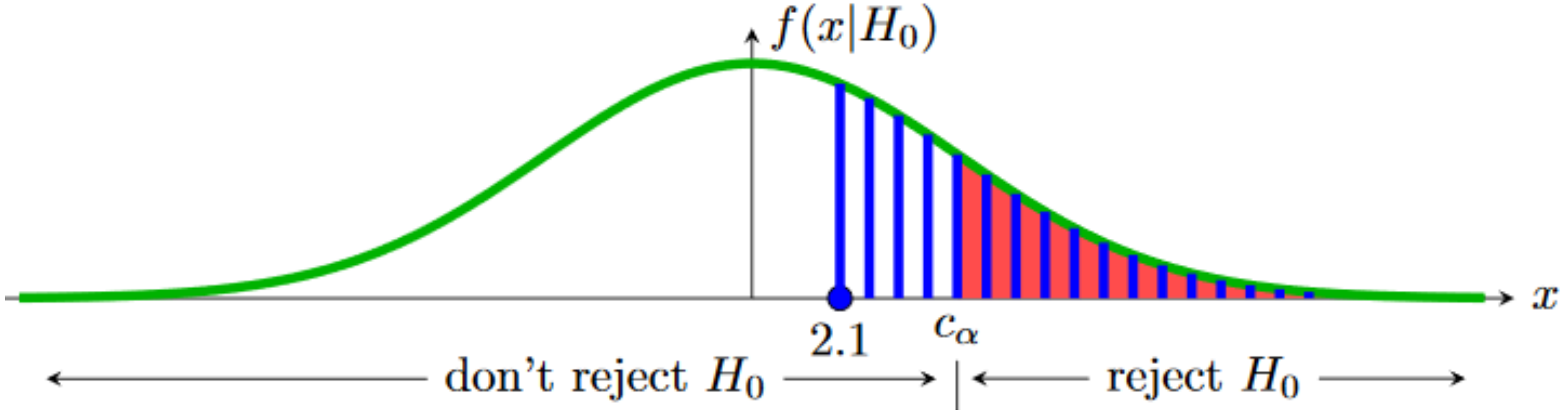

- Significance: $\alpha = P(T \in R|H_0)$ = red area.
- p-value: $p = P(\text{data at least as extreme as T}|H_0)$ = blue area

Since $p > \alpha$, do not reject H_0

P-VALUES

- Similarly, with the other one-sided alternative:
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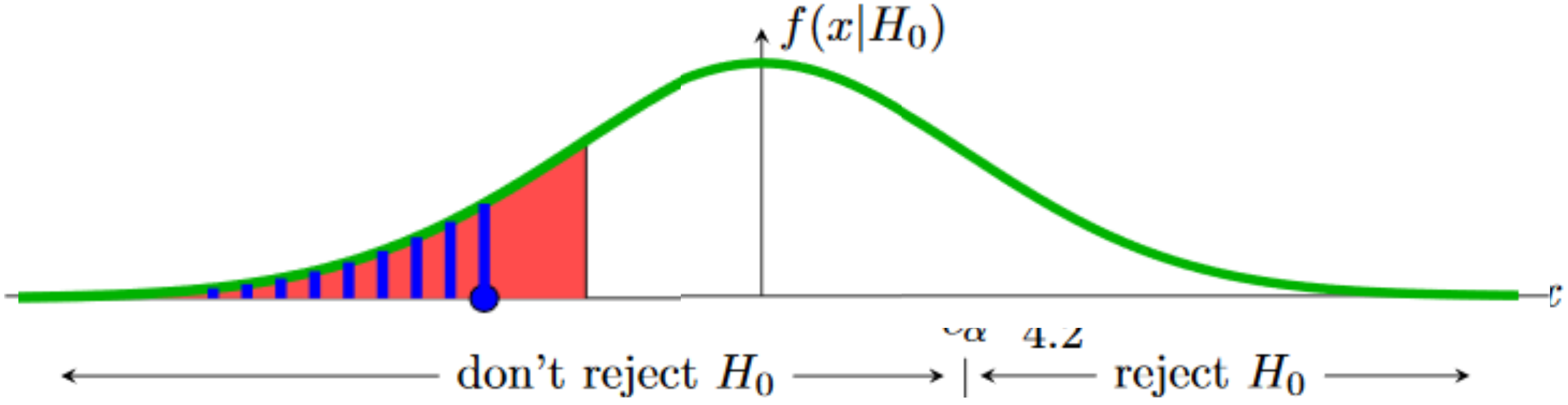

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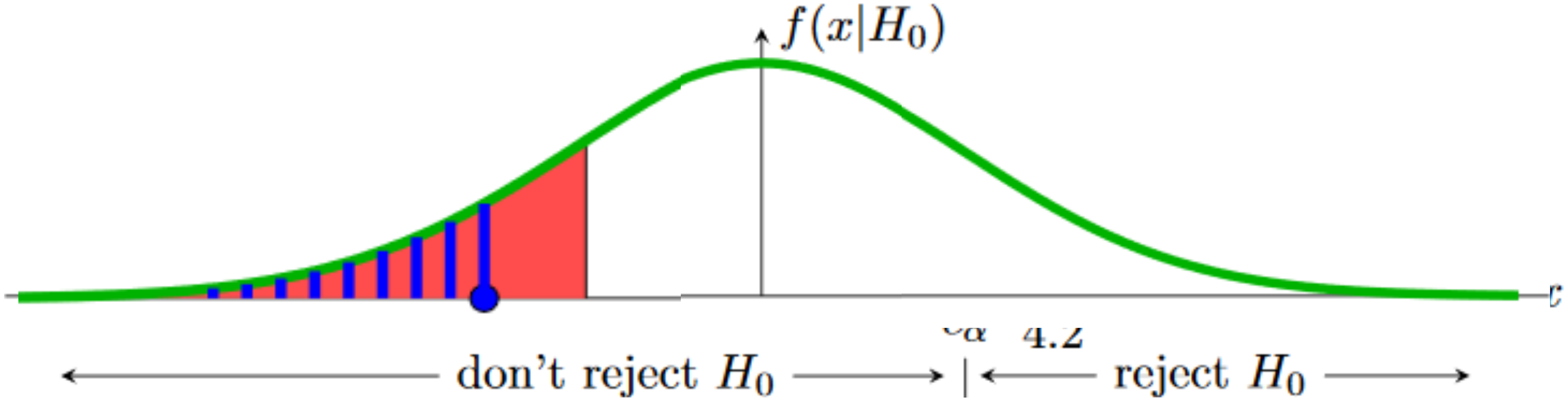

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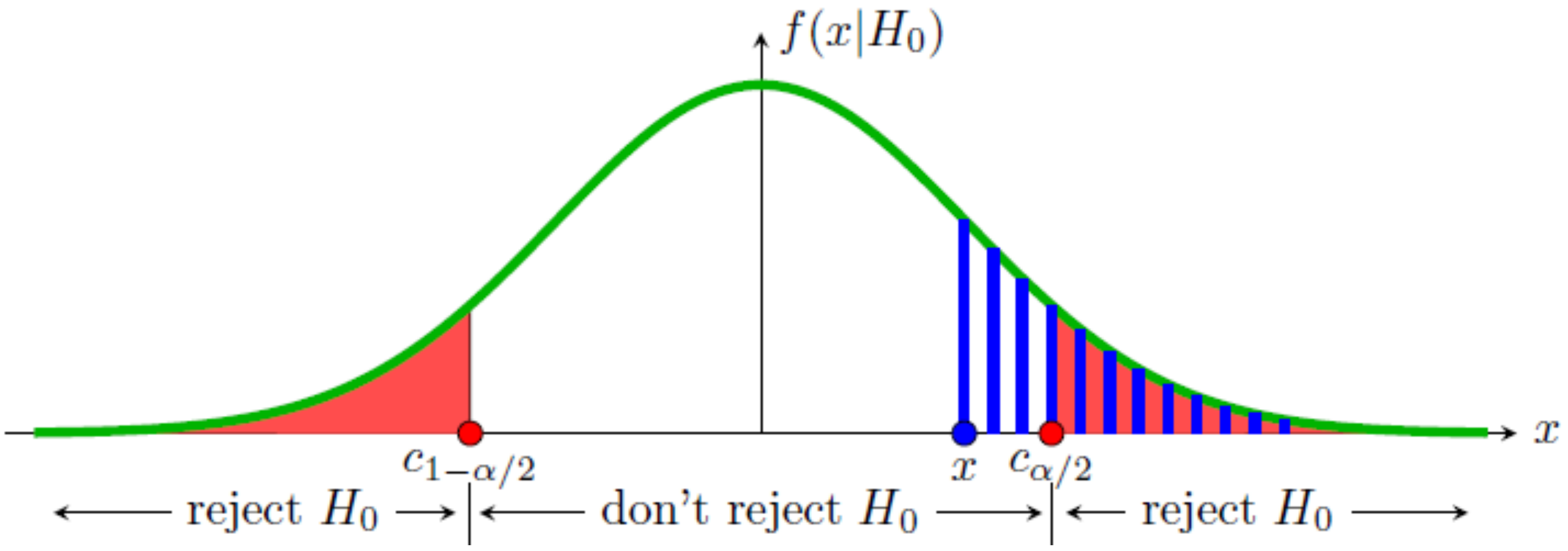

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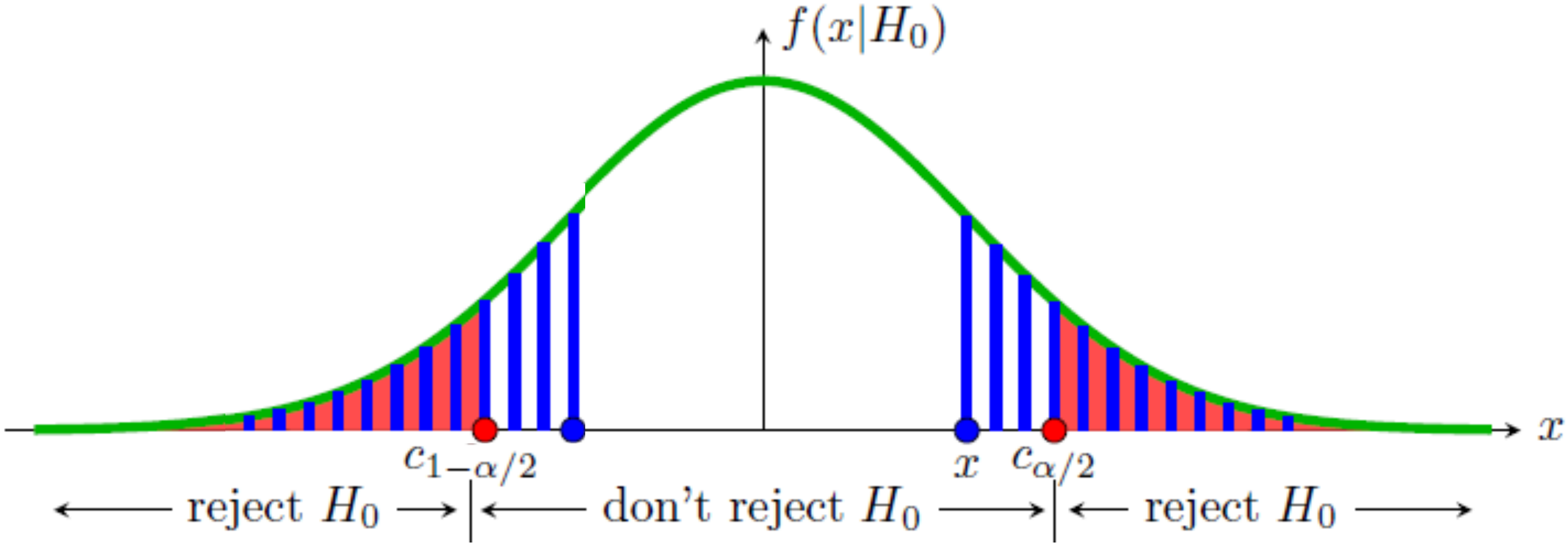

P-VALUES

- Two-sided p-values are a bit trickier:
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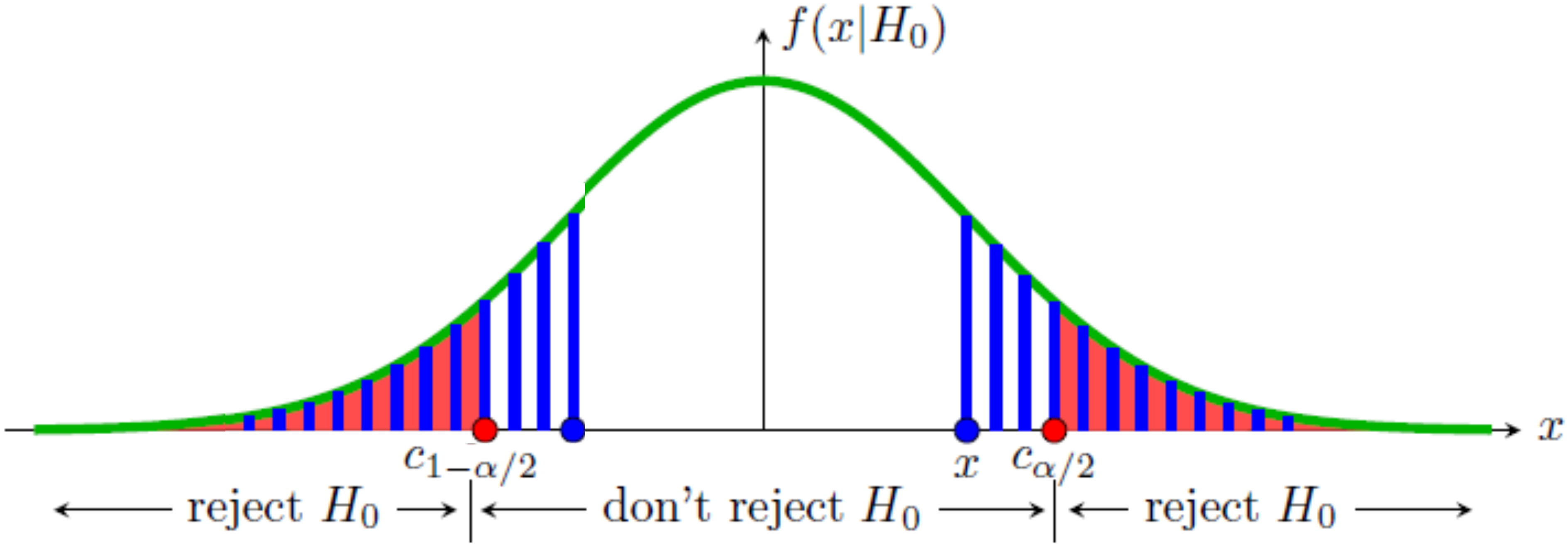
$$p = \mathbf{2}\min\{P(T > t), P(T \leq t)\}$$



P-VALUES

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$$p < \alpha \implies \textbf{Reject } H_0$$

MORE TESTS

MORE TESTS

- There exist a lot of tests.

- Don’t try to memorize all the tests – there’re too many of them.

- Your task is to find the right test when you need it.

- Here are some of the most used ones **(but not all)**.

TWO-SAMPLE TESTS

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- Consider the task of comparing the means of two **independent** samples.

TWO-SAMPLE TESTS

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- Examples:
  - comparing the mean efficacies of two medical treatments;
  - comparing academic performance in two different groups;

TWO-SAMPLE T-TESTS

Test difference in the means of two populations
with unknown but equal variances

TWO-SAMPLE T-TEST

- Two sets of data drawn from normal distribution (i.i.d.):

$$X_1, X_2, \dots, X_n \sim N(\mu_X, \sigma^2)$$

$$Y_1, Y_2, \dots, Y_m \sim N(\mu_Y, \sigma^2)$$

- Means μ_X and μ_Y and the variance σ^2 are **unknown.**
  - Note that the two distributions have **the same variance**.

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- Null hypothesis: H_0: $\mu_X - \mu_Y = 0$
- Alternatives:
    - Two-sided: H_1: $\mu_X - \mu_Y \neq 0$
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TWO-SAMPLE T-TEST

- Test statistic:

$$T(X,Y) = \frac{\bar{X}-\bar{Y}}{s_p}, \text{ where}$$

$$s_p^2 = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2} \cdot \left(\frac{1}{n} + \frac{1}{m}\right)$$

s_X^2, s_Y^2 - sample variances of X and Y.

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- Null distribution:

$$\frac{\bar{X}-\bar{Y}}{s_p} \sim t(n+m-2)$$

TWO-SAMPLE T-TEST: EXAMPLE

- Women admitted to maternity hospital, weeks of pregnancy measured in two groups.
  - Medical: $n = 775, \;\; \bar{X} = 39.08, \;\; s_X^2 = 7.77$
  - Emergency: $m = 633, \;\; \bar{Y} = 39.60, \;\; s_Y^2 = 4.95$

Set up and run two-sample t-test to check if the mean duration of pregnancy differs in the two groups.

Which assumptions do you have to make?

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H_0:

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$$T = \frac{\bar{X} - \bar{Y}}{s_p} \sim t(n + m - 2)$$

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$$s_p^2 = \frac{774 \cdot 7.77 + 632 \cdot 4.95}{775 + 633 - 2}\left(\frac{1}{775} + \frac{1}{633}\right) = 0.0187$$

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$$T \sim t(1408) \approx N(0,1)$$

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$$H_0\colon \mu_x - \mu_Y = 0 - \textbf{rejected}$$

$$H_1\colon \mu_x - \mu_Y \neq 0$$

TWO-SAMPLE T-TEST: EXAMPLE

- Assumptions made:
  - Length of pregnancy is normally distributed
  - Variances are the same in both groups – *unrealistic*

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  - Variances are the same in both groups – *unrealistic*

- **GOOD TO KNOW:**
  - There're special tests to check if the data is normal and if two groups have the same variances.
  - In practice, it's good to apply them first to check the validity of the assumptions.

WELCH'S TESTS

The case of unequal variances

WELCH'S TEST

- Two sets of data drawn from normal distribution (i.i.d.):

$$X_1, X_2, \ldots, X_n \sim N(\mu_X, \sigma_X^2)$$

$$Y_1, Y_2, \ldots, Y_m \sim N(\mu_Y, \sigma_Y^2)$$

- Means μ_X and μ_Y and the variances σ_X^2 and σ_Y^2 are **unknown.**
  - Note that the variances aren't assumed to be equal.

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- Alternatives:
  - Two-sided: $H_1\colon \mu_X - \mu_Y \neq 0$
  - One-sided: $H_1\colon \mu_X - \mu_Y > 0$ or $H_1\colon \mu_X - \mu_Y < 0$

WELCH'S TEST

- Test statistic:

$$T(X,Y) = \frac{\bar{X}-\bar{Y}}{s_p}, \text{ where}$$

$$s_p^2 = \frac{s_X^2}{n} + \frac{s_Y^2}{m}, \qquad df = \frac{(s_X^2/n + s_Y^2/m)^2}{(s_X^2/n)^2/(n-1) + (s_Y^2/m)^2/(m-1)}$$

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s_X^2, s_Y^2 - sample variances of X and Y.

- Null distribution:

$$\frac{\bar{X}-\bar{Y}}{s_p} \sim t(df)$$

PAIRED SAMPLE T-TEST

Difference between the means when data comes in pairs

PAIRED DATA

- Sometimes, data naturally comes in pairs $(X_i, Y_i), i = 1, \ldots, n$.

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- "Repeated measures":
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  - performance of two different algorithms on the same set of benchmarks.
- *Was there any effect?*

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  - performance of two different algorithms on the same set of benchmarks.
- *Was there any effect?*
- Samples are paired = dependent.

PAIRED SAMPLE T-TEST

- Two **dependent** sets of data drawn from normal distribution:

$$X_1, X_2, \ldots, X_n, \qquad E(X_i) = \mu_X$$

$$Y_1, Y_2, \ldots, Y_n, \qquad E(Y_i) = \mu_Y$$

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- Let's introduce $D = X_i - Y_i$ Assumption: $D \sim N(\mu, \sigma^2)$

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FROM NOW, IT'S JUST A ONE-SAMPLE T-TEST IN TERMS OF D

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- Let's introduce $D = X_i - Y_i$ Assumption: $D \sim N(\mu, \sigma^2)$
- Null hypothesis:

$$H_0: E(D) = \mu = 0$$

- Alternatives:
  - Two-sided: $H_1: E(D) = \mu \neq 0$
  - One-sided: $H_1: \mu > 0$ or $H_1: \mu < 0$

PAIRED SAMPLE T-TEST

- Test statistic:

$$T(X,Y) = \frac{\bar{D}\sqrt{n}}{s},$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2$$

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$$T(X,Y) = \frac{\bar{D}\sqrt{n}}{s} \sim t(n-1)$$

NON-PARAMETRIC TESTS

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- Up till now: **parametric** models
  - Example: mean μ and variance σ^2

NON-PARAMETRIC TESTS

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  - Example: mean μ and variance σ^2

- **Non-parametric statistics** doesn’t rely on data belonging to any parametric family of probability distributions:
  - distribution-free or
  - having a specified distribution but with the distribution's parameters unspecified.

NON-PARAMETRIC TESTS

PARAMETRIC HYPOTHESES	NON-PARAMETRIC HYPOTHESES
• Data comes from the normal distribution with specified mean and variance.	
• Data comes from the normal distribution with specified mean and unspecified variance.	

NON-PARAMETRIC TESTS

PARAMETRIC HYPOTHESES	NON-PARAMETRIC HYPOTHESES
• Data comes from the normal distribution with specified mean and variance.	• Data comes from a normal distribution form with both mean and variance unspecified.
• Data comes from the normal distribution with specified mean and unspecified variance.	• Two unspecified continuous distributions are identical.

MANN-WHITNEY U TEST

An alternative to the two-sample t-test
when the distribution of the data cannot be assumed to be normal

MANN-WHITNEY U TEST

- Two independent i.i.d. sets of data:

$$X_1, X_2, \dots, X_n$$

$$Y_1, Y_2, \dots, Y_m$$

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H_0: For randomly selected values X and Y from two populations,

$$P(X > Y) = P(X < Y)$$

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- Null hypothesis:

H_0: For randomly selected values *X* and *Y* from two populations,

$$P(X > Y) = P(X < Y)$$

- Alternatives
  - Two-sided: $P(X > Y) \neq P(X < Y)$
  - One-sided: $P(X > Y) > P(X < Y)$, $\quad P(X > Y) < P(X < Y)$

MANN-WHITNEY U TEST

- Test statistic:

$$U = \sum_{i=1}^{n} \sum_{j=1}^{m} S(X_i, Y_j)$$

where $S(X,Y) = \begin{cases} 1, & Y < X \\ 1/2, & Y = X \\ 0, & Y > X \end{cases}$

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- Null distribution:

For small n, tabulated

For large n, $(n \geq 30)$ $U \sim \text{normal}$

WILCOXON SIGNED RANK TEST

An alternative to the paired t-test
when the distribution of the differences cannot be assumed to be normal

WILCOXON SIGNED RANK TEST

- Two paired i.i.d. sets of data:

$$X_1, X_2, \dots, X_n$$

$$Y_1, Y_2, \dots, Y_m$$

WILCOXON SIGNED RANK TEST

- Two paired i.i.d. sets of data:

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- Null hypothesis:

H_0: difference between the pairs follows a symmetric distribution around zero

WILCOXON SIGNED RANK TEST

- Two paired i.i.d. sets of data:

$$X_1, X_2, \dots, X_n$$

$$Y_1, Y_2, \dots, Y_m$$

- Null hypothesis:

H_0: difference between the pairs follows a symmetric distribution around zero

- Alternatives
  - Two-sided: difference between the pairs doesn't follow a symmetric distribution around zero
  - One-sided: distribution is skewed to one particular side

WILCOXON SIGNED RANK TEST

- Test statistic:

WILCOXON SIGNED RANK TEST

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- Null distributions:
  - some specific tabulated distribution.

χ^2-TEST

Independence of two categorical variables based on contingency table

χ^2-TEST

- Motivating example: consider the following **contingency table**:

Education	Married once	Married multiple times	Total
College	550	61	611
No college	681	144	825
Total	1231	205	1436

χ^2-TEST

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- Are education levels and number of marriages (one / many) independent?

χ^2-TEST

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Total	1231	205	1436

- Are education levels and number of marriages (one / many) independent?

If so, cell probabilities are the product of the marginal ones:

Education	Married once	Married multiple times	Total
College			$611/1436$
No college			$825/1436$
Total	$1231/1436$	$205/1436$	1

χ^2-TEST

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- Are education levels and number of marriages (one / many) independent?

If so, cell probabilities are the product of the marginal ones:

Education	Married once	Married multiple times	Total
College	0.365		$611/1436$
No college			$825/1436$
Total	$1231/1436$	$205/1436$	1

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If so, cell probabilities are the product of the marginal ones:

Education	Married once	Married multiple times	Total
College	0.365	0.061	$611/1436$
No college	0.492		$825/1436$
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χ^2-TEST

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If so, cell probabilities are the product of the marginal ones:

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No college	0.492	0.082	825/1436
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χ^2-TEST

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College	0.365	0.061	611/1436
No college	0.492	0.082	825/1436
Total	1231/1436	205/1436	1

- From this, we can get ***expected counts*** *(multiplying cell probabilities by the total number of women surveyed):*

Education	Married once	Married multiple times
College	550, 523.8	61,
No college	681,	144,

χ^2-TEST

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- From this, we can get ***expected counts*** *(multiplying cell probabilities by the total number of women surveyed):*

Education	Married once	Married multiple times
College	550, 523.8	61, 87.2
No college	681,	144,

χ^2-TEST

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χ^2-TEST

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H_0: cell probabilities are the product of the marginal ones, the difference, so the difference between them should be small

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H_1: the difference between observed and expected counts is large **(!!! one-sided)**

χ^2-TEST

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College	550, 523.8	61, 87.2
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- Test statistic: Pearson's chi-square statistic

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(df), \qquad df = (n-1)(m-1)$$

where O_i - observed count, and E_i - expected count

χ^2-TEST

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- In this case

$$\chi^2 =$$

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$$\chi^2 = 16.01, \ df = 1$$

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$$\chi^2_{1-\alpha}(1) = \chi^2_{0.95}(1) = 7.879$$

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$$16.01 > 7.879 \Longrightarrow \text{reject } H_0$$

NON-PARAMETRIC TESTS

- Fewer assumptions, wider applicability.

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- This comes with the cost: when parametric tests are applicable, non-parametric ones have less *power*
  - *a larger sample size can be required to draw conclusions with the same degree of confidence.*