PROBABILITY AND STATISTICS

Lecture 5 – Discrete distributions

LAST TIME

- Discrete random variables
  - PMF
  - CDF
  - Expectation and variance
- Practice problems

TODAY

- Two random variables
- More on expected value and variance
- Standard discrete distributions and their properties

PROBLEM FROM YESTERDAY

- You toss a coin untill you get Heads. What is the expected number of trial that you will need?

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$$P(X = k) =$$

$$EX =$$

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$$P(X = k) = \begin{cases} \dfrac{1}{2^k}, & k \geq 1 \\ 0, & otherwise \end{cases}$$

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$$EX = \sum_{k=1}^{+\infty} k \cdot P(X = k) = \sum_{k=1}^{+\infty} \frac{k}{2^k} = \dots = 2$$

STANDARD DISCRETE DISTRIBUTIONS

Part I

Bernoulli distribution

BERNOULLI DISTRIBUTION

- Consider a random experiment with two possible outcomes: "success" (with probability p) or "failure" (with probability $1-p$)
  - tossing a coin: H or T;
  - a new child: a boy r a girl;
  - you take an exam: pass or fail.

BERNOULLI DISTRIBUTION

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- Consider a random variable X

x	0	1
$P(X = x)$		

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- Consider a random variable $X \sim \boldsymbol{Bernoulli}(\boldsymbol{p})$

x	0	1
$P(X=x)$	$1-p$	p

A TOURIST

- An English-speaking tourist visits a country in which 30% of the population speaks English. He needs to ask someone directions.

  Find the probability that the first person he encounters will be able to speak English.

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$$X \sim Bernoulli(\quad) \rightarrow$$

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$$X \sim Bernoulli(0.3) \rightarrow$$

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$$X \sim Bernoulli(0.3) \rightarrow P(X = 1) = p = 0.3$$

BERNOULLI

- Consider $X \sim Bernoulli(p)$. What is $EX = ?$

x	0	1
$P(X = x)$	$1 - p$	p

$$EX =$$

BERNOULLI

- Consider $X \sim Bernoulli(p)$. What is $EX = ?$

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$$EX = 0 \cdot (1 - p) + 1 \cdot p = p$$

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Example: algorithm finds a correct solution with $p = 0.8$. If we run it very many times, what % of the trials, on average, be successful?

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<u>Example:</u> algorithm finds a correct solution with $p = 0.8$. If we run it very many times, what % of the trials, on average, be successful?

Each trial: success or fail (Bernoulli random variable X) -> EX = 0.8

BERNOULLI

- Consider $X \sim Bernoulli(p)$. What is $Var[X] = ?$

x	0	1
$P(X = x)$	$1 - p$	p

$$EX = 0 \cdot (1 - p) + 1 \cdot p = p$$

$$Var[X] = EX^2 - (EX)^2 =$$

BERNOULLI

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$P(X = x)$	$1 - p$	p

$$EX = 0 \cdot (1 - p) + 1 \cdot p = p$$

$$Var[X] = EX^2 - (EX)^2 = p - p^2 = p(1 - p)$$

Poisson distribution

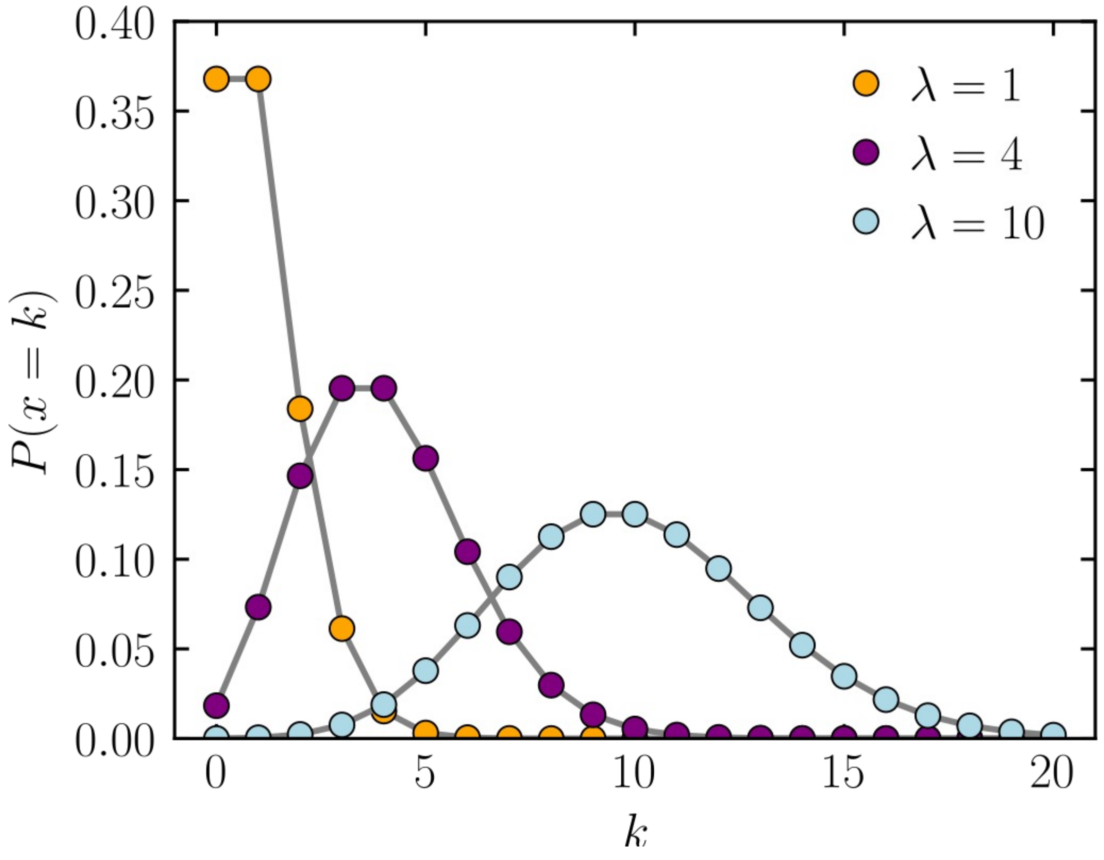
POISSON DISTRIBUTION

$X \sim Po(\lambda), \quad \lambda > 0$

$$P(X = k) = \begin{cases} \frac{e^{-\lambda} \cdot \lambda^k}{k!}, k \geq 0 \\ 0, \text{otherwise} \end{cases}$$

$E(X) = ?, \quad Var(X) = ?$

- Models number of events occurring in a fixed interval of time.
- Assumptions: events occur
  - with a known constant mean rate;
  - independently of the time since the last event.



HOW DOES THIS DISTRIBUTION LOOK LIKE?

1. Plot PMF for different values of the parameter $\lambda > 0$.

2. What is the expected value of $X \sim Poisson(\lambda)$?

$$EX =$$

3. What is the variance of $X \sim Poisson(\lambda)$?

HOW DOES THIS DISTRIBUTION LOOK LIKE?

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$$EX = \sum_{k=0}^{+\infty} k \cdot \frac{e^{-\lambda} \lambda^k}{k!} = \cdots ? \ldots =$$

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$$EX = \sum_{k=0}^{+\infty} k \cdot \frac{e^{-\lambda}\lambda^k}{k!} = \cdots ? \ldots = \lambda$$

3. What is the variance of $X \sim Poisson(\lambda)$?

$$Var(X) = \lambda$$

POISSON DISTRIBUTION: EXAMPLE

$$P(X = k) = \begin{cases} \frac{e^{-\lambda} \cdot \lambda^k}{k!}, k \geq 0 \\ 0, \text{otherwise} \end{cases}$$

- A given restaurant receives an average of 100 customers per day.
- Random variable $X \sim Poisson(100)$ – number of customers the restaurant will receive today.
- Find the probability that the restaurant receives
  - 110 customers:
  - 90 customers:
  - less than 100 customers:

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$$P(X = 110) = \frac{e^{-100} 100^{110}}{110!} \approx 0.0234$$

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    $$P(X = 110) = \frac{e^{-100} 100^{110}}{110!} \approx 0.0234$$
  - 90 customers:
    $$P(X = 90) = \frac{e^{-100} 100^{90}}{90!} \approx 0.025$$
  - less than 100 customers:

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  - 110 customers:
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  - 90 customers:
$$P(X = 90) = \frac{e^{-100} 100^{90}}{90!} \approx 0.025$$
  - less than 100 customers:
$$P(X < 100) = \sum_{k=0}^{99} \frac{e^{-100} 100^k}{k!} \approx 0.4867$$

Two random variables

TWO RANDOM VARIABLES

- Let X and Y be two random variables.
- **Joint** distribution of X and Y: $P_{XY}(x, y) = P(X = x \ \boldsymbol{and} \ Y = y)$

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
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$$P(X = 1, Y = 1) =$$

$$P(X = 0, Y \leq 1) =$$

$$P(X = 0) =$$

$$P(X = 0 \mid Y = 1) =$$

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$$P(X = 1, Y = 1) = \frac{1}{6}$$

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$$P(X = 1, Y = 1) = \frac{1}{6}$$

$$P(X = 0, Y \leq 1) = \frac{1}{6} + \frac{1}{4}$$

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$$P(X = 0) = \frac{1}{6} + \frac{1}{4} + \frac{1}{8}$$

$$P(X = 0 \mid Y = 1) = \frac{1/4}{\frac{1}{4} + \frac{1}{6}}$$

TWO RANDOM VARIABLES

- Let X and Y be two random variables.
- **Marginal distribution**

$$P_X(x) = P(X = x) = \sum_i P(X = x, Y = y_i),$$

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

TWO RANDOM VARIABLES

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- Let X and Y be two random variables.
- **Marginal distribution**

$P_X(x) = P(X = x) = \sum_i P(X = x, Y = y_i)$, $\quad P_Y(y) = P(Y = y) = \sum_j P\left(X = x_j, Y = y\right)$

$$P_X(x) = \begin{cases} \frac{13}{24} & x = 0 \\ \frac{11}{24} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

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$$P_X(x) = \begin{cases} \frac{13}{24} & x=0 \\ \frac{11}{24} & x=1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{7}{24} & y=0 \\ \frac{5}{12} & y=1 \\ \frac{7}{24} & y=2 \\ 0 & \text{otherwise} \end{cases}$$

TWO RANDOM VARIABLES

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

- Let X and Y be two random variables.
- **Conditional distribution**

$$P(X|Y=0):$$

x	0	1
$P(X=x \mid Y=0)$		

TWO RANDOM VARIABLES

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- Let X and Y be two random variables.
- **Conditional distribution**

$$P(X|Y=0):$$

x	0	1
$P(X=x \mid Y=0)$	$\dfrac{1/6}{1/6+1/8}$	$\dfrac{1/8}{1/6+1/8}$

$$P(Y|X=1):$$

TWO RANDOM VARIABLES

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

- Let X and Y be two random variables.
- **Conditional distribution**

$$P(X|Y=0):$$

x	0	1
$P(X=x \mid Y=0)$	$\dfrac{1/6}{1/6+1/8}$	$\dfrac{1/8}{1/6+1/8}$

$$P(Y|X=1):$$

x	0	1	2
$P(X=x \mid Y=0)$			

TWO RANDOM VARIABLES

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x	0	1
$P(X=x \mid Y=0)$	$\dfrac{1/6}{1/6+1/8}$	$\dfrac{1/8}{1/6+1/8}$

$$P(Y|X=1):$$

x	0	1	2
$P(X=x \mid Y=0)$	$\dfrac{1/8}{1/8+1/6+1/6}$	$\dfrac{1/6}{1/8+1/6+1/6}$	$\dfrac{1/6}{1/8+1/6+1/6}$

TWO RANDOM VARIABLES

- Let X and Y be two random variables.
- X and Y are independent if and only if

$$P(X = x \, and \, Y = y) = P_X(x) \cdot P_y(y) \quad \forall x \in R_x, y \in R_Y$$

TWO RANDOM VARIABLES

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- X and Y are independent if and only if

$$P(X = x \; and \; Y = y) = P_X(x) \cdot P_y(y) \quad \forall x \in R_x, y \in R_Y$$

- Are X and Y independent?

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

$$P_X(x) = \begin{cases} \frac{13}{24} & x = 0 \\ \frac{11}{24} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{7}{24} & y = 0 \\ \frac{5}{12} & y = 1 \\ \frac{7}{24} & y = 2 \\ 0 & \text{otherwise} \end{cases}$$

TWO RANDOM VARIABLES

- Let X and Y be two random variables.
- X and Y are independent if and only if

$$P(X = x \text{ and } Y = y) = P_X(x) \cdot P_y(y) \quad \forall x \in R_x, y \in R_Y$$

- Are X and Y independent? **No!**

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

$$P_X(x) = \begin{cases} \frac{13}{24} & x = 0 \\ \frac{11}{24} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{7}{24} & y = 0 \\ \frac{5}{12} & y = 1 \\ \frac{7}{24} & y = 2 \\ 0 & \text{otherwise} \end{cases}$$

E.g., $P(X = 0, Y = 0) = \frac{1}{6} \neq P_X(0) \cdot P_Y(1) = \frac{13}{24} \cdot \frac{7}{24}$

Expected value and variance

EXPECTED VALUE

- Let X be a discrete random variable that takes values from R_X.
- Expected value of X is defined as

$$EX = \sum_{x_k \in R_X} x_k \cdot P(X = x_k)$$

- Let a and b be some numbers. Then

$$E(aX + b) = aEX + b$$

EXAMPLE

- Let X be a random variable with $EX = 10$.
- Let $Y = 5X + 1$

$$E(Y) =$$

- Let $Z = 3X - 2$

$$E(Z) =$$

EXAMPLE

- Let X be a random variable with $EX = 10$.
- Let $Y = 5X + 1$

$$E(Y) = 5 \cdot 10 + 1 = 51$$

- Let $Z = 3X - 2$

$$E(Z) =$$

EXAMPLE

- Let X be a random variable with $EX = 10$.
- Let $Y = 5X + 1$

$$E(Y) = 5 \cdot 10 + 1 = 51$$

- Let $Z = 3X - 2$

$$E(Z) = 3 \cdot 10 - 3 = 28$$

EXPECTED VALUE

- Let $X_1, \dots, X_n$ be some random variables such that $EX_i = \mu_i$.
- It is *always* true that

$$E[X_1 + X_2 + \cdots + X_n] = EX_1 + \cdots + EX_n = \mu_1 + \cdots + \mu_n$$

VARIANCE

- Let X be a random variable with expected value EX.
- Variance is defined as

$$Var(X) = E[(X - EX)^2]$$

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$$Var(X) = E[(X - EX)^2] =$$
$$E[X^2 - 2X \cdot EX + (EX)^2] =$$

VARIANCE

- Let X be a random variable with expected value EX.
- Variance is defined as

$$Var(X) = E[(X - EX)^2] =$$
$$E[X^2 - 2X \cdot EX + (EX)^2] =$$
$$EX^2 - 2(EX^2) + (EX)^2 =$$
$$EX^2 - (EX)^2$$

VARIANCE OF A LINEAR COMBNATION

- X is a random variable, a and b are some numbers.

$$Var(aX + b) =$$

VARIANCE
OF A LINEAR COMBNATION

- X is a random variable, a and b are some numbers.

$$Var(aX+b)=$$

$$=E\big(aX+b-E(aX+b)\big)^2=$$

$$=E(aX+b-aEX-b)^2=E(aX-aEX)^2=$$

$$=a^2E(X-EX)^2=$$

$$=a^2Var(X)$$

VARIANCE
OF A LINEAR COMBNATION

- If X is a random variable with $Var(X) = 1$, compute

$$Var(-5X + 10) =$$

VARIANCE OF A LINEAR COMBNATION

- If X is a random variable with $Var(X) = 1$, compute

$$Var(-5X + 10) =$$
$$25 \cdot Var(X) = 25$$

VARIANCE OF A SUM

- Let's consider two random variables, X with variance $Var(X)$ and Y with variance $Var(Y)$.

$$Var(X+Y) =$$

VARIANCE OF A SUM

- Let's consider two random variables, X with variance $Var(X)$ and Y with variance $Var(Y)$.

$$Var(X + Y) =$$

$$E[X + Y - E(X + Y)]^2 =$$

VARIANCE OF A SUM

- Let's consider two random variables, X with variance $Var(X)$ and Y with variance $Var(Y)$.

$$Var(X+Y) =$$
$$E[X+Y-E(X+Y)]^2 =$$
$$E[(X-EX)-(Y-EY)]^2 =$$
$$E[(X-EX)^2-2(X-EX)(Y-EY)+(Y-EY)^2] =$$
$$Var(X)+Var(Y)-2E(X-EX)(Y-EY) =$$

VARIANCE OF A SUM

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$$E[(X-EX)^2-2(X-EX)(Y-EY)+(Y-EY)^2] =$$
$$Var(X)+Var(Y)-2E(X-EX)(Y-EY) =$$
$$Var(X)+Var(Y)-2Cov(X,Y).$$

COVARIANCE

- **Covariance** is a measure of *joint variability* between two random variables.

$$Cov(X,Y) = E[(X - EX)(Y - EY)]$$

COVARIANCE

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- There is also a shortcut formula

$$Cov(X,Y) = E(XY) - EX \cdot EY$$

COVARIANCE AND INDEPENDENCE

- When random variables X and Y are independent,

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COVARIANCE AND INDEPENDENCE

- When random variables X and Y are independent,

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COVARIANCE AND INDEPENDENCE

- When random variables X and Y are independent,

$$Cov(X,Y) = E(XY) - EX \cdot EY = EX \cdot EY - EX \cdot EY = 0$$

- Therefore, when X and Y are independent,

$$Var(X,Y) = Var(X) + Var(Y).$$

CORRELATION

- Let X and Y be two random variables with variances $Var(X)$ and $Var(Y)$.
- Correlation between X and Y is defined as

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}} = \frac{Cov(X,Y)}{std(X) \cdot std(Y)}$$

- What is the difference between covariance and correlation?
- Covariance is scale-dependent. Correlation isn't.

Programming exercise

Google classroom -> Day 5