PROBABILITY & STATISTICS

Lecture 3 – Conditional probability

LAST TIME

- Frequentist interpretation of probability
  - Python
  - Practice problems

TODAY

- Multiple events
- Conditional probability
  - The law of total probability
  - Bayes Rule

FLIPPING A COIN 10 TIMES

- Which outcome you think is more likely?

HEADS, TAILS, HEADS, TAILS, TAILS,
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Independent events

INDEPENDENT EVENTS

- Two events A and B from the same sample space S are independent if

$$P(A\&B) = P(A) \cdot P(B)$$

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$$P(EF) = P(E) \cdot P(F) = \frac{1}{36} \rightarrow E\&F \text{ are independent!}$$

Practice problems

Google classroom -> Day 3

FLIPPING A COIN

- A coin is flipped 6 times. Let E be the event that heads and tails come up an equal number of times. Let F be the event that heads and tails come up once each during the first two flips of the coin. Are E and F independent events?

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$\rightarrow E$ & F aren't independent

FLIPPING A COIN 2

- Let E be the event that when a coin is flipped three times, we don't get all heads or all tails. Let F be the event that when a coin is flipped three times, heads comes up at most once. Are E and F independent events?

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$\rightarrow E \;\&\; F$ are independent!

FACULTY BOARD

- Faculty board, consisting of 15 professors (10 men and 5 women) and 10 students (5 men and 5 women), is randomly choosing a candidate to represent the faculty at the visiting day.

  Are the following events independent?

$$E_1 = \{man\}, \qquad E_2 = \{professor\}$$

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$$P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2) \rightarrow$$

the events are not independent

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Conditional probability

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- 14% of the population aged 65+.
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Note: $P(M \,\&65+) = P(M|65+) \cdot P(65+)$

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FLIPPING A COIN 1

- A coin is flipped 4 times. What is the probability that heads comes up exactly twice given that heads comes up on the first flip?

A – H comes up twice $\qquad P(A) = \frac{C(4,2)}{2^4} = \frac{3}{8}$

B – H comes up on the 1st flip $\qquad P(B) = \frac{2^3}{2^4} = \frac{1}{2}$

AB – H comes up twice incl. on the 1st flip $\qquad P(AB) = \frac{C(3,1)}{2^4} = \frac{3}{16}$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{3 \cdot 2}{16 \cdot 1} = \frac{3}{8}$$

BOWLS AND BALLS

- There are two bowls: bowl A contains 2 blue and 3 red balls, and bowl B contains 1 blue and 4 red balls. You randomly chose a bowl and then pick one ball from it.
- What is the probability of picking a blue ball?
- What is the probability of picking a blue ball if we chose bowl A?
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$$P(Blue|A) =$$

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$$P(Blue|A) = \frac{2}{5}$$

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$$P(Blue|A) = \frac{2}{5}$$

- What is the probability of picking a blue ball if we chose bowl B?

$$P(Blue|B) =$$

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- What is the probability of picking a blue ball?
- What is the probability of picking a blue ball if we chose bowl A?

$$P(Blue|A) = \frac{2}{5}$$

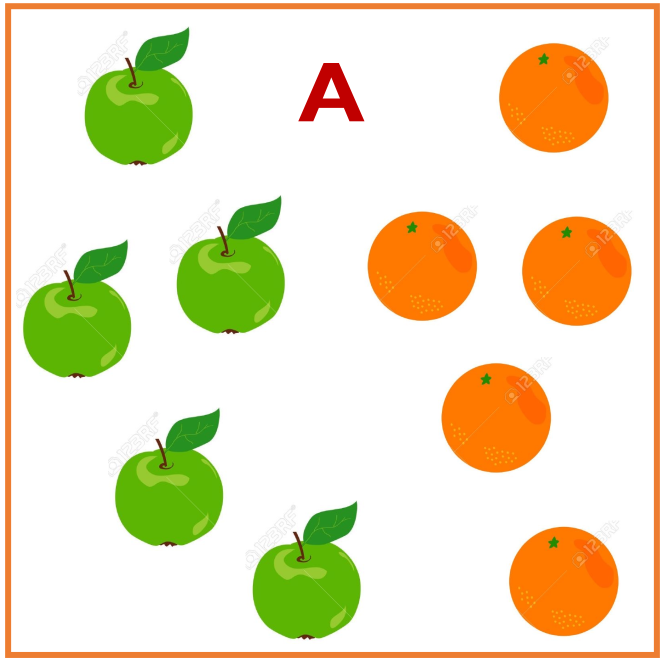
- What is the probability of picking a blue ball if we chose bowl B?

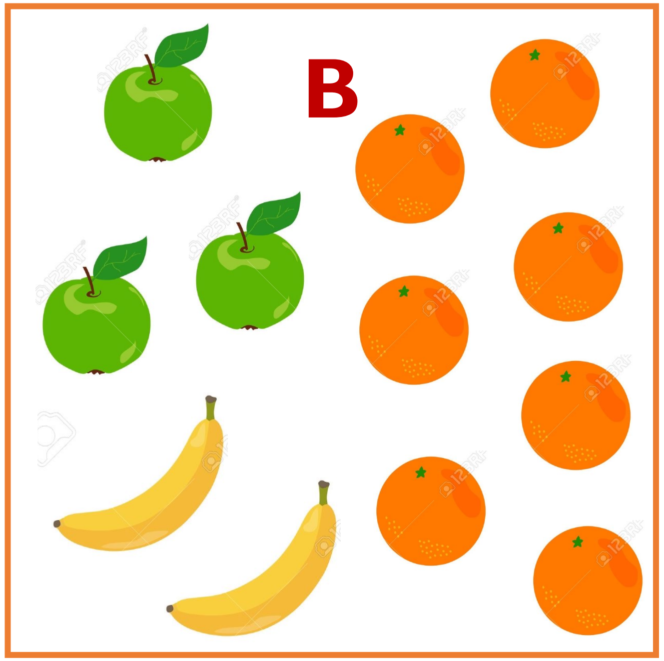
$$P(Blue|B) = \frac{1}{5}$$

The law
of total probability

https://youtu.be/U3_783xznQI

FRUITS





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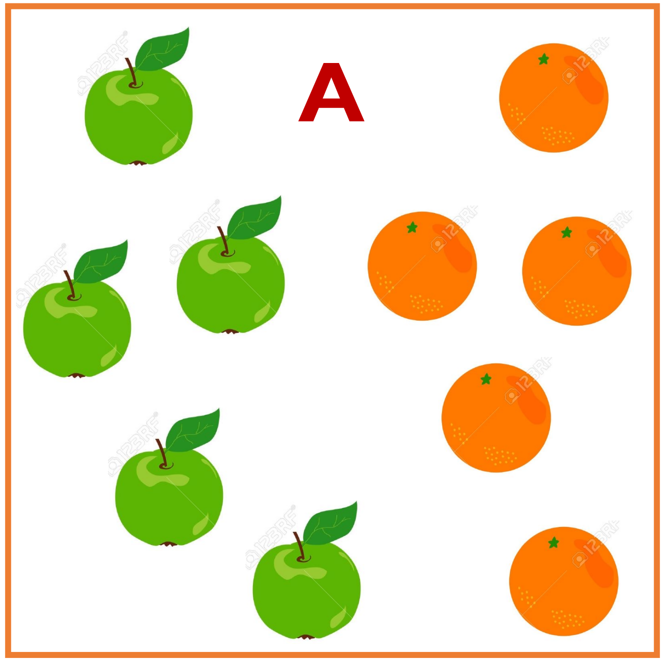

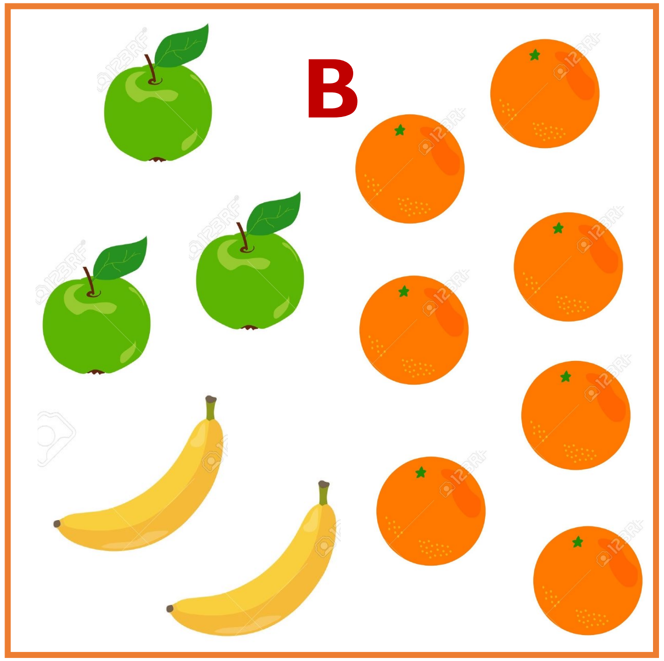

- What is the probability to pick an <u>apple</u> given that you've chosen <u>box A</u>?

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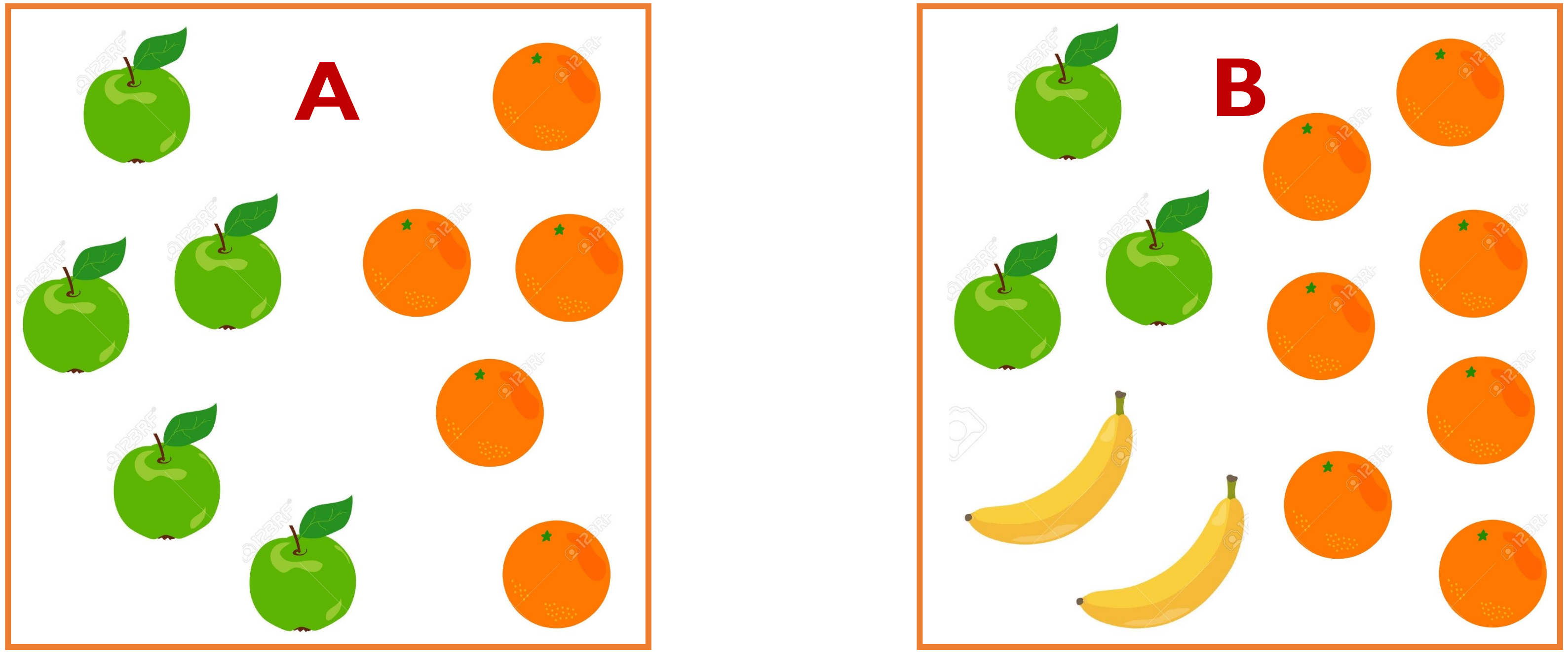

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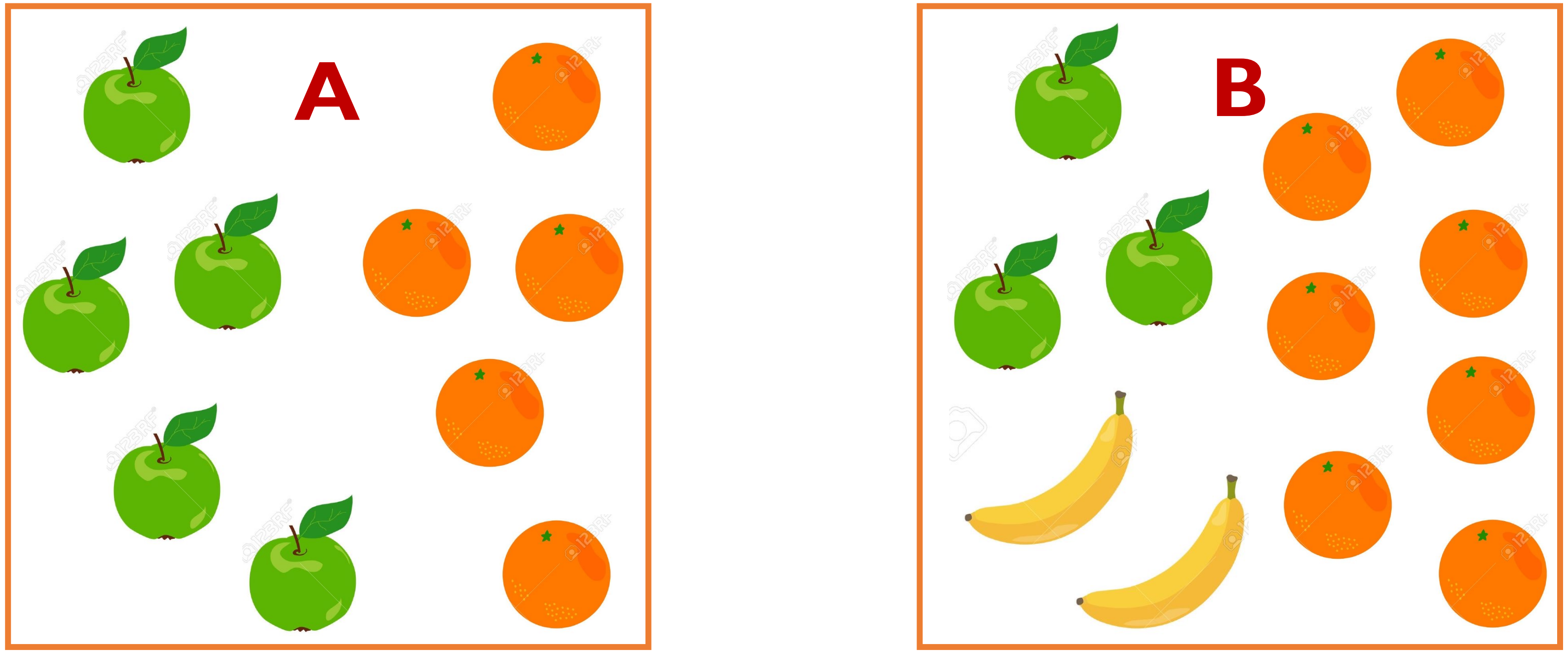
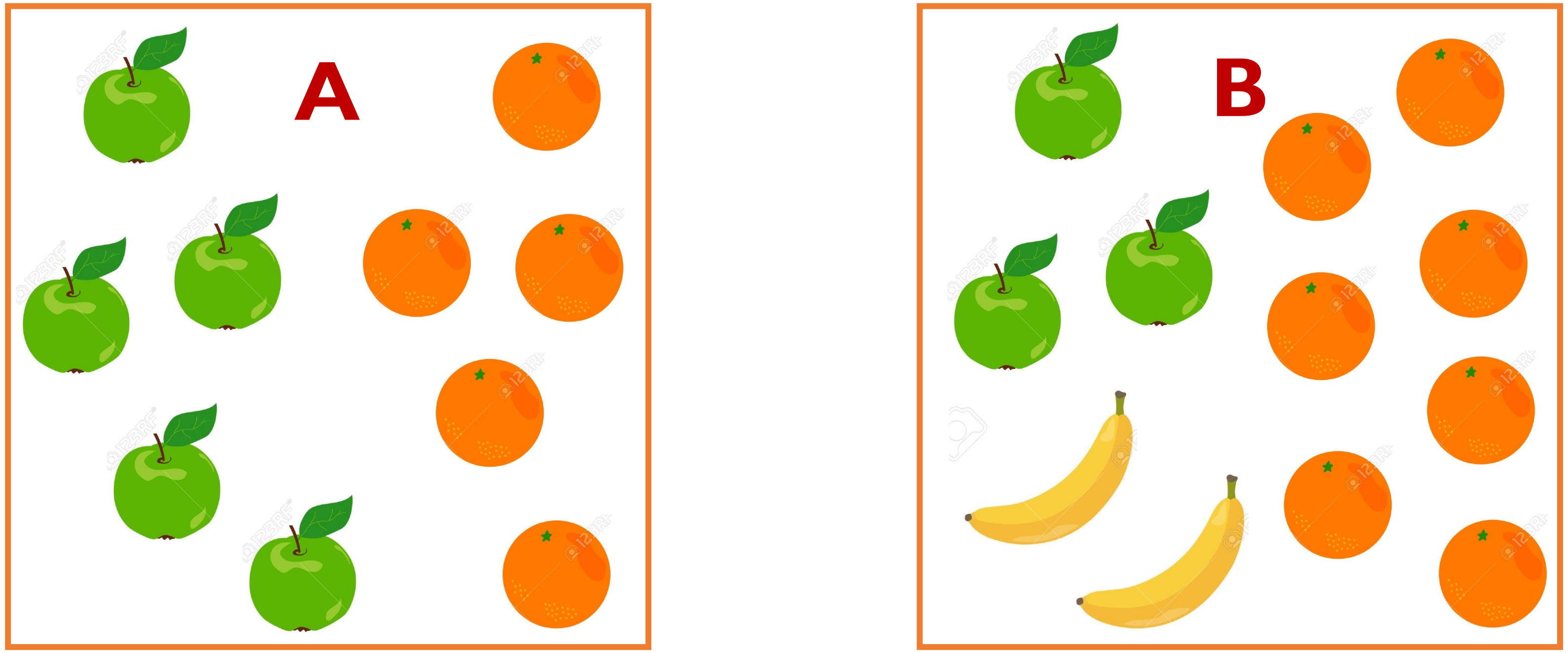
FRUITS



- What is the probability to pick an <u>apple</u> given that you've chosen <u>box A</u>?

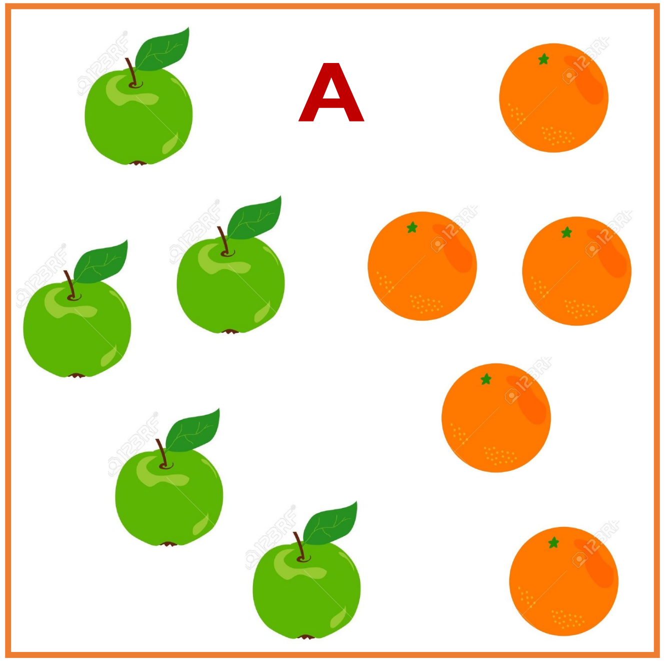
$$P(apple|A) = \frac{5}{10} = 0.5$$

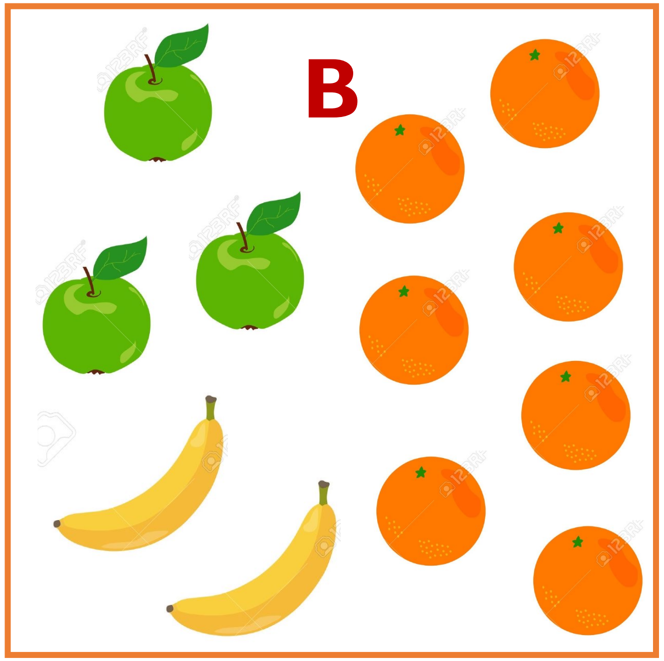
FRUITS



- What is the probability to pick an apple given that you've chosen box B?

FRUITS

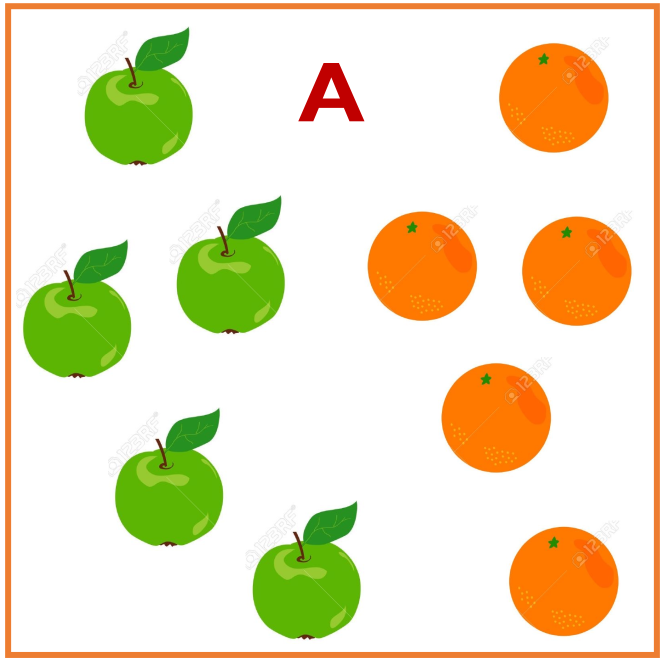


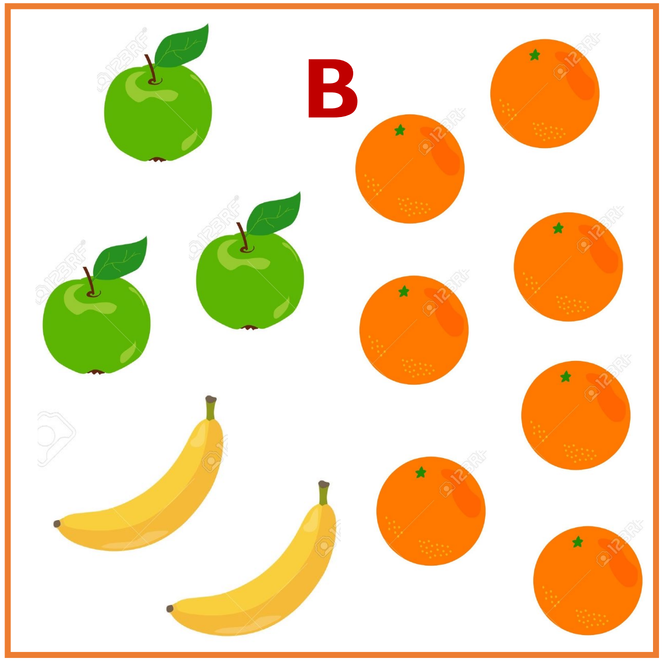


- What is the probability to pick an apple given that you've chosen box B?

$$P(apple|B) = \frac{3}{12} = 0.25$$

FRUITS





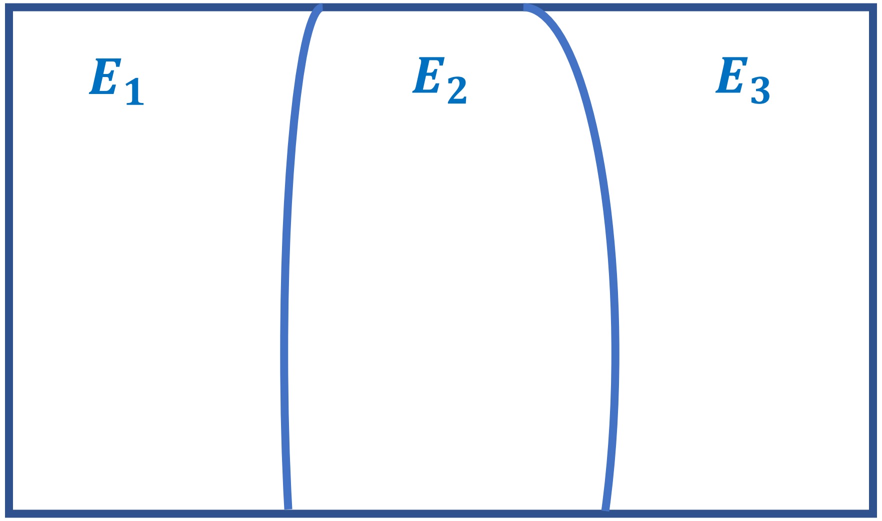
- What is the probability to pick <u>an apple</u>?

THE LAW OF TOTAL PROBABILITY

Suppose that the sample space S is split into n disjoint events:

$$S = E_1 \cup E_2 \cup \cdots \cup E_n,$$

$$E_1 \cap E_2 \cap \cdots \cap E_n = \emptyset$$



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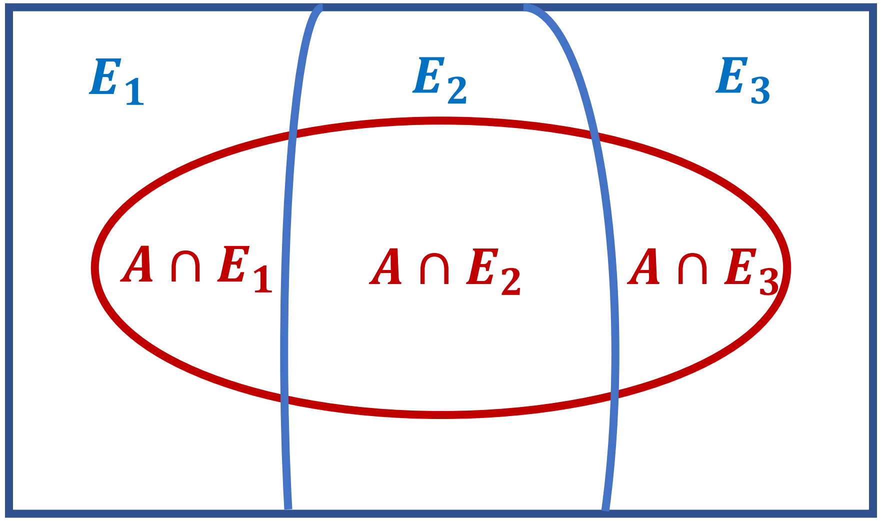


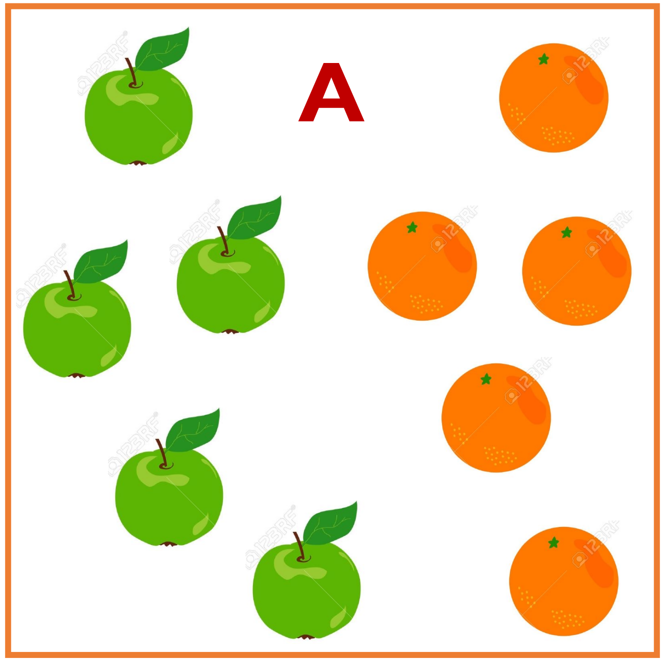
Then $\mathrm{P}(A)$ can be computed as follows:

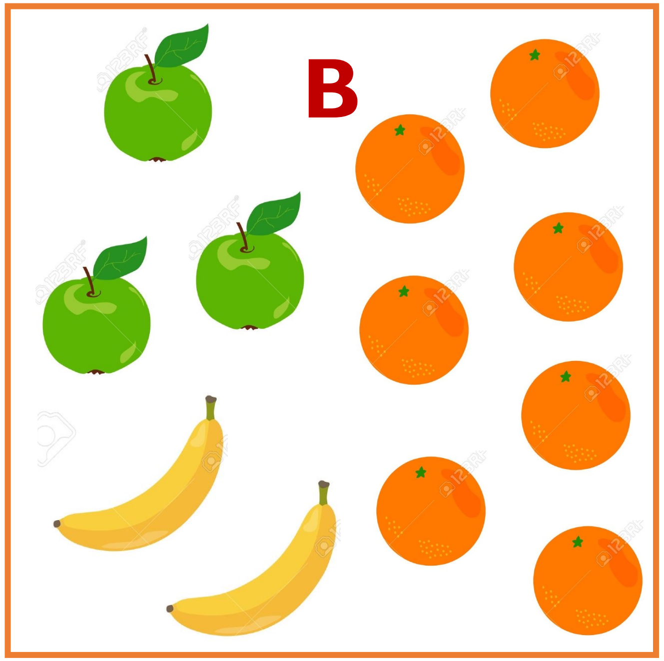
$$P(A) = P(A, E_1) + P(A, E_2) + \cdots + P(A, E_n) =$$

$$= P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) +$$

$$+ \cdots + P(A|E_n) \cdot P(E_n)$$

FRUITS

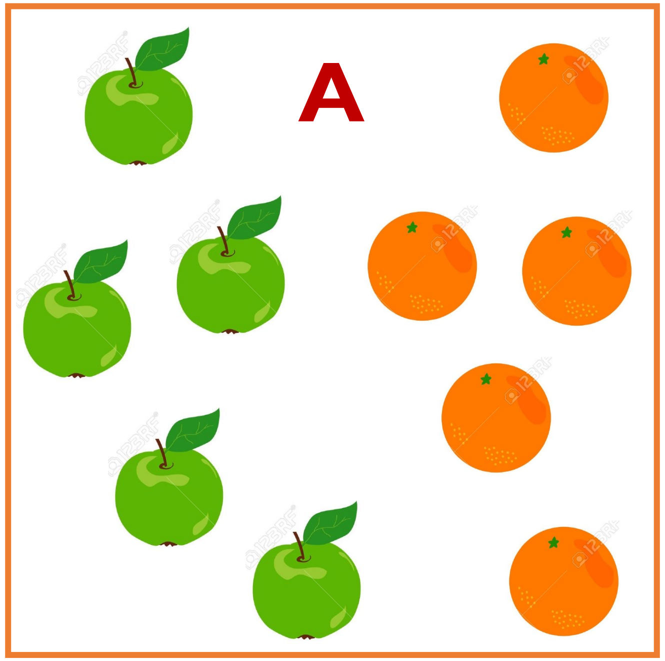


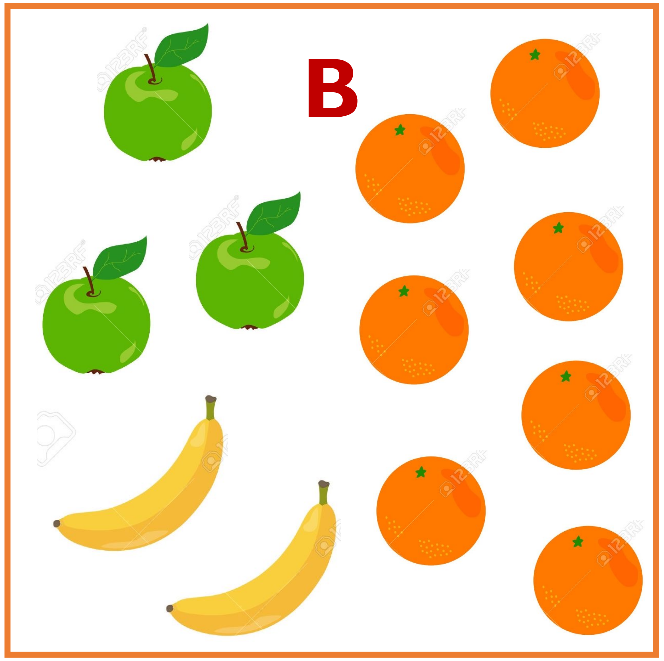


- What is the probability to pick <u>an apple</u>?

$$P(apple) =$$

FRUITS

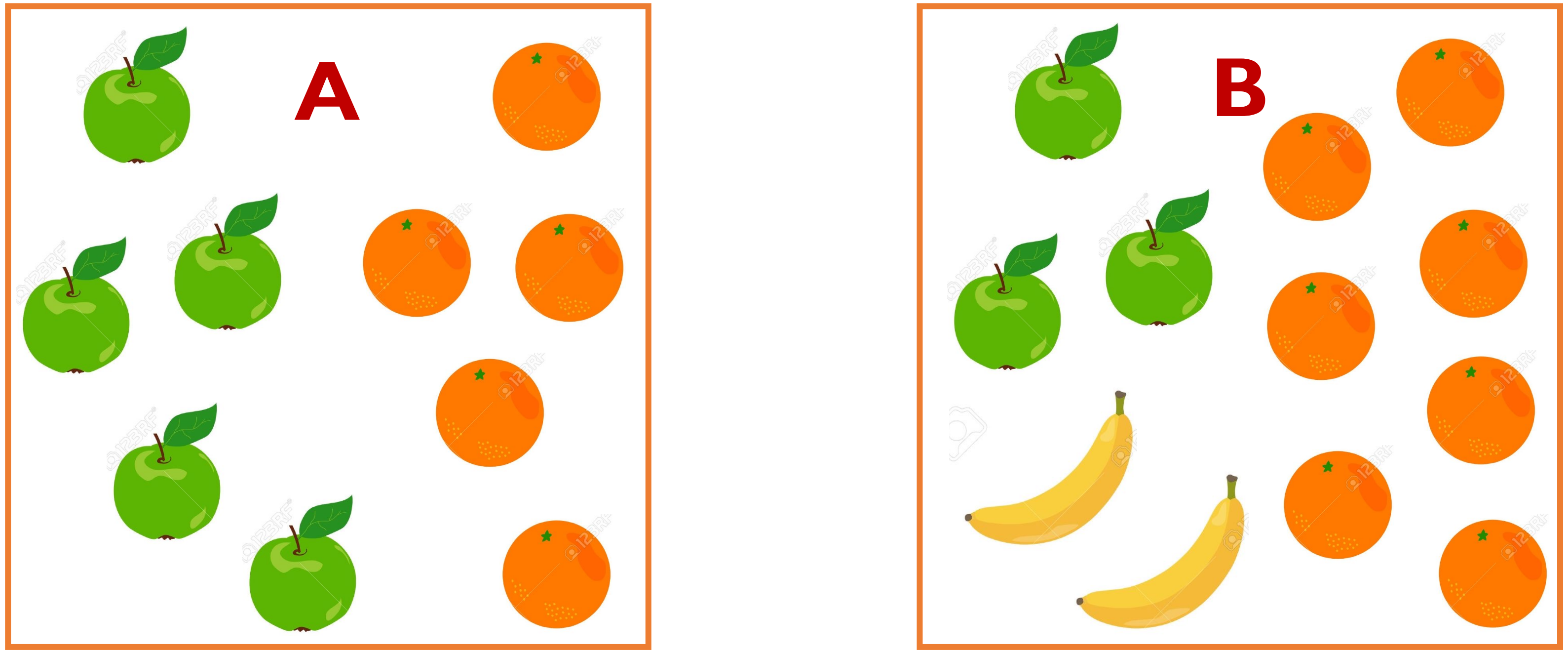




- What is the probability to pick <u>an apple</u>?

$$P(apple) = P(apple|A) \cdot P(A) + P(apple|B) \cdot (B) =$$

FRUITS

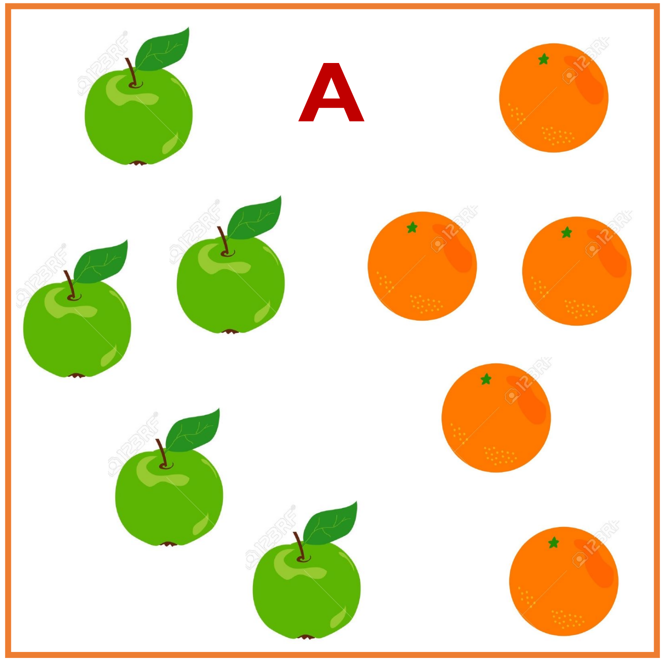


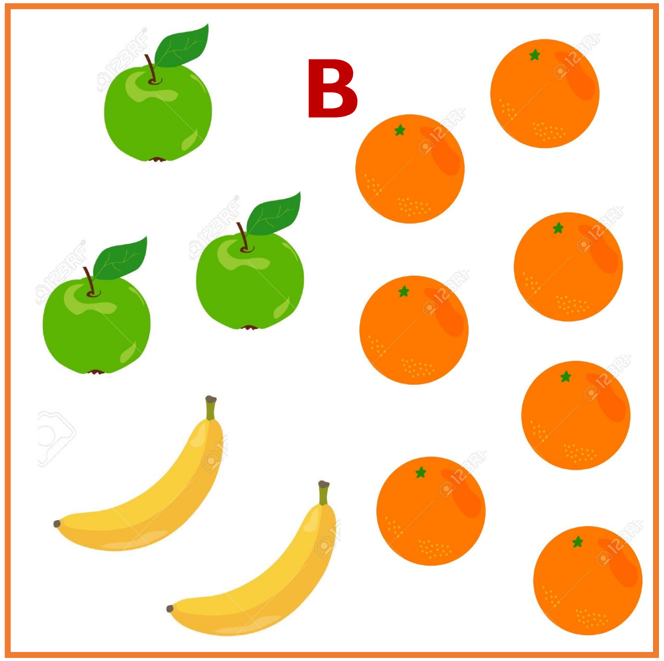
- What is the probability to pick <u>an apple</u>?

$$P(apple) = P(apple|A) \cdot P(A) + P(apple|B) \cdot (B) =$$

$$= 0.5 \cdot 0.5 + 0.25 \cdot 0.5 = 0.375$$

FRUITS





- What is the probability to pick <u>an orange</u>?

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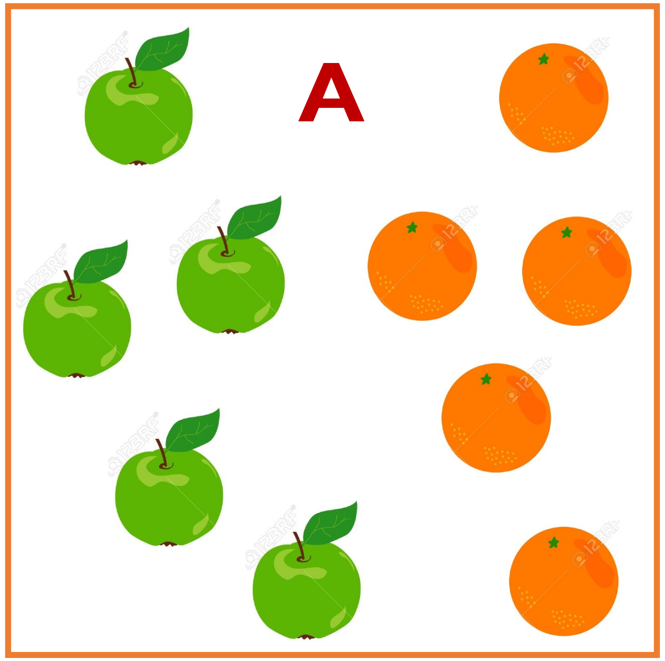

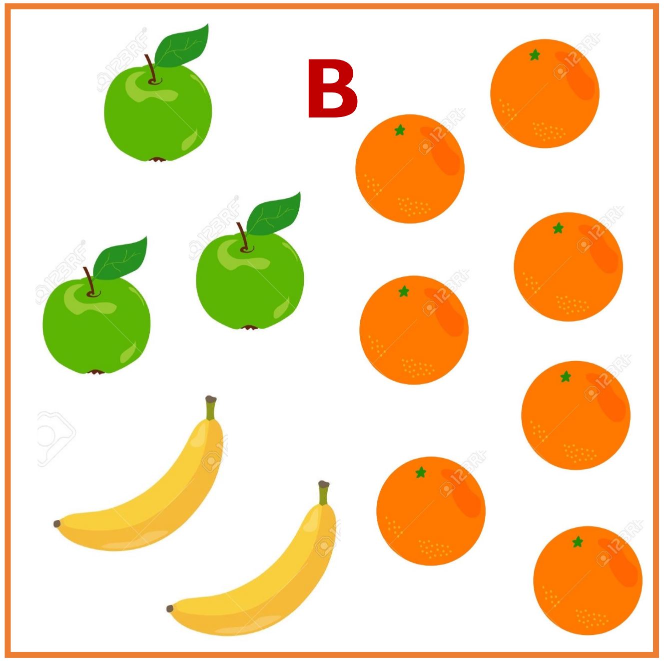

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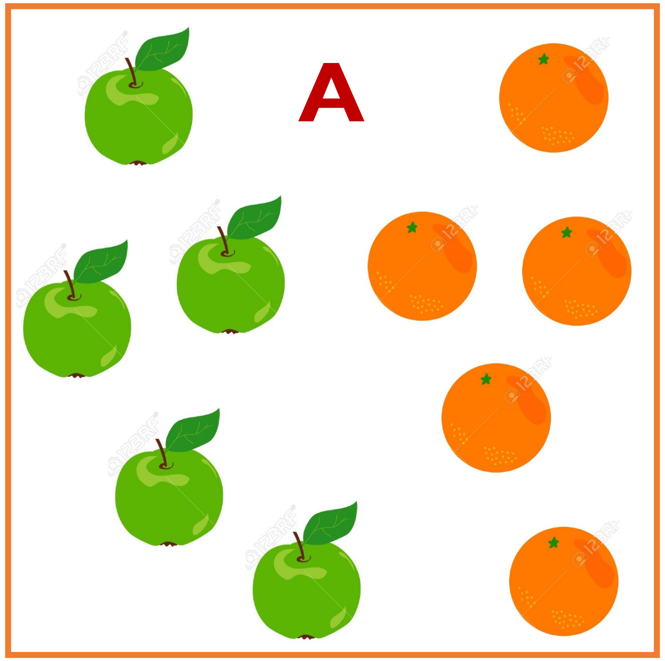
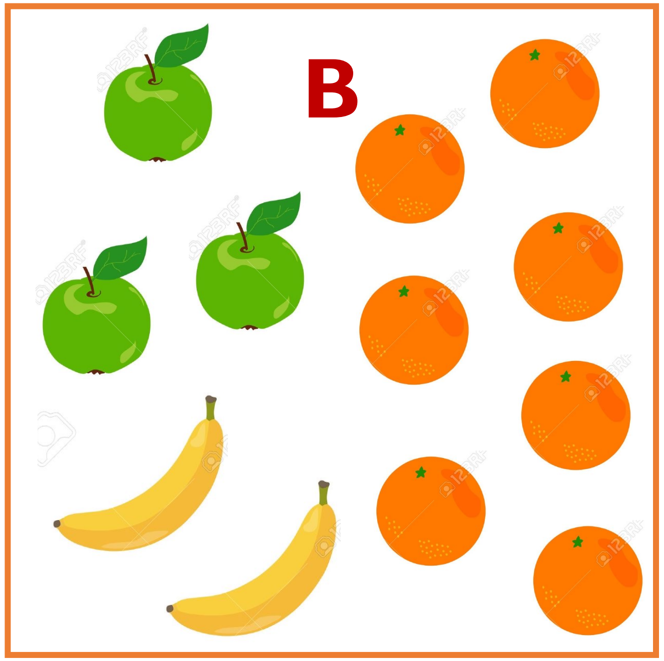
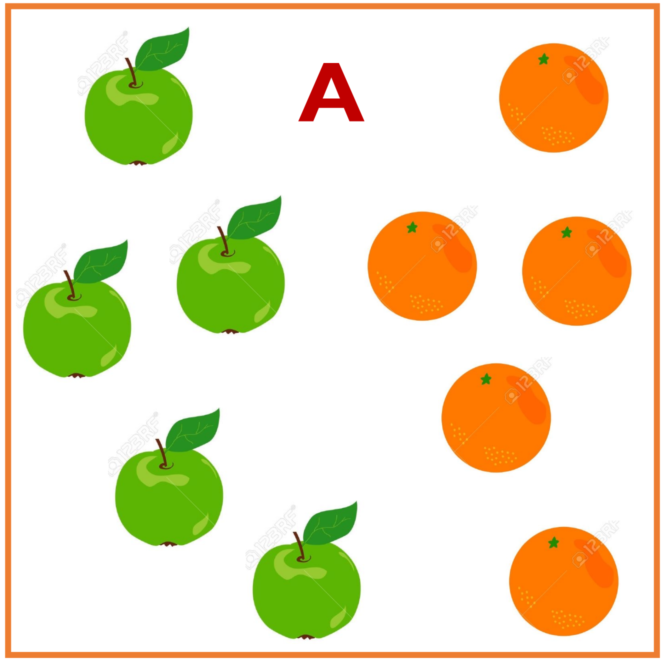


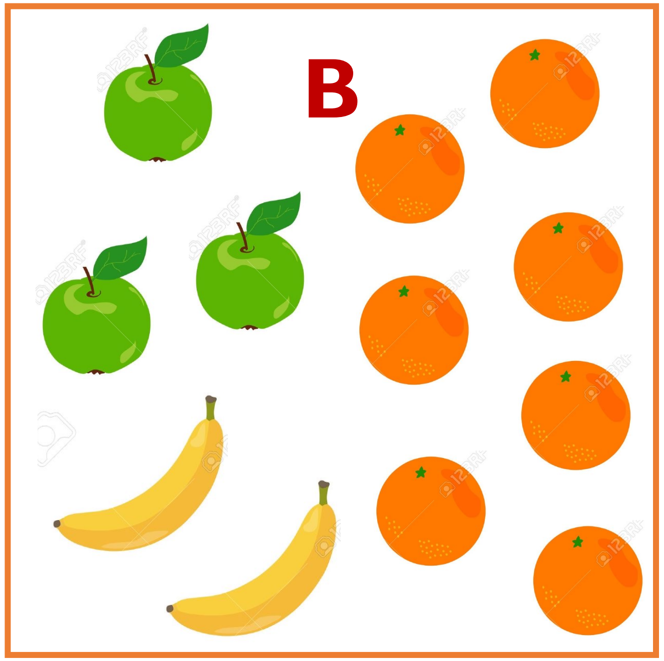


- What is the probability to pick <u>an orange</u>?

$$P(orange) = P(orange|A) \cdot P(A) + P(orange|B) \cdot P(B) =$$

FRUITS



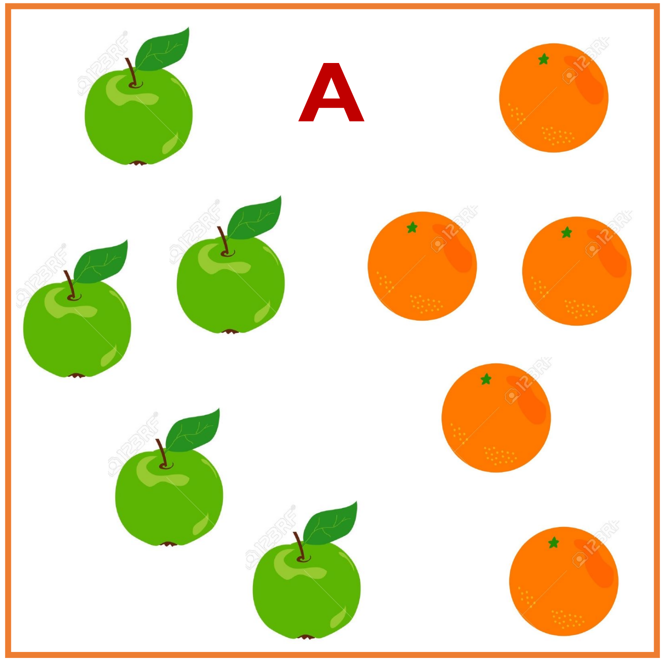


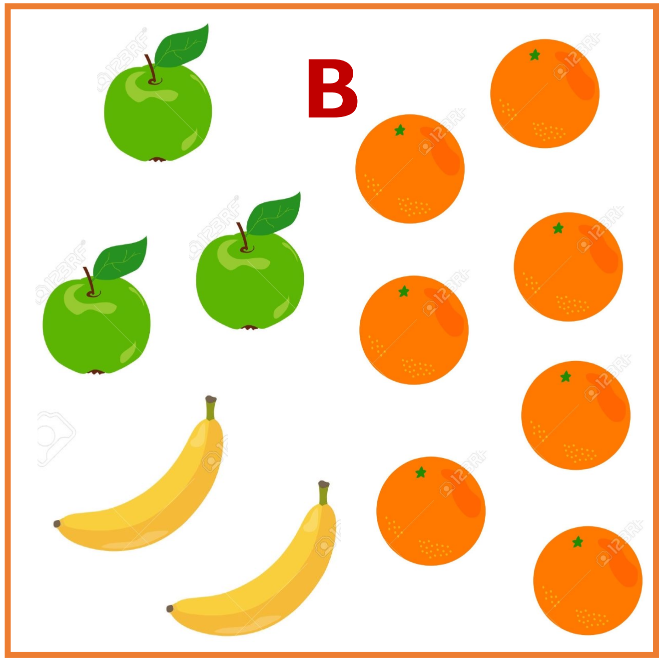
- What is the probability to pick <u>an orange</u>?

$$P(orange) = P(orange|A) \cdot P(A) + P(orange|B) \cdot P(B) =$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{7}{12} \cdot \frac{1}{2} = \frac{1}{4} + \frac{7}{24} = \frac{13}{24}$$

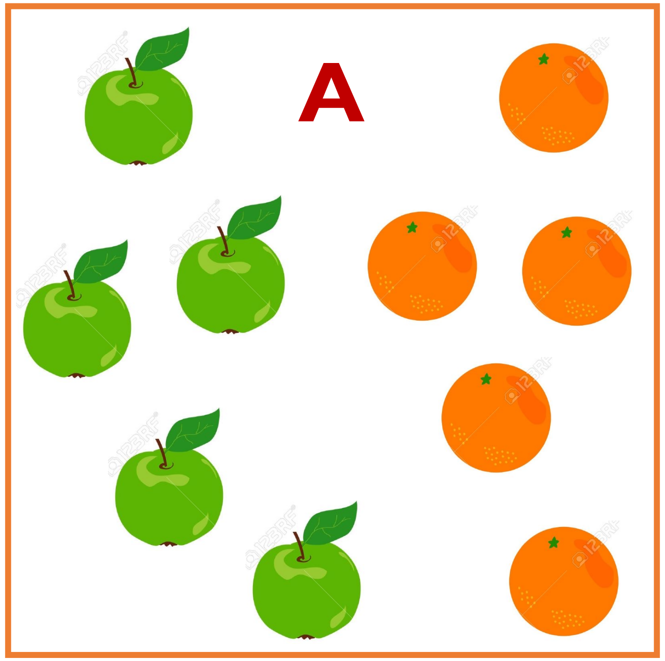
FRUITS

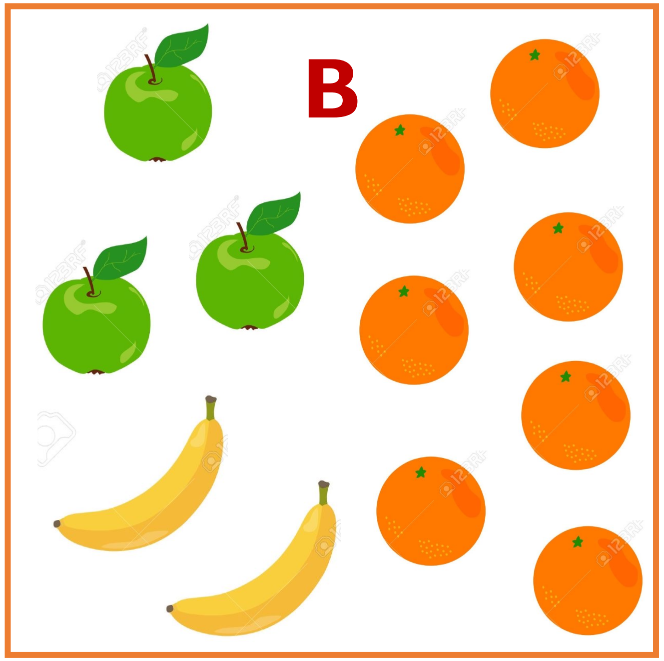




- What is the probability to pick a banana?

FRUITS

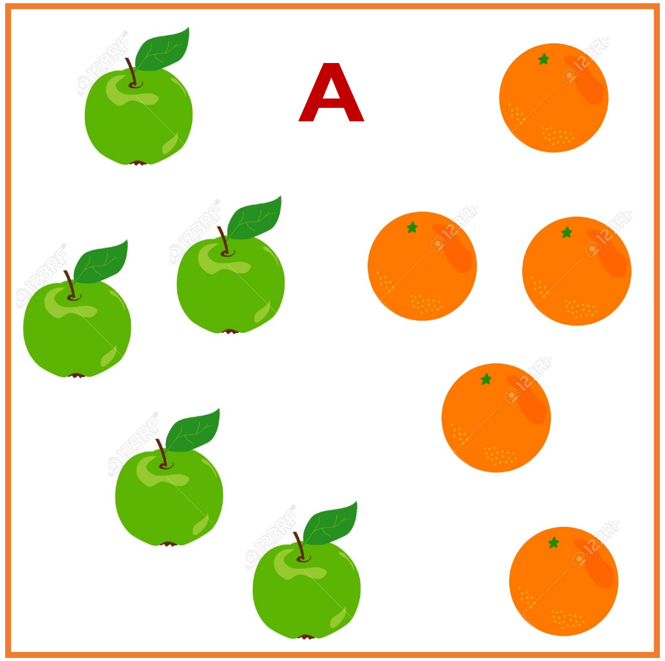


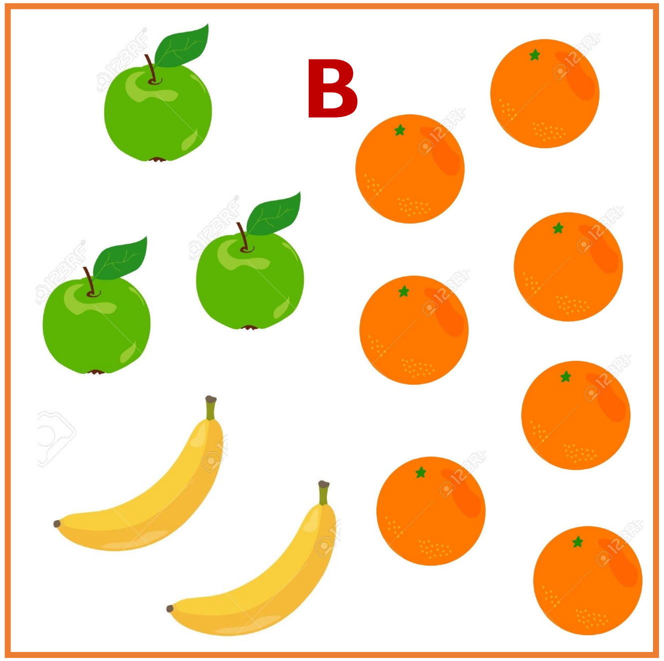


- What is the probability to pick a banana?

$$P(banana) =$$

FRUITS

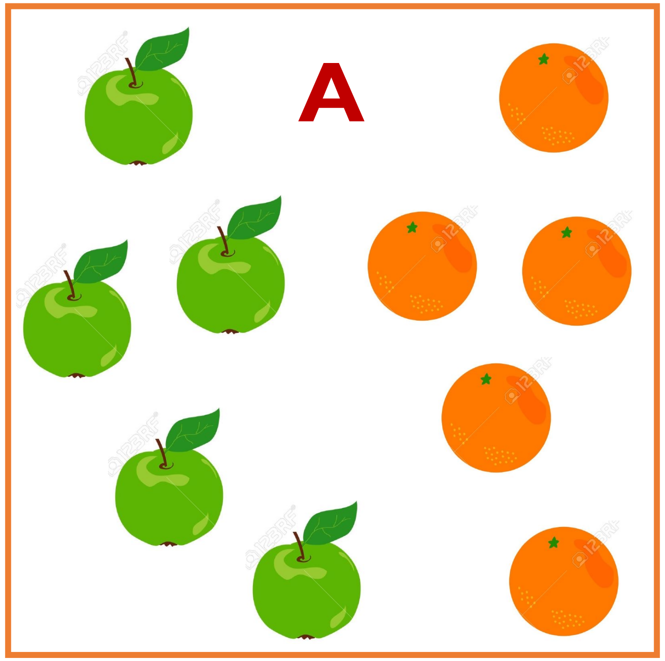


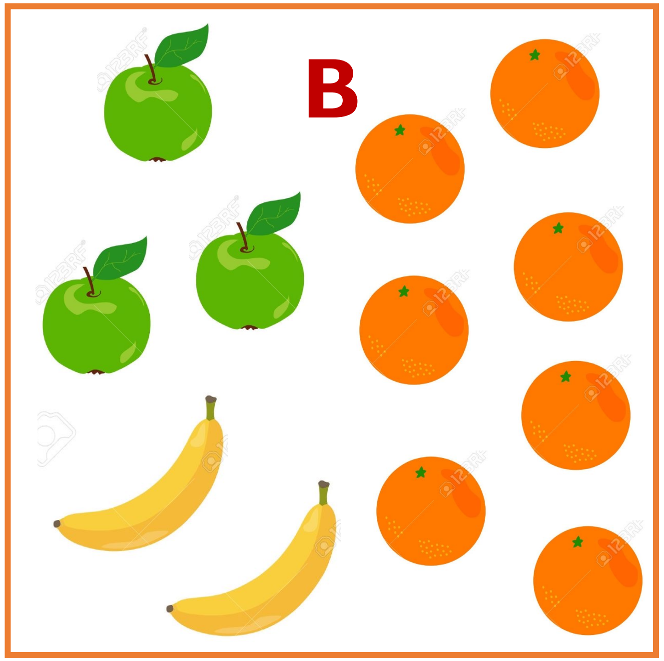


- What is the probability to pick a banana?

$$P(banana) = P(banana|A) \cdot P(A) + P(banana|B) \cdot P(B) =$$

FRUITS





- What is the probability to pick a banana?

$$P(banana) = P(banana|A) \cdot P(A) + P(banana|B) \cdot P(B) =$$

$$= 0 \cdot \frac{1}{2} + \frac{2}{12} \cdot \frac{1}{2} = \frac{1}{12}$$

HEIGHTS

- In a class, 40% of students are male and 60% are female. It's known that 60% of the males and 10% of the females are taller than 6 feet. What % of the class is <u>not taller than</u> 6 feet?

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$$= (1 - 0.6) \cdot 0.4 + (1 - 0.1) \cdot 0.6 = 0.7$$

COINS

- A box contains 3 coins with a head on each side, 4 coins with a tail on each side, and 2 fair coins. One of these nine coins is selected at random and tossed once. What is the probability that a head will be obtained?

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$$P(E_H) = \frac{3}{3+4+2} = \frac{1}{3}$$

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E_F – a fair coin is selected

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$$= 1 \cdot \frac{1}{3} + 0 + \frac{1}{2} \cdot \frac{2}{9} =$$

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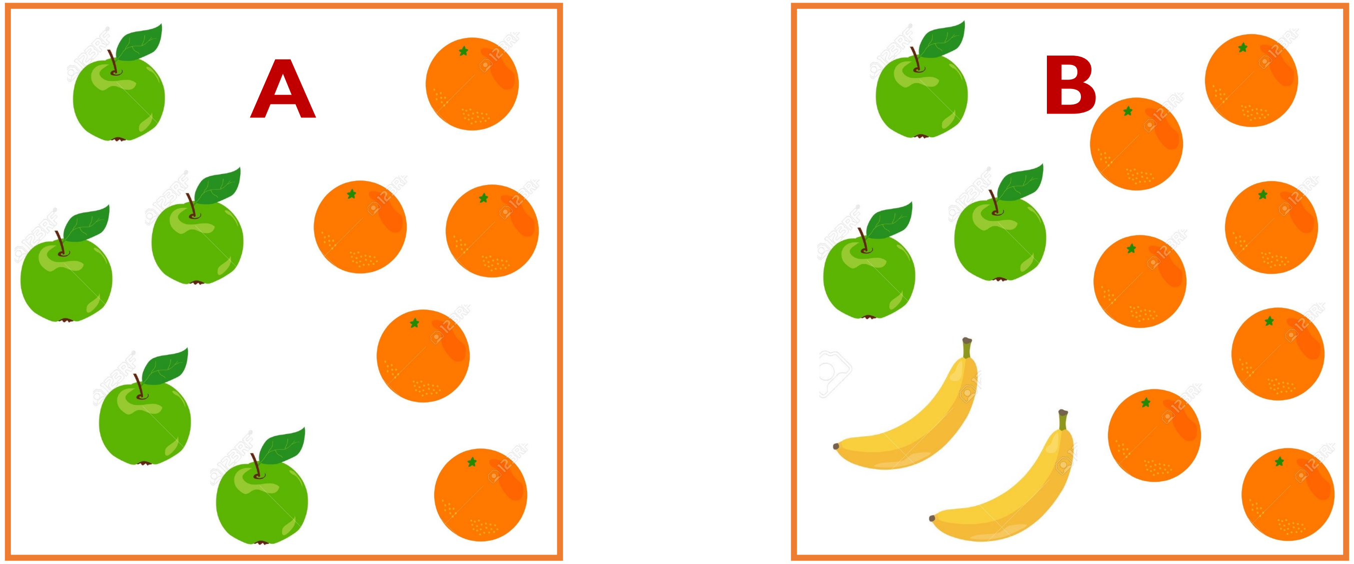
$$P(H) = P(H|E_H) \cdot P(E_H) + P(H|E_T) \cdot P(E_T) + P(H|E_F) \cdot P(E_F) =$$

$$= 1 \cdot \frac{1}{3} + 0 + \frac{1}{2} \cdot \frac{2}{9} = \frac{1}{3} + \frac{1}{9} = \frac{4}{9}$$

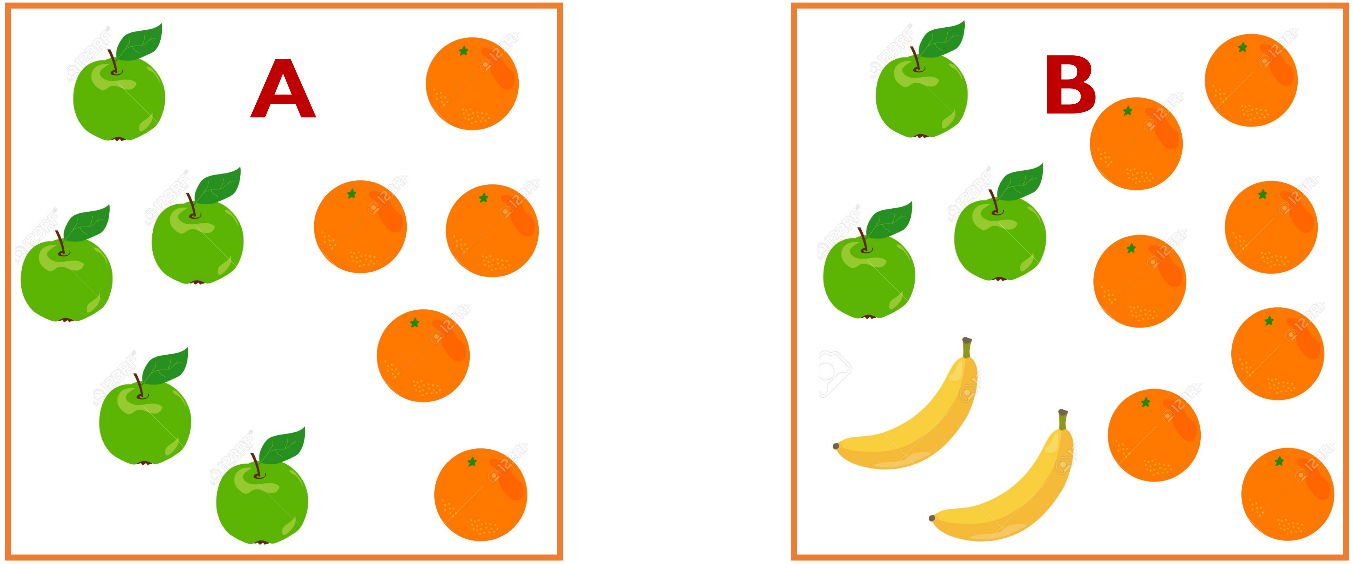
Bayes' rule

FRUITS



- What is the probability that you chose <u>box A</u> given that you picked <u>an apple</u>?

FRUITS



- What is the probability that you chose box A given that you picked an apple?

$$P(A|apple) =$$

BAYES' RULE

$$\boldsymbol{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{P(AB)}{P(B)} = \frac{\boldsymbol{P}(\boldsymbol{B}|\boldsymbol{A}) \cdot \boldsymbol{P}(\boldsymbol{A})}{\boldsymbol{P}(\boldsymbol{B})}$$

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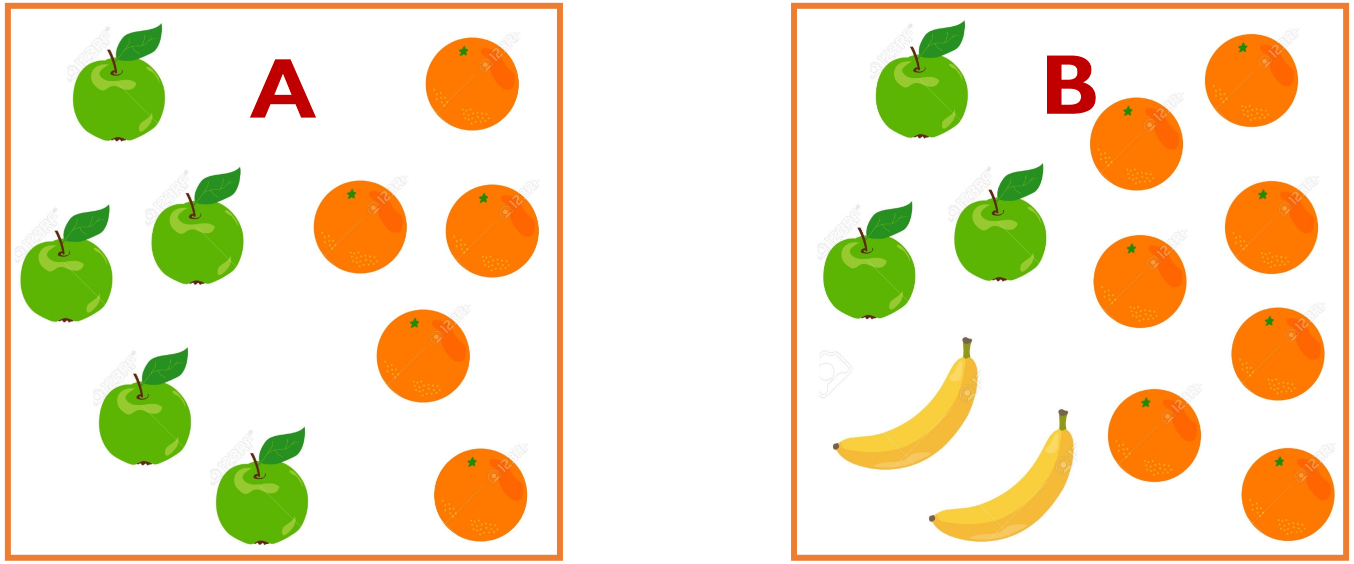

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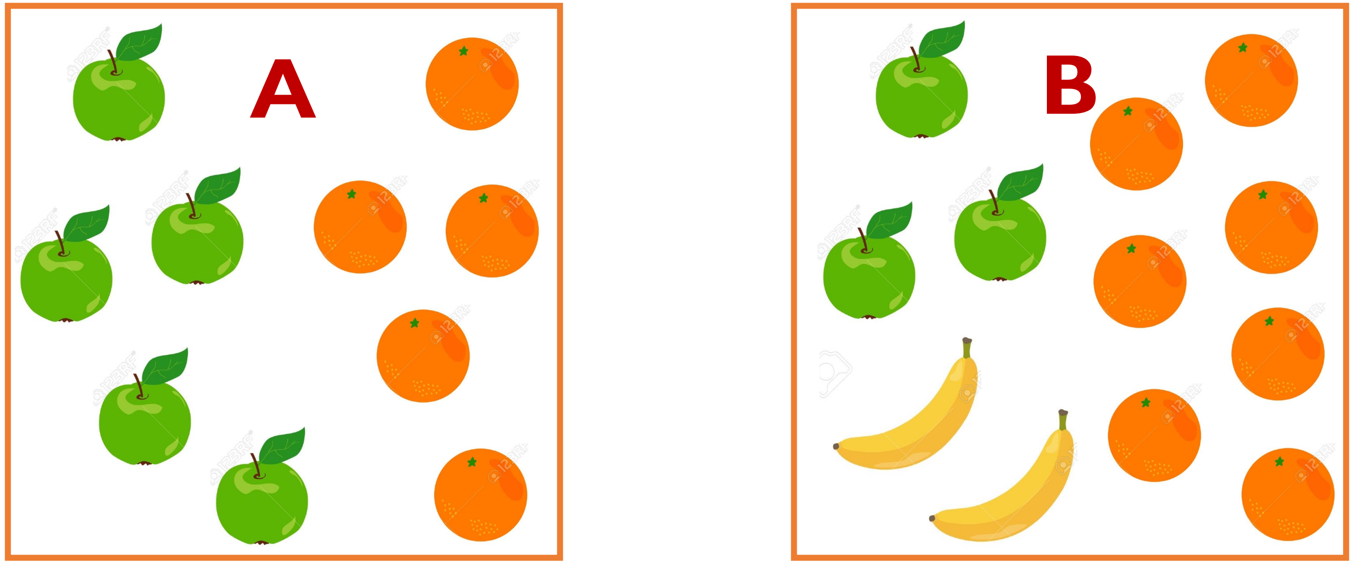

- What is the probability that you chose <u>box A</u> given that you picked <u>an apple</u>?

$$P(A|apple) = \frac{P(apple|A) \cdot P(A)}{P(apple)} =$$

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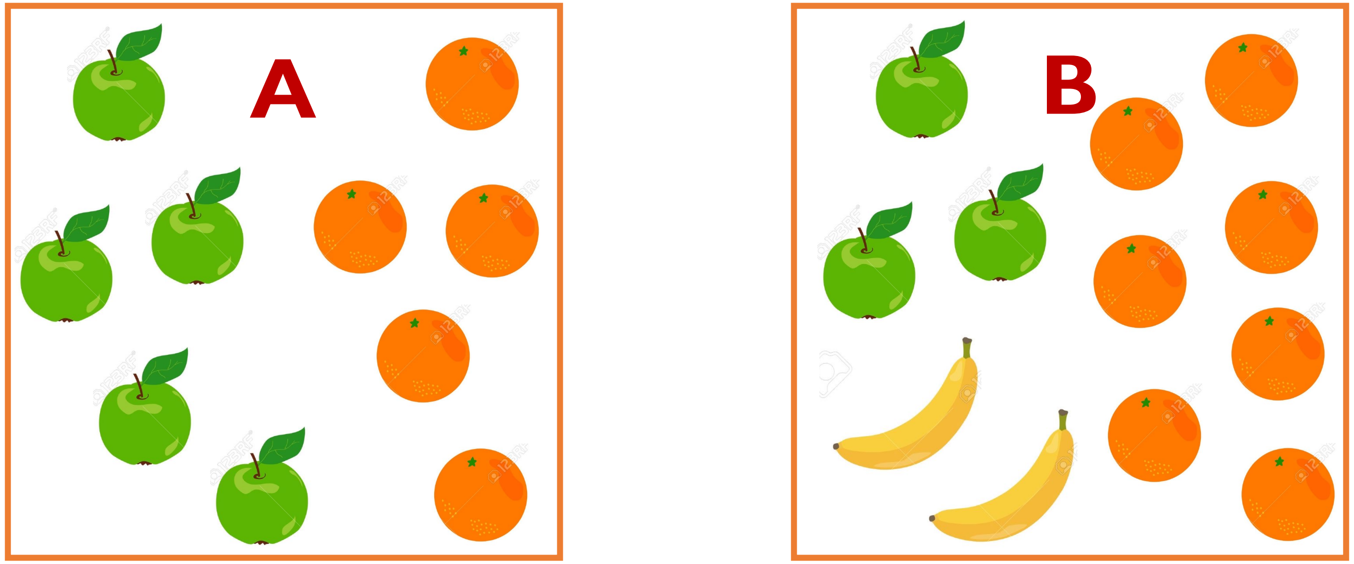

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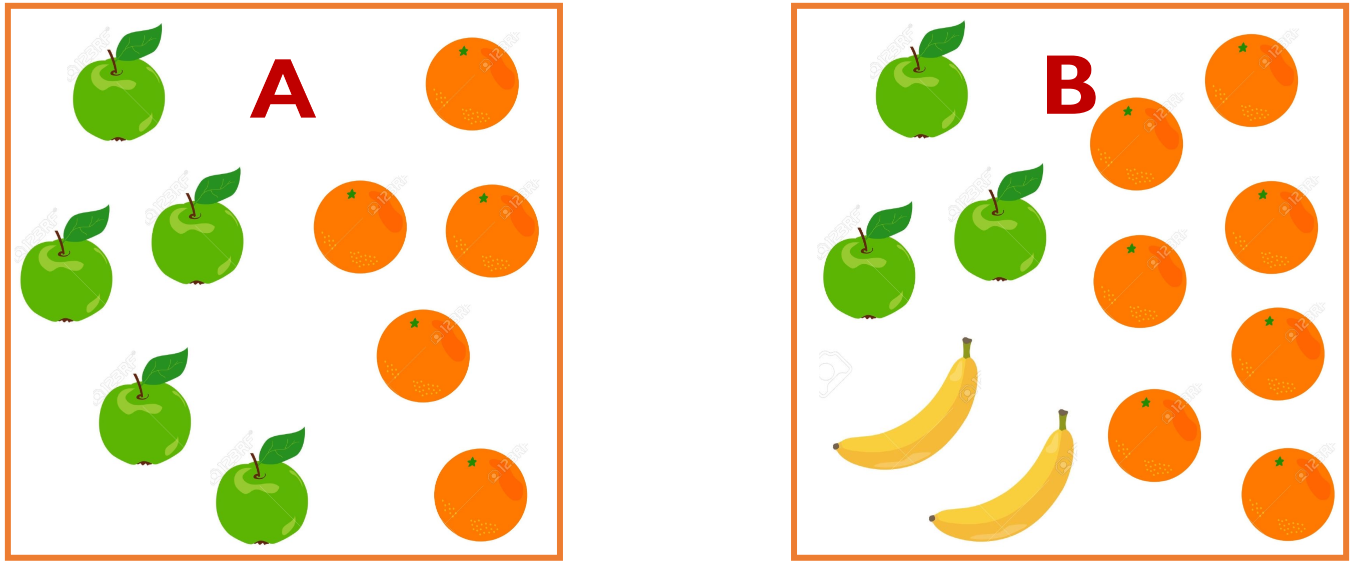

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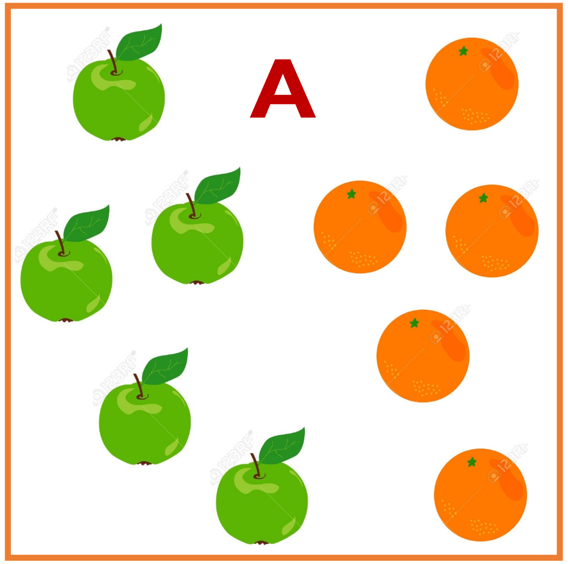
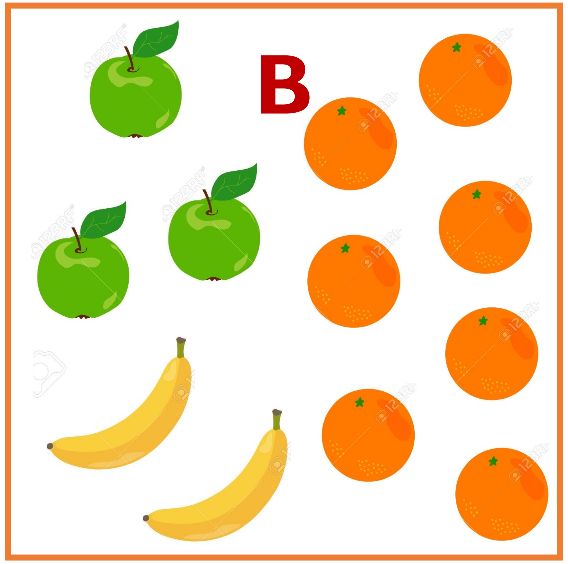
FRUITS

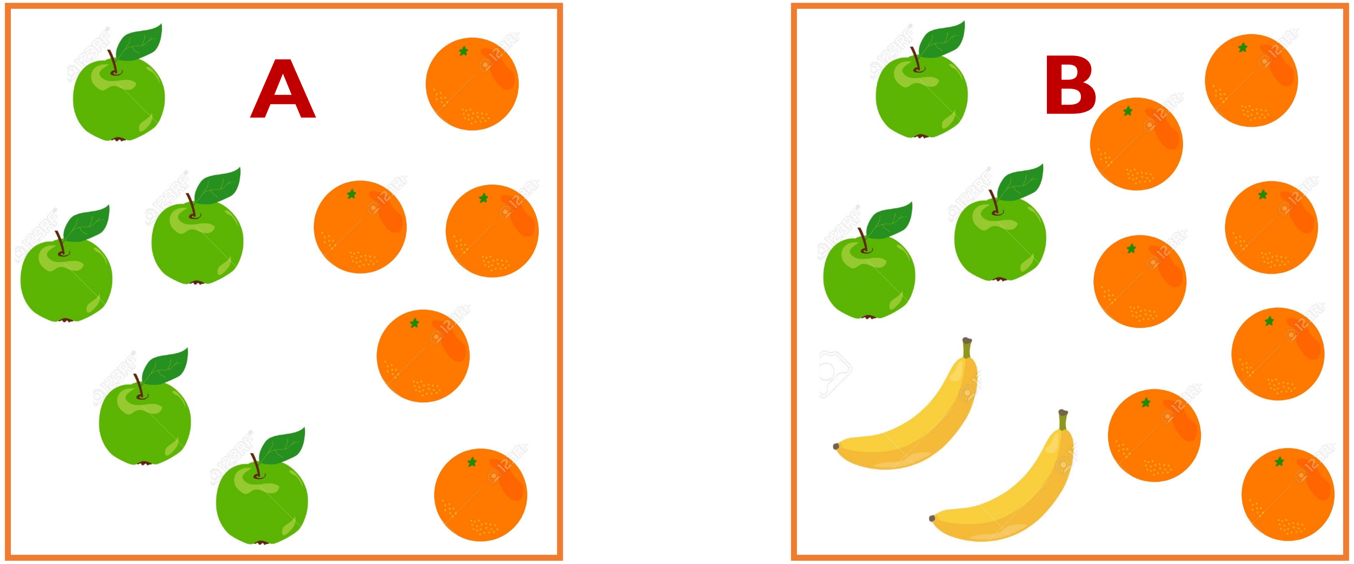




- What is the probability that you chose <u>box B</u> given that you picked <u>a banana</u>?

$$P(B|banana) =$$

FRUITS



- What is the probability that you chose <u>box B</u> given that you picked <u>a banana</u>?

$$P(B|banana) = \frac{P(banana|B) \cdot P(B)}{P(banana)} =$$

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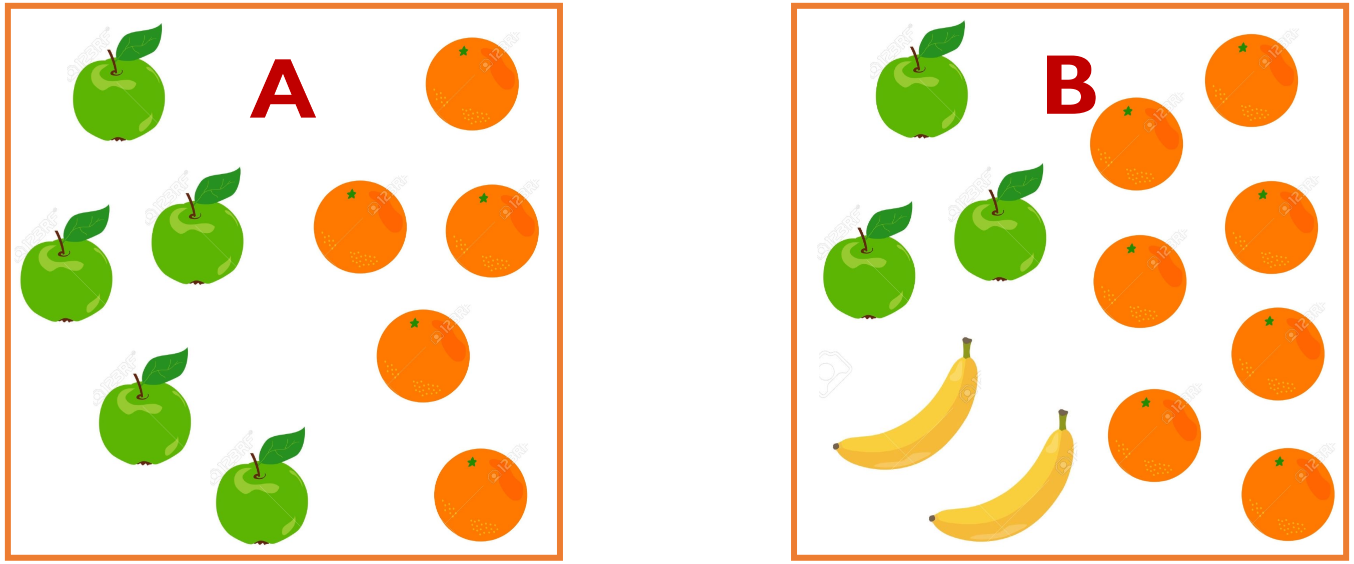

- What is the probability that you chose box B given that you picked a banana?

$$P(B|banana) = \frac{P(banana|B) \cdot P(B)}{P(banana)} =$$

$$= \frac{P(banana|B) \cdot P(B)}{P(banana|A) \cdot P(A) + P(banana|B) \cdot P(B)} =$$

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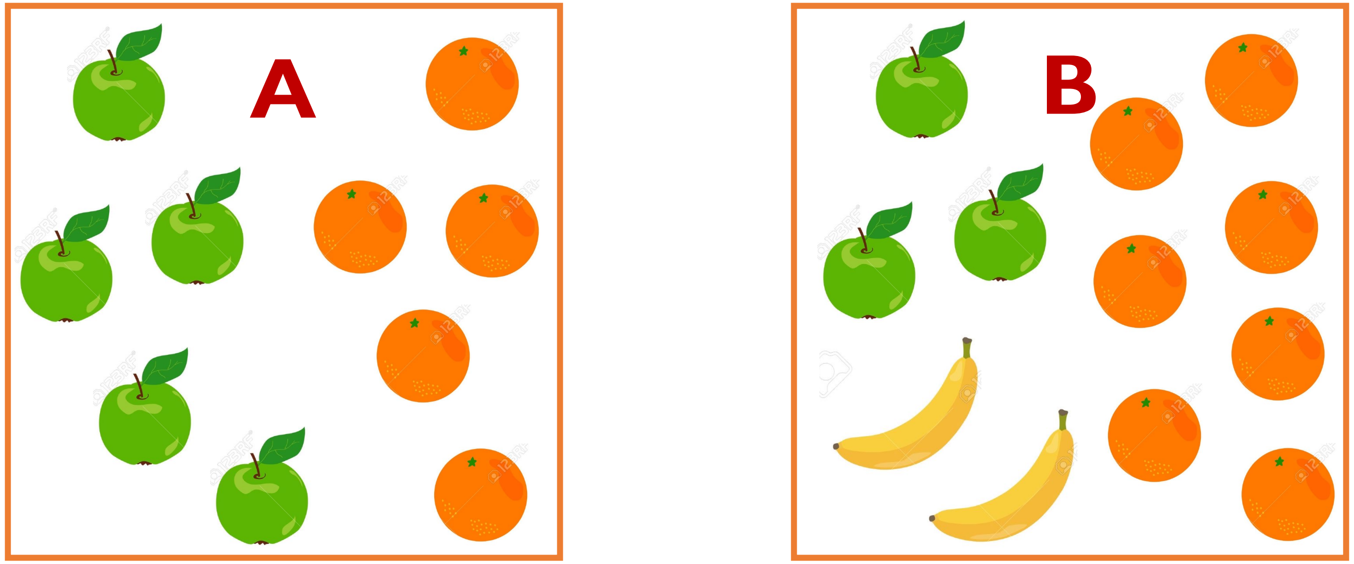

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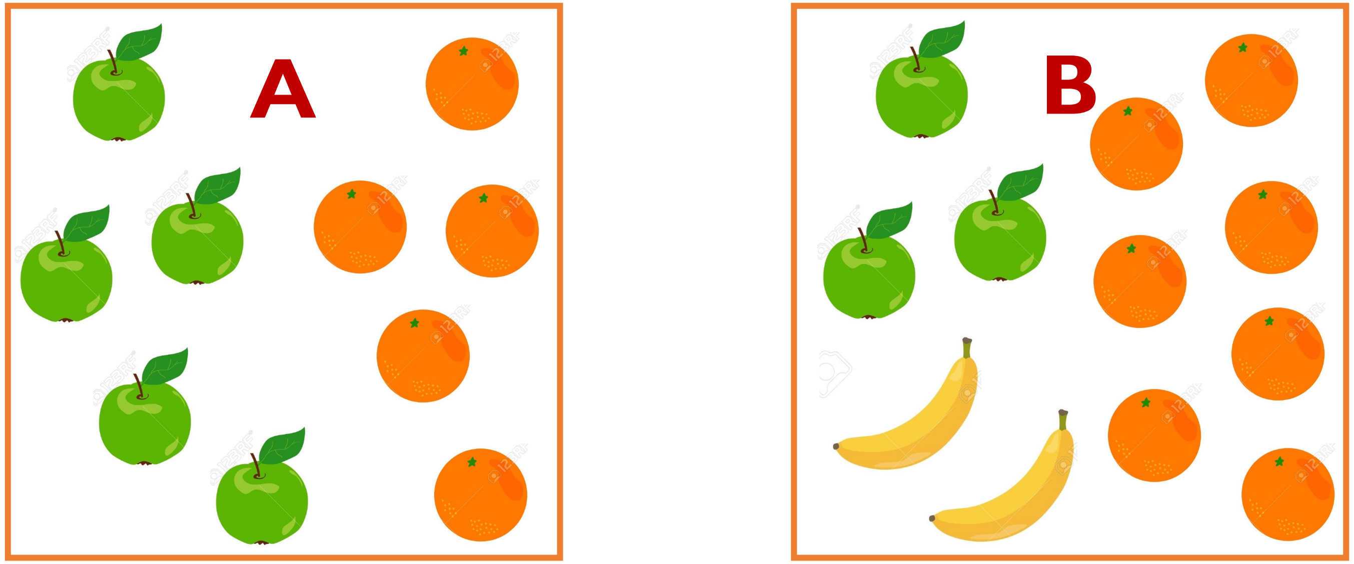

- What is the probability that you chose box A given that you picked an orange?

$$P(A|orange) =$$

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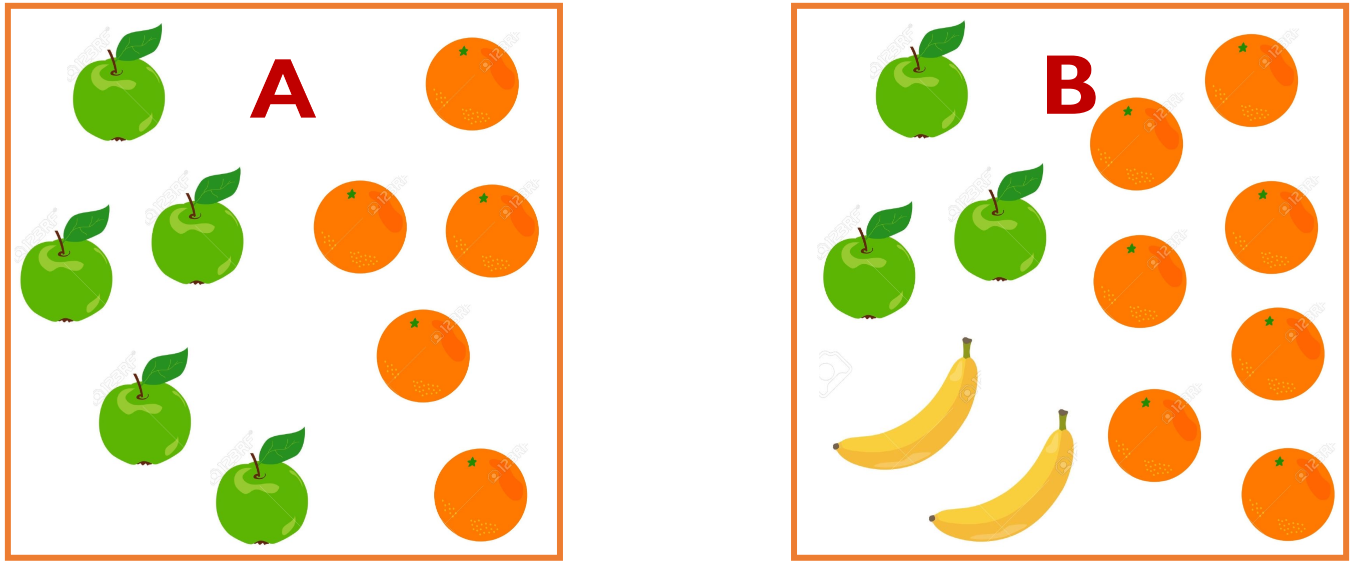

- What is the probability that you chose box A given that you picked an orange?

$$P(A|orange) = \frac{P(orange|A) \cdot P(A)}{P(orange)} =$$

$$=$$

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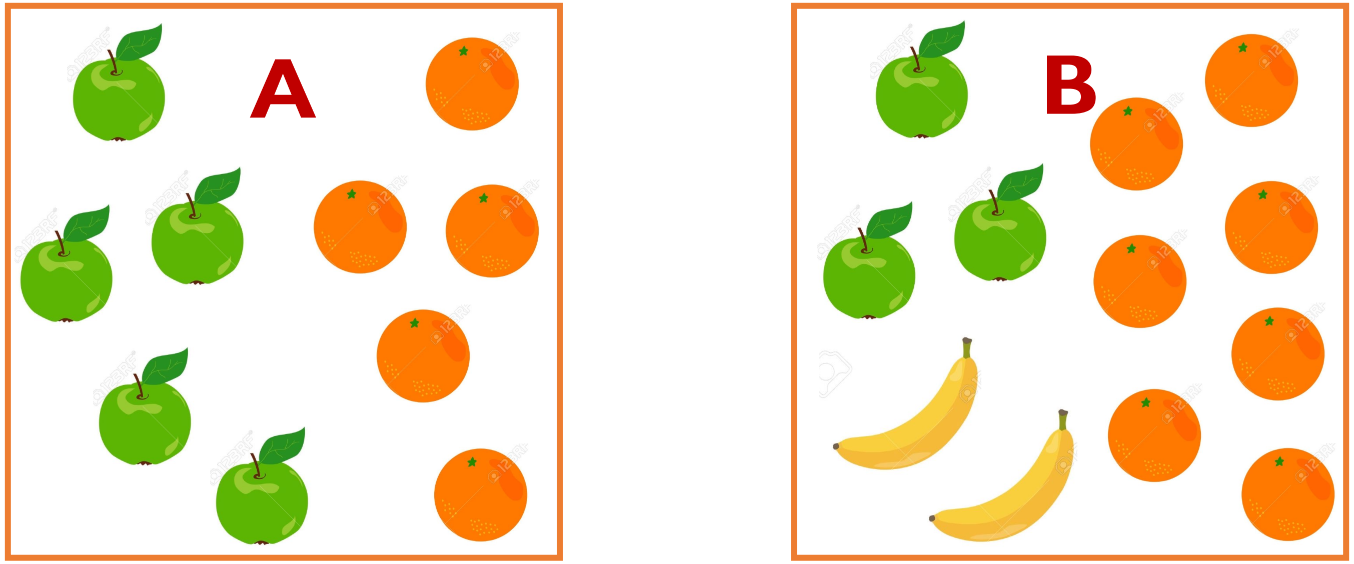

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$$= \frac{P(orange|A) \cdot P(A)}{P(orange|A) \cdot P(A) + P(orange|B) \cdot P(B)} = \frac{0.5 \cdot 0.5}{0.5 \cdot 0.5 + 7/12 \cdot 0.5}$$

DRUG TEST

- 0.5% of the population are using drugs. If a person taking the test has used drugs, the test will show a positive result with a 98% chance, and if they haven't used drugs, the test will be negative with a 98% probability. A random person tests positive. What is the probability that this person has actually used drugs?

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- $P(drugs|+) =?$

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$$P(drugs|+) = \frac{P(drugs \text{ and } +)}{P(+)} =$$

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$$P(drugs|+) = \frac{P(+|drugs) \cdot P(drugs)}{P(+|drugs) \cdot P(drugs) + P(+|no\ drugs) \cdot P(no\ drugs)} =$$

$$= \frac{0.98 \cdot 0.005}{0.98 \cdot 0.005 + (1 - 0.98) \cdot (1 - 0.005)} \approx 0.2$$

Practice Problems

FLIPPING A COIN 2

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A – equal number of H and T

B – different results on the first and last flips $\qquad P(B) = \frac{2^3}{2^4} = \frac{1}{2}$

AB – different results on the first and last flips

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equal number of H and T

$$P(A|B) = \frac{P(AB)}{P(B)} =$$

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AB – different results on the first and last flips $\quad P(AB) = \frac{2}{2^4} = \frac{1}{8}$

equal number of H and T

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1}{2}$$

ROLLING A DIE

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$$P(X_1 = 2 \; and \; X_1 + X_2 = 5) =$$

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$$P(X_1 = 2 \; and \; X_1 + X_2 = 5) = P(X_1 = 2 \; and \; X_2 = 3) =$$

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$$P(X_1 = 2\ and\ X_1 + X_2 = 5) = P(X_1 = 2\ and\ X_2 = 3) = \frac{1}{6 \cdot 6}$$

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$$P(X_1 = 2\ and\ X_1 + X_2 = 5) = P(X_1 = 2\ and\ X_2 = 3) = \frac{1}{6 \cdot 6}$$

$$P(X_1 + X_2 = 5) =$$

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$$P(X_1 = 2 \; and \; X_1 + X_2 = 5) = P(X_1 = 2 \; and \; X_2 = 3) = \frac{1}{6 \cdot 6}$$

$$P(X_1 + X_2 = 5) = \frac{4}{36} = \frac{1}{9}$$

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$$P(X_1 = 2|X_1 + X_2 = 5) = \frac{P(X_1 = 2, X_1 + X_2 = 5)}{P(X_1 + X_2 = 5)} = \frac{1}{4}$$

$$P(X_1 = 2 \; and \; X_1 + X_2 = 5) = P(X_1 = 2 \; and \; X_2 = 3) = \frac{1}{6 \cdot 6}$$

$$P(X_1 + X_2 = 5) = \frac{4}{36} = \frac{1}{9}$$

MARKETING

- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

$$P(P) =$$

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$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$

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$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$

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$$P(M \cup T) =$$

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$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$
$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) =$$

MARKETING

- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$
$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$$

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- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

$$P(P) = P(P|A) \cdot P(A) + P(P|\bar{A}) \cdot P(\bar{A}) =$$
$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$
$$= \frac{1}{3} \cdot 0.21 + 0.1 \cdot (1 - 0.21) =$$

$$P(M \cup T) = P(M) + P(T) - \mathrm{P}(M \cap T) = 0.02 + 0.2 - 0.01 = 0.21$$

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- 1 in 50 potential buyers sees a magazine ad (M), 1 in 5 sees a TV ad $(T)$and 1 in 100 sees both. One in 3 purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product (P)?

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$$= P(P|M \cup T) \cdot P(M \cup T) + P(P|\overline{M \cup T}) \cdot P(\overline{M \cup T}) =$$
$$= \frac{1}{3} \cdot 0.21 + 0.1 \cdot (1 - 0.21) = 0.07 + 0.079 = 0.149$$

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BALLS IN TWO BOWLS

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$$= \frac{P(blue|A) \cdot P(A)}{P(blue|A) \cdot P(A) + P(blue|B) \cdot P(B)} = \frac{0.4 \cdot 0.5}{0.4 \cdot 0.5 + 0.2 \cdot 0.5} =$$

$$= \frac{0.2}{0.3} = \frac{2}{3}$$

COINS

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