PROBABILITY & STATISTICS

Lecture 11 – Confidence Intervals

CONFIDENCE INTERVALS

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- How confident can we be in our estimation?
- CI proposes a range of plausible values.

DEFINITION

A $1 - \alpha$ confidence interval for a parameter θ is an interval $C_n = (a, b)$ such that $\mathrm{T}_1 = t_1(X_1, \dots, X_n)$, $T_2 = t_2(X_1, \dots, X_n)$ and

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- **Random** intervals: T_1 and T_2 are functions of random samples.
- θ is unknown, but fixed
  T_1 and T_2 are random

INTERPRETATION

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- Common interpretation:

If I repeat the experiment many times, the interval will contain the true value of θ 95% of the time (α=0.05).

CI FOR NORMAL DATA

CI for μ, *known* σ

CI FOR μ, KNOWN σ

$X_1, X_2, \dots, X_n$ − samples from $N(\mu, \sigma^2)$, σ is known.
How to construct a CI for μ?

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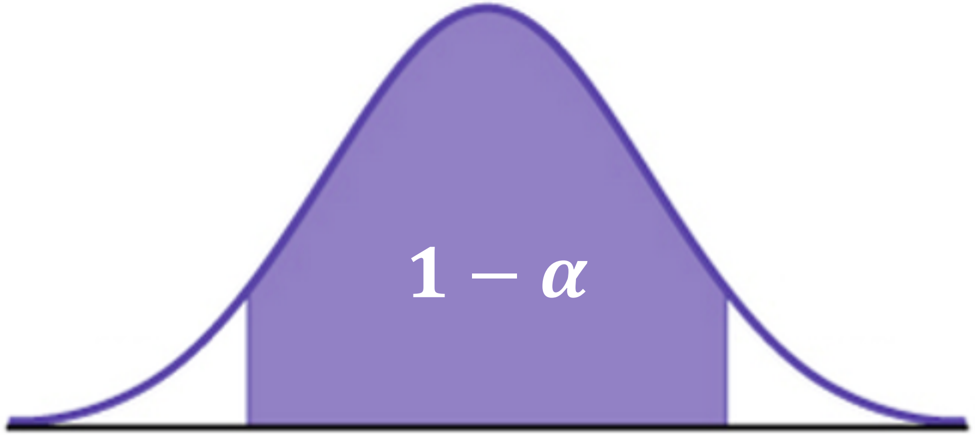

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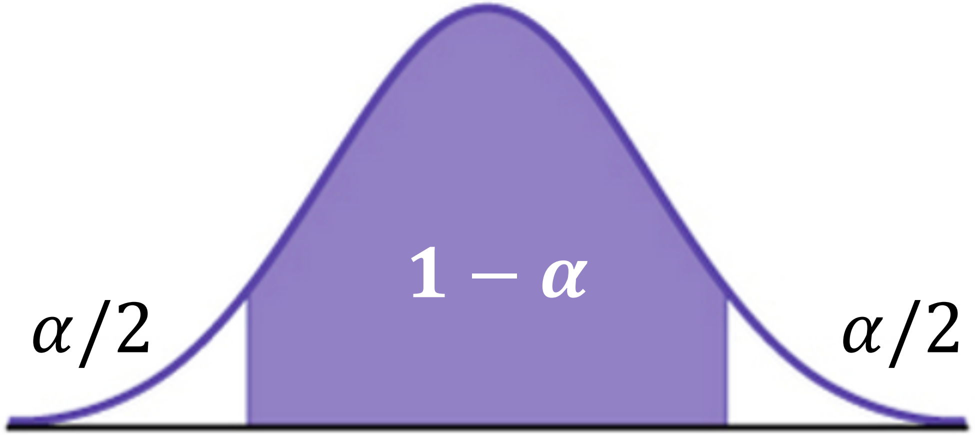

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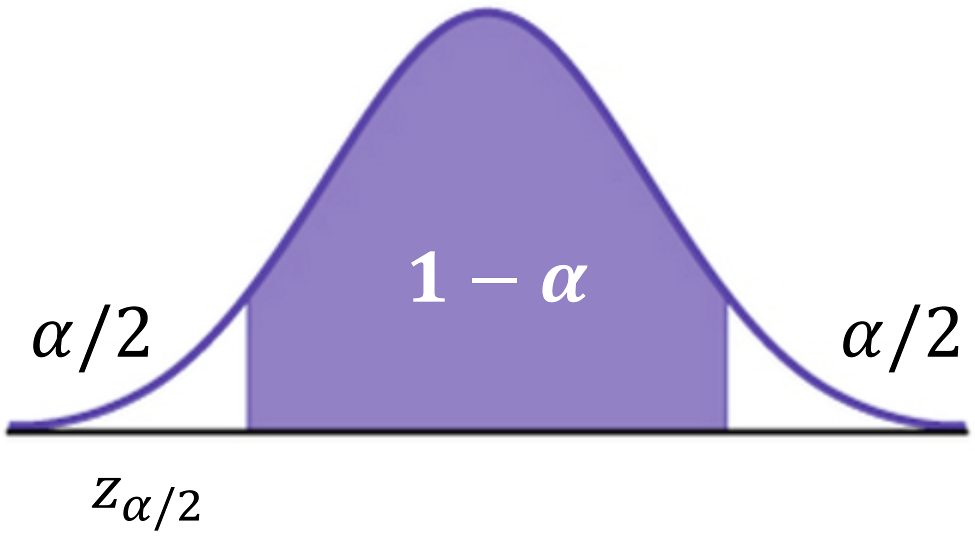

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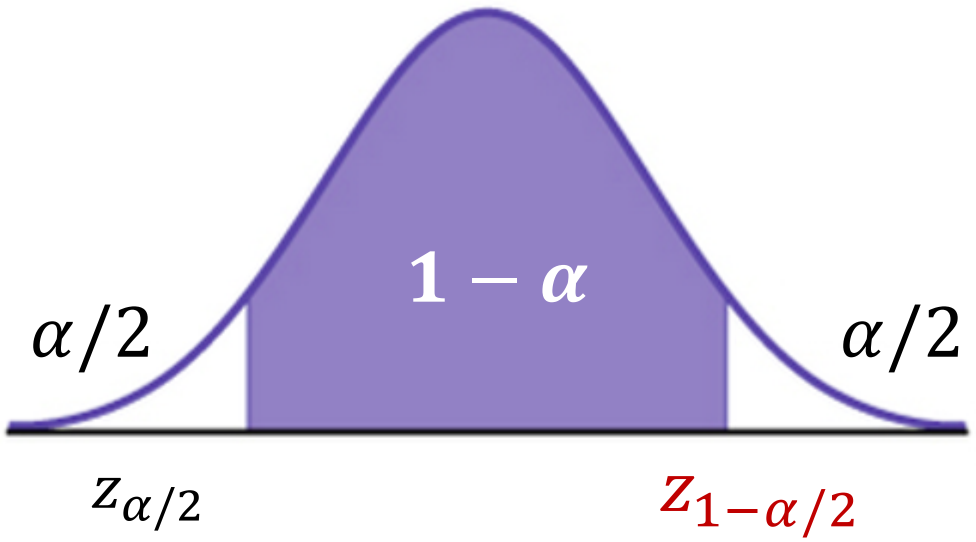

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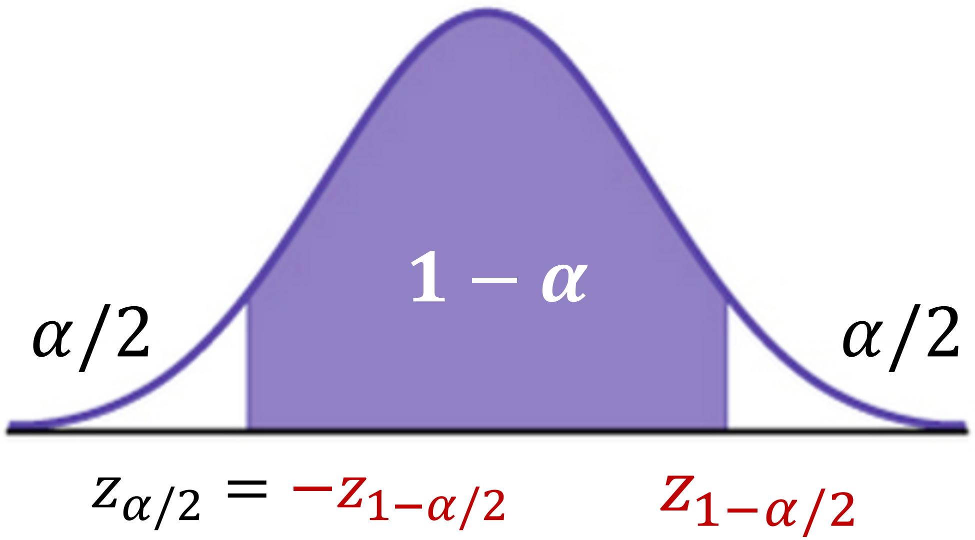

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0.995	2.58
0.99	2.33
0.975	1.96
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$$5 \pm \frac{1}{\sqrt{100}} \cdot 1.64$$

PRACTICE!

Google Classroom -> z-intervals

CONFIDENCE INTERVALS: RECAP

- NORMAL DISTRIBUTION:

$$X_1, X_2, \dots, X_n - \text{i.i.d. samples}$$

  - CI for μ, σ is known: **z-interval**

$$\bar{X} \pm \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2}, \quad z_{1-\alpha/2} - \text{quantile from } N(0,1)$$

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STUDENT DISTRIBUTION

William Sealy Gosset

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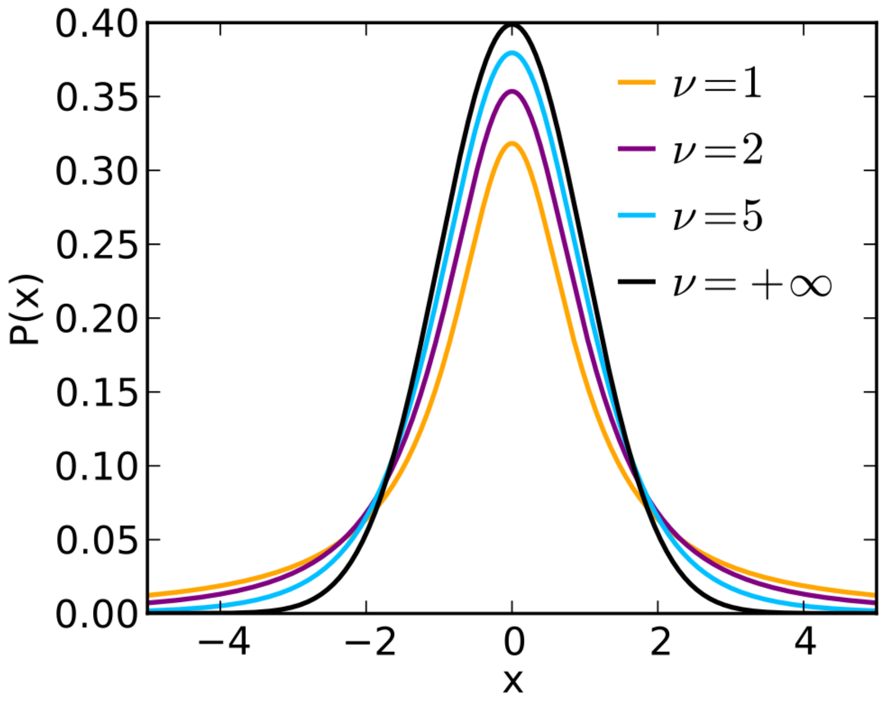

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CI FOR μ, UNKNOWN σ: EXAMPLE

- $X_1, X_2, \dots, X_{20}$ – samples from $N(\mu, \sigma^2)$, σ is unknown.
- $\bar{X} = 42, \quad s^2 = 36$

- Give the 95%-CI for μ.

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$$\mu = 42 \pm \frac{6}{\sqrt{20}} \cdot 2.093$$

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$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1) -$$

Chi-squared distribution

CHI-SQUARED DISTRIBUTION

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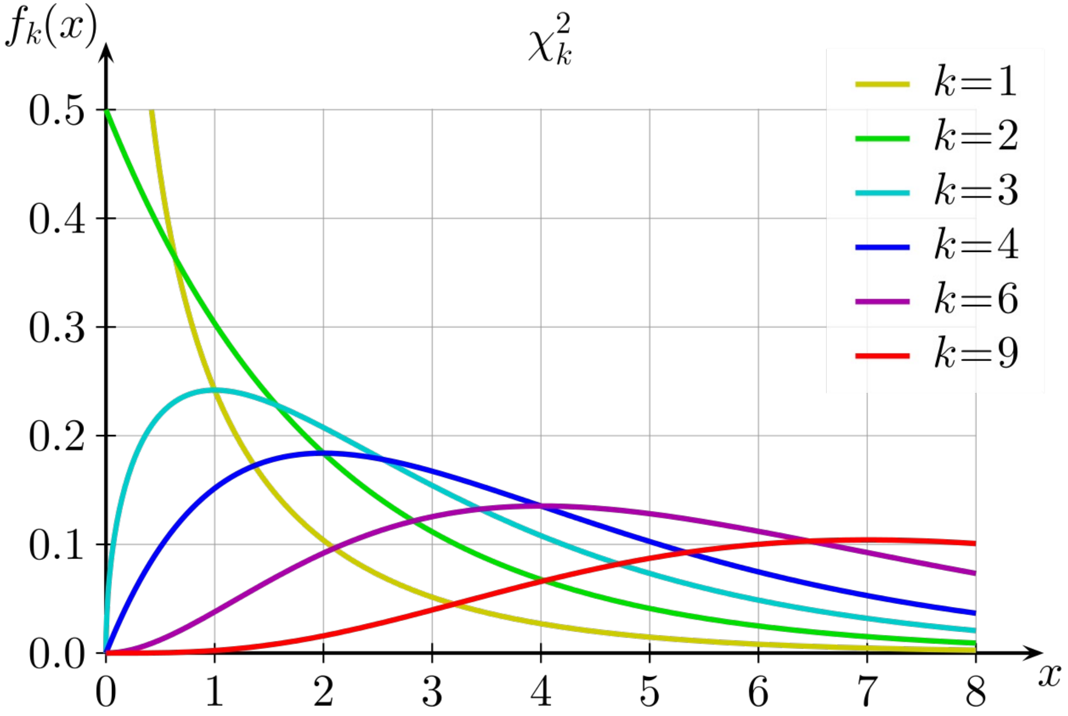

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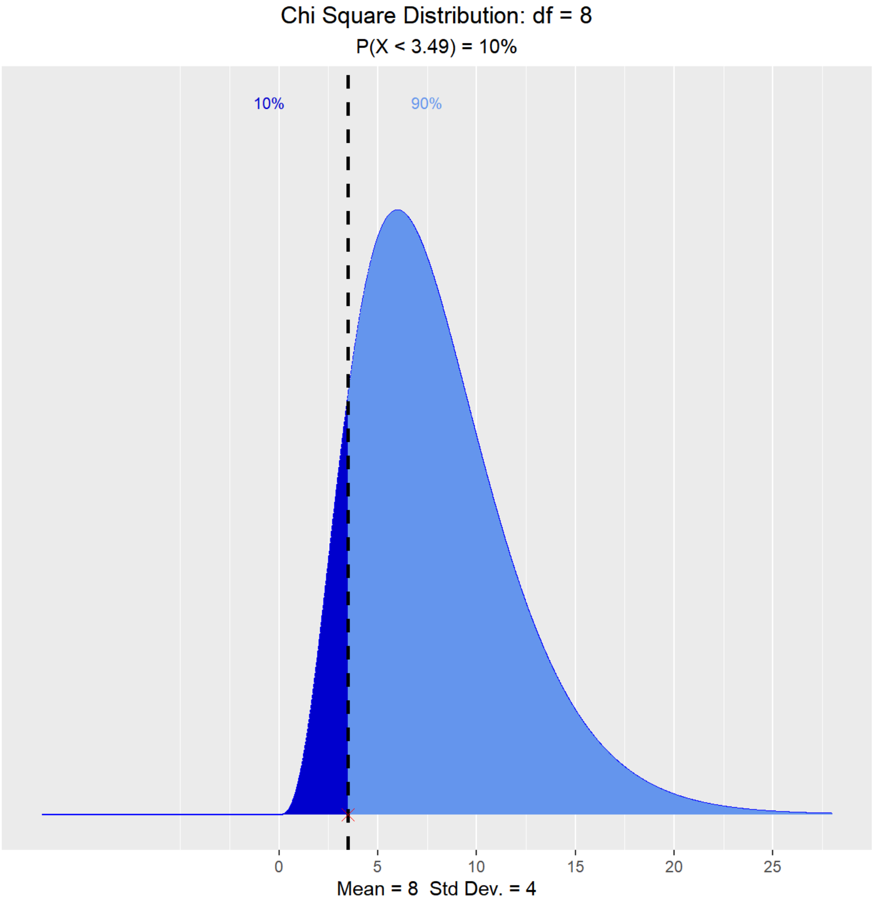

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LARGE SAMPLES

Using Central Limit Theorem

CI FOR LARGE SAMPLES

- Typical task: estimating the mean of a distribution.
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If $\mu, \sigma^2 < \infty$ and if n is sufficiently large, then:

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CI FOR
BERNOULLI DISTRIBUTION

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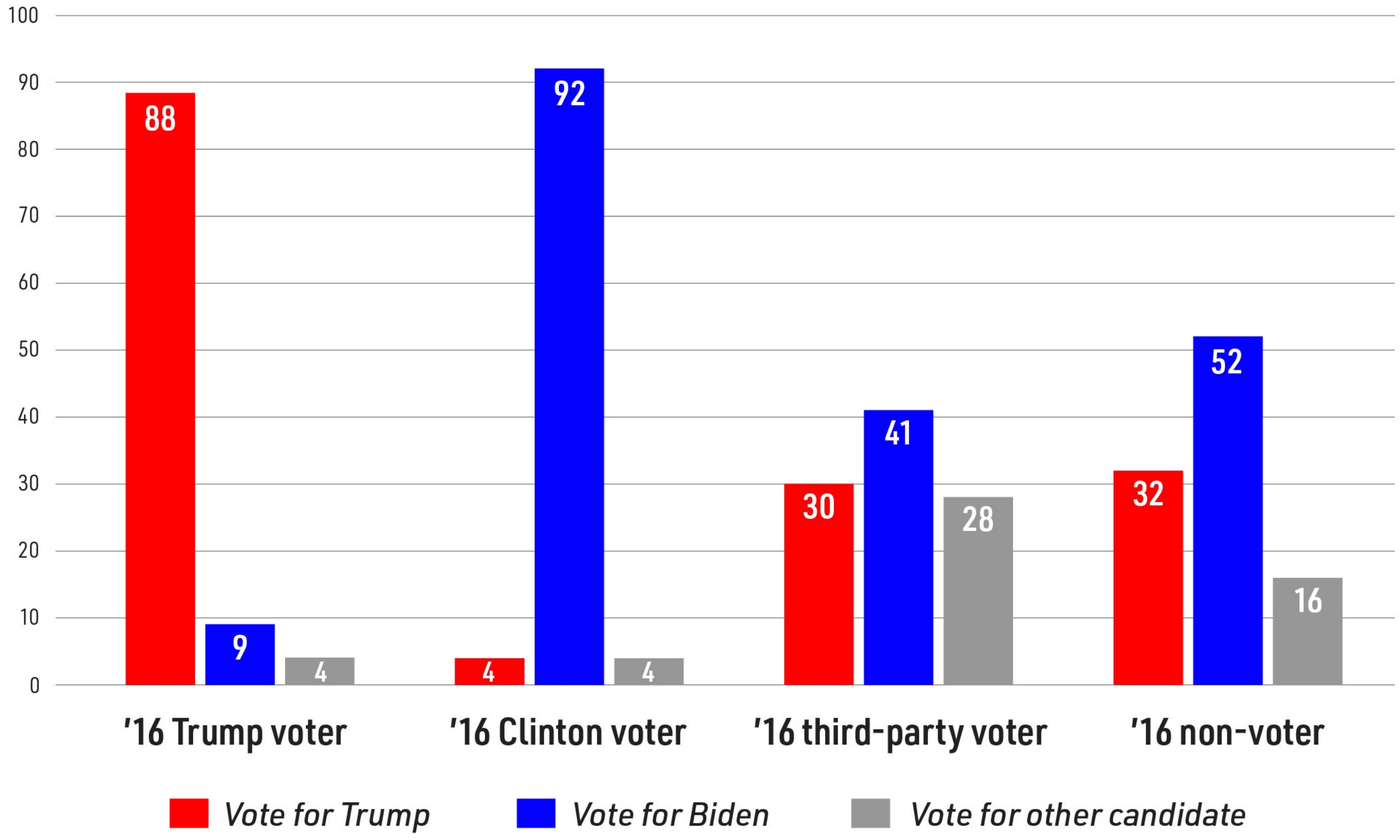
52% favor candidate A with a margin-of-error of ±5%.

Presidential Election Preview:

How Clinton, Trump Voters from 2016 Plan to Vote in 2020



A total of 1,510 eligible voters, who are adult members of the USC Dornsife Center for Economic and Social Research's Understanding America Study internet panel, participated from August 11 – 16, 2020. Margin of sampling error for this preliminary sample is +/-3 percentage points. Tracking graphs will be updated every day. For full question text, methodology, and other information, visit https://uasdata.usc.edu/index.php

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- Similar reasoning applies to the CI for Bernoulli.

- *How many customers should be surveyed in order to estimate the share of satisfied customers to within 3% at the 95% confidence level?*

$$\hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}$$

Let $p_0 = 0.5$ – the worst possible variance

$$1.96\sqrt{\frac{0.25}{n}} \leq 0.03 \iff n \geq 1067.1$$