# Weakly dumped coupled oscillator model for lab experiment design

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## Abstract

A forced weakly dumped harmonic oscillator having two degrees of freedom is considered. In Laboratory 3 it is implemented on air-track experiment set. The goal of the experiment is to find oscillator's natural frequencies and compare it with the model. Having natural frequencies found, the system response to forced oscillations is tested. It is expected, that the system will response on forced oscillations frequencies close to natural frequencies. The system response is considered in terms of energy stored or power consumed, not in terms of oscillations amplitude, since the system has more than one degree of freedom.

In this document the coupled oscillator model is described.

## 1 Natural frequencies

A weakly dumped system with $n$ degrees of freedom can be represented as a decoupled system by coordinates rotation.

Let's suggest we have found somehow $n$ natural frequencies $\omega_i$ and changed coordinates. (this step now skipped, to be done).

## 2 Steady-state solution stored energy

In transformed coordinate system $x_i$ the equations can be written in a decoupled form (see eq.1 Ch.3 p.103 Berkeley 3)

$$M_i\ddot{x}_i(t) = -M_i\omega_i^2 x_i(t) - M_i\Gamma_i\dot{x}_i(t) + F_i\cos(\omega t) \tag{1}$$

Driving force $F_i$ is applied with frequency $\omega$.

For each coordinate $x_i$ the stored energy $E_i(\omega)$ is

$$E_i = \frac{1}{4}A_i(\omega)^2 M_i(\omega^2 + \omega_i^2), \tag{2}$$

where amplitude $A_i(\omega)$ is (see eq. 16, 17, 23 Ch3 Berkeley 3 p.105-106):

$$A_i = \frac{F_i}{M_i} \frac{1}{(\omega_i^2 - \omega^2)^2 + \Gamma_i^2 \omega^2}$$

The full stored energy $E$ of the forced dumped oscillator will be a sum of energies $E_i$

$$E(\omega) = \sum_i E_i(\omega). \tag{4}$$