To say before the experiment

1. Plateau is not really a plateau
2. No need to measure too many point while searching for the plateau
3. The sources should be placed upside down.

In the manual you are required to measure background radiation 150 times, and compare the results with Poissonian and Gaussian distributions. I found that (i) no clear criterion to distinguish between the two is presented to students, and (ii) 150 measurements is not enough to confidently do it. Therefore I want you to do a modified experiment with a stronger, but still very weak source. In this way you will be able to measure every second instead of 10 second as with background radiation. And, as a result, to do some 1000 measurements during reasonable time.

Poisson vs Gaussian process

1. Notations: mean value – overbar: $\bar{x}$. Average value: $\langle x \rangle$. $\bar{x} \to \langle x \rangle$ for infinite number of trials.
2. Consider independent events during a fixed time interval τ. The probability for n events during τ is described by the Poisson process:
$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{1}$$
From this: the average is $\langle n \rangle = \lambda$, the variance is $\sigma^2 = \langle (n-\lambda)^2 \rangle = \langle n^2 \rangle - \lambda^2 = \lambda$, the STD of n is $\sigma = \sqrt{\lambda}$.
3. Consider skewness of the distribution $\kappa_3 = \langle (n - \langle n \rangle)^3 \rangle$. For the Poisson distribution $\kappa_3 = \langle n \rangle = \lambda$, and for the Gaussian one $\kappa_3 = 0$. We therefore shall look at $K_3 = (1/(m-1)) \sum_{i=1}^{m} (n_i - \bar{n})^3$ in order to distinguish between the distributions; $K_3 \to \kappa_3$ as $m \to \infty$.

Measurements

We want you to distinguish between these two distributions. For this let us do m measurements and look at the skewness, K_3.

1. Take a relatively weak source. I took Cesium, and put it label up.
2. Set the measurement time to 1 s, and take, say 10 measurements. Calculate $\bar{n}$. You want it to be relatively small, say ≈ 5. You can play with the source-to-counter distance to get the counts in this ballpark.

3. Take a set of at least $m = 150\bar{n}$ measurements. For $\bar{n} = 5$ it will take some 12 min–not that long. The reason for such a large m is that we want uncertainty of K_3 to be reasonably small. Should we have chosen a larger $\bar{n}$, we would need to do more measurements, and the measurement time would be too long.

4. Calculate $\bar{n}$ and $\mathrm{STD}(\bar{n}) = \mathrm{STD}(n)/\sqrt{m-1}$ for the new, large set of data.

5. Calculate K_3. Compare it with $\bar{n}$. For Poisson process we expect $\bar{n} \to \lambda$ and $K_3 \to \lambda$ for $m \to \infty$.

6. We need to check if the value of K_3 that we got is statistically significant. For this it worth to compare K_3 with expectation for the Gaussian statistics. In the Gaussian case, $\kappa_3 = 0$. But a legitimate question to ask: what is the expected uncertainty of the K_3. We don't have the luxury to repeat sets of m measurements many times.

   Let us consider $\xi_i = (n_i - \bar{n})^3$ as a new stochastic variable. Calculate the variance $\mathrm{Var}(\xi)$. Since $K_3 = \sum \xi_i/(m-1)$ is the average of a large number m of independent variables, the Central Limiting Theorem is applicable to it. Therefore, $\mathrm{Var}(K_3) = \mathrm{Var}(\xi)/(m-1)$. Calculate $\mathrm{STD}(K_3) = \sqrt{\mathrm{Var}(K_3)}$. Compare K_3, $\mathrm{STD}(K_3)$, and $\bar{n}$. Can we conclude that the distribution of n is rather Poissonian than Gaussian?

Required number of measurements

Below I estimate the number of measurements required to confidently distinguish between the distributions. For this we need to look at the expectation value of $\mathrm{Var}(K_3) = \mathrm{Var}(\xi)/(m-1)$ for Poisson distribution of n_i. We start from the variance of $\xi_i = (n_i - \bar{n})^3$. For simplicity we shall look at $\xi_i = (n_i - \lambda)^3$. This will give us *underestimation* for $\mathrm{Var}(\xi)$.

$$\langle \xi \rangle = \lambda$$

$$\langle (\xi - \bar{\xi})^2 \rangle = \langle \xi^2 \rangle - \lambda^2$$

We need to calculate $\langle \xi^2 \rangle$. This can be done straightforwardly using Poisson distribution. However, one can estimate it by approximating $n - \lambda$ distribution by the Gaussian one to get:

$$\langle \xi^2 \rangle = \langle (n-\lambda)^6 \rangle \approx 5!! \langle (n-\lambda)^2 \rangle^3 = 15\lambda^3 \tag{2}$$

You may wonder why we can do it for Poisson distribution. This is because the main contribution to the even central moments comes from the respective power of the second one, the variance.

$$\langle((n-\lambda)^3-\lambda)^2\rangle = \langle(n-\lambda)^6\rangle + \lambda^2 \approx 5!!\lambda^3 + \lambda^2$$

From this

$$\mathrm{Var}(K_3) \approx 5!!\lambda^3/m = 15\lambda^3/m,$$

We want $\mathrm{STD}(K_3)$ to be less that $\lambda/3$, so

$$\mathrm{Var}(K_3) \approx 15\lambda^3/m < \lambda^2/9$$

From this we get $m > 9 \cdot 15\lambda$.

Sources

Thallium Tl_{81}^{104} decays into Tl_{80}^{104} with energy 0.7 MeV.
Strontium-90 ($^{90}_{38}Sr$) decays into $^{90}_{39}Y$ with emission of 0.546 MeV distributed to an electron, an antineutrino, and the yttrium isotope with half-life of 28.8y. This in turn undergoes β- decay with a half-life of 64 hours and a decay energy of 2.28 MeV distributed to an electron, an antineutrino, and ^{90}Zr (zirconium), which is stable. Note that 90Sr/Y is almost a pure beta particle source; the gamma photon emission from the decay of ^{90}Y is so infrequent that it can normally be ignored. $\mu(2.28\,MeV) = 5 \cdot 10^3 cm^2/g \to l = 0.74\,mm$

Tutorial improvement

1. Need to add some theory regarding decay.

2. Most of the attenuation, and it is attenuation and not absorption.

3. Emphasize that one should not measure during the same time interval. Rather to get the same or similar number of counts, and calculate counts/second. T low counting rate it can be impossible, so the errors should increase at low counting rates.

4. Put decay energies into the instructions.