3320. Пусть $x^2 = vw$, $y^2 = uw$, $z^2 = uv$ и

$$f(x, y, z) = F(u, v, w).$$

Доказать, что

$$xf'_x + yf'_y + zf'_z = uF'_u + vF'_v + wF'_w.$$

Предполагая, что произвольные функции φ, ψ и т. п. дифференцируемы достаточное число раз, проверить следующие равенства:

3321. $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$, если $z = \varphi(x^2 + y^2)$.

3322. $x^2\dfrac{\partial z}{\partial x} - xy\dfrac{\partial z}{\partial y} + y^2 = 0$, если $z = \dfrac{y^2}{3x} + \varphi(xy)$.

3323. $(x^2 - y^2)\dfrac{\partial z}{\partial x} + xy\dfrac{\partial z}{\partial y} = xyz$,

если $z = e^y\varphi\left(ye^{\frac{x^2}{2y^2}}\right)$.

3324. $x\dfrac{\partial u}{\partial x} + \alpha y\dfrac{\partial u}{\partial y} + \beta z\dfrac{\partial u}{\partial z} = nu$,

если $u = x^n\varphi\left(\dfrac{y}{x^\alpha}, \dfrac{z}{x^\beta}\right)$.

3325. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = u + \dfrac{xy}{z}$,

если $u = \dfrac{xy}{z}\ln x + x\varphi\left(\dfrac{y}{x}, \dfrac{z}{x}\right)$.

3326. $\dfrac{\partial^2 u}{\partial t^2} = a^2\dfrac{\partial^2 u}{\partial x^2}$, если $u = \varphi(x - at) + \psi(x + at)$.

3327. $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x\,\partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0$,

если $u = x\varphi(x + y) + y\psi(x + y)$.

3328. $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$,

если $u = \varphi\left(\dfrac{y}{x}\right) + x\psi\left(\dfrac{y}{x}\right)$.

3329. $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = n(n-1)u$,

если $u = x^n\varphi\left(\dfrac{y}{x}\right) + x^{1-n}\psi\left(\dfrac{y}{x}\right)$.

3330. $\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial x\,\partial y} = \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x^2}$, если $u = \varphi[x + \psi(y)]$.