ASP3262 Introduction to coding
Week 11 Lab

Numerical Solutions to Nuclear Reaction Networks

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Contents

Task 1

Task1

$$3\,{}^4He \longleftrightarrow {}^{12}C + \gamma$$

$$2\,{}^{12}C \longleftrightarrow {}^{24}Mg + \gamma$$

Forward reaction rates

$$\frac{d}{dt} Y_{^4He} = \left(\lambda_{3^4He \to {}^{12}C}\right)\left(\frac{0-3}{3!\,0!}\right) Y_{^4He}^3 \; Y_{^{12}C}^0$$

$$= \left(\lambda_{3^4He \to {}^{12}C}\right)\left(-\frac{1}{2}\right)\left(Y_{^4He}^3\right)$$

$$\frac{d}{dt} Y_{^{12}C} = \left(\lambda_{3^4He \to {}^{12}C}\right)\left(\frac{1-0}{3!\,0!}\right) Y_{^4He}^3 \; Y_{^{12}C}^0$$

$$+ \left(\lambda_{2^{12}C \to {}^{24}Mg}\right) \frac{0-2}{2!\,0!} Y_{^{12}C}^2 \; Y_{^{24}Mg}^0$$

$$= \left(\lambda_{3^4He \to {}^{12}C}\right)\left(\frac{1}{6}\right) Y_{^4He}^3 + \left(\lambda_{2^{12}C \to {}^{24}Mg}\right)\left(-1\right) Y_{^{12}C}^2$$

$$\frac{d}{dt} Y_{^{24}Mg} = \lambda_{2^{12}C \to {}^{24}Mg} \; \frac{1-0}{2!\,0!} \; Y_{^{12}C}^2 \; Y_{^{24}Mg}^0$$

$$= \left(\lambda_{2^{12}C \to {}^{24}Mg}\right)\left(\frac{1}{2}\right) Y_{^{12}C}^2$$

Reverse reaction rates

$^{12}C \rightarrow 3\,^{3}He$

$^{24}Mg \rightarrow 2\,^{12}C$

$$\frac{d}{dt} Y_{^4He} = \left(\lambda_{^{12}C \rightarrow 3^4He}\right)\left(\frac{3-0}{1!\,0!}\right) Y_{^{12}C}^{1}\, Y_{^3He}^{0}$$
$$= \left(\lambda_{^{12}C \rightarrow 3^4He}\right) (3)\, Y_{^{12}C}^{1}$$

$$\frac{d}{dt} Y_{^{12}C} = \left(\lambda_{^{12}C \rightarrow 3^4He}\right)\left(\frac{0-1}{1!\,0!}\right) Y_{^{12}C}^{1}\, Y_{^3He}^{0}$$
$$+ \left(\lambda_{^{24}Mg \rightarrow 2^{12}C}\right)\left(\frac{2-0}{1!\,0!}\right) Y_{^{24}Mg}^{1}\, Y_{^{12}C}^{0}$$
$$= -\left(\lambda_{^{12}C \rightarrow 3^4He}\right)(1)\, Y_{^{12}C} + \left(\lambda_{^{24}Mg \rightarrow 2^{12}C}\right)(2)\left(Y_{^{24}Mg}\right)$$

$$\frac{d}{dt} Y_{^{24}Mg} = \left(\lambda_{^{24}Mg \rightarrow 2^{12}C}\right)\left(\frac{0-1}{1}\right) Y_{^{24}Mg}$$
$$= -\left(\lambda_{^{24}Mg \rightarrow 2^{12}C}\right)\left(Y_{^{24}Mg}\right)$$

Task 2

The rates of the nuclear reactions are shown on Figures 1 - 4. We can see that the rates for reverse reactions are identical for the two different pressures. In addition, we can see that reverse reactions only happen at temperatures higher than about 1×10^9 K, while forward reactions happen at lower temperatures.

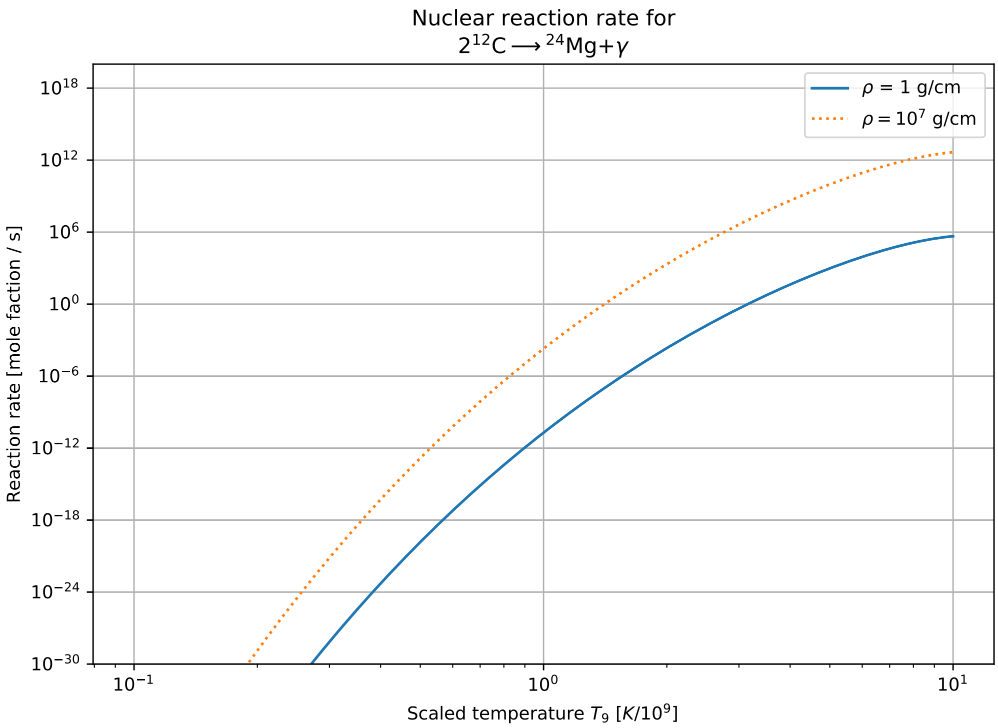

Figure 1: Nuclear reaction rates for $2^{12}\mathrm{C} \longrightarrow {}^{24}\mathrm{Mg}$.

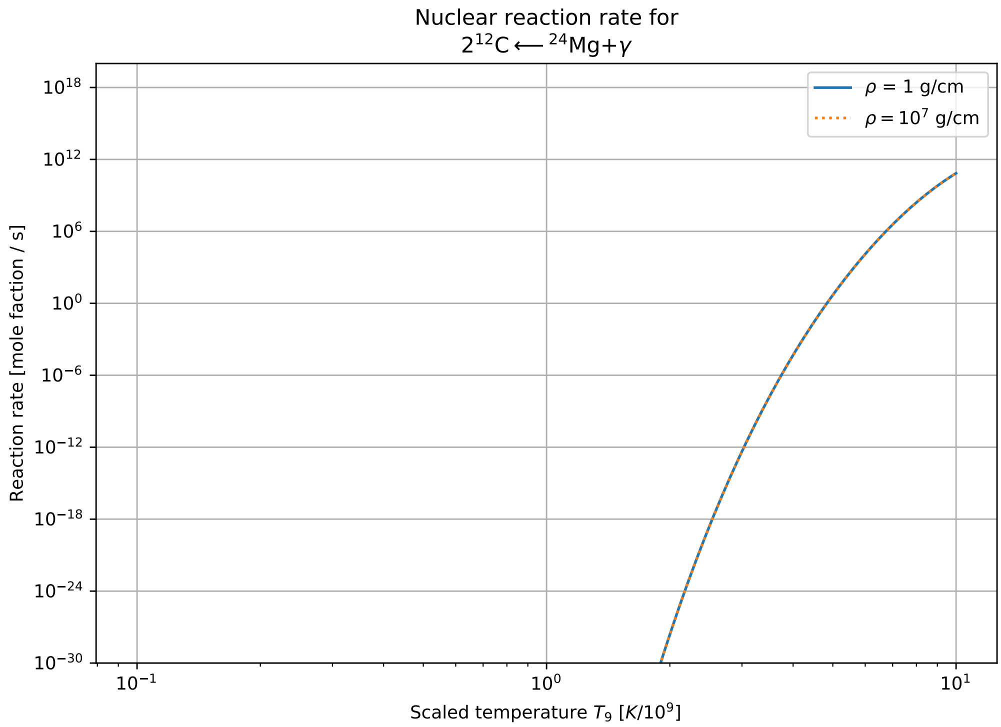

Figure 2: Nuclear reaction rates for $2^{12}C \longleftarrow {}^{24}Mg$.

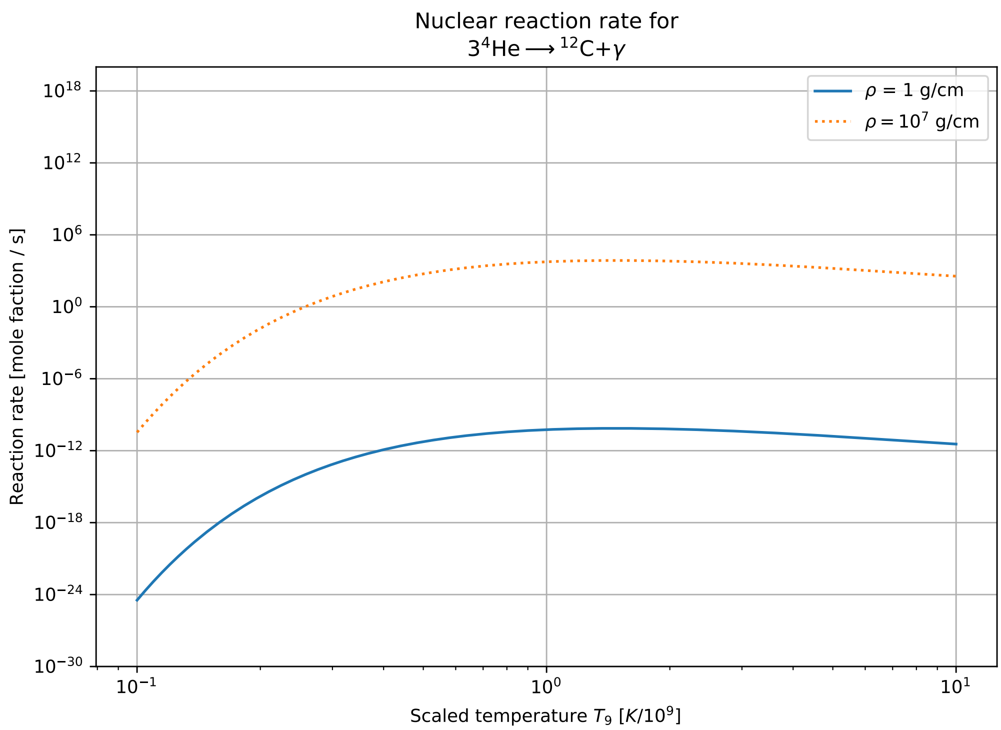

Figure 3: Nuclear reaction rates for $3^4He \longrightarrow {}^{12}C$.

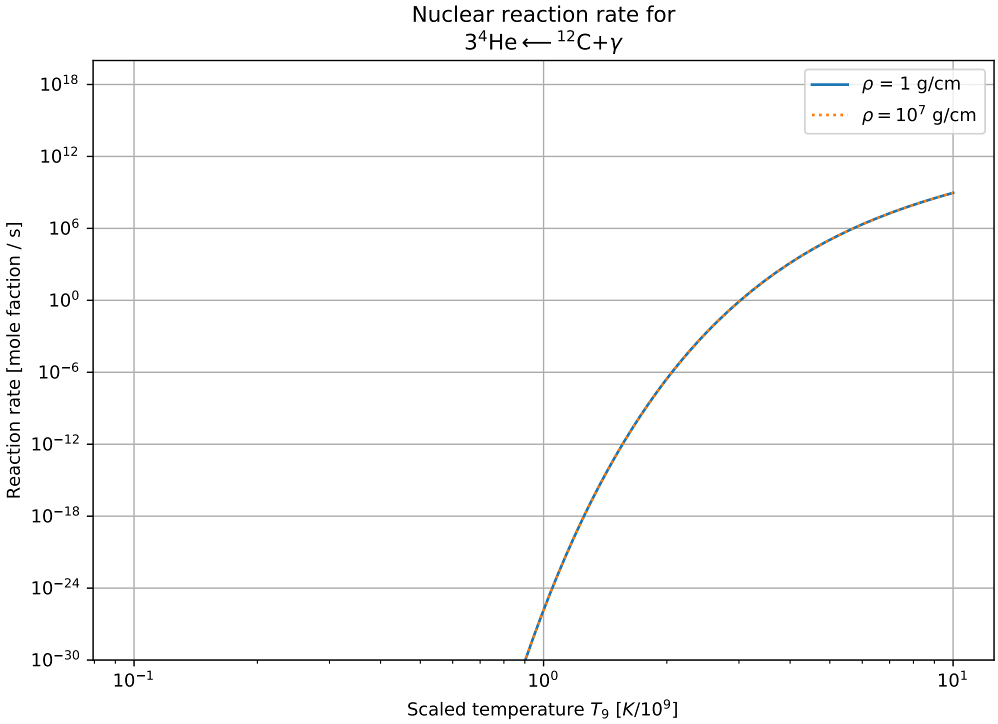

Figure 4: Nuclear reaction rates for $3^4\mathrm{He} \longleftarrow {}^{12}\mathrm{C}$.

Task 3

The changes of mass fraction over time for a range of temperatures are shown on Figures 5 - 14.

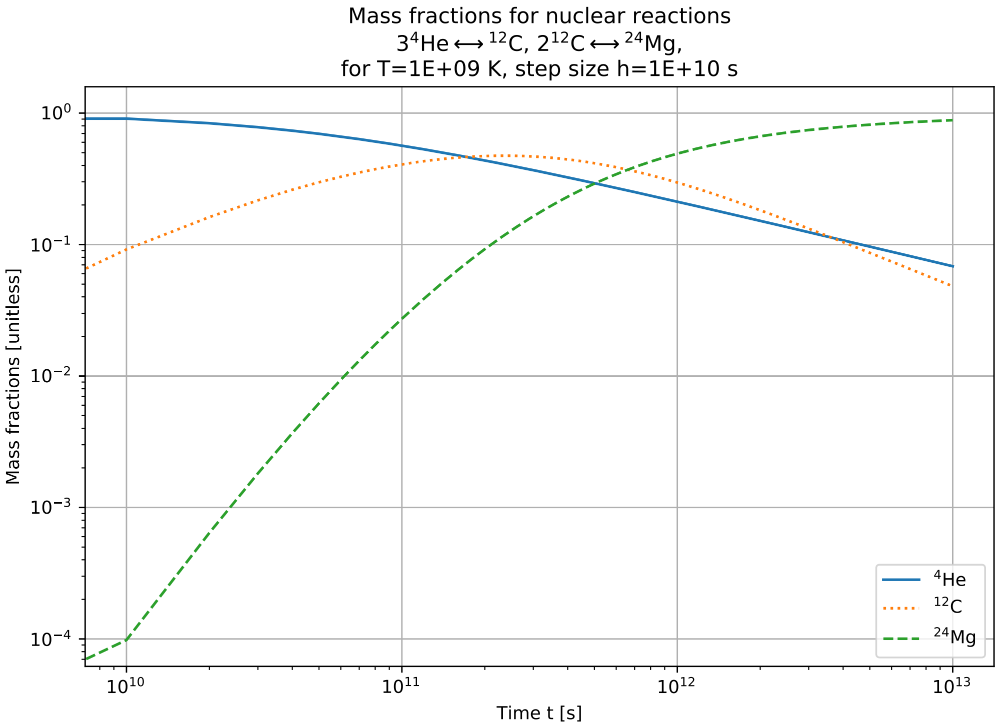

Figure 5: Changes of mass fraction over time for temperature $T = 1 \times 10^9$ K.

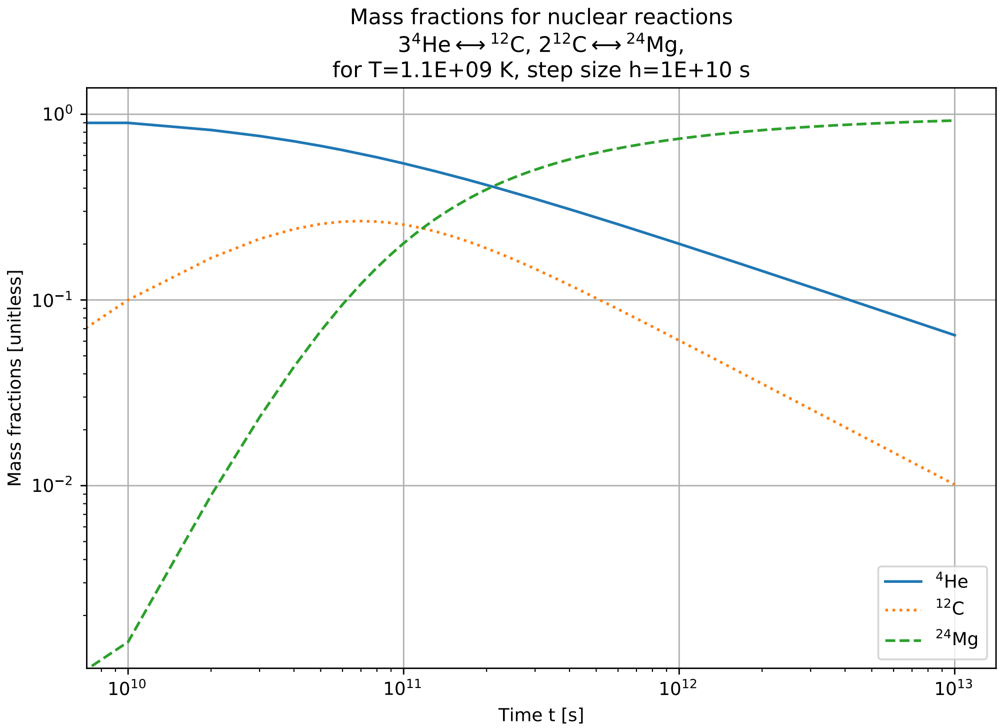

Figure 6: Changes of mass fraction over time for temperature $T = 1.1 \times 10^9$ K.

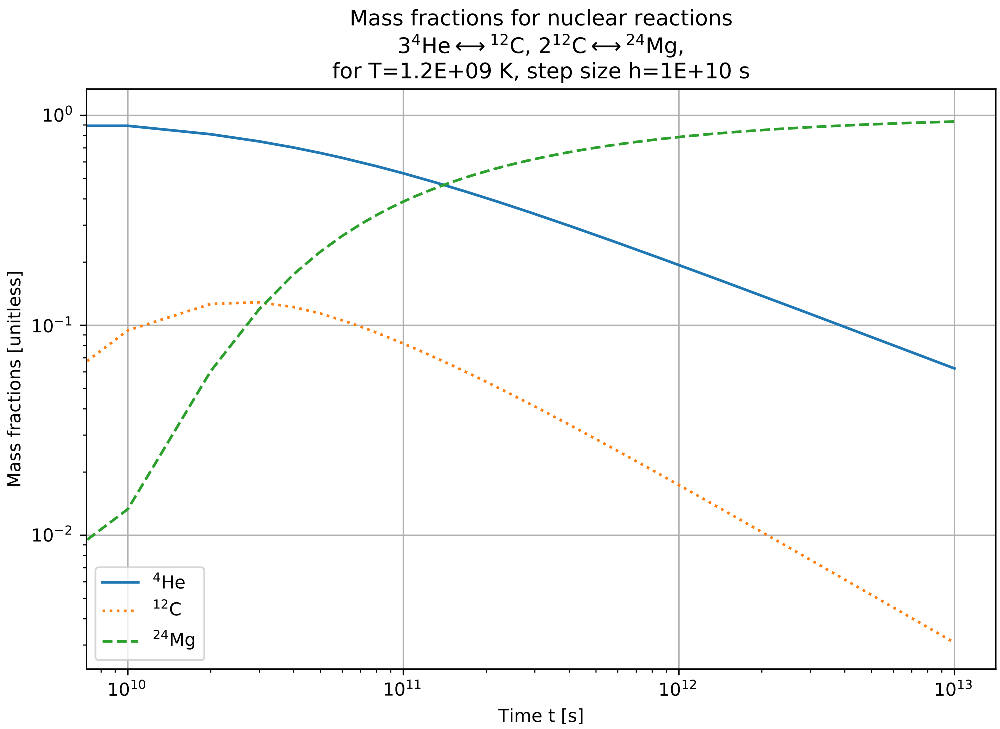

Figure 7: Changes of mass fraction over time for temperature $T = 1.2 \times 10^9$ K.

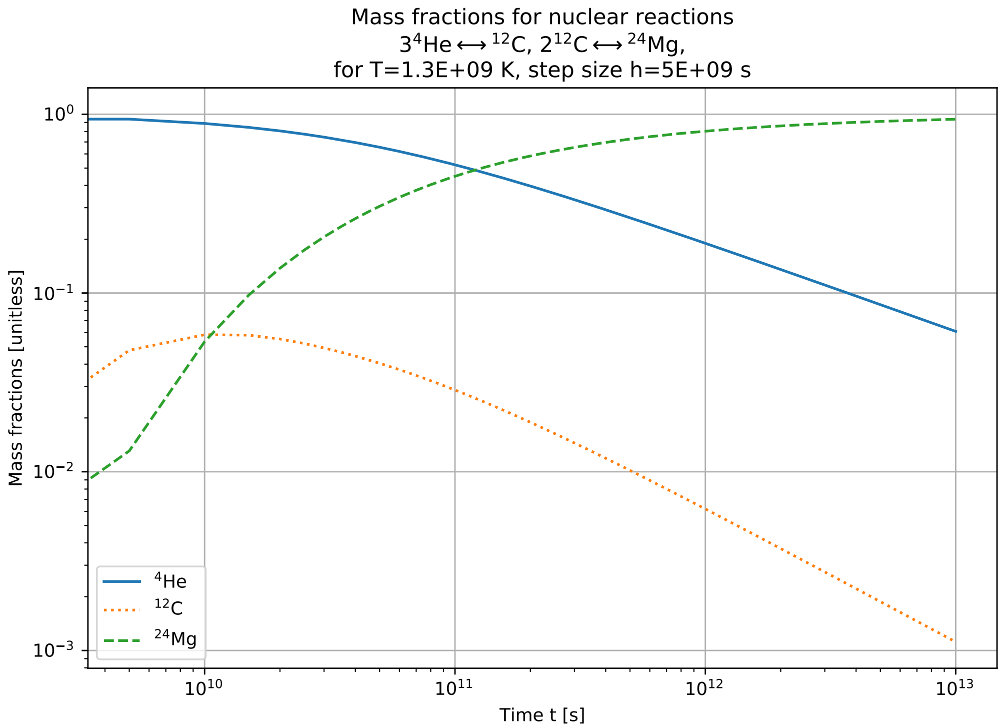

Figure 8: Changes of mass fraction over time for temperature $T = 1.3 \times 10^9$ K.

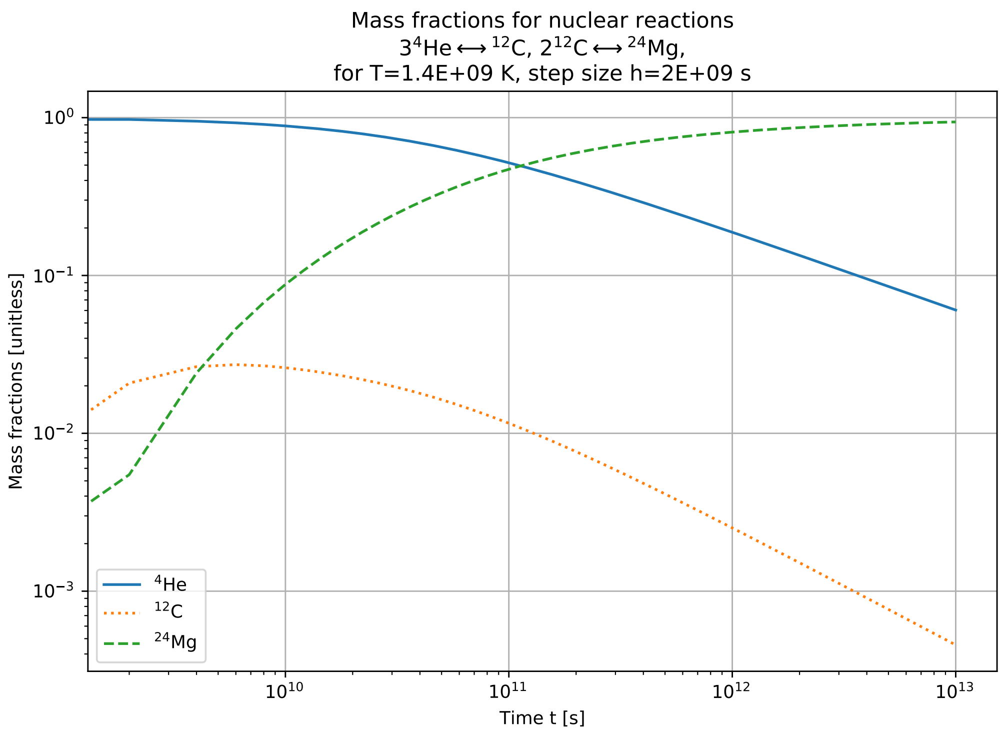

Figure 9: Changes of mass fraction over time for temperature $T = 1.4 \times 10^9$ K.

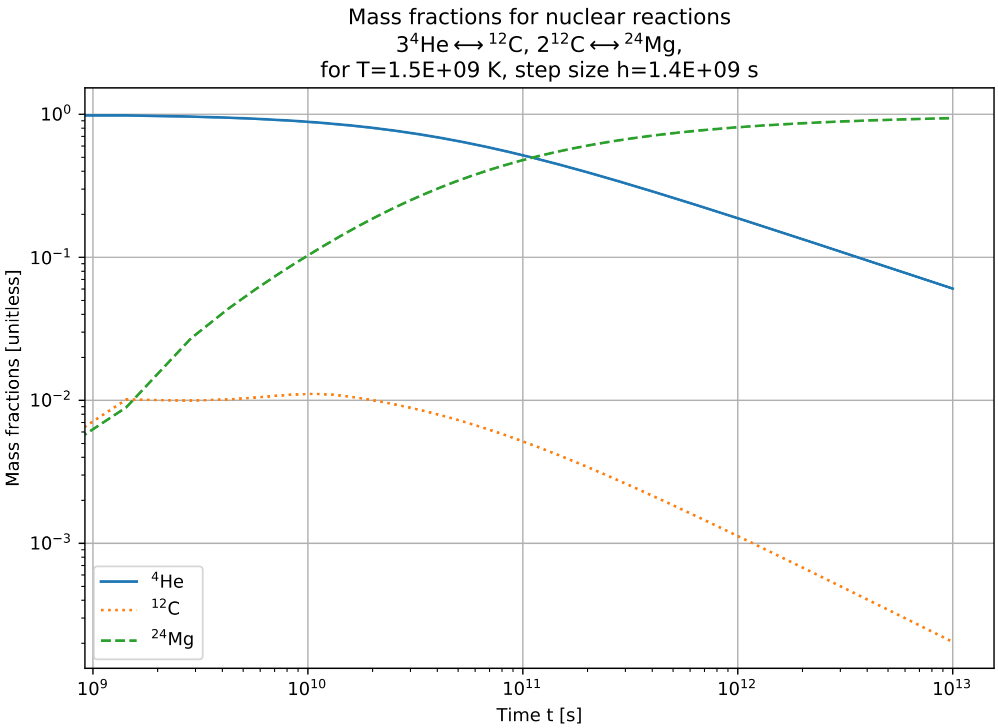

Figure 10: Changes of mass fraction over time for temperature $T = 1.5 \times 10^9$ K.

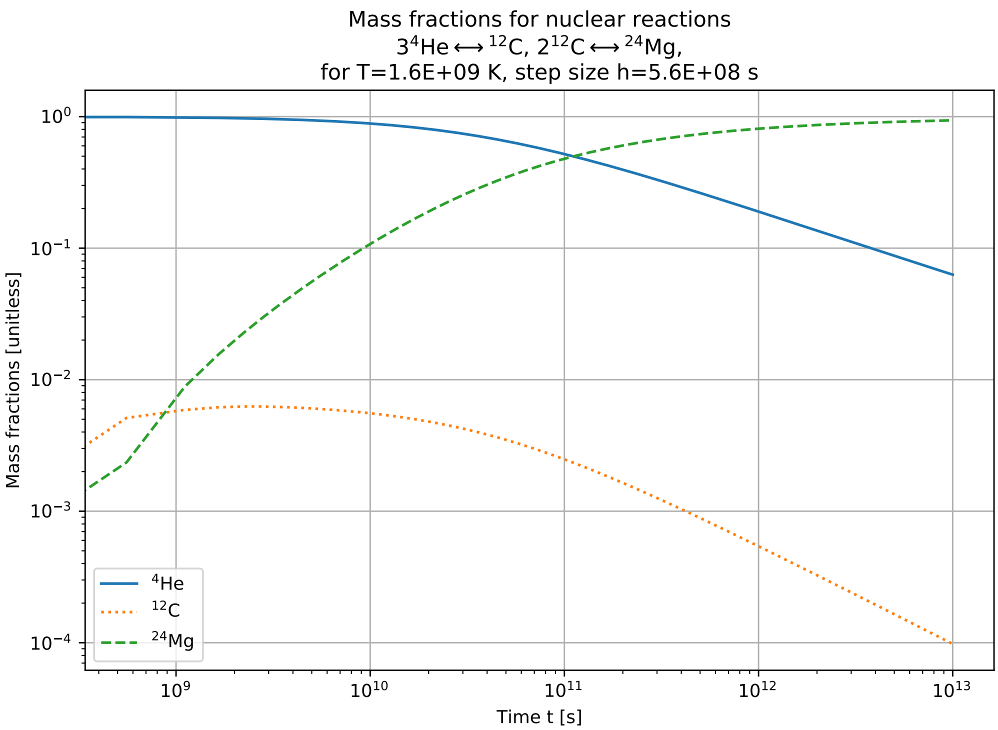

Figure 11: Changes of mass fraction over time for temperature $T = 1.6 \times 10^9$ K.

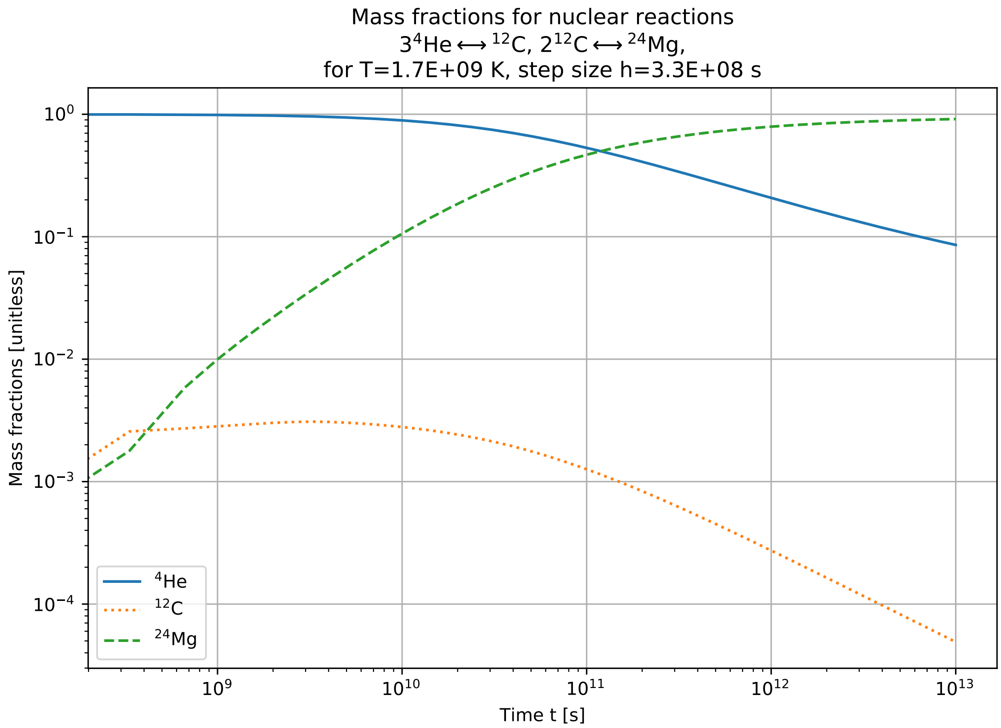

Figure 12: Changes of mass fraction over time for temperature $T = 1.7 \times 10^9$ K.

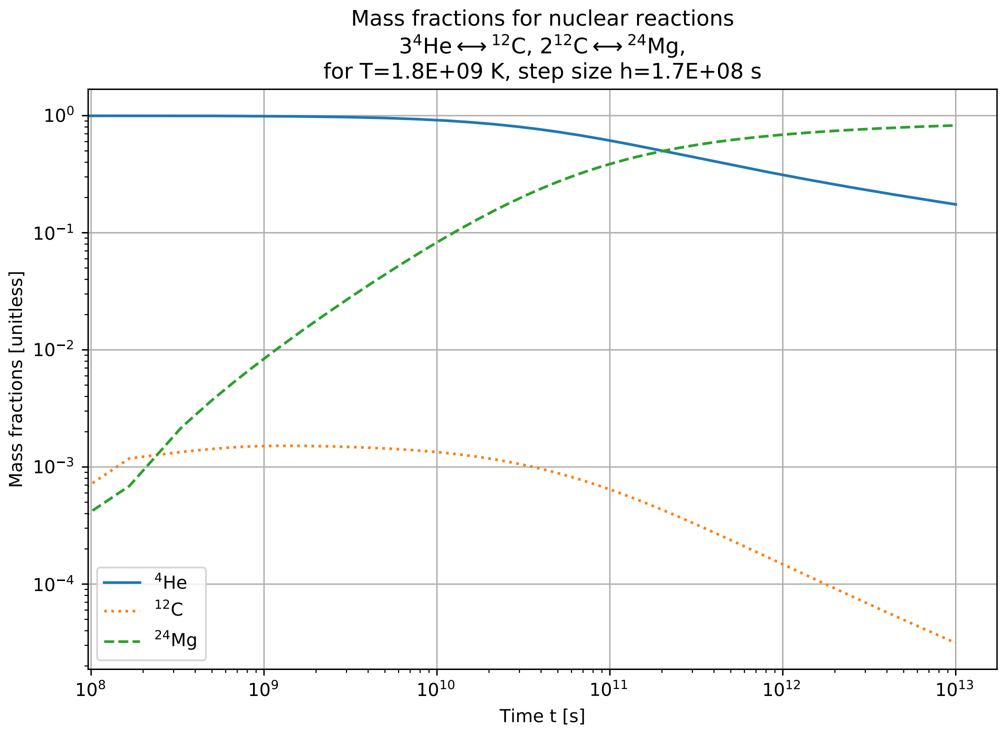

Figure 13: Changes of mass fraction over time for temperature $T = 1.8 \times 10^9$ K.

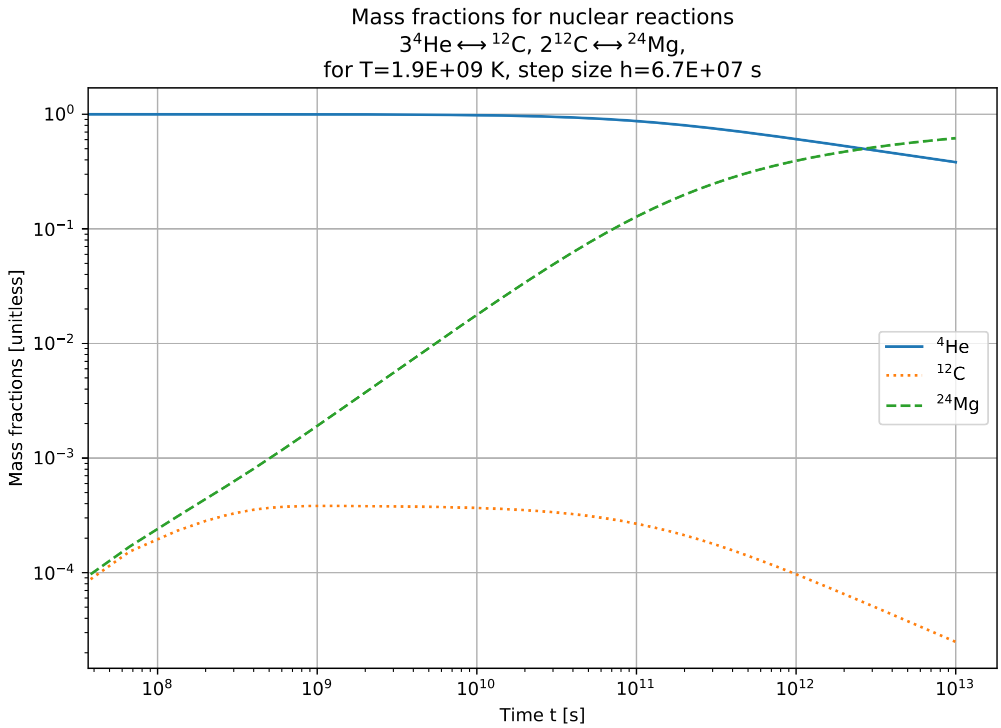

Figure 14: Changes of mass fraction over time for temperature $T = 1.9 \times 10^9$ K.

Task 4

Here we use an integrator with adaptive time step. The changes of mass fraction over time are shown on Figure 15.

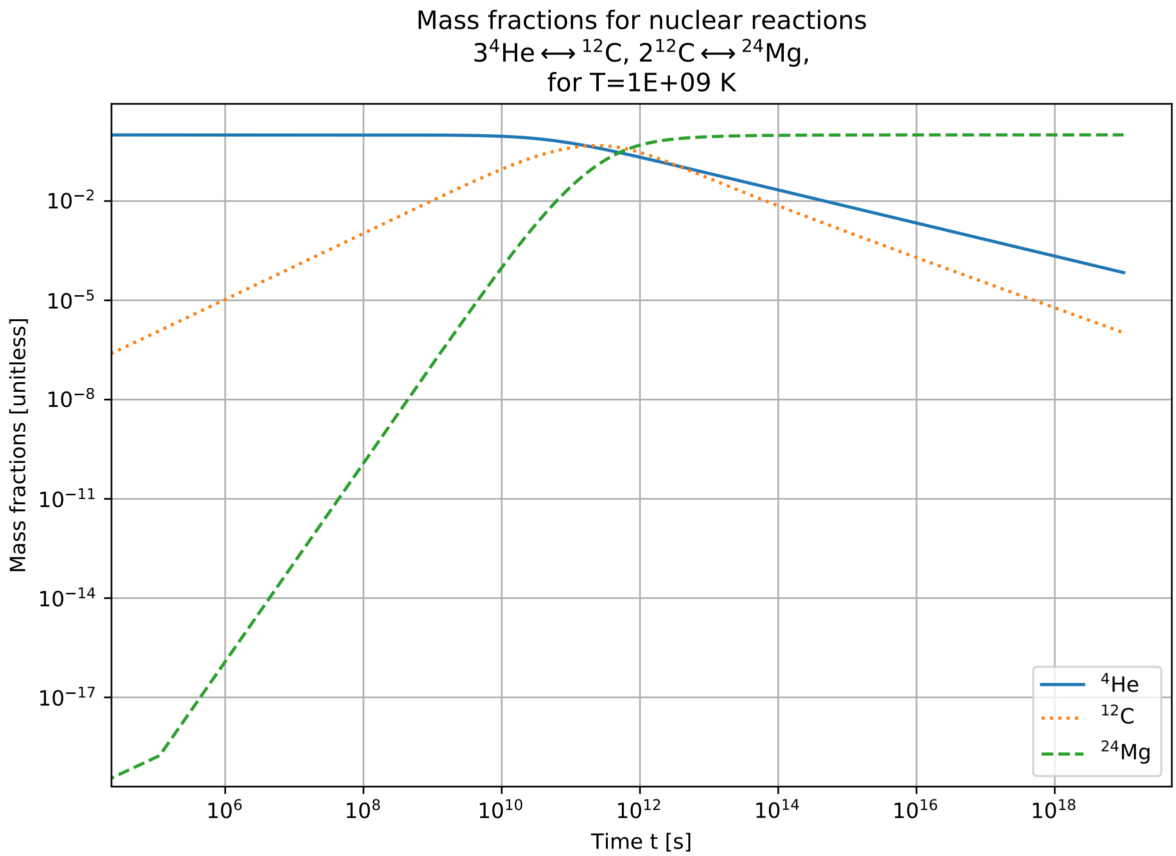

Figure 15: Changes of mass fraction over time for temperature $T = 1 \times 10^9$ K.

Task 5

Solutions obtained with an implicit solver for higher temperatures are shown on Figure 16 and Figure 17.

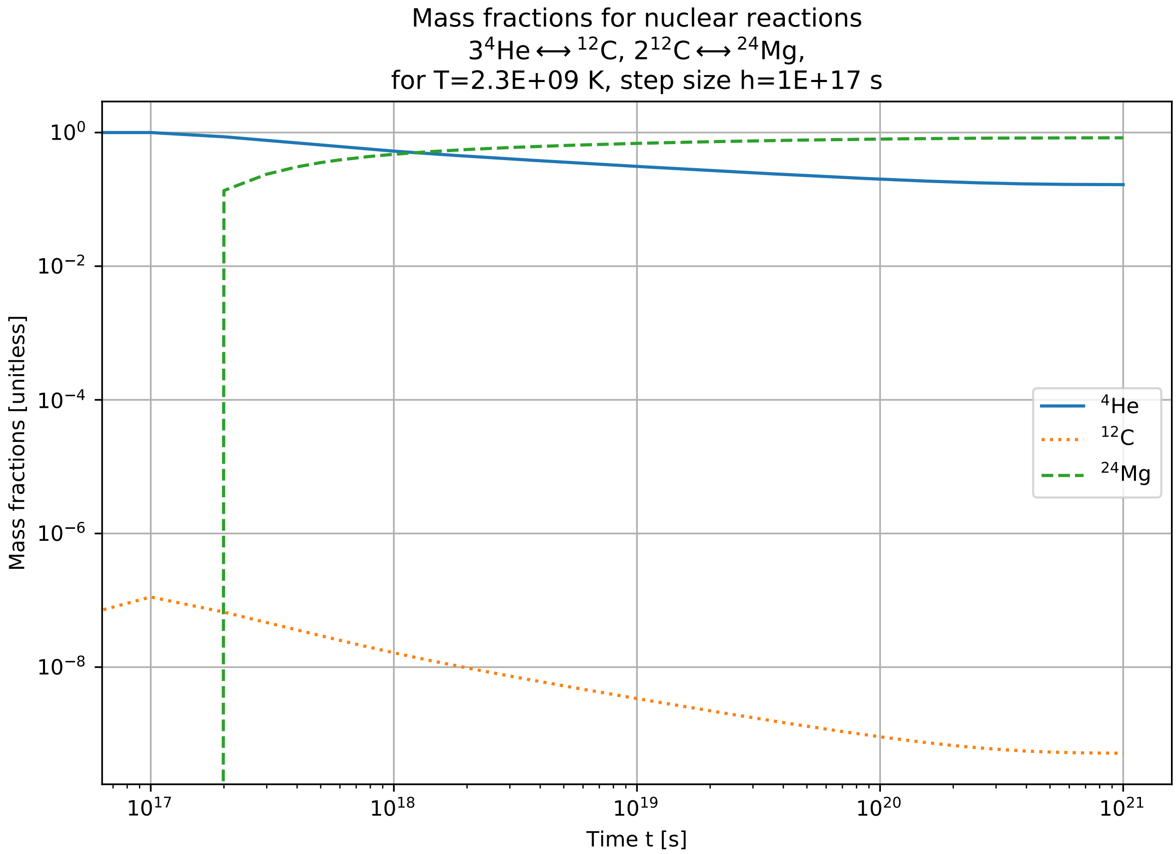

Figure 16: Changes of mass fraction over time for temperature $T = 2.3 \times 10^9$ K.

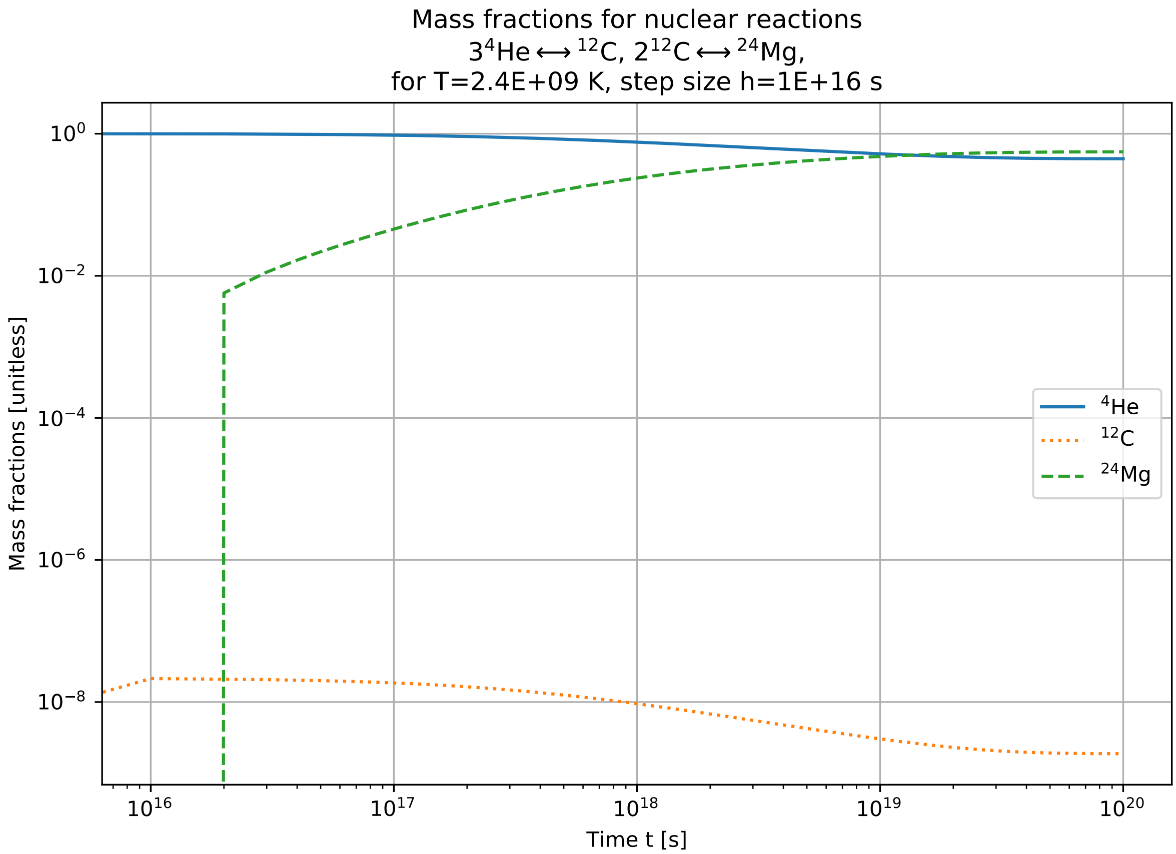

Figure 17: Changes of mass fraction over time for temperature $T = 2.4 \times 10^9$ K.

Task 6

Solutions obtained with an adaptive implicit solver are shown on Figure 18. The final mass fractions for Helium, Carbon and Magnesium are 0.0156, 0 and 0.984 respectively.

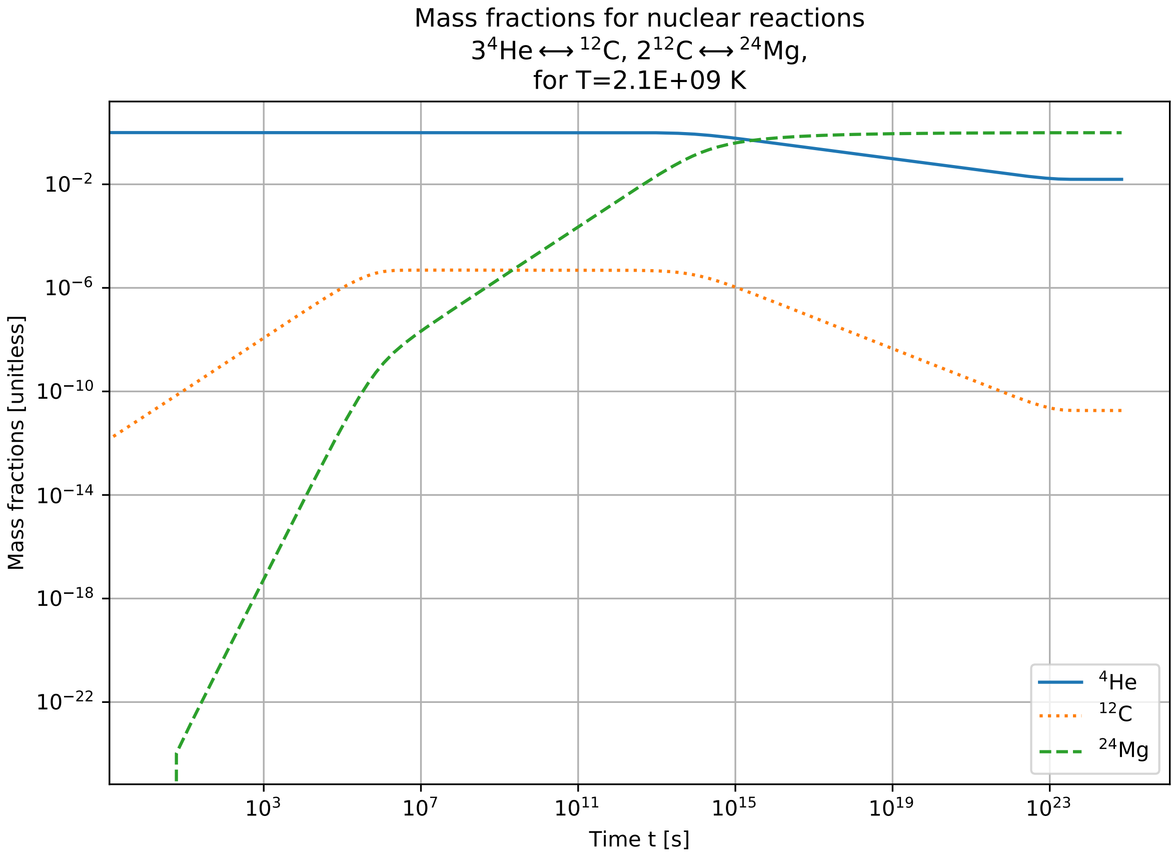

Figure 18: Changes of mass fraction over time for temperature $T = 2.1 \times 10^9$ K calculated with adaptive implicit solver.

Task 7

Solutions obtained using variable temperatures are shown on Figure 19 and Figure 20. For the last solution we had to decrease the temperature to 7×10^9 K in order for the plot to show up.

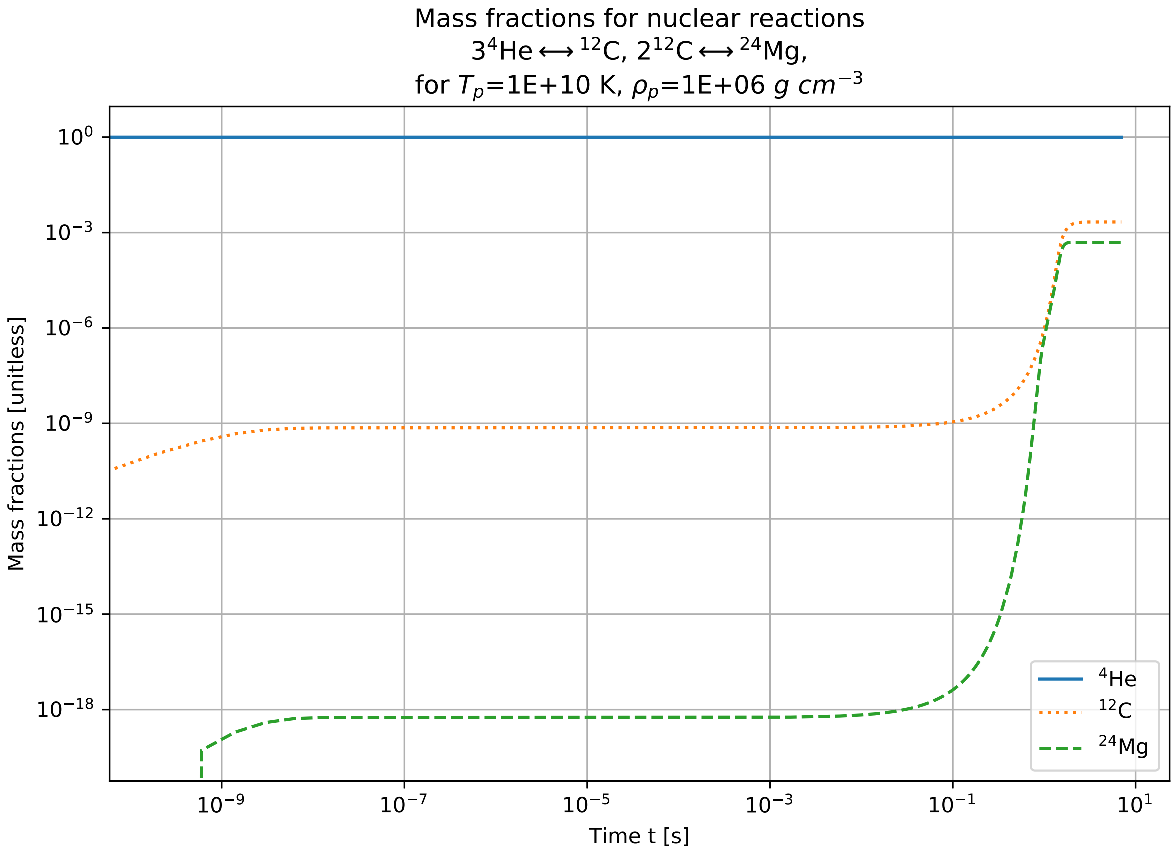

Figure 19: Changes of mass fraction over time for variable temperature.

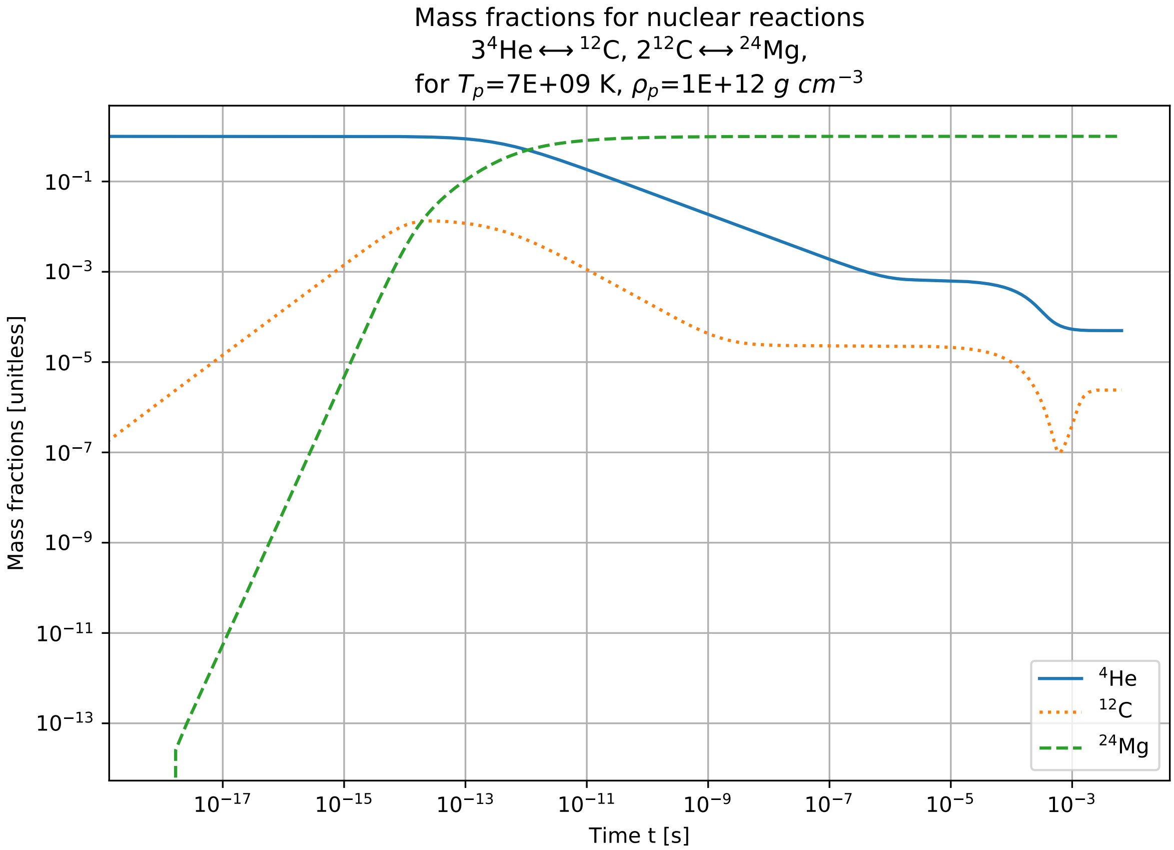

Figure 20: Changes of mass fraction over time for variable temperature.