Estimating magnitude uncertainties

This document contains a list of steps for approximating uncertainties of measured magnitudes in ASP 3231 project.

1 Find uncertainty of star's flux

- Measure twenty background fluxes by choosing apertures in the regions without stars.

- Note that the radius of the apertures for measuring fluxes should be equal to the radius used to measure the fluxes of the stars in that image.

- Calculate standard deviation of these fluxes. We will use it to approximate the uncertainty of the flux u(f) from the stars for that image (I don't know why we can do this, but ok).

2 Calculate uncertainty of reference star's magnitude in photometric image

- The magnitude m_2 of a reference star is calculated from photometric calibration equation:

$$m_2 = z - 2.5 \log_{10}(f/t) + k\,A, \tag{1}$$

where z, and k are constants, f is flux of a reference star measured in photometric image, A is the average air mass index of the stacked images.

- Next we calculate $u(m_2)$ (uncertainty of m_2) by propagating uncertainties $u(f)$ and $u(A)$. The uncertainty of the air mass $u(A)$ is standard deviation of air mass indices of the stacked images.

- Assuming f and A are independent, we calculate uncertainty of m2 using first order approximation:

$$u(m_2) = \sqrt{\left[\frac{2.5}{f}\frac{1}{\ln(10)}u(f)\right]^2 + \left[a\,u(A)\right]^2}. \tag{2}$$

3   Calculate uncertainty of star's magnitude in non-photometric image for one reference star

- The magnitude m_1 of a star in non-photometric image is calculated from equation

$$m_1 = m_2 - 2.5 \log_{10}\left(\frac{f_1}{f_2}\right), \tag{3}$$

where m_2 is magnitude of a reference star, f_1 is the flux of the star we want, and f_2 is flux of a reference star. Both fluxes f_1 and f_2 are measured in non-photometric image.

- We calculate the uncertainty of m_1, assuming all variables in Equation 3 are independent, and thus using first order approximation

$$u(m_1) = \sqrt{[u(m_2)]^2 + \left[2.5\frac{u(f)}{\ln(10)}\right]^2\left(\frac{1}{f_1^2} + \frac{1}{f_2^2}\right)}, \tag{4}$$

where u(f) is the uncertainty of flux measurement in non-photometric image for the star we want.

4   Calculate final uncertainty of star's magnitude

- We use multiple reference stars to calculate a star's magnitudes using Equation 3. So if we have three reference stars a, b and c, we will calculate three magnitudes for each star: m_a, m_b, m_c.

- We calculate the final magnitude of the star m by taking the middle magnitude of m_a, m_b, m_c.

- For the uncertainty of the magnitude we take the uncertainty of the middle magnitude. Not that we do not calculate the average of m_a, m_b, m_c, because if we do so and use the simple error propagation formula we will get:

$$u(m) = \frac{\sqrt{u(m_a)^2 + u(m_b)^2 + u(m_c)^2}}{3}.$$

This approach assumes m_a, m_b, m_c are independent measurements. But this m_a, m_b, m_c are not independent, since in each measurement the star's flux f_1 is the same (Equation 3), and m_2 and f_2 are also the same for all stars. Therefore, if we did that, our uncertainty would be significantly underestimated.

- We report star's magnitude m, its uncertainty $u(m)$ and publish in The International Journal Of Tourism Sciences.