Towards Complex Understanding of Turbulent Convection in Stellar Interiors Using ransX Analysis Framework

Proposal For Post-Doctoctoral Research Position

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1 Introduction

Contemporary ground- and space-based telescopes provide us precise stellar data leading to challenging questions and forcing us to reconsider our basic assumptions regarding turbulent convection and mixing in stars. Properties of supernova explosions studied by HST or Keck can not be linked to their progenitors conclusively [Smartt, 2009]. Such progenitors are known to have a structure interleaved by turbulent convection shells [Hirschi et al., 2004]. VLT is observing massive stars with unexplained chemical peculiarities, where rotational mixing was considered to be enough to explain observations [Evans et al., 2008]. Kepler spacecraft finds unexplained pulsations of δ Scuti and γ Doradus stars [Uytterhoeven, 2011], which depend heavily on properties of sub-surface stellar convection [Guzik et al., 2000]. Explanation of observed element abundances in AGB stars requires physically motivated but still inconclusive tuning for mixing between turbulent envelope convection and underlying hydrogen-free core [Herwig, 2005].

Turbulence is during stellar evolution one of the most fundamental processes and before taking into account binarity, magnetism or rotation of a star to explain observations, we should understand stellar turbulence well first. It is arguably the greatest weakness in the modern theory of stellar evolution, which is mostly derived from one-dimensional calculations approximating dynamic turbulent processes by simplified theories [Kippenhahn and Weigert, 1990, Weiss et al., 2004]. In reality, turbulent flows are multidimensional and driven by non-linear terms of the hydrodynamic Navier-Stokes equations.

2 Aims

We will analyze three-dimensional (3D) hydrodynamic simulations of stellar convection within the context of Reynolds-Averaged Navier Stokes (RANS) approach pursued by Besnard et al. [1992], Livescu et al. [2009], Schwarzkopf et al. [2011]. It is a unique way of learning about turbulence based on budget analysis of hydrodynamic equations averaged in space and time, by which complexity of every term is reduced to a one-dimensional mean field.

Using this methodology, we derived RANS evolution equations for transport/flux/variance of mass, momenta, kinetic/internal/total energy, temperature, enthalpy, pressure and composition densities [Mocák et al., 2014] and implemented them to analysis framework, that we call rans(eXtreme) or ransX[1] for short. It should be noted here, that it is only one of many possible sets of equations relevant to closure problems in turbulence and there are many other formulations, which then need to approximate different terms [Canuto, 1992, 1993, Canuto et al., 2001, Hanjalic, 2002, Alfonsi, 2009, Garaud et al., 2010, Canuto, 2011, Biferale et al., 2011].

RANS approach introduces into the averaged equations many correlations of various thermodynamic fluctuations which are essentially new unknown variables. Hence, to solve them, we need either to design appropriate closures or derive and close evolution equations for them. Either of the tasks is dificult, because stellar turbulence is anisotropic, compressible and embedded in highly stratified environment where external forces like gravity and mean background flow play an important role. But we hope that this approach could in the future allow us to study stellar evolution using solution of the mean fields hydrodynamic equations, move away from canonical form of stellar structure equations and most importantly allow for a comprehensive synergy between engineering turbulence modeling and stellar astrophysics.

Our aims encompass the following targets (the tasks partially overlap with each other and the estimated time of completion is stated in brackets):

- publish our RANS mean-field equations implemented within ransX framework [Mocák et al., 2014] in high-impact referred journal and validate them with new high-resolution 3D hydrodynamic simulations (2+ year)
- help to implement the ransX framework to all stellar hydrodynamic codes capable of simulating stellar core and envelope convection or stellar atmospheres and make it an analysis standard (3+ years)

[1] ransX is free for download and test on https://github.com/mmicromegas/ransX

- validate our new hydrodyanmic stellar structure equations for turbulent regions without MLT inspired by preliminary results delivered by the framework based on low-resolution oxygen burning convective shell of a supernova progenitor (2+ years)

The new hydrodynamic stellar structure equations for stellar turbulence are listed below as equations (1),(2),(3),(4),(5),(6). They appear to work well (Fig.1) but the first four equations still lack a theory that explains them and the last equation (5) requires a proper model for transport of composition density ($\nabla_r f_\alpha$) commonly treated in stars as difussion.

$$\partial_r \overline{M} = -\overline{\rho}\ \overline{M}\ \overline{g}_r / \Gamma_1 \overline{P} + 4\pi r^2 \overline{\rho} \quad (1)$$

$$\partial_r \overline{P} = -\overline{\rho}\ \overline{g}_r \quad (2)$$

$$\partial_r \widetilde{L} = -4\pi r^2 \overline{\rho}\ \overline{g}_r / \Gamma_1 + \widetilde{\epsilon}_t \partial_r 4\pi r^2 \overline{\rho} \widetilde{u}_r \quad (3)$$

$$\partial_r \overline{T} = -(\Gamma_3 - 1)\ \overline{\rho}\ \overline{T}\ \overline{g}_r / \Gamma_1 \overline{P} \quad (4)$$

$$\partial_t \widetilde{X}_i = \widetilde{\dot{X}}_i^{nuc} - (1/\overline{\rho}) \nabla_r f_i - \widetilde{u}_r \partial_r \widetilde{X}_i \quad (5)$$

$$\widetilde{u}_r = \dot{\overline{M}} / 4\pi r^2 \overline{\rho} \quad (6)$$

Partial results from our mean-field RANS analysis related mostly to turbulent kinetic energy and transport of few chemical elements based on oxygen burning shell in massive stars have been already published e.g. Meakin and Arnett [2007], Arnett et al. [2009], Meakin and Arnett [2010], Viallet et al. [2013], Mocák et al. [2018]. In order to cover wider range of conditions present in stars like Schwarzschild and Ledoux stable/unstable regions, electron degeneracy and multiplicity of convection zones, we plan to extend our library of ransX mean fields calculated at runtime of 3D high-resolution hydrodynamic simulations of:

- single convection zone during core helium flash in low-mass stars with Ledoux unstable region at its bottom [Mocák et al., 2008, 2009, 2011] (1+ year)

- dual convection zone during core helium flash in metallicity free stars [Mocák et al., 2010] (1+ year)

- single convection zone resulting from core carbon flash in intermediate stars with Ledoux unstable region at its bottom [Mocák et al., 2011] (1+ years)

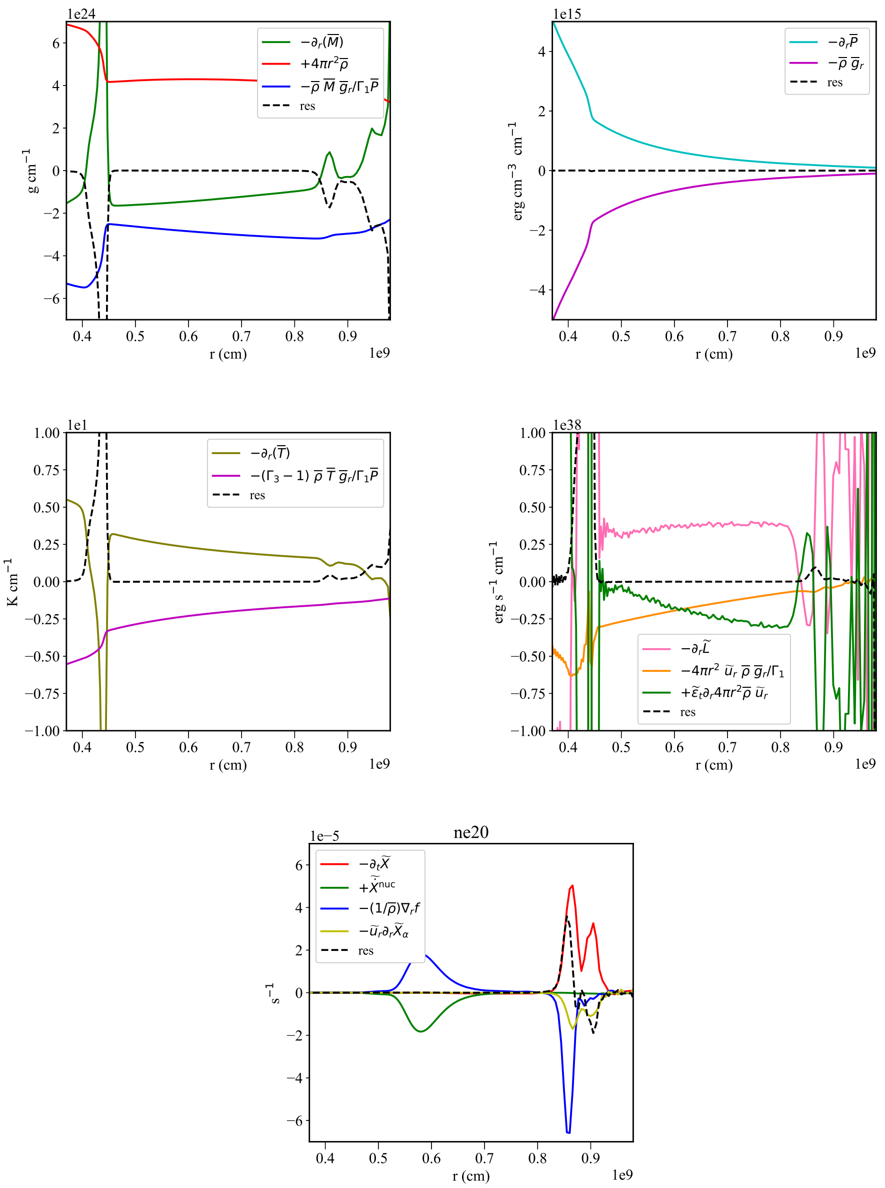

Figure 1: Hydrodynamic stellar structure equations without MLT validated by 3D low-resolution oxygen burning convective shell simulation. Initial model is described more in detail in Mocák et al. [2018].

- O-Ne-C-rich stellar interior in massive pre-supernova progenitor with multiple interacting convection zones [Meakin and Arnett, 2006] (3+ years)

The setups are already prepared in our MPI parallelized multi-species compressible fluid dynamics code PROMPI [Meakin and Arnett, 2007]. Anticipated problems encompass computational time required to perform high-resolution 3D simulations, that may require 100k CPU hours for a single convective turnover timescale. In order to get statistically robust mean-fields from our framework, we need to simulate at least three such timescales per model after initial transient behaviour.

3 Summary

- establish comprehensive description and validation of the ransX framework
- extend our library of 3D hydrodynamic simulations
- validate and close new 1D hydrodynamic stellar structure equations without MLT for all sorts of conditions within stellar interior
- make ransX a standard analysis tool in as many hydrodynamic codes as possible

4 Collaboration

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