

1 Discrete Fourier transform in 2D

$$\begin{array}{ll} \textbf{Direct} & F(u, v) = \frac{1}{MN} \sum_{y=0}^{M-1} \sum_{z=0}^{N-1} f(y, z) \exp \left[-2\pi i \left(\frac{uy}{M} + \frac{vz}{N} \right) \right] \\ \textbf{Inverse} & f(y, z) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp \left[2\pi i \left(\frac{uy}{M} + \frac{vz}{N} \right) \right] \end{array} \quad (1)$$

1.1 Parseval's theorem

$$\frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \overline{G(u, v)} = \int_0^N \int_0^M f(y, z) \overline{g(y, z)} dydz \quad (2)$$

if f and g are real-valued functions and $F(u, v) = A_{u,v} + iB_{u,v}$ and $G(u, v) = C_{u,v} + iD_{u,v}$, then:

$$\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \overline{G(u, v)} = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} (A_{u,v}C_{u,v} + B_{u,v}D_{u,v}) \quad (3)$$

1.2 Fourier spectra

if $k = \sqrt{u^2 + v^2}$ then:

$$E = \int_0^N \int_0^M f(y, z) \overline{g(y, z)} dydz = \frac{1}{MN} \int_k (A_{u,v}C_{u,v} + B_{u,v}D_{u,v}) dk \quad (4)$$