

1 Full Turbulence Velocity Field Hypothesis based on Dilatation Flux Relations

1.1 Dilatation Flux Relations inferred from Mean Field Acoustic Flux Equations in r, θ, ϕ

$$\overline{u'_r d'} \sim \frac{\bar{\rho} \bar{R}_{rr} \bar{g}_r}{\Gamma_1 \bar{P}} \quad (1)$$

$$\overline{u'_\theta d'} \sim \frac{\bar{\rho} \bar{R}_{\theta r} \bar{g}_r}{\Gamma_1 \bar{P}} \quad (2)$$

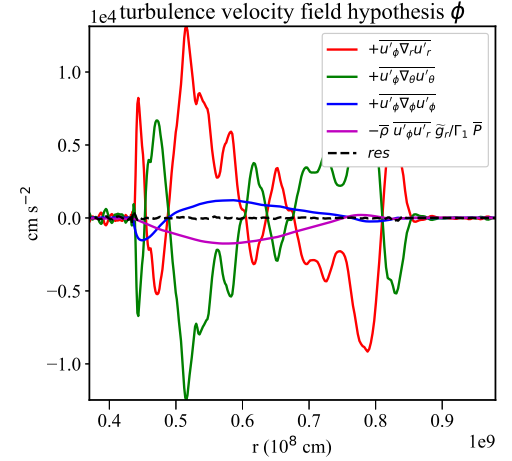
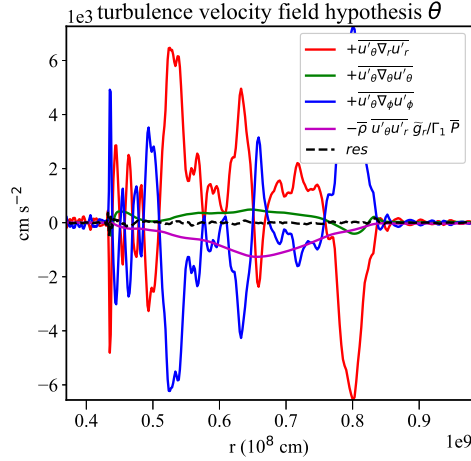
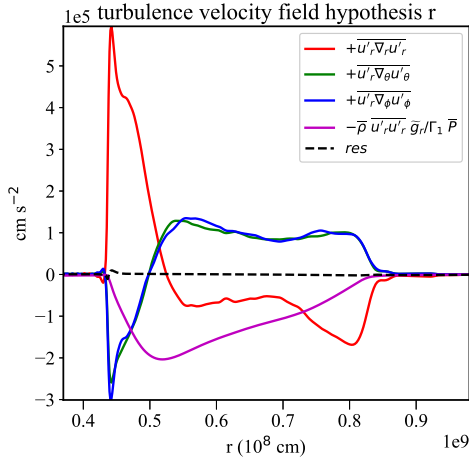
$$\overline{u'_\phi d'} \sim \frac{\bar{\rho} \bar{R}_{\phi r} \bar{g}_r}{\Gamma_1 \bar{P}} \quad (3)$$

1.2 Full Turbulence Velocity Field Hypothesis

$$\overline{u'_r \nabla_r u'_r} + \overline{u'_r \nabla_\theta u'_\theta} + \overline{u'_r \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_r u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (4)$$

$$\overline{u'_\theta \nabla_r u'_r} + \overline{u'_\theta \nabla_\theta u'_\theta} + \overline{u'_\theta \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_\theta u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (5)$$

$$\overline{u'_\phi \nabla_r u'_r} + \overline{u'_\phi \nabla_\theta u'_\theta} + \overline{u'_\phi \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_\phi u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (6)$$



These equations will give us full turbulence velocity field in convection zone in hydrostatic equilibrium. The hypothesis is formulated using Reynolds fluctuations but its formulation using Favrian fluctuations is almost the same.

1.3 Simplified Turbulence Velocity Field Hypothesis

$$\overline{u'_r \nabla_r u'_r} + \overline{u'_r \nabla_\theta u'_\theta} + \overline{u'_r \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_r u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (7)$$

$$\overline{u'_\theta \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_\theta u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (8)$$

$$\overline{u'_\phi \nabla_\phi u'_\phi} \sim \bar{\rho} \overline{u'_\phi u'_r} \bar{g}_r / \Gamma_1 \bar{P} \quad (9)$$

Definitions:

$$\nabla_r u'_r = \partial_r u'_r$$

$$\nabla_\theta u'_\theta = (\partial_\theta u'_\theta + u'_r) / r$$

$$\nabla_\phi u'_\phi = (\partial_\phi u'_\phi + u'_r \sin \theta + u'_\theta \cos \theta) / r \sin \theta$$

$$d' = \nabla \cdot u' = \nabla_r u'_r + \nabla_\theta u'_\theta + \nabla_\phi u'_\phi \quad \text{dilatation: trace of covariant derivative}$$

$$\Gamma_1 = \partial \ln P / \partial \ln \rho|_s$$

ρ density

u_r velocity in r

u_θ velocity in θ

u_ϕ velocity in ϕ

$$u = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_\phi \mathbf{e}_\phi$$

g_r gravity

P pressure

$$R_{rr} = u'_r u'_r$$

$$R_{\theta r} = u'_\theta u'_r$$

$$R_{\phi r} = u'_\phi u'_r$$