# Towards Complex Understanding of Turbulent Convection in Stellar Interiors Using ransX Analysis Framework

Proposal For Post-Doctoctoral Research Position

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## 1 Introduction

Contemporary ground- and space-based telescopes provide us precise stellar data leading to challenging questions and forcing us to reconsider our basic assumptions regarding turbulent convection and mixing in stars. Properties of supernova explosions studied by HST or Keck can not be linked to their progenitors conclusively [Smartt, 2009]. Such progenitors are known to have a structure interleaved by turbulent convection shells [Hirschi et al., 2004]. VLT is observing massive stars with unexplained chemical peculiarities, where rotational mixing was considered to be enough to explain observations [Evans et al., 2008]. Kepler spacecraft finds unexplained pulsations of $\delta$ Scuti and $\gamma$ Doradus stars [Uytterhoeven, 2011], which depend heavily on properties of sub-surface stellar convection [Guzik et al., 2000]. Explanation of observed element abundances in AGB stars requires physically motivated but still inconclusive tuning for mixing between turbulent envelope convection and underlying hydrogen-free core [Herwig, 2005].

Turbulence is during stellar evolution one of the most fundamental processes and before taking into account binarity, magnetism or rotation of a star to explain observations, we should understand stellar turbulence well first. It is arguably the greatest weakness in the modern theory of stellar evolution, which is mostly derived from one-dimensional calculations approximating dynamic turbulent processes by simplified theories [Kippenhahn and Weigert, 1990, Weiss et al., 2004]. In reality, turbulent flows are multidimensional and driven by non-linear terms of the hydrodynamic Navier-Stokes equations.

## 2 Aims

I will analyze three-dimensional (3D) hydrodynamic simulations of stellar convection within the context of Reynolds-Averaged Navier Stokes (RANS) approach pursued by Besnard et al. [1992], Livescu et al. [2009], Schwarzkopf et al. [2011]. It is a unique way of learning about turbulence based on budget analysis of hydrodynamic equations averaged in space and time, by which complexity of every term is reduced to a one-dimensional mean field.

Using this methodology, we derived RANS evolution equations for transport/flux/variance of mass, momenta, kinetic/internal/total energy, temperature, enthalpy, pressure and composition densities (no magnetic fields, no rotation) [Mocák et al., 2014] and implemented them to analysis framework, that we call rans(eXtreme) or ransX[1] for short. It should be noted here, that it is only one of many possible sets of equations relevant to closure problems in turbulence and there are many other formulations, which then need to approximate different terms [Canuto, 1992, 1993, Canuto et al., 2001, Hanjalic, 2002, Alfonsi, 2009, Garaud et al., 2010, Canuto, 2011, Biferale et al., 2011].

RANS approach introduces into the averaged equations many correlations of various thermodynamic fluctuations which are essentially new unknown variables. Hence, to solve them, we need either to design appropriate closures or derive and close evolution equations for them. Either of the tasks is difficult, because stellar turbulence is anisotropic, compressible and embedded in highly stratified environment where external forces like gravity and mean background flow play an important role. But we hope that this approach could in the future allow us to study stellar evolution using solution of the mean fields hydrodynamic equations, move away from canonical form of stellar structure equations and most importantly allow for a comprehensive synergy between engineering turbulence modeling and stellar astrophysics.

My aims encompass the following targets (their content partially overlap with each other and the estimated time of completion is stated in brackets):

- publish our RANS mean-field equations implemented within ransX framework [Mocák et al., 2014][2] in high-impact referred journal and validate them with new high-resolution 3D hydrodynamic simulations (2+ year)

[1] ransX is free for download and test on https://github.com/mmicromegas/ransX

[2] More up-to-date equation content of the ransX framework can be found here https://github.com/mmicromegas/ransX/blob/master/DOCS/ransXtheoryGuide.pdf

- help to implement the ransX framework to all hydrodynamic codes capable of simulating stellar core and envelope convection (e.g. MUSIC [Viallet et al., 2011], PROMETHEUS [Fryxell et al., 1991, Mueller et al., 1991]) or stellar atmospheres in 3D and make it an analysis standard (3+ years)

Partial results from our mean-field RANS analysis related mostly to turbulent kinetic energy and transport of some chemical elements based on oxygen burning shell in massive stars have been already published e.g. Meakin and Arnett [2007], Arnett et al. [2009], Meakin and Arnett [2010], Viallet et al. [2013], Mocák et al. [2018].

In order to cover wider range of conditions present in stars like Schwarzschild and Ledoux stable/unstable regions, electron degeneracy and multiplicity of convection zones, I also plan to extend our library of ransX mean fields calculated during 3D high-resolution hydrodynamic simulations of:

- single convection zone during core helium flash in low-mass stars with Ledoux unstable region at its bottom [Mocák et al., 2008, 2009, 2011] (1+ year)
- dual convection zone during core helium flash in metallicity free stars [Mocák et al., 2010] (1+ year)
- single convection zone resulting from core carbon flash in intermediate stars with Ledoux unstable region at its bottom [Mocák et al., 2011] (1+ years)
- O-Ne-C burning stellar interior in massive pre-supernova progenitor with multiple interacting convection zones [Meakin and Arnett, 2006] (3+ years)

The setups are already prepared in our MPI parallelized multi-species compressible fluid dynamics code PROMPI [Meakin and Arnett, 2007]. Anticipated problems encompass computational time required to perform high-resolution 3D simulations, that may require 100k CPU hours for a single convective turnover timescale. In order to get statistically robust mean-fields from our framework, we need to simulate at least three such timescales per model after initial transient behaviour.

Besides general understading of the time-dependency, non-local and compressibility effects of turbulent convection in stars, these simulations will also serve as test beds for turbulence models inspired by work of Rogers et al. [1989], Lazeroms et al. [2013] and Biferale et al. [2011].

## 3 Summary

- publish comprehensive description and validation of the ransX framework in referred journal
- extend our library of 3D hydrodynamic simulations with core helium flash, core carbon flash, dual core flash and O-Ne-C burning shell simulations (setups already prepared in our hydrodynamic code PROMPI)
- develop turbulence models suitable for 1D stellar evolution calculations inspired by engineering turbulence literature with focus on turbulent composition flux, which in reactive flow controls nuclear reaction rates
- make ransX a standard analysis tool in as many hydrodynamic codes as possible

## 4 Intended Collaboration and Topic

- Simon Campbell (Monash Centre for Astrophysics, Australia)
  - 3D simulations of dual core flashes and nucleosynthesis in low-mass stars
- Casey Meakin (Meakin Technologies, Pasadena, California)
  - turbulence modelling, ransX development, hydrodynamic stellar structure equations
- Dave Arnett (Steward Observatory, University of Arizona)
  - turbulence modelling, hydrodynamic stellar structure equations
- Cyril Georgy (Geneva Observatory, University of Geneva, Switzerland)
  - ransX development, hydrodynamic stellar structure equations
- Ewald Mueller (Max-Planck-Institut für Astrophysik, Germany)
  - search for origin of gravitational wave signals during 3D hydrodynamic simulations of core-collapse supernovas using ransX framework

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