ransX framework

Implementation Guide

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March 25, 2019

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1 Introduction

- present ransX as a framework with 3 main parts

 1. calc of running averages 2. calc of avarages over t avg 3. calc of terms for rans equations and their plotting

 - mean fields calculated at run-time

2 Calculation of ransX mean fields by hydrodynamic code

- describe running averages, rans avg.f90, implementation in PROMPI, and their connection to tseries ransX.py

3 Post-processing of ransX mean fields

- describe rans tseries.py

4 Implementation of ransX equations

- describe python post processing code

4.1 Continuity Equation with Mass FLux

ContinuityEquationWithMassFlux.py

4.2 Continuity Equation with Favrian Dilatation

ContinuityEquationWithFavrianDilatation.py

$$\widetilde{D}_t \overline{\rho} = - \overline{\rho} \widetilde{d} \tag{1}$$

$$\partial_t \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} = - \overline{\rho} \widetilde{d} \tag{2}$$

$$\partial_t dd + ddux/dd \ \partial_r dd = - dd * \nabla_r \ ddux/dd \tag{3}$$

$$\partial_t dd + fht\_ux \ \partial_r dd = - dd * \nabla_r \ fht\_ux \tag{4}$$

$$\tag{5}$$

4.3 Momentum Equation X

4.4 Momentum Equation Y

4.5 Momentum Equation Z

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4.7 Reynolds Stress YY

4.8 Reynolds Stress ZZ

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4.21 Pressure Flux Equation

4.22 Pressure Variance Equation

4.23 Temperature Equation

4.24 Temperature Flux Equation

4.25 Temperature Variance Equation

4.26 Enthalpy Equation

4.27 Enthalpy Flux Equation

$$
\begin{aligned}
\overline{\rho}\widetilde{D}_t\widetilde{X}_i &= -\nabla_r f_i + \overline{\rho}\widetilde{\dot{X}}_i^{\mathrm{nuc}} \\
\overline{\rho}\partial_t\widetilde{X}_i + \overline{\rho}\widetilde{u}_r\partial_r\widetilde{X}_i &= -\nabla_r\overline{\rho}\widetilde{X_i''u_r''} + \overline{\rho}\widetilde{\dot{X}}_i^{\mathrm{nuc}} \\
\overline{\rho}\partial_t\widetilde{X}_i + \overline{\rho}\widetilde{u}_r\partial_r\widetilde{X}_i &= -\nabla_r\overline{\rho}(\widetilde{X_i u_r} - \widetilde{X}_i\widetilde{u}_r) + \overline{\rho}\widetilde{\dot{X}}_i^{\mathrm{nuc}} \\
\overline{\rho}\partial_t\overline{\rho X_i}/\overline{\rho} + \overline{\rho u_r}\partial_r\overline{\rho X_i}/\overline{\rho} &= -\nabla_r\left(\overline{\rho X_i u_r} - \overline{\rho X_i}\,\overline{\rho u_r}/\overline{\rho}\right) + \overline{\rho\dot{X}_i^{\mathrm{nuc}}} \\
dd\ \partial_t\ ddxi/dd + ddux\ \partial_r\ ddxi/dd &= -\nabla_r\left(ddxiux - ddxi*ddux/dd\right) + ddxidot
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
dd\ \partial_t(ddxiux/dd - ddxi*ddux/dd*dd) + ddux\ \partial_r(ddxiux/dd - ddxi*ddux/dd*dd) = \\
-\nabla_r(ddxiuxux - ddxi/dd*dduxux - 2*ddux/dd*ddxiux + 2*ddxi*ddux*ddux/dd*dd) \\
-(ddxiux - ddxi*ddux/dd)*\partial_r ddux/dd - (dduxux - ddux*ddux/dd)*\partial_r ddxi/dd \\
-(xi\ \partial_r\ pp - ddxi/dd\ \partial_r\ pp) - (xigradxpp - xi\ \partial_r\ pp) + (ddxidotux - ddux/dd*ddxidot) \\
-(ddxiuyuy - ddxi/dd*dduyuy - 2*dduy/dd*ddxiuy + 2*ddxi*dduy*dduy/dd*dd)/r \\
-(ddxiuzuz - ddxi/dd*dduzuz - 2*dduz/dd*ddxiuz + 2*ddxi*dduz*dduz/dd*dd)/r \\
+(ddxiuyuy - ddxi/dd*dduyuy)/r \\
+(ddxiuzuz - ddxi/dd*dduzuz)/r
\end{aligned}
$$

$$\overline{\rho}\widetilde{D}_t\sigma_i = -\nabla_r f_i^r - 2f_i\partial_r\widetilde{X}_i + 2\overline{X_i''\rho\dot{X}_i^{\rm nuc}} \tag{8}$$

$$\overline{\rho}\widetilde{D}_t\widetilde{X_i''X_i''} = -\nabla_r(\overline{\rho X_i''X_i''u_r''}) - 2\overline{\rho}\widetilde{X_i''u_r''}\partial_r\widetilde{X}_i + 2\overline{X_i''\rho\dot{X}_i^{\rm nuc}}$$

$$\overline{\rho}\partial_t(\widetilde{X_iX_i} - \widetilde{X_i}\widetilde{X_i}) + \overline{\rho}\widetilde{u}_r\partial_r(\widetilde{X_iX_i} - \widetilde{X_i}\widetilde{X_i}) = -\nabla_r(\overline{\rho X_iX_iu_r} - 2\widetilde{X_i}\overline{\rho X_iu_r} - \widetilde{u}_r\overline{\rho X_iX_i} + 2\widetilde{X}_i\widetilde{X}_i\overline{\rho u_r})$$
$$- 2\overline{\rho}(\widetilde{X_iu_r} - \widetilde{X}_i\widetilde{u}_r)\partial_r\widetilde{X}_i + (\overline{X_i\rho\dot{X}_i} - \widetilde{X}_i\overline{\rho\dot{X}_i})$$

$$dd\ \partial_t\ (ddxisq/dd - ddxi*ddxi/dd*dd)$$
$$+\ ddux\ \partial_r\ (ddxisq/dd - ddxi*ddxi/dd*dd) = -\nabla_r(ddxisqux - 2*ddxi/dd*ddxiux - ddux/dd*ddxisq + 2*ddxi*ddxi*ddux/dd*dd)$$
$$-\ 2*dd\ (ddxiux/dd - ddxi*ddux/dd*dd)*\partial_r\ ddxi/dd$$
$$+2*(ddxixidot - ddxi/dd*ddxidot)$$

4.34 Composition Variance Equation

4.35 Density-specific Volume Covariance

$$\overline{D}_t b = +\overline{v}\nabla_r\overline{\rho}\overline{u_r''} - \overline{\rho}\nabla_r(\overline{u_r'v'}) + 2\overline{\rho}\overline{v'd'} \tag{9}$$

$$\partial_t b + \overline{u}_r\partial_r b = \overline{v}\nabla_r\overline{\rho}(\overline{u}_r - \widetilde{u}_r) - \overline{\rho}\nabla_r(\overline{u_rv} - \overline{u}_r\overline{v}) + 2\overline{\rho}(\overline{vd} - \overline{v}\overline{d}) \tag{10}$$

$$\partial_t\overline{v'\rho'} + \overline{u}_r\partial_r(\overline{v'\rho'}) = \overline{v}\nabla_r\overline{\rho}(\overline{u}_r - \widetilde{u}_r) - \overline{\rho}\nabla_r(\overline{u_rv} - \overline{u}_r\overline{v}) + 2\overline{\rho}(\overline{vd} - \overline{v}\overline{d}) \tag{11}$$

$$\partial_t(\underbrace{\overline{v\rho}}_{1} - \overline{v}\ \overline{\rho}) + \overline{u}_r\partial_r(\underbrace{\overline{v\rho}}_{1} - \overline{v}\ \overline{\rho}) = \overline{v}\nabla_r\overline{\rho}(\overline{u}_r - \widetilde{u}_r) - \overline{\rho}\nabla_r(\overline{u_rv} - \overline{u}_r\overline{v}) + 2\overline{\rho}(\overline{vd} - \overline{v}\overline{d}) \tag{12}$$

$$-\partial_t(\overline{v}\ \overline{\rho}) - \overline{u}_r\partial_r(\overline{v}\ \overline{\rho}) = \overline{v}\nabla_r\overline{\rho}(\overline{u}_r - \widetilde{u}_r) - \overline{\rho}\nabla_r(\overline{u_rv} - \overline{u}_r\overline{v}) + 2\overline{\rho}(\overline{vd} - \overline{v}\overline{d}) \tag{13}$$

$$-\partial_t(sv*dd) - ux\partial_r(sv*dd) = sv*\nabla_r(dd*ux - ddux/dd) - dd\nabla_r(svux - sv*ux) + 2*dd(svdivu - sv*divu) \tag{14}$$

4.36 Density Variance Equation

$$
\begin{aligned}
\widetilde{D}_t \sigma_\rho = &- \nabla_r \overline{(\rho' \rho' u''_r)} - 2\overline{\rho}\ \overline{\rho' d''} - 2\overline{\rho' u''_r} \partial_r \overline{\rho} - 2\widetilde{d}\ \sigma_\rho - \overline{\rho' \rho' d''} && (15)\\
\partial_t \overline{\rho' \rho'} + \widetilde{u}_r \partial_r \overline{\rho' \rho'} = &- \nabla_r \overline{(\rho' \rho' u''_r)} - 2\overline{\rho}\ \overline{\rho' d''} - 2\overline{\rho' u''_r} \partial_r \overline{\rho} - 2\widetilde{d} \overline{\rho' \rho'} - \overline{\rho' \rho' d''} && (16)\\
\partial_t (\overline{\rho\rho} - \overline{\rho}\ \overline{\rho}) + \widetilde{u}_r \partial_r (\overline{\rho\rho} - \overline{\rho}\ \overline{\rho}) = &- \nabla_r (\overline{\rho\rho u_r} - 2\overline{\rho u_r}\ \overline{\rho} + \overline{\rho\rho}\ \overline{u_r} - \overline{\rho\rho} \widetilde{u}_r + \overline{\rho}\ \overline{\rho} \widetilde{d}) && (17)\\
&- 2\overline{\rho} (\overline{\rho d} - \overline{\rho} \widetilde{d} - \overline{\rho}\ \overline{d} + \overline{\rho} \widetilde{d}) && (18)\\
&- 2(\overline{\rho u_r} - \overline{\rho} \widetilde{u}_r - \overline{\rho}\ \overline{u}_r + \overline{\rho} \widetilde{u}_r) \partial_r \overline{\rho} && (19)\\
&- 2\widetilde{d} (\overline{\rho\rho} - \overline{\rho}\ \overline{\rho}) - (\overline{\rho\rho u_r} - 2\overline{\rho u_r}\ \overline{\rho} + \overline{\rho}\ \overline{\rho u}_r - \overline{\rho\rho} \widetilde{u}_r + \overline{\rho}\ \overline{\rho} \widetilde{d}) && (20)\\
\partial_t (ddsq - dd * dd) & && (21)\\
+ddux/dd \partial_r (ddsq - dd * dd) = & \\
&- \nabla_r (ddddux - 2 * ddux * dd + ddsq * ux - ddsq * ddux/dd + dd * dd * ddux/dd) \\
&- 2 * dd * (-dd * divu + dddivu) - 2 * (-dd * ux + ddux) \partial_r dd \\
&- 2 * dddivu/dd * (ddsq - dd * dd) \\
&- (ddddivu - 2 * dddivu * dd + ddsq * divu - ddsq * dddivu/dd + dd * dddivu)
\end{aligned}
$$

4.37 Internal Energy Variance Equation

$$
\begin{aligned}
\overline{\rho} \widetilde{D}_t \sigma_{\epsilon I} = &- \nabla_r (\overline{\rho \epsilon''_I \epsilon''_I u''_r}) - 2 f_I \partial_r \widetilde{\epsilon}_I - 2\overline{\epsilon''_I}\ \overline{P}\ \widetilde{d} - 2\overline{P}\ \overline{\epsilon''_I d''} - 2\widetilde{d}\ \overline{\epsilon''_I P'} - 2\overline{\epsilon''_I P' d''} + 2\overline{\epsilon''_I \mathcal{S}} && (22)\\
\overline{\rho} \widetilde{D}_t \widetilde{\epsilon''_I \epsilon''_I} = &- \nabla_r (\overline{\rho \epsilon''_I \epsilon''_I u''_r}) - 2\overline{\rho} \widetilde{\epsilon''_I u''_r} \partial_r \widetilde{\epsilon}_I - 2\overline{\epsilon''_I}\ \overline{P}\ \widetilde{d} - 2\overline{P}\ \overline{\epsilon''_I d''} - 2\widetilde{d}\ \overline{\epsilon''_I P'} - 2\overline{\epsilon''_I P' d''} + 2\overline{\epsilon''_I \rho \varepsilon_{nuc}} && (23)\\
\overline{\rho} \partial_t \widetilde{\epsilon''_I \epsilon''_I} + \overline{\rho} \widetilde{u}_r \nabla_r (\widetilde{\epsilon''_I \epsilon''_I}) = &- \nabla_r (\overline{\rho \epsilon''_I \epsilon''_I u''_r}) - 2\overline{\rho} \widetilde{\epsilon''_I u''_r} \partial_r \widetilde{\epsilon}_I - 2\overline{\epsilon''_I}\ \overline{P}\ \widetilde{d} - 2\overline{P}\ \overline{\epsilon''_I d''} - 2\widetilde{d}\ \overline{\epsilon''_I P'} - 2\overline{\epsilon''_I P' d''} + 2\overline{\epsilon''_I \rho \varepsilon_{nuc}} && (24)
\end{aligned}
$$

$$
\begin{aligned}
dd * \partial_t(ddeiei/dd - ddei * ddei/(dd * dd)) + \\
ddux * \nabla_r(ddeiei/dd - ddei * ddei/(dd * dd)) = \\
& - \nabla_r(ddeieiux/dd - 2 * ddei/dd * ddeiux/dd - ddux/dd * ddeiei/dd \\
& + 2 * ddei * ddei * ddux/(dd * dd * dd)) \\
& - 2 * dd * (ddeiux/dd - ddei * ddux/(dd * dd))\partial_r ddei/dd \\
& - 2 * (ei - ddei/dd) * pp * dddivu/dd \\
& - 2 * pp * (eidd - ei * dddivu/dd - ddei/dd * divu + ddei * dddivu/dd) \\
& - 2 * dddivu/dd * (eippdivu - eidivu * pp - ddei/dd * ppdivu \\
& + ddei/dd * pp * dd - eipp * dddivu/dd + ei * pp * dddivu/dd) \\
& + 2 * (eiddenuc - ddei/dd * (ddenuc1 + ddenuc2))
\end{aligned} \quad (25)
$$

4.38 Mean Number of Nucleon per Isotope a.k.a Abar Equation

4.39 Mean Number of Nucleon per Isotope Flux a.k.a Abar Flux Equation

4.40 Mean Charge per Isotope a.k.a Zbar Equation

4.41 Mean Charge per Isotope Flux a.k.a Zbar Flux Equation

4.42 Hydrodynamic Stellar Structure Equations

Continuity Equation

Momentum Equation

Luminosity Equation

Temperature Equation

Composition Equation

4.43 MLT Velocity

$$u_{MLT} \equiv (u'_{rms}) = \frac{F_c}{\alpha_E c_P (T'_{rms})} = \frac{\overline{\rho}\widetilde{h''u''_r}}{\alpha_E \widetilde{c_P} (\widetilde{TT} - \widetilde{T}\widetilde{T})^{1/2}} \sim \frac{\overline{\rho}\overline{h'u'_r}}{\alpha_E \overline{c_P} (\overline{TT} - \overline{T}\ \overline{T})^{1/2}} ? \tag{26}$$

$$u_{MLT} \equiv (u'_{rms}) = \frac{\overline{\rho}(\widetilde{hu_r} - \widetilde{h}\widetilde{u_r})}{\alpha_E \widetilde{c_P} (\widetilde{TT} - \widetilde{T}\widetilde{T})^{1/2}} \sim \frac{\overline{\rho}(\overline{hu_r} - \overline{h}\overline{u_r})}{\alpha_E \overline{c_P} (\overline{TT} - \overline{T}\ \overline{T})^{1/2}}$$

$$u_{MLT} \equiv (u'_{rms}) = \frac{ddhhux - ddhh * ddux/dd}{\alpha_E * ddcp/dd\ (ddttsq/dd - ddtt * ddtt/dd * dd)^{1/2}} \sim \frac{dd * hhux - dd * hh * ux}{\alpha_E * cp\ (ttsq - tt * tt)^{1/2}}$$

4.44 Usefull Identities

$$\overline{a''} = \overline{a - \widetilde{a}} = \overline{a} - \widetilde{a} \tag{27}$$

$$\widetilde{a''b''} = \widetilde{(a - \widetilde{a}) * (b - \widetilde{b})} = \widetilde{ab} - \widetilde{a}\widetilde{b} \tag{28}$$

$$\overline{a'b'} = \overline{(a - \overline{a}) * (b - \overline{b})} = \overline{ab} - \overline{a}\overline{b} = \overline{a'b''} \tag{29}$$

$$\widetilde{a''b''c''} = \widetilde{(a - \widetilde{a}) * (b - \widetilde{b}) * (c - \widetilde{c})} = \widetilde{abc} - \widetilde{a}\widetilde{bc} - \widetilde{b}\widetilde{ac} - \widetilde{c}\widetilde{ab} + 2\widetilde{a}\widetilde{b}\widetilde{c} \tag{30}$$

$$\overline{a'b'c''} = \overline{(a - \overline{a}) * (b - \overline{b}) * (c - \widetilde{c})} = \overline{abc} - \overline{ac}\,\overline{b} - \overline{a}\,\overline{bc} + \overline{a}\,\overline{b}\,\overline{c} - \overline{ab}\,\widetilde{c} + \overline{a}\,\overline{b}\widetilde{c} \tag{31}$$

$$\overline{a''b'c''} = \overline{(a - \widetilde{a}) * (b - \overline{b}) * (c - \widetilde{c})} = \overline{abc} - \overline{ac}\overline{b} - \widetilde{a}\overline{bc} + \widetilde{a}\overline{b}\overline{c} - \overline{ab}\widetilde{c} + \overline{a}\overline{b}\widetilde{c} \tag{32}$$

$$\overline{a''bc} = \overline{(a - \widetilde{a})bc} = \overline{abc} - \widetilde{a}\overline{bc} \tag{33}$$

$$\overline{a''\partial_r b'} = \overline{(a - \widetilde{a})\partial_r b'} = \overline{a\partial_r b'} - \cancelto{0}{\widetilde{a}\partial_r \overline{b'}} = \overline{a\partial_r b} - \overline{a}\partial_r \overline{b} \tag{34}$$

5 Definitions

- only for basic quantities, without overbar e.g dd is density, ux is x velocity, ei internal energy etc.