1 Linearized Continuity Equation

$$\partial_t \rho + \nabla \cdot \rho \mathbf{u} = 0 \tag{1}$$

Let us assume, that $\rho(\mathbf{r}, t) = \rho_0(\mathbf{r}) + \rho'(\mathbf{r}, t)$ where $\rho_0(\mathbf{r})$ is time-independent background density state around which we'll linearize the continuity equation. We get:

$$\partial_t \left[\rho_0(\mathbf{r}) + \rho'(\mathbf{r}, t)\right] + \nabla \cdot \left[\rho_0(\mathbf{r}) + \rho'(\mathbf{r}, t)\right] \mathbf{u}(\mathbf{r}, t) = 0 \tag{2}$$

$$\cancelto{0}{\partial_t \rho_0(\mathbf{r})} + \partial_t \rho'(\mathbf{r}, t) + \nabla \cdot \left[\rho_0(\mathbf{r}) \mathbf{u}(\mathbf{r}, t)\right] + \nabla \cdot \left[\rho'(\mathbf{r}, t) \mathbf{u}(\mathbf{r}, t)\right] = 0 \tag{3}$$

And because we also assume that $\rho' << \rho_0$, we have that $\nabla \cdot [\rho_0(\mathbf{r})\mathbf{u}(\mathbf{r}, t)] >> \nabla \cdot [\rho'(\mathbf{r}, t)\mathbf{u}(\mathbf{r}, t)]$ and get:

$$\partial_t \rho' + \nabla \cdot (\rho_0 \mathbf{u}) \sim 0 \tag{4}$$

Furthermore, we know that $\mathbf{u} \sim \mathbf{u}'$ and therefore we can write:

$$\partial_t \rho' + \nabla \cdot (\rho_0 \mathbf{u}') \sim 0 \tag{5}$$

This is equation 23 from Viallet et al, 2013.

2 Mean linearized continuity equation

$$\partial_t \rho' + \nabla \cdot (\rho_0 \mathbf{u}) \sim 0 \tag{6}$$

After space-time averaging of this linearized continuity equation, we get:

$$\overline{\partial_t \rho'} + \overline{\nabla \cdot (\rho_0 \mathbf{u})} \sim 0 \tag{7}$$

$$\cancelto{0}{\overline{\partial_t \rho'}} + \overline{\nabla \cdot (\rho_0 \mathbf{u})} \sim 0 \tag{8}$$

$$\nabla_r \overline{\rho_0 u_r} \sim 0 \tag{9}$$

$$\tag{10}$$

But because $\rho_0 \equiv \overline{\rho}$, we can write:

$$\nabla_r \overline{\rho}\ \overline{u}_r \sim 0 \tag{11}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{u}_r \partial_r \overline{\rho} \sim 0 \tag{12}$$

$$\overline{\rho} \nabla_r \overline{u}_r + (\overline{u''}_r + \widetilde{u}_r) \partial_r \overline{\rho} \sim 0 \tag{13}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{u''}_r \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{14}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{\rho' u'_r} / \overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{15}$$

$$\overline{\rho} \nabla_r \overline{u}_r - f_\rho / \overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{16}$$

But from my latest analysis of the full (non-linearzed) continuity equation, it turns out that:

$$\overline{\rho} \nabla_r \overline{u}_r - f_\rho / \overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \neq 0 \tag{17}$$

Or, when multiply this by -1, I get the equation in terms of quantities shown on the figure I sent you, where color of the term coresponds to color of the line it represents:

$$-\overline{\rho} \nabla_r \overline{u}_r + f_\rho / \overline{\rho}\ \partial_r \overline{\rho} - \widetilde{u}_r \partial_r \overline{\rho} \neq 0 \tag{18}$$

Linearization relies on time-independent background state around which you can linearize, which turns out not to be our case and $\partial_t \rho_0$ or in other words $\partial_t \overline{\rho} \neq 0$ (green curve). Also it gets rid of the transport of turbulent density field $\nabla_r f_\rho$ (cyan curve), see figure below:

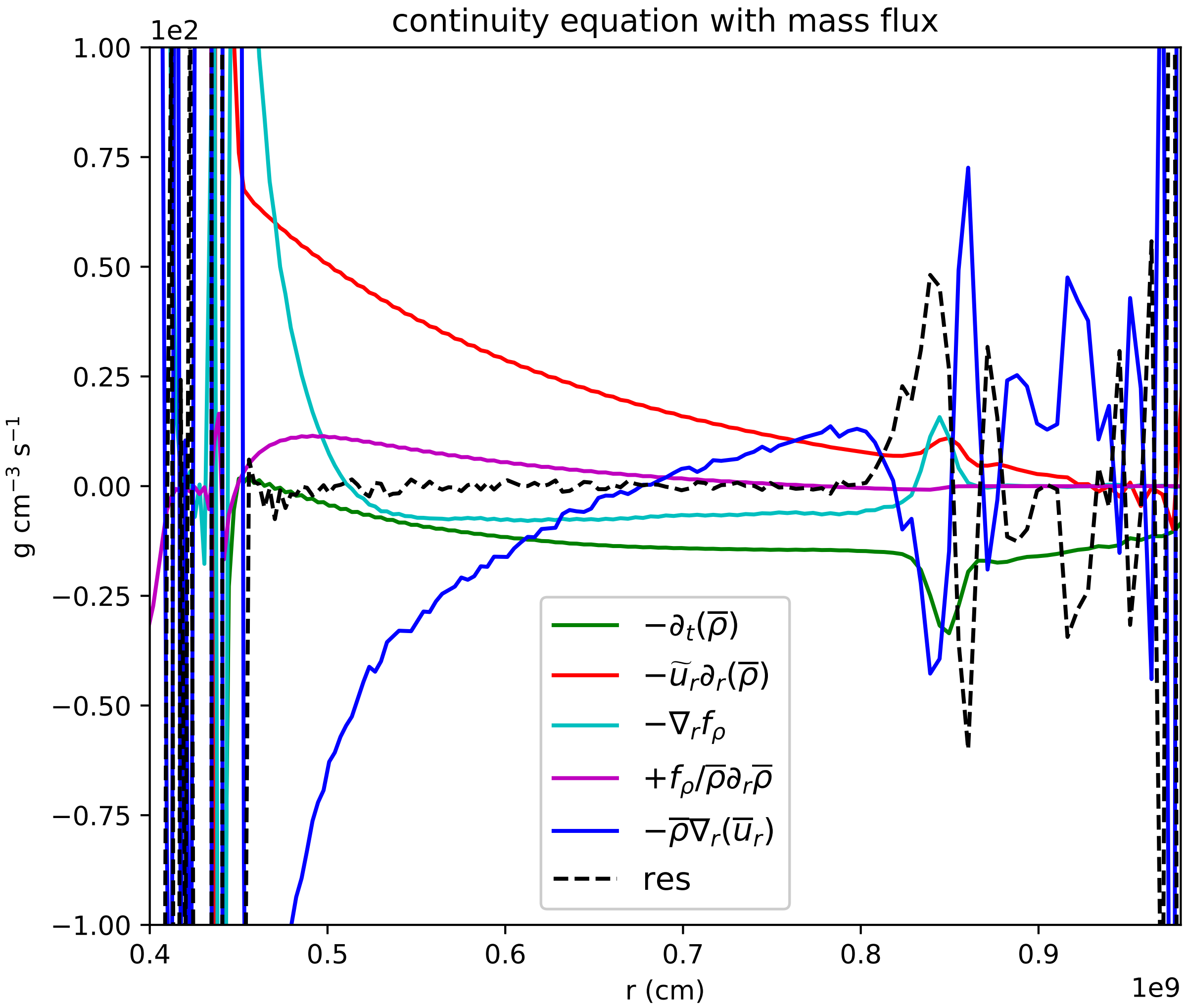

Figure 1: Oxygyn burning shell, low-rez