Math 40 - Section —
HW 9 - Eigenvalues and Eigenvectors
Tuesday, February 23, 2016

4.3.{6, 8, 22, 24, 32, 34}

4.3.6 Compute (a) the characteristic polynomial of A, (b) the eigenvalues of A, (c) a basis for each eigenspace of A, and (d) the algebraic and geometric multiplicity of each eigenvalue.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$$

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4.3.8 Compute (a) the characteristic polynomial of A, (b) the eigenvalues of A, (c) a basis for each eigenspace of A, and (d) the algebraic and geometric multiplicity of each eigenvalue.

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

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4.3.22 If $\mathbf{v}$ is an eigenvector of A with corresponding eigenvalue λ and c is a scalar, show that $\mathbf{v}$ is an eigenvector of $A - cI$ with corresponding eigenvalue $\lambda - c$.

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4.3.24 Let A and B be $n \times n$ matrices with eigenvalues λ and μ, respectively.

(a) Give an example to show that $\lambda + \mu$ need not be an eigenvalue of $A + B$.

(b) Give an example to show that $\lambda\mu$ need not be an eigenvalue of AB.

(c) Suppose λ and μ correspond to the *same* eigenvector $\mathbf{x}$. Show that, in this case, $\lambda + \mu$ is an eigenvalue of $A + B$ and $\lambda\mu$ is an eigenvalue of AB.

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4.3.32
Let $p(x)$ be the polynomial

$$p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

The **companion matrix** of $p(x)$ is the $n \times n$ matrix

$$C(p) = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \dots & -a_1 & -a_0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}.$$

(a) Use mathematical induction to prove that, for $n \geq 2$, the companion matrix $C(p)$ of $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has characteristic polynomial $(-1)^n p(\lambda)$. [*Hint*: Expand by cofactors along the last column. You may find it helpful to introduce the polynomial $q(x) = (p(x) - a_0)/x$.]

(b) Show that if λ is an eigenvalue of the companion matrix $C(p)$ in Equation (4), then an eigenvector corresponding to λ is given by

$$\begin{bmatrix} \lambda^{n-1} \\ \lambda^{n-2} \\ \vdots \\ \lambda \\ 1 \end{bmatrix}$$

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4.3.34 THM: (Cayley-Hamilton) If $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ and A is a square matrix, we can define a square matrix $p(A)$ by

$$p(A) = A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 I.$$

If $c_A(\lambda)$ is the characteristic polynomial of the matrix A, then $c_A(A) = O$. In words, every matrix satisfies it's characteristic equation.

Verify the Cayley-Hamilton Theorem for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

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