Math 40
FP1 - Vectors and Vector Spaces
Fri. 1/20, 2016

1.2{ 17, 18, 52, 56, 58, 60 }

17 If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, $n \geq 2$, and c is a scalar, explain why the following expressions make no sense:

(a) $\|\mathbf{u} \cdot \mathbf{v}\|$

(b) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

(c) $\mathbf{u} \cdot \mathbf{v} + \mathbf{w}$

(d) $c \cdot (\mathbf{u} + \mathbf{w})$

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18 Determine whether the angle between $\mathbf{u} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is acute, obtuse or a right angle.

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52 Under what conditions are the following true for vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$ or $\mathbb{R}^3$?

(a) $\|\mathbf{u}+\mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$

(b) $\|\mathbf{u}+\mathbf{v}\| = \|\mathbf{u}\| - \|\mathbf{v}\|$

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56 Prove $d(\mathbf{u}, \mathbf{w}) \leq d(\mathbf{u}, \mathbf{v}) + d(\mathbf{v}, \mathbf{w})$ for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

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58 Prove that $\mathbf{u} \cdot c\mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$ for all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ and all scalars c.

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60 Suppose we know that $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$. Does it follow that $\mathbf{v} = \mathbf{w}$?
If it does, give a proof that is valid in $\mathbb{R}^n$; otherwise, give a *counterexample* (i.e., a *specific* set of vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ for which $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ but $\mathbf{v} \neq \mathbf{w}$).

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