Math 40 - Section —
HW 8 - Determinants, Eigenvectors and Eigenvalues
Friday, February 19, 2016

4.1.{26, 35, 36}, 4.2.{14, 38, 54}

4.1.26 In Exercises 23-26, use the method of Example 4.5 to find all of the eigenvalues of the matrix A. Give bases for each of the corresponding eigenspaces. Illustrate the eigenspaces and the effect of multiplying eigenvectors by A as in Figure 4.8.

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$$

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4.1.35

(a) Show that the eigenvalues of the 2×2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

are the solutions of the quadratic equation

$$\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0$$

where tr(A) is the trace of A.

(b) Show that the eigenvalues of the matrix A in part (a) are

$$\lambda = \frac{1}{2}\left(a + d \pm \sqrt{(a-d)^2 + 4bc}\right)$$

(c) Show that the trace and determinant of the matrix A in part (a) are given by

$$\mathrm{tr}(A) = \lambda_1 + \lambda_2$$

and

$$\det(A) = \lambda_1 \lambda_2$$

where λ_1 and λ_2 are the eigenvalues of A.

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4.1.36 Consider again the matrix A in Exercise 35. Give conditions on a, b, c, and d such that A has

(a) two distinct real eigenvalues,

(b) one real eigenvalue, and

(c) no real eigenvalues.

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4.2.14 Compute the determinant using cofactor expansion along any row or column.

$$\begin{vmatrix} 2 & 0 & 3 & -1 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 4 \\ 2 & 0 & 1 & -3 \end{vmatrix}$$

■

4.2.38 Find the determinant, assuming that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$$

of,

$$\begin{vmatrix} a-c & b & c \\ d-f & e & f \\ g-i & h & i \end{vmatrix}.$$

■

4.2.54 Let A and B be $n \times n$ matrices. If B is invertible, prove that

$$\det(B^{-1}AB) = \det(A).$$

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