Name:

Physics 51

Homework #23

December 5, 2016

Townsend 1.{25*, 27, 29, 34}

Townsend 1.25* Suppose that a thin film of acetone (index of refraction $n = 1.25$) of thickness d is coating a thick plate of glass (index of refraction = 1.50). Take the magnitude of the amplitude for reflection of a photon from the top or the bottom surface of the acetone at normal incidence to be r and assume that there is an additional phase change of π in the reflection from the top and the bottom surface of the acetone, since at each of these surfaces light is passing from a medium with a lower index of refraction to one with a higher index of refraction. Calculate the probability that a photon of wavelength λ reflected. Assume that amplitudes that involve multiple reflections at the bottom surface of the film can be neglected in your calculation. Express your answer in terms λ and r as well as the thickness d and the index of refraction n of the acetone. What is the minimum thickness of the coating necessary to produce zero reflection? *Note*: For the air-acetone and acetone-glass surfaces $r \cong 0.1$.

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Townsend 1.27 Figure 1.43 shows a Michelson interferometer with a movable mirror M_1, a fixed mirror M_2, and a beam splitter M_s which is a half-silvered mirror that transmits one-half the light and reflects one-half the light incident upon it independent of the direction of the light. The source emits monochromatic light of wavelength λ. There are two paths that light can follow from the source to the detector. as indicated in the figure. Note that path 1 includes travel from the beam splitter M_s to the movable mirror M_1 and back to the beam splitter, while path 2 includes travel from the beam splitter M_s to the fixed mirror M_2 and back to the beam splitter. Assume the beam splitter introduces a phase change of π for light that follows path 1 from the source to the detector relative to light that follows path 2 from the source to the detector. Also assume the mirrors M_1 and M_2 reflect 100% of the light incident upon them and the photodetector PM (a photomultiplier) is 100% efficient as well.

(a) Use the principles of quantum mechanics to determine the probability that a photon entering the interferometer is detected by the photodetector. Express your answer in terms of the lengths l_1, l_2 and λ.

(b) Find an expression for l_1 in terms of l_2 and λ such that there is 100% probability that the photon is detected by the photodetector.

(c) Suppose that the movable mirror is shifted upward by a distance $\lambda/6$ from the position(s) that you determined in part(b). Find the probability that the photon is detected at the photodetector in this case.

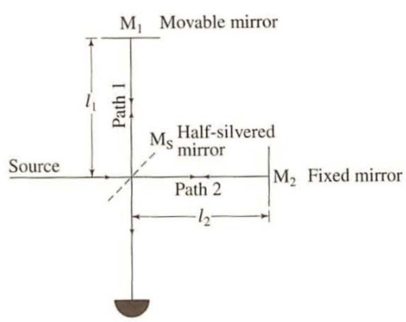

Figure 1.43 The Michelson interferometer.

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Townsend 1.29 Suppose that the two very narrow slits (widths $\ll \lambda$) in the double-slit experiment are not the same width and that the probability amplitude for a photon of wavelength λ to strike a photomultiplier centered at a particular point P in the detection plane that makes an angle θ with the horizontal from one of the slits is larger by a factor of $\sqrt{2}$ than for the other slit. Determine the visibility

$$V = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}$$

of the interference fringes, where P_{max} is the maximum probability and P_{min} is the minimum probability that a photon is detected.

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Townsend 1.34 Starting from first principles, show that the probability that a photon of wavelength λ hits a photomultiplier centered on a point P in the detection plane that makes an angle θ with the horizontal for a grating composed of three very narrow slits each separated by a distance d is given by

$$\text{Prob} = r^2 \left(1 + 4\cos\phi + 4\cos^2\phi\right)$$

where r^2 is the probability that the photon would strike the photomultiplier with a single slit open and $\phi = kd\sin\theta = 2\pi d\sin\theta/\lambda$.

$\blacksquare$