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Physics 24a - Section 1
Work, Energy, and Dynamics
Monday, March 7, 2016

5.12, 6.{3,8,13}

Reading: Chapters 5.6–5.11 and 6.1–6.3, 6.5

Lennard-Jones potential - KK 5.12 A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones 6-12 potential given by

$$U = \epsilon \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^{6} \right]$$

1. Find the position of the potential minimum and its value.
2. Near the minimum the atoms execute simple harmonic motion. Find the frequency of oscillation.

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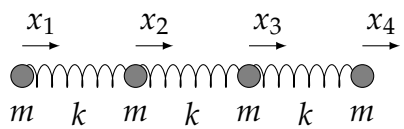
Normal modes and symmetry - KK 6.3

Four identical masses m are joined by three identical springs, of spring constant k, and are constrained to move on a line, as shown. There is a high degree of symmetry in this problem, so that one can guess the normal mode motions by inspection, without a lengthy calculation. Once the relative amplitudes of the normal mode motions are known, the normal mode vibrational frequencies follow directly.



Because of the symmetry, the normal mode amplitudes must obey $x_1 = \pm x_4$ and $x_2 = \pm x_3$. Another condition is that the center of mass must remain at rest. The possibilities are:

1. $(x_4 = x_1)$ and $(x_3 = x_2)$
2. $(x_4 = -x_1)$ and $(x_3 = -x_2)$

The normal mode equations lead to three possible non-trivial vibrational frequencies and three corresponding normal modes. Find the normal mode frequencies. It is convenient to use the dimensionless parameter $\beta = \omega^2/\omega_0^2$, where ω is a frequency to be found and $\omega_0 \equiv \sqrt{k/m}$.

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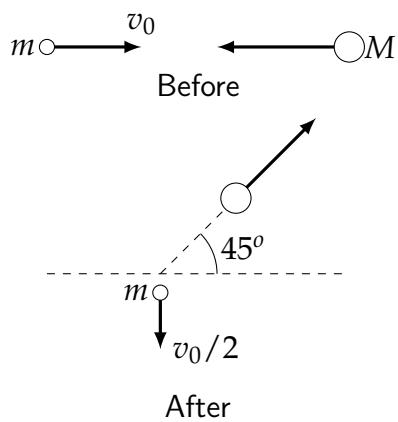
Collision of m and M - KK 6.8

A particle of mass m and initial velocity v_0 collides elastically with a particle of unknown mass M coming from the opposite direction as shown in the sketch. After the collision, m has velocity $v_0/2$ at right angles to the incident direction, and M moves off in the direction shown in the sketch. Find the ratio M/m.



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Nuclear reaction of α-rays with lithium - KK 6.13 A thin target of lithium is bombarded by helium nuclei (α-rays) of energy E_0. The lithium nuclei are initially at rest in the target but are essentially unbound. When an α-ray enters a lithium nucleus, a nuclear reaction can occur in which the compound nucleus splits apart into a boron nucleus and a neutron. The collision is inelastic, and the final kinetic energy is less than E_0 by 2.8MeV. The relative masses of the particles are: helium, mass 4; lithium, mass 7; boron, mass 10; neutron, mass 1. The reaction can be symbolized

$$^{7}\mathrm{Li} + {}^{4}\mathrm{He} \longrightarrow {}^{10}\mathrm{B} + {}^{1}\mathrm{n} - 2.8\mathrm{MeV}$$

1. What is $E_{0,\mathrm{threshold}}$, the minimum value of E_0 for which neutrons can be produced?
2. Show that if the incident energy falls in the range $E_{0,\mathrm{threshold}} < E_0 < E_{0,\mathrm{threshold}} + 0.27\mathrm{MeV}$, the neutrons ejected in the forward direction do not all have the same energy but must have either one or the other of two possible energies. (You can understand the origin of the two groups by looking at the reaction in the center of mass frame.)

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