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DISUNIFICATION IN THE DESCRIPTION LOGIC $\mathcal{EL}$:

Open Problem and Partial Solutions

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Overview

Disunification (mod E)

Connection to Admissibility

Description Logic $\mathcal{EL}$

Disunification in $\mathcal{EL}$

Partial Results

Conclusions

Disunification (mod E)

Given: E a set of universally quantified equations between first order terms.

Disunification Problem

Input:
$$\Gamma := \{\underbrace{s_1 =^? t_1, \quad \ldots, \quad s_n =^? t_n}_{\Gamma^+}, \quad \underbrace{s_{n+1} \neq^? t_{n+1}, \quad \ldots, \quad s_{n+m} \neq^? t_{n+m}}_{\Gamma^-}\}$$

Question: does a substitution σ exists, such that

$E \models \sigma(s_1) = \sigma(t_1) \wedge \ldots \wedge \sigma(s_n) = \sigma(t_n)$
and
$E \not\models \sigma(s_{n+1}) = \sigma(t_{n+1}), \ldots, E \not\models \sigma(s_{n+m}) = \sigma(t_{n+m})$?

Connection to Admissibility

Given: a logic L

Admissibility

A rule $\Gamma \Rightarrow \Delta$ is admissible iff for every subsitution σ,
$L \models \sigma(\Gamma)$ implies $L \models \sigma(\Delta')$ for a non-empty set Δ', $\Delta' \subseteq \Delta$.

or

A rule $\Gamma \Rightarrow \Delta$ is not admissible iff there is a subsitution σ,
such that $L \models \sigma(\phi_1) \wedge \cdots \wedge \sigma(\phi_n)$ and $L \not\models \sigma(\psi_1), \ldots L \not\models \sigma(\psi_m)$,
where $\Gamma := \{\phi_1, \ldots, \phi_n\}, \quad \Delta := \{\psi_1, \ldots, \psi_m\}$.

Disunification Problem

$\ldots \phi_1 \wedge \cdots \wedge \phi_m \equiv_L^? \top$ and $\psi_1 \not\equiv^? \top, \ldots \psi_m \not\equiv_L^? \top$.

Description Logic $\mathcal{EL}$

Syntax of concept terms

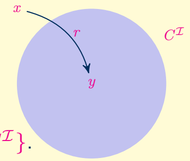
Signature: concept names and role names. **Constructors:** $\top, \sqcap, \exists r$

$\mathcal{EL}$ is a fragment of $\mathbf{K}_m$:

t	$\leftrightarrow$	$\top$	
p	$\leftrightarrow$	A	A is a concept name
$C \wedge D$	$\leftrightarrow$	$C \sqcap D$	
$\Diamond_r.C$	$\leftrightarrow$	$\exists r.C$	r is a role name.

Semantics of $\mathcal{EL}$: $(\Delta^{\mathcal{I}}, \mathcal{I})$

- top: $\top^{\mathcal{I}} := \Delta^{\mathcal{I}}$,
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$,
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$,
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$,
- $(\exists r.C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, (x, y) \in r^{\mathcal{I}}, y \in C^{\mathcal{I}}\}$.



Equivalence and subsumption of concepts

Subsumption & equivalence

$C \sqsubseteq D$ iff for every interpretation $\mathcal{I}$, $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.
$C \equiv D$ iff $C \sqsubseteq D$ and $D \sqsubseteq C$

Equational theory of $\mathcal{EL}$

Associativity of $\sqcap$:	$C_1 \sqcap (C_2 \sqcap C_3) \equiv (C_1 \sqcap C_2) \sqcap C_3$
Commutativity of $\sqcap$:	$C_1 \sqcap C_2 \equiv C_2 \sqcap C_1$
Idempotence of $\sqcap$:	$C_1 \sqcap C_1 \equiv C_1$
Identity for $\sqcap$:	$C_1 \sqcap \top \equiv C_1$
Monotonicity of roles:	$\exists r.C_1 \sqcap \exists r.C_2 \equiv \exists r.C_1$ iff $C_1 \sqcap C_2 \equiv C_1$

Deciding subsumption between concepts

- Polynomial time decision procedure for a subsumption (word problem) in $\mathcal{EL}$.
- Unification in $\mathcal{EL}$ is NP-complete.
- Disunification in $\mathcal{EL}$?

Disunification in $\mathcal{EL}$: example

Two definitions of a Patient with severe head injury

- $\mathsf{C}_1 \equiv \mathsf{Patient} \sqcap \exists \mathsf{finding}.(\mathsf{Head\_injury} \sqcap \exists \mathsf{severity}.\mathsf{Severe})$,
- $\mathsf{C}_2 \equiv \mathsf{Patient} \sqcap \exists \mathsf{finding}.(\mathsf{Severe\_finding} \sqcap \mathsf{Injury} \sqcap \exists \mathsf{finding\_site}.\mathsf{Head})$

Multiple unifiers

- $\sigma_1 := \{\ \mathsf{Head\_injury} \equiv \mathsf{Injury} \sqcap \exists \mathsf{finding\_site}.\mathsf{Head},$
  $\mathsf{Severe\_finding} \equiv \exists \mathsf{severity}.\mathsf{Severe}\}$
- $\sigma_2 := \{\ \mathsf{Head\_injury} \equiv \mathsf{Patient} \sqcap \mathsf{Injury} \sqcap \exists \mathsf{finding\_site}.\mathsf{Head},$
  $\mathsf{Severe\_finding} \equiv \mathsf{Patient} \sqcap \exists \mathsf{severity}.\mathsf{Severe}\}$

To filter out σ_2 add a negative constraint:

$\mathsf{Head\_injury} \not\sqsubseteq^{?} \mathsf{Patient}$

Disunification in $\mathcal{EL}$: definition

Given disjoint sets of variables and constants and $\mathcal{EL}$-concept terms $C_1, \ldots, D'_n$ constructed over these sets,

Problem: $\Gamma := \{\underbrace{C_1 \sqsubseteq^? D_1, \ldots, C_m \sqsubseteq^? D_m}_{\Gamma^+}\} \cup \{\underbrace{C'_1 \not\sqsubseteq^? D'_1, \ldots, C'_n \not\sqsubseteq^? D'_n}_{\Gamma^-}\}$

Solution: a substitution σ, such that:
$\sigma(C_1) \sqsubseteq \sigma(D_1), \ldots, \sigma(C_m) \sqsubseteq \sigma(D_m)$ and $\sigma(C'_1) \not\sqsubseteq \sigma(D'_1), \ldots, \sigma(C'_n) \not\sqsubseteq \sigma(D'_n)$.

Prefered is a ground solution: no variables in the range of σ.

Disunification in $\mathcal{EL}$: Local solutions

Atoms

Atoms: $A, \exists r.C, \exists r.\top, \dots,$
where A is a variable or a constant, C is a concept, r is a role name.

Every concept term is a conjunction of atoms or is $\top$.

Given a (dis)unification problem Γ, we define:

Local atoms

Local atoms are atoms of concept terms in Γ.

For example: $C :=$ Patient $\sqcap$ $\exists$finding.(Head_injury $\sqcap$ $\exists$severity.Severe).
Atoms of C:
Patient, $\exists$finding.(Head_injury $\sqcap$ $\exists$severity.Severe), Head_injury, $\exists$severity.Severe

A substitution σ for variables in Γ is local
if atoms of concept terms in $range(\sigma)$ are local.

An idea of a procedure for disunification in $\mathcal{EL}$

Let Γ be a disunification problem in $\mathcal{EL}$.
Let γ be a ground solution of Γ.

Theorem

There is a local solution σ of Γ^+ such that for each variable in Γ,

$$\gamma(X) \sqsubseteq \sigma(X)$$

Idea

1. Guess or compute σ.
2. Modify σ such that it solves Γ^-.

Important property

$C_1 \sqcap \cdots \sqcap C_m \not\sqsubseteq D_1 \sqcap \cdots \sqcap D_n$ iff there is D_j, such that for each C_i, $C_i \not\sqsubseteq D_j$

Disunification procedure: example

Example

Let A, B, C be constants.

Unification part

$\Gamma^+ := \{X \sqsubseteq^? B, \quad A \sqcap B \sqcap C \sqsubseteq^? X, \quad \exists r.X \sqsubseteq^? Y\}$

Let $\sigma = \{X \mapsto B, \quad Y \mapsto \top\}$.

Disunification part

$\Gamma^- := \{\top \not\sqsubseteq^? Y, \quad Y \not\sqsubseteq^? \exists r.B\}$

Disunification procedure: example

Unsolved dissubsumption

$\top \not\sqsubseteq^? Y$

- $\sigma_1 = \{X \mapsto B, \boldsymbol{Y \mapsto \exists r.U}, U \mapsto \top\}$
- add $\top \not\sqsubseteq^? \exists r.U$ to Γ^-

Extension of σ

Condition: The rule applies to $C \not\sqsubseteq Y \in \Gamma^-$.

Action:

- adds $\exists r.U$ to $\sigma(Y)$, for a fresh variable U,
- adds $C \not\sqsubseteq \exists r.U$ to Γ^-.
- triggers **Expansion of** Γ

Disunification procedure: example

Initially solved (dis)subsumptions

$\exists r.X \sqsubseteq^? Y \in \Gamma^+$,
$Y \not\sqsubseteq^? \exists r.B \in \Gamma^-$

- adds $\exists r.X \sqsubseteq^? \exists r.U$ to Γ^+.
  It follows that $X \sqsubseteq^? U$ is added to Γ^+.
- adds $\exists r.U \not\sqsubseteq \exists r.B$ to Γ^-.
  It follows that $U \not\sqsubseteq B$ is added to Γ^-.

Expansion of Γ

Condition: Applies when an atom $\exists r.U$ is added to $\sigma(Y)$.

Action:

- for $C \sqsubseteq^? Y \in \Gamma^+$, adds $C \sqsubseteq^? \exists r.U$ to Γ^+.
- for $Y \not\sqsubseteq D \in \Gamma^-$, adds $\exists r.U \not\sqsubseteq D$ to Γ^-.

$\sigma_1 = \{X \mapsto B, Y \mapsto \exists r.U, U \mapsto \top\}$
σ_1 solves
$\Gamma = \{X \sqsubseteq^? B, \quad A \sqcap B \sqcap C \sqsubseteq^? X, \quad \exists r.X \sqsubseteq^? Y\} \cup \{\top \not\sqsubseteq^? Y, \quad Y \not\sqsubseteq^? \exists r.B\}$.

Disunification procedure: example of non-termination

Let $\Sigma = \{r\}$.
$\Gamma_0 = \{\boldsymbol{Y \not\sqsubseteq^? X}, \quad \boldsymbol{X \not\sqsubseteq^? Y}\}$
$\sigma_0 := \{X \mapsto \top, \quad Y \mapsto \top\}$

$Y \not\sqsubseteq^? X$ triggers **Extension** of σ_0: $\boldsymbol{\sigma_1(X) = \exists r. U_1}$.

$\Gamma_1 = \Gamma_0 \cup \{Y \not\sqsubseteq^? \exists r. U_1\}$
$\sigma_1 := \{X \mapsto \exists r. U_1, \quad Y \mapsto \top, \quad U_1 \mapsto \top\}$

$X \not\sqsubseteq^? Y$ triggers **Extension** of σ_1: $\boldsymbol{\sigma_2(Y) = \exists r. U_2}$
and this triggers **Expansion** of Γ_1:.

$\Gamma_2 = \Gamma_0 \cup \{Y \not\sqsubseteq^? \exists r. U_1\} \cup \{X \not\sqsubseteq^? \exists r. U_2\} \cup \{\exists r. U_1 \not\sqsubseteq^? \exists r. U_2, \exists r. U_2 \not\sqsubseteq^? \exists r. U_1\}$

By **Decomposition**, $\Gamma_3 = \Gamma_2 \cup \{\boldsymbol{U_1 \not\sqsubseteq^? U_2, U_2 \not\sqsubseteq^? U_1}\}$

$\sigma_2 := \{X \mapsto \exists r. U_1, \quad Y \mapsto \exists r. U_2, \quad U_1 \mapsto \top, \quad U_2 \mapsto \top\}$

Local disunification

Restriction of solutions to local substitutions

σ is local iff for every variable X, $\sigma(X)$ is a conjunction of local atoms, i.e., atoms in Γ.

Restricted **Extension** rule

Local Extension:

Condition: The rule applies to $C \not\sqsubseteq^? Y \in \Gamma^-$.
Action:

- adds D to $\sigma(Y)$, for a local atom D (not a variable),
- adds $C \not\sqsubseteq^? D$ to Γ^-.
- triggers **Expansion of** Γ

Dismatching: definition

Given disjoint sets of variables and constants and $\mathcal{EL}$-concepts $C_1, \ldots, D'_n$ constructed over these sets.

Problem: $\Gamma := \{\underbrace{C_1 \sqsubseteq^? D_1, \ldots, C_m \sqsubseteq^? D_m}_{\Gamma^+}\} \cup \{\underbrace{C'_1 \not\sqsubseteq^? D'_1, \ldots, C'_n \not\sqsubseteq^? D'_n}_{\Gamma^-}\}$, such that **for each** $i \in \{1, \ldots, n\}$**,** C'_i **or** D'_i **is ground (contains no variables).**

Solution: a substitution σ, such that:
$\sigma(C_1) \sqsubseteq \sigma(D_1), \ldots, \sigma(C_m) \sqsubseteq \sigma(D_m)$ and $\sigma(C'_1) \not\sqsubseteq \sigma(D'_1), \ldots, \sigma(C'_n) \not\sqsubseteq \sigma(D'_n)$.

Example

Head_injury $\not\sqsubseteq^?$ Patient

Dismatching: reduction to local disunification

Getting rid of left-ground dissubsumptions

$\mathfrak{s} = C \not\sqsubseteq^? X$ (C is a ground term)

- Guess a "witness" atom D for $\mathfrak{s}$: constant or $\exists r.U$ (new variable)
- Add $X \sqsubseteq^? D$ to Γ^+
- Add $C \not\sqsubseteq^? D$ to Γ^-.

$C \not\sqsubseteq^? \exists r.U \quad \rightsquigarrow \quad C' \not\sqsubseteq^? U$, where $|C'| < |C|$.
(Only polynomially many new variables needed.)

Output: $\Gamma' := \{C_1 \sqsubseteq^? D_1, \ldots, C_m \sqsubseteq^? D_m\} \cup \{X_1 \not\sqsubseteq^? D'_1, \ldots, X_n \not\sqsubseteq^? D'_n\}$

Dismatching: reduction to local disunification

Theorem

A dismatching problem Γ has a solution iff a disunification problem Γ' has a local solution.

Idea of a proof

- If γ is a ground solution of Γ, then it is a solution of Γ'.
- γ induces a local solution σ of the positive part of Γ', $\gamma(X) \sqsubseteq \sigma(X)$.
- Since

  $$\gamma(X_i) \not\sqsubseteq D_i'$$

  and

  $$\gamma(X_i) \sqsubseteq \sigma(X_i)$$

  then (by transitivity of subsumption)

  $$\sigma(X_i) \not\sqsubseteq D_i'$$

  Hence σ is a solution of Γ', and thus also of Γ.

Conclusions

We have seen:

- local disunification – as a way to reduce the number of local unifiers,
- in practice we use dismatching problems and these can be reduced to local disunification,
- we have implemented local disunification in the unifier for $\mathcal{EL}$, UEL.

UEL http://uel.sourceforge.net

- Given two ontologies and a set of variables, computes a local unifier for two concepts.
- Includes SAT reduction of local disunification.
- Rules-based local disunification still needs to be implemented.

Open problem

Disunification in $\mathcal{EL}$