Łukasiewicz Public Announcement Logic

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This contribution reports on ongoing joint work with Ricardo Rodríguez (University of Buenos Aires, Argentina) and Leonardo Cabrer (TU-Wien, Austria). We develop a logic of public announcements in a many-valued setting: we provide an algebraic and a Kripke-style semantics, proving completeness with respect to both. We show how our approach can be extended to other systems of dynamic epistemic logic. Dynamic logics are language expansions of (classical) modal logic designed to reason about changes induced by actions of different kinds, e.g. updates on the memory state of a computer, displacements of a moving robot, belief-revisions changing the common ground among different cognitive agents, knowledge updates. Semantically, an action is represented by a transformation of a model describing a given state of affairs: this transformation shows how the state of affairs is modified after the action has been performed. The logic of public announcements [8, 1] is a dynamic logic that models the epistemic change on the cognitive state of a group of agents resulting from a given fact (expressed by some proposition, say α) becoming publicly known. To each α one associates a *dynamic* modal operator $\langle\alpha\rangle$ such that $\langle\alpha\rangle\varphi$ is a proposition in the language of public announcement logic which is interpreted as saying that, *after* α *has been publicly announced*, the proposition φ holds. Our work combines the logic of [1] with the family of finite-valued Łukasiewicz modal logics studied in [5]. The recent [7, 6, 9] introduce dynamic epistemic expansions of several logics weaker than classical logic, providing a semantic definition to introduce the extended logic. The main methodological novelty of these works is a dual characterization of epistemic updates via Stone-type dualities: here we adopt the same approach, further extending it from a mathematical point of view. Epistemic updates induced by public announcements are formalized in relational (Kripke-style) models by means of the relativization construction, which creates a submodel of the original model. The corresponding submodel injection map is dually represented in [7] as a quotient construction between the complex algebras of the original model and of the updated one. This construction allows us to study epistemic updates within mathematical environments having a support that is more general than classical logic, as long as we can exploit an existing duality between algebraic and Kripke-style semantics. We further generalize [7, 6] taking as propositional base the family of finite-valued Łukasievicz logics, while epistemic (i.e. *static*) modalities are modeled within the framework of the modal Łukasievicz logics of [5]. We can thus build propositions such as $\langle\alpha\rangle\Box_i\varphi$ expressing that, after α has been announced, the agent i knows that φ holds. Each static modal operator of Łukaziewicz modal logic $\Box_i$ ($\Diamond_i$) is used to represent the knowledge (beliefs) of an agent i. The need for such a framework, which combines a many-valued approach with modalities, has been forcefully argued for in a number of recent works [3, 2]. The main idea is that, while modal operators proved to be an invaluable tool in a variety of logical applications, classical logic turned out to be unsuitable in many reasoning contexts, especially those involving partial or contradictory information [9], constructive reasoning [7] and vagueness. Especially this last issue, namely the problem of graded properties and vague predicates, can be best dealt with in the many-valued framework of Łukasiewicz logic. In this contribution we lay the mathematical foundations for such an approach. We generalize the pseudo-quotient construction of [7, 9] to the algebraic semantics of Łukasiewicz modal logic (*modal MV-algebras*), which allows us to define a natural interpretation of the language of public announcement logic on these algebraic structures. We establish which interaction axioms between dynamic modalities and the other connectives of

the logic are sound with respect to our intended semantics. The resulting calculus defines a many-valued version of public announcement logic, which we prove to be complete with respect to our algebra-based semantics. We also introduce an equivalent relational semantics based on many-valued Kripke frames, which is obtained from the algebraic semantics via a Stone-type duality. From a technical point of view, the main difficulties, and hence the novelty, of our approach stem from two sources. Firstly, the existing definitions of pseudo-quotient (which models the epistemic update on algebraic models of the logic) as such do not work on modal MV-algebras. This led us to restrict our attention (at least for the time being) to algebras having a non-modal reduct that belongs to a finitely-generated variety of MV-algebras, which allowed us to exploit the fact that any such algebra is k-potent for some finite natural number k. Our construction also provides insight on the possibility of defining pseudo-quotients on more general algebras, such as residuated lattices endowed with modal operators (corresponding to the logics considered e.g. in [2]). Essentially, what one needs is to be able to provide a "simple" equational characterization (such as that of [7, Section 3.2]) of the logical filter generated by any given element in the algebra. Secondly, the algebraic approach to dynamic epistemic logics relies on the existence of a well-developed duality for the algebras corresponding to the logic, which in all existing studies is a Stone- or Priestley-type duality. MV-algebras are not easily understood from the point of view of Stone- or Priestley-type duality theory (see, e.g., [4]). In this work, since we restrict ourselves to k-potent MV-algebras, we have at hand the natural duality for finitely generated varieties of MV-algebras developed in [10] and its extension to modal MV-algebras [5]. We expand their algebraic and Kripke-style semantics to account for dynamic modalities. As potential directions for future research, we mention the issue of extending our treatment to arbitrary modal MV-algebras, and also that of extending our algebraic account of updates in order to be able to introduce and axiomatize a Łukasiewicz version of the logic of epistemic action and knowledge considered in [6].

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