Towards a General Theory of Effective Paraconsistent First-order Logics

Arnon Avron, Ofer Arieli and Anna Zamansky

Paraconsistent reasoning is by now a very active area of research, with a lot of interesting results and applications. However, the *mathematical* depth and breadth of what has been achieved are not well established, especially in the CS and AI communities. One of the reasons for this is the fact that the propositional fragment is the heart of every paraconsistent logic ever studied, and so from the philosophical point of view (which was the main concern of the founders), the first-order level has been much less interesting. Indeed, the main conceptual problems involved in developing useful paraconsistent logics arise already on the propositional level, and all the various ideas about how to tackle them are connected with this level as well. As a result, the subject of first-order paraconsistent logic is not well-developed. However, when it comes to practical applications, logics do need to be extended (at least) to the first-order level, and it turned out that doing this in the case of paraconsistent logics involves complex technical problems.

In this talk we first survey our unification and systematization of the theory of *effective* propositional paraconsistent logics using the semantic framework of multi-valued (deterministic and non-deterministic) matrices. Among other things, this includes providing precise definitions of the notions of negation and paraconsistency, characterization of desirable properties of paraconsistent logics (such as maximality and containment in classical logic), and modular construction of corresponding analytic proof systems[1]. After that we will discuss the problems involved in extending these results to the first-order level, and propose some directions for

[1] Many, but not all, of the results here have been presented in previous papers, like [1, 2, 3, 4, 5].

handling them, based on our previous investigation in [6] of a particular large family of first order paraconsistent logics: the Logics of Formal Inconsistency (LFIs [7]).

References

[1] O. Arieli and A. Avron. Three-valued paraconsistent propositional logics. In *Proceedings of the 5th World Congress on Paraconsistency*, 2014. To appear.

[2] O. Arieli, A. Avron, and A. Zamansky. Ideal paraconsistent logics. *Studia Logica*, 99(1–3):31–60, 2011.

[3] O. Arieli, A. Avron, and A. Zamansky. Maximal and premaximal paraconsistency in the framework of three-valued semantics. *Studia Logica*, 97(1):31–60, 2011.

[4] A. Avron, B. Konikowska, and A. Zamansky. Cut-free sequent calculi for C-systems with generalized finite-valued semantics. *Logic and Computation*, 23(3):517–540, 2013.

[5] A. Avron, B. Konikowska, and A. Zamansky. Efficient reasoning with inconsistent information using C-systems. *Information Sciences*, 296:219–236, 2015.

[6] A. Avron and A. Zamansky. Many-valued non-deterministic semantics for first-order logics of formal (in)consistency. In S. Aguzzoli, A. Ciabattoni, B. Gerla, C. Manara, and V. Marra, editors, *Algebraic and Proof-theoretic Aspects of Non-classical Logics*, number 4460 in LNAI, pages 1–24. Springer, 2007.

[7] W. A. Carnielli, M. E. Coniglio, and J. Marcos. Logics of formal inconsistency. In D. M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 14, pages 1–95. Springer, 2007. Second edition.