Expressivity of many-valued modal logics, coalgebraically

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Abstract theory of coalgebras has recently became one of most important bridges between modal logic and computer science: from a logician's point of view it provides techniques and a new level of generality for studying various modal logics, while from a computer-scientist's point of view it provides a general framework for designing expressive (modal) languages describing behavior of abstract transition systems modeled as coalgebras. It is natural to ask what are benefits of a coalgebraic approach to many-valued modal logics: from a logician's point of view, we can generalize logics of many-valued Kripke-style relational semantics to the coalgebraic level, offering in particular a new perspective to the question what is the minimal modal logic over a given residuated lattice $\mathscr{V}$, in which valuations and the accessibility relation take values (cf. [3]). From a computer-scientist's point of view it allows to generalize logics of abstract transition systems modeled as coalgebras to the many-valued setting, allowing for a many-valued observable phenomena to be captured in the framework. A notion of behavioral equivalence is central in studying coalgebras, and, in case the coalgebra functor preserves weak pullbacks, it coincides with bisimilarity. We outline two approaches of designing a logical language for coalgebras, parametric in the coalgebra functor, and apply them in a many-valued setting. Our goal is to give sufficient conditions (both on the coalgebra functor and the algebra of truth values) for the resulting language being expressive for bisimilarity. In this respect, we are generalising results of Metcalfe and Martí on Hennessy-Milner property for language with box and diamond over crisp many-valued image-finite Kripke frames where $\mathscr{V}$ is a complete MTL-chain, obtained in [6]. Since we are interested in this talk in bisimilarity and in finitary languages, we restrict ourselves to weak pullback preserving finitary set functors.

Moss' coalgebraic logic. A coalgebraic language introduced in Moss' pioneering paper [7], extends the underlying propositional logic with a single modality whose arity is given by the coalgebra functor and whose semantics is given by a lifting of the local satisfaction relation with the coalgebra functor. The relation lifting is available provided the coalgebra functor preserves weak pullbacks, in which case also behavioral equivalence coincides with bisimilarity. The nonstandard syntax allows for a simple proof of the Hennessy-Milner property — the language is expressive for bisimilarity. We will investigate possibilities of extending the results beyond the boolean setting. We first consider set-based coalgebras for weak pullback preserving finitary functors with many-valued valuations, of which many-valued image-finite Kripke frames are a special case, and show that the resulting Moss' many-valued logic is adequate and expressive for a crisp notion of bisimilarity based on many-valued bisimulations. Both semantics of the language and the definition of bisimulations use the concept of a many-valued relation lifting developed in [1], which works well provided $\mathscr{V}$ is a complete

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Heyting algebra. The expressivity result is available whenever $\mathscr{V}$ is moreover a chain, plus an additional condition on $\mathscr{V}$, similar to one considered in [6].

Many-valued predicate liftings. The approach based on a *logical connection* [2, 5, 4] between modal algebras and coalgebras has a benefit of providing one with an abstract machinery producing, for a given functor, a language of *all* modalities, corresponding to *predicate liftings*. Languages based on predicate liftings were developed and their expressivity investigated by Pattinson in [9, 8] and further by Schröder in [10], where a sufficient condition on a subset of predicate liftings (namely, being separating) is given to ensure that the resulting logic is expressive for behavioral equivalence (and this works for all accessible set functors). We consider many-valued logical connections and adapt the notion of a separating set of predicate liftings to the many-valued setting, including now a condition on the algebra $\mathscr{V}$ (of which the one given in [6] is a special case). We prove that, given a separating set of modalities and the additional condition on $\mathscr{V}$, the resulting logic is expressive for bisimilarity. We will also discuss further generalisation for all accessible functors and behavioral equivalence instead of bisimilarity. This part is based on an ongoing joint-work with Matěj Dostál.

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