Hájek's Fuzzy Probability Logic Revisited

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Fuzzy Probability Logic FP(Ł) over infinite-valued Łukasiewicz logic Ł was introduced and studied by Hájek in [2, Chapter 8.4]. The purpose of FP(Ł) is to formalize reasoning about properties of finitely-additive probability measures on (possibly infinite) Kripke-style models. Probability assessments are syntactically represented in the language by a unary modality P, which applies to Boolean formulas φ only. The intended meaning of the modal formula $P\varphi$ is: "φ is probable". The logic FP(Ł) was substantially generalized by Cintula and Noguera. Their work [1] applies to uncertainty measures more general than probabilities, and includes completeness theorems for the corresponding logics.

Following Hájek, let us say that the triple (W, e, μ) is a *probability Kripke frame* if W is a set of possible worlds, $e\colon W \times Form \to \{0, 1\}$ is a Boolean evaluation of Boolean formulas $\varphi \in Form$ for each possible world $w \in W$, and μ is a finitely-additive probability measure on a Boolean algebra of subsets of W containing every set $\{w \in W \mid e(w, p) = 1\}$, for each of the finitely-many propositional variables p. Probability Kripke frames provide a complete semantics for FP(Ł). In our contribution we show how to interpret FP(Ł) using a different class of structures. We consider triples (B, e, μ), where B is a Boolean algebra, $e\colon Form \to B$ is an evaluation of Boolean formulas in $Form$ into B, and μ is a finitely-additive probability measure — alias a *state*, following Mundici and others — on B. For each Boolean formula φ we define the truth value of the modal formula $P\varphi$ to be $\mu(e(\varphi))$. We show that FP(Ł) is complete with respect to the class of all models (B, e, μ), where B is a Boolean σ-algebra, and μ is a (σ-additive) probability measure on B. This follows from a much stronger statement that seems to have escaped attention so far: FP(Ł) has the finite model property, that is, it is complete with respect to the class of models (B, e, μ) with B a finite Boolean algebra. If time allows we indicate how these results may be extended to the broader setting of many-valued events, and discuss directions for further research.

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References

[1] P. Cintula and C. Noguera. Modal logics of uncertainty with two-layer syntax: A general completeness theorem. In U. Kohlenbach, P. Barceló, and R. de Queiroz, editors, *Logic, Language, Information, and Computation*, volume 8652 of *Lecture Notes in Computer Science*, pages 124–136. Springer Berlin Heidelberg, 2014.

[2] P. Hájek. *Metamathematics of fuzzy logic*, volume 4 of *Trends in Logic—Studia Logica Library*. Kluwer Academic Publishers, Dordrecht, 1998.