International Collegiate Programming Contest 2024

Gran Premio de México - Primera Fecha

May 11th, 2024

Contest Session

This problem set contains 12 problems; pages are numbered from 1 to 14.

General information

Unless otherwise stated, the following conditions hold for all problems.

Program name

1. Your solution must be called *codename*`.c`, *codename*`.cpp`, *codename*`.java`, *codename*`.kt`, *codename*`.py3`, where *codename* is the capital letter which identifies the problem.

Input

1. The input must be read from standard input.
2. The input consists of a single test case, which is described using a number of lines that depends on the problem. No extra data appear in the input.
3. When a line of data contains several values, they are separated by *single* spaces. No other spaces appear in the input. There are no empty lines.
4. The English alphabet is used. There are no letters with tildes, accents, diaereses or other diacritical marks (ñ, Ã, é, Ì, ô, Ü, ç, etcetera).
5. Every line, including the last one, has the usual end-of-line mark.

Output

1. The output must be written to standard output.
2. The result of the test case must appear in the output using a number of lines that depends on the problem. No extra data should appear in the output.
3. When a line of results contains several values, they must be separated by *single* spaces. No other spaces should appear in the output. There should be no empty lines.
4. The English alphabet must be used. There should be no letters with tildes, accents, diaereses or other diacritical marks (ñ, Ã, é, Ì, ô, Ü, ç, etcetera).
5. Every line, including the last one, must have the usual end-of-line mark.
6. To output real numbers, round them to the closest rational with the required number of digits after the decimal point. Test case is such that there are no ties when rounding as specified.

Problem A – Arrayland's Challenge

In the digital realm of Arrayland, where data structures shape the foundations of society, the wise council has posed a challenging task to sharpen the minds of its citizens. The challenge involves understanding the subtle differences within the ordered structures, specifically within subarrays.

You are given an array of integers A of size N. Accompanying the array, you receive Q queries. Each query specifies a subarray defined by two indices L and R (where $1 \leq L < R \leq N$), and you are tasked with finding the smallest absolute difference between any two distinct elements within this subarray.

For each query, determine the minimum absolute difference between any two distinct elements contained in the subarray from index L to R.

Input

The first line of input contains an integer N ($1 \leq N \leq 10^4$), the size of the array. The second line contains N integers a_i separated by spaces ($1 \leq a_i \leq 10^6$), the elements in the array.

The third line contains an integer Q ($1 \leq Q \leq 10^5$), the number of queries to answer.

Each of the next Q lines contains two integers L, and R ($1 \leq L < R \leq N$) representing the indices of the subarray for the query.

Output

Print Q lines, where the i-th line contains a sintle integer representing the answers for to the i-th query.

Sample input 1	Sample output 1
10 4 5 1 3 2 1 4 6 7 8 3 1 10 3 6 7 10	0 0 1

Sample input 2	Sample output 2
8 3 45 1 2 3 4 4 4 2 1 8 1 2	0 42

Problem B – Bacterial Sampling

There is a new bacteria that grows very fast. One of the local universities in town wants to breed some of this bacteria, their particular interest is to study the bacteria as an option to revert environmental damage because it is supposed to eat plastic! In order to calculate the required production, the university wants to know how much bacteria they can have in a container at minute N.

Several studies have shown the bacteria grows in the following way:

1. A bacteria needs 2 minutes to mature.
2. When a bacteria is mature, every 4 minutes it produces by meiosis 3 brand new bacteria.
3. When a bacteria is 20 minutes old, it dies.

To start the experiment, the university at minute 0 will put just 1 "new born" bacteria in the container. The university needs your help to know how much bacteria will be held in the container at minute N.

Input

The first line of input contains a number T ($1 \leq T \leq 50$) indicates the number of test cases. The next T lines contains a single integer N ($1 \leq N \leq 10^9$), where the i-th line represents the minutes the university will let the experiment run on the i-th test case

Output

Print T lines, where the i-th line contains a single integer, the amount of bacteria that will be on the container at minute N, since this number can be huge print it modulo $10^9 + 7$

Sample input 1	Sample output 1
4 1 2 3 12	1 1 1 16

Problem C – Chocolate Packing

In preparation for Mother's Day, you are tasked with packing chocolate boxes into a larger shipping box for delivery. Each chocolate box is a small rectangular box, and you need to determine the maximum number of these chocolate boxes that can be packed into the shipping box. The chocolate boxes, just like your love for your mom, come in identical shapes, and they need to be packed efficiently into the shipping box, which is also a rectangular box.

However, there's a catch! To maintain the aesthetic appeal of the gift, the chocolate boxes must all be packed in the same orientation, with their sides parallel to the sides of the shipping box. Also, the chocolate boxes cannot be broken or resized to fit into the shipping box.

Your task is to write a program that calculates the maximum number of chocolate boxes that can be packed inside the shipping box.

Input

The first line contains three integers L, W, and H, representing the dimensions of the shipping box ($1 \leq L, W, H \leq 1000$). The second line contains three integers l, w, and h, representing the dimensions of a chocolate box ($1 \leq l, w, h \leq 1000$).

Output

Output a line with a single integer representing the maximum number of chocolate boxes that can be packed inside the shipping box.

Sample input 1	Sample output 1
1 1 1 3 3 3	0

Sample input 2	Sample output 2
6 5 4 3 2 1	20

Sample input 3	Sample output 3
21 11 31 10 3 2	105

Problem D – Different Triangles

In the quiet hours of a long evening, you find yourself with a box of matchsticks and a growing curiosity. With these simple sticks, you begin constructing triangles, starting with a classic 3-4-5 triangle. After counting 12 matchsticks to form this first shape, a question sparks in your mind: How many distinct triangles can be formed using at most N matchsticks? Each side of these triangles must be made up of an integer number of matchsticks, and the triangles must have a positive area.

To add to the complexity, triangles that are merely rotations or reflections of each other are considered the same. For example, triangles with sides 3-4-5, 4-5-3, and 5-3-4 are indistinguishable under these rules.

Because this number can be very large print it modulo $10^9 + 7$

Input

A single integer N ($1 \leq N \leq 10^6$) representing the total matchsticks available

Output

Output a line with a single integer number, the number of distinct triangles that can be formed using at most N matchsticks modulo $10^9 + 7$.

Sample input 1	**Sample output 1**
`5`	`2`

Sample input 2	**Sample output 2**
`12`	`18`

Problem E – Evaluating Linear Expressions

In the realm of Algebraica, a land governed by the laws of arithmetic and algebra, citizens often challenge each other with mathematical puzzles. One popular challenge involves exploring linear expressions of the form $ax + b$.

You are given a linear expression in the form $ax + b$. Your task is to generate the first k terms of the sequence generated by this expression, starting with $x = 1$ and ending with $x = k$.

Given the coefficients a and b of the linear expression, along with an integer k, compute and output the first k terms of the sequence resulting from substituting $x = 1, 2, 3, ..., k$ into the expression.

Input

The first line contains three integers, a, b, and k $(1 \leq a, b, k \leq 100)$ K integers on a line representing the first K elements of the sequence.

Sample input 1	**Sample output 1**
`1 1 5`	`2 3 4 5 6`

Sample input 2	**Sample output 2**
`1 10 7`	`11 12 13 14 15 16 17`

Sample input 3	**Sample output 3**
`3 1 4`	`4 7 10 13`

Problem F – Factory TikTak Trend

Maria Jose has recently started working at a new company in her town that is internationally renowned for printing cool, yet nonsensical text on T-shirts. These shirts have become a sensation on a platform called TikTak (For legal purposes of this statement). Maria Jose is also very popular on TikTak, but lately anyone using the "Pedro Pedro" trend seems to get more views than her. Seeking to stand out, she has decided to breach company policy by creating a behind-the-scenes video at the factory that makes these trending shirts.

At the start of each month, the factory selects two long strings, s and t, to print on a single T-shirt. Machine S is exclusively assigned string s and machine T is exclusively assigned string t. Each machine has a specific operation that it performs on its string after printing a shirt in order to create various patterns.

Machine S can be described in its i-th state, S_i, where it performs the following operation i times:

- Removes the first character of s and appends it to the end of s.

Machine T can be described in its i-th state, T_i, where it performs the following operation i times:

- Removes the last character of t and appends it to the beginning of t.

Maria Jose needs to give the final touch to her TikTak to get the maximum views possible, for that she has to add a curious cool fact about the most popular shirts in the world, and as is well known nothing is cooler than knowing what a string is lexicographically less than or equal to another string.

Maria Jose needs your help to find the number of distinct pairs (i, j) such that $0 \leq i, j \leq N - 1$ where S_i is lexicographically less than or equal to T_j.

Input

The first line contains an integer N ($1 \leq N \leq 2 \times 10^5$), the length of the strings s and t.
The second line contains the string s, consisting of lowercase English letters.
The third line contains the string t, consisting of lowercase English letters.

Output

Print a single integer indicating the number of unique (i, j) pairs for which $S_i \leq T_j$.

Sample input 1	**Sample output 1**

`3`
`bec`
`dbc`
	`4`

Problem G – Granitus Stone Towers

In the ancient lands of Granitus, two rival architects, Alicius and Bobius, are competing in a legendary contest. The plains are scattered with N stone towers, each varying in height. The architects take turns with a unique challenge: in each turn, the player in turn must choose a positive integer X and remove X stones from each tower that still has stones. The chosen X cannot exceed the number of stones in the shortest tower that still has stones remaining. A player loses the game when he cannot make a valid move.

Given N towers of stones, determine which architect, Alicius or Bobius, will win if both play optimally. Alicius always takes the first turn.

Your task is to predict the winner of the game assuming optimal play by both competitors.

Input

The first line contains a single integer N ($1 \leq N \leq 10^6$) representing the amount of towers. The second line contains N integers a_i ($1 \leq a_i \leq 10^6$) representing the amount of stones in the $i-th$ tower.

Output

Print a line containing the name of the architect which will win the game: "Alicius", or "Bobius"

Sample input 1	Sample output 1
4 5 10 5 3	Alicius

Sample input 2	Sample output 2
5 1 7 8 10 15	Bobius

Problem H – Highest Score APPQ

Juan's birthday is coming soon, and Luis wants to give him a special present. Juan is a Number Theory enthusiast, so Luis thinks the best present would be an *anti-prime-power-quotient* set of numbers (APPQ set from now on). An APPQ set is defined as a set S such that, for any $x, y \in S$ where $x < y$, either x is not a divisor of y, or the quotient $\frac{y}{x}$ has more than one distinct prime factor.

Luis has a number generator machine. The power of the prime factors of the numbers that his machine can generate is limited by an array $A = [a_1, a_2, \ldots a_n]$ of size n. More formally, the machine can generate only numbers of the form:

$$\prod_{i=1}^{n} p_i^{e_i}$$

where p_i is the i-th smallest prime number and e_i satisfies $0 \le e_i \le a_i$.

Luis aims to craft the best APPQ set S utilizing his number generator machine. However, he's aware that Juan has discerning tastes when it comes to numbers. Juan assigns a score to each number based on the total count of its prime factors, **counting repetitions**. For a number $x = \prod_{i=1}^{k} p_i^{e_i}$, its score is defined as

$$2024!^{2024! \cdot \sum_{i=1}^{k} e_i}$$

The score of a set is the sum of the scores of each of its elements.

Help Luis find the APPQ set with the greatest score, and print how many elements the best APPQ set has and the sum of its elements. Since the answer may be huge, print it modulo 998244353.

Input

The first line contains an integer n ($1 \le n \le 500$).

The second line contains n integers $a_1, a_2, \ldots a_n$ ($0 \le a_i \le 500$).

Output

Print two integers – the number of elements in the best APPQ set and the sum of its elements. Since the answer may be huge, print it modulo 998244353. If there are multiple answers, print any of them.

Sample input 1	Sample output 1
1 1	1 2

Sample input 2	Sample output 2
2 1 1	2 7

Sample input 3	Sample output 3
3 2 0 2	3 111

Problem I – Inspecting Merge Algorithm

Sergio is analyzing the merge algorithm for his algorithms assignment. His professor has written the following algorithm:

```
std::vector<int> merge(std::vector<int> A, std::vector<int> B) {
    std::vector<int> C;
    int i=0, j=0;
    while (i < A.size() && j < B.size()) {
        if (A[i] <= B[j]) {
            C.push_back(A[i]);
            i += 1;
        } else {
            C.push_back(B[j]);
            j += 1;
        }
    }
    while (i < A.size()) {
        C.push_back(A[i]);
        i += 1;
    }
    while (j < B.size()) {
        C.push_back(B[j]);
        j += 1;
    }
    return C;
}
```

To do this, he has M **non-empty** sequences S_1, S_2, ..., S_M, and performs the following operation $S_1 = \text{merge}(S_1, S_i)$ for i from 2 to M. Thus, finally, to carry out his analysis, he wants to count the number of distinct valid initial sequences that produce the same final sequence S_1.

Two initial sequences (sequences of sequences) $S_1, S_2, \ldots, S_M$ and $S'_1, S'_2, \ldots, S'_m$ are distinct if there exists an index i $(1 \leq i \leq M)$ such that $S_i \neq S'_i$. Similarly, two sequences A and B are distinct if they differ in their number of elements or there exists an index j $(1 \leq j \leq |A_i|)$ such that $A_j \neq B_j$.

Note that the final S_1 sequence is not necessarily sorted.

Input

The first line of input contains two integers M and N $(1 \leq N, M \leq 3000)$, indicating the number of initial sequences and the size of the final sequence S_1, respectively.

The second line contains N positive integers, S_{1i} $(1 \leq S_{1i} \leq N)$, separated by spaces, representing the final sequence S_1,

Output

Print a line with a single integer modulo $10^9 + 7$, indicating the number of possible initial sequences.

Sample input 1	Sample output 1
3 6 1 2 3 4 5 6	150

Sample input 2	Sample output 2
3 8 1 3 4 2 5 2 7 8	150

Sample input 3	Sample output 3
4 8 2 4 1 4 1 2 3 2	0

Problem J – Journey To Stringland

In the mystical kingdom of Stringland, where characters and sequences hold the secrets to ancient magic, there exists a revered challenge that tests the wisdom and skill of the realm's scribes. This challenge, known as "The Quest for the Palindromic Subsequence," involves transforming ordinary sequences of letters into powerful palindromic symbols.

You, a scribe of Stringland, are given a sequence of characters S and must undertake a quest to create a palindromic subsequence of length K. Palindromic sequences are believed to hold magical properties as they read the same forward and backward. To aid in your quest, you are allowed to change any character in S to any other character. The fewer changes you make, the stronger the resulting magic.

Determine the minimum number of character changes needed for S to contain at least one palindromic subsequence of length K.

Input

The first line contains two integers N and K ($1 \leq K \leq N \leq 500$), representing the length of the string S and the size of the palindromic subsequence. The second line contains the string S, composed of lowercase English letters.

Output

Output one line with an integer, the minimum number of character changes needed for S to contain at least one palindromic subsequence of length K.

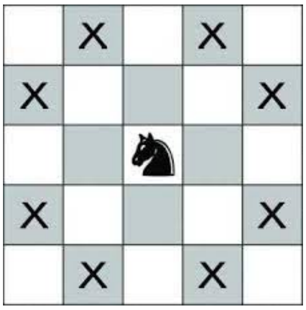
Sample input 1	Sample output 1
`3 3` `abc`	`1`

Sample input 2	Sample output 2
`10 4` `abcdcaefgj`	`0`

Sample input 3	Sample output 3
`5 2` `abcde`	`1`

Problem K – Knights In The Board

In the world of chess, the knight moves in an L-shape: two squares in one direction and then one square perpendicular, or one square in one direction and then two squares perpendicular. This unique movement allows it to jump over other pieces, reaching eight possible squares from a central position on an empty board.



Possible moves of a knight in a chess board

Given a chessboard of size $N \times N$ and K knights placed on it, determine the minimum number of knights that must be removed from the board so that no two knights are attacking each other. A knight is considered to be attacking another knight if it can move to the square occupied by the second knight in one turn based on its L-shaped movement pattern.

Input

The first line of input contains two integer numbers separated by a space N ($3 \leq N \leq 25$), and K ($1 \leq K \leq N * N$), representing the size of the board, and the number of knights in the board.

Each of the following K lines contains two integer numbers r_i, c_i, representing the i-th knight is placed on row r_i and column c_i in the board. No two knights will be places on the same position.

Output

Output a line with an integer number, the minimum number of knights that should be removed from the board so that no two knights are attacking each other.

Sample input 1	**Sample output 1**
3 4 1 1 1 2 2 1 2 2	0

Sample input 2	**Sample output 2**
5 9 3 3 1 2 2 1 1 4 2 4 4 1 5 2 4 5 5 4	2

Sample input 3	Sample output 3
3 9 1 1 2 3 3 1 3 2 3 3 1 2 1 3 2 1 2 2	4

Problem L – Lost Land of Numeralia

In the vibrant land of Numeria, where numbers are not just symbols but the essence of life itself, the annual Festival of Sums is a spectacle of mathematical prowess. This year, the festival introduces a new challenge, captivating the minds of Numerians: to craft summations that not only meet but can also exceed a specified sum S, using the orderly sequence of multiples of a chosen number P. This challenge embodies Numeria's celebration of abundance and precision, inviting participants to demonstrate their skill in navigating the realms of numbers beyond the exactness, into the realm of surplus.

As a contender in this celebrated festival, you are presented with an array of numbers from 1 to N. Your quest involves solving Q puzzles, for each puzzle you must find how many unique ways you can select two distinct positive integers i, and j such that $j - i \leq 10^3$, and sum all numbers kP where $i \leq k \leq j$ and $kP \leq N$ for a given prime number P such that the total sum is at least S. Two selections are different if any of the values for i and j differ.

Input

The first line of input contains an integer N ($1 \leq N \leq 10^9$), indicating the array of numbers in the festival contains the values 1 to N.

The following line introduces Q ($1 \leq Q \leq 10^4$), the number of puzzles to solve in the celebration.

Each of the next Q lines contains two integers S and P ($1 \leq S \leq 10^9$, $1 \leq P \leq N$), with S being the target sum and P the specified multiple.

Output

Output Q lines, where the i-th line contains the answer to the i-th puzzle of the input.

Sample input 1	**Sample output 1**
20 3 10 2 10 3 15 5	48 17 6

Sample input 2	**Sample output 2**
20 3 1 2 1 3 1 5	55 21 10