FLUID MECHANICS
First Year Exam Solutions 2013

Q1 Give answers to all of the following questions (5 marks each):

(a) A cylinder of 1m in diameter is made with material of relative density 0.5 . It is moored in fresh water by one end and the water level is 1m above the middle of the cylinder. Find the tension of the mooring cable.

Solution:

When the cylinder is submerged to the middle, the tension is zero ($\sigma = 0.5$). Additional buoyancy due to further submerging is compensated by the cable tension:

$$T = \rho\, g\, \frac{\pi\, d^2}{4}\, l = 1000\, kg/m^3 \times 9.81\, m/s^2 \times \frac{\pi \times 1^2\, m^2}{4} \times 1\, m \approx 7700\, N$$

(b) A Pitot-static tube is place in an air flow (ρ = 1.3kg/m^3). A connected manometer shows pressure difference 20mm of water. Whats is velocity of the flow?

Solution:

Using Bernoulli's equation for a stagnation point we can write:

$$P + \frac{\rho_a\, U^2}{2} = P_0 \quad \Rightarrow \quad U = \sqrt{2\, \Delta\, P/\rho_a}\, .$$

The pressure reading is in mm of water, that is $\Delta\, P = \rho_w\, g\, \Delta\, h$. Then

$$U = \sqrt{2\, \frac{\rho_w}{\rho_a}\, g\, \Delta\, h}$$

and

$$U = \sqrt{2 \times (1000/1.3) \times 9.81\, m/s^2 \times 20 \times 10^{-3}\, m} = 17.4\, m/s$$

(c) A 2mm space between two parallel plates is filled with viscous fluid. One plate is moving with velocity 1m/s. Find the mean velocity of the flow between the plates and the flow rate if plates width is 20cm.

Solution:

The velocity profile is linear, and the mean velocity is the average between velocities on the walls: $U = 0.5\, m/s$. The flow rate is

$$Q = a\, b\, U = 2 \times 10^{-3}\, m \times 0.2\, m \times 0.5\, m/s = 2 \times 10^{-4}\, m^3/s$$

(d) Water ($\mu = 10^{-3}$ Pa s) flows through a pipe of 5mm in diameter with mean velocity 0.2m/s. Can the formula $f = 16/Re$ be applied to calculate the friction factor for this flow?

Solution:

Reynolds number of the flow:

$$Re = \frac{\rho\, V\, d}{\mu} = \frac{1000\, kg/m^3 \times 0.2\, m/s \times 5 \times 10^{-3}\, m}{10^{-3}\, Pa\, s} = 1000\,.$$

$Re < 2000 \Rightarrow$ flow is laminar and we can use formula $f = 16/Re$ for calculating the friction coefficient.

(e) A lock gate is 2m high and 3m wide. It can rotate around a vertical hinge at one side. Calculate the maximal force applied to the gate by water, and maximal moment about the hinge.

Solution:

Maximal possible depth is $h = 2\, m$, the maximal pressure is $P = \rho\, g\, h$, for the triangular distribution the total load is

$$F = \frac{P\, A}{2} = \frac{1}{2}\rho\, g\, h^2\, b = \frac{1}{2}\times 1000\, kg/m^3 \times 9.81\, m/s^2 \times 2^2\, m^2 \times 3\, m = 58860\, N\,.$$

The load is applied in the middle of the width, therefore the moment about the vertical axis is:

$$M = F\, b/2 = 58860\, N \times 1.5\, m = 88290\, N\, m$$

(f) Calculate the minimal head of the pump required to pump 1m^3 of water per second to the height 10m trough a pipe with cross section area 0.1m^2. Assume constant friction coefficient $f = 0.01$ and neglect all losses in fittings.

Solution:

The pump head is the gravity head plus losses, which can be found by Darcy's equation and are minimal when the pipe is vertical:

$$H = Z + h = Z + 4\, f\, \frac{L}{d}\, \frac{Q^2}{2\, g\, A^2}$$

$$H = 10\, m + 4 \times 0.01 \times \frac{10\, m \times (1\, m^3/s)^2}{2 \times 9.81\, m/s^2\, (0.1\, m^2)^2} = 12\, m$$

Q2 A tank 900mm square in plain is filled with water and is drained via a 60mm diameter pipe, 6m in length. The pipe has two bends along its length with head loss coefficients $k = 0.6$ each, and for the entry of the pipe $k = 0.5$. The outlet of the pipe is 230mm below the level of the base of the tank (Fig. 1). The value of a friction factor $f = 0.008$ can be assumed constant.

1. What is the depth h of water in the tank when the rate of flow through the pipe is 7 liters per second?

[7 marks]

2. How fast the pressure at the bottom of the tank changes for this value of h?

[6 marks]

3. What is the value of the flow rate when the tank is half empty?

[5 marks]

4. Justify application of steady-flow equations to this problem.

[2 marks]

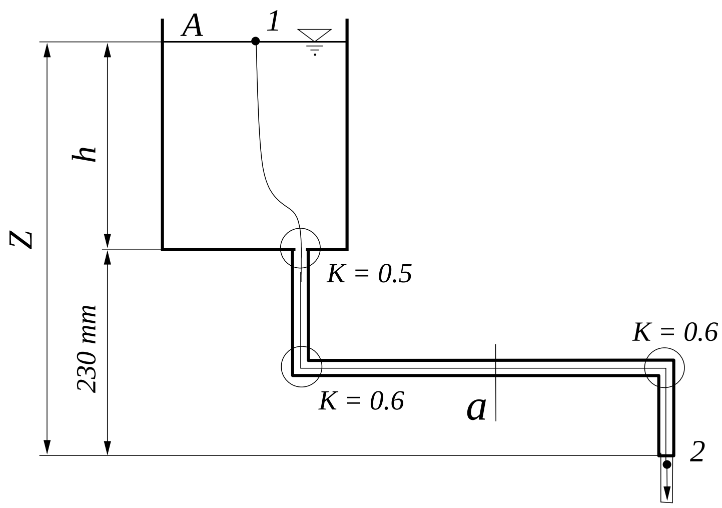

Fig. 1

Solution:

1. Steady flow energy equation along 1–2:

$$Z - \frac{Q^2}{2\,a^2\,g} = \left(4\,f\,\frac{L}{d} + K_\Sigma\right)\,\frac{Q^2}{2\,a^2\,g}\,,$$

where a is the pipe cross section area, $K_\Sigma = \sum_i K_i$ is the sum of all loss coefficients, and $Z(t)$ is the the height of the water surface in the tank above the outlet at time t. For $Z(t)$ we have

$$Z = \left(1 + K_\Sigma + 4\,f\,\frac{L}{d}\right)\,\frac{Q^2}{2\,a^2\,g},$$

and when $Q = 7\,l/s$ the value of Z is:

$$Z = (1 + 0.6 + 0.6 + 0.5 + 4 \times 0.008\,\frac{6m}{0.06m}) \times \frac{7^2 \times 10^{-6}\frac{m^6}{s^2}}{2 \times 9.81\frac{m}{s^2}}\,\frac{4^2}{\pi^2\,0.06^4 m^4} = 1.843m\,,$$

and the corresponding depth $h = 1.843 - 0.230 = 1.613m\,.$

2. The flow rate can be expressed via the rate of change of water level as:

$$Q = -A\,\frac{dh}{dt},$$

where A is the area of the water surface in the tank. Bottom pressure is $P = \rho\, g\, h$ and it changes with the rate

$$\frac{dP}{dt} = \rho\, g\, \frac{dh}{dt} = -\rho\, g\, \frac{Q}{A} = -1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times \frac{7 \times 10^{-3}\frac{m^3}{s}}{0.9^2 m^2} = -84.8\frac{Pa}{s}$$

3.

$$Z \sim Q^2 \quad \Rightarrow \quad \sqrt{\frac{Z_1}{Z_2}} = \frac{Q_1}{Q_2} \quad \Rightarrow \quad Q_2 = Q_1 \sqrt{\frac{Z_2}{Z_1}}$$

$$\frac{Z_2}{Z_1} = \frac{1.613/2 + 0.230}{1.613 + 0.230} = 0.56; \quad Q_2 = 7\, l/s \times \sqrt{0.56} = 5.24\, l/s$$

4. $a \ll A$

Q3 A liquid with $\sigma = 1.15$ flows from a 50mm diameter pipe A through an abrupt enlargement into a 100mm diameter pipe B, the two pipes being coaxial and horizontal. At some distance downstream of the enlargement in pipe B is a Pitot tube which is facing upstream and connected to one limb of a vertical U-tube manometer containing mercury of relative density 13.56. The other limb of the manometer is connected to a static pressure hole in the wall of pipe A. Neglecting frictional effects at the pipe walls, calculate the mass flow rate of the liquid when the difference in manometer levels $h = 46$mm. A head loss coefficient at an abrupt enlargement from area A_1 to area A_2 can be calculated by the formula $k = (1 - A_1/A_2)^2$.

[20 marks]

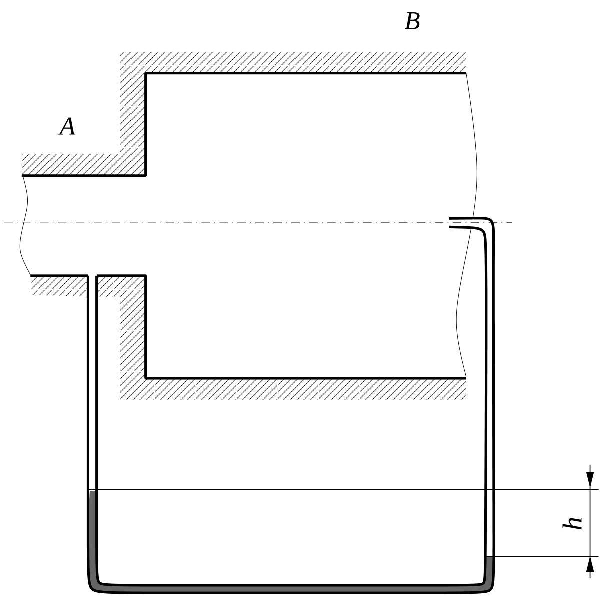

Fig. 2

Solution:

Manometer shows difference between static pressure at A and stagnation pressure at B. Condition of fluid equilibrium in the manometer gives:

$$P_A + \sigma_1 \rho g h = P_B + \frac{\rho U_B^2}{2} + \rho g h \,,$$

where ρ is the density of the liquid, and σ_1 is mercury density relative to the density of the liquid: $\sigma_1 = \sigma_{Hg}/\sigma = 13.56/1.15$. The pressure difference is:

$$P_A - P_B = \frac{\rho U_B^2}{2} + (1 - \sigma_1) \rho g h \,.$$

Head loss in a sudden expansion:

$$H_A - H_B = \frac{U_A^2}{2g} \left(1 - \frac{A_A}{A_B}\right)^2 = \frac{(U_A - U_B)^2}{2g} \,,$$

where the heads are:

$$H_A = \frac{P_A}{\rho g} + \frac{U_A^2}{2g} \quad \text{and} \quad H_B = \frac{P_B}{\rho g} + \frac{U_B^2}{2g} \,.$$

Therefore

$$\frac{P_A - P_B}{\rho g} + \frac{U_A^2 - U_B^2}{2g} = \frac{(U_A - U_B)^2}{2g} \,,$$

and substituting the value of the pressure difference we have:

$$(1 - \sigma_1) h = \frac{(U_A - U_B)^2}{2g} - \frac{U_A^2}{2g} \,.$$

After rearrangement and substitution of $U_A = Q/A_A$ and $U_B = Q/A_B$ we obtain:

$$Q^2 \left(\frac{1}{A_B^2} - \frac{2}{A_B A_A} \right) = 2 (1 - \sigma_1) g h \,,$$

and finally

$$Q^2 = 2 (\sigma_1 - 1) g h \frac{A_A}{2 A_B - A_A} A_B^2 \,;$$

$$Q = \sqrt{2 \times (\frac{13.56}{1.15} - 1) \times 9.81 \frac{m}{s^2} \times 0.046 m \times \frac{50^2}{2 \times 100^2 - 50^2} \times \frac{\pi^2 \, 0.1^4}{4^2} m^4} = 0.00926 \frac{m^3}{s} \,.$$

Mass flow rate:

$$\dot{m} = \rho Q = 1.15 \times 1000 \frac{kg}{m^3} \times 0.00926 \frac{m^3}{s} = 10.65 \frac{kg}{s} \,.$$