Elliptically Polarized Light

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Abstract

In our Capstone Project, we explored properties of elliptically polarized light, which we produced with an elliptical polarizer composed of a linear polarizer and a quarter-wave plate. Our objectives were to

1. Determine and verify the relationship between the relative angle of the linear polarizer and the quarter-wave plate of our elliptical polarizer and the eccentricity and inclination of the elliptical light produced.
2. Derive and verify Malus's Law for elliptically polarized light (i.e. an equation that gives the intensity of elliptically polarized light after passing through a linear polarizer.)

Motivation

Following the lab experiments on linear and circularly polarized light, we were interested in elliptically polarized light. Specifically, we wanted to generalize the lab about circularly polarized light by exploring the relationship between the angle of polarizers used to construct the light and the intensity of the light.

A key intermediate value in this relationship is the eccentricity of the light, or how “uncircular” it is. We were also interested in calculating this value from experiment.

Upon further research we had trouble finding an equivalent to Malus’s Law. This was something we wanted to investigate ourselves.

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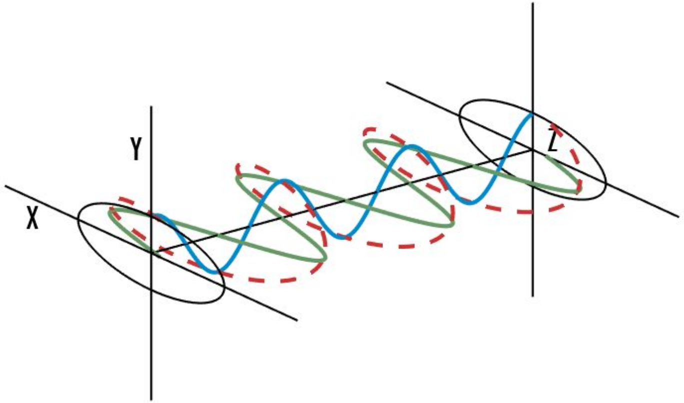
Theory



- Elliptically polarized light is light composed of two perpendicular components of unequal amplitude. Elliptical light can be produced by passing unpolarized light through a linear polarizer and then through a quarter-wave plate.

- If the relative angle between the polarization axis of the linear polarizer and the fast axis of the quarter wave plate, φ, is ≤ 45°, the inclination of the polarization ellipse, θ = φ and the eccentricity of the polarization ellipse, $e = \sqrt{1 - tan^2\varphi}$. If φ > 45°, θ = φ – 90° and $e = \sqrt{1 - cot^2\varphi}$.

- If we pass elliptically polarized light through a linear polarizer where ψ is the relative angle between the linear polarization axis and the major axis of the polarization ellipse, *e* is eccentricity, and *I* is the intensity of the unfiltered elliptical light, the intensity is given by

$$I(e,\ \psi) = \frac{I}{2-e^2}(1 - e^2 sin^2\psi)$$

Methodology Exp 1

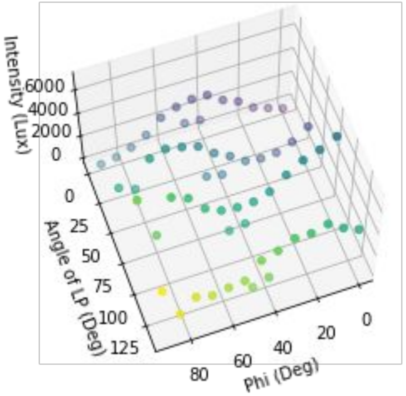
For this experiment, we set up the laser with a linear polarizer set vertically, a quarter wave plate, and a second linear polarizer with a photodetector taking the reading on the other end.

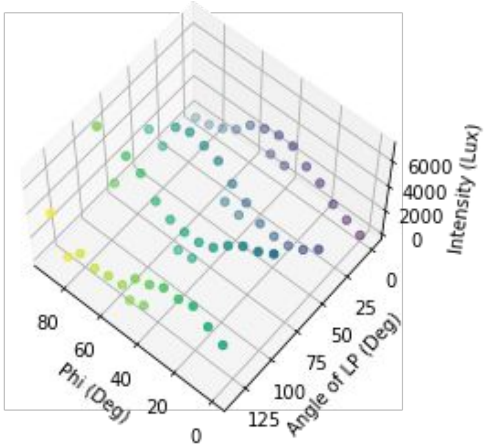
1. Determine the locations of the axes of each polarizer
2. Determine the fast axis of the quarter wave plate
3. Test the intensity of the light with the second linear polarizer at 0°, 90°, 45°, and 135° to find Stokes' parameters
4. Vary φ in order to produce different eccentricities and inclinations

$$I = I_0 + I_{90} \qquad Q = \frac{I_0 - I_{90}}{I} \qquad U = \frac{I_{45} - I_{135}}{I}$$

Data Reduction

φ (°)	α_φ (°)	I_0 (lux)	$\alpha_{I,0}$ (lux)	I_{90} (lux)	$\alpha_{I,90}$ (lux)	I_{45} (lux)	$\alpha_{I,45}$ (lux)	I_{135} (lux)	$\alpha_{I,135}$ (lux)
15	2.8	961.1	0.12	5792	0.29	1592	0.1	5202	0.3
30	2.8	2451	0.12	3903	0.18	1724	0.12	4864	0.17
45	2.8	3363	0.13	3411	0.16	3327	0.12	3780	0.089
60	2.8	2585	0.1	4239	0.17	4734	0.09	2170	0.13
75	2.8	754.2	0.054	6176	0.14	4597	0.13	2277	0.1
90	2.8	79.65	0.018	6851	0.21	2866	0.099	3893	0.16
0	2.8	39.43	0.053	6954	0.18	3063	0.097	3741	0.22
7.5	2.8	480.6	0.069	6253	0.13	2130	0.1	4364	0.12
22.5	2.8	1911	0.12	4947	0.18	1504	0.11	4831	0.13
37.5	2.8	3294	0.37	3447	0.11	2601	0.11	4194	0.15
52.5	2.8	3091	0.19	3589	0.13	4351	0.18	2319	0.15
67.5	2.8	1455	0.081	5636	0.14	5011	0.12	1910	0.13
82.5	2.8	285.6	0.071	3189	0.72	2299	0.55	1097	0.29
48.75	2.8	1446	0.3	1446	0.62	1416	0.26	1428	0.27
41.25	2.8	1301	0.35	1641	0.21	1147	0.16	1900	0.19





For Data Reduction, we calculate modified Stokes' Parameters (set of variables that fully characterize polarized light):

$$I = I_0 + I_{90} \qquad Q = \frac{I_0 - I_{90}}{I} \qquad U = \frac{I_{45} - I_{135}}{I}$$

We can use these parameters to determine the eccentricity of the light, e, and its inclination, θ.

Analysis: Eccentricity

From the Stokes' Parameters, we can calculate *e* using $e = \dfrac{2\sqrt{Q^2+U^2}}{1+\sqrt{Q^2+U^2}}$

We can also calculate our expected value for *e* using the relation from theory. If φ ≤ 45°,

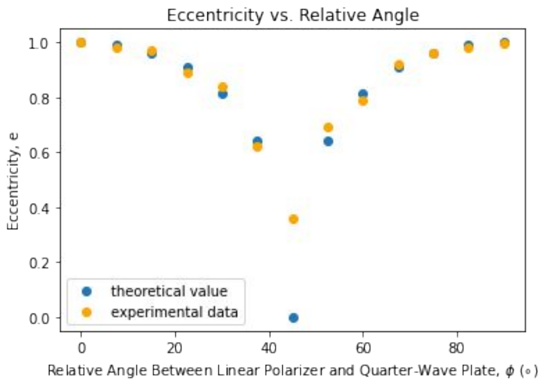
$e = \sqrt{1-\tan^2\phi}$ Otherwise, $e = \sqrt{1-\cot^2\phi}$

φ (°)	$e_{experiment}$	$\alpha_{e,\ experiment}$	e_{theory}	$\alpha_{e,\ theory}$	Agreement?
15	0.973709	3.10E-05	0.963	0.015	TRUE
30	0.842345	3.10E-05	0.816	0.047	TRUE
45	0.356532	4.00E-05	0.00E+00	N/A	N/A
60	0.788588	2.60E-05	0.816	0.047	TRUE
75	0.961229	1.60E-05	0.963	0.015	TRUE
90	0.9992845	2.20E-05	1	N/A	N/A
0	1.000596	1.90E-05	1	N/A	N/A
7.5	0.981092	1.50E-05	0.9913	0.0067	TRUE
22.5	0.892854	2.30E-05	0.91	0.026	TRUE
37.5	0.621768	4.10E-05	0.641	0.094	TRUE
52.5	0.693112	4.30E-05	0.641	0.094	TRUE
67.5	0.92248	1.80E-05	0.91	0.026	TRUE
82.5	0.97919	0.00015	0.9913	0.0067	TRUE

Testing the Data:

10 of the 13 experimental data points agree with their accepted values. The other three points cannot be tested formally, but they seem to fit with what we expect.

Performing a R^2 test to the full data set, we get an R^2 value of 0.856 (OK). If we exclude the outlier point at φ = 45° degrees, we get an R^2 value of 0.972 (much better).



In general, our data confirms our theoretical expectations.

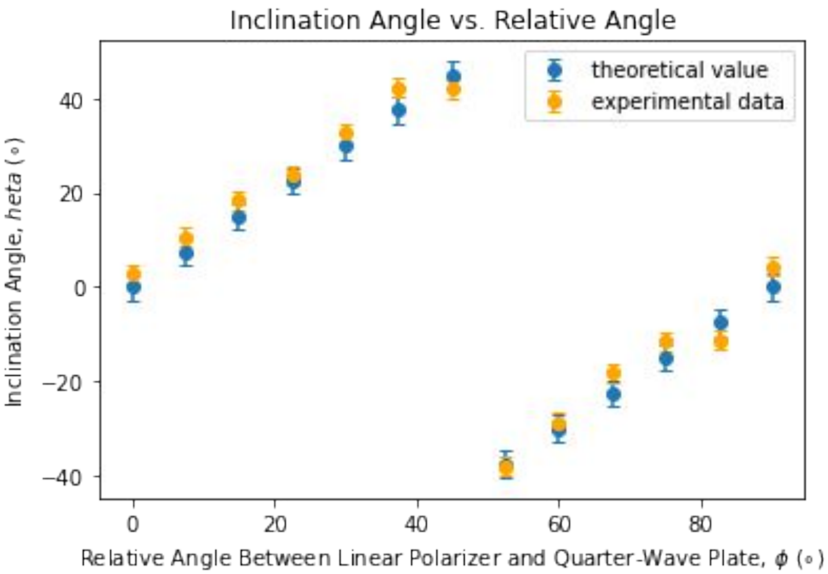
Analysis: Inclination

From the Stokes' Parameters, we can calculate θ using

$$\theta = arctan\left(\frac{U}{Q}\right)$$

We can also calculate our expected value for θ using the relation from theory. If $\varphi \leq 45°$, $\theta = \varphi$. If not, $\theta = \varphi - 90°$.

φ (°)	$\theta_{experiment}$ (°)	$\alpha_{\theta,\ experiment}$ (°)	θ_{theory} (°)	$\alpha_{\theta,\ theory}$ (°)	Agreement?
15	18.4	2	15	2.8	TRUE
30	32.6	2	30	2.8	TRUE
45	42	2	45	2.8	TRUE
60	-28.6	2	-30	2.8	TRUE
75	-11.6	2	-15	2.8	TRUE
90	4.3	2	0	2.8	TRUE
0	2.8	2	0	2.8	TRUE
7.5	10.6	2	7.5	2.8	TRUE
22.5	23.8	2	22.5	2.8	TRUE
37.5	42.3	2	37.5	2.8	TRUE
52.5	-38.1	2	-37.5	2.8	TRUE
67.5	-18.3	2	-22.5	2.8	TRUE
82.5	-11.2	2	-7.5	2.8	TRUE

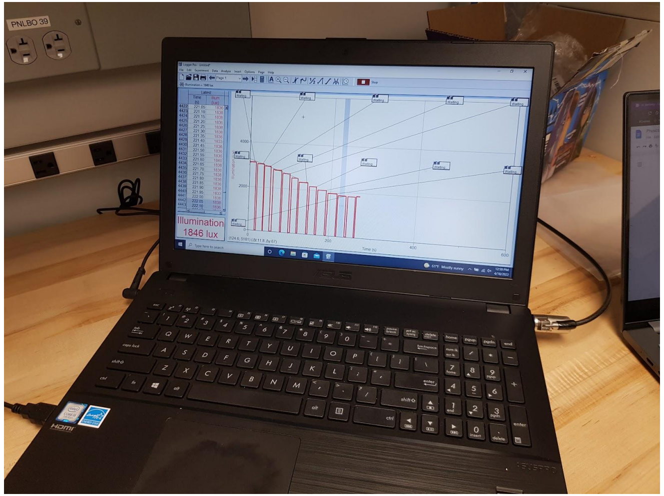


All 13 data points agree with their expected values, confirming our theoretical expectations.

Methodology: Exp 2

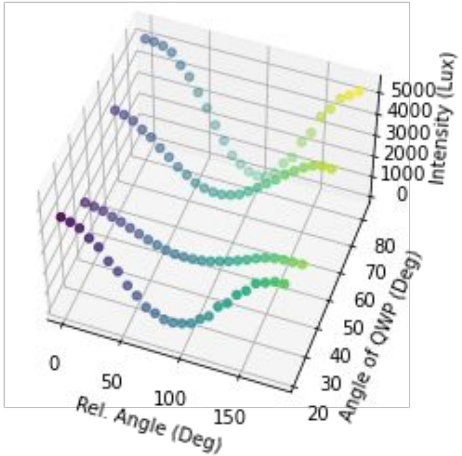
Our lab bench setup for experiment 2 was the same as experiment 1.

1. Calibrate all the polarizers and quarter wave plates.
2. Measure the intensity against several relative angles between the second linear polarizer and the major axis of the elliptically polarized light (psi) to confirm Malus’ Law.
3. Vary phi in order to produce different eccentricities and inclinations.

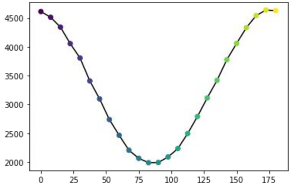


Experiment 2 Raw Data

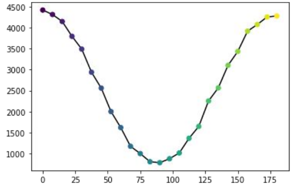
psi (Deg)	psi_err (Deg)	I_60 (Lux)	I_60_err (Lux)	I_82_5 (Lux)	I_82_5_err (Lux)	I_22_5 (Lux)	I_22_5_err (Lux)	I_37_5 (Lux)	I_37_5_err (Lux)
0	2.8	4612	0.094	5217	0.16	4422	0.21	3115	0.28
7.5	2.8	4512	0.14	5205	0.17	4316	0.18	3048	0.14
15	2.8	4341	0.094	5114	0.25	4149	0.16	2960	0.1
22.5	2.8	4055	0.077	4923	0.2	3795	0.25	2802	0.14
30	2.8	3805	0.1	4440	0.18	3490	0.2	2674	0.13
37.5	2.8	3407	0.077	4015	0.18	2938	0.22	2500	0.094
45	2.8	3095	0.094	3391	0.14	2559	0.14	2326	0.08
52.5	2.8	2736	0.075	2765	0.12	1999	0.11	2166	0.11
60	2.8	2463	0.07	2052	0.17	1617	0.1	2030	0.086
67.5	2.8	2203	0.068	1405	0.13	1170	0.082	1914	0.099
75	2.8	2063	0.087	799.5	0.075	996.1	0.1	1836	0.12
82.5	2.8	1987	0.07	465.4	0.12	801.2	0.065	1822	0.11
90	2.8	1990	0.093	206	0.063	778.9	0.085	1847	0.1
97.5	2.8	2087	0.083	108.7	0.063	873.4	0.078	1897	0.14
105	2.8	2233	0.086	211.9	0.056	1011	0.12	1982	0.17
112.5	2.8	2493	0.091	502.9	0.066	1361	0.13	2115	0.12
120	2.8	2789	0.1	1090	0.1	1645	0.14	2235	0.1
127.5	2.8	3112	0.074	1497	0.13	2252	0.088	2400	0.22
135	2.8	3417	0.14	2357	0.1	2565	0.11	2565	0.083
142.5	2.8	3775	0.13	2923	0.13	3103	0.17	2754	0.088
150	2.8	4058	0.06	3799	0.13	3434	0.16	2910	0.12
157.5	2.8	4332	0.084	4264	0.12	3917	0.13	3009	0.11
165	2.8	4542	0.084	4847	0.14	4077	0.16	3068	0.11
172.5	2.8	4638	0.088	5107	0.16	4257	0.15	3081	0.13
180	2.8	4628	0.071	5397	0.12	4280	0.12	3108	0.09



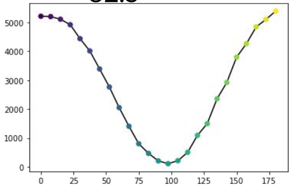
60.0°



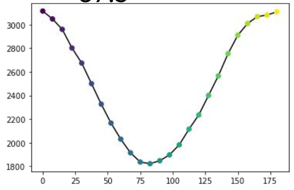
22.5°



82.5°



37.5°



An immediate visual check confirms that the intensity of the light follows a sinusoidal pattern in relation to the relative angle between the linear polarizer and the major axis of the elliptically polarized light.

Data fitting: Intensity

We focus on phi = 60° for the cleanest data

Fit to $I(e, \psi) = \frac{I}{2-e^2}(1 - e^2 sin^2\psi) \Rightarrow m * sin^2(\psi + p) + b$

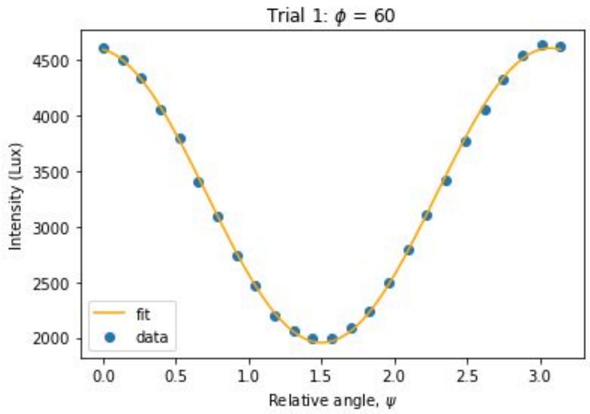
where m, p, and b are fitted parameters

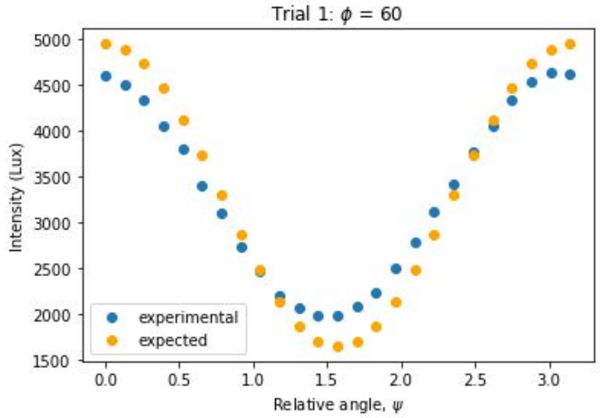
m, *b* are dependent on I_0 and *e* (dependent on *phi*)

p is a correcting parameter for the angle alignment

Our data fits very well to the Malus' Law

The strictly expected values seem to have an angle shift and a wider range of intensity magnitudes

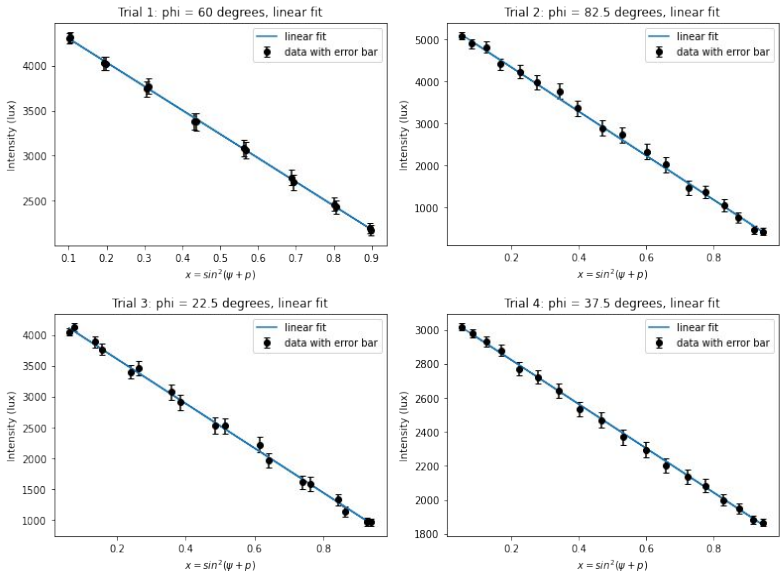




Data Fitting: Eccentricity

First fit $x = \sin^2(\psi + p)$ to $I(e,\ \psi) = \frac{I}{2-e^2}(1 - e^2 \sin^2 \psi)$
to a linear model since $y = I$ is a linear function of x.

Note: the correcting angle shift, p, is added to measured ψ



Solving our fitted linear parameters gives us the following eccentricities

Points with arbitrarily low errors were removed from the weighted fit

φ Angle (degrees)	p Phase Shift (degrees)	Calculated Eccentricity	Expected Eccentricity	Reduced Chi-Squared	Agreement Test (e)	I_{exp} (lux)	I_{the} (lux)	Agreement Test (I)
60	3.94	0.762±0.0089	0.8165±0.047	0.0654	Pass	6481±91	6539.76±0.13	Pass
82.5	-5.86	0.9872±0.0095	0.9913±0.0067	0.309	Pass	5430±130	5360.76±0.17	Pass
22.5	0.77	0.9127±0.0086	0.9102±0.026	0.324	Pass	5002±5002	5138.66±0.23	Pass
37.5	5.7	0.6491±0.0055	0.6413±0.094	0.136	Pass	4859±4859	4899.76±0.30	Pass

Conclusion

- In experiment 1, we find that observed eccentricity and inclination values closely compare to expected values. Excluding outliers, we find that over 97% of the variance in expected eccentricity values can be explained by values obtained from experiment. Our outlier point came from the region of angles near $\varphi=45°$, which is where the derivative is greatest. Here, any small error in φ becomes a much larger error in eccentricity.

- In experiment 2, we find that observed data fits very well with an equation in the form of Malus's law. However, we again find both an angle difference with the exact expected values for intensity. This is likely due to an imperfect calibration process. We had a similar problem when calculating eccentricity.

Future Studies

- Our **recommendation** for future students attempting this project would be to take great care to make sure the linear polarizers and the quarter-wave plates are calibrated well. We also advise avoiding “poorly-behaved” points in the analysis to keep the math clean.

- We could **improve** the project by taking more data (i.e. testing negative angles for Experiment 1, or trying more eccentricities for Experiment 2). We could **expand** the project by also testing the relationship between φ and the handedness of the elliptically polarized light.