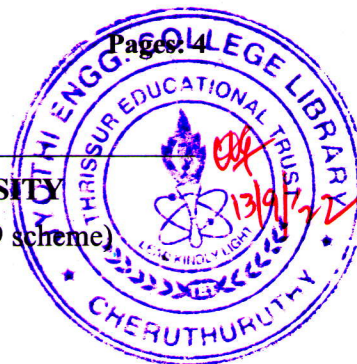


Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
 Fourth Semester B.Tech Degree Examination June 2022 (2019 scheme)



Course Code: MAT206

Course Name: GRAPH THEORY

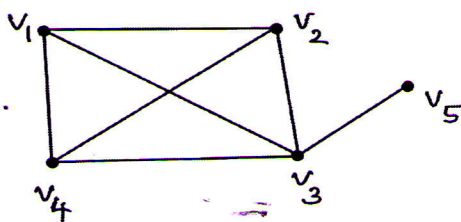
Max. Marks: 100

Duration: 3 Hours

**PART A***(Answer all questions; each question carries 3 marks)*

Marks

- |   |  |   |
|---|--|---|
| 1 | Prove that the maximum number of edges in a simple graph with $n$ vertices is $\frac{n(n-1)}{2}$ .                   | 3 |
| 2 | Define walk, path and circuit with examples.   | 3 |
| 3 | Draw a graph which is Eulerian but not Hamiltonian   | 3 |
| 4 | Distinguish between strongly connected digraphs and weakly connected graphs with examples.                           | 3 |
| 5 | Prove that there is one and only one path between every pair of vertices in a tree.                                  | 3 |
| 6 | Draw all unlabelled trees with 5 vertices.   | 3 |
| 7 | Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in $G$ . | 3 |
| 8 | Define planar graph and non-planar graph with examples.  | 3 |
| 9 | Write the adjacency matrix for the following graph.  | 3 |

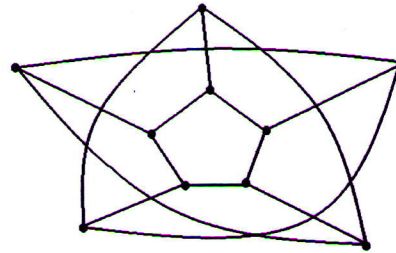
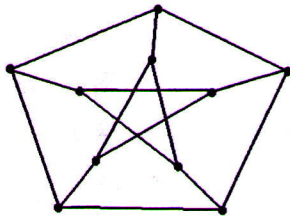


- |    |   |   |
|----|---|---|
| 10 | Prove that the chromatic polynomial of a complete graph with 4 vertices is $\lambda(\lambda - 1)(\lambda - 2)(\lambda - 3)$ . | 3 |
|----|---|---|

**PART B***(Answer one full question from each module, each question carries 14 marks)***Module -1**

- |       |   |   |
|-------|---|---|
| 11 a) | Prove that the number of vertices of odd degree in a graph is always even | 7 |
|-------|---|---|

- b) If a connected graph  $G$  is decomposed into two subgraphs  $g_1$  and  $g_2$ , then prove that there must be at least one vertex common between  $g_1$  and  $g_2$  7
- 12 a) Determine whether the following graphs are isomorphic or not. 7



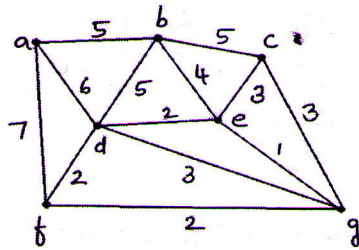
- b) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. 7

#### Module -2

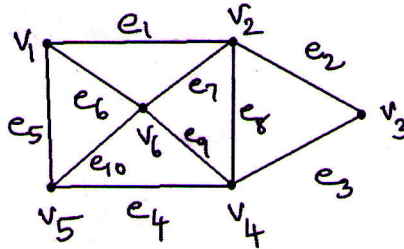
- 13 a) In a complete graph with  $n$  vertices, prove that there are  $\frac{n-1}{2}$  edge-disjoint Hamiltonian circuits, if  $n$  is an odd number  $\geq 3$ . 7
- b) 1) For a binary relation "is greater than" on the set  $X = \{3, 4, 7, 5, 8\}$  7
- Draw the digraph representing the above relation
  - Write its relation matrix
- 2) Define equivalence digraph with an example
- 14 a) Prove that a connected graph  $G$  is an Euler graph if and only if all vertices of  $G$  are of even degree. 7
- b) Define Hamiltonian circuit and Hamiltonian path. Give an example for each. 7
- Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit.

#### Module -3

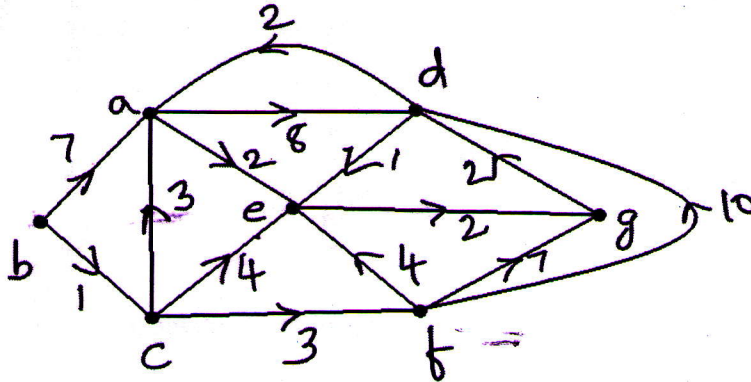
- 15 a) Prove that every tree has either one or two centers. 7
- b) Apply Kruskal's algorithm to find the minimal spanning tree for the following weighted graph. 7



- 16 a) For any spanning tree of a connected graph with  $n$  vertices and  $e$  edges, prove that there are  $n-1$  tree branches and  $e-n+1$  chords. For the following graph find two spanning trees and hence show that an edge that is a branch of one spanning tree can be a chord with respect to another spanning tree of same graph. 7

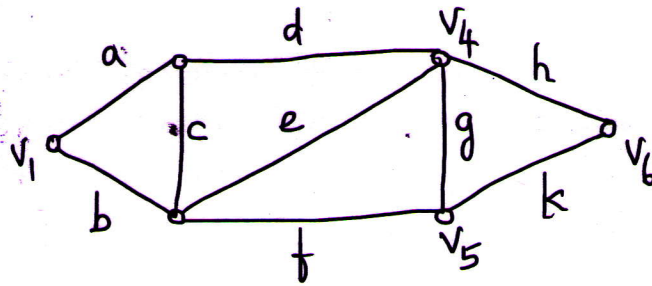


- b) Use Dijkstra's algorithm to find the shortest path for the following weighted digraph and find the shortest distance from vertex a to other vertices. 7

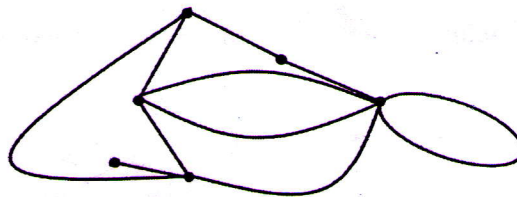


#### Module -4

- 17 a) Illustrate the statement: "The ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets", in the following graph. 7

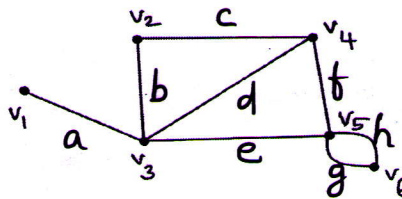


- b) Define edge connectivity, vertex connectivity separable and non-separable graph. 7  
Give an example for each.
- 18 a) Prove that the complete graph on 5 vertices is non-planar 7  
b) Draw the geometric dual of the following graph 7



## Module -5

- 19 a) For the following graph find the 7  
i. Incidence matrix  
ii. Path matrix between  $v_2$  and  $v_5$   
iii. Circuit matrix



- b) Draw a connected graph and show that the rank of its incidence matrix is one less than the number of vertices. 7
- 20 a) Prove that every tree with two or more vertices is 2-chromatic 7  
b) Prove that a covering  $g$  of a graph is minimal if and only if  $g$  contains no path of length three or more. 7

\*\*\*