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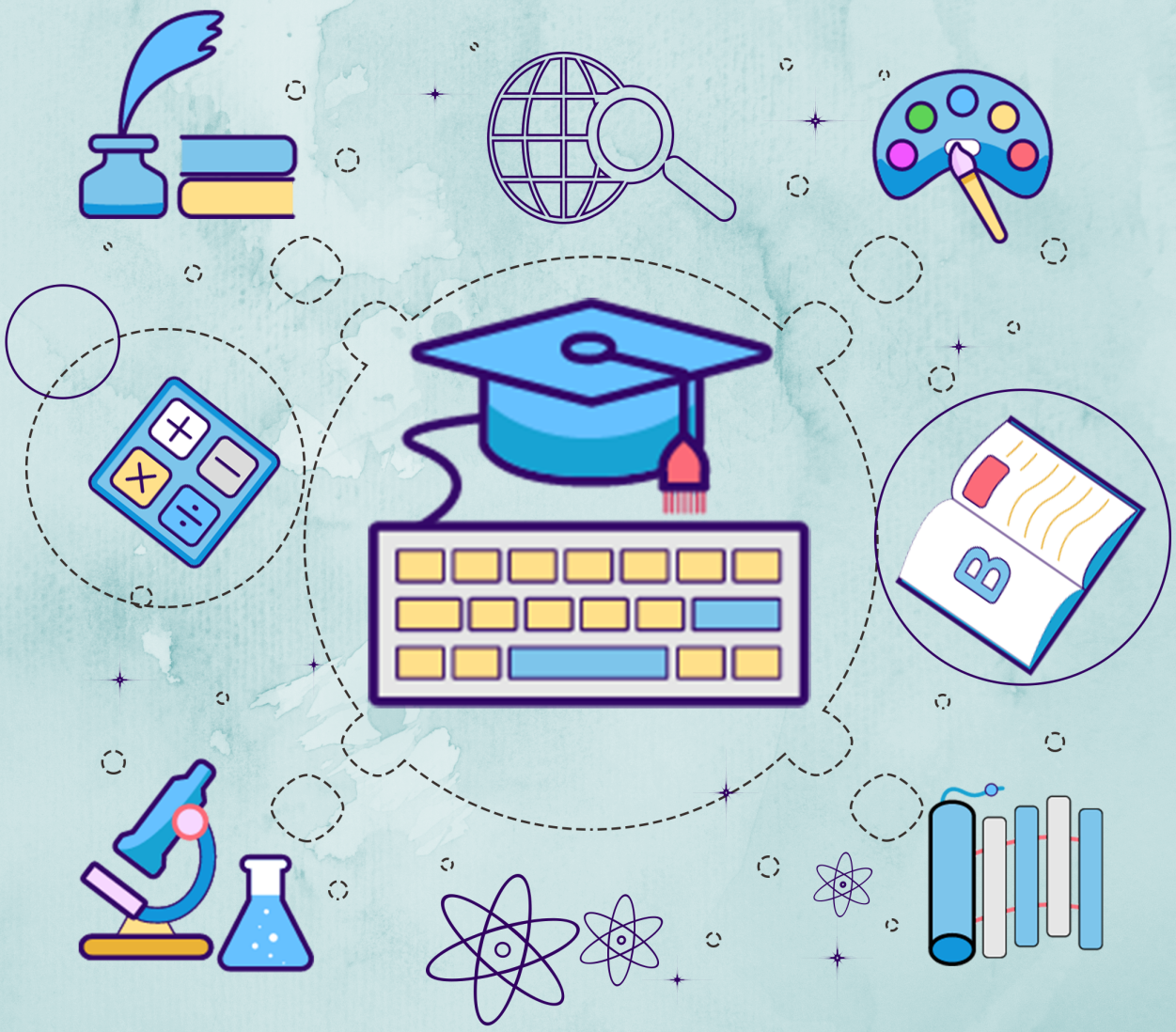

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GRAPH THEORY (MAT206)

Module 1

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Module 1

1. Differentiate a walk, path and circuit in a graph.

   Soln :

   Walk : A walk in a graph G is a sequence w = v1e1v2e2…. Vk-1ekvk whose terms alternate b/w vertices (can repeat) and edges (distinct).

   Path : A path is a walk such that all of the vertices and edges are distinct.

   Circuit : A circuit is a path in which initial and final vertices are the same.

   Go through class notes for examples and further details

2. Is it possible to construct a graph with 12 vertices such that two of the vertices have degree 3 and the remaining vertices have degree 4? Justify

   Soln : We have sum of degree of all vertices is twice the number of edges.

$$\deg(V_i) = 2e$$

   No. of. Vertices, n= 12

   2 Vertices have degree = 3

   10 Vertices have degree = 4

   2*3+10*4 = 2e

   6+40 = 2e

   46 = 2e

   No of Edges, e = 23

   Therefore it is possible to construct a graph.

3. Prove that a simple graph with n vertices must be connected, if it has more than (n-1)(n-2)/2 edges.

   Soln: We know that a simple graph with n vertices and k connected components has atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

   The value of $\frac{(n-k)(n-k+1)}{2}$ decreases when k becomes larger.

   If a simple graph with n vertices is not connected, it will contain at least 2 connected components. Therefore, the value of k in the above expression is,k ≥ 2. Then, there are at most (n − 2)(n − 2 + 1)/2 edges in the graph, which contradicts to the condition that the graph has more than (n − 1)(n − 2)/2 edges.

   Therefore, the graph is connected.

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4. Prove the statement: If a graph (connected or disconnected) has exactly two odd degrees, then there must be a path joining these two vertices.

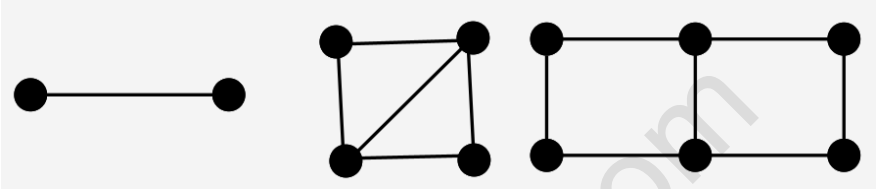
Soln:

- By handshaking lemma, sum of node/vertex degrees is even.

  That is Σdeg=2e=Σodd+Σeven

  From here you have Σodd=2e−Σeven.

  From here you can see that sum of odd nodes is even, that can happen only if there are a couple (or some even number) of odd nodes (and there is a path to join them if graph is connected).



- Let G be a disconnected finite graph with exactly two vertices, u and v, with odd degrees and two components. Suppose that u and v belong to different components of G.

  Then either of the two possibilities given below must hold such that number of edges in G is a positive integer.

  1) There exists at least one other vertex with odd degree in each component of G such that the number of edges in each component is a positive integer. A contradiction with our assumption that G has exactly two vertices with odd degree.

  2) There exists an edge joining the two components. Another contradiction, as we have said previously that G is disconnected.

5. Construct a simple graph of 12 vertices with two of them having degree 1, three having degree 3 and the remaining seven having degree 10.

Soln : We have sum of degree of all vertices is twice the number of edges.

$$\deg(V_i) = 2e$$

No. of. Vertices, n= 12

2 Vertices have degree = 1

3 Vertices have degree = 3

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7 Vertices have degree = 10

2*1+3*3+7*10 = 2e

2+9+70 = 2e

81 = 2e

No of Edges, e = 40.5

Therefore it is not possible to construct a graph.

6. What is the largest number of vertices in a graph with 35 edges, if all vertices are of degree at least 3?

Soln :

The Handshaking theorem states that,

$$\sum \deg(V_i) = 2e$$

Which means, Each edge contributes twice to the sum of the degrees of all vertices.

Sum of degree of vertices = 2 ×Edges

Now, the graph has 35 Edges.

∴ 2×Edges=2×35=70 , is the Maximum Sum of degree of the vertices.

Assume, the total number of vertices will be n.

Condition given is "All vertices are of degree at least 3"

Sum of the degree of vertices = 3×n [∵ All vertices are of at least degree 3]

∴ Minimum sum of degree of vertices will be 3n which will be less than or equal to 70 because 70 is the Maximum Sum of degree of vertices.

∴ 3×n ≤ 2×(35)

Or, n ≤ 70/3

Or, n ≤ 23.3

∴ Largest possible number of vertices in the graph must be 23 as we can't take the fractions & will take only the whole number.

7. Prove that for any simple graph with at least two vertices have two vertices of the same degree.

Soln: Let G be any finite simple graph with more than one vertex and |VG| = n ≥ 2.

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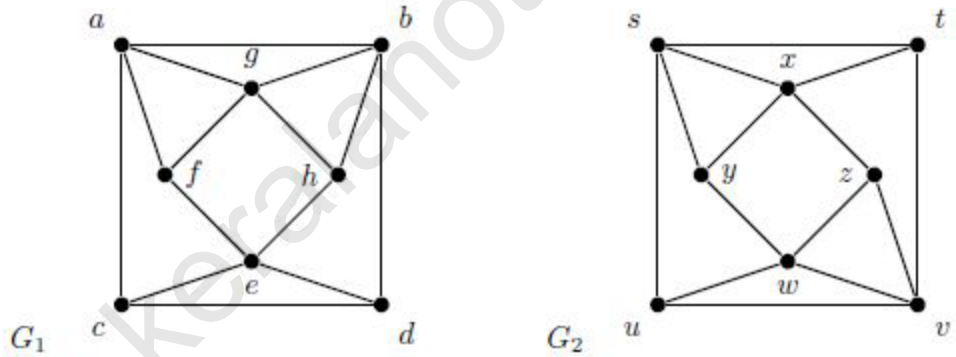
First, we notice that the maximal degree of any vertex in G is less than equal n − 1. Also, if our graph G is not connected, then the maximal degree is strictly less than n − 1.

Case 1: Assume that G is connected. We cannot have a vertex of degree 0 in G, so the set of vertex degrees is a subset of S = {1, 2, · · · , n − 1}. Since the graph G has n vertices, by pigeon-hole principle we can find two vertices of the same degree in G.

Case 2: Assume that G is not connected. G has no vertex of degree n − 1, so the set of vertex degrees is a subset of S ′ = {0, 1, 2, · · · , n − 2}. By pigeon-hole principle again, we can find two vertices of the same degree in G.

If there are n people who can shake hands with one another (where n > 1), the pigeonhole principle shows that **there is always a pair of people who will shake hands with the same number of people**.

8. Determine whether the following graphs ***G1 = (V1, E1)*** and ***G2 = (V2, E2)*** are isomorphic or not. Give justification.



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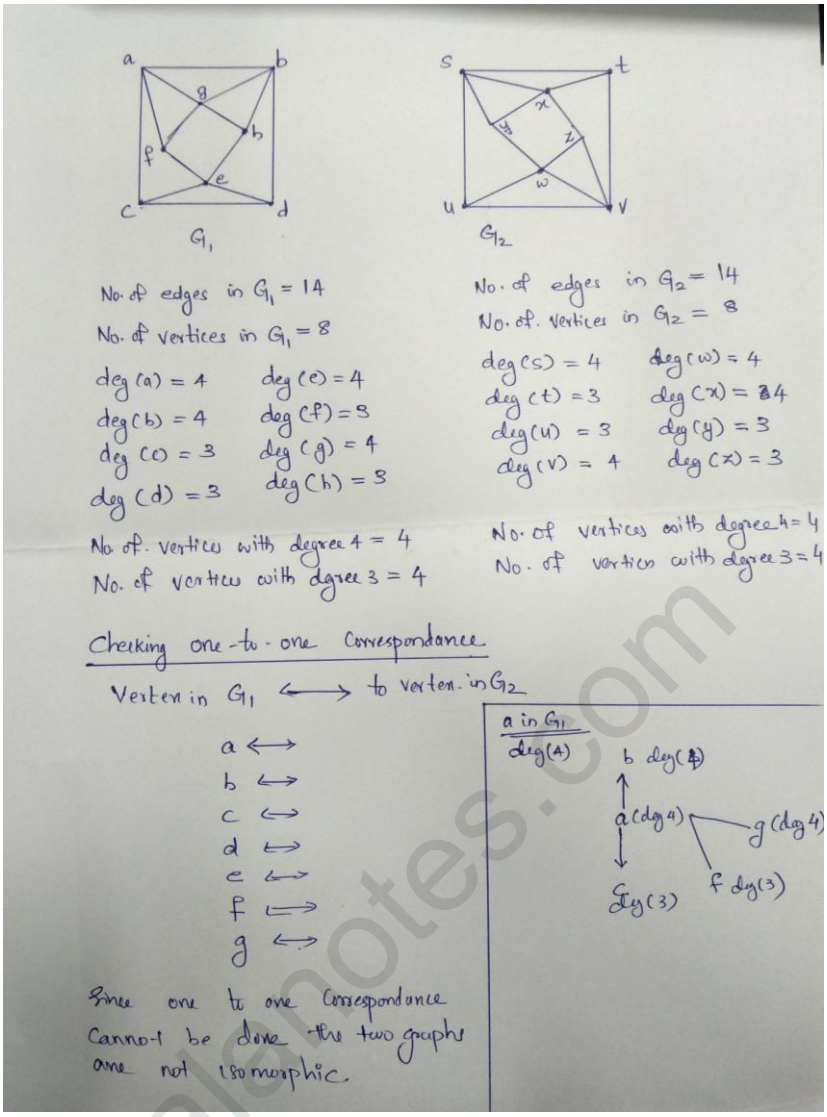

9. Prove that a simple graph with ***n*** vertices and ***k*** components can have at most ***(n-k) (n-k+1)/2*** edges.

   Soln

- Let the no. of vertices in each component be named as n1,n2,.....nk
- Then n1+ n2+ n3+ ..... + nk = n ∀ni≥1
- The proof is based on the algebraic inequality

$$\sum_{i=1}^{k} n_i^2 \leq n^2 - (k-1)(2n-k)$$

- The total no. of edges possible in the 1st component is

$$\frac{n_1(n_1-1)}{2}$$

- The total no. of edges possible in the 2nd component is

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$$\frac{n_2(n_2-1)}{2}$$

- Similarly in the kth component is

$$\frac{n_k(n_k-1)}{2}$$

- Then the maximum no. of edges possible in G is the sum of maximum no. of edges possible in each component

$$E_{max} = \frac{n_1(n_1-1)}{2} + \frac{n_2(n_2-1)}{2} + \ldots + \frac{n_k(n_k-1)}{2}$$

$$= \frac{1}{2}\sum_{i=1}^{k} n_i(n_i-1)$$

$$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - n_i$$

$$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - \frac{1}{2}\sum_{i=1}^{k} n_i$$

$$\leq \frac{1}{2}(n^2 - (k-1)(2n-k)) - \frac{n}{2}$$

$$\leq \frac{1}{2}(n^2 - (2nk - k^2 - 2n + k)) - \frac{n}{2}$$

$$\leq \frac{1}{2}(n-k)^2 - (k-n)$$

$$\leq \frac{1}{2}(n-k)(n-k+1)$$

$$\leq \frac{(n-k)(n-k+1)}{2}$$

E_{max} can be atmost$(n-k)(n-k+1)/2$

Hence the proof.

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