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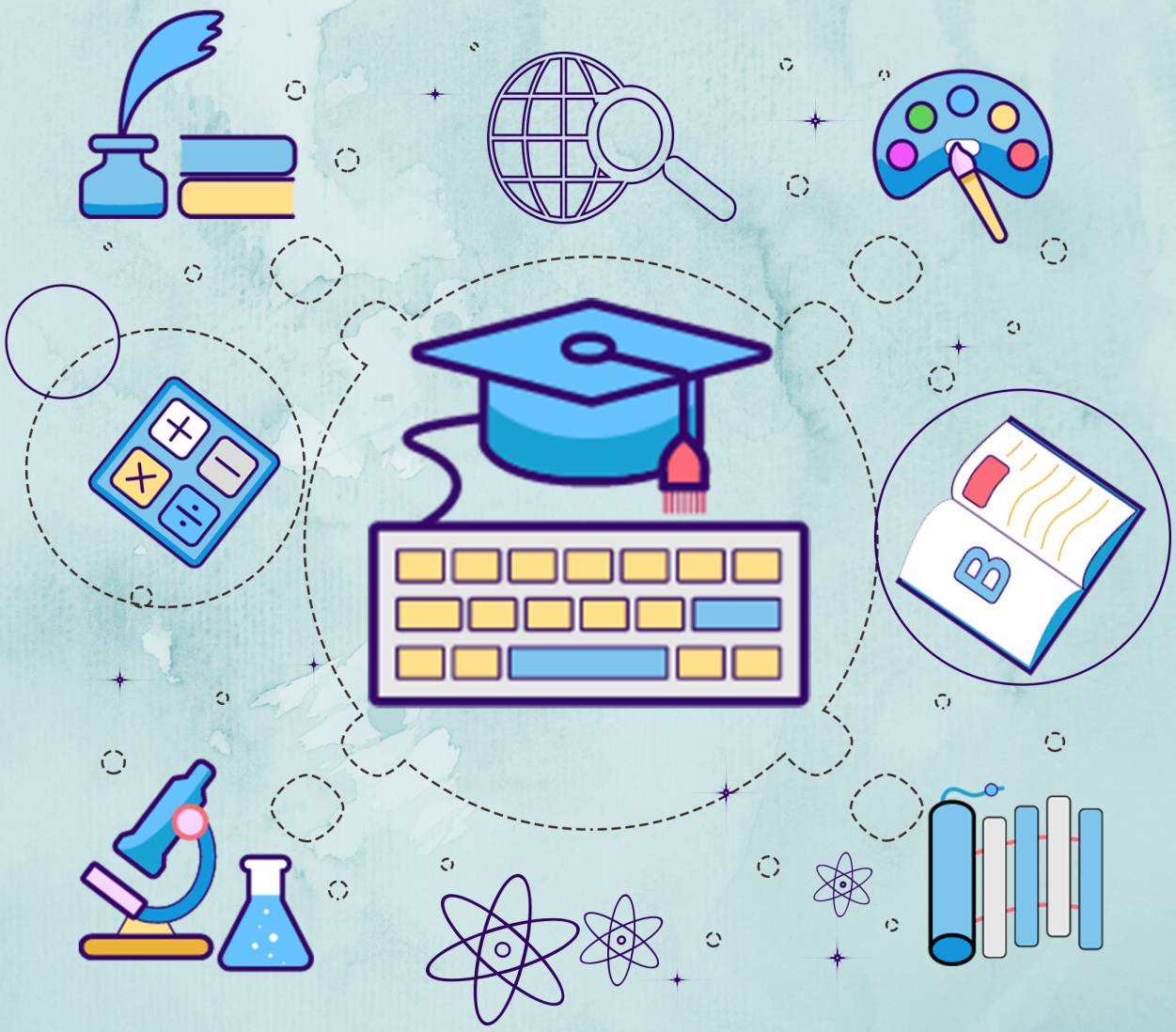

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# KTU S4 CSE QUESTION BANK

# GRAPH THEORY (MAT206)

# Module 2

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# Module 2

## Eulerian and Hamiltonian graphs :

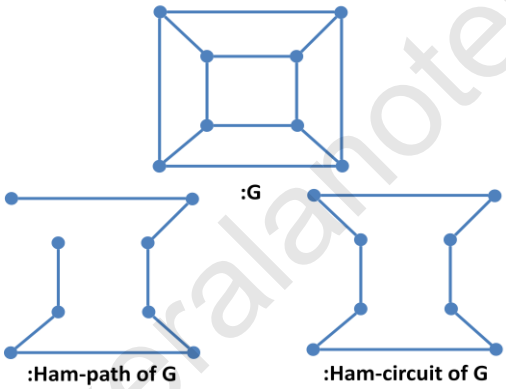
Euler graphs, Operations on graphs, Hamiltonian paths and circuits, Travelling salesman problem. Directed graphs – types of digraphs, Digraphs and binary relation, Directed paths, Fleury's algorithm.

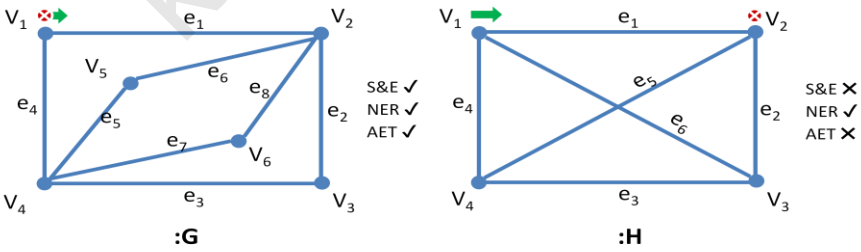
1. Define Hamiltonian circuit and Euler graph. Give one example for each.

   Soln : A **Hamiltonian circuit**: A circuit in a connected graph that traverses all its vertices exactly once.

   **Euler Graph** is a graph that consists of an Euler line

   - An Euler line: A closed walk that contains all the edges of the graph exactly once
     - Closed - starting and ending at same vertex (S&E)
     - Walk - no edge repetition, vertex may repeat (NER)
     - Euler - all edges traversed exactly once (AET)




- Graph G is an Euler graph as we can get an Euler line,
- $V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_7\ V_6\ e_8\ V_2\ e_6\ V_5\ e_5\ V_4\ e_4\ V_1$
- Note that starting from any vertex, we can get an Euler line
- But, Graph H is not an Euler graph, as from none of the vertices, we are able to get an Euler line



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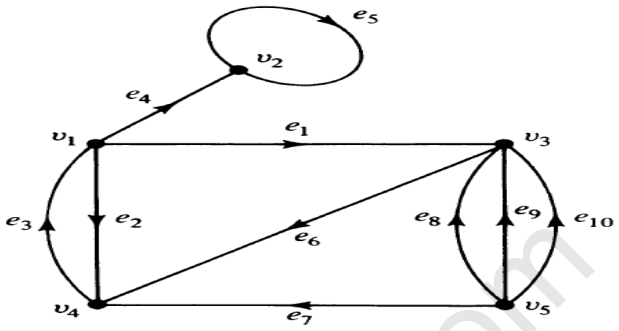
2. Define directed graphs. Differentiate between symmetric digraphs and asymmetric digraphs.

   Soln:

   A directed graph (digraph) G consist of a set of vertices V = { v1,v2,....}, a set of Edges E = { e1,e2....} and a mapping that maps every edge onto an ordered pair of vertices ($v_i$, $v_j$).




- Asymmetric Digraphs : Digraphs that has at most one directed edge between a pair of vertices, but are allowed to have self-loops are called Asymmetric Digraphs or Anti symmetric.
- Symmetric Digraphs : Digraphs in which for every edge (a,b) (i.e, from vertex a to b) there is also an edge (b,a).

3. Prove that a connected graph G is an Euler graph if all vertices of G are of even degree.

   Soln :

   An Eulerian circuit is a traversal of all the edges of a simple graph once and only once, staring at one vertex and ending at the same vertex.

   You can repeat vertices as many times as you want, but you can never repeat an edge once it is traversed.

   The degree of a vertex is the number of edges incident with that vertex.

   So let G be a graph that has an Eulerian circuit. Every time we arrive at a vertex during our traversal of G, we enter via one edge and exit via another. Thus there must be an even number of edges at every vertex. Therefore, every vertex of G has even degree.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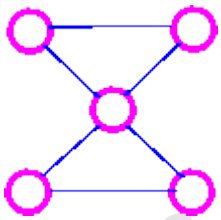
4. Prove that a graph G of n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices $Vi$, $Vj$ in G satisfies the condition $d(Vi) + d(Vj) = n - 1$.

   Soln :

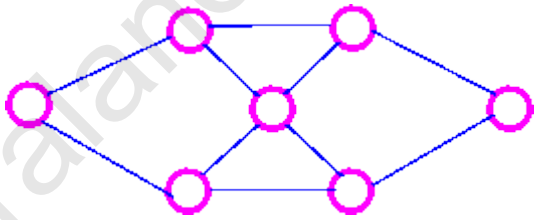
5. Define a Euler graph. Give an example of Eulerian graph which is not Hamiltonian.

   Soln : **Euler Graph** is a graph that consists of an Euler line

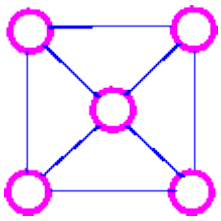
   - An Euler line: A closed walk that contains all the edges of the graph exactly once



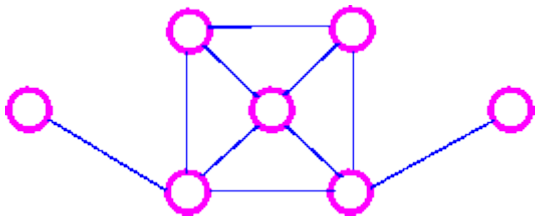
This graph is Eulerian, but **NOT** Hamiltonian.



This graph is **BOTH** Eulerian and Hamiltonian.



This graph is an Hamiltionian, but **NOT** Eulerian.



This graph is **NEITHER** Eulerian **NOR** Hamiltionian

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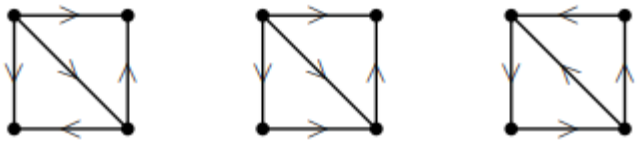
6. Give an example of a strongly connected simple digraph without a directed Hamiltonian path.

   Sol: A directed path in a digraph D is called a Hamilton directed path if it contains all the vertices of D.

   A directed cycle in D is called a Hamilton directed cycle if contains all the vertices of D.

   A directed graph D is called a Hamilton digraph if it contains a directed Hamilton cylcle.



$D_1$ $D_2$ $D_3$

- ✓ D1 contains no directed Hamilton path.
- ✓ D2 contains a directed Hamilton path but contains no directed Hamilton cycle.
- ✓ D3 contains a directed Hamilton cycle and hence it is a directed Hamilton graph.

7. Let $S$ be a set of 5 elements. Construct a graph $G$ whose vertices are subsets of $S$ of size 2 and two such subsets are adjacent in $G$ if they are disjoint.
   i. Draw the graph $G$.
   ii. How many edges must be added to $G$ in order for $G$ to have a Hamiltonian cycle?
8. Let $G$ be a graph with exactly two connected components, both being Eulerian. What is the minimum number of edges that need to be added to $G$ to obtain an Eulerian graph?
9. Show that a $k$-connected graph with no Hamiltonian cycle has an independent set of size $k + 1$.
10. i. Let $G$ be a graph that has exactly two connected components, both being Hamiltonian graphs. Find the minimum number of edges that one needs to add to $G$ to obtain a Hamiltonian graph.

    ii. For which values of $n$ the graph $Q_n$ (hyper-cube on $n$ vertices) is Eulerian.



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11. Prove that in a complete graph with ***n*** vertices there are **(n-1)/2** edge disjoint Hamiltonian circuits and ***n >= 3***

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