Reg No.:_____

Name:

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSETY

Fourth Semester B. Tech Degree Examination July 2021 (2019 Scher

CHERUFH!

Course Code: MAT206
Course Name: GRAPH THEORY

PART A

Max. Marks: 100

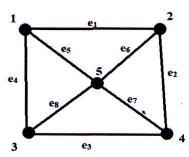
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Duration: 3 Hours

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(Answer all questions; each question carries 3 marks) What is the maximum number of edges in a simple graph with n vertices? 3 Justify your answer.

- There are 25 telephones in Metropolis. Is it possible to connect them with wires 3 so that each telephone is connected with exactly 7 others? Why?
- 3 Show that all vertices of an Euler graph G are of even degree 3
- Explain strongly connected and weakly connected graphs with the help of 3 examples.
- 5 Prove that a connected graph G with n vertices and n-1 edges is a tree. 3
- How many labelled trees are there with n vertices? Draw all labelled trees with 3 vertices.
- Define planar graphs. Is K_4 , the complete graph with 4 vertices, a planar graph? 3 Justify.
- 8 Define fundamental circuits and fundamental cut-sets. 3
- 9 Construct the adjacency matrix and incidence matrix of the graph.



Define chromatic number. What is the chromatic number of a tree with two or more vertices?

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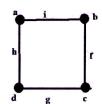
PART B

(Answer one full question from each module, each question carries 14 marks) Module -1

- 11 a) Define complete graph and complete bipartite graph. Draw a graph which is a complete graph as well as a complete bipartite graph.
 - b) Explain walks, paths and circuits with the help of examples.
- 12 a) Define isolated vertex, pendant vertex, even vertex and odd vertex. Draw a graph that contains all the above.
 - b) Prove that simple graph with n vertices and k components can have at most 7 (n-k)(n-k+1)/2 edges.

Module -2

13 a)



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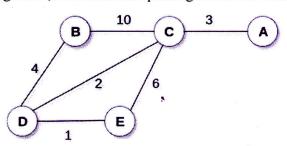
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Find the union, intersection and ring sum of the above graphs.

- State travelling salesman problem. How it is related to Hamiltonian circuits?
- 14 a) Prove that in a complete graph with n vertices there are (n-1)/2 edge disjoint 7 Hamiltonian circuits, if n is an odd number and $n \ge 3$.
 - b) For which values of m, n is the complete graph $K_{m,n}$ an Euler graph? Justify your answer.

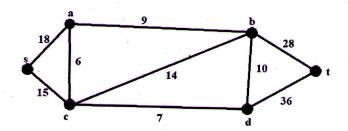
Module -3

- 15 a) Prove that a binary tree with n vertices has (n+1)/2 pendant vertices.
 - b) Using Prims algorithm, find a minimal spanning tree for the following graph.



16 a) Write down Dijkstra's algorithm and use it to find the shortest path from s to t.

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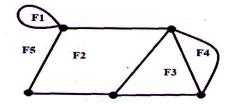
b) Prove that every tree has either one or two centers.

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Module -4

- 17 a) Define cut-set. Prove that every circuit in G has an even number of edges in common with any cut-set.
 - b) Construct the geometric dual of the graph below

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- 18 a) Prove that a connected planar graph with n vertices and e edges has e-n+2 9 regions.
 - b) Let G be a connected graph and e an edge of G. Show that e is a cut-edge if and only if e belongs to every spanning tree.

Module -5

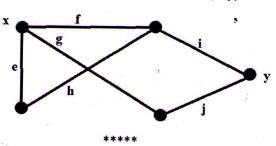
19 a) Explain four colour problem using the concept of chromatic number.

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- b) Let B and A be the circuit matrix and the incidence matrix of a graph G which is free from loops, whose columns are arranged using the same order of edges. Show that AB^T=BA^T=0 (mod 2).
- 20 a) Show that chromatic polynomial of a tree with n vertices is $P_n(\lambda) = 7$ $\lambda(\lambda - 1)^{n-1}$
 - b) Define path matrix of a graph. Find the path matrix P(x, y) for the graph below. 7



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