PHYS 605 Lab #3

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(Dated: February 18, 2016)

I. INTRODUCTION AND THEORY

A. Purpose

In this lab an oscilloscope was used to examine the behavior of a circuit made up of a resistor and a capacitor in series (an "RC circuit") powered by an AC source. The resistor, capacitor, and frequency will be altered in order to gain intuition for the behavior of the circuit. Then the circuit's behavior as a function of frequency will be measured more closely to allow for quantitative analysis.

B. Background / Theory

The behavior of a circuit with an AC power source can be described as a function of its angular frequency, ω. The impedance of a resistor is given by

$$Z_R(\omega) = R \tag{1}$$

and the impedance of a capacitor, by

$$Z_C(\omega) = \frac{1}{j\omega C}\,. \tag{2}$$

Total impedance is written as

$$Z = X + jY \tag{3}$$

where X and Y are real numbers in units of resistance.

Impedances are added when they are in series, so in the RC circuit this gives

$$Z(\omega) = Z_R + Z_C = R - \frac{j}{\omega C}\,. \tag{4}$$

The phase shift, ϕ describes the the difference between the phase of the voltage and the phase of the current that occurs due to complex impedances. When $\phi_v = \phi_i$, they are in phase, the same way they would be in a resistor.

The impedance of a capacitor can also be written as

$$Z_C(\omega) = \frac{1}{\omega C} e^{-j\frac{\pi}{2}}\,. \tag{5}$$

Where the $-\frac{\pi}{2}$ is $\phi_v - \phi_i$, showing the voltage lagging the current by 90°.

Gain, or attenuation, is a ratio of output to input amplitude or power.

$$G(\omega) \equiv \frac{v_{out}}{v_{in}} = \frac{Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)}\,. \tag{6}$$

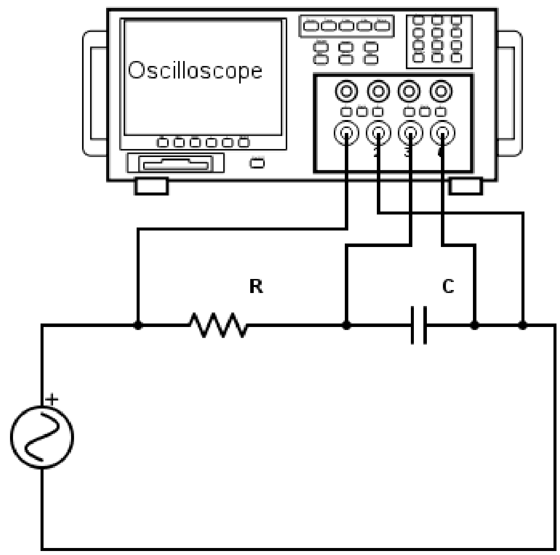

FIG. 1: A circuit built so that the voltage across the capacitor, v_C, can be compared with the voltage from the source, v_{in}. The phase shift ϕ can also be observed.

Phase shift is also given by the relation

$$\phi = \arctan\frac{Y}{X}\,. \tag{7}$$

A *bel* is a factor of 10 change in power. This can be used to measure gain and attenuation. More commonly used is the decibel, a tenth of a bel. Attenuation in decibels would be given as

$$A_{decibel} = 20\log_{10}\left(G(\omega)\right)\,. \tag{8}$$

II. METHODOLOGY

1. Construct RC circuit with oscilloscope as show in figure (1).
2. Adjust the circuit in order to gain some intuition about how the it behaves— vary the resistor, frequency, and capacitor and note the effects.
3. Choose a circuit design such that a range of behavior can be observed while keeping the frequency below 100 kHz.
4. Measure and record the voltage drop, V_C, and the phase shift with respect to input, ϕ_C across the capacitor.

5. Measure and record the voltage drop, V_R, and the phase shift with respect to input, ϕ_R across the resistor.

6. Repeat steps (4) and (5) for several (about ten) frequencies spanning multiple orders of magnitude.

III. RESULTS AND ANALYSIS

A. Data

For steps (1) and (2), a 10V peak-to-peak AC voltage supply was used, along with a 10 kΩ resistor and 4.7 µF capacitor in series. It was observed that adjusting the

FIG. 2: The screen of oscilloscope used to collect data. Here, at 100 Hz, showing a 300mV voltage across the capacitor and a 90° phase shift.

frequency seemed to impact the phase shift, and it was eventually found that as the frequency got very large, the rate at which the phase shift changed decreased. The capacitor was switched to a 0.518 nF capacitor. This time when the frequency was adjusted, there seemed to be much less change in the phase shift; it stayed below 5° the entire time. The voltage across the capacitor was 2.58V. Lastly, the resistor was changed to a 98.4 kΩ resisotr. This increased the voltage across the capacitor but did not seem to change the phase shift noticably.

For steps (3) through (5), the peak-to-peak voltage provided by the source was 12.2V. A 10 kΩ resistor and 4.7 µF capacitor were used.

f [Hz]	v_C [V]	ϕ [°]
10.92	3.68	78
24.04	1.76	90
71.73	0.72	97
97.66	0.44	95
337.8	0.134	90
757.4	0.07	92
1066	0.0438	80
5102	0.02	87
12990	0.016	52

The voltage and phase shift across the resistor were also measured, and found to be 12.2V and 0°. These did not change with frequency.

B. Calculations

The impedance of the capacitor, Z_C, and the impedance of the resistor, Z_R, could be calculated using equations (1) and (2).

$$Z_R(\omega) = 10\,000\,\Omega$$

$$Z_C(\omega) = \frac{1}{j\omega\ 4.7 \times 10^{-6}}$$

Using equation (6), the gain was calculated, first with the ratio of the voltages, then using the impedances where Z_2 was the impedance of the capacitor and Z_1 was the impedance of the resistor. This allowed us to calculate error.

f [Hz]	$Gain_{measured}$	$Gain_{expected}$	% Error
10.92	-10.4	-12.5	16.8
24.04	-16.8	-18.17	7.53
71.73	-24.6	-26.9	6.92
97.66	-28.9	-29.5	2.03
337.8	-39.2	-40.1	2.24
757.4	-48.9	-47.0	4.68
1 066	-48.9	-50.0	2.20
5 102	-55.7	-63.6	12.4
12 990	-57.6	-72.7	20.8

C. Analysis

Observing the circuit's behavior qualitatively demonstrated that increasing resistance and decreasing capacitance causes an increase in the voltage across the capacitor. It also showed that a change in capacitance will change the phase shift, but a change in resistance will not.

The variation in error for the attenuation suggests that a substantial portion of the error was due to the oscilloscope. There was also more error at the lowest frequencies and highest frequencies. This could be due to less accurate measurements being made when the voltage across the capacitor was very small.

Using the data collected, a Bode plot was drawn. It is apparent that all of the measured values are after the so-called "-3 dB Point" as the gain plot is definitively negative.

The phase shift plot seemed slightly more erratic. Because the plot is after the -3 dB point, it is expected that the phase will begin to approach 90°.

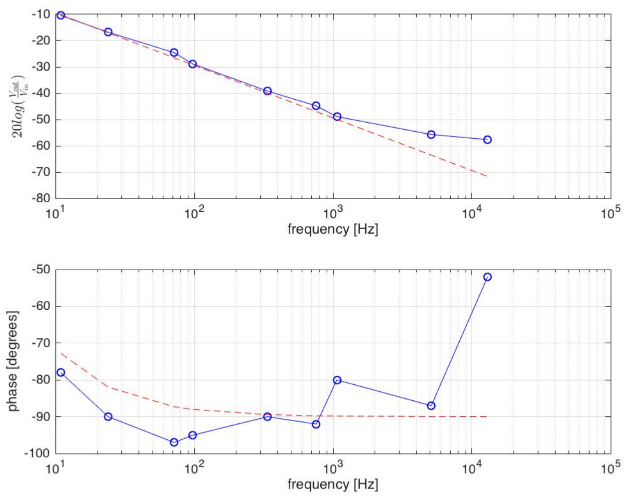

FIG. 3: A Bode plot of the data. The measured values are in blue, while the expected values from equations (6) and (7) are in red.

A line was fit to the gain plot, and the slope showed that the roll off of the transfer function is 16.24 dB/decade

IV. CONCLUSION

The relationship between frequency, impedance, and voltage was made obvious, and observations matched expectations based on known equations.

Measured values were compared to calculated values with some error, which seemed to increase at extreme values. The roll off was calculated, and the group was able to observe the behavior of a RC circuit in frequencies that were a part of the filtered frequencies.