Understanding Spin Hall Magnetoresistance

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Magnetoresistance research has vast applications in spintronics. In order to advance spintronics and technology, all types of magnetoresistance must be understood. Spin Hall magnetoresistance is one of many types of magnetoresistance. From its discovery to its understanding, it is clear that SMR is a subtle and complex phenomena. The goals of this paper are to introduce SMR and convey how it occurs, explain how it was discovered, and discuss what further experiments will investigate.

I. INTRODUCTION

Condensed matter physics has shown that it is an extremely useful area of physics as it has been key in the developement of computer technology. One such important phenomena that has been studied is magnetoresistance. The most notable type of magnetoresistance is giant magnetoresistance (GMR). After its discovery in 1988 by Fert and Grünberg, *et al*[1], GMR was used in many technologies, such as hard drives, for data storage. Developments of such utility attest to the importance of further solid state research as well as the utility of spintronics. The goal in futher magnetoresistance research is to develope data strorage abilities.

Magnetoresistance research is important to the developement of spintronics as the effects of an extrenal magnetic field on a spintronic device must be understood in order to create stable devices. If there are unexpected properties of the materials used in spintronic devices it could mean that devices will not operate as expected or will not work at all. Research relating to magnetoresistance aims to understand all of the properties of materials that could be used in spintronic devices and will therefore help the developement of spintronic technology.

Magnetoresistance (MR) is the dependence of a material's resistivity on an external magnetic field. Since MR was first discovered there have been a multitude of classifications of effects fitting this broad description. One such effect is known as spin Hall magnetoresistance (SMR).

SMR was discovered after spin transport effects in bilayers of a normal metal and a ferromagnetic insulator (denoted $N|F$) were measured and misidentified as a consequence of static magnetic proximity effects (MPE)[2]. Soon after, it was demonstrated that this was not the correct cause of the spin transport phenomena. After ruling out MPE, spin Hall magnetoresistance (SMR) was identified as the magnetoresistance measured in the bilayer[3]. SMR is magnetoresistance that occurs because of a combination of both the spin Hall effect (SHE) and the inverse spin Hall effect (ISHE).

This paper will give a phenomenological explanation of SMR. The paper will also highlight the various phenomena that, togehter, cause SMR. The mathematics presented will be the minimum needed to explain the phenomena of SMR. Any details that will not help the reader understand the origin of SMR may be omitted.

II. THEORY

This section aims to explain SMR as well as present ideas such as spin currents and SHE/ISHE that are needed to understand SMR.

A. Theoretical Background

This section will prepare the reader with an understanding of spin currents and SHE/ISHE. This will be important as these effects are the cause of SMR.

1. Spin Currents

We need to be define electrical currents and spin currents in order to begin the discussion of SMR. We start with the equation for electrical currents (Ohm's Law):

$$\vec{j}_c = \sigma \vec{E} \tag{1}$$

Where $\vec{j}_c$ denotes the electrical current. In ferromagnets where there is spin dependence of energy and conductivity, we can "keep track" of all of the spin up ($\uparrow$) and spin down ($\downarrow$) electrons. We can therefore re-write Ohm's law in terms of the current of spin up and spin down electrons and their spin dependent conductivities $\sigma_{\uparrow/\downarrow}$[4]:

$$\vec{j}_c = \vec{j}_\uparrow + \vec{j}_\downarrow = (\sigma_\uparrow + \sigma_\downarrow)\vec{E} \tag{2}$$

Now, we can define spin current $\vec{j}_s$ as net spin up that is moving by subtracting $\vec{j}_\uparrow$ and $\vec{j}_\downarrow$[4]:

$$\vec{j}_s = \vec{j}_\uparrow - \vec{j}_\downarrow = (\sigma_\uparrow - \sigma_\downarrow)\vec{E} \tag{3}$$

Now we define the conductance spin polarization P to relate $\vec{j}_s$ and $\vec{j}_c$[4]:

$$P = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} \tag{4}$$

$$\overrightarrow{j}_s = P\sigma\overrightarrow{E} \tag{5}$$

Spin transport refers to transfer of spin via spin currents from one material to another.

With a clear definition of spin current in mind, we can look at the SHE and ISHE.

2. Spin Hall Effect and Inverse Spin Hall Effect

The spin Hall effect (SHE) is the phenomena where strong spin-orbit coupling and a charge current causes a spin current perpendicular to the charge current. This is shown in Figure (1).

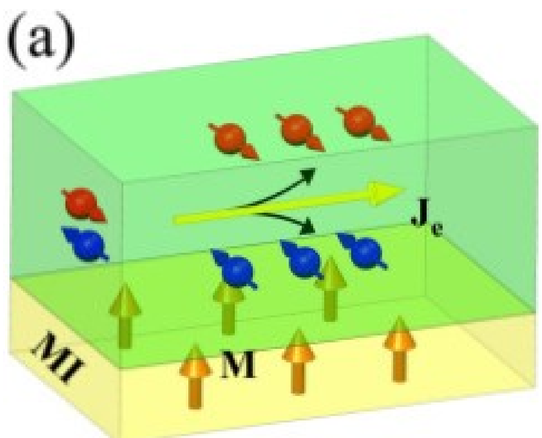

FIG. 1. The charge current is denoted by J_e and the up and down spins are denoted by the red and blue colored electrons respectively. Therefore, the spin current is up through the solid[8].

The inverse is also true and is therefore called the inverse spin Hall effect (ISHE); a spin current will induce a charge current that is perpendicular to the spin current. The governing equations of the SHE and ISHE are[4]:

$$\overrightarrow{j}_{s\hat{\imath}}^{SHE} = \theta_{SH}\hat{\imath} \times \overrightarrow{j}_c \tag{6a}$$

$$\overrightarrow{j}_c^{ISHE} = \theta_{SH}\hat{\imath} \times \overrightarrow{j}_{s\hat{\imath}} \tag{6b}$$

Where θ_{SH} is the spin Hall angle and $\hat{\imath} = \hat{x}, \hat{y}, \hat{z}$. The spin Hall angle is a measured quantity and for the purposes of this paper can be thought of as a material parameter. Typical values are around $0.01-0.1$[2].

The SHE originates from scattering theory in solids[5] and ISHE is demanded by thermodynamics according to Onsager reciprocity[4]; however, these points are not critical for understanding how the SHE and ISHE cause SMR.

B. SMR Theory

We will now introduce the theory of SMR outlined by Chen, *et al*[46]. This theory is consistent with the explaination of SMR given by of Nakayama, *et al.*

1. (Spin) Currents in Ferromagnets without the SHE/ISHE

Spin Hall magnetoresistance occurs in a bilayer of a normal metal (N) and a ferromagnetic insulator (F) as shown in Figure (2). To begin, we will formulate how electric fields and spin accumulations cause spin and charge currents in absence of the SHE and ISHE.

A chemical potential will be associated with an applied electric field and a spin buildup. A chemical potential is a form of potential energy. Since particles tend to lower their potential energy if possible, electrons will to move relative to the gradient of the chemical potential. This chemical potential can be a combination of both electric and spin potentials. The chemical potential for spin up/spin down electrons will be[4]:

$$\mu_{\uparrow/\downarrow F} = e\phi + \delta\mu_{\uparrow/\downarrow F} \tag{7}$$

Where ϕ is the electric potential $-\nabla\phi = E$ and e is the charge on the electron $e = -|e|$, and $\delta\mu_{\uparrow/\downarrow F}$ is the chemical potential due to a spin accumulation. A concentration of one type of spin in a region will cause the electrons of that spin to be repelled from the region. This is analogous to charge accumulation which repels like charge from the area.

At the boundaries of the bilayer, it is expected that there are spin dependent conductivities. Therefore, there is a difference in the electrical currents of the two spin types. The current for each spin type will be along the gradient of the chemical potential for that spin type[4]:

$$\overrightarrow{j}_{\uparrow/\downarrow F} = -\frac{\sigma_{\uparrow/\downarrow F}}{e}\nabla\mu_{\uparrow/\downarrow F} \tag{8}$$

We can now write the electrical chemical potential and the spin chemical potential in order to create a linear system of coupled equations for the charge and spin currents. The charge chemical potential is the sum of the up and down chemical potentials. This is because the electrical current is the sum of the currents of spin up and spin down electrons[4].

$$\mu_{cF} = \frac{\mu_{\uparrow F} + \mu_{\downarrow F}}{2} \tag{9}$$

The spin chemical potential is the difference of the spin up and spin down chemcial potentials as the spin current is the difference of spin up and spin down currents[4].

$$\mu_{sF} = \mu_{\uparrow/F} - \mu_{\downarrow F} \tag{10}$$

Notice the consistency of the addition of the chemical potentials creating the charge chemical potential as the charge current will be generated from the sum of the spin up and spin down electrons moving in the same direction. The spin chemical potential is the difference of the two chemical potentials as a spin current will generate from electrons of opposite spins moving in opposite directions. The importance that $\delta\mu_{\uparrow F} \neq \delta\mu_{\downarrow F}$ is clear now for if it were the case that $\delta\mu_{\uparrow F} = \delta\mu_{\downarrow F}$ then $\mu_{sF} = 0$. This is good, as without the spin dependence, there should only be a charge current.

There are two key processes occuring here. First, since there is a spin dependence of the conductivities ($\sigma_\uparrow \neq \sigma_\downarrow$), there will be a difference in the current of up and down spin electrons. This is what we have defined as a spin current in equation (3). Second, a spin current will cause a nonuniform distribution of spins. Areas with a buildup of a certain type of spin will cause $\delta\mu_{\uparrow F} \neq \delta\mu_{\downarrow F}$ which in turn means $\mu_{\uparrow F} \neq \mu_{\downarrow F}$. The chemical potential will be different for the two spins, which will drive a current. These two processes occur simultaneously. This is a coupling of the spin and charge currents. By combining equations (5), (8), (9), and (10), we can re-write Ohm's law to include this coupling[4]:

$$\begin{pmatrix} \overrightarrow{j}_{cF} \\ \overrightarrow{j}_{sF} \end{pmatrix} = \sigma_F \begin{pmatrix} 1 & P \\ P & 1 \end{pmatrix} \begin{pmatrix} -\nabla\mu_{cF}/e \\ -\nabla\mu_{sF}/(2e) \end{pmatrix} \tag{11}$$

The chemical potentials can be found by diffusion equations; however, we will not explicitly write down or solve the diffusion equations. It is more important that the reader focus on the fact that spin accumulations and electric potential in the solids will be the source of the chemical potential. In SMR, these chemical potentials will couple the spin and charge currents as in equation (11), but will also cause the SHE and ISHE. The SHE and ISHE will add terms to the coupling of spin and charge currents and the combined effect is SMR.

2. (Spin) Currents in Normal Metals without the SHE/ISHE

In the normal metal, unlike in the ferromagnet, there is no dependence on spin. This has the implication that conduction of all electrons is governed by one conductivity denoted as σ_N as well as the implication that the spin chemical potential should not be dependent on spin type. The spin chemical potential in the normal metal will be denoted in individual components as $\mu_{s\hat{\imath}}$. Where $\hat{\imath} = \hat{x}, \hat{y}, \hat{z}$ will denote the direction of the spin current that is associated with the chemical potential. The resultant current equations are given by Fick's laws[47]:

$$\overrightarrow{j}_{cN} = -\frac{\sigma_N}{e}\nabla\mu_{cN} \tag{12a}$$

$$\overrightarrow{j}_{s\hat{\imath}} = -\frac{\sigma_N}{2e}\nabla\mu_{s\hat{\imath}} \tag{12b}$$

Here, the spin contribution to the chemical potential will arise from the SHE or the effects of the ferromagnet at the interface. In a normal metal alone (not in the bilayer) and without the SHE there should be no spin current and only electric potential. This is the case of a normal conducting metal which obeys the form of Ohm's law given in Equation (1).

Again, the chemical potential can be found from diffusion equations, but this will not be done in this paper but is done by Chen, *et al*.

3. SMR and Incorporation of SHE/ISHE

SMR occurs in a bilayer of a normal metal (N) and a ferromagnetic insulator (F). The set up of the bilayer is shown in Figure (2).

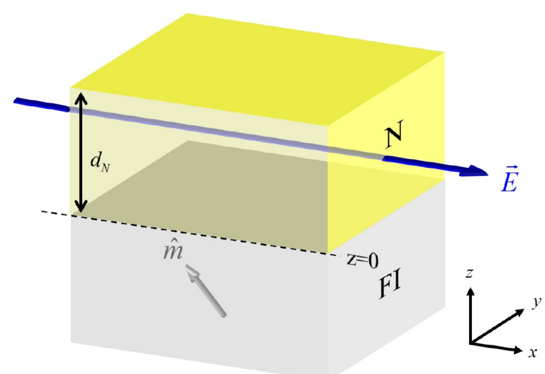

FIG. 2. The $N|F$ bilayer. N is the normal metal and F/FI is the ferromagnetic insulator. The electric field is aligned with the $\hat{x}$ direction and the magnetization of the ferromagnet is in the x-y plane.[4]

The boundary conditions at the $N|F$ interface are important to understanding the nature of SMR. The formulation of Chen, *et al* ignores the spin-orbit coupling and proximity effects at the interface and only considers the spin accumulation at the interface[46]. This spin accumulation is generated by the SHE in the normal metal. This spin accumulation is the cause of the spin current from the normal metal to the ferromagnet[4]. The spin current from the normal metal to the feromagnetic insulator is the spin transport that was initially measured by Haung, *et al*.

Though the boundary conditions for spin accumulations at the interface are important for formulating the theory of spin Hall magnetoresistance, they will not be mathematically formulated here as they will not be needed for the phenomenological explanation of this paper.

In equations (12a) and (12b), the source for the currents is the chemical potential. When we include the SHE and ISHE, we will now have that the sources for $\overrightarrow{j}_c^{ISHE}$ and $\overrightarrow{j}_s^{SHE}$ will be $\theta_{SH}\hat{\imath} \times -\frac{\sigma_N}{2e}\nabla\mu_{s\hat{\imath}}$ and $\theta_{SH}\hat{\imath} \times -\frac{\sigma_N}{e}\nabla\mu_{cN}$ respectively. If equations (6), (12a) and (12b) are combined so that the net current is the sum of the source terms from these equations, a general matrix equation can be formed for the new Ohm's law[4]:

$$\begin{pmatrix} \overrightarrow{j}_{cN} \\ \overrightarrow{j}_{sx} \\ \overrightarrow{j}_{sy} \\ \overrightarrow{j}_{sz} \end{pmatrix} = \sigma_N \begin{pmatrix} 1 & \theta_{SH}\hat{x}\times & \theta_{SH}\hat{y}\times & \theta_{SH}\hat{z}\times \\ \theta_{SH}\hat{x}\times & 1 & 0 & 0 \\ \theta_{SH}\hat{y}\times & 0 & 1 & 0 \\ \theta_{SH}\hat{z}\times & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\nabla\mu_{cN}/e \\ -\nabla\mu_{sx}/(2e) \\ -\nabla\mu_{sy}/(2e) \\ -\nabla\mu_{sz}/(2e) \end{pmatrix} \tag{13}$$

Here we will skip the details of formulating the boundary conditions and the equations for the chemical potential, solving for the chemical potential, and solving the resulting equations. The results are the resistivities in terms of the important parameters such as the normal metal thickness, Fermi velocity, and spin-flip/relaxation times. At the boundary, it is assumed there is no charge accumulation as the ferromagnet is an insulator.

The resultant resistivities corresponding to SMR in the weak SHE and small θ_{SH} limit are[4]:

$$\rho_{long} = \sigma_{long}^{-1} = (\frac{\overline{j_{c,long}}}{E_x})^{-1} \approx \rho + \Delta\rho_0 + \Delta\rho_1(1 - m_y^2) \tag{14}$$

$$\rho_{trans} = -\frac{\sigma_{trans}}{\sigma_{long}^2} \approx -(\frac{\overline{j_{c,long}}/E_x}{\sigma_N^2}) = \Delta\rho_1 m_x m_y + \Delta\rho_2 m_z^2 \tag{15}$$

with

$$\frac{\Delta\rho_0}{\rho} = -\theta_{SH}^2 \frac{2\lambda}{d_N} tanh^2 \frac{d_N}{2\lambda} \tag{16}$$

$$\frac{\Delta\rho_1}{\rho} \approx -\theta_{SH}^2 \frac{\lambda}{d_N} \frac{2G_r tanh^2 \frac{d_N}{2\lambda}}{\sigma_N + 2\lambda G_r coth \frac{d_N}{\lambda}} \tag{17}$$

$$\frac{\Delta\rho_2}{\rho} \approx -\theta_{SH}^2 \frac{\lambda}{d_N} \frac{2G_i tanh^2 \frac{d_N}{2\lambda}}{(\sigma_N + 2\lambda G_r coth \frac{d_N}{\lambda})^2} \tag{18}$$

G is the spin-mixing conductance at the interface. This is determined by the scattering of the electrons at the interface[4]. Here, G_r and G_i are the real and imaginary parts of G and m is the magnetization of the ferromagnet shown in Figure (2). In this case, the approximations assume that $G_r >> G_i$. The spin-mixing conductance will affect the spin build up at the interface and will thereby change the SMR; however, the ISHE that causes SMR will occur as long as there is a spin accumulation at the interface.

III. EXPERIMENT

This section will discuss the experiments that discovered SMR, other experiments that have been done relating to SMR, and what remaining questions need to be addressed experimentally.

A. Discovery and Identification of SMR

SMR was first discovered in Platinum (Pt) grown on Yttrium Iron Garnet (YIG) by Huang, et al[2]. The experiment aimed to measure spin effects in a bilayer of a normal metal and a ferromagnet. A $Pt|YIG$ bilayer was used in the experiment as Pt is commonly used in studying spin currents in normal metals and YIG is a well known ferromagnetic insulator[2]. The magnetic field in the YIG layer in this experiment was set in the x-y plane, as shown in Figure (2), at an angle θ with respect to the x-axis when making MR measurements[2].

The experiment showed that there are spin transport effects at the $Pt|YIG$ interface, but Huang, et al misidentified the cause as MPE rather than the SHE[23]. Since the transport effects were believed to be caused by MPE, the MR was misidentified as anisotropic magnetoresistance (AMR)[23]. Since AMR was already a known phenomena, its measurement was assumed and not the main conclusion of the experiment.

The same bilayer was soon after used in an experiment by Nakayama, *et al.* They realized that the correct cause of the MR effects measured by Huang, et al was a combination of the SHE and ISHE[3]. Nakayama, *et al* demonstrated that the spin transport in the $Pt|YIG$ bilayer was caused by the SHE and not MPE. This led to the realization that the MR measured by Huang, et al was not AMR and instead was the new phenomena they called SMR[3].

In order to show that the spin transport measured was not due to MPE, Nakayama, *et al* made a trilayer of $Pt|Cu|YIG$. There should not be MPE shown in the Cu layer (as Cu is far from the Stoner instability[3]) and the MPE should not occur over the 6 nm thickness of the Cu layer. When SMR was measured even after the Cu layer was added, it showed that there was not MPE or AMR in this system.

Nakayama, *et al* also measured that the dependence of the MR on the angle θ was not as expected for AMR, but did follow the expectation for SMR. This is because the MR effect should be a peak centered around $\theta = 45$, but AMR has a more sinusoidal nature[3].

Since the measurements made by Nakayama, *et al* discovered SMR, there have been many more experiments and some controversy over the effects. The review paper of Chen, *et al*, based on the theory they presented, still coincides well with Nakayama, *et al.*

B. Further Experimentation

1. Unresolved Questions

Though there is a model for SMR, there is still more to be explained. The biggest question surrounds the dependence of SMR on temperature. It is unclear if SMR simply decreases with temperature or if AMR plays an important role in this regime[4].

The implications of roughness on the surface of the $N|F$ interface is not well understood as it has mostly been ignored even though it will affect the spin transport at the interface[4].

As mentioned earlier, Chen, *et al* ignore spin-orbit coupling at the interface even though it is clear that this may have implications on the spin transport properties[4].

2. SMR Without a Ferromagnetic Insulator

An interesting experiment was conducted where SMR was claimed to be measured in a bilayer of a normal metal and a non-magnetic insulator[8]. In this experiment they used a bilayer of Pt and Co_2FeAl (CFA), where CFA is the non-magnetic insulator.

The main finding of this paper was that MR fitting the curve for SMR was found in the bilayer. The SMR measured was also much stronger than measured in other experiments. Zhang, *et al* state that since SMR is observed in this new system, it should be considered that it may occur in more systems than just the $N|F$ bilayers[8].

From the theory presented by Chen, *et al*, if the SHE still occurs in the normal metal (as assumed by Zhang, *et al*) SMR would be observed in this system. The interesting observation is that the SMR measured was greater than in other systems. This is reasonable if there is no spin transport across the interface. If this is the case, there will be a larger spin accumulation at the interface. This increase in spin accumulation would clearly cause a greater SMR effect.

IV. CONCLUSION

SMR was first thought to be AMR caused by MPE. This was later shown not to be the case and has since been given its name SMR. There will continue to be experiments to determine unknown properties of SMR such as its temperature dependence. New developments such as the measurement of SMR in a bilayer without a ferromagnetic insulator will continue to advance the understanding of the effect.

In this paper we discussed spin and charge currents, the SHE and ISHE, and the chemical potentials that couple the spin and charge currents. The combination of these ideas brings about spin Hall magnetoresistance. The generation of SMR comes from the action of both the SHE and ISHE. In the normal metal, the SHE will create a spin accumulation near the bilayer. This accumulation causes a spin chemical potential which will generate a source for a charge current via the ISHE. The combination of these effects creates the interesting type of MR, SMR.

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