Scattering of the Higgs Field off of Quantum Gravity Topological Fluctuations

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Any work of communication should include a focus on the audience of the piece. In this case, the presenter is not more knowledgable than the audience. This means that the presentation will potentially include blatant errors. This is difficult to combat, however, the learning experience of attempting to formulate a description of concepts that are very unfamiliar and are at a level above the presente's knowledge is still useful. Here, I attempt to outline the results of qualitative calculations of scattering of scalar fields off of quantum gravity topological fluctuations. I then look to interpret the work that was done before the discovery of the Higgs Boson (a scalar field) into an interpretation of these calculations given the modern knowledge of the Higgs boson.

CONTENTS

I. INTRODUCTION

An important part of quantum gravity research is calculating the interactions between known fields and gravity in the theory. Quantum gravity would include (at least in principle) a way of integrating over virtual quantum gravity "particles". In the papers to be discussed the quanta of the theory are thought of as topological fluctuations of the metric. Stephen Hawking was an author on several papers that investigate different topological fluctuations such as "virtual black hole loops" [1] and "spacetime foam" [2]. This work has been to investigate the scattering of scalar fields off of these topological fluctuations [1] [3]. The results of these investigations led Stephen Hawking think the Higgs boson would not be discovered or that it would not be a fundamental particle (it would be made of multiple elementary spin 0 particles). Given the measurements of the Higgs boson [4] which have led us to believe that it is a fundamental particle, may instead imply that the topological fluctuations that Hawking has described are not physical. These calculations are done with particular metrics such as the done with the C metric ("virtual black holes"). These calculations can give qualitative results that we may expect to indicate results obtained from properly integrating the Greens functions over all possible metrics. If topological fluctuations are the correct way to formulate quantum gravity it would mean that arbitrary calculations would not be feasible. Physicist hope that calculations can be done in the correct theory of quantum gravity. Therefore, it would be very enlightening if these calculations indicate problems with fromulating quantum gravity as these topological fluctuations.

II. QUANTUM FIELD THEORY

A field is an object (usually a linear operator) that is defined at every point in spacetime. In the framework we define the field as a combination of the creation and annihilation operators [5]

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} \left[a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right] \tag{1}$$

interpreted as raiseing and lowering operators for the number of quanta (or particles) of the field. The way to use this theory for calculations is to calculate the Hamiltonian and assert the usual commutation relationship:

$$[\phi(\vec{x}),\, \pi(\vec{y})] = i\hbar\, \delta(\vec{x} - \vec{y}) \tag{2}$$

know as “cannonical quantization” [6]. The Hamiltonian for the system istaken to be the integral over all number operators:

$$H = \int d^4p\ a^\dagger_{\vec{p}}\, a_{\vec{p}} \tag{3}$$

Where we have neglected the other contributions to the Hamiltonian (this is how we define the ground state [5]). The Hamiltonian is the total energy of the system. Therefore the Hamiltonian is the number of particles times the energy per particle, which is the total energy.

In quantum field theory we use this framework and formulate our fields as combinations of fundamental particles that are described by operators that represent creation and anihilation of that particle type. In quantum gravity we look for an analogy that applies to ”fluctuations” relating to the topology of metric.

III. THE HIGGS BOSON

The Higgs boson is a fundamental scalar field with mass ~126GeV [4]. It is responsible for the emass of the electroweak bosons. The measurement of the Higgs boson informs us of constrants on the interactions of scalar fields in quantum gravity theories. In particular a quantum gravity result that would cause scalar fields to have a mass greater than the mass of the Higgs boson would be ruled out as acceptable theories.

IV. CONFORMAL TRANSFORMATIONS

Conformal transformations are functions of coordinates that are scalar multiplied by the metric to obtain a new metric:

$$g'_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu} \tag{4}$$

There is investigation into conformal field theories in which symmetries under continuous conformal transformations determine the associated gauge fields. This is similar to the standard model approach. Here, the associated gauge fields are considered to be the graviational fluctuations.

Conformal field theories face immense difficulties because there is an infinite-demensional algrbra of such local conformal transformations.

V. METRIC FLUCTUATIONS

The term metric fluctuations refers to the contribution to the quantum gravity path integral of the integral over all metrics of the Greens functions (the fluctuations, analogous to standard model “particles”, thought of as “spacetime foam”, “virtual black hole pairs”) for each metric. This infinite-dimensional integral is not feasbile, but it is possible to demonstrate qualitative results using the C metric [1] (or other metrics [3]) and the associated field solutions.

VI. GREENS FUNCTIONS FOR TOPOLOGICAL FLUCTUATIONS

The Euclidean Greens Functions are the set of metric topologies that would be integrated over in a path integral formulation of quantum gravity. Integrating over all metrics in this case is analogous to the integral over all paths in QFT.

$$H = \int d^4x \mathcal{H} \tag{5}$$

There is a quantum state obtained from the Euclidean green functions by this integral. This can be done for certain metrics with field solutions such as the C metric. This would be one contribution to the path integral.

VII. SCATTERING OFF VIRTUAL BLACK HOLES

In [1] a calculation was done with the Euclidian C metric (C metric), which has a toplogy of $S^2 \times S^2 - \{point\}$. The calculation for scattering of scalar fields off virtual black hole loops uses a finite dimensional approximation to an asymptotically Euclidean metric, in this case the C metric. The C metric is a solution of the conformal field equation.

The result of the calculation is a non-zero number of particles at infinity which is a loss of quantum coherence (a violation of unitarity) in the qualitative “semi-classical calculation”. The calculation obtains a transmission factor such that the dominant contribution to the particle production is in the s-wave which is surpressed for scattering of higher-spin fields off such virtual black hole loops relative to that of scalar fields because scalar fields cannot radiate in the s—wave.

This calculation led the authors to the conclusion that fundamental scalar fields would not exist. The measurement of the Higgs Boson would therefore imply that if the conclusions from the calculation are correct, these topological fluctuations would not be physical as they would violate unitarity.

VIII. SCATTERING OFF SPACETIME FOAM

The "gravitational bubbles" of the "spacetime foam" can be thouhgt of as being built out of three units of "topological fluctuation", CP^2, $S^2 \times S^2$ and K3. The results of a qualitative analysis demonstrated for the "quantum gravity bubbles" [8] associated with the "spacetime foam" is that the scattering amplitudes are very small for fermions and vectors but would predict scalar particles to have a mass on the order of the Plank mass (M_P~1.2×10^{19} GeV) [3].

The scattering amplitudes of the fluctuations investigated in [3] are of order unity for scalar particles and could give rise to an effective mass of the order of the Planck mass; however, fermions and vectors will not acquire a mass.

This calculation led the authors to a similar conclusion to the case of virtual black hole loops, which is that scaar fields either would not be fundamental, would have a large mass, or would not be discovered at all. This concusion is fundamentally incorrect given the measurements of the Higgs boson.

IX. DISCUSSION

Since the Higgs Boson has been discovered and its measurements have indicated it is a fundmental particle, this may mean that the topological fluctuations proposed do not exist. It may be nice if nature is not a quanum gravity theory of topological fluctuations of the metric since there is no way to do the infinite—dimentional integral over all metric fluctuations.

Small successes that add to the constraints of our physical models are important. Though it is not clear if these qualitative calculations are indicative of the results of a proper conformal field theory calculation, it they are, then the topological fluctuations proposed such as virtual black hole loops and gravitational bubbles, could not be physically correct.

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