- Algebraic circuit over some ring R is a circuit which computes polynomial in $R[x_1, \ldots, x_n]$.
- Algebraic circuit has addition and multiplication gates, starting from the input variables $\vec{x}$ and constants from R.
- The size of an algebraic circuit C is the number of nodes in it.
- Constant-free algebraic circuit is an algebraic circuit in which the only constants used are $0, 1, -1$.

Definition. Let $f \in \mathbb{Z}[\overline{x}]$ be a multivariate polynomial over $\mathbb{Z}$. Then $\tau(f)$ is the minimal size of a constant-free algebraic circuit that computes f (that is, a circuit where the only possible constants that may appear on leaves are $1, 0, -1$).

Definition. A sequence (f_n) of integer polynomials belongs to the complexity class VP^0 iff there exists a sequence (C_n) of division-free and constant-free algebraic circuits such that C_n computes f_n and the size and the formal degree of C_n are polynomially bounded in n.

Definition. A sequence $(f_n(X_1, \ldots, X_{u(n)}))$ of integer polynomials belongs to the complexity class VNP^0 iff there exists a sequence $(g_n(X_1, \ldots, X_{v(n)}))$ in VP^0 such that

$$f_n(X_1, \ldots, X_{u(n)}) = \sum_{e \in \{0,1\}^{v(n)-u(n)}} g_n(X_1, \ldots, X_{u(n)}, e_1, \ldots, e_{v(n)-u(n)}).$$

where $u(n)$ and $v(n)$ are polynomially bounded functions of n.

We recall that *permanent* of the matrix $[X_{ij}]$ is defined as

$$Per_n = \sum_{\pi \in S_n} X_{1\pi(1)} \cdots X_{n\pi(n)}$$

Theorem. If $\tau(n!) \neq (\log n)^{O(1)}$ then $\tau(PER_n) \neq n^{O(1)}$. If $\tau(PER_n) \neq n^{O(1)}$ then $(PER_n) \notin VP^0$. If $(PER_n) \notin VP^0$ then $VP^0 \neq VNP^0$.