Definition 1. *The tensor product $G \times H$ of finite simple graphs G and H has vertex set $V(G) \times V(H)$, and pairs (g,h) and (g',h') are adjacent if and only if $\{g,g'\} \in E(G)$ and $\{h,h'\} \in E(H)$.*

It's obvious that

$$\chi(G \times H) \leq \min\{\chi(G), \chi(H)\}.$$

Main result 1.

$$\exists G, H : \chi(G \times H) < \min\{\chi(G), \chi(H)\}.$$

Some tools for proof:

Definition 2. *Let c be a positive integer, and let Γ be a finite graph that we allow to contain loops; the exponential graph $\mathcal{E}_c(\Gamma)$ has all mappings $V(\Gamma) \to \{1,\ldots,c\}$ as vertices, and two distinct mappings φ, ψ are adjacent if, and only if, the condition $\varphi(x) \neq \psi(y)$ holds whenever $\{x,y\} \in E(\Gamma)$.*

The relevance of $\mathcal{E}_c(\Gamma)$ to the problem is easy to see because the graph $\Gamma \times \mathcal{E}_c(\Gamma)$ has the proper c-coloring $(h,\psi) \to \psi(h)$.

Definition 3. *For a simple graph G, we define the strong product $G \boxtimes K_q$ as the graph with vertex set $V(G) \times \{1,\ldots,q\}$ and edges between (u,i) and (v,j) when, and only when, $\{u,v\} \in E(G)$ or $(u = v)\&(i \neq j)$.*

Lemma 1. *Let G be a finite simple graph with finite girth ≥ 6 Then, for sufficiently large q, one has $\chi(\mathcal{E}_c(G \boxtimes K_q)) > c$ with $c = \lceil 3.1q \rceil$.*

Some other things from report:

Definition 4. *Given positive integers $p \geq q$, a (p,q)-colouring of a graph G is a mapping $c : V(G) \to \{0,1,\ldots,p-1\}$ such that for any edge (x,x') of G, $q \leq |c(x) - c(x')| \leq p - q$. The circular chromatic number $\chi_c(G)$ of G is the infimum of $\frac{p}{q}$ for which G has a (p,q)-colouring.*

It's obvious that

$$\chi_c(G \times H) \leq \min\{\chi_c(G), \chi_c(H)\}.$$

The question about inequality is open.

Definition 5. *A fractional colouring of a graph G is a mapping f which assigns to each independent set I of G a real number $f(I) \in [0,1]$ such that for any vertex x, $\sum_{x \in I} f(I) = 1$ The total weight $w(f)$ of a fractional colouring f of G is the sum of $f(I)$ over all the independent sets I of G. The fractional chromatic number $\chi_f(G)$ of G is the minimum total weight of a fractional colouring of G.*

It's obvious that

$$\chi_f(G \times H) \leq \min\{\chi_f(G), \chi_f(H)\}$$

and it's true that

$$\chi_f(G \times H) = \min\{\chi_f(G), \chi_f(H)\}.$$