Separation Logic language is incomplete

The Hoare logic is a common tool for software verification. Its statements are written as triples like $\{P\}instr\{Q\}$. Such notation states that if P holds then after execution of instruction $instr$ Q will hold.

Unfortunately such approach complicates reasoning about complex programs. For example, one can easily prove that a single instruction doesn't overwrite all available memory but it's hard to prove the same for a large program.

We note though that a program can be usually split into functions that modify separate memory parts and therefore don't interfere with each other. It's easier to prove that if a property holds for such a function then it also holds for a whole program.

This idea underlies the separation logic. This is a Hoare logic extension which allows to express the fact that the memory is split between different functions. For example, the statement "the memory consists of exactly two locations with addresses x and y with values 1 and 2 correspondingly"can be rephrased as "the memory can be split into two disjoint parts, each consisting of a single location x (or y) and value 1 (or 2)". The latter statement can be written in the separation logic language as $(x \mapsto 1) * (y \mapsto 2)$.

The satisfiability of this language's formula depends on memory state. Notation $h \models \phi$ denotes that if memory is described with function $h : Addresses \to Values$ then the formula ϕ holds. It can be seen that the tautological formulas of FOL are similar to this language's valid statements — those that hold with any memory state.

For separation logic the validity problem is undecidable. It follows from the non-recursively-enumerability of this logic's language [1].

The non-r.e. is proved by reduction from other non-r.e. language. It is the set of such formulas of FOL language with only one binary predicate that hold in every finite structure. The non-r.e. of this language has been shown by Trakhtenbrot.

For simpler versions of this language the validity problem is decidable though. For example, the validity for language without quantifiers and $\mapsto$ predicate is NP-complete.

[1] by Post theorem