The summary for the article of Michal Kunc "The Power of Commuting with Finite Sets of Words".

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1 The main definitions and notations.

The finite set A will be the alphabet in following definitions.

The set A^+ is the set of non-empty finite words over the alphabet A.

The set A^* is the set of all words over the alphabet A; $A^* = A^+ \cup \{\varepsilon\}$, where ε is the empty word.

The languages over the alphabet A are arbitrary subsets of A^*.

The main operations on languages:

- Union. For languages K and L we can consider their union $K \cup L$.
- Concatenation. The concatenation of K and L is $KL = \{uv \mid u \in U, v \in V\}$.
- The Kleene star. This operation is denoted as L^* for language L, and $L^* = \cup_{m \in \mathbb{N}_0} L^m$, where $L^m = LL \dots L$ (m times), $L^0 = \{\varepsilon\}$.
- Complementation. The complementation of L is the set $A^* \setminus L$.

Regular languages are languages definable by finite automata, or equivalently, by rational expressions. Every regular language can be obtained from finite languages using union, concatenation and the Kleene star.

The language is called star-free if it can be obtained from finite languages using the operations of union, complementation and concatenation.

Let us consider the commutation equation $XL = LX$ for language L.
The language $C(L)$ is the largest language over A, which commutes with L. ($C(L)$ is the largest solution of the equation $XL = LX$, all other solutions are subsets of $C(L)$).
The language $C^+(L)$ is the largest ε-free language (without empty word) which commutes with L.

2 The main results.

Theorem 1. *There exists a star-free language L such that:*

1. *The largest language commuting with L is not recursively enumerable.*
2. *The difference between the largest language commuting with L and the largest ε-free language commuting with L is not recursively enumerable.*

The following theorem is the main result of the paper.

Theorem 2. *There exists a finite language L such that the largest language commuting with L and the largest ε-free language commuting with L are not recursively enumerable.*

These theorems are important results in theory of language equations, the proofs of these theorems are very difficult and technical.