$$\frac{4\pi}{n}\sum_{k=1}^{n}\sin\left(\frac{4\pi}{n}\cdot k\right) = \left[t = 2k\right] = \frac{4\pi}{n}\sum_{t=2,\ t\vdots 2}^{2n}\sin\left(\frac{2\pi}{n}\cdot t\right) =$$

$$= \frac{4\pi}{n}\left(\sum_{t=2,\ t\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot t\right) + \sum_{t=n+2,\ t\vdots 2}^{2n}\sin\left(\frac{2\pi}{n}\cdot t\right)\right) = \Big[\ p = t-n;\ t = p+n$$

$$t_0 = n+2,\ p_0 = 2$$

$$t_e = 2n,\ p_e = n \Rightarrow$$

$$\Rightarrow \sum_{t=n+2;\ t\vdots 2}^{2n}\sin\left(\frac{2\pi}{n}\cdot t\right) = \sum_{p=2,\ p\vdots 2}^{n}\sin\left(\frac{2\pi}{n}(p+n)\right) = \sum_{p=2,\ p\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot p + 2\pi\right) = \sum_{p=2,\ p\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot p\right)\Big]$$

$$= \frac{4\pi}{n}\cdot 2\cdot\sum_{t=2,\ t\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot t\right) = A$$

а) n – чётное $\Rightarrow A = \frac{4\pi}{n}\cdot 2\cdot\left(\sum_{t=2,\ t\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\cdot t\right) + \sum_{t=\frac{n}{2}+2,\ t\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot t\right)\right) =$

$$= \Big[\ p = t - \frac{n}{2},\ p_0 = 2,\ p_e = \frac{n}{2},\ t = p + \frac{n}{2} \Rightarrow \sum_{t=\frac{n}{2}+2,\ t\vdots 2}^{n}\sin\left(\frac{2\pi}{n}\cdot t\right) =$$

$$= \sum_{p=2,\ p\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\left(p+\frac{n}{2}\right)\right) = \sum_{p=2,\ p\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\cdot p + \pi\right) = -\sum_{p=2,\ p\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\cdot p\right)\Big] =$$

$$= \frac{4\pi}{n}\cdot 2\left(\sum_{t=2\ t\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\cdot t\right) - \sum_{p=2,\ p\vdots 2}^{\frac{n}{2}}\sin\left(\frac{2\pi}{n}\cdot p\right)\right) = \frac{4\pi}{n}\cdot 2\left(\sum_{t=2,\ t\vdots 2}^{\frac{n}{2}}\underbrace{\left(\sin\left(\frac{2\pi}{n}\cdot t\right) - \sin\left(\frac{2\pi}{n}\cdot t\right)\right)}_{0}\right)$$

$$\Rightarrow A = 0$$

б) n – нечётное $\Rightarrow A = \frac{4\pi}{n}\cdot 2\cdot\left(\sum_{t=2,\ t\vdots 2}^{\lfloor\frac{n}{2}\rfloor}\sin\left(\frac{2\pi}{n}\cdot t\right) + \sum_{t=\lfloor\frac{n}{2}\rfloor+2,\ t\vdots 2}^{n-1}\sin\left(\frac{2\pi}{n}\cdot t\right) +\right.$

$$\left. + \sin\left(\frac{2\pi}{n}\left(\left\lfloor\frac{n}{2}\right\rfloor + 1\right)\right)\right) = \Big[\ n = 2q+1 \Rightarrow \left|\left[2;\ \left\lfloor\frac{n}{2}\right\rfloor\right]\right| = \left|\left[2;\ q\right]\right| =$$

$= q-2+1 = q-1;\quad \left|\left[\left\lfloor \frac{n}{2} \right\rfloor+2; n-1\right]\right| = \left|\left[q+2; n-1\right]\right| = n-1-q-2+1 =$

$= n-q-2 =$

$= 2q+1-q-2 = q-1 = \left|\left[2; \left\lfloor \frac{n}{2} \right\rfloor\right]\right| \Rightarrow$ в суммах

одинаковое число элементов $\Rightarrow$ они

„самоуничтожатся", как в п. а) $\Big] =$

$$= \frac{4\pi}{n} \cdot 2 \cdot \left(0 + \sin\left(\frac{2\pi}{n}\left(\left\lfloor \frac{n}{2} \right\rfloor+1\right)\right)\right) = \frac{8\pi}{n} \cdot \sin\left(\frac{2\pi}{n}\left(\left\lfloor \frac{n}{2} \right\rfloor+1\right)\right);$$

$\lim\limits_{n\to\infty} A \to 0$, т.к. $\frac{8\pi}{n}$ – б.м., $\sin\left(\frac{2\pi}{n}\left(\left\lfloor \frac{n}{2} \right\rfloor+1\right)\right)$ – огр. $\Rightarrow$

$\Rightarrow A \to 0$