Replication Information and Results

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November 25, 2025

1 Introduction

This document accompanies the GitHub repository[1] that illustrates and provides code for the estimation procedure developed in the paper *"Jump Contagion among Stock Market Indices: Evidence from Option Markets"* written by H. Peter Boswijk, Roger J. A. Laeven, Andrei Lalu and Evgenii Vladimirov.

2 Synthetic, Simulated Data

The code illustrates the estimation procedure developed in the paper given a synthetic, simulated data-set. The original data-set used in the paper has been purchased from the Thomson Reuters Tick History database, hence public sharing of this proprietary data is not possible. Instead, we provide a synthetic, simulated data-set, which allows analogous versions of the tables and figures in the paper, included in this document, to be replicated. This follows the Guidance for Authors provided by the *Journal of the American Statistical Association*; due to the proprietary nature of the data not all tables and figures in the paper can be considered for replication. The simulated data, stored as a MATLAB `.mat` file in the data folder, have been generated using the estimated bivariate option pricing model proposed in the paper. More specifically, the state vectors (stored in the variables `mY`, `mV`, and `mLambda`) are simulated us-

[1] `https://github.com/evladimirov/Jump-Contagion`

ing an Euler discretization, and the option prices (stored in the variables `mOptPriceIV1` and `mOptPriceIV2`) are computed using the COS method, see Fang and Oosterlee (2008). For further details concerning the simulations, see Section 4 of the main text and Appendix C.4 of the Supplementary Material.

3 Main Building Blocks of the Code

The main code (in the file `main.m`) first reads the simulated options data, and then estimates the parameters of the bivariate model proposed in the paper based on the partial-information implied-state C-GMM procedure.

The estimation procedure minimizes the criterion function:
`./code/mSVhatHJ_crit_inst4.m`,
which in turn involves the implied state procedure
`./code/mSVhatHJ_ImpIntens.m`,
and four numerical integrations of criterion functions based on the marginal states
`./code/mSVhatHJ_int_inst4.m`.
Given the estimated parameters, the standard errors are calculated using the function
`./code/mSVhatHJ_std4.m`
MATLAB 2018b is required for successful replication of the results below.

4 Overview of the Output and Function Dependences

Successfully running the `main.m` file will produce the following results.

4.1 Table 3

After running lines 3–40 of the code in the main file, `main.m`, the estimated parameters are displayed as the result of the optimization procedure based on the simulated data. These results are provided in Table R.1 below and are analogous to Table 3 in the paper.

The following functions are used to produce this table:

- `./code/mSVhatHJ_crit_inst4.m`
- `./code/mSVhatHJ_ImpIntens.m`
- `./code/mSVhatHJ_ODE_cos.m`
- `./code/mSVhatHJ_int_inst4.m`
- `./lib/fwtpts.m`

- ./lib/affineODE.m
- ./lib/my_ode45.m
- ./lib/calcBSImpVol.m
- ./code/mSVhatHJ_fmin_constr.m
- ./lib/FMINSEARCHBND/fminsearchcon.m
- ./code/mSVhatHJ_std4.m
- ./code/mSVhatHJ_ccf4.m
- ./code/mSVhatHJ_mom4.m

Table R.1: Parameter estimates based on simulated data

	$\mu^{\mathbb{Q}}$	σ	κ	$\overline{\lambda}$	δ^s	δ^c	μ	η
index-1	-0.127	0.027	5.627	0.944	3.201	1.151	-0.035	2.986
	(0.0005)	(0.0009)	(0.0158)	(0.0030)	(0.0181)	(0.0096)	(0.0118)	(4.7736)
index-2	-0.122	0.031	4.568	0.788	2.135	3.288	-0.036	4.182
	(0.0008)	(0.0030)	(0.0075)	(0.0024)	(0.0116)	(0.0126)	(0.0102)	(4.7159)

This table reports bivariate model parameter estimates of the partial-information implied-state C-GMM procedure using simulated data. Standard errors are reported in parentheses.

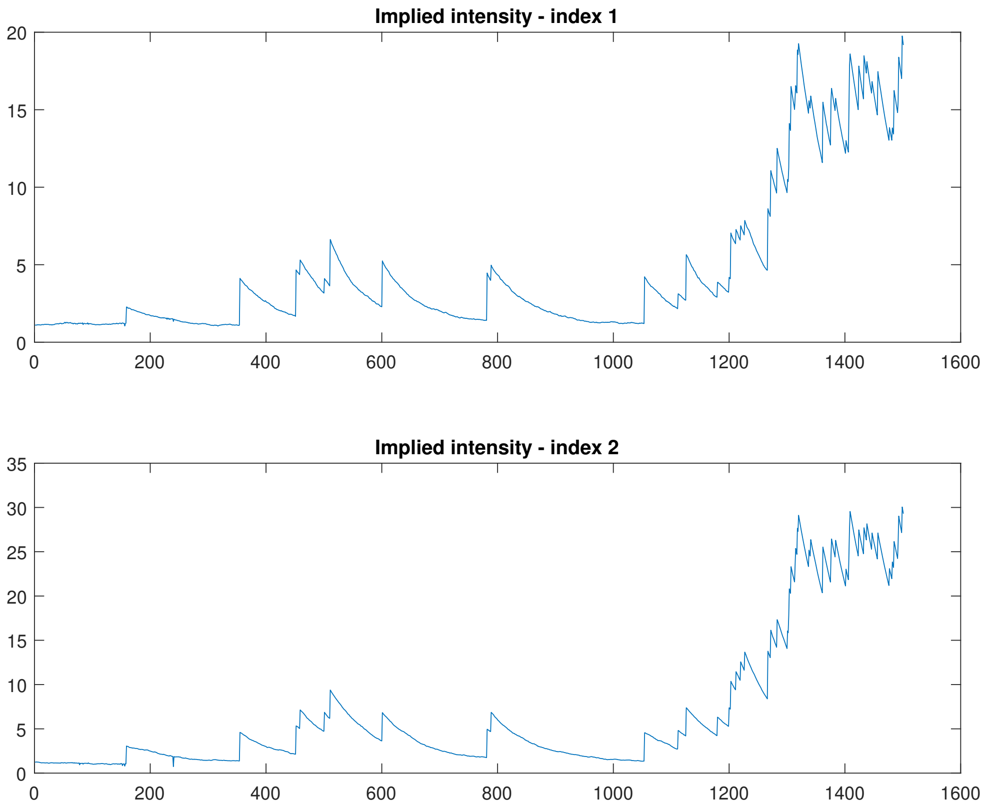
4.2 Figure 2

Running lines 42–51 of the code in the main file, `main.m`, generates a figure illustrating the implied intensities, similar to Figure 2 in the paper. This figure is depicted in Figure R.1 below.

The following functions are used to produce this figure:

- ./code/mSVhatHJ_ImpIntens.m
- ./code/mSVhatHJ_ODE_cos.m
- ./lib/affineODE.m
- ./lib/my_ode45.m
- ./lib/calcBSImpVol.m

Figure R.1: Implied intensities based on simulated data



4.3 Table 4

Running lines 54–75 of the code in the main file, `main.m`, produces the option pricing fit table given the parameter estimates from the simulated data. The output is analogous to Table 4 in the paper and is provided in Table R.2 below.

The following functions are used to produce this table:

- `./code/mSVhatHJ_price_opt.m`
- `./code/mSVhatHJ_ImpIntens.m`
- `./code/mSVhatHJ_ODE_cos.m`
- `./lib/affineODE.m`
- `./lib/my_ode45.m`
- `./lib/calcBSImpVol.m`

- ./lib/Opt_price_cos.m

Table R.2: Option prices: fit of simulated data

k	0.80	0.85	0.90	0.95	1.00	1.05	1.10	Total
index-1	0.22	0.25	0.21	0.29	0.43	0.76	1.71	0.75
index-2	0.31	0.29	0.27	0.27	0.24	0.69	1.3	0.6

This table reports the root mean square errors (RMSEs, displayed as a percentage) of the simulated option prices, expressed in terms of the market-observed and model-implied Black-Scholes implied volatility, as a function of the strike-to-forward ratio $k = K/F$, using the bivariate model and parameter estimates.

4.4 Table 5

Running lines 77–94 of the code in the main file, `main.m`, provides the parameter estimates for the first index from the univariate model. The results are also reported in Table R.3 and are analogous to Table 5 in the main text.

The following functions are used to produce this table:

- ./code/SVhatHJ/SVhatHJ_crit_inst.m
- ./code/SVhatHJ/SVhatHJ_ImpIntens.m
- ./code/SVhatHJ/SVhatHJ_ODE_cos.m
- ./code/SVhatHJ/SVhatHJ_ccf2.m
- ./code/SVhatHJ/SVhatHJ_int_inst.m
- ./lib/affineODE.m
- ./lib/my_ode45.m
- ./lib/calcBSImpVol.m
- ./lib/FMINSEARCHBND/fminsearchcon.m

Table R.3: Univariate model estimation result

	$\mu^{\mathbb{Q}}$	σ	κ	$\overline{\lambda}$	δ	μ	η
index-1	-0.120	0.031	4.645	1.341	3.009	-0.031	3.124

This table reports univariate model parameter estimates for the first index.

4.5 Table 6

Running line 99 of the code in the main file, `main.m`, calls the function `Table6.m`, which reproduces Table 6 from the paper. Table R.4 duplicates this table here for convenience.

The following functions are used to produce this table:

- `./code/Table6.m`
- `./code/mSVhatHJ_sim.m`

Table R.4: Descriptive statistics for the conditional log-return distribution (simulated using model parameter estimates, horizon $h = 10$ days).
The table corresponds to Table 6 in the paper.

	0.1%	1%	5%	25%	50%	75%	95%	S	K	$\mathbb{E}[N_t \mid \lambda_0]$
	(a) Base Case: $\lambda_{1,0} = \overline{\lambda}_1, \lambda_{2,0} = \overline{\lambda}_2$									
Bivariate - FTSE	-7.51	-3.17	-2.11	-0.78	0.11	1.00	2.27	-0.49	6.04	0.0071
Univariate - FTSE	-6.22	-3.11	-2.09	-0.77	0.12	1.01	2.29	-0.28	4.55	0.0077
Bivariate - S&P	-7.85	-3.21	-2.09	-0.77	0.13	1.02	2.31	-0.59	7.21	0.0075
Univariate - S&P	-8.57	-3.20	-2.08	-0.75	0.14	1.04	2.33	-0.72	8.74	0.0077
	(b) Euro Debt Crisis: $\lambda_{1,0} = \lambda_{2,0} = 5$									
Bivariate - FTSE	-12.77	-7.56	-3.03	0.32	1.37	2.33	3.68	-2.15	12.07	0.1248
Univariate - FTSE	-10.63	-6.14	-2.11	0.32	1.34	2.29	3.65	-1.69	10.28	0.1234
Bivariate - S&P	-13.26	-7.57	-2.66	0.51	1.54	2.50	3.88	-2.17	12.39	0.1227
Univariate - S&P	-14.53	-8.27	-2.56	0.67	1.70	2.65	4.08	-2.29	14.06	0.1227
	(c) S&P Shock: $\lambda_{1,0} = 20, \lambda_{2,0} = \overline{\lambda}_2$									
Bivariate - FTSE	-9.27	-3.73	-2.10	-0.68	0.24	1.15	2.46	-0.93	8.41	0.0194
Univariate - FTSE	-6.22	-3.11	-2.09	-0.77	0.12	1.01	2.29	-0.28	4.55	0.0077
Bivariate - S&P	-15.91	-8.41	-2.95	3.77	5.90	7.12	8.78	-1.74	7.53	0.4872
Univariate - S&P	-17.17	-8.96	-3.04	4.51	6.57	7.80	9.63	-1.74	7.89	0.4906
	(d) FTSE Shock: $\lambda_{1,0} = \overline{\lambda}_1, \lambda_{2,0} = 20$									
Bivariate - FTSE	-15.80	-8.94	-3.65	2.77	5.13	6.36	7.93	-1.68	6.82	0.4838
Univariate - FTSE	-12.07	-6.38	-2.03	3.34	5.16	6.36	7.99	-1.54	6.71	0.4906
Bivariate - S&P	-8.53	-3.27	-2.07	-0.73	0.17	1.06	2.36	-0.75	8.34	0.0105
Univariate - S&P	-8.57	-3.20	-2.08	-0.75	0.14	1.04	2.33	-0.72	8.74	0.0077
	(e) 2008 Global Financial Crisis: $\lambda_{1,0} = 20, \lambda_{2,0} = 15$									
Bivariate - FTSE	-15.19	-8.81	-3.88	2.28	4.00	5.16	6.69	-1.81	7.70	0.3760
Univariate - FTSE	-11.88	-6.56	-2.49	2.45	3.89	5.01	6.54	-1.65	7.62	0.3688
Bivariate - S&P	-15.93	-8.39	-2.95	3.78	5.93	7.15	8.81	-1.74	7.52	0.4895
Univariate - S&P	-17.17	-8.96	-3.04	4.51	6.57	7.80	9.63	-1.74	7.89	0.4906

This table displays the empirical quantiles (in percentages), skewness (S), kurtosis (K), and expected number of jumps implied by the conditional distribution of simulated log-returns for S&P 500 ("index 1") and FTSE 100 ("index 2"). The stock index price paths are simulated using bivariate and univariate model parameter estimates, conditional upon different values ("scenarios") of the latent jump intensities. The return horizon is $h = 10$ days. Volatilities are assumed to be constant throughout the horizon and are set to $v_{i,s} = 8.36\%$ for both indices, and the instantaneous Brownian correlation is set to be 0.6.

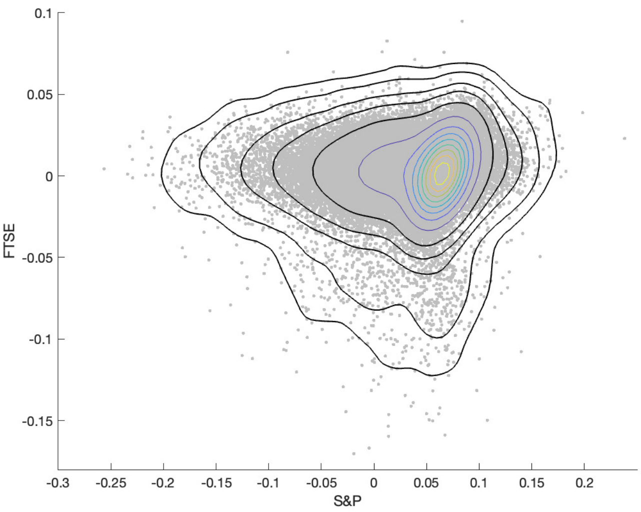
4.6 Figure 3

Running line 102 in the main file, `main.m`, calls the function `Figure3.m`, which generates Figure 3 from the paper. Figure R.2 duplicates this figure here for convenience.

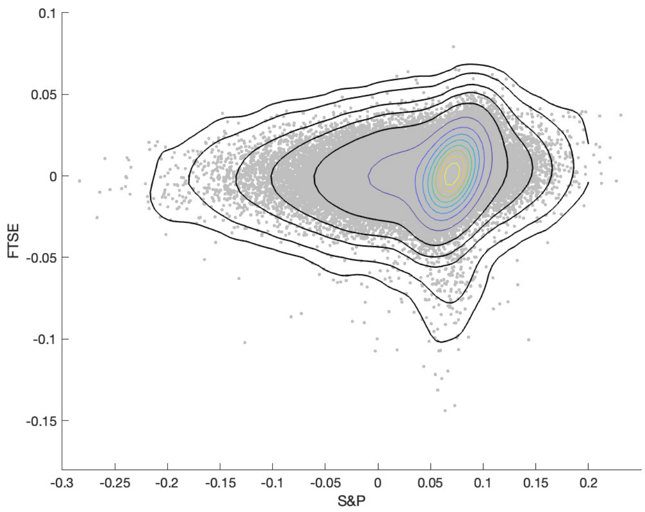
The following functions are used to produce this figure:

- `./code/Figure3.m`
- `./code/mSVhatHJ_sim.m`

Figure R.2: Contour plots, with $\lambda_{1,0} = 20, \lambda_{2,0} = \overline{\lambda}_2, h = 10$.
The figure corresponds to Figure 3 in the paper.



(a) Bivariate estimates



(b) Univariate estimates

Note: Contour plots overlayed on top of scatter plots of log-return data simulated using parameter estimates for the bivariate model (panel (a)) and the two univariate models (panel (b)). Return horizon set to $h = 10$ days. Initial jump intensities set to $\lambda_{1,0} = 20$ for S&P 500 and $\lambda_{2,0} = \overline{\lambda}_2$ for FTSE 100. Volatilities are assumed to be constant throughout the horizon and are set to $v_{i,s} = 8.36\%$ for both indices, and the instantaneous correlation between Brownian increments is set to be 0.6.

4.7 Table 2 and Table C.1

Finally, running line 107 in the main file, `main.m`, calls the function `mSVhatHJ_monte_carlo.m`, which runs the Monte Carlo simulation and reproduces Table 2 from the main text as well as Table C.1 from the Supplementary Appendix. Table R.5 duplicates the simulation results here for convenience. Note that the Monte Carlo simulation requires a significant amount of time (i.e., several days) to complete.

The following functions are used to produce this table:

- `./code/mSVhatHJ_monte_carlo.m`

- ./code/MSVHJ_sim.m
- ./code/MSVHJ_sim_opt.m
- ./code/MSVHJ_ODE_cos.m
- ./lib/Opt_price_cos.m
- ./lib/fwtpts.m
- ./code/mSVhatHJ_crit_inst4.m
- ./code/mSVhatHJ_ImpIntens.m
- ./code/mSVhatHJ_ODE_cos.m
- ./code/mSVhatHJ_int_inst4.m
- ./lib/affineODE.m
- ./lib/my_ode45.m
- ./lib/calcBSImpVol.m
- ./code/mSVhatHJ_fmin_constr.m
- ./lib/FMINSEARCHBND/fminsearchcon.m

References

Fang, F., & Oosterlee, C. W. (2008). A novel pricing method for European options based on Fourier-cosine series expansions. *SIAM Journal on Scientific Computing*, *31*(2), 826–848.

Table R.5: Simulation results for the bivariate model.
The table corresponds to Table C.1 and Table 2 in the paper.

	$\mu_1^{\mathbb{Q}_1}$	σ_1	κ_1	$\overline{\lambda}_1$	δ_{11}	δ_{12}	μ_1	η_1
true	-0.130	0.030	6.000	1.000	3.000	1.000	-0.040	2.000
mean	-0.129	0.032	5.816	1.005	2.909	1.035	-0.038	1.966
std	0.010	0.008	0.464	0.193	0.335	0.186	0.007	1.972
25%	-0.133	0.027	5.520	0.924	2.685	0.925	-0.042	1.560
50%	-0.129	0.031	5.872	1.043	2.901	1.050	-0.038	2.467
75%	-0.125	0.034	6.116	1.086	3.070	1.131	-0.035	2.957

	$\mu_2^{\mathbb{Q}_2}$	σ_2	κ_2	$\overline{\lambda}_2$	δ_{22}	δ_{21}	μ_2	η_2
true	-0.130	0.030	5.000	1.000	2.000	3.000	-0.040	2.000
mean	-0.128	0.030	4.895	1.071	2.010	3.052	-0.039	1.676
std	0.008	0.006	0.281	0.243	0.244	0.410	0.008	2.240
25%	-0.132	0.028	4.729	0.945	1.835	2.803	-0.043	1.333
50%	-0.127	0.030	4.925	1.083	2.002	3.074	-0.039	2.237
75%	-0.123	0.033	5.073	1.175	2.135	3.323	-0.036	2.667

This table provides Monte Carlo results for the bivariate model using the partial-information criterion function, and the semi-nonparametric approximation. Each iteration consists of 1500 time points including simulated stock prices and 8 option prices for each time observation. True parameters and Monte Carlo sample means, standard deviations and 25%, 50%, 75% quantiles are presented on separate rows. The following parameters are used to simulate the stochastic volatility processes: $\nu_1 = \nu_2 = 4.8$, $\overline{\xi_1^2} = \overline{\xi_2^2} = 0.015$, $\sigma_{\xi,1} = \sigma_{\xi,2} = 0.22$, $\rho_{\xi,1} = \rho_{\xi,2} = -0.6$.