Electric Circuits I

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Project 1: Optimal Power Delivery

In the circuit below the objective is to deliver power to resistors R_9 and R_{10}.

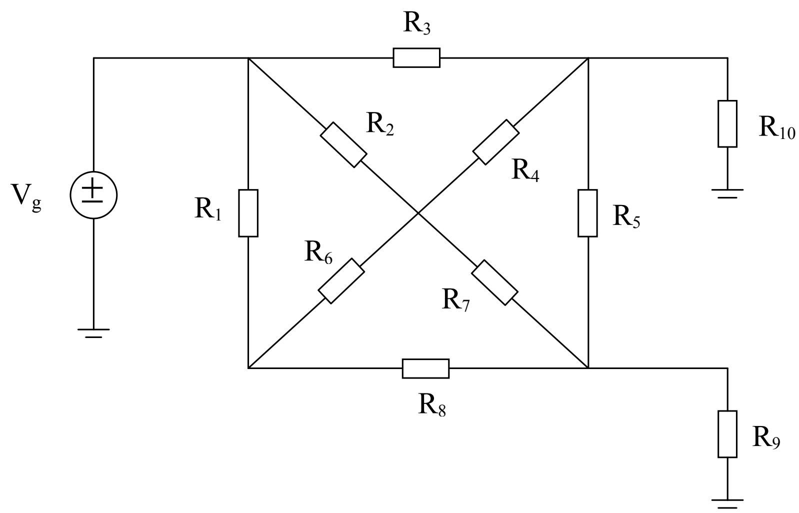

Fig. 1. A power distribution network.

All resistors except R_9 should be chosen randomly, and V_g is assumed to be $12\,V$.

Problem 1. Find a Thevenin equivalent with respect to R_9. Use this result to determine conditions under which R_9 absorbs *maximal* power.

Problem 2. Create an m-file in Matlab that will compute powers P_9, P_{10} and the efficiency coefficient

$$\eta = \frac{(P_9 + P_{10})}{P_G} \tag{1}$$

for different values of R_9. Plot $P_9(R_9)$ and $P_{10}(R_9)$ on one graph and $\eta(R_9)$ on another using a logarithmic scale for R_9.

NOTE: Your plot for $P_9(R_9)$ should be consistent with the results obtained in Problem 1.

Problem 3. Based on the plots obtained in Problem 2, select R_9 so that η is maximized with the following additional constraint:

$$0.7 \leq P_9/P_{10} \leq 1.3 \tag{2}$$

Problem 4. Assemble the circuit in Fig. 1 and measure P_9, P_{10} and η over an appropriate range of values for R_9. Plot the measured values in Matlab and compare with your simulation results. Have the requirements for P_9, P_{10} and η been satisfied with the choice of R_9 obtained in Problem 3?

NOTE: Think about how you are going to measure the generated power P_G *before* you come to lab!

Project Tutorial for ELEN 50

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Consider the following circuit

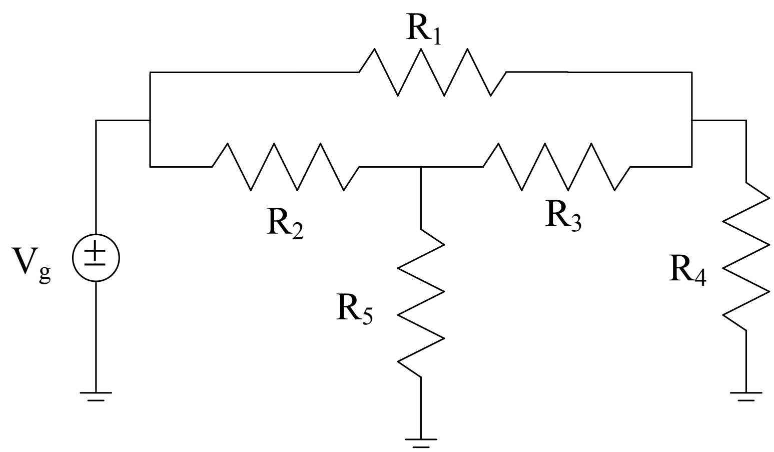

Fig. 1. A simple power transmission system.

in which $R_1 = 500\,\Omega$, $R_2 = 1,000\,\Omega$, $R_3 = 500\,\Omega$, $R_5 = 400\,\Omega$ and $V_g = 12\,V$ are given. If we view resistors R_1, R_2 and R_3 as transmission lines and R_4 and R_5 as loads to which energy must be supplied, this circuit looks very much like a simple power system model. In such a model, loads are used to represent a wide variety of power consuming entities, ranging from simple appliances to industrial plants and even entire cities. This diversity suggests that some of the loads in the system can be controlled, while others can not. To reflect this fact, we will assume that resistance R_4 is controllable, and our objective will be to choose its value so that power is delivered to both loads in the most efficient manner possible.

Before proceeding, it is important to understand that the analogy between the circuit in Fig. 1 and a realistic power system should not be taken too far. There are a number of very important differences, two of which are singled out below:

(i) The circuit in Fig. 1 is a DC circuit, while a power system is characterized by *sinusoidal* currents and voltages.

(ii) Loads and transmission lines in a realistic power system cannot be adequately modeled by resistors. A proper representation requires inductors and capacitors as well (you will learn much more about this in ELEN 105).

In order to get a sense for the practical issues that arise in the process of power delivery, we will examine two possible strategies for selecting resistor R_4.

Design Strategy 1. Choose R_4 so that the power absorbed by this resistor is maximized (this type of requirement is common in communication systems).

The implementation of this strategy is best understood by considering the simple circuit in Fig. 2, in which V_g and R_S are *fixed* and R_L can be adjusted. Since the current through R_L is given as

$$i = \frac{V_g}{R_S + R_L} \tag{1}$$

the power absorbed by this resistor can be expressed as

$$P_{R_L} = R_L i^2 = \frac{R_L}{(R_S + R_L)^2} V_g^2 \tag{2}$$

It is easily verified that this power is maximal when

$$\frac{dP_{R_L}}{dR_L} = 0 \tag{3}$$

which occurs for $R_L = R_S$. Such a choice for R_L produces

$$P_{\max} = \frac{1}{4} \frac{V_g^2}{R_S} \tag{4}$$

which is the largest amount of power that this resistor can absorb.

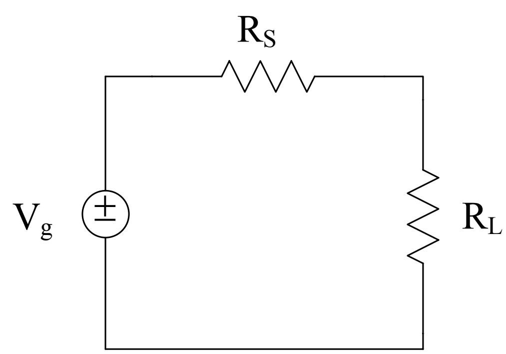

Fig. 2. A simplified circuit for studying maximal power delivery.

The above result can be easily extended to the circuit in Fig. 1 if we compute a Thevenin equivalent with respect to resistor R_4. In that case our problem reduces to the circuit in Fig. 2, in which V_g and R_S are replaced by V_{th} and R_{th}, respectively. For the given element values, it is easily verified that $R_{th} = 305.5\,\Omega$ and $V_{th} = 8.67\,V$, so the optimal value for R_4 is $306\,\Omega$.

Maximizing the power absorbed by load R_4 is not always the best design strategy. In fact, in many cases it is more important to minimize the energy that is lost in the course of transmission (such a requirement is very common in power systems). From that standpoint, it would be appropriate to maximize the so-called *efficiency ratio*, which is defined as

$$\eta = \frac{\text{Total load power}}{\text{Total generated power}} \tag{5}$$

In our circuit, this ratio can be expressed as

$$\eta = \frac{(P_{R4} + P_{R5})}{P_G} \tag{6}$$

To evaluate η for Design Strategy 1, we must first solve the circuit in Fig. 1 with $R_4 = 306\,\Omega$. The KCL equations for this circuit are

$$\begin{aligned} &1)\ \ i_g + i_{R1} + i_{R2} = 0 \\ &2)\ \ -i_{R1} + i_{R3} + i_{R4} = 0 \\ &3)\ \ -i_{R2} - i_{R3} + i_{R5} = 0 \end{aligned} \qquad \begin{aligned} i_{R1} &= (V_1 - V_2)/R_1 \\ i_{R2} &= (V_1 - V_3)/R_2 \\ i_{R3} &= (V_2 - V_3)/R_3 \\ i_{R4} &= V_2/R_4 \\ i_{R5} &= V_3/R_5 \\ i_g &= \,? \end{aligned} \tag{7}$$

Equations (7) can be rewritten in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/R_1 & (1/R_1 + 1/R_3 + 1/R_4) & -1/R_3 \\ -1/R_2 & -1/R_3 & (1/R_2 + 1/R_3 + 1/R_5) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_g \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

and their solution is

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12.000\,V \\ 4.3365\,V \\ 3.7587\,V \end{bmatrix} \tag{9}$$

Based on these results, we can conclude that resistors R_4 and R_5 absorb $P_{R4} = 0.0615\,W$ and $P_{R5} = 0.0353\,W$, respectively. Since the power delivered by the generator is $P_G = V_g i_g$, we also need to compute current i_g. We can do this using the first KCL equation, which implies

$$i_g = -i_{R1} - i_{R2} = 0.0236\,A \tag{10}$$

From (10) it follows that the generator delivers

$$P_G = 0.2828\,W \tag{11}$$

and that the corresponding efficiency coefficient is $\eta = 0.3422$.

The obtained result points to two potential weaknesses of Design Strategy 1:

(i) Only 34% of the generated power reaches the loads, while the rest is lost in the transmission system.

(ii) The power absorbed by resistor R_4 is almost twice as large as the one absorbed by R_5. A more balanced distribution would be preferable.

In order to resolve these problems, we will now consider an alternative strategy:

Design Strategy 2. Choose R_4 to maximize the efficiency coefficient, while keeping the power distribution reasonably balanced. In practial terms, the latter requirement can be expressed as an inequality constraint, such as

$$0.8 \le P_{R4}/P_{R5} \le 1.2 \tag{12}$$

or something similar.

This problem is much harder than the previous one, and has no analytic solutions (Thevenin equivalents can't help us here). One possible approach is to repeatedly solve system (8) for different values of R_4, and plot the corresponding function $\eta(R_4)$. This is not difficult to do in Matlab if we rewrite equation (8) as

$$(G_1 + \frac{1}{R_4} G_2)V = b \tag{13}$$

where

$$G_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1/R_1 & (1/R_1 + 1/R_3) & -1/R_3 \\ -1/R_2 & -1/R_3 & (1/R_2 + 1/R_3 + 1/R_5) \end{bmatrix} \tag{14}$$

and

$$G_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{15}$$

are *fixed* matrices, V is the vector of unknown voltages, and

$$b = \begin{bmatrix} V_g \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The Matlab program for this problem is provided in Appendix 1, and the simulation results are shown in Figs. 3 and 4.

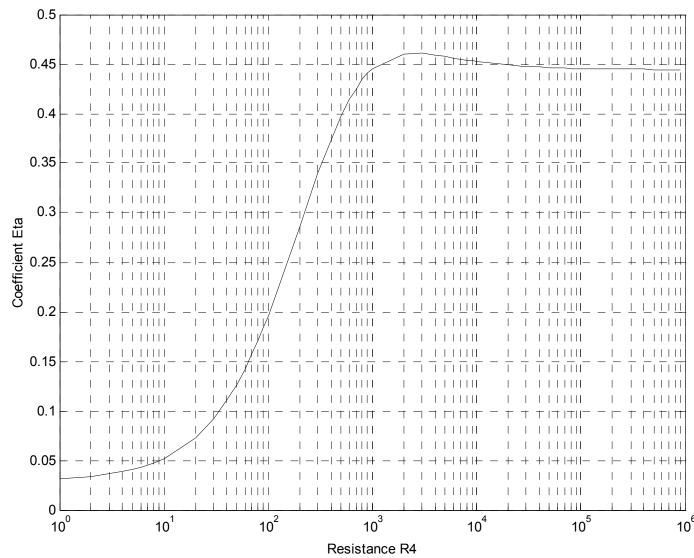

Fig. 3. The efficiency coefficient η as a function of R_4.

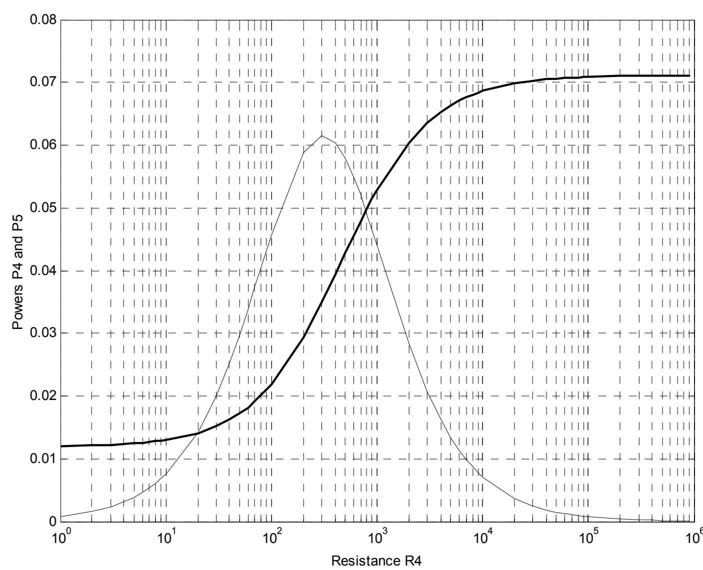

Fig. 4. Powers P_4 and P_5 (heavier line) as functions of R_4.

The plot in Fig. 3 suggests that $R_4 = 2,000\,\Omega$ is the best choice from the standpoint of maximizing η, producing $\eta = 0.46$. However, Fig. 4 shows that for such a choice the power P_{R5} is almost twice as large as P_{R4}. A better balance is achieved by selecting $R_4 = 1,000\,\Omega$, in which case η is only slightly reduced (to $\eta = 0.447$), but $P_{R4}/P_{R5} = 0.85$.

Appendix 1

```
function [eta,P4,P5]=powerdeliv(G1,G2,b,R)
% G1, G2 and b are known vectors and matrices. R is the
% vector of resistance values for which you want to calculate
% the power in the circuit.

R1=500; R2=1000;
R3=500; R5=400; VG=12;
% These element values are fixed in the circuit.

for i=1:length(R)
   % We have to solve the circuit once for each different value of R4.
   % Since all these different values are stored in vector R, length(R)
   % determines how many times we must execute the loop.

   R4=R(i);
   % R4 takes the next value from vector R.

   A=G1+(1/R4)*G2;
   V=A\b;
   % V contains the solution of the node voltage equations.

   PR4=(V(2)^2)/R4;
   PR5=(V(3)^2)/R5;
   IR1=(V(1)-V(2))/R1;
   IR2=(V(1)-V(3))/R2;
   IG=IR1+IR2;
   PG=VG*IG;
   % Computation of the powers associated with R4, R5 and the
   % generator.

   P4=[P4 PR4];
   P5=[P5 PR5];
   % Powers PR4 and PR5 computed for different values of R4 are
   % stored in vectors P4 and P5.

   W=(PR4+PR5)/PG;
   eta=[eta W];
   % Eta represents the percentage of the generated power that is
   % absorbed by resistors R4 and R5.
end
```