Note that this report should cover from labs 1 to 4

# Lab 3 & 4 Fourier series for Low pass filter and high pass filter

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ECE3101L - Andrew Pagnon
October 6, 2022

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# Summary

In the following Lab we observe the different wave signals (sine square and sawtooth when they are to 2V with the period of .1ms) and we observe their output when passing through first and third order low pass or a first and third order high pass

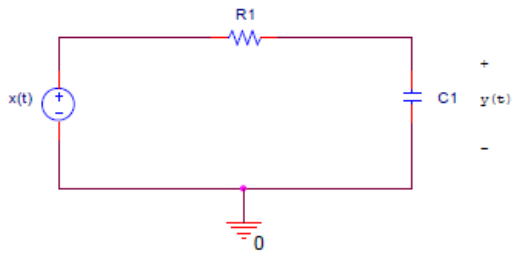
# Objective

- Understand the Low and High pass filter, and the their response when they are at high and low frequency
- Understand how the composition of the filter could affect the
- Calculate the complex exponential Fourier Series coefficient Xn of the sinusoid, square and sawtooth when it is affected by a low pass filter and a high pass filter

# Math

## Filter transfer function

### Low pass filter



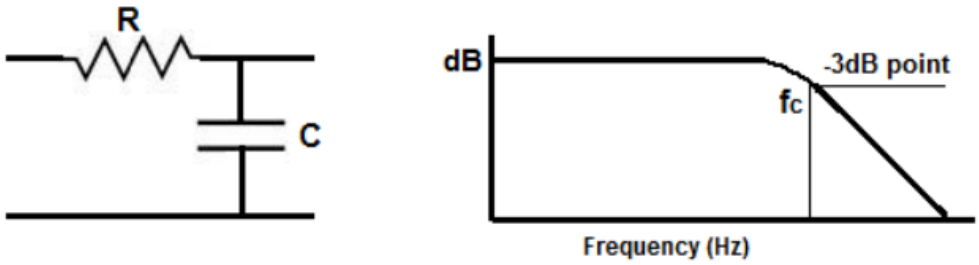

In a low pass filter Vout is the Voltage through the Capacitor

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{V_{out}}{V_{in}} = \frac{V_c}{V_r + V_c}$$

$$H(jw) = \frac{I_c * Z_c}{I_c * Z_r + I_c * Z_c} = \frac{Z_c}{Z_r + Z_c}$$

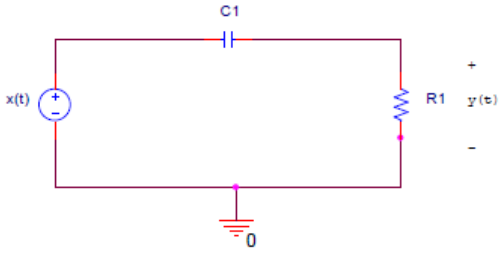
$$H(jw) = \frac{\frac{1}{jwC}}{R + (\frac{1}{jwC})} * \frac{jwC}{jwC}$$

$$H(jw) = \frac{1}{jwRC + 1}$$




The low pass filter allows for the low frequencies while blocking high frequencies

## High pass filter



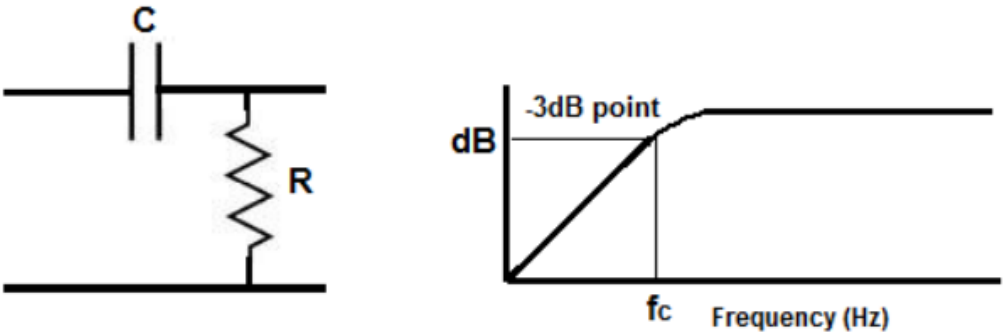

In a High pass filter Vout is the Voltage through the Capacitor

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{V_{out}}{V_{in}} = \frac{V_r}{V_r + V_c}$$

$$H(jw) = \frac{I_r * R}{I_r * R + I_r * Z_c} = \frac{R}{R + Z_c}$$

$$H(jw) = \frac{R}{R + (\frac{1}{jwC})} * \frac{jwC}{jwC}$$

$$H(jw) = \frac{jwRC}{jwRC + 1}$$




The high pass filter allows the high frequencies through while blocking or reducing the low frequencies

## Find the 3dB cutoff frequency ω3dB as a function of R1 and C1

$$filter\ gain\ (dB) = 10\ *\ (linear\ power\ gain)\ = \left(\frac{1}{2}\right)\ = -\ 3dB$$

$$Power = \frac{Voltage^2}{R} \Rightarrow Power\ Gain = \frac{\left(\frac{Vout}{R}\right)^2}{\left(\frac{Vin}{R}\right)^2} = \left(\frac{Vout}{Vin}\right)^2 = (Voltage\ Gain)^2$$

$$3dB\ corner\ frequency\ Power\ Gain = \frac{1}{2} = (Voltage\ Gain)^2$$

$$Voltage\ Gain = \frac{1}{\sqrt{2}}$$

3dB cutoff happens when: $|H(w)| = \frac{1}{\sqrt{2}}$

Low pass filter transfer function= $\frac{1}{\sqrt{1+(jwRC)^2}}$

$$|H(w)| = \frac{1}{\sqrt{1+(jwRC)^2}} = \frac{1}{\sqrt{2}}$$

$$\sqrt{1 + (jwRC)^2} = \sqrt{2}$$

$$1 + (jwRC)^2 = 2$$

$$(jwRC)^2 = 1$$

$$-w^2R^2C^2 = 1$$

$$-w^2 = \frac{1}{R^2C^2}$$

$$w(3dB) = \frac{1}{R_1C_1}$$

$$3dB\ corner\ or\ cutoff\ frequency\ c = \frac{1}{R_1C_1}$$

Calculate R1 for ω3dB = 2π3000 rad/s and C1 = 0.01μF

$$w(3dB) = \frac{1}{R_1C_1}$$

$$R = \frac{1}{w(3dB)*C1} = \frac{1}{2\pi3000*0.01\mu F} = 5305\Omega = 5.3K\Omega$$

# Fourier Coefficient expressions of filtered sine, square & sawtooth waves using Matlab

## Low pass filter

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{V_{out}}{V_{in}} = \frac{V_c}{V_r+V_c}$$

$$H(jw) = \frac{I_c*Z_c}{I_c*Z_r+I_c*Z_c} = \frac{Z_c}{Z_r+Z_c}$$

$$H(jw) = \frac{\frac{1}{jwC}}{R+(\frac{1}{jwC})} * \frac{jwC}{jwC}$$

$$H(jw) = \frac{1}{jwRC+1}$$

## High pass filter

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{V_{out}}{V_{in}} = \frac{V_r}{V_r+V_c}$$

$$H(jw) = \frac{I_r * R}{I_r * R + I_r * Z_c} = \frac{R}{R + Z_c}$$

$$H(jw) = \frac{R}{R + (\frac{1}{jwC})} * \frac{jwC}{jwC}$$

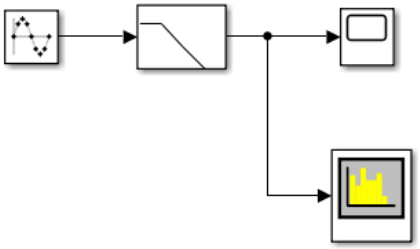
$$H(jw) = \frac{jwRC}{jwRC + 1}$$

Good theory summary

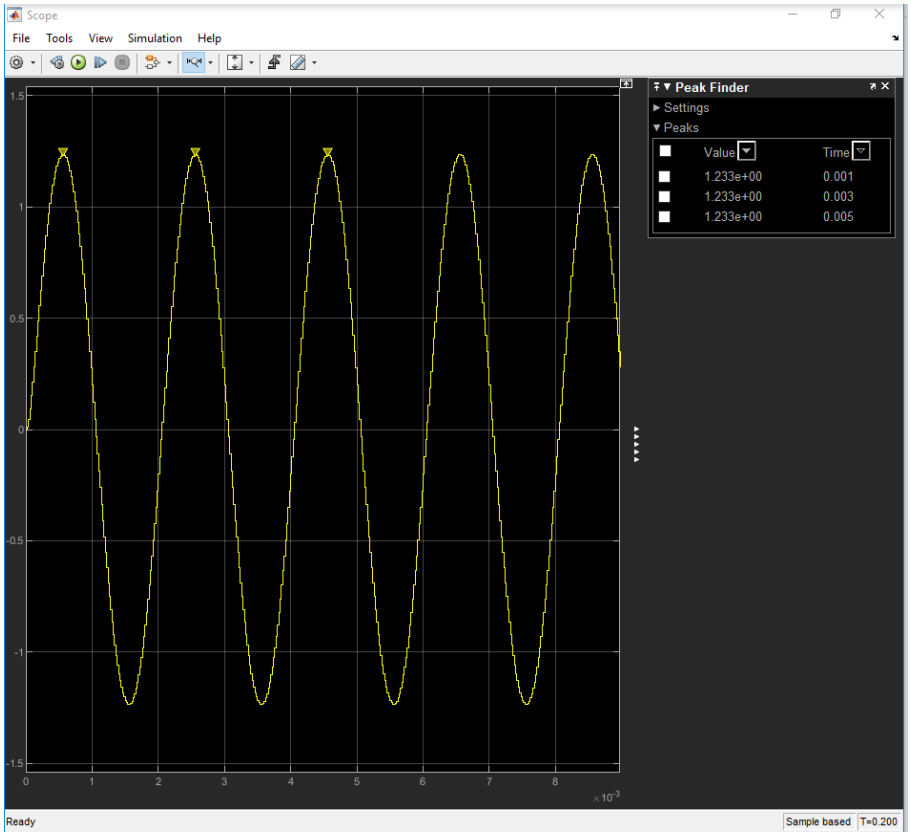
# Simulink

## Low pass filtered sine wave

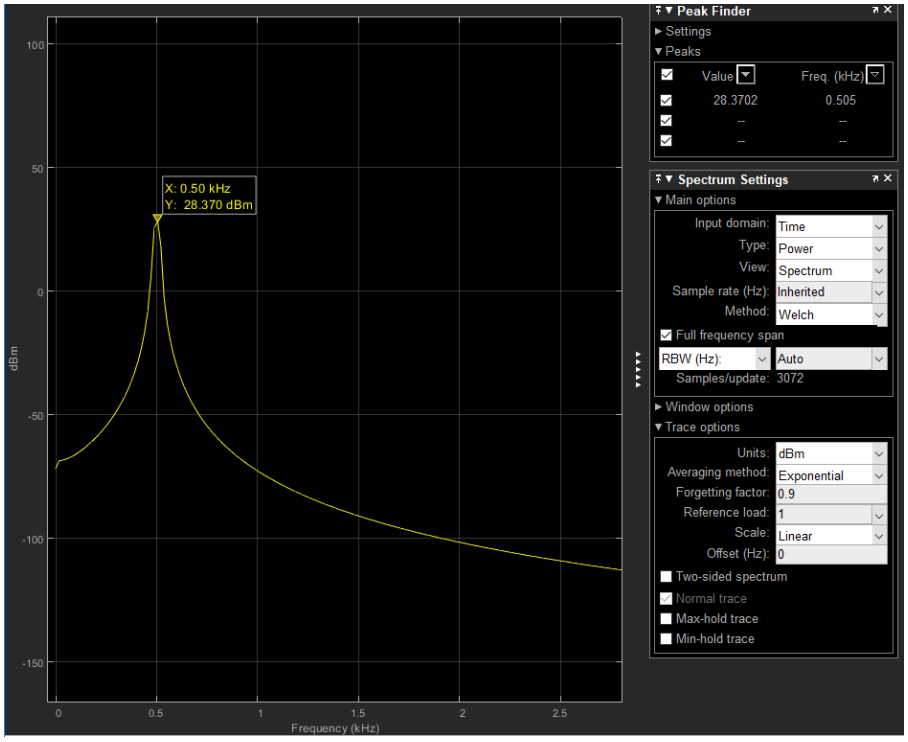
Sinewave Vpp=2, Vp=1, fo=1000, 1st order LPF fc=3000, sample time=20us, run time=0.2s



Sinewave wave Vpp=V, Vp=V, fo=Hz, 1st order LPF fc=KHz, scope time domain

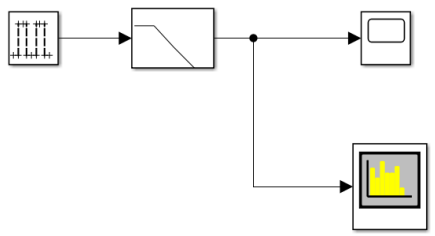



Sinewave wave Vpp=V, Vp=V fo=Hz, 1st order LPF fc=KHz Spectrum Analyzer frequency domain

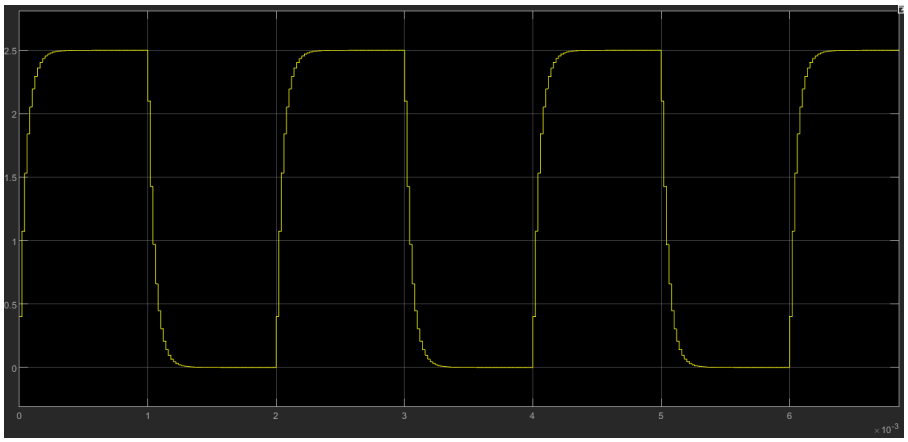


# 1st order low pass filtered square wave

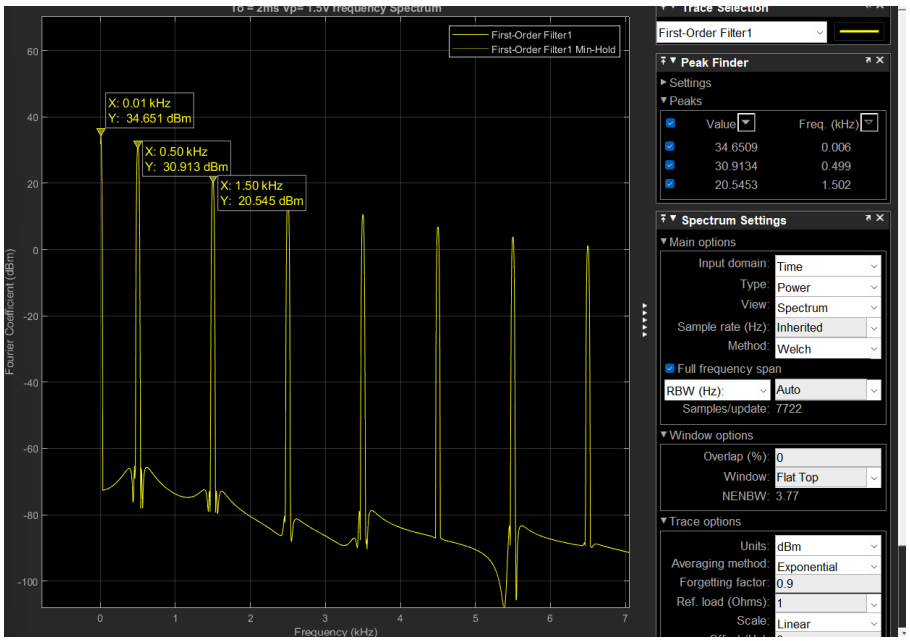
## Square wave Vpp=V, Vp=V, fo=Hz, dc=%, 1st order LPF fc=KHz, Ts=20us, run time=0.2s



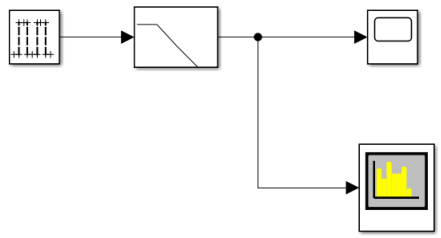
## Square wave Vpp=V, Vp=V, fo=Hz, dc=% 1st order LPF fc=KHz, scope time domain



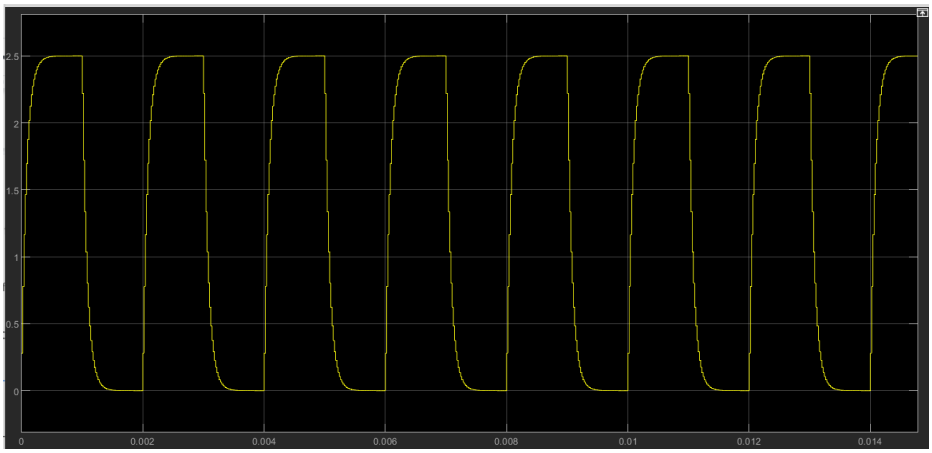

## Square wave Vpp=V, Vp=V, fo=Hz, dc=%, 1st order LPF fc=KHz Spectrum Analyzer frequency domain



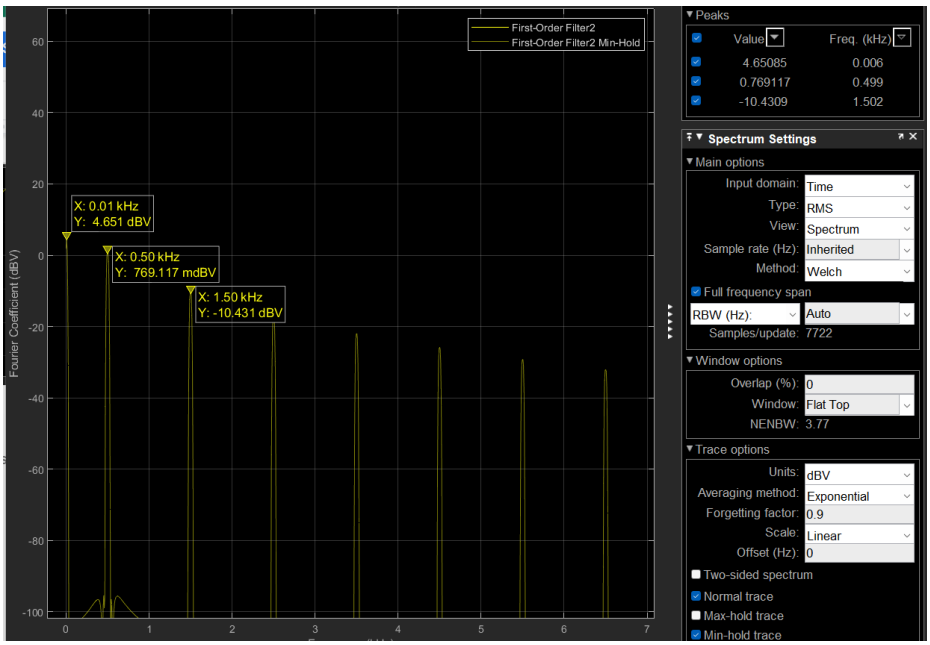
Square wave Vpp=V, Vp=V, fo=Hz, 1st order LPF fc=KHz, Ts=20us, run time=0.2s



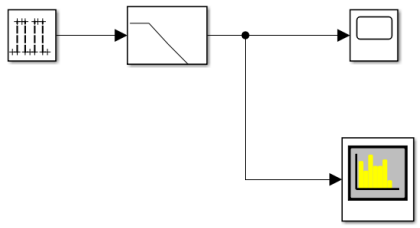
Square wave Vpp=V, Vp=V, fo=Hz, dc=% 1st order LPF fc=KHz, scope time domain



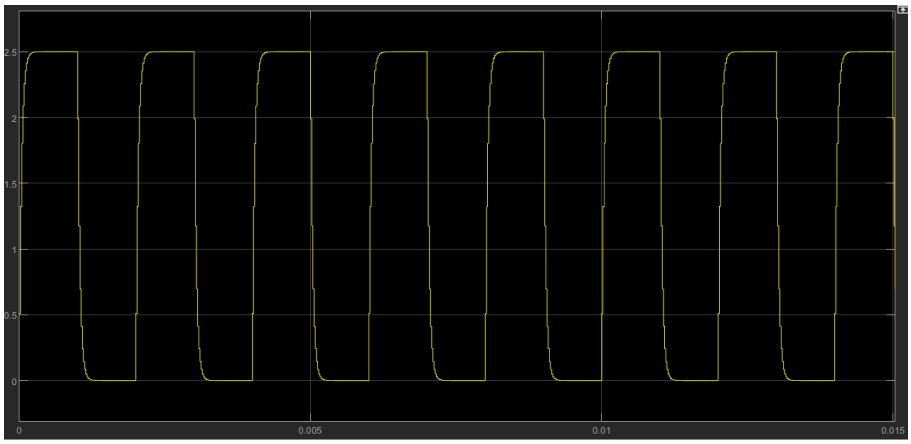

Square wave Vpp=V, Vp=V fo=Hz, dc=% 1st order LPF fc=KHz Spectrum Analyzer frequency domain



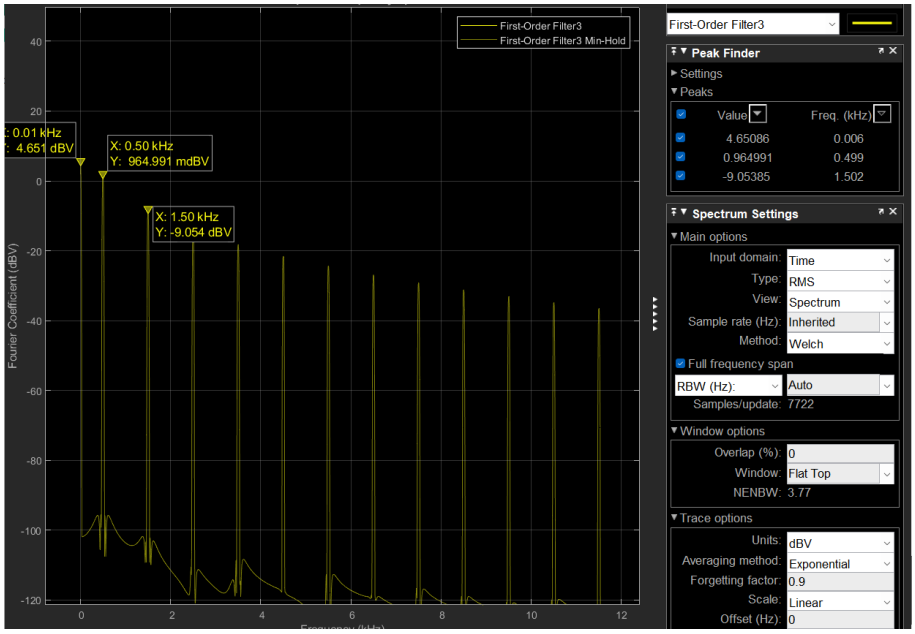
Square wave Vpp=V, Vp=V, fo=Hz, dc=% 1st order LPF fc=KHz, Ts=20us, run time=0.2s



Square wave Vpp=V, Vp=V, fo=Hz, dc=% 1st order LPF fc=KHz, scope time domain

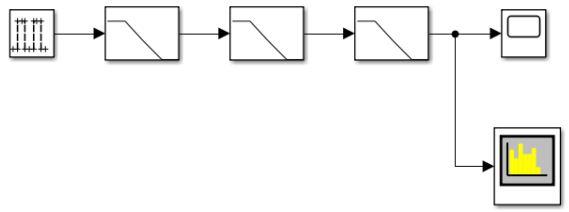



Square wave Vpp=V, Vp=V fo=Hz, dc=% 1st order LPF fc=KHz Spectrum Analyzer frequency domain

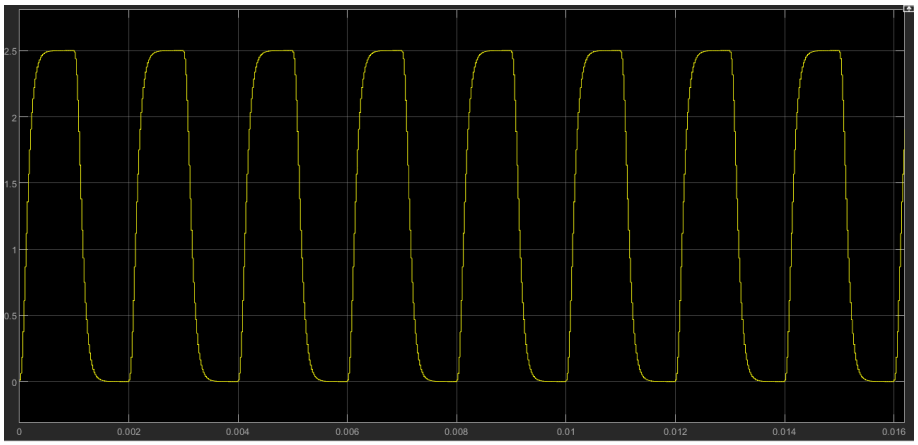


# 3rd order low pass filtered square wave

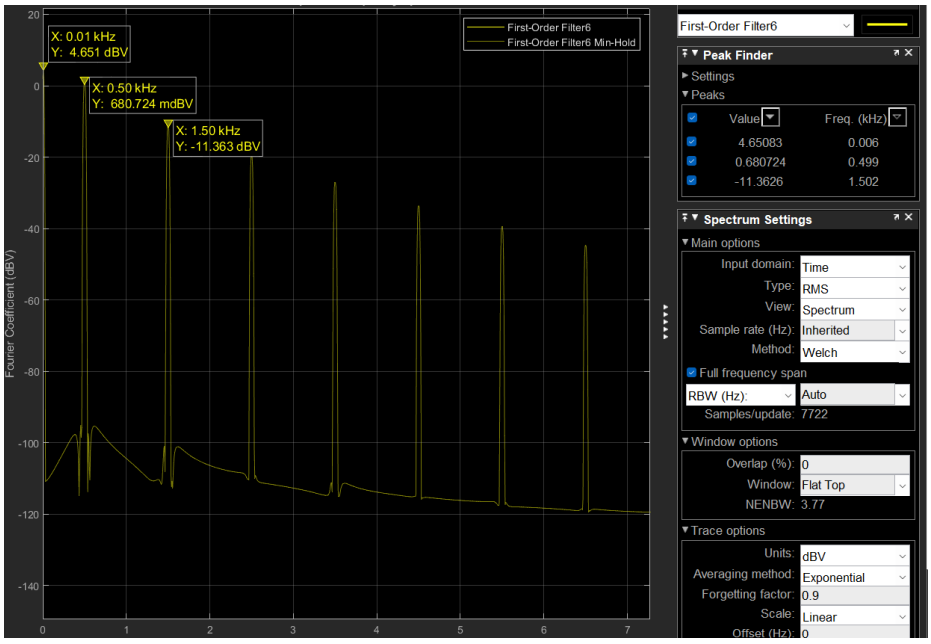
Square wave Vpp=V, Vp=V, fo=Hz, dc=%, 3rd order LPF fc=KHz, Ts=20us, run time=0.2s



Square wave Vpp=V, Vp=V, fo=Hz, 3rd order LPF fc=KHz, scope time domain

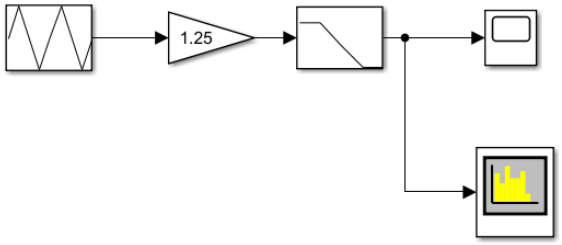



Square wave Vpp=V, Vp=V fo=Hz, 3rd order LPF fc=KHz Spectrum Analyzer frequency domain

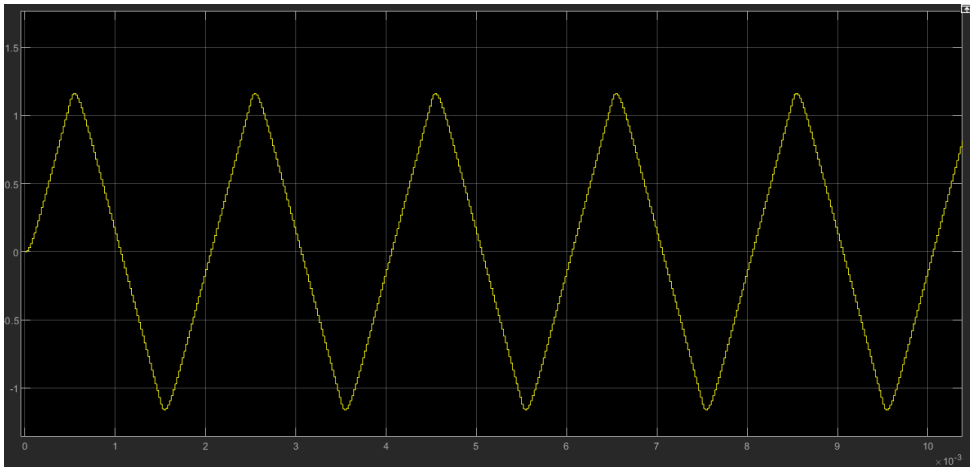


# 1st and 3rd order low pass filtered sawtooth wave

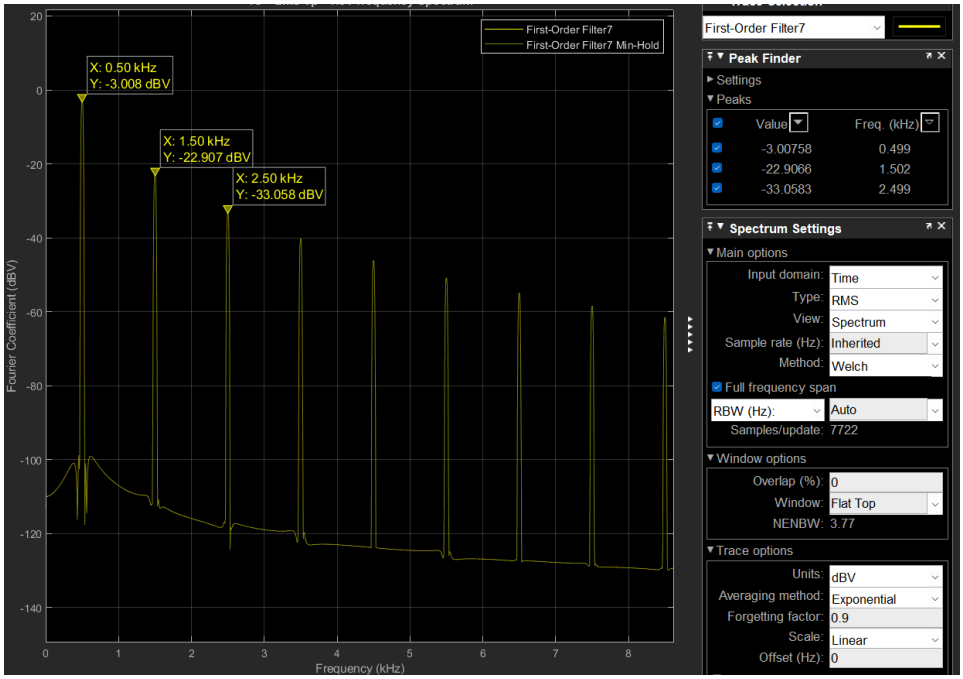
Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 1st order LPF fc=KHz, Ts=20us, run time=0.2s



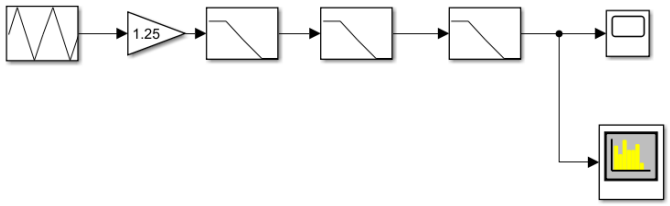

Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 1st order LPF fc=KHz, scope time domain



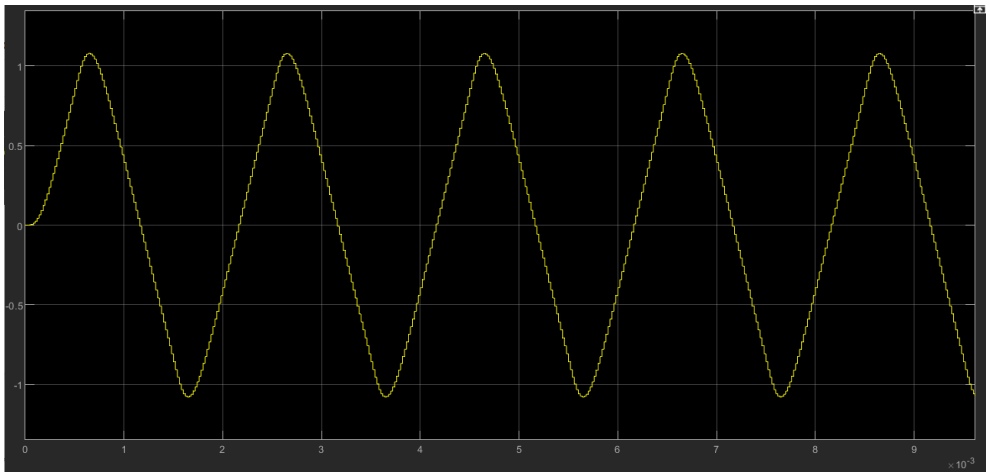
Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 1st order, LPF fc=KHz Spectrum Analyzer frequency domain



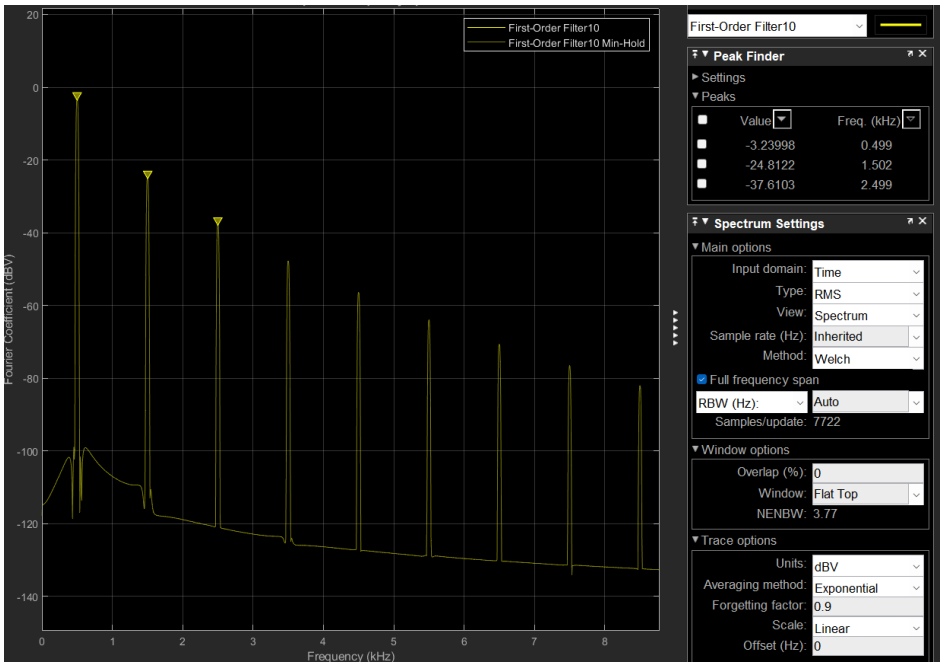

Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 3rd order LPF fc=KHz, Ts=20us, run time=0.2s




Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 3rd order LPF fc=KHz, scope time domain

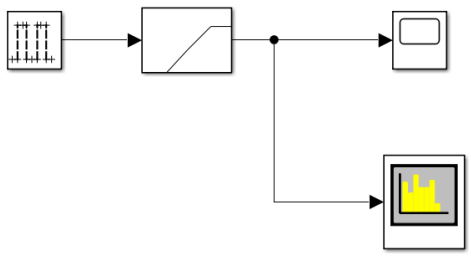


Sawtooth Vpp=V, Vp=V, fo=Hz, sym=%, 1st order, LPF fc=KHz Spectrum Analyzer frequency domain

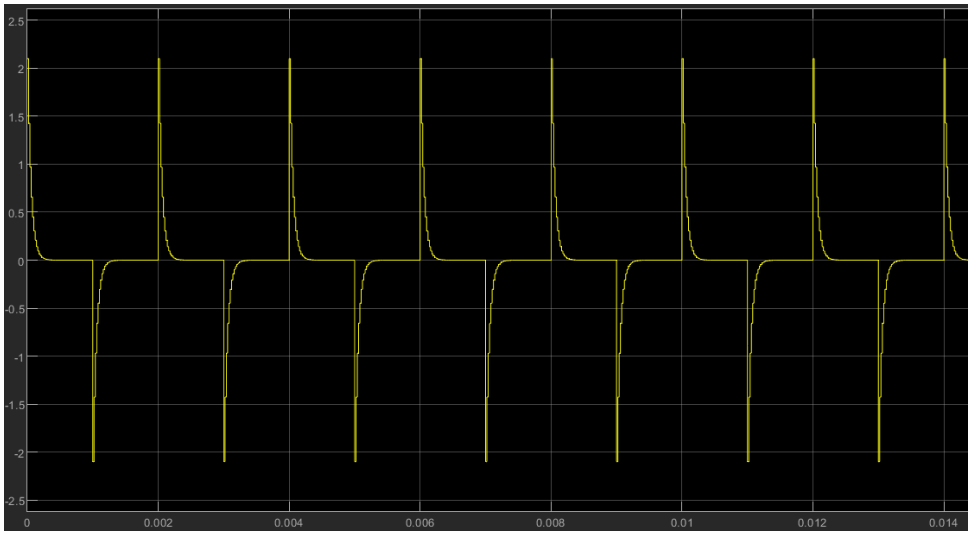



## 1st order high pass filtered square wave with different cutoff frequencies

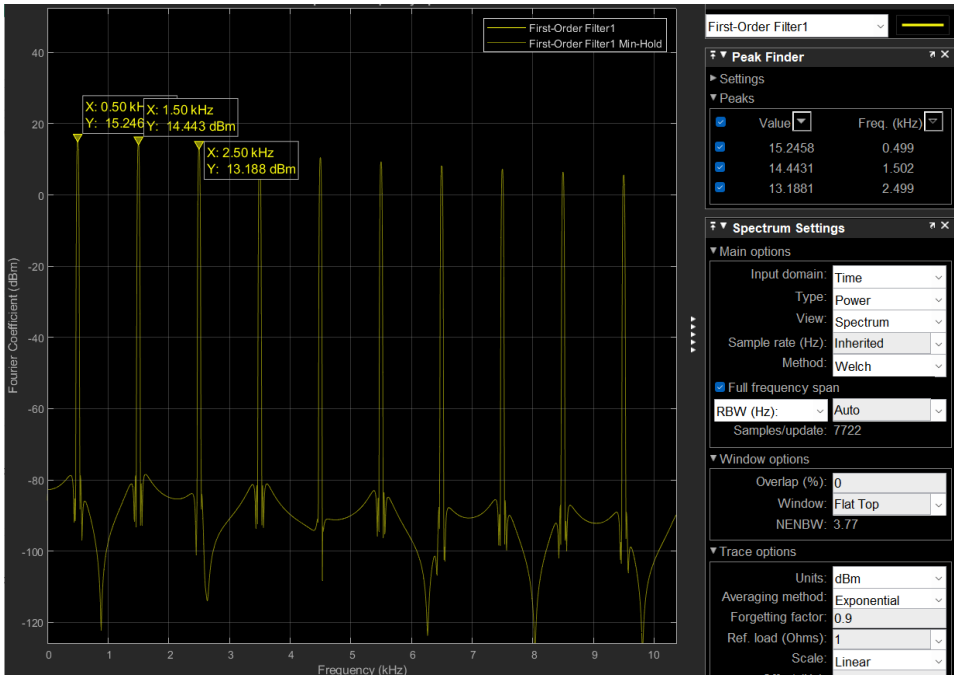
Square wave Vpp, Vp, fo, dc=%, 1st order HPF fc=KHz, Ts=20us, run time=0.2s



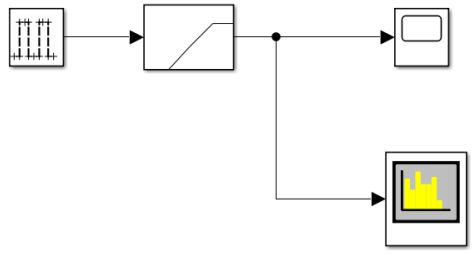
Square wave Vpp, Vp=V, fo, dc=%, 1st order HPF fc=KHz, scope time domain



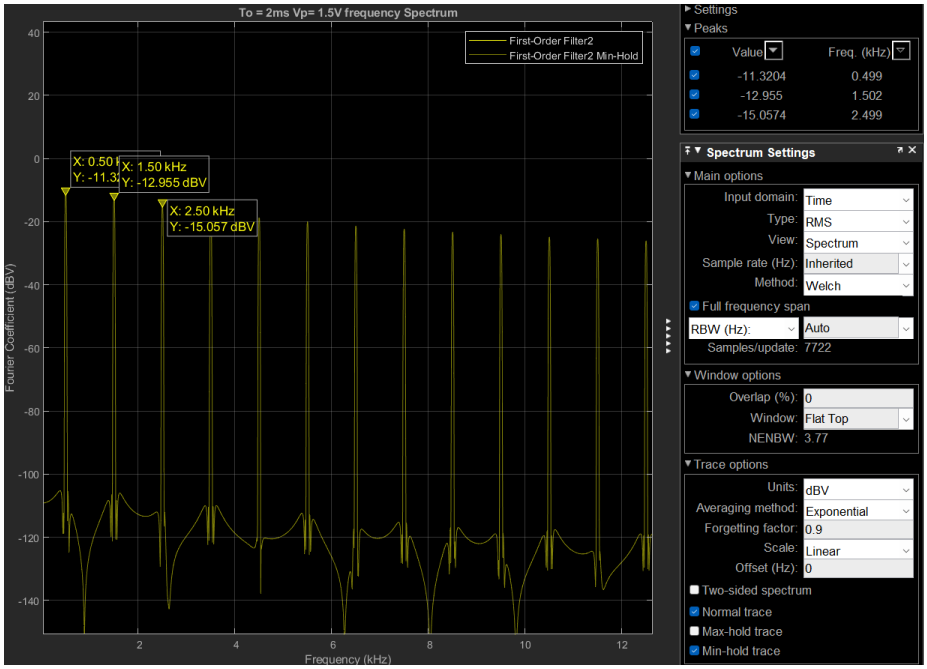

Square wave Vpp, Vp fo, dc=%, 1st order, HPF fc=KHz Spectrum Analyzer frequency domain



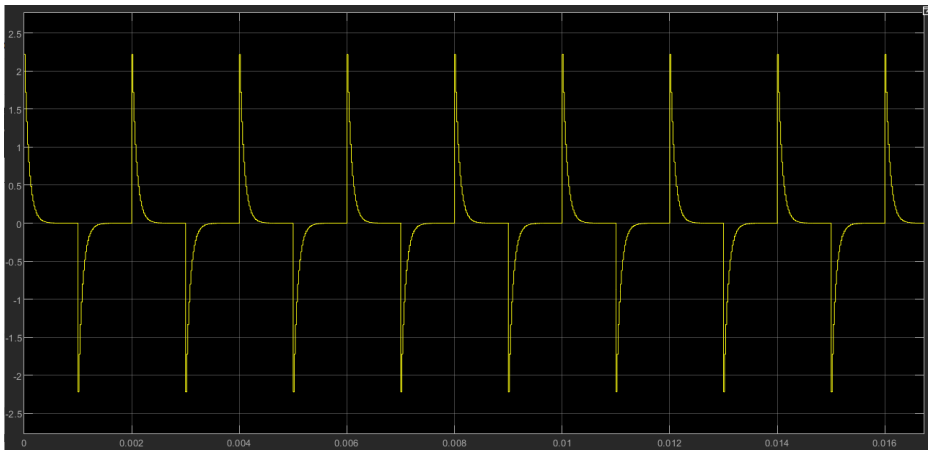
Square wave Vpp, Vp, fo, dc=%, 1st order HPF fc=KHz, Ts=20us, run time=0.2s



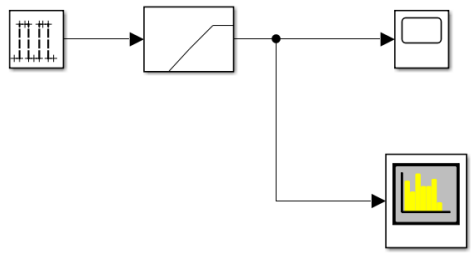
Square wave Vpp, Vp, fo, dc=%, 1st order HPF fc=KHz, scope time domain



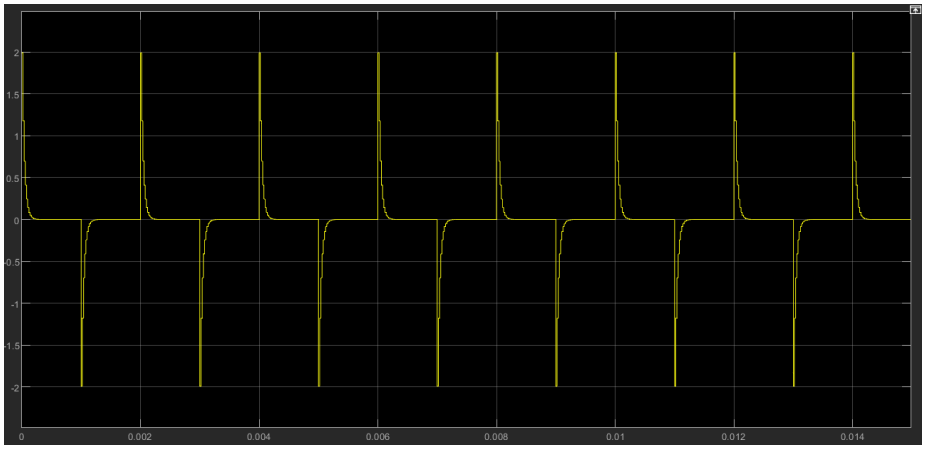

Square wave Vpp, Vp fo, dc=%, 1st order, HPF fc=KHz Spectrum Analyzer freq domain



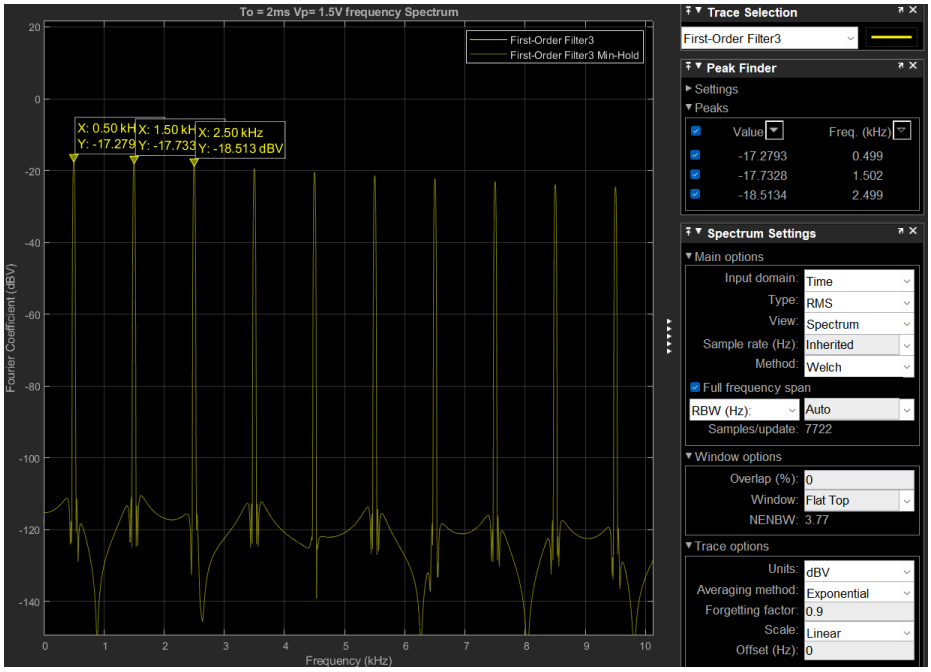
Square wave Vpp, Vp, fo, dc=%, 1st order HPF fc=KHz, Ts=20us, run time=0.2s



Square wave Vpp, Vp, fo, dc=%, 1st order HPF fc=KHz, scope time domain

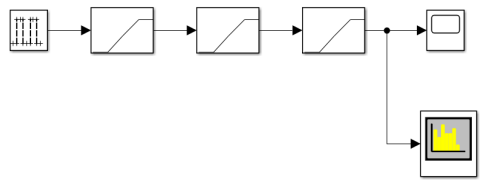



Square wave Vpp, Vp fo, dc=%, 1st order, HPF fc=KHz Spectrum Analyzer frequency domain



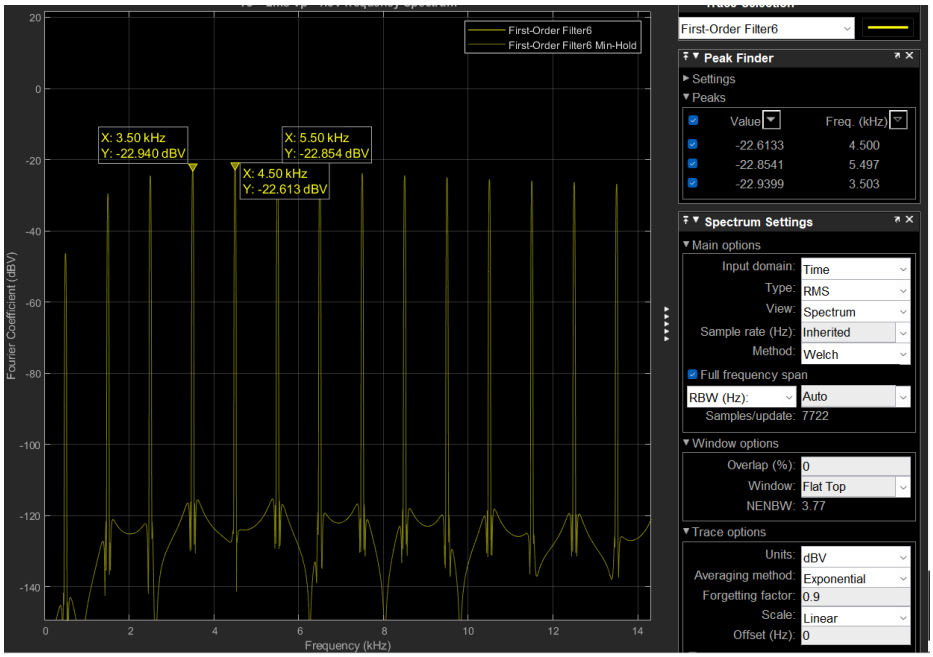
# 3rd order high pass filtered square wave

## Square wave Vpp, Vp, fo, dc=%, 3rd order HPF fc=KHz, Ts=20us, run time=0.2s



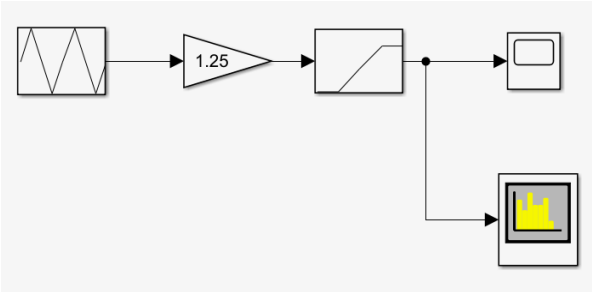
## Square wave Vpp, Vp, fo, dc=%, 3rd order HPF fc=KHz, scope time domain


## Square wave Vpp, Vp fo, dc=%, 3rd order, HPF fc=KHz Spectrum Analyzer frequency domain

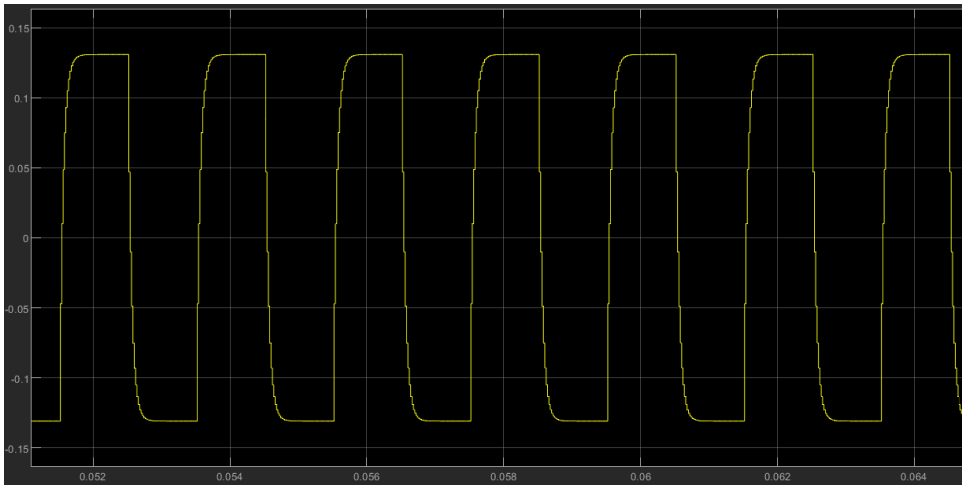


# 1st and 3rd order high pass filtered sawtooth wave

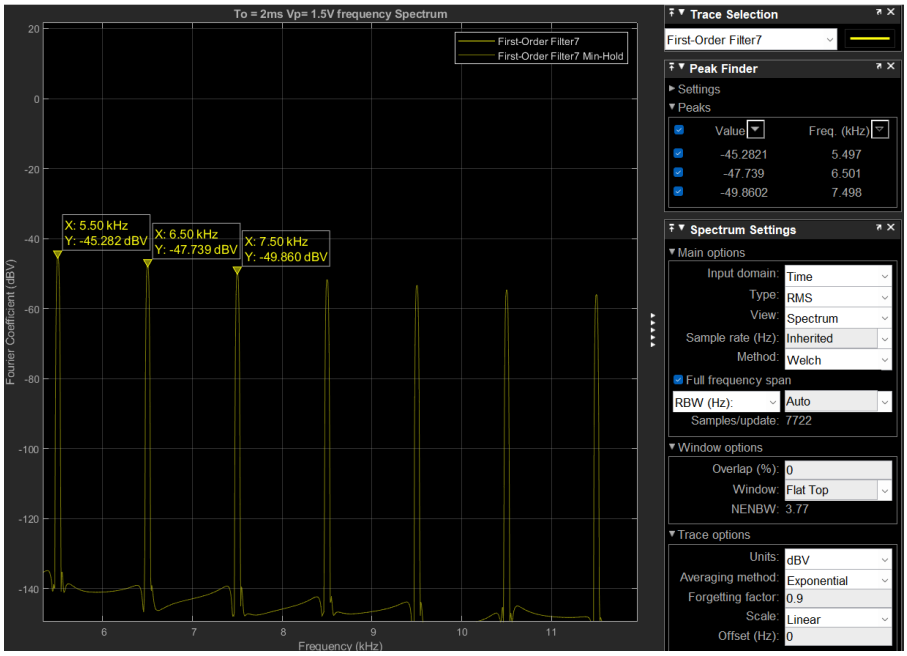
## Sawtooth Vpp, Vp, fo, sym=%, 1st order HPF fc=KHz, Ts=20us, run time=0.2s



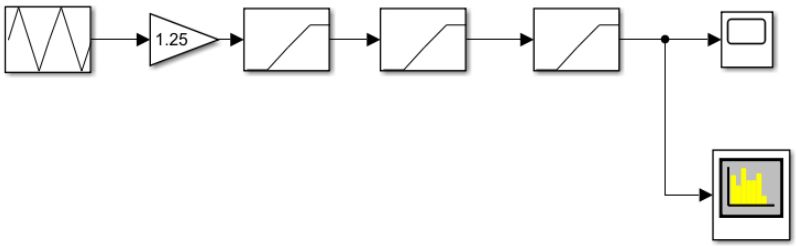

Sawtooth Vpp, Vp, fo, sym=%, 1st order HPF fc=KHz, scope time domain



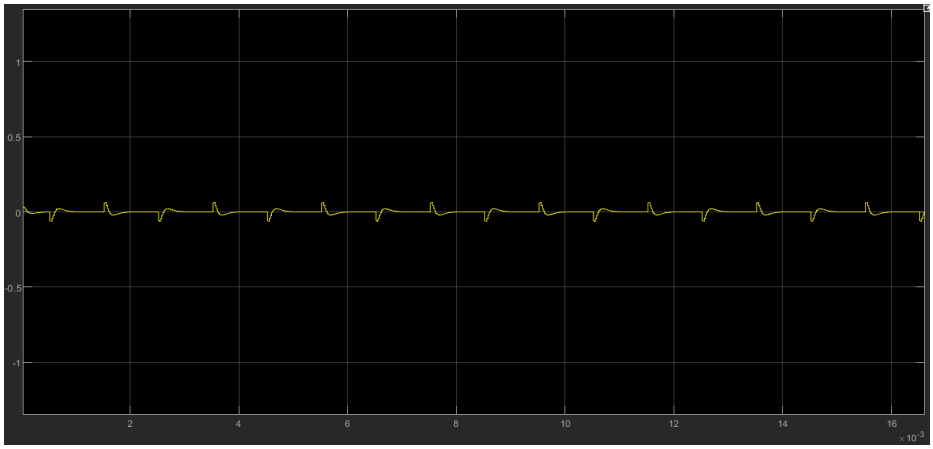
Sawtooth Vpp, Vp, fo, sym=%, 1st order, HPF fc=KHz Spectrum Analyzer freq domain



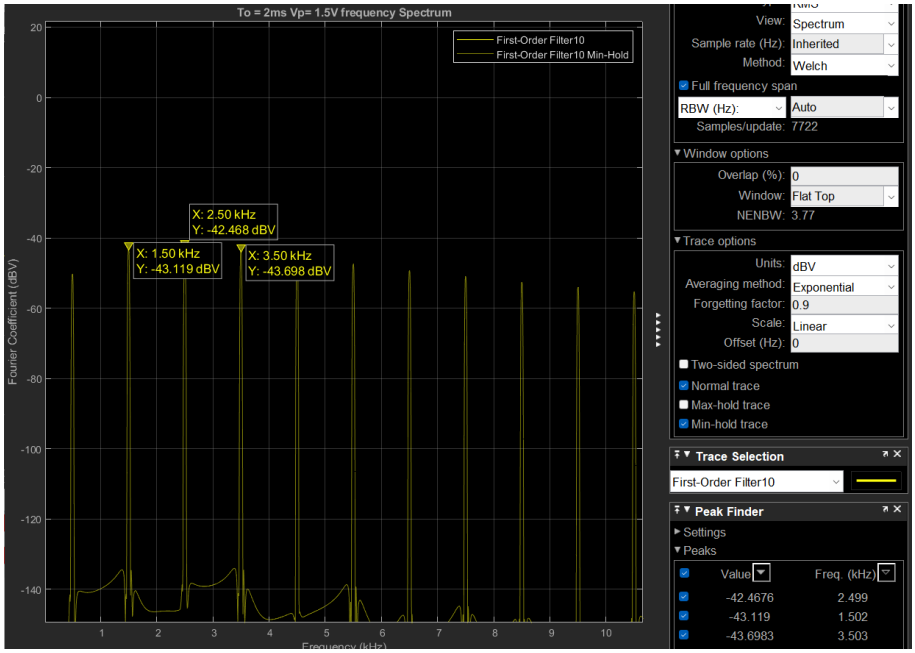

Sawtooth Vpp, Vp, fo, sym=%, 3rd order HPF fc=KHz, Ts=20us, run time=0.2s



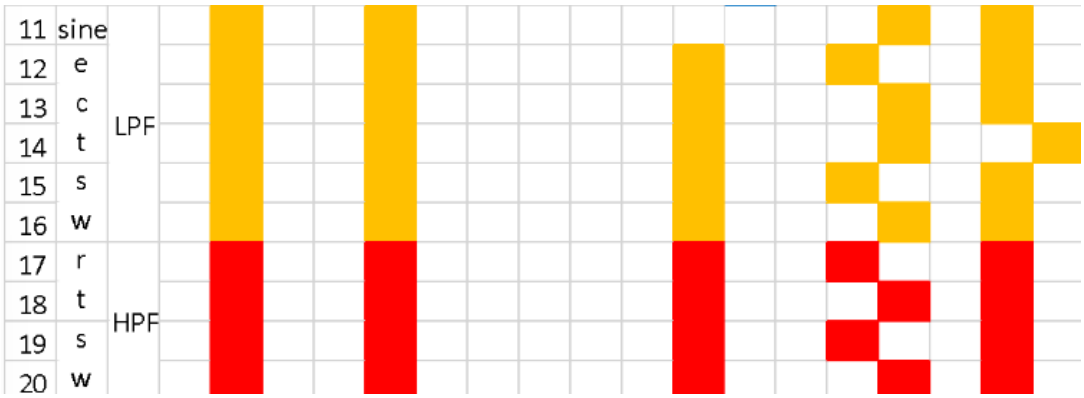

Sawtooth Vpp, Vp, fo, sym=%, 3rd order HPF fc=KHz, scope time domain



Sawtooth Vpp, Vp, fo, sym=%, 3rd order, HPF fc=KHz Spectrum Analyzer frequency domain




# Lab

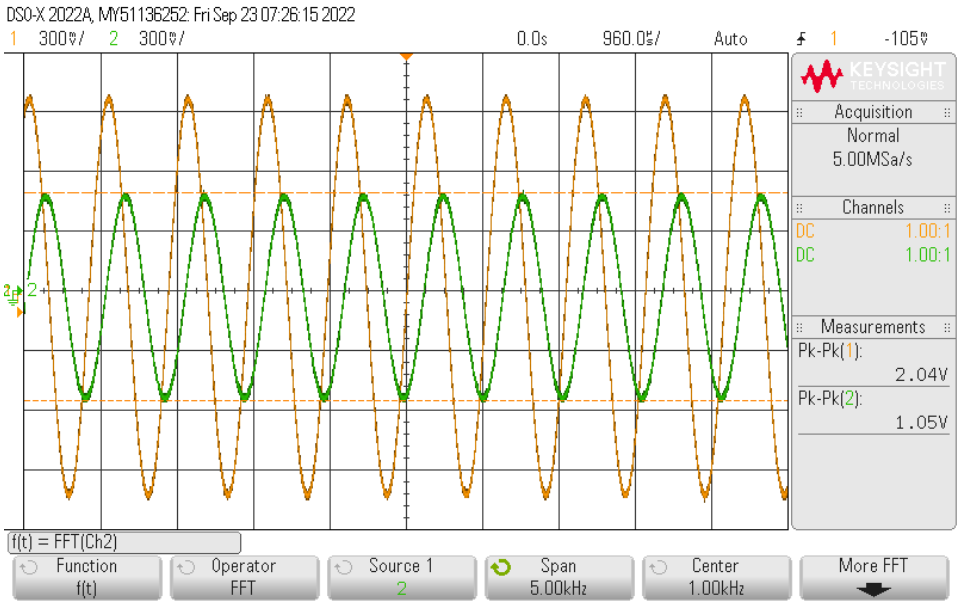


# Low pass filter

## Sine

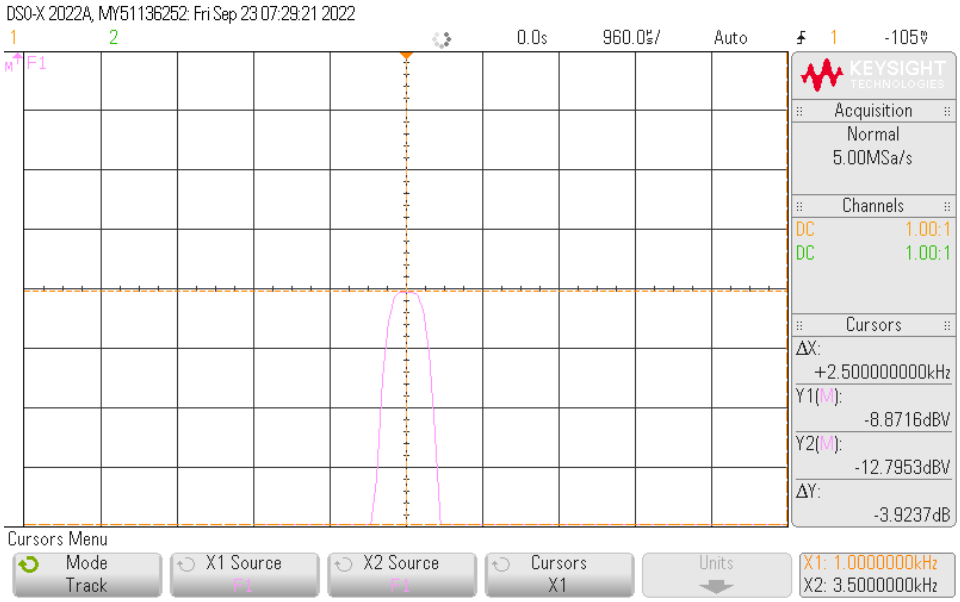
### 2sine(2*pi* 1000)  in a 3 order circuit, cut off frequency =3000

#### Voltage input and output




*In the following image, the yellow wave represents Vin (the voltage applied to the low pass filter) and the green wave represents Vo (the voltage exiting the low pass filter), we could see how the voltage decreases after going through the circuit because ____. The following image also shows that Vo shifts because of the capacitor in the low pass filter that introduces complex __.*

#### Fourier Transform



*The following image shows there is only one peak in the Fourier transform. This is because there is only one sine wave with one frequency in the system. Due to the low pass filter, we can see how the coefficient decreases compared to our previous lab*

## Square

### 2square(2*pi* 1000) 50% duty cycle in a 1 order circuit, cut off frequency =3000

#### Voltage input and output


| | |
|---|---|
| Voltage input | 2.01 |
| Voltage Output | 2.006 |

#### Fourier Transform

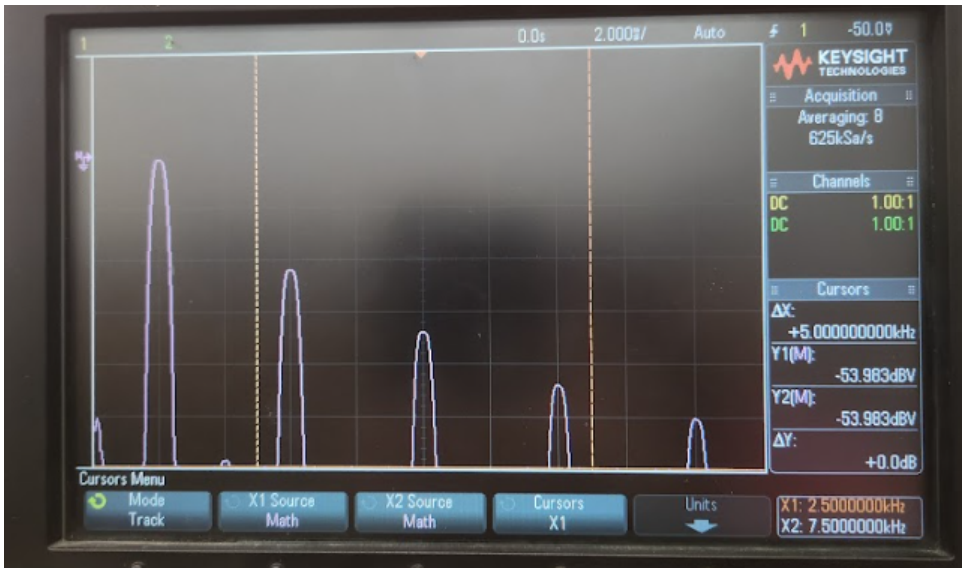
| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -1.33 |
| 2 | 3000 | -13.3 |
| 3 | 5000 | -20.5 |
| 4 | 7000 | -25.8 |
| 5 | 9000 | -29.8 |

2square(2*pi* 1000) 50% duty cycle in a 3 order circuit, cut off frequency =3000

Voltage input and output


| Voltage input | 2.02 |
|---|---|
| Voltage Output | 1.39 |

Fourier Transform

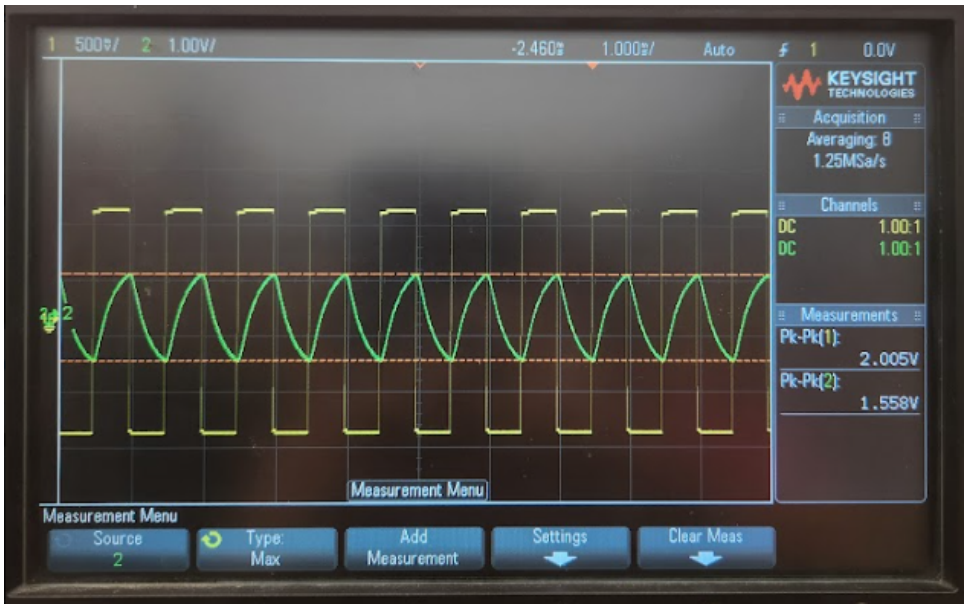



| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -6.8 |
| 2 | 3000 | -26.1 |
| 3 | 5000 | -37.4 |
| 4 | 7000 | -44.5 |
| 5 | 9000 | -50.5 |

You can consolidate tables or even better plot them

2square (2*pi* 1000) 50% duty cycle in a 3 order circuit, cut-off frequency =3500

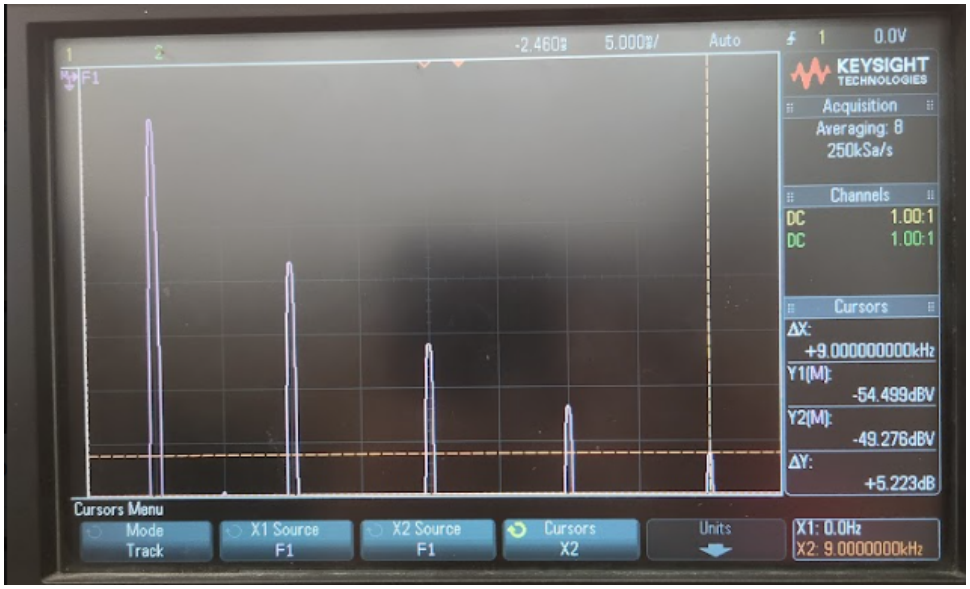
Voltage input and output




use the same scale for input an

| | |
|---|---|
| Voltage input | 2.02 |
| Voltage Output | 1.55 |

Fourier Transform

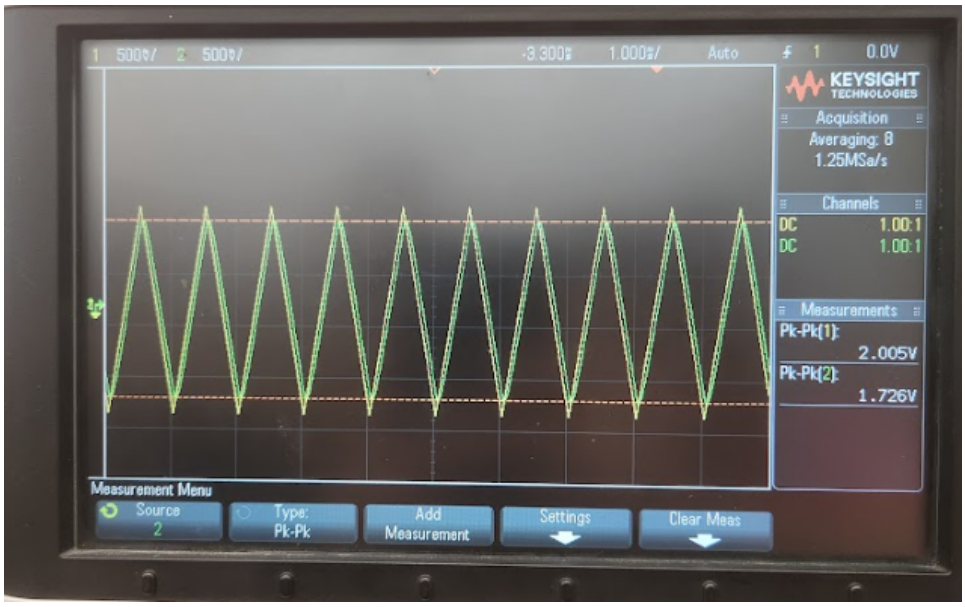


| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -5.7 |
| 2 | 3000 | -24.3 |
| 3 | 5000 | -35.1 |
| 4 | 7000 | -43.2 |
| 5 | 9000 | -49.2 |

## Sawtooth

2sawtooth(2*pi* 1000) 50% duty cycle in a 1 order circuit, cut-off frequency =3000

Voltage input and output




| Voltage input | 2.00 |
|---|---|
| Voltage Output | 1.76 |

Fourier Transform


| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -5.23 |
| 2 | 1500 | -66.4 |
| 3 | 2500 | -26.8 |
| 4 | 3500 | -63.2 |
| 5 | 4500 | -38.1 |
| 6 | 6500 | -46.6 |
| 7 | 8500 | -52.7 |

2sawtooth (2*pi* 1000) 50% duty cycle in a 3 order circuit, cut-off frequency =3000

Voltage input and output


| | |
|---|---|
| Voltage input | 2.00 |
| Voltage Output | .834 |

Fourier Transform


| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -10.57 |
| 2 | 1500 | -61 |
| 3 | 2500 | -40 |

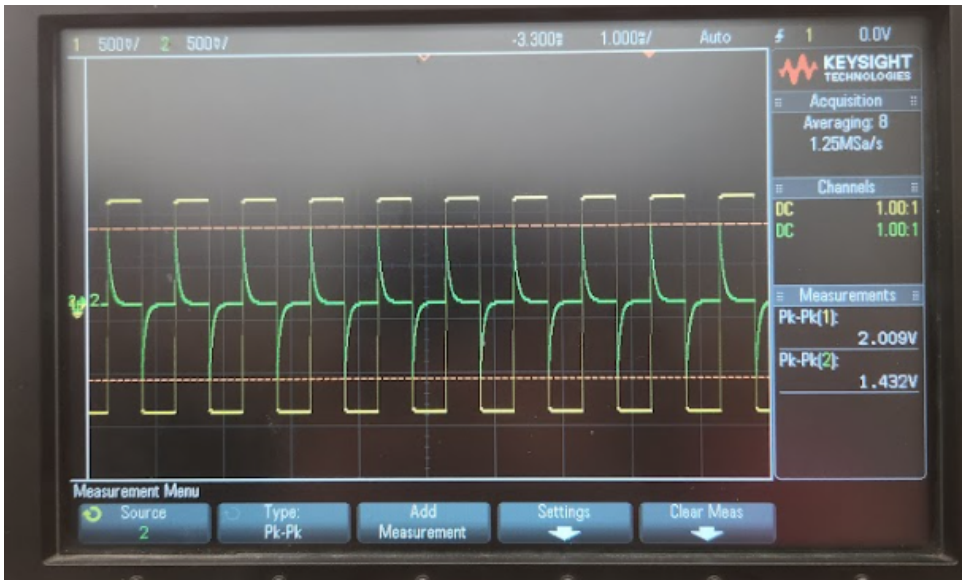
| | | |
|---|---|---|
| 4 | 4500 | -55 |
| 5 | 6500 | -67 |
| 6 | 8500 | -75 |

# High pass filter

## Square

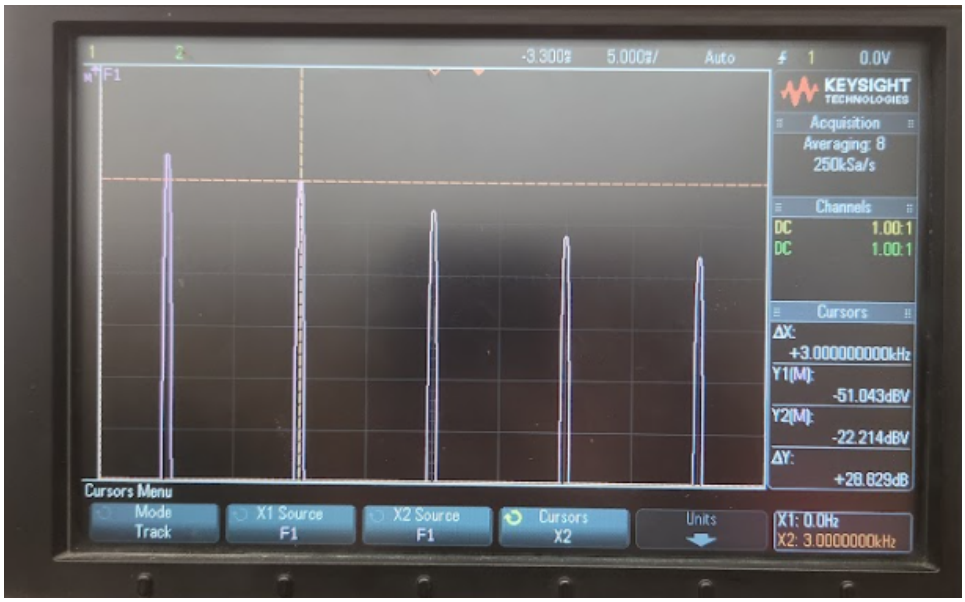
2square(2*pi* 1000) 50% duty cycle in a 1 order circuit, cut off frequency =3000

Voltage input and output




| | |
|---|---|
| Voltage input | 2.00 |
| Voltage Output | 1.43 |

Fourier Transform




| N | Frequency | dBv |
|---|---|---|
| 1 | 1000 | -19.7 |
| 2 | 3000 | -22.2 |
| 3 | 5000 | -24.9 |
| 4 | 7000 | -27.2 |
| 5 | 9000 | -29.2 |

2square(2*pi* 1000) 50% duty cycle in a 3 order circuit, cut-off frequency =3000

Voltage input and output


| | |
|---|---|
| Voltage input | 2.00 |
| Voltage Output | 3.19 |

Fourier Transform


| | | | |
|---|---|---|---|
| N | | | |

| | | | |
|---|---|---|---|
| 1 | 1000 | | |
| 2 | 3000 | | |
| 3 | 5000 | | |
| 4 | 7000 | | |
| 5 | 9000 | | |

## Sawtooth

2 sawtooth(2*pi* 1000) 50% duty cycle in a 1 order circuit, cut off frequency =3000

Voltage input and output


| | |
|---|---|
| Voltage input | 2.00 |
| Voltage Output | .152 |

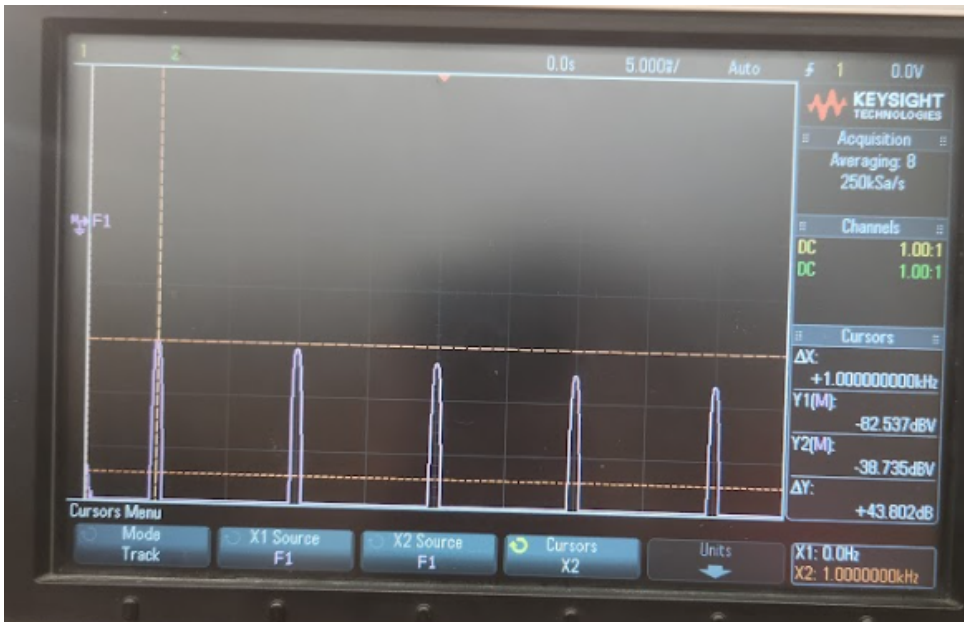
Fourier Transform


| N | | | |
|---|---|---|---|
| 1 | 1000 | -10.57 | |
| 2 | 1500 | -61 | |
| 3 | 2500 | -40 | |
| 4 | 4500 | -55 | |
| 5 | 6500 | -67 | |
| 6 | 8500 | -75 | |

2 sawtooth(2*pi* 1000) 50% duty cycle in a 3 order circuit, cut off frequency =3000

Voltage input and output

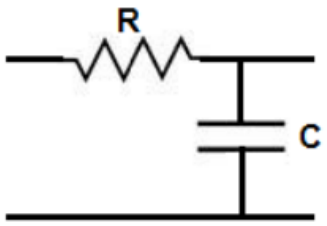

Fourier Transform

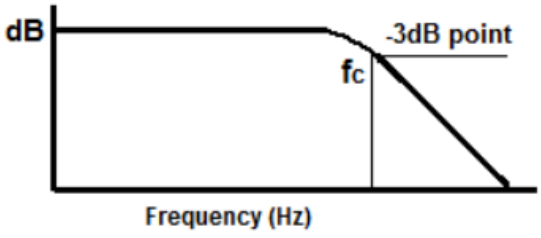



# Conclusion

On the following we were able to see the affects of a RC low pass filter and a RC high pass filter in a sine, square and sawtooth wave both of the filter reduce the voltage coming out of the system therefore aswell reducing the voltage gain however they affected the frequency of the original signal differectly.

In the following sections we notice the importance of having the right component (such as resistor values and the capacitor values) to get the right cutoff frequency that we are looking for. For the following lab  we used the example cut-off frequency =3000Hz and cut-off frequency =3500Hz. The cut of frequency at 3000Hz is really important because it represents 3dB point, the cutoff frequency of an electronic amplifier stage at which the output power has dropped to half of its mid-band level. The cut-off frequency is directly proportional to 1/RC therefore the values of the resistor and the capacitor most compliment each other to get the right frequency of the transfer function. The lab we used 5300Ω and .01μ but we could have used other values to gave the same cut-off frequency.
The low pass filter mainly blocks the high frequencies while maintain the low frequencies to go through the system, while still reducing the voltage output and the gain



The high pass filter mainly blocks the high frequencies while maintain the low frequencies to go through the system. while still reducing the voltage output and the gain.

Something interested that when notice about the high pass filter, was when it interacted with the square wave the the peak voltage output was higher then the original signal peak however the voltage output is still reduce in the system because there is a lower volgate output through out the wave