Lab number, date, note your partner

Unfiltered sine, square, and Sawtooth

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ECE 3101 3101L

Objective

Be able to calculate the complex exponential Fourier Series coefficient Xn of periodic filtered and unfiltered sinusoid, rectangular, and triangle signals and from them obtain Cn's Dn's and θn's before and after the filters. Be able to calculate how the Fourier Coefficients of periodic signals like pulse trains are affected by changes in their periods, magnitudes, duty cycles, or symmetries. Be able to measure the time & frequency spectrum of the filtered sinusoid, rectangular, and triangle pulse train using a time domain Oscilloscope and a frequency domain spectrum analyzer. Be able to calculate how the Fourier Coefficients of periodic signals like pulse trains are affected by filters

Fourier Coefficient expressions of unfiltered sine, square & sawtooth waves using

Sinewave

Sine with Vpp = 2V, To= 1ms
Fourier Coefficient expressions of unfiltered sine
For the Vpp=2V

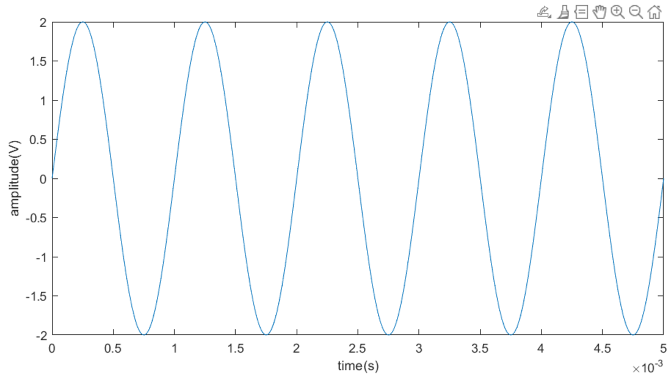
$V_{RMS} = \frac{V_{PP}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.414$ units

power

THE VRMS VALUES HELPS US FIND THE EQUIVALENT VOLTAGE DISSIPATION

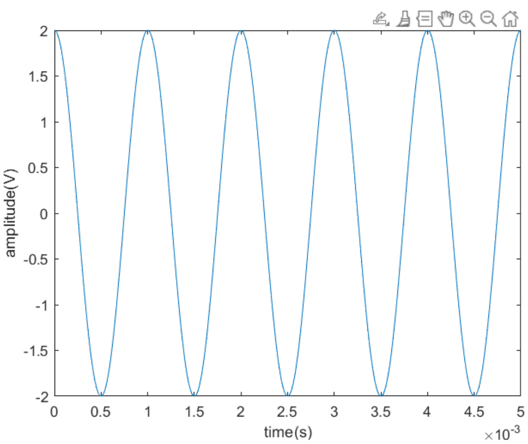
We need the amplitude, Period and there vertical shift to generate a sine wave in our scenario amplitude is 2v, the period is 1ms and there is no vertical shift. $V = 2sin(2\pi 1000)$
Mathematically we could change the amplitude, phase, and shift, however, each sinusoid could only have 1 frequency component.

```
t=0:0.00001:.005
To= 1*10^(-3)
f=1/To
amp=2
V= amp*sin((2*pi*f*t));
plot(t,V)
xlabel('time(s)')
ylabel('amplitude(V)')
```

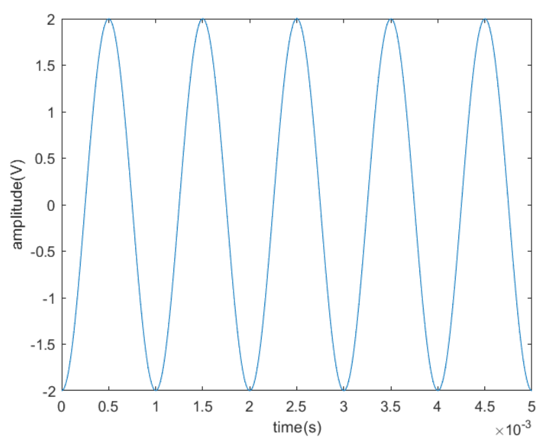


To make the sine even symmetrical we need to introduce the phase angle $\pm\frac{\pi}{2}$ for cosines

$V = 2sin(2\pi 1000 + \frac{\pi}{2}) = 2cos(2\pi 1000)$



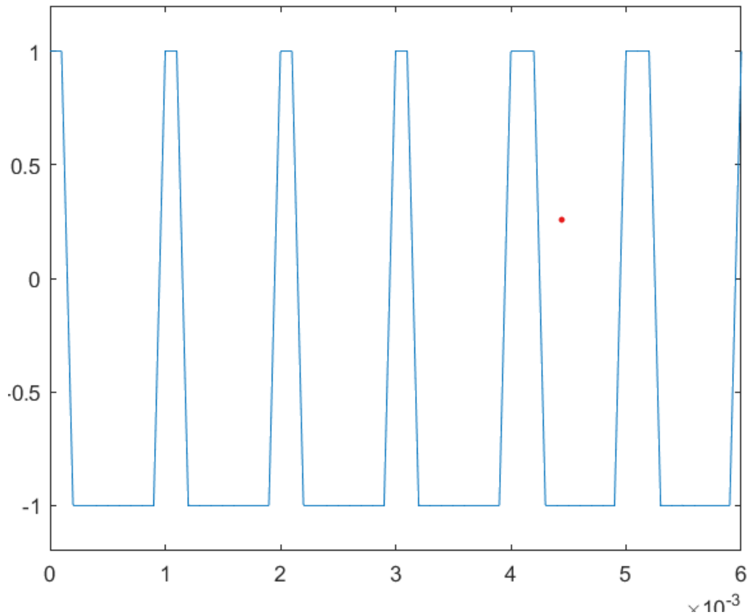
$V = 2sin(2\pi 1000 - \frac{\pi}{2}) = 2cos(2\pi 1000 + \pi)$



Square pulse train

2V Square 20% duty cycle, period 1ms

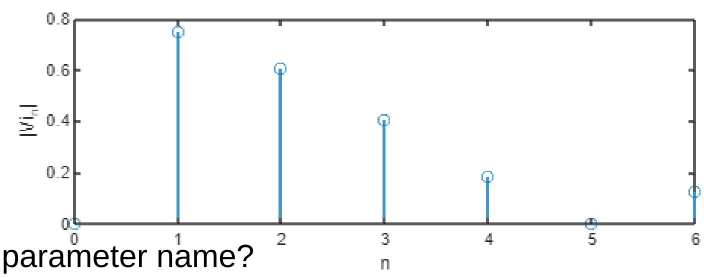
They don't look quite square



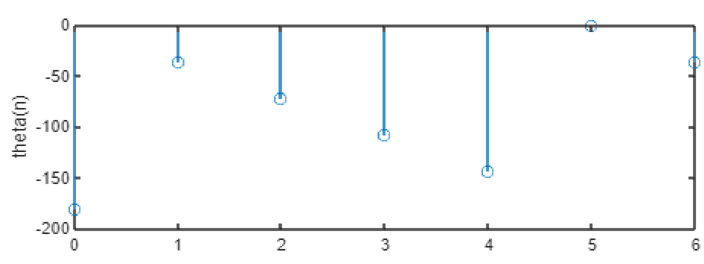

```
t=0:.0001:.006
f=1000;
x = 1*square(2*pi*f*t,20)

plot(t,x)
ylim([-1.2 1.2])
```

```
syms x

n=1:6;
t0=1*10^-3;
w=(2*pi)/t0;
A1=1;
A2=-1;
duty=1/5; %1/5=20 percent
t1=t0*duty;
fn1=(1/t0)*A1*exp(-1j*n*w*x);
fn2=(1/t0)*A2*exp(-1j*n*w*x);

fn=int(fn1,x,0,t1)+int(fn2,x,t1,t0);
fn0=int(A1,x,0,t1)+int(A2,x,t1,t0);
if fn0 < 0
 fn0=abs(fn0);
 theta0=-180;
else
    theta0=0;
end

X0=2.5;
vi=2*abs(fn);
theta=angle(fn)*180/pi;
VI=[fn0 vi];
thetat=[theta0 theta];
fprintf(' %4.3f ',VI);
fprintf(' %4.3f ',thetat);

nn=0:6;
figure
subplot(2,1,1)
stem(nn,VI)
xlabel('n')
ylabel('|Vi_n|')
figure(1)
subplot(2,1,2)
stem(nn,thetat)
xlabel('n')
ylabel('theta(n)')
```

```
0.001  0.748  0.605  0.404  0.187  0.000  0.125
-180.000  -36.000  -72.000  -108.000  -144.000  0.000  -36.000
```



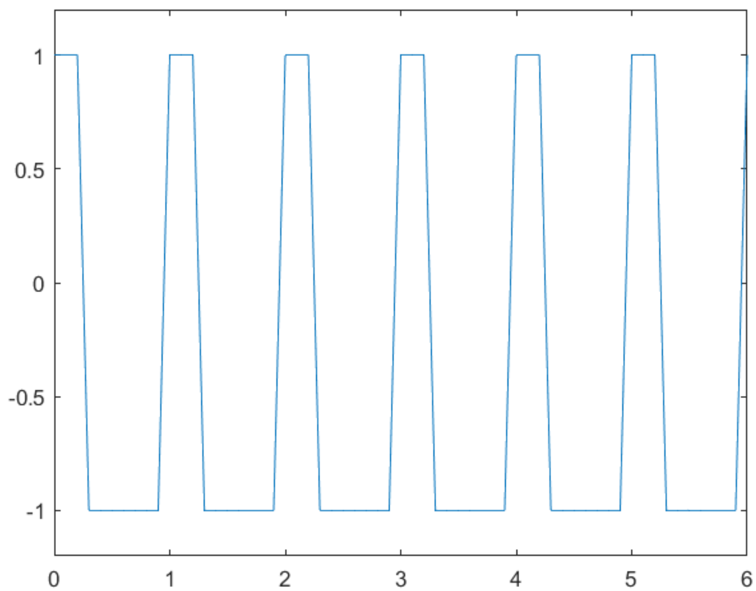
parameter name?



$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = c_0 + \sum_{n=1}^{\infty} c_n \cos\cos\left(nw_0 t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

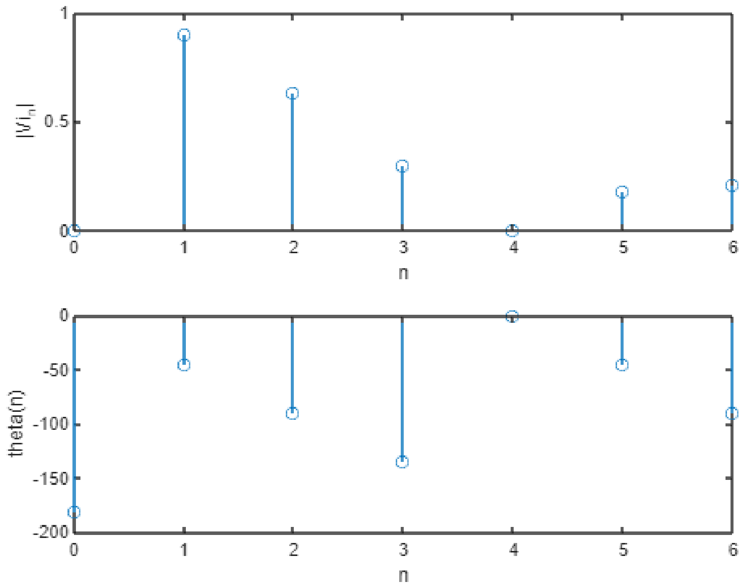
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = .001cos\left(0w_0 t - 180\right) + .748cos\left(w_0 t - 36\right) + .605cos\left(2w_0 t - 72\right)$$
$$+ .404cos\left(3w_0 t - 108\right) + .187cos\left(4w_0 t - 144\right) + .125cos\left(6w_0 t - 36\right), \quad w_0 = \frac{2\pi}{1ms}$$

2V Square 25% duty cycle, period 1ms



Don't forget to label your axis with the parameter name and units

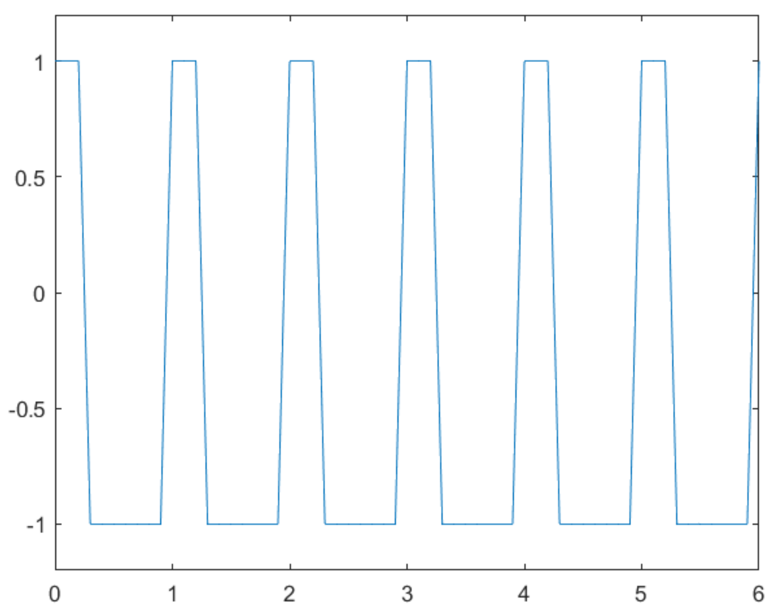
```
0.001  0.900  0.637  0.300  0.000  0.180  0.212
-180.000  -45.000  -90.000  -135.000  0.000  -45.000  -90.000
```



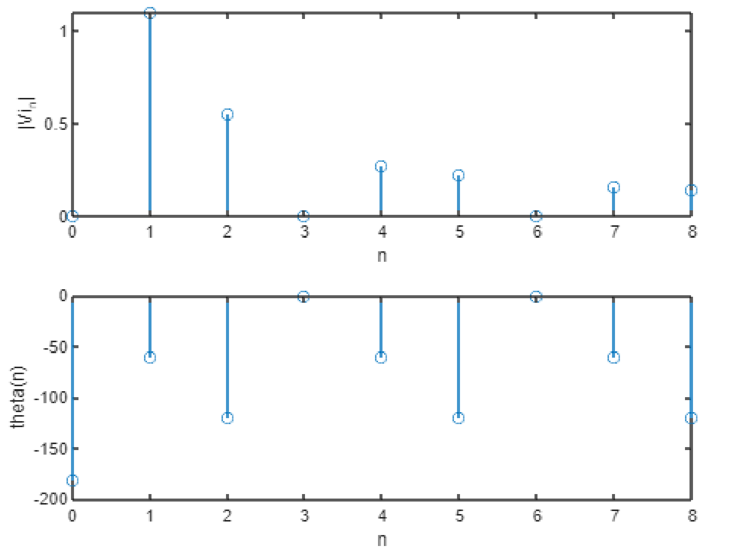
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = c_0 + \sum_{n=1}^{\infty} c_n \cos cos\left(nw_0 t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = .001cos\left(0w_0 t - 180\right) + .900cos\left(w_0 t - 45\right) + .637cos\left(2w_0 t - 90\right)$$
$$+ .300cos\left(3w_0 t - 135\right) + .180cos\,(5t - 45) + .212cos\left(6w_0 t - 90\right), \quad w_0 = \frac{2\pi}{1ms}$$

2V Square 33% duty cycle, period 1ms



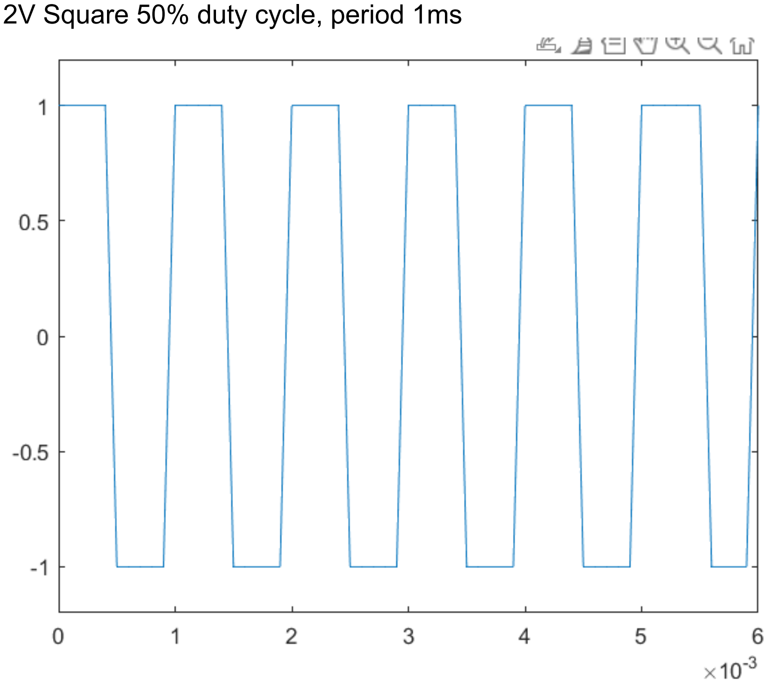
```
0.000   1.103   0.551   0.000   0.276   0.221   0.000   0.158   0.138
-180.000   -60.000   -120.000   0.000   -60.000   -120.000   0.000   -60.000   -120.000
```



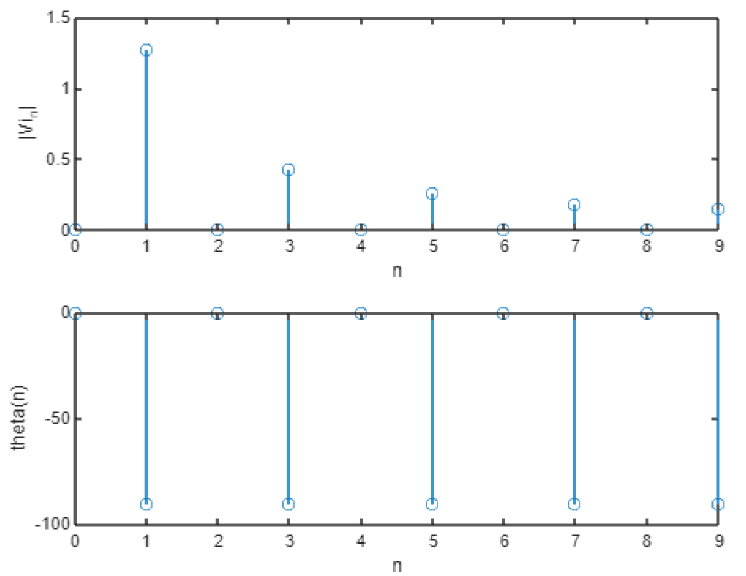
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0t} = c_0 + \sum_{n=1}^{\infty} c_n \cos\cos\left(nw_0t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0t} = .001cos\left(0w_0t - 180\right) + 1.0103cos\left(w_0t - 60\right) + .551cos\left(2w_0t - 120\right)$$
$$+ .276cos\left(4w_0t - 60\right) + .221cos\left(5t - 120\right) + .158cos\left(7w_0t - 60\right) + .138cos\left(8w_0t - 120\right), \quad w_0 = \frac{2\pi}{1ms}$$

2V Square 50% duty cycle, period 1ms



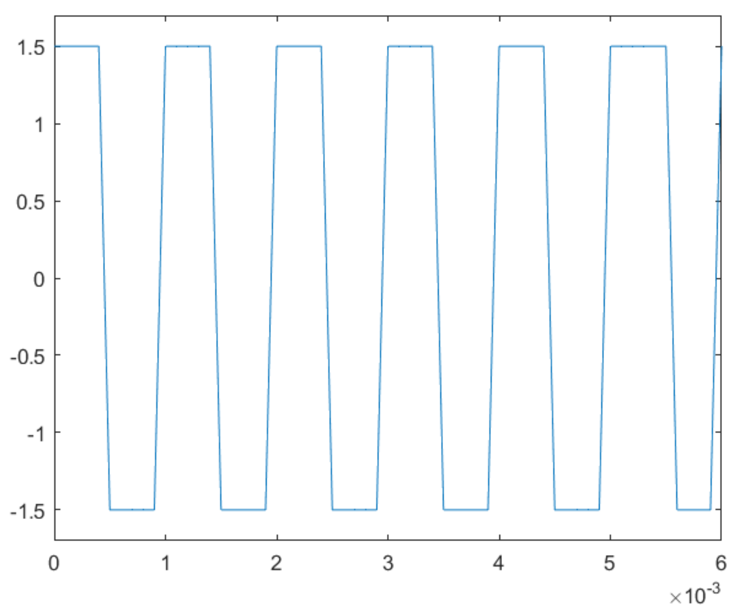
```
0.000  1.273  0.000  0.424  0.000  0.255  0.000  0.182  0.000  0.141
0.000  -90.000  0.000  -90.000  0.000  -90.000  0.000  -90.000  0.000  -90.000
```



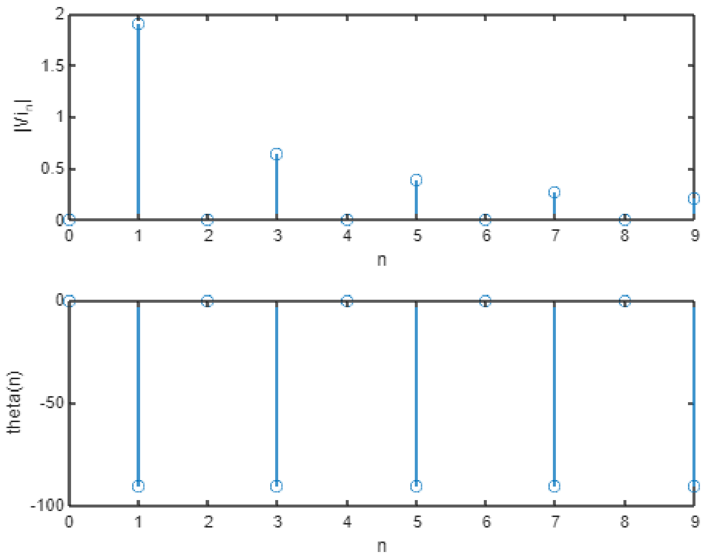
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = c_0 + \sum_{n=1}^{\infty} c_n \cos\cos\left(nw_0 t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = .001\cos\left(0w_0 t - 180\right) + 1.273\cos\left(w_0 t - 45\right) + .424\cos\left(2w_0 t - 90\right)$$
$$+ .255\cos\left(3w_0 t - 135\right) + .182\cos\left(5t - 45\right) + .141\cos\left(6w_0 t - 90\right), \quad w_0 = \frac{2\pi}{1ms}$$

3V Square 50% duty cycle, period 1ms



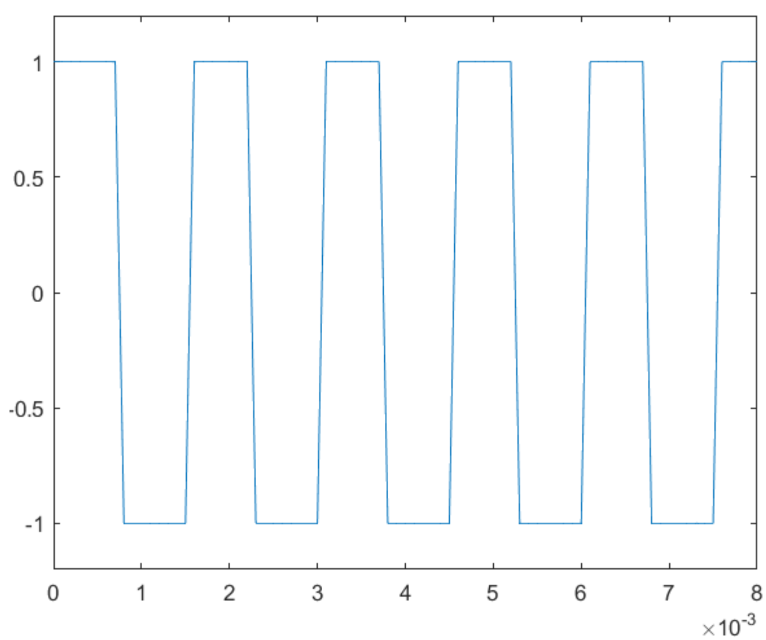
```
0.000  1.910  0.000  0.637  0.000  0.382  0.000  0.273  0.000  0.212
0.000  -90.000  0.000  -90.000  0.000  -90.000  0.000  -90.000  0.000  -90.000
```



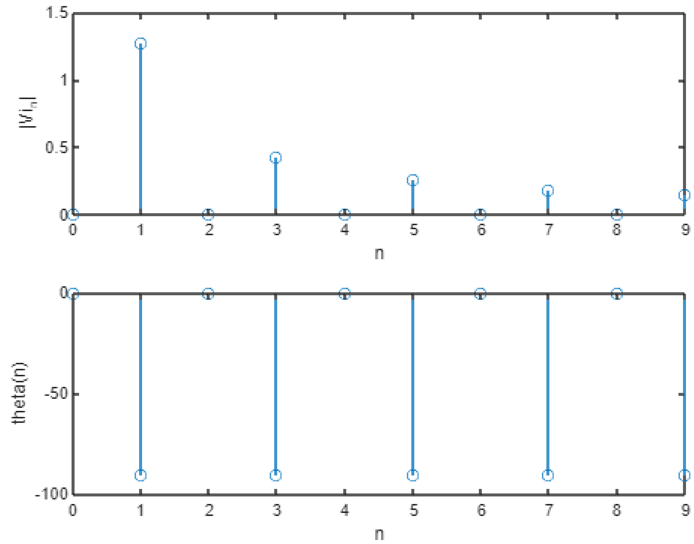
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = c_0 + \sum_{n=1}^{\infty} c_n \cos cos \left(nw_0 t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = 1.910cos\left(w_0 t - 90\right) + .637cos\left(2w_0 t - 90\right)$$
$$+ .382cos\left(3w_0 t - 90\right) + .273cos\left(5t - 90\right) + .212cos\left(6w_0 t - 90\right), \quad w_0 = \frac{2\pi}{1ms}$$

2V Square 50% duty cycle, period 1.5ms



```
0.000   1.273   0.000   0.424   0.000   0.255   0.000   0.182   0.000   0.141
0.000   -90.000   0.000   -90.000   0.000   -90.000   0.000   -90.000   0.000   -90.000
```



$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = c_0 + \sum_{n=1}^{\infty} c_n \cos cos \left(nw_0 t + \theta_n\right), \quad w_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} = .001cos\left(0w_0 t - 180\right) + 1.273cos\left(w_0 t - 45\right) + .424cos\left(2w_0 t - 90\right)$$
$$+ .255cos\left(3w_0 t - 135\right) + .182cos\,(5t - 45) + .141cos\left(6w_0 t - 90\right), \quad w_0 = \frac{2\pi}{1ms}$$

Sawtooth pulse train

50% symmetry

$$X_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} x(?)e^{-jn??}dt\,, \quad n = \mp 1, \mp 2, \mp 3, \ldots = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(?) \cos cos\,(n??)\,dt$$

$$X_n = \frac{2}{T_0} \int_{0}^{T_0/2} x(?) \cos cos\,(n??)\,dt$$

Between 0 and $T_0/2$, as x(?) is symmetrical about the x and y axis, it can be represented by:

$$x(?) = \frac{Vpp}{2} - \frac{4*\left(\frac{Vpp}{2}\right)}{T_0} *?$$

Substituting x(t) into X_n we can find the expression for the complex exponential Fourier coefficient

$$X_n = \frac{2}{T_0} \int_{0}^{T_0/2} \left(\frac{?}{2} - \frac{4*\left(\frac{?}{2}\right)}{T_0} *?\right) \cos cos\,(n??)\,dt = \frac{?}{T_0} \int_{0}^{T_0/2} \cos cos\,(n??)\,dt - \frac{4}{T_0} \int_{0}^{T_0/2} t \cos cos\,(n??)\,dt$$

Integrating by parts or with a table of integrals or by computer

$$X_n = \frac{Vpp}{T_0} * \left(\frac{T_0 \text{sinsin}\, \pi n}{2\pi n} + \frac{4}{T_0}\left(\frac{T_0^2 * (2*\text{sinsin}\left(\frac{\pi n}{2}\right)^2 - \pi n \text{sinsin}\,(\pi n))}{4\pi^2 n^2}\right)\right)$$

As $\sin sin\,(\pi n) = 0$

$$X_n = \frac{Vpp}{T_0} * \frac{4}{T_0} \frac{T_0^2 * 2 * \text{sinsin}\left(\frac{\pi n}{2}\right)^2}{4\pi^2 n^2} = \frac{4Vpp * \text{sinsin}\left(\frac{\pi n}{2}\right)^2}{\pi^2 n^2} * \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{Vpp}{2} * \frac{\text{sinsin}\left(\frac{\pi n}{2}\right)^2}{\left(\frac{\pi n}{2}\right)^2}$$

$$X_n = \frac{?}{2} * \left(\frac{?n}{2}\right)$$

$$C_n = 2\left|X_n\right| = ? * \left(\frac{?n}{2}\right), \quad n = 1, 2, 3, \ldots$$

$$C_n(dBV) = 20log\left(\frac{? * \left(\frac{?n}{2}\right)}{\sqrt{2}}\right), \quad n = 1, 2, 3, \ldots$$

100% symmetry

We phase shift the 100% symmetry Sawtooth waveform x(?) by half a period so that it is odd.
That way x(?) times a cosine function which has even symmetry has odd symmetry and its integral is 0.
The Fourier coefficient is then reduced to

$$X_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} x(?) e^{-jn??} dt, \quad n = \mp 1, \mp 2, \mp 3, \ldots = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(?) \sin sin\,(n??)\, dt$$

Between $-T_0/2$ and $T_0/2$, x(?) can be represented as an increasing ramp:

$$x(?) = \frac{Vpp}{T_0} * ?$$

Substituting x(?) into X_n we can find the expression for the complex exponential Fourier coefficient

$$X_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left(\frac{?}{T_0} * ?\right) \sin sin\,(n??)\, dt = \frac{Vpp}{T_0^2} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} (n??)\, dt$$

Integrating by parts or with a table of integrals or by computer

$$X_n = \frac{Vpp}{2} * \frac{\text{sinsin}\,(\pi n) - \pi n \text{coscos}\,(\pi n)}{\pi^2 n^2}$$

As $\sin sin\,(\pi n) = 0$ and $\cos cos\,(\pi n) = (-1)^n$

$$X_n = \frac{-\frac{Vpp}{2}}{\pi n}(-1)^n$$

$$7 = 2\left|X_n\right| = \frac{-?}{\pi n}(-1)^n, \quad n = 1, 2, 3, \ldots$$

$$C_n(dBV) = 20log\left(\frac{-?}{\pi n\sqrt{2}}(-1)^n\right), \quad n = 1, 2, 3, \ldots$$

0% symmetry

$$X_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} x(?) e^{-jn??} dt, \quad n = \mp 1, \mp 2, \mp 3, \ldots = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(?) \sin sin\,(n??)\, dt$$

Between $-T_0/2$ and $T_0/2$, x(?) can be represented as a decreasing ramp:

$$x(t) = -\frac{Vpp}{T_0} *?$$

Substituting x(t) into X_n we can find the expression for the complex exponential Fourier coefficient

$$X_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left(-\frac{?}{T_0} *?\right) \sin sin\,(n??)\,dt = -\frac{Vpp}{T_0^2}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} (n??)\,dt$$

Integrating by parts or with a table of integrals or by computer

$$X_n = -\frac{Vpp}{2} * \frac{sinsin\,(\pi n) - \pi n coscos\,(\pi n)}{\pi^2 n^2}$$

As $\sin sin\,(\pi n) = 0$ and $\cos cos\,(\pi n) = (-1)^n$

$$X_n = \frac{\frac{?}{2}}{?n}(-1)^n$$

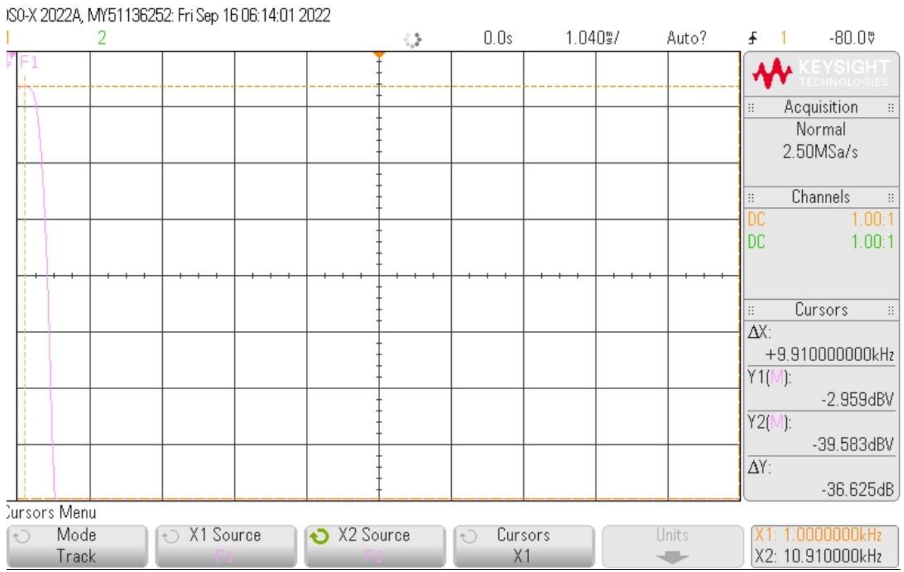
You should work on the question marks

$$C_n = 2\left|X_n\right| = \frac{?}{?n}(-1)^n, \quad n = 1, 2, 3, ...$$

Laboratory

			Vp/p		To		dutycycle/symmetry				
			V		ms		%				
			low	high	low	high	20	25	33	50	100
1	sine										
2		N									
3		o									
4	r										
5	e	f									
6	c	i									
7	t	l									
8	s	t									
9	w	e									
10	t	r									

Sinewave

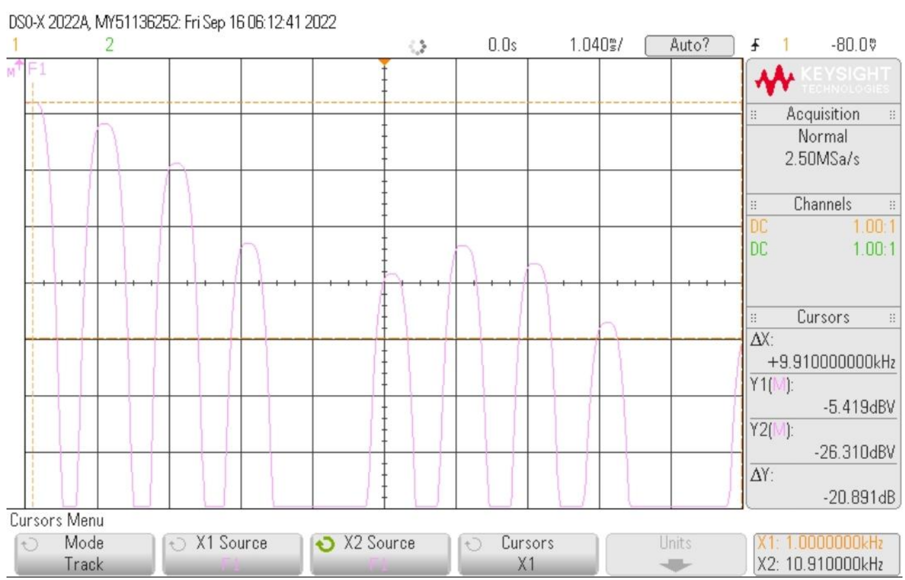


In the for

Peaks	Frequency	dBv
1	1KHz	-2.96

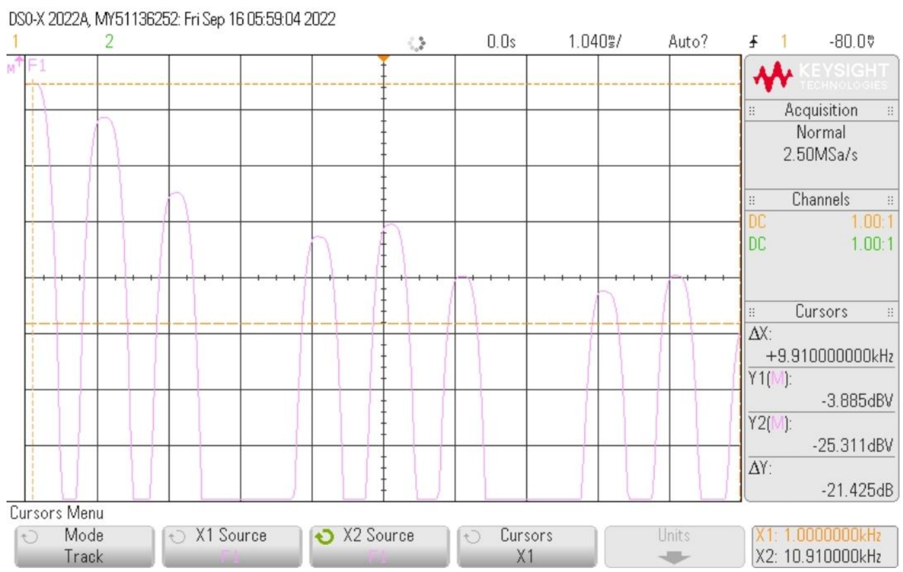
Square pulse train

2V Square 20% duty cycle, period 1ms



	VPP	N	FREQUENCY	dBv
2v square % 20	2.00	1.00	1,000.00	-5.5
	2.00	2.00	2,000.00	-7.3
	2.00	3.00	3,000.00	-10.9
	2.00	4.00	4,000.00	-17.7
	2.00	5.00	5,000.00	0
	2.00	6.00	6,000.00	-20.6

2V Square 25% duty cycle, period 1ms

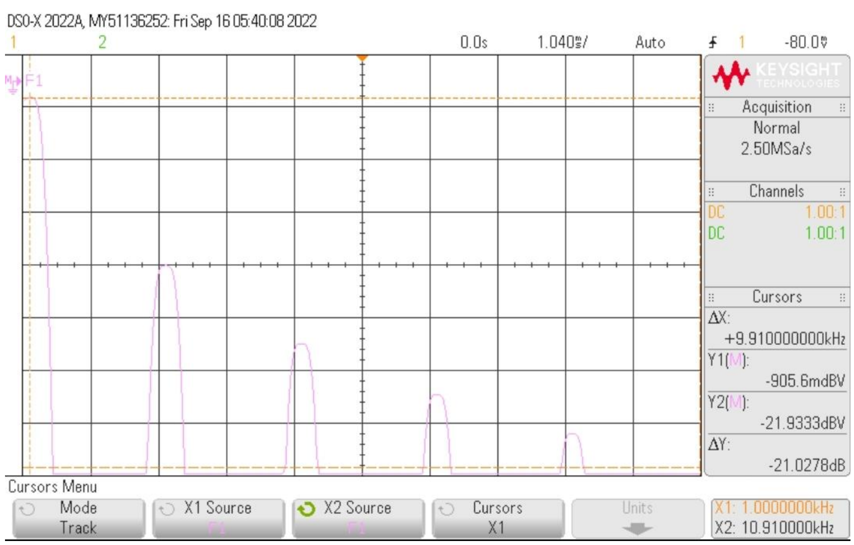


2v square % 25	2.00	1.00	1,000.00	-3.83
	2.00	2.00	2,000.00	-6.95
	2.00	3.00	3,000.00	-13.5
	2.00	4.00	4,000.00	0
	2.00	5.00	5,000.00	-17.7
	2.00	6.00	6,000.00	-16.4
	2.00	7.00	7,000.00	-21.3

2V Square 33% duty cycle, period 1ms

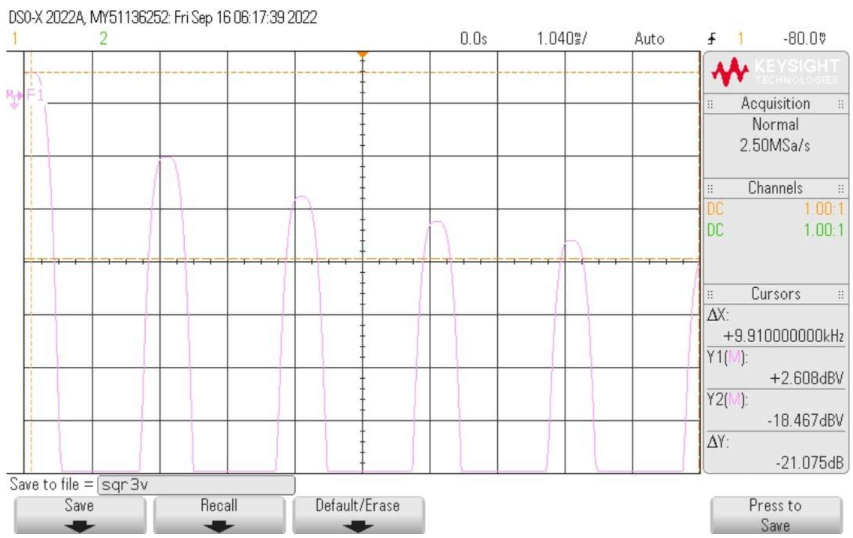
2v square % 33	2.00	1.00	1,000.00	-2.16
	2.00	2.00	2,000.00	.8.08
	2.00	3.00	3,000.00	0
	2.00	4.00	4,000.00	-14.3
	2.00	5.00	5,000.00	-15.9
	2.00	6.00	6,000.00	0
	2.00	7.00	7,000.00	-19.2

2V Square 50% duty cycle, period 1ms



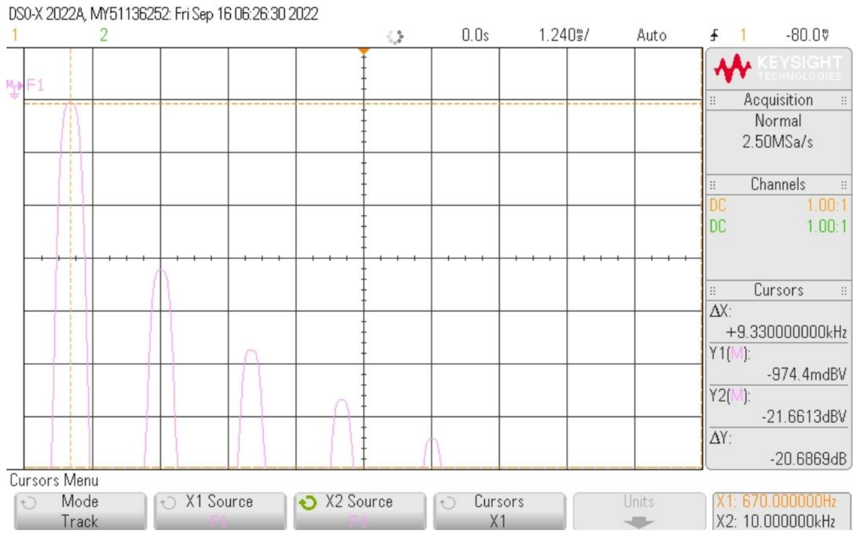
2v square % 50	2.00	1.00	1,000.00	-.899.4
	2.00	2.00	2,000.00	0
	2.00	3.00	3,000.00	-10.43
	2.00	4.00	4,000.00	0
	2.00	5.00	5,000.00	-14.88
	2.00	6.00	6,000.00	0
	2.00	7.00	7,000.00	-17.7
	2.00	8.00	8,000.00	0
	2.00	9.00	9,000.00	-20

3V Square 50% duty cycle, period 1ms



3v square % 50	3.00	1.00	1,000.00	-2.6
	3.00	2.00	2,000.00	0
	3.00	3.00	3,000.00	-6.9
	3.00	4.00	4,000.00	0
	3.00	5.00	5,000.00	-11.4
	3.00	6.00	6,000.00	0
	3.00	7.00	7,000.00	-14.3
	3.00	8.00	8,000.00	0
	3.00	9.00	9,000.00	-16.5

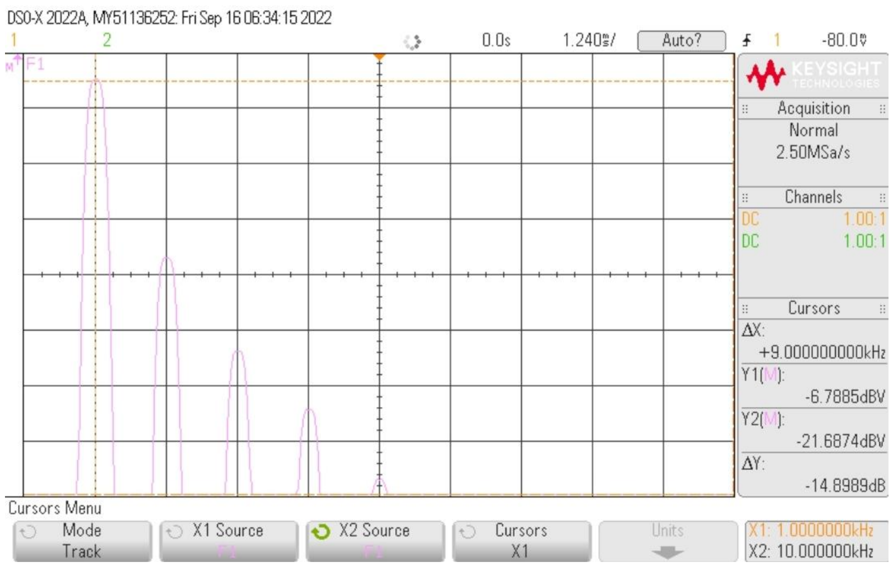
2V Square 50% duty cycle, period 1.5ms



2v square % 50	2.00	1.00	666.00	-0.939
to=1.5ms	2.00	2.00	1,332.00	0
	2.00	3.00	1,998.00	-10.4
	2.00	4.00	2,664.00	0
	2.00	5.00	3,330.00	-14.9
	2.00	6.00	3,996.00	0
	2.00	7.00	4,662.00	-17.8
	2.00	8.00	5,328.00	0
	2.00	9.00	5,994.00	-20.1

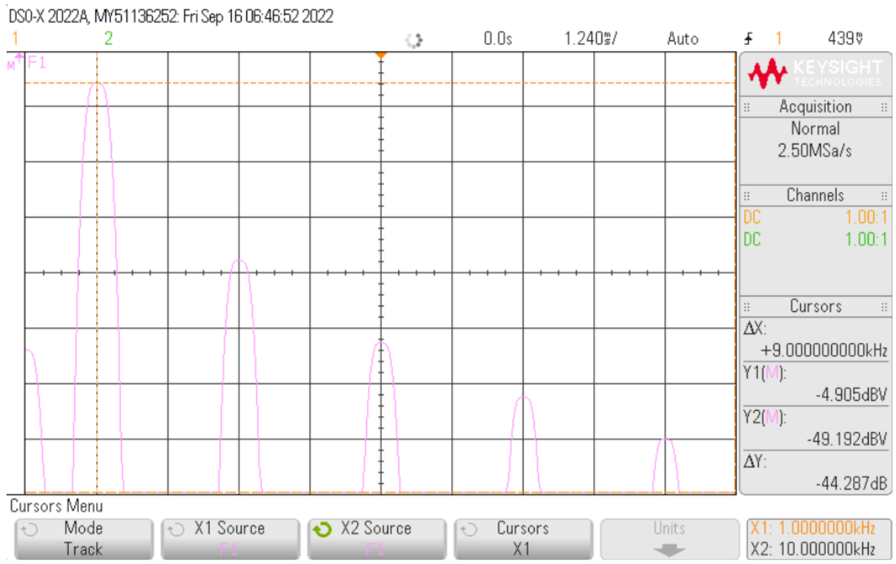
Sawtooth pulse train

sawthooth 0%



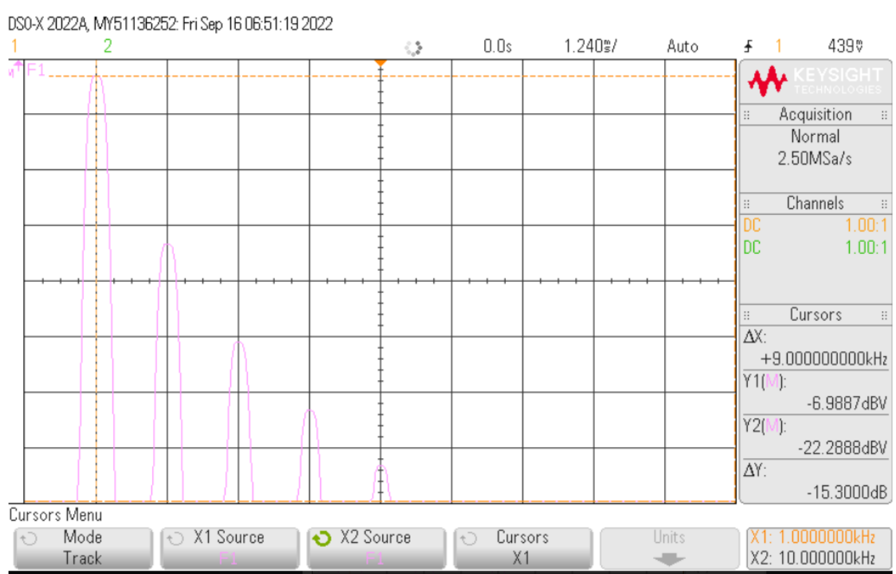
sawthooth 0%	2.00	1.00	1,000.00	-6.7
	2.00	2.00	2,000.00	-12.9
	2.00	3.00	3,000.00	-16.4
	2.00	4.00	4,000.00	-18.6
	2.00	5.00	5,000.00	-21.3

sawthooth 50%



sawthooth 50%	2.00	1.00	1,000.00	-4.9
	2.00	2.00	2,000.00	-24
	2.00	3.00	3,000.00	-33
	2.00	4.00	4,000.00	-38.8
	2.00	5.00	5,000.00	-43.2

sawthooth 100%



sawthooth 100%	2.00	1.00	1,000.00	-6.9

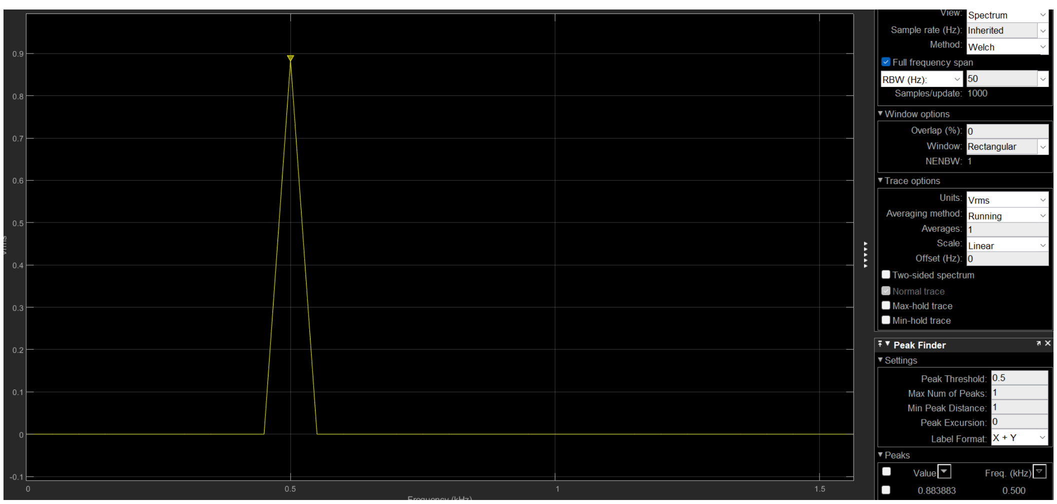
	2.00	2.00	2,000.00	-13
	2.00	3.00	3,000.00	-16.5
	2.00	4.00	4,000.00	-19
	2.00	5.00	5,000.00	-21

Understanding the relationships of the Sine,Square Wave and Sawtooth

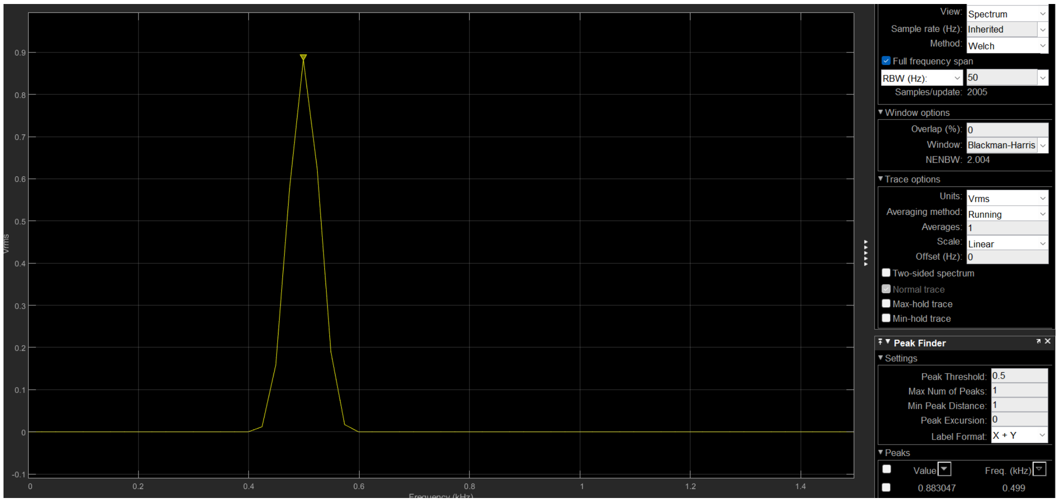
Simulation of unfiltered sine, square & sawtooth waves using Simulink

Sinewave

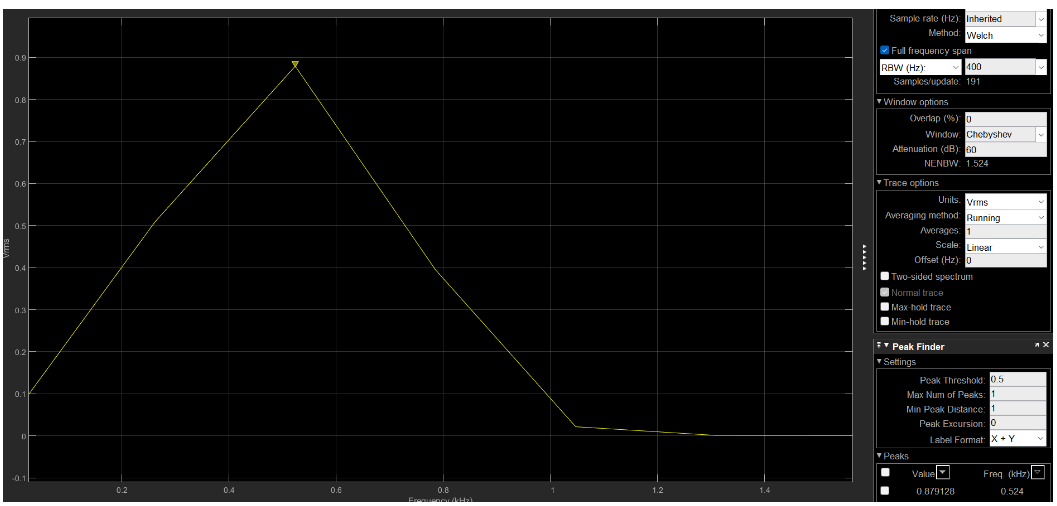
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Rectangular window (RBW=50)



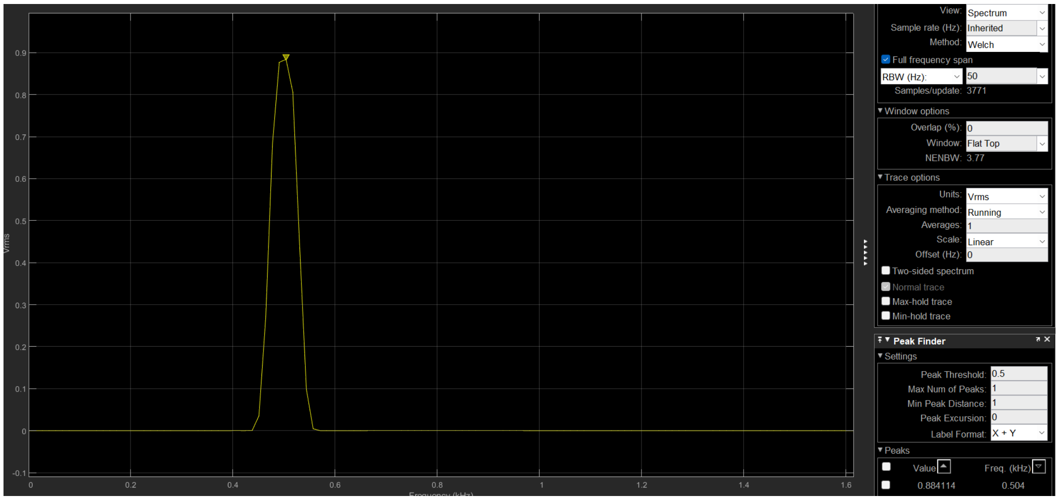
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Blackman-Harris window (RBW=50)



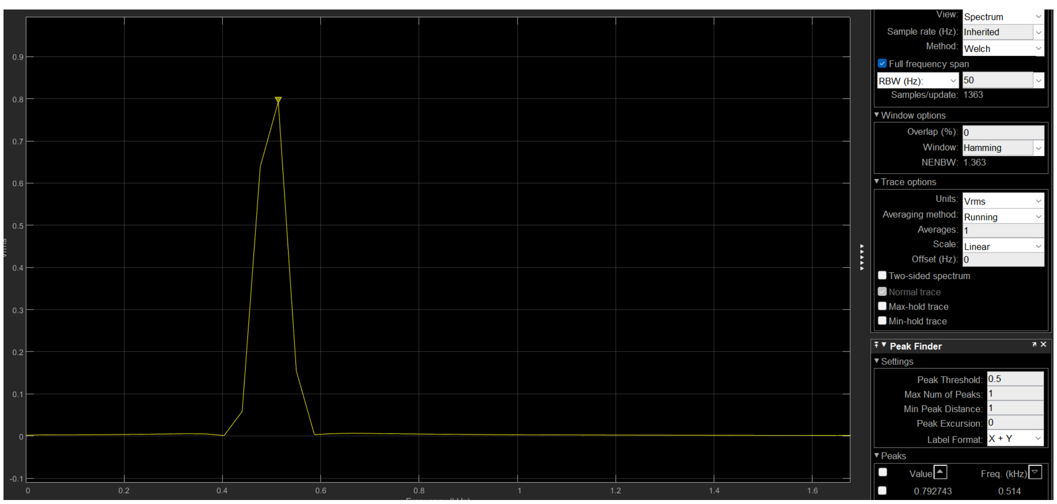
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Chebyshev window (RBW=400)



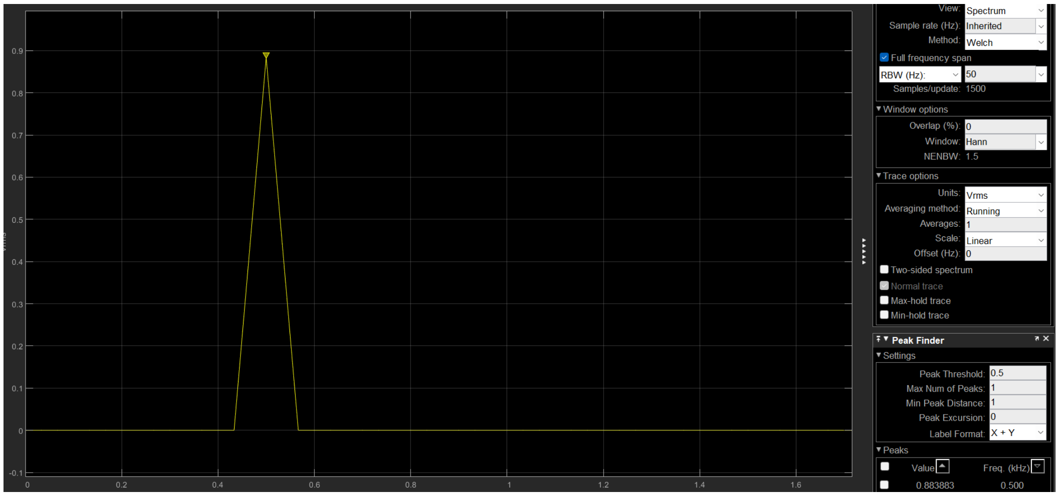
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Flat-Top window (RBW=50)



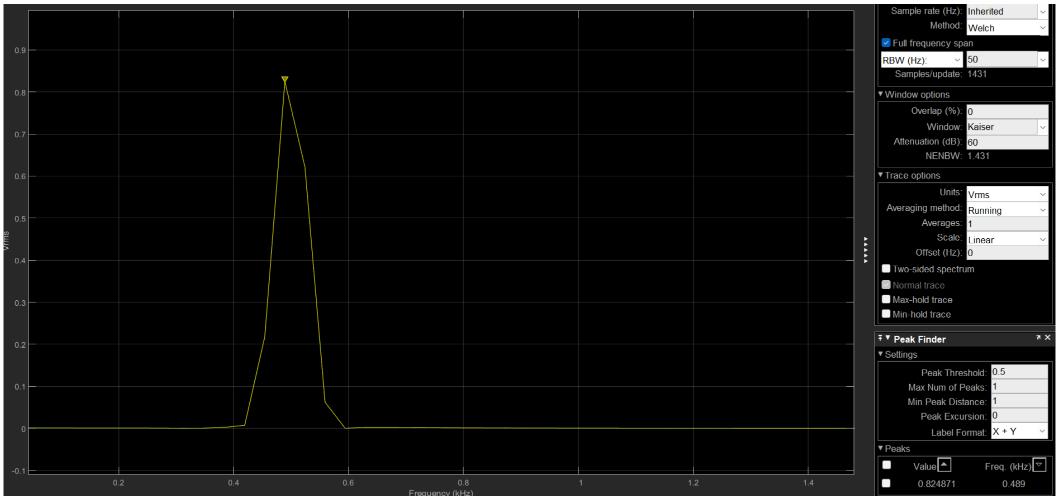
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Hamming window (RBW=50)



Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Hahn window (RBW=50)



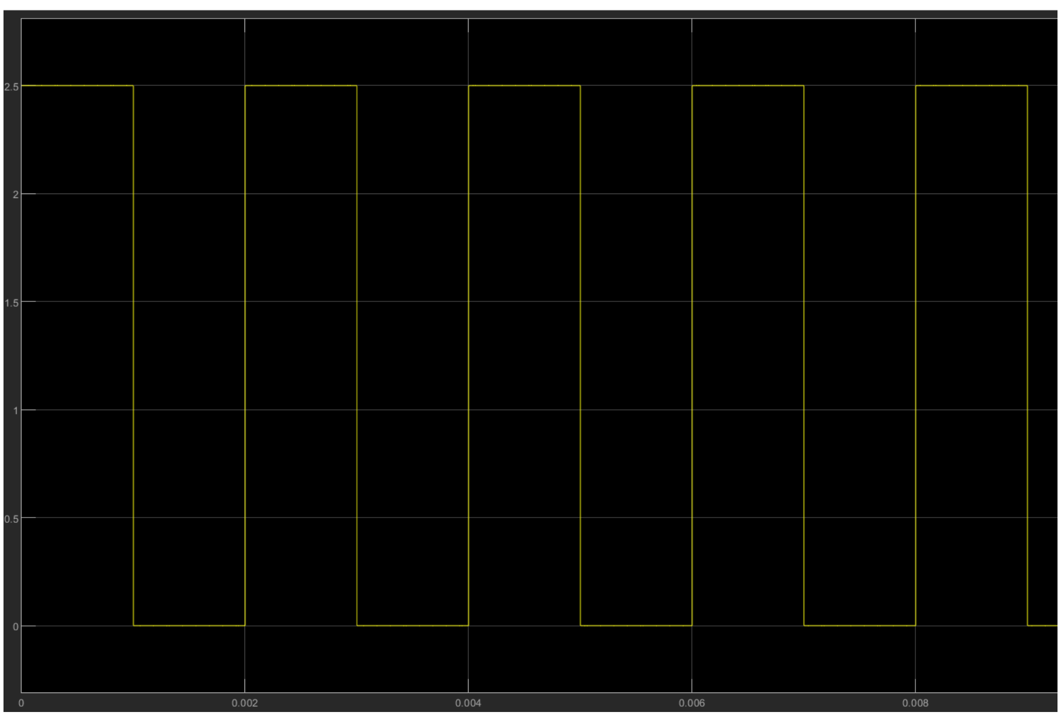
Sinewave Vpp=V, fo=Hz Vrms (dBV) Spectrum analyzer Kaiser window (RBW=50)



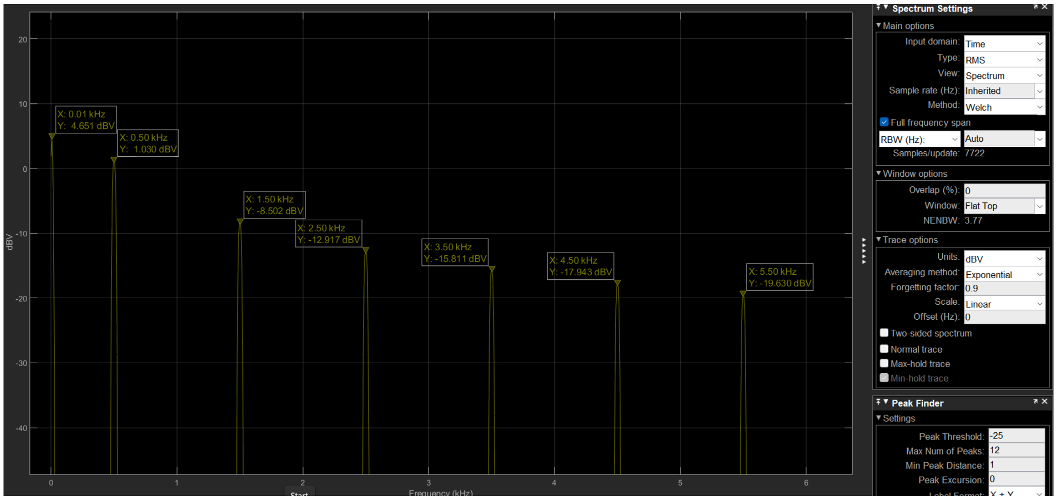
Square pulse train

Square wave Vpp=V, Vp=V, frequency=Hz, duty cycle=1/2, Ts=20us, run time=0.2s

Square wave Vpp=V, fo=Hz duty cycle=50% scope time domain

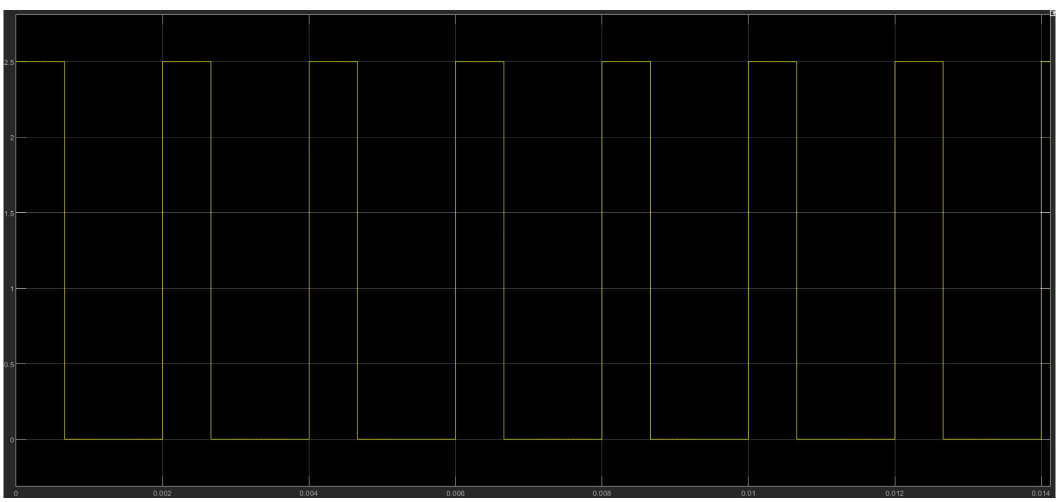


Square wave Vpp=V, fo=Hz duty cycle=50% Spectrum Analyzer frequency domain

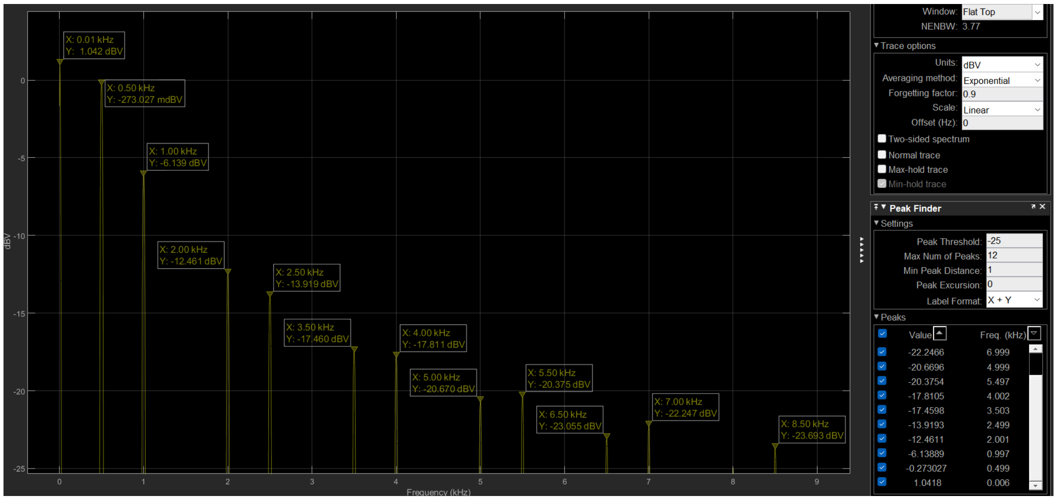


Square wave Vpp=V, Vp=V, frequency=Hz, duty cycle=1/3, sample time=20us, run time=0.2s

Square wave Vpp=V, fo=Hz duty cycle=33% scope time domain

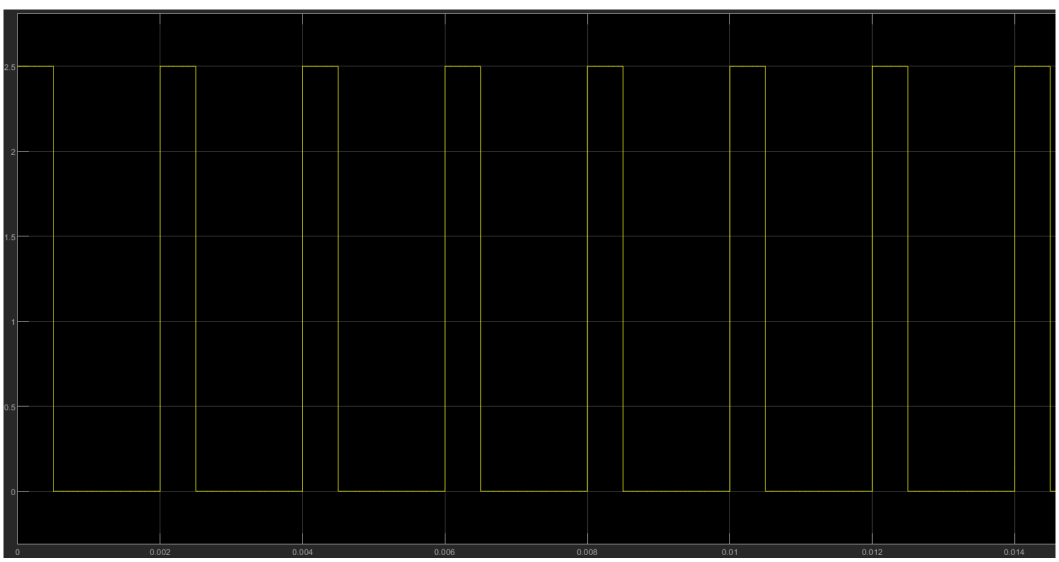


Square wave Vpp=V, fo=Hz duty cycle=33% Spectrum Analyzer frequency domain

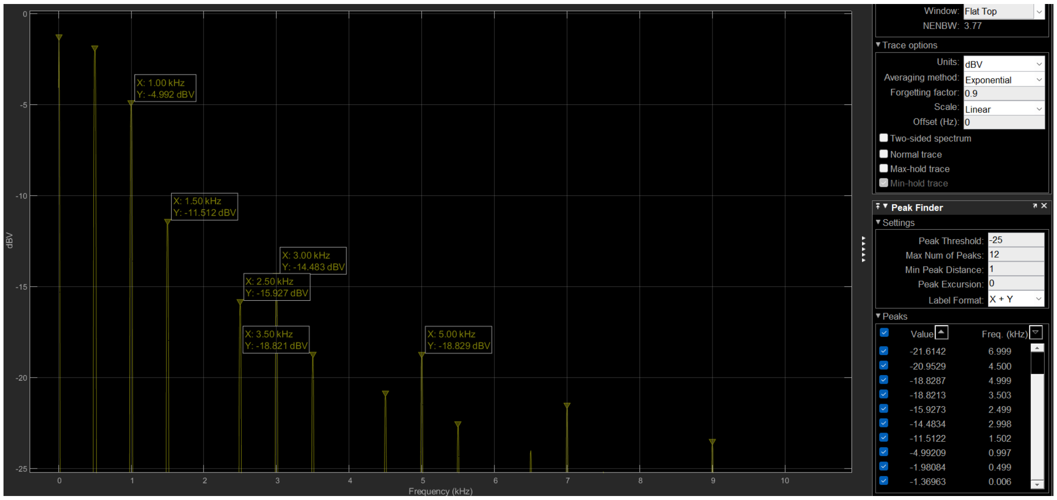


Square wave Vpp=V, fo=Hz, duty cycle=25%, sample time=20us, run time=0.2s

Square wave Vpp=V, fo=Hz duty cycle=25% scope time domain

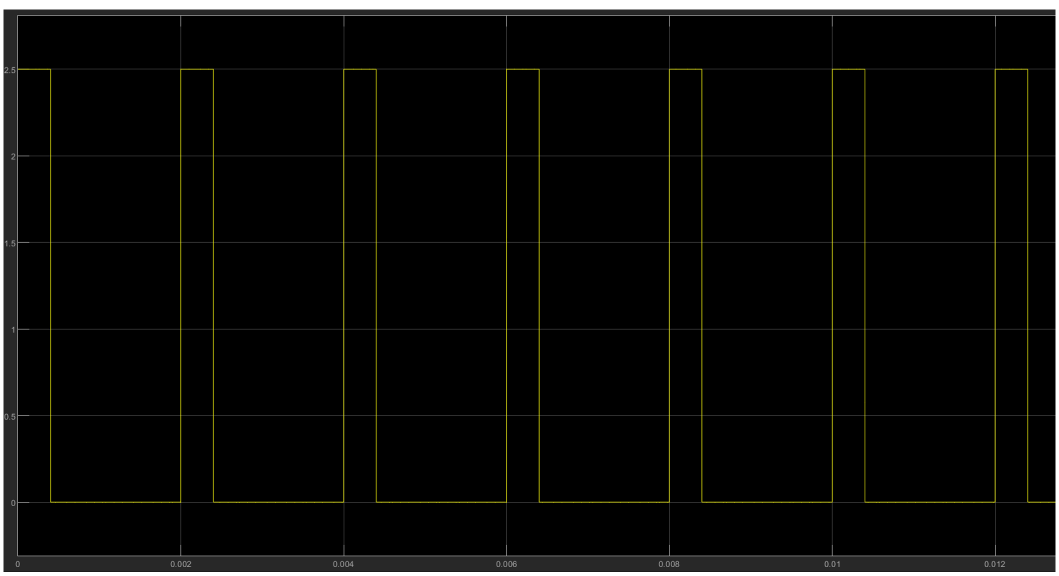


Square wave Vpp=V, fo=Hz duty cycle=25% Spectrum Analyzer frequency domain

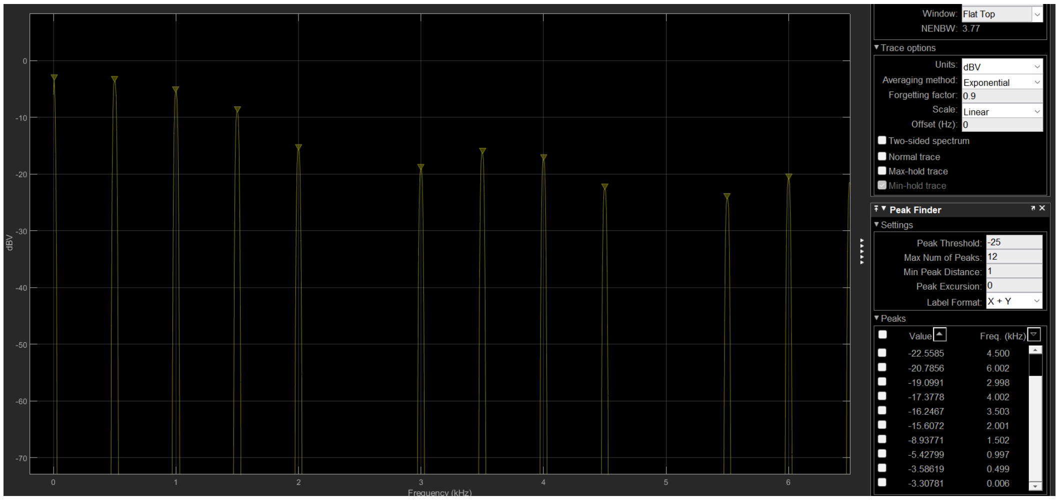


Square wave Vpp=V, Vp=V, fo=Hz, duty cycle=%, sample time=20us, run time=0.2s

Square wave Vpp=V, fo=Hz duty cycle=20% scope time domain



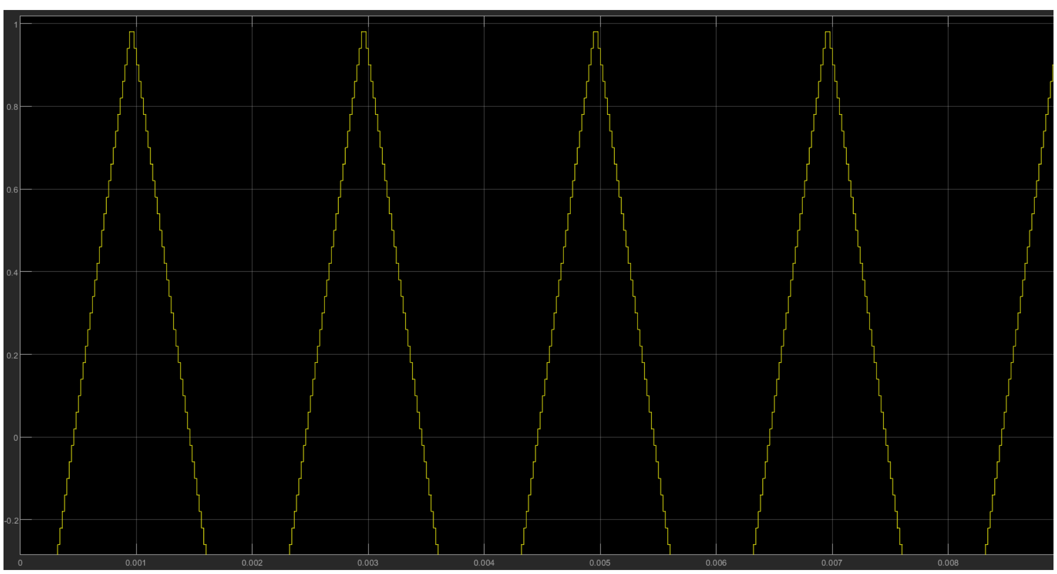
Square wave Vpp=V, fo=Hz duty cycle=20% Spectrum Analyzer frequency domain



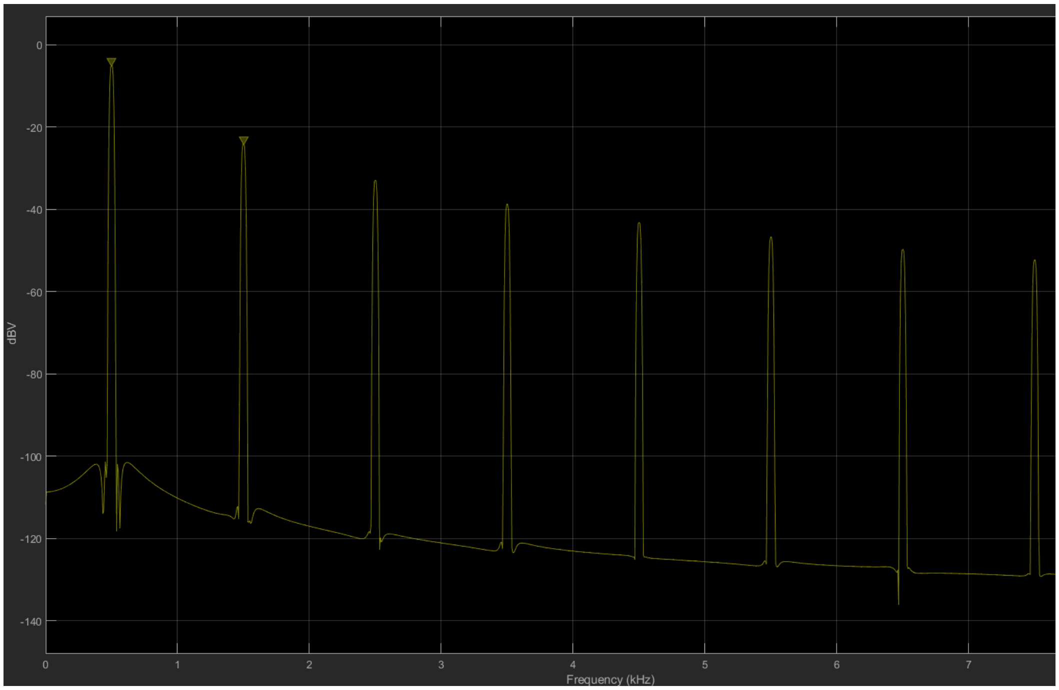
Sawtooth pulse train

Sawtooth Vpp=V, Vp=V, frequency=Hz, symmetry=50%, sample time=20us, run time=0.2s

Sawtooth wave Vpp=V, Vp=V, fo=Hz symmetry=% scope time domain

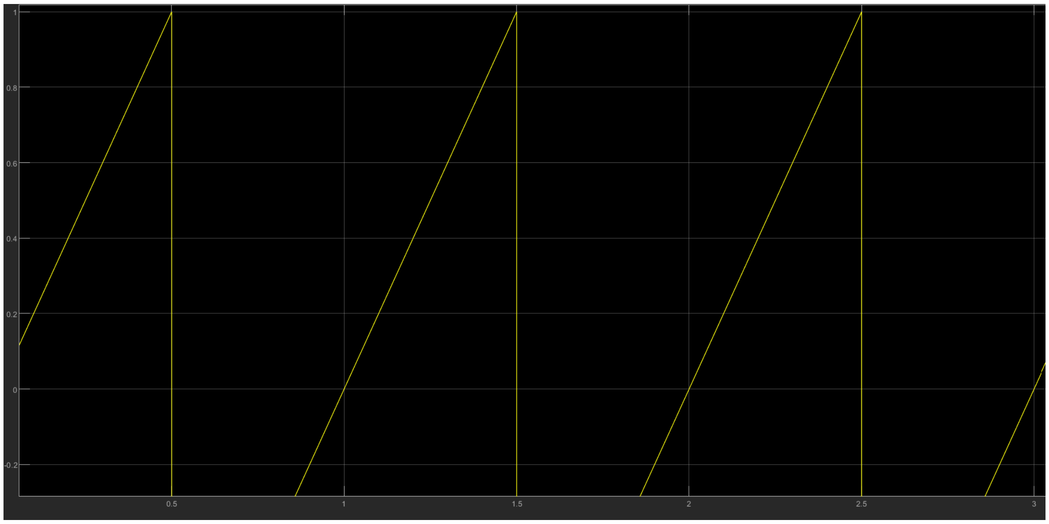


Sawtooth wave Vpp=V, Vp=V fo=Hz symmetry=% Spectrum Analyzer frequency domain

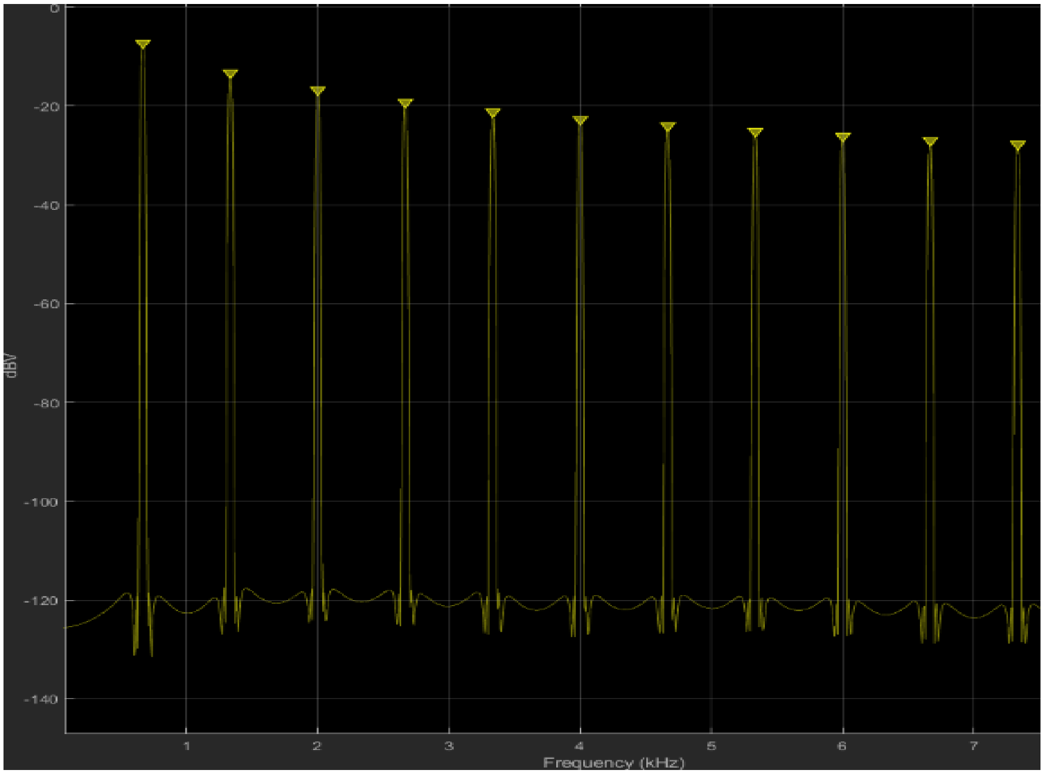


Sawtooth Vpp=V, Vp=V, fo=Hz, symmetry=100%, sample time=20us, run time=0.2s

Sawtooth wave Vpp=V, Vp=V, fo=Hz symmetry=% scope time domain

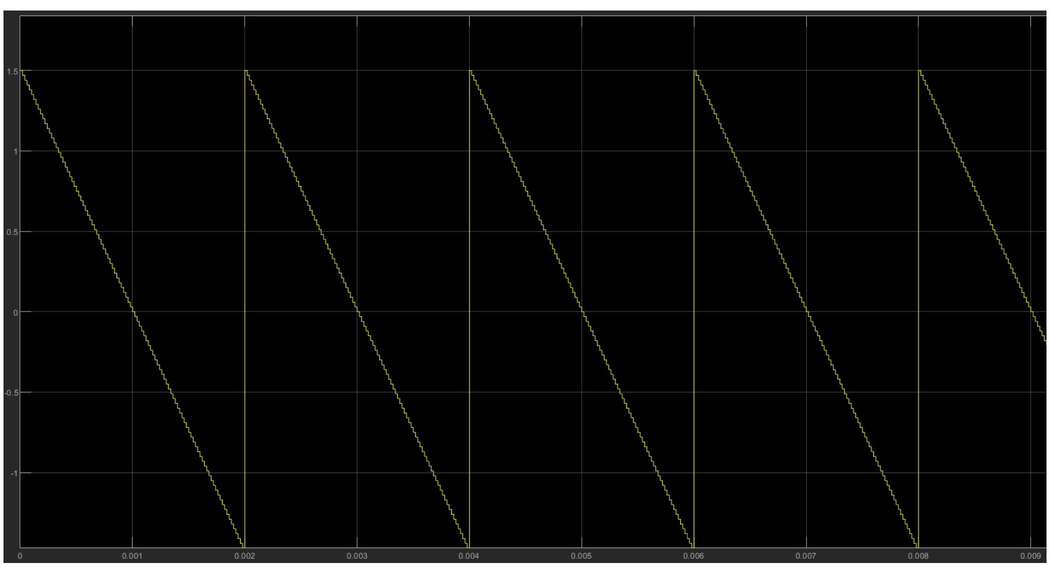


Sawtooth wave Vpp=V, Vp=V fo=Hz symmetry=% Spectrum Analyzer frequency domain



Sawtooth Vpp=V, Vp=V, fo=Hz, symmetry=0%, sample time=20us, run time=0.2s

Sawtooth wave Vpp=V, Vp=V, fo=Hz symmetry=% scope time domain



Sawtooth wave Vpp=V, Vp=V fo=Hz symmetry=% Spectrum Analyzer frequency domain