CURVE TRACING

SPAR

For Cartesion Curves -

Rule 1 :- Symmetry.

1) If all powers of y are even $\Rightarrow$ Curve is Symmetric about X axis.

2) If all powers of x are even $\Rightarrow$ Curve is Symmetric about Y axis.

3) Replace $x \to y$ & $y \to x$. equation is unchanged. Curve is Symmetric about line $y = x$.

Rule 2 :- Pts of Intersection

Origin :- put $x = 0$ & $y = 0$ if equation is satisfied then Curve passes through Origin.

Tangent at Origin $\to$ Equate lowest degree term to zero.

Intersection with X axis :- put $y = 0$, find all possible values of x.

Intersection with Y axis :- put $x = 0$, find all possible values of y.

Rule 3 :- $\boxed{\text{Asymptote}}$ (tangent at infinity).

1) Parallel to X-axis :- equate Coefficient of highest power of x to zero.

2) Parallel to Y axis :- equate Coefficient of highest power of y to zero

Note :- If Coefficient is Constant then no asymptote in that case.

Rule 4 :- $\boxed{\text{Region of Absence :-}}$

1) Arrange equation as $y^2 = f(x)$ find x for which y^2 is negative or y is imaginary.

or

2) Arrange equation as $x^2 = f(y)$. find y where x^2 is negative or x is imaginary.

3) Sometimes find x & y and observe whether they are ↑ or ↓
(like $x = (y-1)(y-2)(y-3)$) → example.

Note :- Two points on same line are always Connected with a loop.

POLAR CURVES (in terms of r & θ).

SPTRA

Rule 1:- $\boxed{\text{Symmetry}}$

1) put θ by $-\theta$ if equation is unchanged.
$\Rightarrow$ Curve is Symmetric about initial line $\theta = 0°$.

2) Put $\theta \to -\theta$ & $r \to -r$ if equation is unchanged $\Rightarrow$ Curve is Symmetric about $\theta = \frac{\pi}{2}$ line.

Same Symmetry exist when equation is unchanged replacing θ by $\pi - \theta$.
$\downarrow$

Use it for equations having sin terms as $\sin(\pi - \theta) = \sin\theta$.

Rule 2:- $\boxed{\text{Pole}}$

put $r = 0$ & find $\theta \Rightarrow$ if such θ exists then pole lies on Curve.

$\theta \to$ represents tangent at pole.

Rule 3:- $\boxed{\text{Table for } r \text{ \& } \theta}$

1) for Symmetric about $\theta = 0°$.

θ	0	45	90	135	180
r					

2) for Symmetric about $\theta = \pi/2$.

θ	90	135	180	225	270
r					

Rule 4 :- $\boxed{\text{Region of absence}}$

For $r^2 = f(\theta)$ find values of θ where r^2 is negative then no Curve for that value of θ. If equation is not given in terms of $r^2 = f(\theta)$ then Curve is present for all θ.

Rule 5 :- Angle ϕ between tangent & radius vector.

Solve $\boxed{\tan\phi = r\frac{d\theta}{dr}}$

ROSE CURVES

Type:- $r = a\cos n\theta$ / $r = a\sin n\theta$

SPNDA

Rule 1:- Symmetry

1) put θ as $-\theta$ if equation is unchanged $\Rightarrow$ Curve is Symmetric about initial line $\theta = 0°$.

2) put $\theta \to -\theta$ & $r \to -r$ if equation is unchanged $\Rightarrow$ Curve is Symmetric about $\theta = \frac{\pi}{2}$ line.

Rule 2:- Pole :-

put $r = 0$ & find θ if such θ exists then pole lies on the Curve.

$\theta \to$ represents tangent at pole.

Rule 3:- No of loops :- There are :→

a) 2n loops if n is even

b) n loops if n is odd.

Rule 4:- Divide each quadrant into n equal parts.

If equation is $r = a\cos n\theta$ → first loop starts along $\boxed{\theta = 0°}$

If equation is $r = a\sin n\theta$ → first loop starts along $\boxed{\theta = \frac{\pi/2}{n}}$

For n is even - Draw loops Consecutively in every two sectors.

For n is odd - Draw loops alternatively keeping two sectors vaccant in between them.

Rule 5 :- Angle between tangent & radius vector

$$\tan\phi = r \cdot \frac{d\theta}{dr}.$$

Note :- for $r = a\sin n\theta$

putting $r = 0$

we get $\sin n\theta = 0$

$$n\theta = 0, \pi, 2\pi, 3\pi, \ldots$$

$$\theta = 0, \frac{\pi}{n}, \frac{2\pi}{n}, \frac{3\pi}{n}, \ldots$$

for $r = a\cos n\theta$

for $r = 0$

$$0 = \cos n\theta$$

$$n\theta = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

$$\theta = -\frac{\pi}{2n}, \frac{\pi}{2n}, \frac{3\pi}{2n}, \ldots$$