Navigation System for Low-Cost Lunar Lander with LOS Measurements

ECE/MAE 7560: Optimal Estimation

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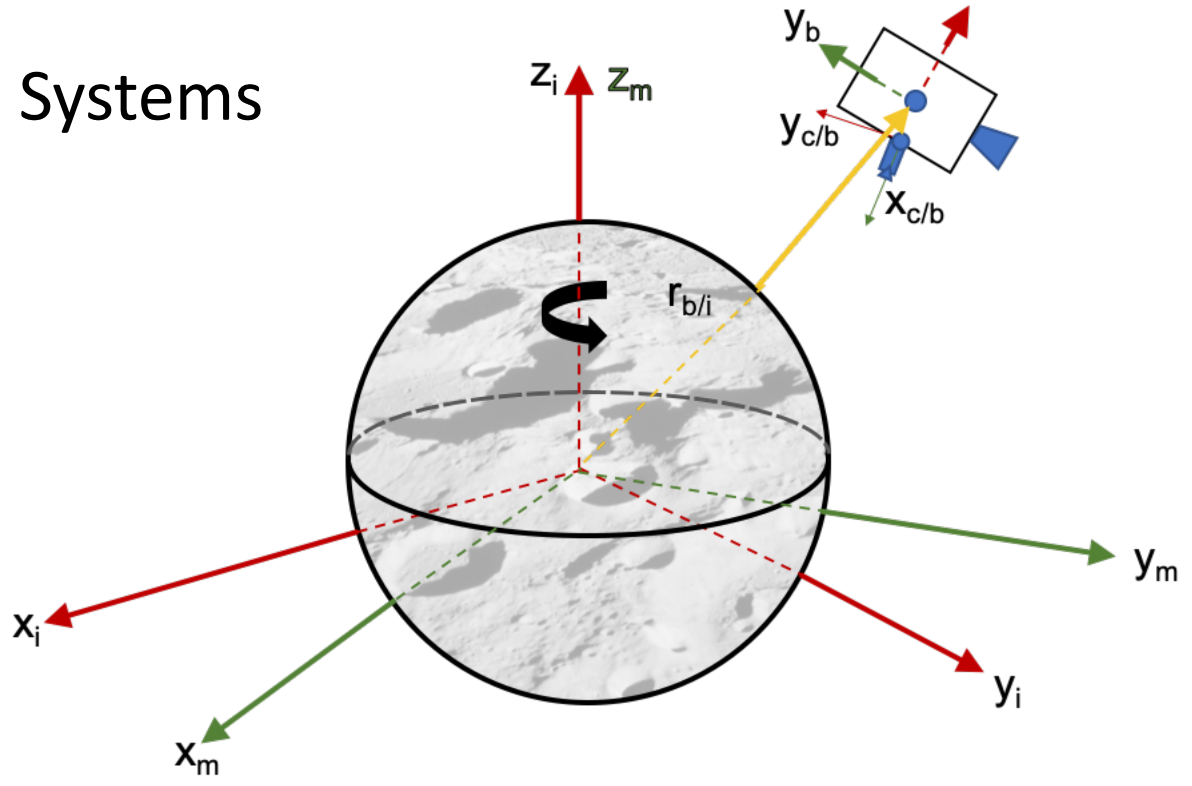
Contents

Introduction

- Returning to the moon is high-priority
- Automation is desirable for establishing a long-term presence
- Any automated lander needs to be able to make precision landings, navigate unmapped areas, and should be as low-cost as possible
- Many autonomous landing and navigation schemes already exist, but the good ones tend to be very expensive (6+ sensors = $$$)
- Trade-off: efficacy vs. cost
- Proposed solution: Minimal sensor suite (monocular camera and accelerometer)

Coordinate Systems

- 4 Coordinate Systems
  - Intertial
  - Moon
  - Body
  - Camera



Truth vs. Design States

- Truth State

$$\boldsymbol{x_t} = \begin{bmatrix} \boldsymbol{r}^i_{b/i} \\ \boldsymbol{v}^i_{b/i} \\ q^m_i \\ q^c_b \\ b_r \\ \boldsymbol{\epsilon}^i_g \\ h_t \\ \boldsymbol{b}^b_a \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_v \\ \boldsymbol{p} \end{bmatrix}$$

$$x_v = \begin{bmatrix} \boldsymbol{r}^i_{b/i} \\ \boldsymbol{v}^i_{b/i} \\ \boldsymbol{q}^m_i \end{bmatrix}$$

$$\boldsymbol{p} = \begin{bmatrix} \boldsymbol{q}^c_b \\ b_r \\ \boldsymbol{\epsilon}^i_g \\ h_t \\ \boldsymbol{b}^b_a \end{bmatrix}$$

- Design State

Truth vs. Design States

- Truth State
- Design State

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{r}}^i_{b/i} \\ \dot{\boldsymbol{v}}^i_{b/i} \\ \dot{b}_r \\ \dot{\boldsymbol{\epsilon}}^i_g \\ \dot{h} \\ \dot{\boldsymbol{b}}_{accl} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}^i_{b/i} \\ T^i_b \left(\tilde{\boldsymbol{a}}^b - \boldsymbol{b}_a - \boldsymbol{n}_a \right) + \boldsymbol{\epsilon}^i_g + \boldsymbol{a}_{grav} \\ -\frac{1}{\tau_r} b_r + w_r \\ -\frac{\|\boldsymbol{v}_\perp\|}{d_g} \boldsymbol{\epsilon}^i_g + \boldsymbol{w}_g \\ -\frac{\|\boldsymbol{v}_\perp\|}{d_h} h + w_h \\ -\frac{1}{\tau_{accl}} \boldsymbol{b}_{accl} + \boldsymbol{w}_{accl} \end{bmatrix}$$

Linearization of Model

- Perturbation Model

$$\begin{bmatrix} \delta \boldsymbol{r}^i_{b/i} \\ \delta \boldsymbol{v}^i_{b/i} \\ \delta b_r \\ \delta \boldsymbol{\epsilon}^i_g \\ \delta h_t \\ \delta \boldsymbol{b}^b_a \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}^i_{b/i} \\ \boldsymbol{v}^i_{b/i} \\ b_r \\ \boldsymbol{\epsilon}^i_g \\ h_t \\ \boldsymbol{b}^b_a \end{bmatrix} - \begin{bmatrix} \hat{\boldsymbol{r}}^i_{b/i} \\ \hat{\boldsymbol{v}}^i_{b/i} \\ \hat{b}_r \\ \hat{\boldsymbol{\epsilon}}^i_g \\ \hat{h}_t \\ \hat{\boldsymbol{b}}^b_a \end{bmatrix} = \boldsymbol{e}(\hat{\boldsymbol{x}}, \boldsymbol{x})$$

- General form

$$\delta \dot{\boldsymbol{x}} = F(\hat{\boldsymbol{x}})\delta \boldsymbol{x} + B\omega$$

- Taylor Series resulting in Jacobian

$$F = \left. \frac{df(x,u)}{dx} \right|_{x=\hat{x}}$$

$$= \begin{bmatrix} \mathbf{0}_3 & I_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 \\ \frac{-\mu(I_3 - 3 i_r i_r^T)}{\|\hat{\boldsymbol{r}}\|^3} & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & I_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \frac{-1}{\tau_r} & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & \frac{\|\hat{\mathbf{v}}_\perp\|}{dg} I_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 & \frac{\|\hat{\mathbf{v}}_\perp\|}{dh} & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{3\times 1} & \mathbf{0}_3 & \mathbf{0}_3 & \frac{-1}{\tau_{accl}} \end{bmatrix}$$

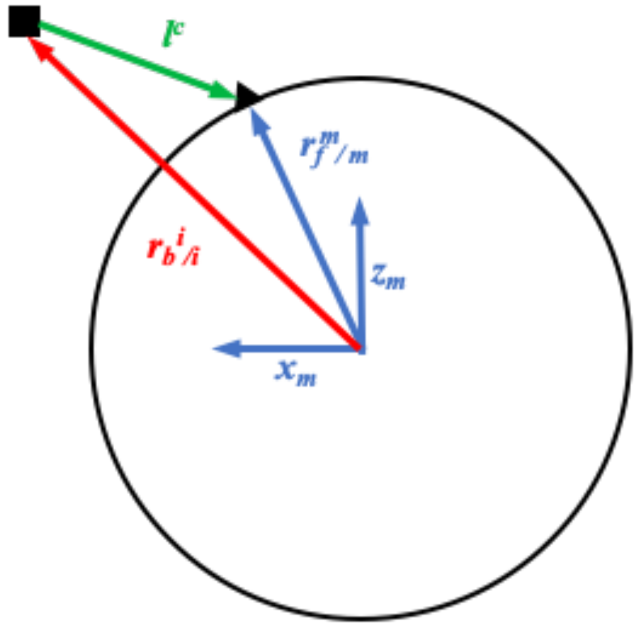
Measurement Model

- Camera measurements

$$\boldsymbol{\ell}^c = T_b^c T_i^b (T_m^i \boldsymbol{r}_{f/m}^m - \boldsymbol{r}_{b/i}^i) = \begin{bmatrix} \ell_x \\ \ell_y \\ \ell_z \end{bmatrix}$$

  - Non-linear Model

$$\tilde{\boldsymbol{z}} = \begin{bmatrix} \frac{\ell_x}{\ell_z} \\ \frac{\ell_y}{\ell_z} \end{bmatrix} + \boldsymbol{v}_c$$



Measurement Model Linearization

- Linearized model $$\delta\tilde{\boldsymbol{z}} = H(\hat{\boldsymbol{x}})\delta\boldsymbol{x} + G\nu$$

- Linearization done in two steps

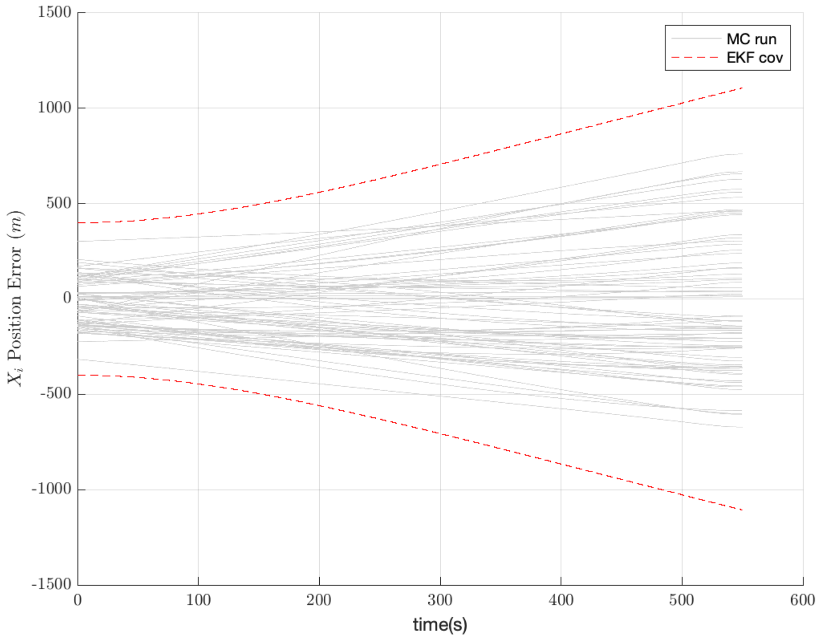
$$H = \frac{dh}{d\boldsymbol{l}}\frac{d\boldsymbol{l}}{d\boldsymbol{x}}\bigg|_{x=\hat{x}}$$

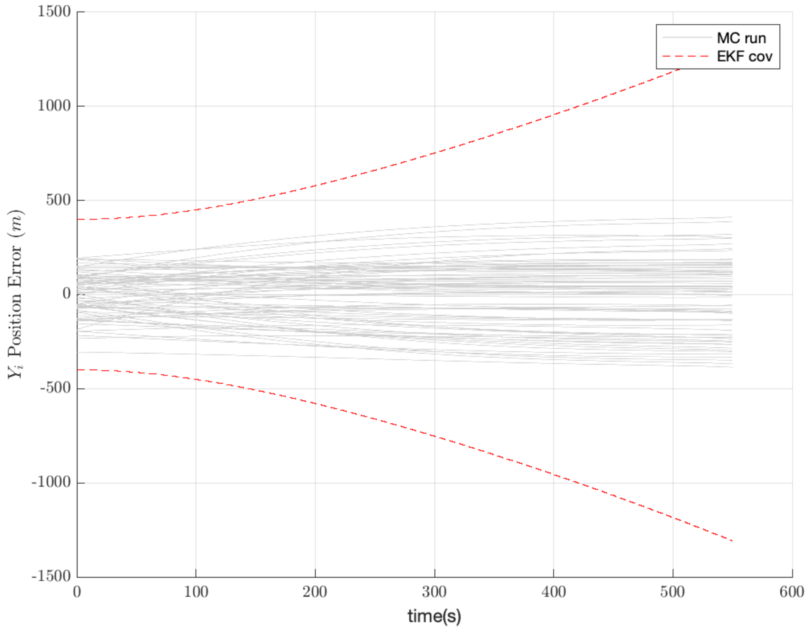
$$\frac{dh}{d\boldsymbol{l}} = \begin{bmatrix} 1/\tilde{l}_z & 0 & -\tilde{l}_x/\tilde{l}_z^2 \\ 0 & 1/\tilde{l}_z & -\tilde{l}_y/\tilde{l}_z^2 \end{bmatrix}$$

$$\frac{d\boldsymbol{l}}{d\boldsymbol{x}} = \begin{bmatrix} -T_b^i T_i^b I_3 & 0_{3\times 11} \end{bmatrix}$$

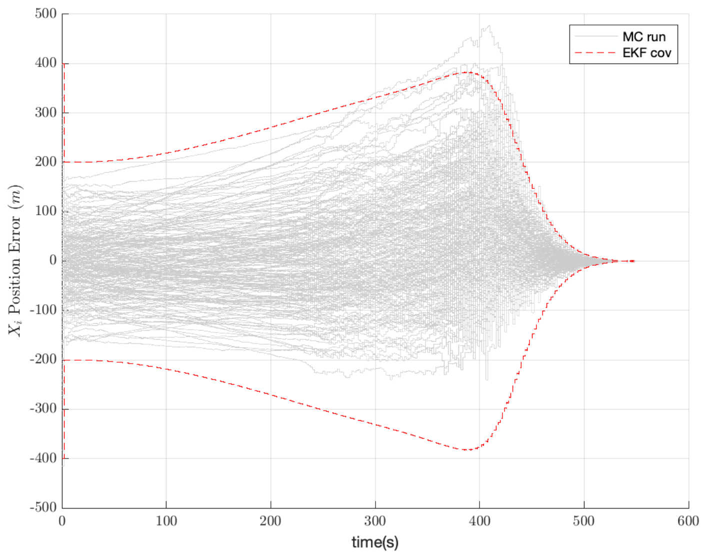
- Result $$H = \begin{bmatrix} 1/\tilde{l}_z & 0 & -\tilde{l}_x/\tilde{l}_z^2 \\ 0 & 1/\tilde{l}_z & -\tilde{l}_y/\tilde{l}_z^2 \end{bmatrix}\begin{bmatrix} -T_b^i T_i^b I_3 & 0_{3\times 11} \end{bmatrix}$$

Estimation plots before filter

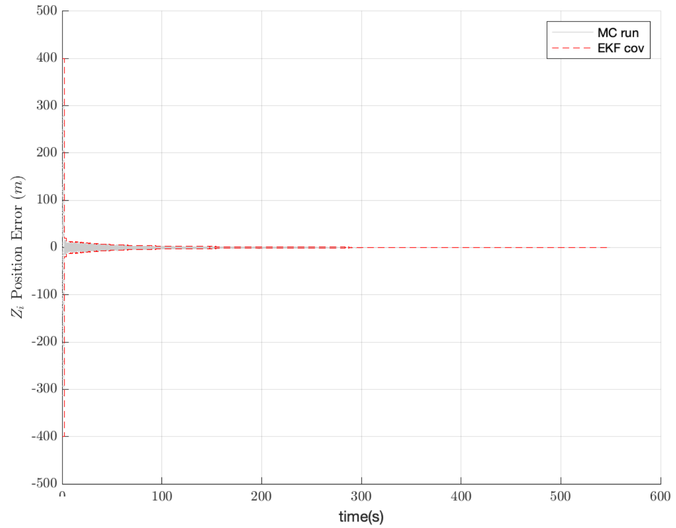


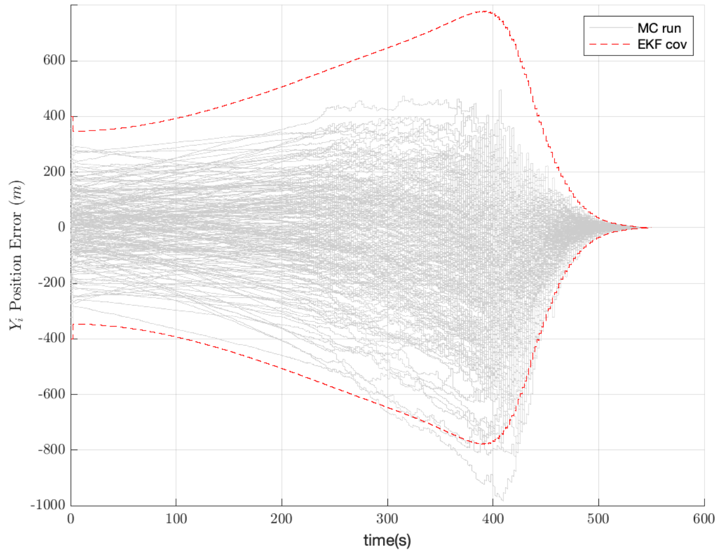


Estimation plots

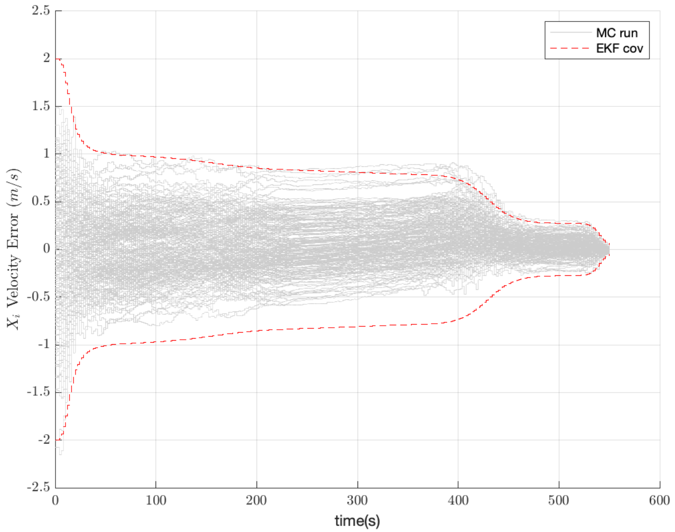


- X position looks okay, though biased
- Y position looks good, though biased
- Z position looks great

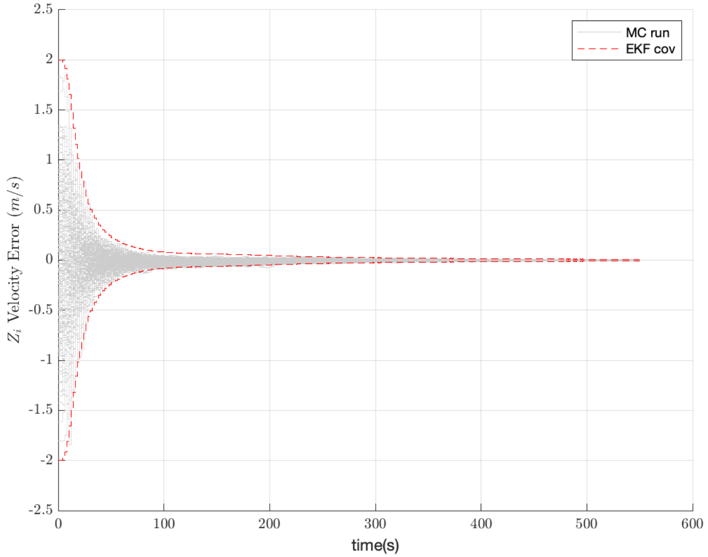


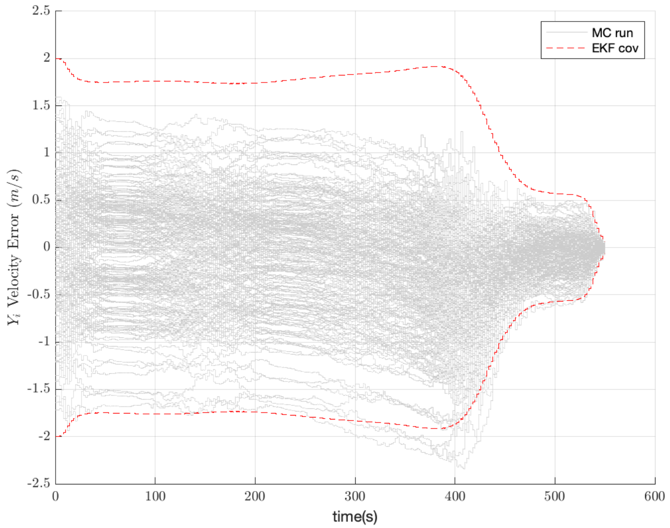


Estimation plots



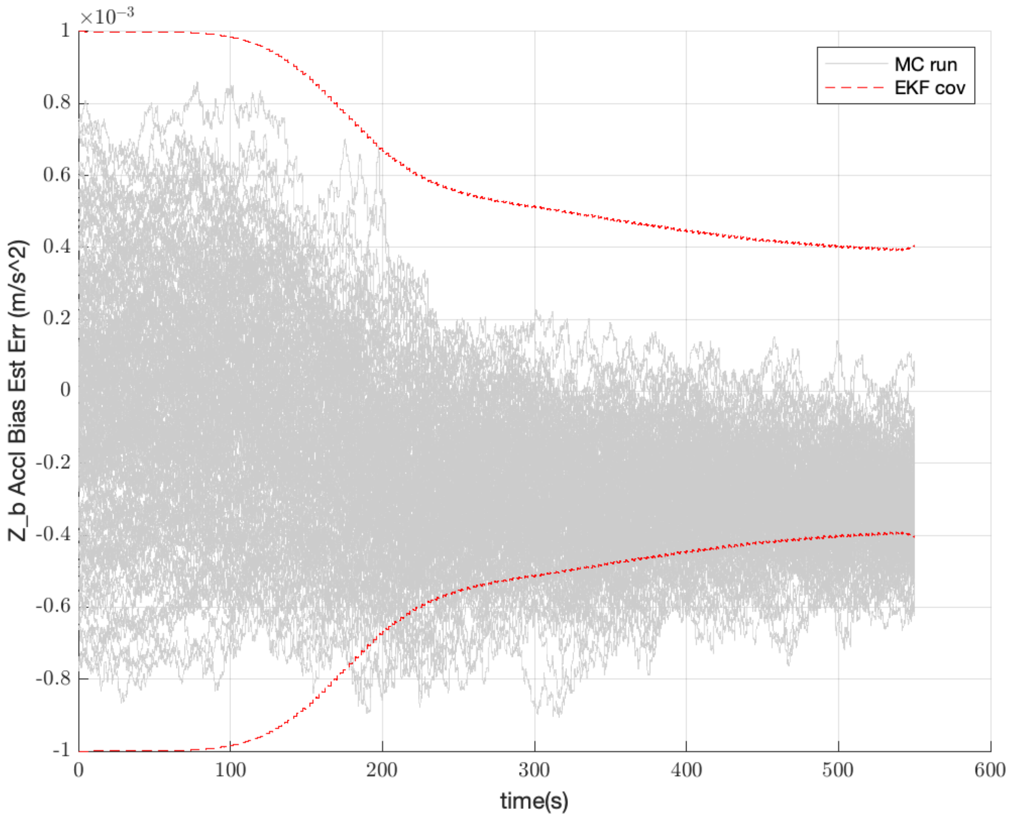
- X velocity looks okay, though biased
- Y velocity looks good, though biased
- Z velocity looks great



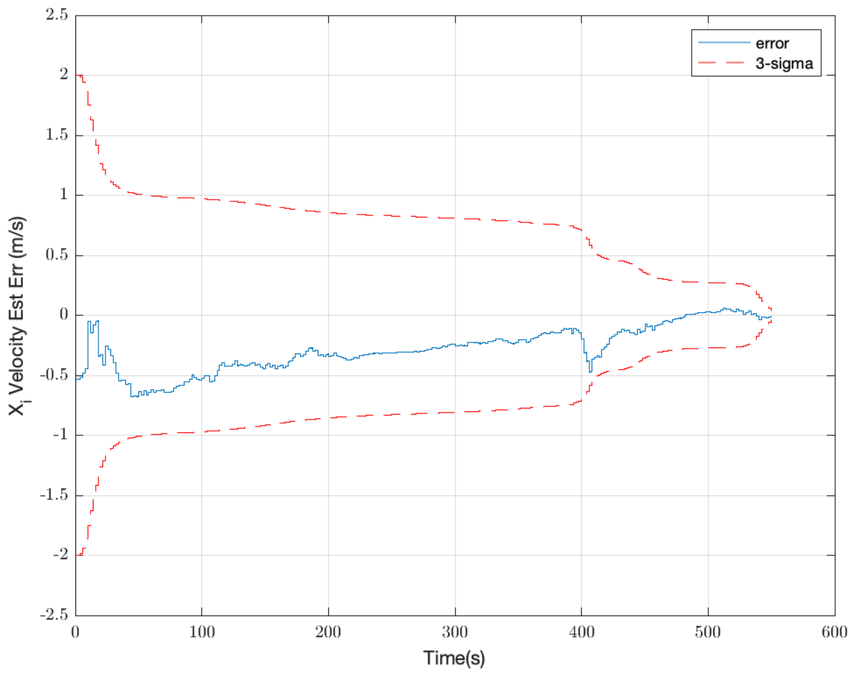


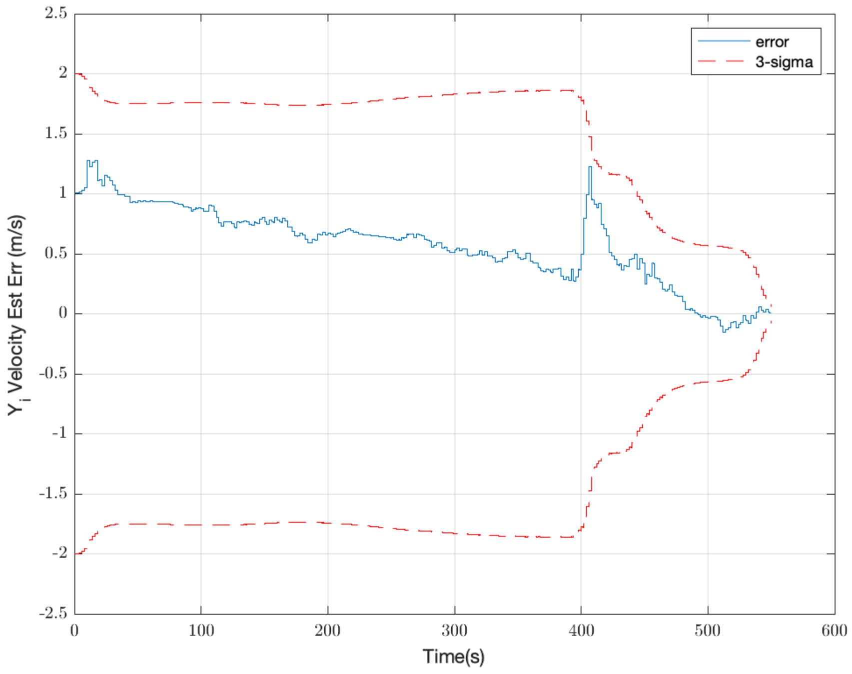
Estimation plots

- Not yet sure what is going on with this
- Probably what is causing the position and velocity problems
- Covariance looks good, probably a bug in the state propagation



Residual Plots from single Monte Carlo





Results

- The lander is very accurate in some states
  - Position within a couple hundred meters
  - Velocity within 0.5 m/s
- Executes a fairly soft landing, although it could be softer
- Using one or two more sensors could greatly improve the system – but it would be more expensive to add those
  - Radio altimeter
  - Gyro
  - Star Tracker

Conclusions

- Lunar lander with a minimal sensor suite is feasible
- Linearizations can occasionally be very difficult
- Communication between different parts of the simulation is key
- EKFs are very hard to debug
  - Isolate as much as possible
  - Debug in small chunks
  - It just takes a long time
  - Validate where possible
  - Coordinate systems are very important
- I could take the class again and learn just as much