CPSC-354 Report

Eleas Vrahnos
Chapman University

October 23, 2022

Abstract

To be written at a later date.

Contents

1 Introduction

This report is written by Eleas Vrahnos. It details all assignments and progress made in the Programming Languages course at Chapman University. It includes weekly homework assigments, programming assignments, and a final project that demonstrate understanding and application in various class topics.

2 Homework

This section will contain my solutions to the weekly homework assignments.

2.1 Week 1

The following is a Python implementation of the Euclidean algorithm:

```
def gcd(a,b):
    while a != b:
        if a > b:
            a = a-b
        else:
            b = b-a
    return a
```

We can test this code by going through the function with a sample input `gcd(9, 33)`, step by step.

1. `gcd(9, 33)`
   - The function is called, assigning 9 to variable `a` and 33 to variable `b`.
2. `while a != b:`
   - The while loop condition returns True, so the loop starts.
3. `else:`
   - `a > b` ($9 > 33$) returns False, so the else block executes.
4. `b = b-a`
   - `b` is now assigned to $33 - 9$, which is 24.
5. `while a != b:`
   - The while loop condition returns True, so the loop starts.
6. `else:`
   - `a > b` ($9 > 24$) returns False, so the else block executes.
7. `b = b-a`
   - `b` is now assigned to $24 - 9$, which is 15.
8. `while a != b:`
   - The while loop condition returns True, so the loop starts.
9. `else:`
   - `a > b` ($9 > 15$) returns False, so the else block executes.
10. `b = b-a`
    - `b` is now assigned to $15 - 9$, which is 6.
11. `while a != b:`
    - The while loop condition returns True, so the loop starts.
12. `if a > b:`
    - `a > b` ($9 > 6$) returns True, so the first block executes.
13. `a = a-b`
    - `a` is now assigned to $9 - 6$, which is 3.
14. `while a != b:`
    - The while loop condition returns True, so the loop starts.
15. `else:`
    - `a > b` ($3 > 6$) returns False, so the else block executes.
16. `b = b-a`
    - `b` is now assigned to $6 - 3$, which is 3.
17. `while a != b:`
    - The while loop condition returns False (`3 == 3`), so the loop ends.
18. `return a`
    - `a` is returned from the function, giving the correct greatest common divisor of **3**.

2.2 Week 2

The following are implementations of various functions in Haskell.

`select_evens`, lists the even-indexed elements of a given list:

```
-- Implementation
select_evens [] = [] -- in the case of a list with even number elements
select_evens (x:[]) = [] -- in the case of a list with odd number elements
select_evens (x:y:xs) = y : select_evens (xs)

-- Execution Sequence with example ["a","b","c","d","e"]
select_evens ["a","b","c","d","e"] =
    "b" : (select_evens["c","d","e"]) =
    "b" : ("d" : (select_evens["e"])) =
    "b" : ("d" : ([])) =
    ["b","d"]
```

`select_odds`, lists the odd-indexed elements of a given list:

```
-- Implementation
select_odds [] = [] -- in the case of a list with even number elements
select_odds (x:[]) = [x] -- in the case of a list with odd number elements
select_odds (x:y:xs) = x : select_odds (xs)

-- Execution Sequence with example ["a","b","c","d","e"]
select_odds ["a","b","c","d","e"] =
    "a" : (select_odds["c","d","e"]) =
    "a" : ("c" : (select_odds["e"])) =
    "a" : ("c" : ("e")) =
    ["a","c","e"]
```

`member`, determines whether an element is part of a given list:

```
-- Implementation
member a [] = False
member a (x:xs)
    | a==x = True
    | otherwise = member a (xs)

-- Execution Sequence with example 2 [5,2,6]
member 2 [5,2,6] =
    member 2 [2,6] =
    True
```

`append`, appends a list to another list:

```
-- Implementation
append [] ys = ys
append (x:xs) ys = x : append xs ys

-- Execution Sequence with example [1,2] [3,4,5]
append [1,2] [3,4,5] =
    1 : (append [2] [3,4,5]) =
    1 : (2 : (append [] [3,4,5])) =
    1 : (2 : ([3,4,5])) =
    [1,2,3,4,5]
```

`revert`, reverses a list:

```
-- Implementation
revert [] = []
revert (x:xs) = append (revert(xs)) [x]

-- Execution Sequence with example [1,2,3]
revert [1,2,3] =
    append (revert [2,3]) [1] =
    append (append (revert [3]) [2]) [1] =
    append (append (append (revert []) [3]) [2]) [1] =
    append (append (append [] [3]) [2]) [1] =
    append (append [3] [2]) [1] =
    append (3 : (append [] [2])) [1] =
    append (3 : [2]) [1] =
    append [3,2] [1] =
    3 : (append [2] [1]) =
    3 : (2 : (append [] [1]) =
    3 : (2 : [1]) =
    [3,2,1]
```

`less_equal`, checks if the element in a list is less than or equal to the same-indexed element in another list:

```
-- Implementation
less_equal [] [] = True
less_equal (x:xs) (y:ys)
    | x > y = False
    | otherwise = less_equal (xs) (ys)

-- Execution Sequence with example [1,2,3] [2,3,2]
less_equal [1,2,3] [2,3,2] =
    less_equal [2,3] [3,2] =
    less_equal [3] [2] =
    False
```

2.3 Week 3

The following investigates the Tower of Hanoi problem. Here is a given Haskell implementation describing moves in the game, as well as the execution sequence for the test input `hanoi 5 0 2`.

```
-- Implementation
hanoi 1 x y = move x y

hanoi (n+1) x y =
    hanoi n x (other x y)
    move x y
    hanoi n (other x y) y

-- Execution Sequence
hanoi 5 0 2
    hanoi 4 0 1
        hanoi 3 0 2
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
            move 0 2
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
        move 0 1
        hanoi 3 2 1
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
            move 2 1
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
    move 0 2
    hanoi 4 1 2
        hanoi 3 1 0
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
            move 1 0
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
        move 1 2
        hanoi 3 0 2
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
            move 0 2
```

```
hanoi 2 1 2
    hanoi 1 1 0 = move 1 0
    move 1 2
    hanoi 1 0 2 = move 0 2
```

From this execution, the moves for a 5-ring Tower of Hanoi game can be seen as follows:

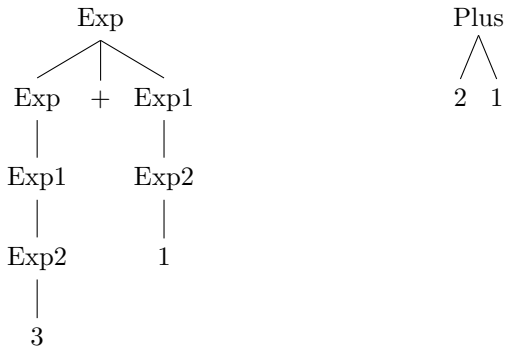
```
0->2, 0->1, 2->1, 0->2, 1->0, 1->2, 0->2, 0->1, 2->1, 2->0, 1->0, 2->1, 0->2, 0->1, 2->1, 0->2,
    1->0, 1->2, 0->2, 1->0, 2->1, 2->0, 1->0, 1->2, 0->2, 0->1, 2->1, 0->2, 1->0, 1->2, 0->2
```

Analysis: From this computation, the word `hanoi` appears exactly 31 times in the execution. Based on executions of the game with a different number of starting rings, the formula $2^n - 1$ can be derived to determine how many times `hanoi` will appear, with `n` being the number of disks in the game.

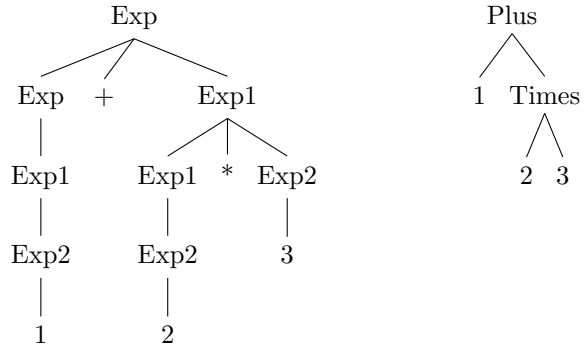
2.4 Week 4

The following compares concrete and abstract syntax trees of various mathematical expressions.

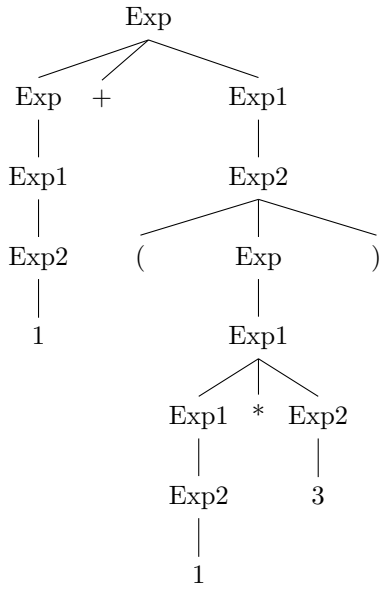
The expression $2 + 1$:

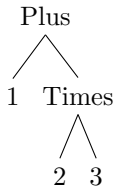


The expression $1 + 2 * 3$:

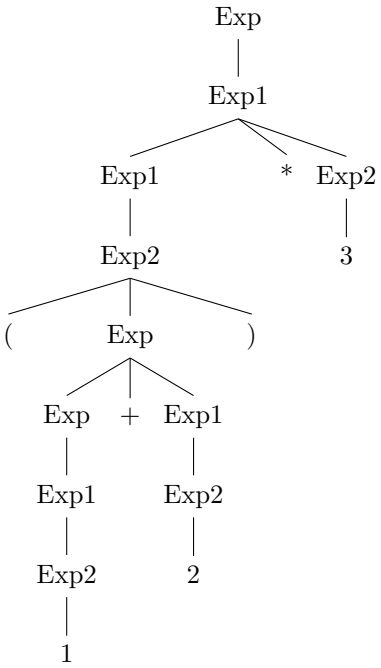


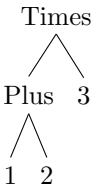
The expression $1 + (2 * 3)$:



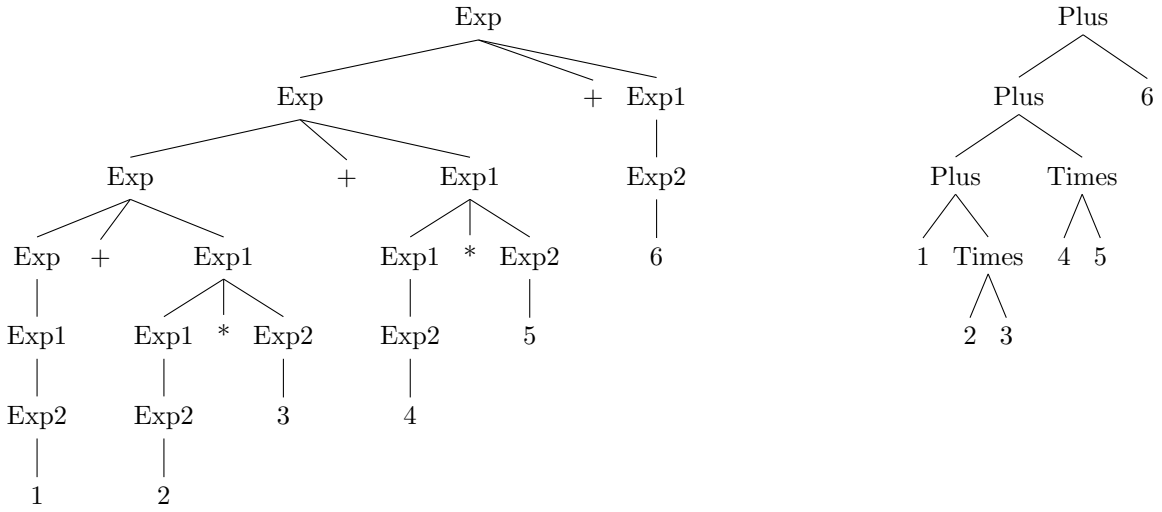


The expression $(1 + 2) * 3$:

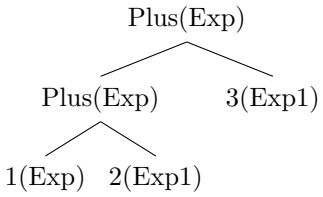




The expression $1 + 2 * 3 + 4 * 5 + 6$:



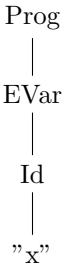
Analysis of the abstract syntax tree of $1 + 2 + 3$: The abstract syntax tree of $1 + 2 + 3$ would match the tree of $(1 + 2) + 3$. This is because the first breakdown of $+$ separates it to `Exp` and `Exp1`, and `Exp1` cannot reduce down to another sum. Therefore, the right side of the tree must become an integer, while the left side reduces down to a sum. The resulting tree would be as follows, which matches $(1 + 2) + 3$ and not $1 + (2 + 3)$.



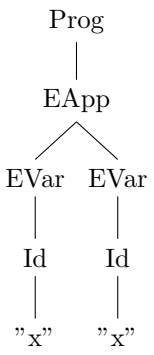
2.5 Week 5

After generating a working parser demonstrating lambda calculus, linearized abstract syntax trees and 2-dimensional notation abstract syntax trees can be generated for the below expressions.

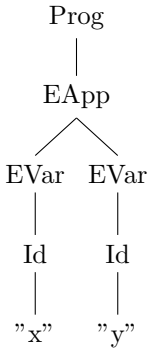
```
-- x
x
Prog (EVar (Id "x"))
```



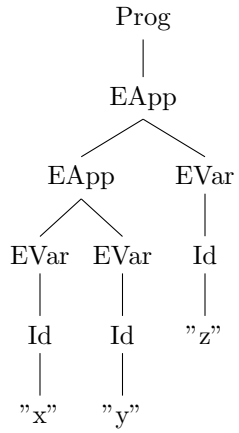
```
-- x x
x x
Prog (EApp (EVar (Id "x")) (EVar (Id "x")))
```



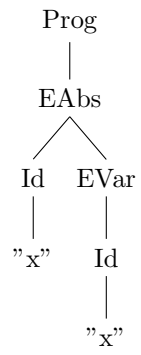

```
-- x y
x y
Prog (EApp (EVar (Id "x")) (EVar (Id "y")))
```



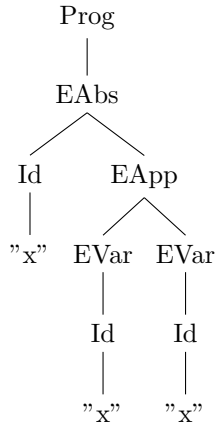

```
-- x y z
x y z
Prog (EApp (EApp (EVar (Id "x")) (EVar (Id "y"))) (EVar (Id "z")))
```



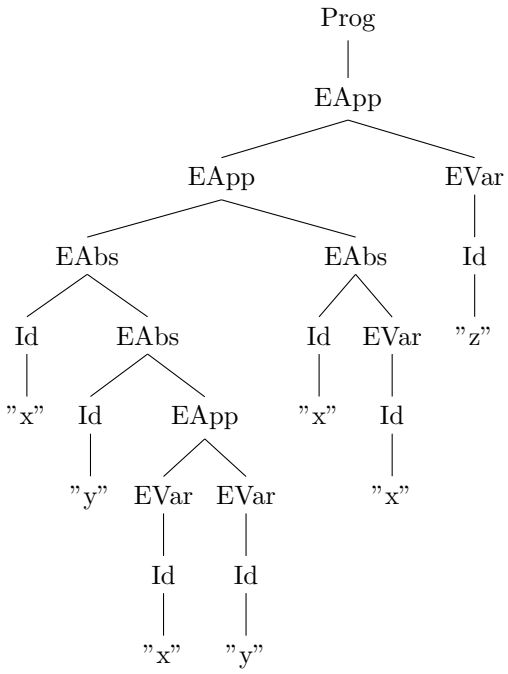
```
-- \ x.x
\ x . x
Prog (EAbs (Id "x") (EVar (Id "x")))
```



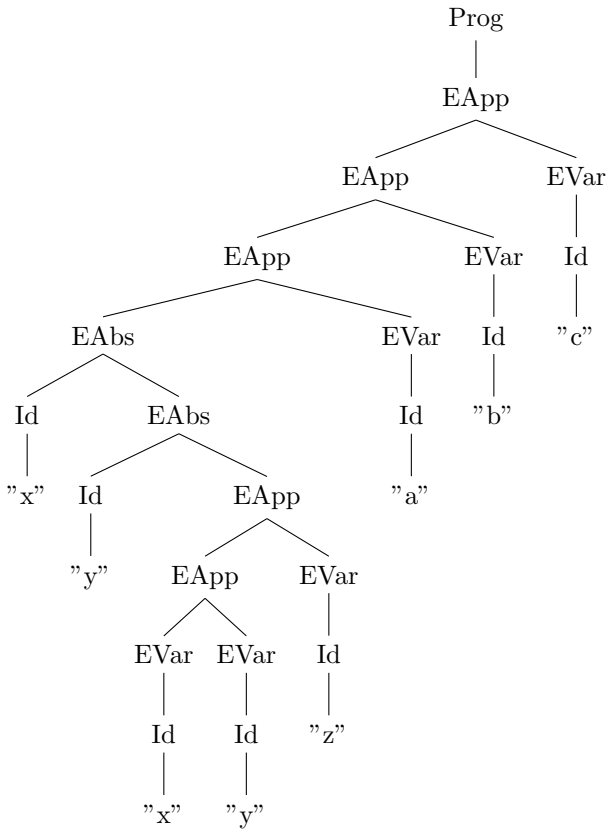

```
-- \ x.x x
\ x . x x
Prog (EAbs (Id "x") (EApp (EVar (Id "x")) (EVar (Id "x"))))
```



```
-- (\ x . (\ y . x y)) (\ x.x) z
\ x . \ y . x y (\ x . x)z
Prog (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EApp (EVar (Id "x")) (EVar (Id "y"))))) (EAbs (Id
    "x") (EVar (Id "x")))) (EVar (Id "z")))
```



```
-- (\ x . \ y . x y z) a b c
\ x . \ y . x y z a b c
Prog (EApp (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EApp (EApp (EVar (Id "x")) (EVar (Id "y")))
    (EVar (Id "z"))))) (EVar (Id "a"))) (EVar (Id "b"))) (EVar (Id "c")))
```



The following will show the reduction of several lambda calculus expressions.

```
(\x.x) a =
   a
```

```
\x.x a =
   \x.x a
```

```
(\x.\y.x) a b =
   (\y.a) b =
   a
```

```
(\x.\y.y) a b =
   (\y.y) b =
   b
```

```
(\x.\y.x) a b c =
   (\y.a) b c =
   a c
```

```
(\x.\y.y) a b c =
   (\y.a) b c =
   b c
```

```
(\x.\y.x) a (b c) =
   (\y.a) (b c) =
   a
```

```
(\x.\y.y) a (b c) =
   (\y.y) (b c) =
   b c
```

```
(\x.\y.x) (a b) c =
   (\y.(a b)) c =
   a b
```

```
(\x.\y.y) (a b) c =
   (\y.y) c =
   c
```

```
(\x.\y.x) (a b c) =
   \y.(a b c)
```

```
(\x.\y.y) (a b c) =
   \y.y
```

```
evalCBN (\x.x)((\y.y)a) =
   evalCBN (EApp (EAbs (Id "x") (EVar (Id "x"))) (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar (Id
       "a")))) = -- converted to concrete format
   evalCBN (subst (Id "x") (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar (Id "a"))) (EVar (Id
       "x"))) = -- line 27
   evalCBN (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar (Id "a"))) = -- reduction of subst in one
       step
   evalCBN (subst (Id "y") (EVar (Id "a")) (EVar (Id "y"))) = -- line 27
   EVar (Id "a") -- reduction of subst in one step
```

2.6 Week 6

The following is an evaluation of a longer lambda calculus expression.

```
-- (\exp . \two . \three . exp two three)
-- (\m.\n. m n)
-- (\f.\x. f (f x))
-- (\f.\x. f (f (f x)))

    = ((\m.\n. m n) (\f.\x. f (f x)) (\f.\x. f (f (f x))))
    = ((\m.\n. m n) (\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2))))
    = ((\n. (\f.\x. f (f x)) n) (\f2.\x2. f2 (f2 (f2 x2))))
    = ((\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2))))
    = (\x. (\f2.\x2. f2 (f2 (f2 x2))) ((\f2.\x2. f2 (f2 (f2 x2))) x))
    = (\x. (\x2. ((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2
        (f2 x2))) x) x2))))
    = (\x. (\x2. (x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
    = (\x. (\x2. (x (x (x (((\x2. x (x (x x2)))) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
    = (\x. (\x2. (x (x (x ((\x2. x (x (x x2))) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
    = (\x. (\x2. (x (x (x (x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))))
    = (\x. (\x2. (x (x (x (x (x (x ((\x2. x (x (x x2))) x2)))))))))
    = \x. \x2. x (x (x (x (x (x (x (x (x x2))))))))
```

2.7 Week 7

This section will investigate bound and free variables in a lambda-calculus interpreter.
The following is a section of the interpreter code.

```
evalCBN (EApp e1 e2) = case (evalCBN e1) of -- line 5
(EAbs i e3) -> evalCBN (subst i e2 e3)      -- line 6
e3 -> EApp e3 e2                            -- line 7
```

In this code, all variables (`e1`, `e2`, `e3`, and `i`) are **bound variables**. This is because the variable names can be changed without changing the function `evalCBN`.

`e1`:
- Binder: `EApp e1 e2`
- Scope: `case (evalCBN e1) of | (EAbs i e3) -> evalCBN (subst i e2 e3) | e3 -> EApp e3 e2`

`e2`:
- Binder: `EApp e1 e2`
- Scope: `case (evalCBN e1) of | (EAbs i e3) -> evalCBN (subst i e2 e3) | e3 -> EApp e3 e2`

`e3`:
- Binder (line 6): `EAbs i e3`
- Scope (line 6): `subst i e2 e3`
- Binder (line 7): `e3`
- Scope (line 7): `EApp e3 e2`

`i`:
- Binder: `EAbs i e3`
- Scope: `subst i e2 e3`

The following is another section of the interpreter code.

```
    subst id s (EAbs id1 e1) =        -- line 18
    let f = fresh (EAbs id1 e1)       -- line 20
        e2 = subst id1 (EVar f) e1 in -- line 21
        EAbs f (subst id s e2)        -- line 22
```

In this code, all variables (`id`, `s`, `id1`, `e1`, `f`, and `e2`) are **bound variables**. This is because the variable names can be changed without changing the function `subst`.

`id`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`s`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`id1`:
Binder: `EAbs id1 e1`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`e1`:
Binder: `EAbs id1 e1`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`f`:
Binder: `f = fresh (EAbs id1 e1)`
Scope: `| e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`
`e2`:
Binder: `e2 = subst id1 (EVar f) e1`
Scope: `EAbs f (subst id s e2)`

Another example of evalCBN is demonstrated here:

```
evalCBN (\x.\y.x) y z =
    evalCBN (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EVar (Id "x")))) (EVar (Id "y"))) (EVar (Id
        "z"))) = -- converted to concrete format
    evalCBN (EApp (EApp (EAbs (Id "x") (EAbs (Id "y0") (EVar (Id "x")))) (EVar (Id "y"))) (EVar
        (Id "z"))) = -- fresh applied in one step
    evalCBN (EApp (EApp (evalCBN (subst (Id "y0") (EVar (Id "y")) (EVar (Id "x")))) (EVar (Id
        "y"))) (EVar (Id "z"))) = -- line 6
    evalCBN (EApp (EAbs (Id "y0") (EVar (Id "y"))) (EVar (Id "z"))) = -- subst reduction
    evalCBN (EApp (evalCBN (subst (Id "y0") (EVar (Id "z")) (EVar (Id "y")))) (EVar (Id "z"))) =
        -- line 6
    evalCBN (EVar (Id "y")) = -- subst reduction
    EVar (Id "y") -- evalCBN x = x
```

The following is an analysis of several abstract reduction systems.

A={}

A picture will not be drawn for this, for it is just an empty set. This ARS is terminating because it cannot reduce any further. It is confluent because every peak converges to the same result. It has a unique normal form (which is the empty set).

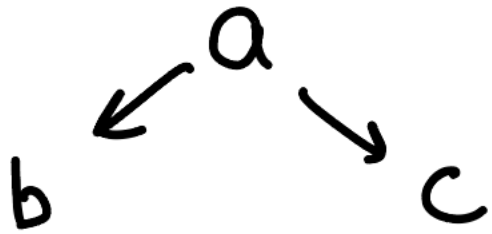
A={a} and R={}

A picture will not be drawn for this, for it is just the letter "a". This ARS is terminating because it cannot reduce any further. It is confluent because every peak converges to the same result. It has a unique normal form (which is "a").

A={a} and R={(a,a)}

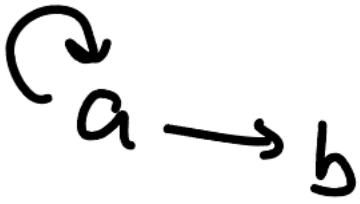
This ARS is not terminating because it can always reduce further. It is confluent because every peak converges to the same result. It has a unique normal form (which is "a").

A={a,b,c} and R={(a,b),(a,c)}



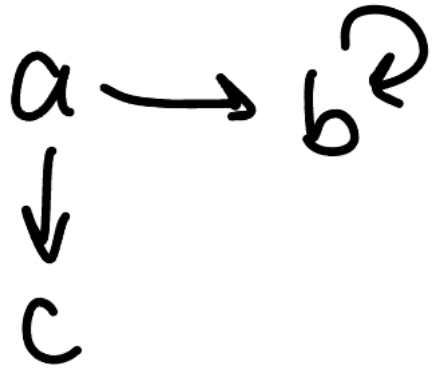
This ARS is terminating because it cannot reduce any further. It is not confluent since the computations do not converge back. It does not have a unique normal form.

A={a,b} and R={(a,a),(a,b)}



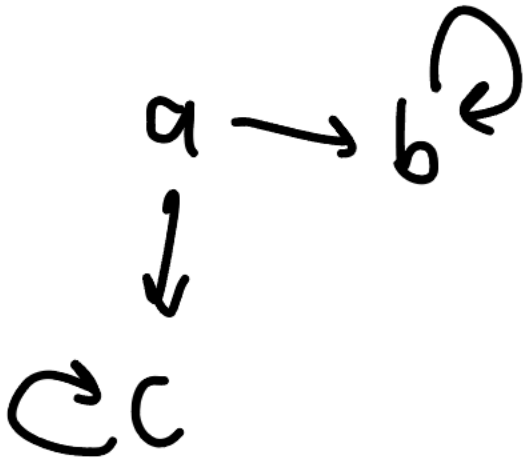
This ARS is not terminating because it can always reduce further. It is not confluent since the computations do not converge back. It does not have a unique normal form.

A={a,b,c} and R={(a,b),(b,b),(a,c)}



This ARS is not terminating because it can always reduce further. It is not confluent since the computations do not converge back. It does not have a unique normal form.

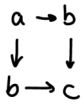
A={a,b,c} and R={(a,b),(b,b),(a,c),(c,c)}



This ARS is not terminating because it can always reduce further. It is not confluent since the computations do not converge back. It does not have a unique normal form.

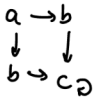
Lastly, the 8 different cases of confluence, termination, and presence of unique normal forms will be shown.

True, True, True:



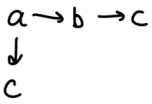
True, True, False: Congruence implies that there are unique normal forms, so there is no example of this case.

True, False, True:



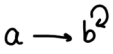
True, False, False: Congruence implies that there are unique normal forms, so there is no example of this case.

False, True, True:

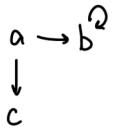


False, True, False:

False, False, True:



False, False, False:



2.8 Week 8

The following will investigate properties of the following rewrite system:

```
aa -> a
bb -> b
ba -> ab
ab -> ba
```

Termination: This ARS does not terminate because of the final two rules. There is a cycle between the strings `ab` and `ba`, meaning that there is no end to the cycle if there exists either of these strings.

Normal forms: The normal forms of this ARS are therefore an empty string, `a`, and `b`. Any string longer than one character with just `ab` and `ba` will be rewritten as another same-length string, as per the rewrite rules.

Unique normal forms: These rewrite rules can be slightly altered so that the ARS can be defined as having unique normal forms. If we treat `a` and `b` as the base-10 digits `0` and `1`, it has a slightly different meaning. Because base-10 numbers inherently have an order (0 before 1), these new rewrite rules will allow constant reduction to a normal form not by length, but by number size.

ARS function: The normal forms of this updated rewrite system would thus be the lowest number possible that can be reduced from the starting string. In terms of `a` and `b`, the resulting string would have all `a`'s to the left and all `b`'s to the right. However, this letter form does not have much meaning, as the letters do not have a predefined comparison order, and thus cannot be counted as a normal form. The normal forms in terms of `0` and `1` has more meaning.

3 Project

This section will contain all details for my final project of this course.

3.1 Specification

For my final project, I plan to learn a new programming language, give a concise but informative tutorial on it, and develop a project that ties in to the course material. For this, I plan to learn either Ruby, Elixir, or PureScript. Ruby is in use more than the other two suggestions, which is why it is listed as a potential candidate. Learning a new popular language could be beneficial for personal purposes. Elixir and PureScript are suggestions because they are similar to the language that is focused on in this course, Haskell. They use similar concepts, such as pattern matching and functional programming. My goal is to both learn a relevant language and utilize some concepts learned in the course to create a meaningful project that represents my knowledge of programming languages. The language and representative project of choice will be determined in the near future.

4 Conclusions

To be written at a later date.