CPSC-354 Report

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Abstract

Every computer needs their own set of instructions and syntax in order to communicate with users and other computers. Programming languages are these sets of instructions that instruct a computer how to operate. Programming languages operate all kinds of computers in the modern world, including those found in automobiles and microwaves. The most popular programming languages used by developers and programmers today include Python, JavaScript, and C++. Even though these languages are different in many ways, they all originated from their own basic building blocks. These building blocks make each programming language unique in their use and functionality. For example, the difference between interpreted vs. compiled languages can lead to tradeoffs between syntax simplicity and performance issues. Each programming language has their own advantages and disadvantages, and the way they are structured determines what strengths and weaknesses it will have. This report aims to highlight what these building blocks are and how they can be used to build a fully functional programming language. This will be done through explorations of various concepts such as syntax trees, abstract reduction systems, and recursion applications. A final software project will also be showcased at the end, investigating how the Ruby programming language compares to other languages.

Contents

1 Introduction

This report is written by Eleas Vrahnos, a third-year undergraduate at Chapman University at the time of this report's completion. It details all assignments and progress made in the Programming Languages course, CPSC 354. It includes weekly homework assigments and a final project that demonstrate understanding and application in various class topics covered that week. Almost all of these exercises are coded in the functional programming language Haskell, which is used to help make creating an example programming language interpreter as simple as possible.

2 Homework

This section will contain my solutions to the weekly homework assignments.

2.1 Week 1

The following is a Python implementation of the Euclidean algorithm, a method for computing the greatest common divisor between two integers. A sample input `gcd(9, 33)` is tested step by step below.

```
def gcd(a,b):
    while a != b:
        if a > b: a = a-b
        else: b = b-a
    return a
```

1. `gcd(9, 33)`
   - The function is called, assigning 9 to variable `a` and 33 to variable `b`.
2. `while a != b:`
   - The while loop condition returns True, so the loop starts.
3. `else:`
   - `a > b` ($9 > 33$) returns False, so the else block executes.
4. `b = b-a`
   - `b` is now assigned to $33 - 9$, which is 24.
5. `while a != b:`
   - The while loop condition returns True, so the loop starts.
6. `else:`
   - `a > b` ($9 > 24$) returns False, so the else block executes.
7. `b = b-a`
   - `b` is now assigned to $24 - 9$, which is 15.
8. `while a != b:`
   - The while loop condition returns True, so the loop starts.
9. `else:`
   - `a > b` ($9 > 15$) returns False, so the else block executes.
10. `b = b-a`
    - `b` is now assigned to $15 - 9$, which is 6.
11. `while a != b:`
    - The while loop condition returns True, so the loop starts.
12. `if a > b:`
    - `a > b` ($9 > 6$) returns True, so the first block executes.
13. `a = a-b`
    - `a` is now assigned to $9 - 6$, which is 3.
14. `while a != b:`
    - The while loop condition returns True, so the loop starts.
15. `else:`
    - `a > b` ($3 > 6$) returns False, so the else block executes.
16. `b = b-a`
    - `b` is now assigned to $6 - 3$, which is 3.
17. `while a != b:`
    - The while loop condition returns False (`3 == 3`), so the loop ends.
18. `return a`
    - `a` is returned from the function, giving the correct greatest common divisor of **3**.

2.2 Week 2

The following are implementations of various functions in Haskell and corresponding examples. For the execution sequences, equational reasoning is shown in the comments by line number of the implementation.

`select_evens`, lists the even-indexed elements of a given list:

```
-- Implementation
select_evens [] = [] -- in the case of a list with even number elements
select_evens (x:[]) = [] -- in the case of a list with odd number elements
select_evens (x:y:xs) = y : select_evens (xs)

-- Execution Sequence with example ["a","b","c","d","e"]
select_evens ["a","b","c","d","e"]
    = "b" : (select_evens["c","d","e"]) -- line 3
    = "b" : ("d" : (select_evens["e"])) -- line 3
    = "b" : ("d" : ([])) -- line 2
    = ["b","d"] -- line 1
```

`select_odds`, lists the odd-indexed elements of a given list:

```
-- Implementation
select_odds [] = [] -- in the case of a list with even number elements
select_odds (x:[]) = [x] -- in the case of a list with odd number elements
select_odds (x:y:xs) = x : select_odds (xs)

-- Execution Sequence with example ["a","b","c","d","e"]
select_odds ["a","b","c","d","e"]
    = "a" : (select_odds["c","d","e"]) -- line 3
    = "a" : ("c" : (select_odds["e"])) -- line 3
    = "a" : ("c" : ("e")) -- line 2
    = ["a","c","e"] -- syntax of lists
```

`member`, determines whether an element is part of a given list:

```
-- Implementation
member a [] = False
member a (x:xs)
    | a==x = True
    | otherwise = member a (xs)

-- Execution Sequence with example 2 [5,2,6]
member 2 [5,2,6]
    = member 2 [2,6] -- lines 2 -> 4
    = True -- lines 2 -> 3
```

`append`, appends a list to another list:

```
-- Implementation
append [] ys = ys
append (x:xs) ys = x : append xs ys

-- Execution Sequence with example [1,2] [3,4,5]
append [1,2] [3,4,5]
    = 1 : (append [2] [3,4,5]) -- line 2
    = 1 : (2 : (append [] [3,4,5])) -- line 2
    = 1 : (2 : ([3,4,5])) -- line 1
    = [1,2,3,4,5] -- syntax of lists
```

`revert`, reverses a list (makes use of `append`):

```
-- Implementation
revert [] = []
revert (x:xs) = append (revert(xs)) [x]

-- Execution Sequence with example [1,2,3]
revert [1,2,3]
    = append (revert [2,3]) [1] -- line 2
    = append (append (revert [3]) [2]) [1] -- line 2
    = append (append (append (revert []) [3]) [2]) [1] -- line 2
    = append (append (append [] [3]) [2]) [1] -- line 1
    = append (append [3] [2]) [1] -- append line 1
    = append (3 : (append [] [2])) [1] -- append line 2
    = append (3 : [2]) [1] -- append line 1
    = append [3,2] [1] -- syntax of lists
    = 3 : (append [2] [1]) -- append line 2
    = 3 : (2 : (append [] [1])) -- append line 2
    = 3 : (2 : [1]) -- append line 1
    = [3,2,1] -- syntax of lists
```

`less_equal`, checks if the element in a list is less than or equal to the same-indexed element in another list:

```
-- Implementation
less_equal [] [] = True
less_equal (x:xs) (y:ys)
    | x > y = False
    | otherwise = less_equal (xs) (ys)

-- Execution Sequence with example [1,2,3] [2,3,2]
less_equal [1,2,3] [2,3,2]
    = less_equal [2,3] [3,2] -- lines 2 -> 4
    = less_equal [3] [2] -- lines 2 -> 4
    = False -- lines 2 -> 3
```

2.3 Week 3

The following investigates the Tower of Hanoi problem in Haskell and the execution of a 5-ring game.

```
hanoi 1 x y = move x y
hanoi (n+1) x y =
    hanoi n x (other x y)
    move x y
    hanoi n (other x y) y
-- Execution Sequence
hanoi 5 0 2
    hanoi 4 0 1
        hanoi 3 0 2
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
            move 0 2
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
        move 0 1
        hanoi 3 2 1
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
            move 2 1
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
    move 0 2
    hanoi 4 1 2
        hanoi 3 1 0
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
            move 1 0
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
        move 1 2
        hanoi 3 0 2
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
            move 0 2
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
```

From this execution, the moves for a 5-ring Tower of Hanoi game can be seen as follows:

```
0->2, 0->1, 2->1, 0->2, 1->0, 1->2, 0->2, 0->1, 2->1, 2->0, 1->0, 2->1, 0->2, 0->1, 2->1, 0->2,
    1->0, 1->2, 0->2, 1->0, 2->1, 2->0, 1->0, 1->2, 0->2, 0->1, 2->1, 0->2, 1->0, 1->2, 0->2
```

From this computation, the word `hanoi` appears exactly 31 times in the execution. Based on executions of the game with a different number of starting rings, the formula $2^n - 1$ can be derived to determine how many times `hanoi` will appear, with `n` being the number of rings in the game. This is also seen to be the optimal amount of moves to solve the Tower of Hanoi problem with `n` starting rings.

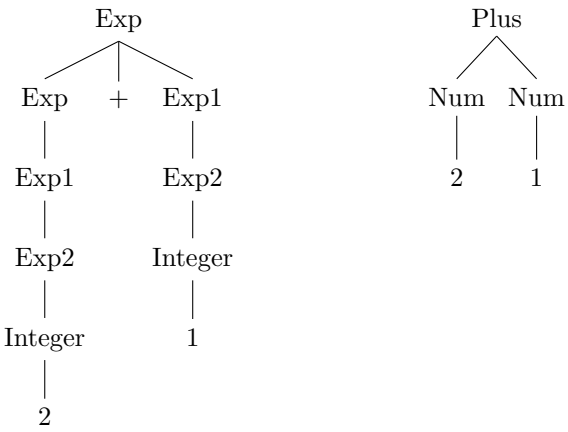
2.4 Week 4

The following compares concrete and abstract syntax trees of various mathematical expressions.

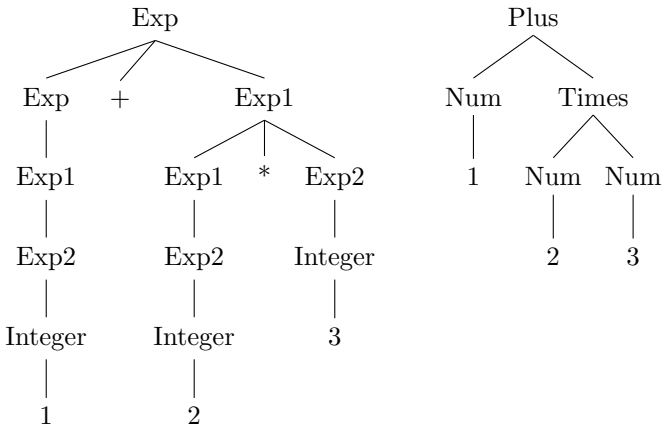
Shown below is the context-free grammar used for the concrete syntax trees.

```
Exp -> Exp '+' Exp1
Exp1 -> Exp1 '*' Exp2
Exp2 -> Integer
Exp2 -> '(' Exp ')'
Exp -> Exp1
Exp1 -> Exp2
```

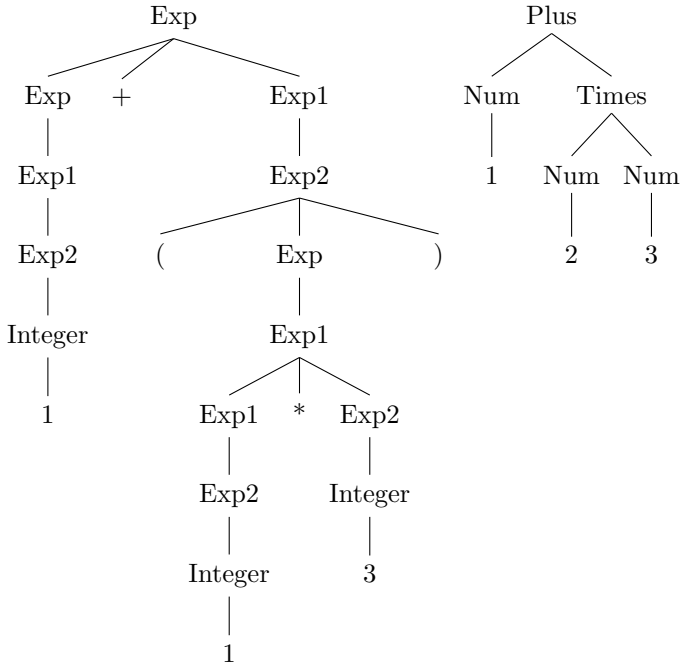
The expression $2 + 1$:



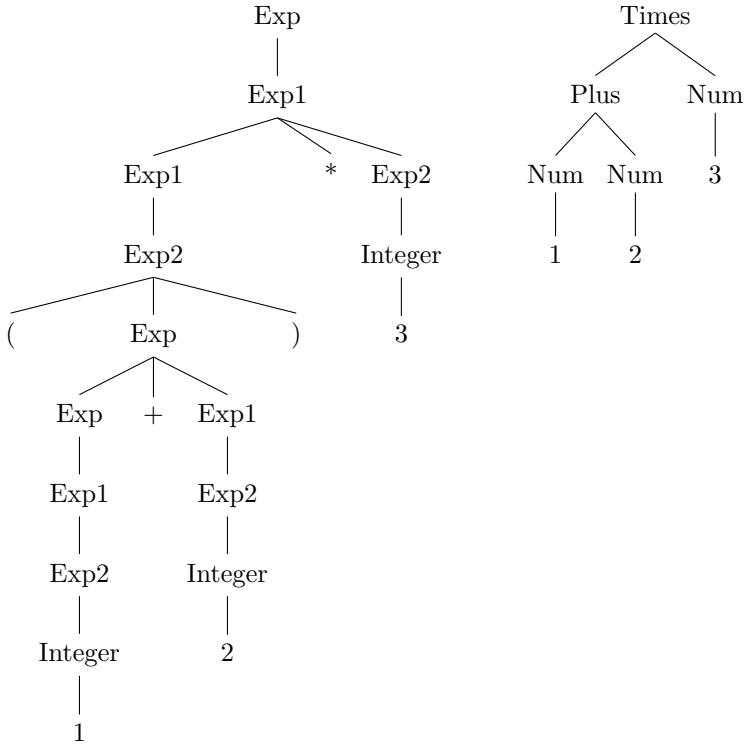
The expression $1 + 2 * 3$:



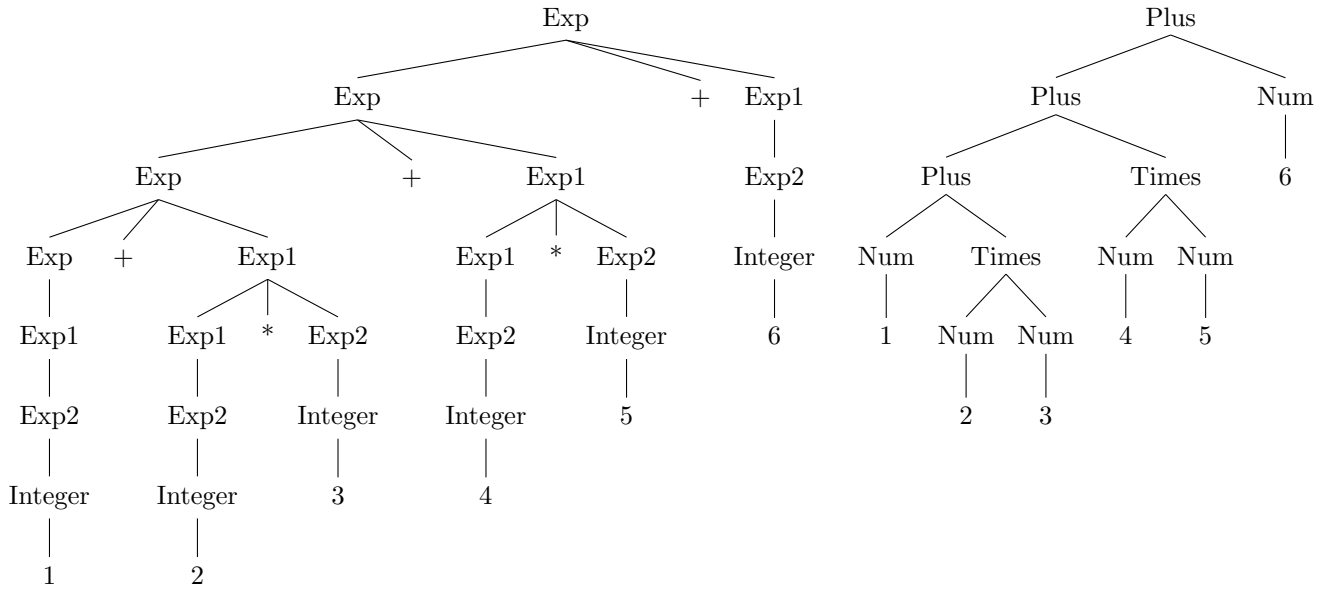
The expression $1 + (2 * 3)$:



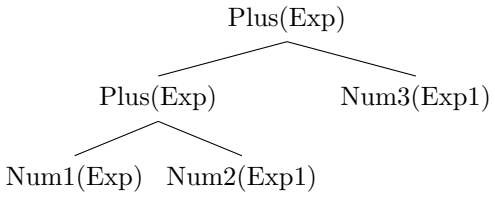
The expression $(1 + 2) * 3$:



The expression $1 + 2 * 3 + 4 * 5 + 6$:



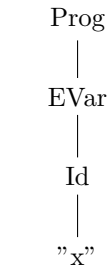
Analysis of the abstract syntax tree of $1 + 2 + 3$: The abstract syntax tree of $1 + 2 + 3$ would match the tree of $(1 + 2) + 3$. This is because the first breakdown of $+$ separates it to `Exp` and `Exp1`, and `Exp1` cannot reduce down to another sum. Therefore, the right side of the tree must become an integer, while the left side reduces down to a sum. The resulting tree would be as follows, which matches $(1 + 2) + 3$ and not $1 + (2 + 3)$.



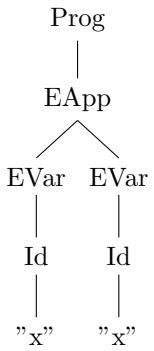
2.5 Week 5

After generating a working parser demonstrating lambda calculus, linearized abstract syntax trees and 2-dimensional notation abstract syntax trees can be generated for the below expressions.

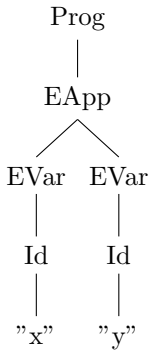
```
-- x
x
Prog (EVar (Id "x"))
```



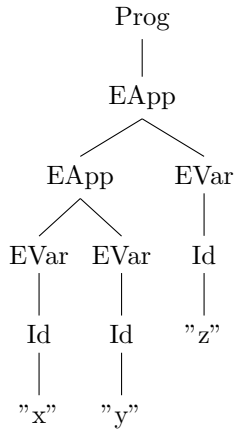

```
-- x x
x x
Prog (EApp (EVar (Id "x")) (EVar (Id "x")))
```



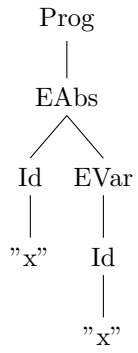

```
-- x y
x y
Prog (EApp (EVar (Id "x")) (EVar (Id "y")))
```



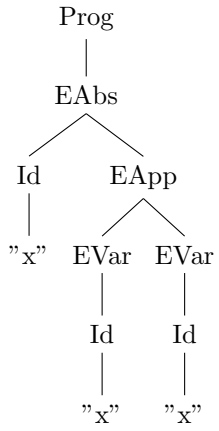
```
-- x y z
x y z
Prog (EApp (EApp (EVar (Id "x")) (EVar (Id "y"))) (EVar (Id "z")))
```



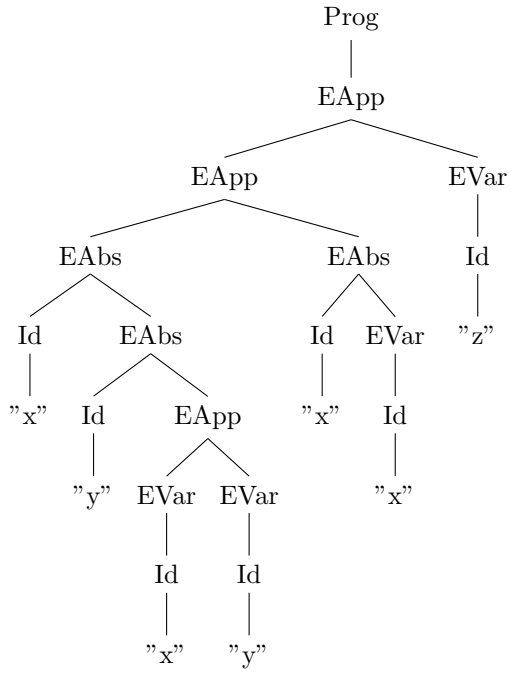

```
-- \ x.x
\ x . x
Prog (EAbs (Id "x") (EVar (Id "x")))
```



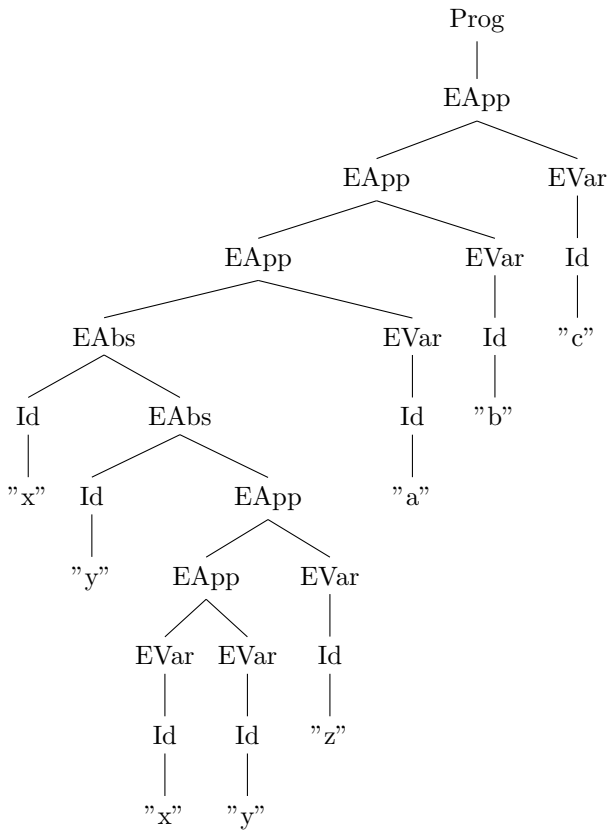
```
-- \ x.x x
\ x . x x
Prog (EAbs (Id "x") (EApp (EVar (Id "x")) (EVar (Id "x"))))
```




```
-- (\ x . (\ y . x y)) (\ x.x) z
\ x . \ y . x y (\ x . x)z
Prog (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EApp (EVar (Id "x")) (EVar (Id "y"))))) (EAbs (Id
    "x") (EVar (Id "x")))) (EVar (Id "z")))
```



```
-- (\ x . \ y . x y z) a b c
\ x . \ y . x y z a b c
Prog (EApp (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EApp (EApp (EVar (Id "x")) (EVar (Id "y")))
    (EVar (Id "z"))))) (EVar (Id "a"))) (EVar (Id "b"))) (EVar (Id "c")))
```



The following will show the reduction of several lambda calculus expressions, with the last reduction using the `evalCBN` method from provided interpreter code.

```
(\x.x) a
   = a
```

```
\x.x a
   = \x.x a
```

```
(\x.\y.x) a b
   = (\y.a) b
   = a
```

```
(\x.\y.y) a b
   = (\y.y) b
   = b
```

```
(\x.\y.x) a b c
   = (\y.a) b c
   = a c
```

```
(\x.\y.y) a b c
   = (\y.a) b c
   = b c
```

```
(\x.\y.x) a (b c)
   = (\y.a) (b c)
   = a
```

```
(\x.\y.y) a (b c)
   = (\y.y) (b c)
   = b c
```

```
(\x.\y.x) (a b) c
   = (\y.(a b)) c
   = a b
```

```
(\x.\y.y) (a b) c
   = (\y.y) c
   = c
```

```
(\x.\y.x) (a b c)
   = \y.(a b c)
```

```
(\x.\y.y) (a b c)
   = \y.y
```

```
evalCBN (\x.x)((\y.y)a)
   = evalCBN (EApp (EAbs (Id "x") (EVar (Id "x"))) (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar
       (Id "a")))) -- converted to concrete format
   = evalCBN (subst (Id "x") (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar (Id "a"))) (EVar (Id
       "x"))) -- provided interpreter line 27
   = evalCBN (EApp (EAbs (Id "y") (EVar (Id "y"))) (EVar (Id "a"))) = -- reduction of subst in
       one step
   = evalCBN (subst (Id "y") (EVar (Id "a")) (EVar (Id "y"))) -- provided interpreter line 27
   = evalCBN (EVar (Id "a")) -- reduction of subst in one step
   = a -- line 32
```

2.6 Week 6

The following is an evaluation of a longer lambda calculus expression.

```
-- (\exp . \two . \three . exp two three)
-- (\m.\n. m n)
-- (\f.\x. f (f x))
-- (\f.\x. f (f (f x)))

   = ((\m.\n. m n) (\f.\x. f (f x)) (\f.\x. f (f (f x))))
   = ((\m.\n. m n) (\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2))))
   = ((\n. (\f.\x. f (f x)) n) (\f2.\x2. f2 (f2 (f2 x2))))
   = ((\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2))))
   = (\x. (\f2.\x2. f2 (f2 (f2 x2))) ((\f2.\x2. f2 (f2 (f2 x2))) x))
   = (\x. (\x2. ((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2
       (f2 x2))) x) x2))))
   = (\x. (\x2. (x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
   = (\x. (\x2. (x (x (x (((\x2. x (x (x x2)))) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
   = (\x. (\x2. (x (x (x ((\x2. x (x (x x2))) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))
   = (\x. (\x2. (x (x (x (x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))))
   = (\x. (\x2. (x (x (x (x (x (x ((\x2. x (x (x x2))) x2)))))))))
   = \x. \x2. x (x (x (x (x (x (x (x (x x2))))))))
```

2.7 Week 7

This section will investigate bound and free variables in a lambda-calculus interpreter.

The following is a section of the interpreter code.

```
evalCBN (EApp e1 e2) = case (evalCBN e1) of -- line 5
    (EAbs i e3) -> evalCBN (subst i e2 e3)  -- line 6
    e3 -> EApp e3 e2                        -- line 7
evalCBN x = x                               -- line 8
```

In this code, all variables (`e1`, `e2`, `e3`, `i`, and `x`) are **bound variables**. This is because the variable names can be changed without changing the function `evalCBN`.

`e1`:
Binder: `evalCBN (EApp e1 e2)`
Scope: `case (evalCBN e1) of | (EAbs i e3) -> evalCBN (subst i e2 e3) | e3 -> EApp e3 e2`

`e2`:
Binder: `evalCBN (EApp e1 e2)`
Scope: `case (evalCBN e1) of | (EAbs i e3) -> evalCBN (subst i e2 e3) | e3 -> EApp e3 e2`

`e3`:
Binder (line 6): `EAbs i e3`
Scope (line 6): `evalCBN (subst i e2 e3)`
Binder (line 7): `e3`
Scope (line 7): `EApp e3 e2`

`i`:
Binder: `EAbs i e3`
Scope: `evalCBN (subst i e2 e3)`

`x`:
Binder: `evalCBN x`
Scope: `x`

The following is another section of the interpreter code.

```
subst id s (EAbs id1 e1) =        -- line 18
let f = fresh (EAbs id1 e1)       -- line 20
    e2 = subst id1 (EVar f) e1 in -- line 21
    EAbs f (subst id s e2)        -- line 22
```

In this code, all variables (`id`, `s`, `id1`, `e1`, `f`, and `e2`) are **bound variables**. This is because the variable names can be changed without changing the function `subst`.

`id`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`s`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`id1`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`e1`:
Binder: `subst id s (EAbs id1 e1)`
Scope: `let f = fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`f`:
Binder: `let f`
Scope: `fresh (EAbs id1 e1) | e2 = subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

`e2`:
Binder: `e2`
Scope: `subst id1 (EVar f) e1 in | EAbs f (subst id s e2)`

Another example of `evalCBN` from a provided interpreter snippet is demonstrated here, now including the above snippet for `fresh`:

```
evalCBN (\x.\y.x) y z =
   = evalCBN (EApp (EApp (EAbs (Id "x") (EAbs (Id "y") (EVar (Id "x")))) (EVar (Id "y"))) (EVar
       (Id "z"))) -- converted to concrete format
   = evalCBN (EApp (evalCBN (EApp (EAbs (Id "x") (EAbs (Id "y") (EVar (Id "x")))) (EVar (Id
       "y")))) (EVar (Id "z"))) -- line 5
   = evalCBN (EApp (evalCBN (subst (Id "x") (EVar (Id "y")) (EAbs (Id "y") (EVar (Id "x")))))
       (EVar (Id "z"))) -- line 6
   = evalCBN (EApp (evalCBN (EAbs (Id "y0") (subst (Id "x") (EVar (Id "y")) (subst (Id "y") (EVar
       (Id "y0")) (EVar (Id "x")))))) (EVar (Id "z"))) -- line 22
   = evalCBN (EApp (evalCBN (EAbs (Id "y0") (subst (Id "x") (EVar (Id "y")) (EVar (Id "x")))))
       (EVar (Id "z"))) -- line 16
   = evalCBN (EApp (evalCBN (EAbs (Id "y0") (EVar (Id "y")))) (EVar (Id "z"))) -- line 15
   = evalCBN (EApp (EAbs (Id "y0") (EVar (Id "y"))) (EVar (Id "z"))) -- line 8
   = evalCBN (subst (Id "y0") (EVar (Id "z")) (EVar (Id "y"))) -- line 6
   = evalCBN (EVar (Id "y")) -- line 16
   = y -- line 8
```

The following is an analysis of several abstract reduction systems.

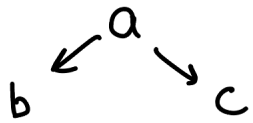
`A={}`
A picture will not be drawn for this, for it is just an empty set. This ARS is terminating because an empty set cannot reduce any further. It is confluent because there is no divergence of elements. It has unique normal forms (which is the empty set).

`A={a} and R={}`
A picture will not be drawn for this, for it is just `a`. This ARS is terminating because `a` cannot reduce any further. It is confluent because there is no divergence of elements. It has unique normal forms (which is `a`).

`A={a} and R={(a,a)}`

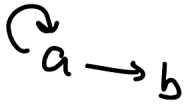
This ARS is not terminating because `a` can always reduce to itself. It is confluent because the divergence of `a` can converge back to itself. It does not have unique normal forms, as `a` can always reduce to itself.

`A={a,b,c} and R={(a,b),(a,c)}`



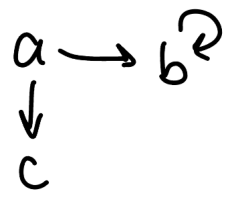
This ARS is terminating because it cannot reduce any further than `b` or `c`. It is not confluent since the divergence to `b` and `c` cannot converge back. It does not have unique normal forms, as there are two normal forms of `a` (`b` and `c`).

`A={a,b} and R={(a,a),(a,b)}`



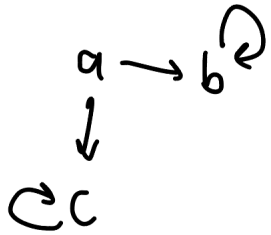
This ARS is not terminating because `a` can always reduce further. It is confluent because the divergence to `a` and `b` can converge back, with the recursive `a` converging back to the original `b`. It has unique normal forms, as `a` has the unique normal form of `b`.

A={a,b,c} and R={(a,b),(b,b),(a,c)}



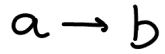
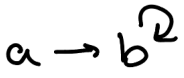
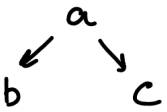
This ARS is not terminating because b can always reduce further. It is not confluent since the divergence to b and c cannot converge back. It does not have unique normal forms, as there are two normal forms of a (b and c).

A={a,b,c} and R={(a,b),(b,b),(a,c),(c,c)}



This ARS is not terminating because b and c can always reduce further. It is not confluent since the divergence to b and c cannot converge back. It does not have unique normal forms, as there are two normal forms of a (b and c).

Lastly, the 8 different cases of confluence, termination, and presence of unique normal forms will be shown.

Confluent	Terminating	Has unique normal forms	Example
True	True	True	
True	True	False	If an ARS is confluent and terminating then all elements reduce to a unique normal form.
True	False	True	
True	False	False	
False	True	True	An ARS has unique normal forms if (and only if) it is confluent and normalising.
False	True	False	
False	False	True	An ARS has unique normal forms if (and only if) it is confluent and normalising.
False	False	False	

2.8 Week 8

The following will investigate properties of the following rewrite system:

```
aa -> a
bb -> b
ba -> ab
ab -> ba
```

Why does the ARS not terminate?: This ARS does not terminate because of the final two rules. There is a cycle between the strings `ab` and `ba`, meaning that there is no end to the cycle if there exists either of these strings.

What are the normal forms?: The normal forms of this ARS are therefore an empty string, `a`, and `b`. Any string longer than one character with just `ab` and `ba` will be rewritten as another same-length string, as per the rewrite rules.

Modify the ARS so that it is terminating, has unique normal forms: These rewrite rules can be slightly altered so that the ARS is terminating and has unique normal forms. By removing the fourth rule, it eliminates the possibility of the original termination problem. What this does is allow a combination of `a` and `b` through only one permutation. This would end up adding another normal form to the previously mentioned three, which is `ab`.

This works because the functionality of the ARS is preserved. Going through the 4 possible combinations of two-letter strings, `aa` and `bb` reduce in the same fashion. In order to prevent termination, the third or fourth rule should be eliminated. The fourth rule was chosen to be eliminated here to keep the alphabetical orderliness of `ab`. Therefore, the third rule reduces the same and the elimination of the fourth rule causes `ab` to be a normal form.

Describe the specification implemented by the ARS: The specification can be found by seeing the patterns that emerge from reduction using this ARS. This can be done by coming up with possible invariants. Looking at the rules, it can be guessed that any string with the normal form `a` or `b` only contained the letter `a`/`b` in the beginning string, respectively. If a string contained both `a` and `b`, the third rewrite rule would be used eventually, leading to the normal form of `ab`.

Therefore, an invariant that can be concluded is that **the original string only contained `a`'s if the normal form of the string is `a`**. The same invariant applies to `b` as well. Likewise, **the original string contained `a`'s and `b`'s if the normal form of the string is `ab`**. Finally, **the original string contained neither `a`'s nor `b`'s if the normal form of the string is an empty string**. All of these invariants assume that the only possible letters that can appear in the string are `a` and `b`.

The specification can then be constructed to describe the ARS as a function that determines the presence of `a`'s and the presence of `b`'s at the same time. The 4 possible combinations of `a`'s and `b`'s presences can be matched up to the 4 possible normal forms of the ARS.

2.9 Week 9

Regarding the final project, the following deadlines will be made to keep track of progress of my programming language exploration:

November 13: Create a tutorial that covers the evolution of Ruby and the fundamentals of the programming language. The tutorial part will cover these categories: basics, control structures, and collections. Exercises and their answers will be provided.

November 20: Have an idea for the project portion of the report laid out in detail.

November 27: Have said project complete and full within the report.

December 4: Have the critical appraisal completed, and therefore the entirety of the Project section of the report complete.

The next section will involve an analysis of the following ARS:

```
ba -> ab
ab -> ba
ac -> ca
ca -> ac
bc -> cb
cb -> bc

aa -> b
ab -> c
ac ->
bb ->
cb -> a
cc -> b
```

The first thing to note is that with this current version of the ARS, it is not terminating, nor does order of the letters matter. This is because the top half of the ARS does not show the entire ARS to be a complete measure function, and that any combination of letters can be reoriented within the entire string.

Also to note are some normal forms, which include the empty string (from Rules 9 and 10) and the one-character strings (a, b, c). The first thing of notice is that any string larger than 1 character can be reduced to a form without any as. This is because of the reordering rules. Say we order our string so that all as appear first, followed by bs and cs. Generally, This would turn every pair of as into bs, with any lone as therefore joining with newly created bs to form cs. This observation holds for smaller strings as well, of course including those strings that don't contain a to begin with.

After noticing all strings (except for a) can be reduced to a string without as, I noticed that if the resulting string has an even number of bs and $Cmod4 == 0$ (where C is the number of cs in the string), then the string will reduce to an empty string. This is dependent on Rule 10 and Rule 12, saying that if all cs eventually reduce to an even number of bs, and there were originally an even number of bs already present in the string, then everything will reduce to an empty string.

2.10 Week 10

The following is a calculation of $fix_F 2$ using equational reasoning.

```
fix_F 2 = (\n. if n == 0 then 1 else n * fix_F (n-1)) 2
        = if 2 == 0 then 1 else 2 * fix_F (2-1)
        = 2 * (fix_F 1)
        = 2 * (\n. if n == 0 then 1 else n * fix_F (n-1)) 1
        = 2 * (if 1 == 0 then 1 else 1 * fix_F (1-1))
        = 2 * (1 * fix_F 0)
        = 2 * (1 * (\n. if n == 0 then 1 else n * fix_F (n-1)) 0)
        = 2 * (1 * (if 0 == 0 then 1 else 0 * fix_F (0-1)))
        = 2 * (1 * (1))
        = 2
```

2.11 Week 11

Some excerpts and my thoughts regarding contracts in the field of financial engineering.

A question I posed:

Q: Reading into "Composing Contracts: An Adventure in Financial Engineering", I was wondering about the global-scale technological opportunities of this new way of creating financial contracts. If this method of composing contracts was more widely known today, how would that affect the process of how contracts are done and agreed on? Could it affect the values of the contract themselves?

Excerpts of conversations I replied to:

Q: For financial traders making trades with this system, would this system be dangerous for those not capable of understanding the grammar thoroughly? What is to stop someone from making a predatory contract, and obfuscating the malicious details with syntax?

A: I was also thinking about the dangerous side of the potential capabilities of creating the entire financial engineering side of contracts. Neutrality in many aspects should be ensured to keep fairness and visibility in this kind of program. For example, a more experienced programmer should not have some kind of advantage over a lesser experienced programmer.

Q: If financial experts took the time to learn the language, how beneficial would it really be? In the article it states "identifying the 'right' primitive combinators is quite a challenge." So would it be worth/necessary for financial experts to try?

A: I was thinking about the non-contract-related opportunities of this financial language. Even if learning the language perfectly had no beneficial effect to using these contracts, it can be beneficial outside of creating contracts. Programming is widely known as an essential skill in many fields, and it can still prove to be useful at least somewhere else, since financial engineering combines two big seemingly-unrelated fields.

2.12 Week 12

The following is an analysis of the following code through the lens of Hoare logic:

```
while (x!=0) do z:=z*y; x:= x-1 done
```

Looking at the short program, the program will terminate only if $x > 0$ before the while loop begins. The program will also calculate $z = y^x$ if $z = 1$ before the while loop.

To find the invariant for this while loop, a table of execution can be made. The example initialization will be $x = 10$, $y = 2$, and $z = 1$. `t` will be used to count the number of iterations so far in the while loop.

t	x	y	z
0	10	2	1
1	9	2	2
2	8	2	4
...	...	...	...
9	1	2	512
10	0	2	1024

These numbers satisfy the following invariants:

- $t + x = 10$
- $z = y^t$

Using algebra, these two equations can be modified to get an invariant consisting of only `x`, `y`, and `z`: $z = y^{(10-x)}$.

This can be formalized through a Hoare triple:

$\{z = y^{(n-x)}\}$ `while (x != 0) do z:=z*y; x:= x-1 done` $\{z = y^{(n-x)}\}$

This can be proved from the intended purpose of this for loop with the following starting Hoare triple:

$\{z = 1 \wedge x = n\}$ `while (x != 0) do z:=z*y; x:= x-1 done` $\{z = y^n\}$

To prove this, it must be shown that the precondition implies the invariant and that the invariant implies the postcondition.

To show that the precondition implies the invariant, it must be shown that $z = 1 \wedge x = n$ implies $z = y^{(n-x)}$. If $z = 1$ and $x = n$, the second equation reduces to $1 = y^0$, or $1 = 1$. This is always true.

To show that the invariant implies the postcondition, it must be shown that $z = y^{(n-x)}$ implies $z = y^n$. This is true because of the end assumption that $x = 0$ when the while loop terminates. Therefore, the first equation reduces to $z = y^n$, making both equations the same and thus the implication true.

3 Project

This section will contain all details for my final project of this course.

3.1 Specification

For my final project, I plan to learn a new programming language, give a concise but informative tutorial on it, and develop a project that ties in to the course material. For this, I plan to learn Ruby. Learning a new popular language could be beneficial for personal purposes. My goal is to both learn a relevant language and utilize some concepts learned in the course to create a meaningful project that represents my knowledge of programming languages. The representative project of choice will be determined in the near future.

3.2 Prototype

Regarding the final project, the following deadlines will be made to keep track of progress of my programming language exploration:

November 13: Create a tutorial that covers the evolution of Ruby and the fundamentals of the programming language. The tutorial part will cover these categories: basics, control structures, and collections. Exercises and their answers will be provided.

November 20: Have an idea for the project portion of the report laid out in detail.

November 27: Have said project complete and full within the report.

December 4: Have the critical appraisal completed, and therefore the entirety of the Project section of the report complete.

The protoype tutorial can be found at the following link: https://hackmd.io/@evrahnos/rJyCCKdri

4 Conclusions

To be written at a later date.