

Large-scale evidence for logarithmic effects of word predictability on reading time

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Abstract

During real-time language comprehension, our minds rapidly decode complex meanings from sequences of words. The difficulty of doing so is known to be related to words' contextual predictability, but what cognitive processes do these predictability effects reflect? In one view, predictability effects reflect facilitation due to anticipatory processing of words that are predictable from context. This view predicts a linear effect of predictability on processing demand. In another view, predictability effects reflect the costs of probabilistic inference over sentence interpretations. This view predicts either a logarithmic or a superlogarithmic effect of predictability on processing demand, depending on whether it assumes pressures toward a uniform distribution of information over time. The empirical record is currently mixed. Here we revisit this question at scale: we analyze six reading datasets, estimate next-word probabilities with diverse statistical language models, and model reading times using recent advances in nonlinear regression. Results support a logarithmic effect of word predictability on processing difficulty, which favors probabilistic inference as a key component of human language processing.

Comprehending language involves continuously integrating new input with context in order to rapidly form an interpretation of the meanings of the utterances we hear and read. Precisely how the mind achieves this goal is unknown, but a wealth of prior studies offer an important clue: the difficulty of processing a word is related to its predictability in context. This claim is supported by diverse evidence, including self-paced reading (1–3), eye-tracking during reading (4–6), electrophysiology (7–9), and neuroimaging (10–12), using both naturalistic stimuli (4) and stimuli specifically designed to manipulate predictability (3). But what cognitive processes do predictability effects reflect? The answer to this question is tied to a major open debate about the cognitive architecture of human language comprehension (1, 3, 13–15).

Some contend that predictability effects reflect *facilitation* due to anticipatory processing (e.g., lexical retrieval and structural integration) of future words (e.g., 3, 16). In this FACILITATION view, the primary work of sentence processing is to build a mental representation of language structure and meaning, with processing demand proportional to the difficulty of the cognitive operations required to build this representation (e.g., recognizing words, retrieving their representations from the mental lexicon, and integrating those representations into existing syntactic and semantic structures). Prediction facilitates this process by allowing the processor to deal with some of this burden in advance when words are highly predictable from context, thus making more efficient use of processing resources. This view thus predicts a linear effect of contextual probability: a word can be partially processed in advance in proportion to the probability with which it can be correctly guessed (in a serial processor, see e.g., 1, 6, for discussion) or in proportion to the processor resources probabilistically allocated to it (in a parallel processor, 3). A consequence of the FACILITATION view is that predictability effects should be driven primarily by highly predictable words, since these are the words for which predictions are likely to be correct and can therefore confer a substantive benefit. Small absolute differences in low probability should have little practical impact on processing demand, since little advance processing is possible. In the limit of total prediction failure (i.e. encountering a word with contextual probability 0), processing simply proceeds without any anticipatory benefit, resulting in no facilitation.

Others contend that predictability effects primarily reflect a processing *cost*, namely, the cost of probabilistic inference. This COST view draws from information theory in framing prediction as an intrinsic

feature of a generative, probabilistic mental processor whose primary work is incremental *probabilistic inference* over a vast (even infinite) space of possible analyses of the unfolding sentence (17, 18). In this view, an interpretation is a probability distribution, and processing demand is determined by the size of the change in the interpretation: in particular, the Kullback-Leibler (KL) divergence between the interpreter states before and after observing a word. This divergence can be shown to be equivalent to the *surprisal* (negative log probability, also known as *Shannon information*) of a word in context (18). Thus, this position predicts a *logarithmic* effect of contextual predictability (or, equivalently, a linear effect of surprisal) on processing difficulty (for discussion of possible mechanisms underlying this predicted logarithmic relationship, see e.g., 1, 19, 20). A consequence of the COST view is that predictability effects should be driven primarily by small absolute differences in low probability, since these differences are large on a logarithmic (surprisal) scale. In the limit of total prediction failure, catastrophic processing failure (infinite processing cost) ensues—by consequence, under this view, next-word probability is assumed to never be truly zero.

A variant of the COST view is the uniform information density (UID) hypothesis (21, 22), in which probabilistic inference trades off with a bias against word-by-word variation in surprisal (thus smoothing processing load over time). While some versions of the COST view, like *surprisal theory* (e.g., 18), are indifferent to the temporal arrangement of information in the linguistic message, the UID view posits additional pressures toward a more even distribution of information over time, in service of communicative efficiency (23). To the extent that these hypothesized pressures derive from constraints on comprehenders' information processing, one natural basis for UID pressures would be a superlogarithmic relationship between contextual probability and processing cost: if highly surprising words (i.e., spikes in information content) are disproportionately difficult to process, uniform information density is favored (13). Although early UID proposals did not specify a processing mechanism, recent work has shown that some inferential processing algorithms have superlogarithmic time complexity in predictability, thus potentially grounding UID pressures in comprehension processes (14).

The hypothesized relationships between predictability and processing demand under each of these three views are schematized in **Figure 1**, which shows all three sets of predictions both on a probability scale (left) and a surprisal scale (right). As shown, the FACILITATION view (blue) predicts a linear fall-off in processing demand as predictability drops to zero. On a surprisal scale, this prediction appears as a plateau in which the slope of the change in processing demand decreases rapidly on surprisal. By contrast, the COST view (green) predicts a skyrocketing increase in processing demand as predictability drops to zero, since surprisal is climbing to infinity. On a surprisal scale, this prediction appears as a straight line. The UID view predicts an even steeper increase in processing difficulty (red). The UID view is most easily differentiated from the COST view on a surprisal scale, where, as shown, the slope of the change in processing demand also increases on surprisal.

The FACILITATION, COST, and UID views thus make testably different predictions about the relationship between word predictability and processing demand. However, the empirical record on this question is currently mixed, with some studies reporting a linear predictability effect (3), others reporting a logarithmic predictability effect (1, 15, 24, 25), and still others reporting a superlogarithmic predictability effect (13, 14). These differences in results plausibly derive from methodological differences, some of which concern experimental design. For example, a key challenge in studying the construct of human subjective predictability is that it is not observable and must be approximated using a model of contextual probability, and studies differ in how they implement this approximation. For example, Smith and Levy (1) quantified contextual word probabilities using statistical language models, whereas Brothers and Kuperberg (3) used probabilities derived from a *cloze task* (26) in which humans predicted the next word based on preceding context. The advantages of cloze estimates are that (i) they directly reflect human subjective probabilities and (ii) they have been shown to be superior to corpus-based estimates in predicting human reading patterns (27); although both of these purported advantages are under debate (see **Discussion** and **SI 1**). The disadvantage of cloze estimates is the inherent practical difficulty in accurately estimating degrees of low contextual probability—millions of samples per context would be needed to reach the precision of statistical language models. Unfortunately, these are precisely the probabilities that most strongly differentiate the empirical predictions of the hypotheses reviewed above.

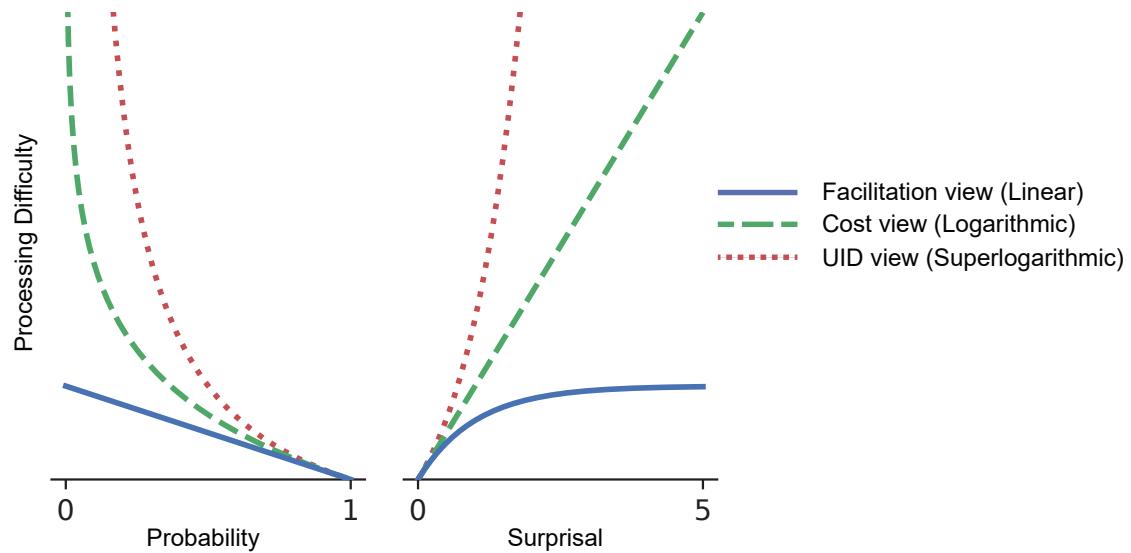


Figure 1: Expected relationships between word predictability (x -axis) and processing demand (y -axis) according to the FACILITATION, COST, and UID views of predictability effects in language comprehension. Hypothesized effects are represented on both a probability scale (left) and a surprisal scale (right). If, per the FACILITATION view, prediction serves to facilitate advance processing of highly predictable words, then processing gains will be proportional to probability. By contrast, the COST and UID views derive predictability effects from a process that updates a probability distribution over sentence interpretations, where the cost of this update is logarithmic or superlogarithmic on word predictability. Thus, as shown in the left plot, both the COST and UID views predict rapidly increasing (asymptotically infinite) processing demand as probability goes to 0, and differ in the rate of this predicted increase. Equivalently, as shown in the right plot, the FACILITATION view predicts a plateau in processing cost as surprisal increases, where as the COST and UID views respectively predict a linear or superlinear increase in processing cost as a function of surprisal.

Studies also show design differences in their use of constructed vs. naturalistic language materials. Brothers and Kuperberg used constructed materials, which they justify in light of the problems for causal inference presented by observational (naturalistic) data. However, these inferential gains come at the cost of (i) limited coverage of the critical low-probability interval of the contextual probability spectrum, (ii) data loss due to restricted focus on a critical region, rather than word-by-word modeling, and (iii) ecological validity (see also e.g., 28–31, SI 1). In addition, the theoretically predicted patterns should at minimum hold in observational data, even if the existence of such patterns is insufficient to establish causal effects. Perhaps in light of these considerations, most other studies of the functional form of predictability effects use naturalistic data (e.g., 1, 13–15, 25).

Design differences aside, all previous studies share a reliance on standard analysis methods that enforce implausible simplifying assumptions when applied to complex continuous-time processes like language comprehension. These assumptions include: linearity and/or additivity of effects, discrete-time dynamics (i.e., *spillover* effects at the word level), time-invariance, and constant error. All of these assumptions are likely unwarranted for human language comprehension, and a failure to account for their violations can substantially influence effect estimates and hypothesis tests, especially in naturalistic data (32, 33). Although some studies (e.g., 1, 25) relax the linearity assumption through generalized additive models (GAMs, 34), which can flexibly infer nonlinear effects, they still rely on implausible dynamical and distributional assumptions (i.e., a homoscedastic, additive, discrete-time stationary model).

In light of these concerns, we revisit the functional form of word predictability effects by analyzing the largest collection of naturalistic reading data to-date (six large-scale public English-language datasets with a combined total of over 2.2 million data points across three different reading modalities), combining recent advances in statistical language modeling with statistical analyses based on the recently-introduced continuous-time deconvolutional regressive neural network (CDRNN, 32, 33). In brief, CDRNNs leverage the power of deep learning to infer a highly expressive *impulse response function* (IRF) that relates features of fixated words to measured reading times as a function of their distance in continuous time. For example, the fitted model will contain an estimate of how a given surprisal value at a given fixated word will affect reading behavior 500ms in the future, thus directly taking into account the possibility of nonlinear and continuously delayed effects. The architecture of CDRNNs allows them to relax all of the aforementioned simplifying assumptions: predictors can exert arbitrary nonlinear and interactive influences on the response, the response function can change over the course of the experiment (non-stationarity), and the predictors can influence all parameters of the predictive distribution, not just the mean (heteroscedasticity). CDRNNs thus provide a more flexible analysis approach that substantially improves fit to reading behavior (32, 33).

To anticipate our results: even though CDRNNs are expressive enough to learn any of the functional forms discussed above, they emergently discover a logarithmic effect of word predictability, as predicted by the COST view (17, 18). Detailed model comparisons show that this logarithmic effect is better supported by our results than either the linear effect predicted by the FACILITATION view or the superlogarithmic effect predicted by the UID view.

Results

We evaluate predictability effects in six publicly available naturalistic reading datasets: the Brown self-paced reading (SPR) dataset (1), the Dundee eye-tracking (ET) dataset (35), the monolingual English version of the GECO eye-tracking dataset (36), the Natural Stories self-paced reading dataset (37), the Natural Stories Maze dataset (38), and the Provo eye-tracking dataset (39). In each case, the critical response variable is how long participants spent reading each word in a running text (for supplemental analyses of predictability effects on word skipping in the three eye-tracking datasets, see SI 2).

We consider word predictability estimates derived from diverse statistical *language models*, computational models that define a probability distribution over the next word given its linguistic context. Specifically, we consider an n -gram model that predicts the next word from a table of counts of word sequences in a text corpus (40), a probabilistic context-free grammar (PCFG) model that predicts the next word given a set of hypotheses about the sentence's structure (syntactic tree, 41), and three pre-trained deep neural network

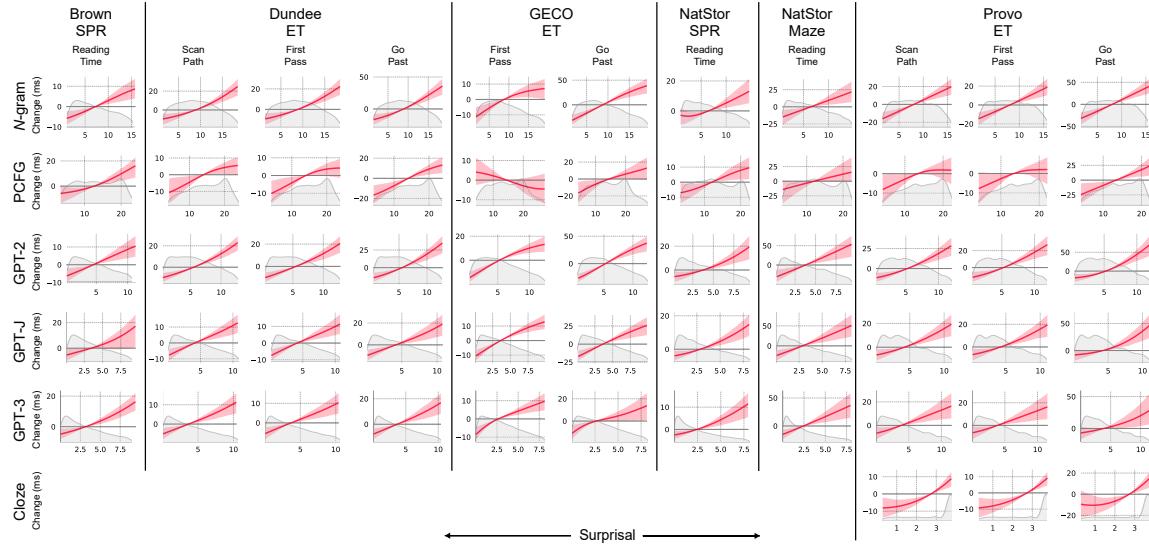


Figure 2: CDRNN-estimated functional form of surprisal (x -axis) effects on reading times (y -axis) across language model types (n-gram, PCFG, GPT-2, GPT-J, GPT-3, and human cloze) with no delay (i.e. at the surprising word). Plots cover the interdecile range of values in each training dataset (for plots covering the full empirical range, see **Figure S13**). Kernel density plots show the distribution of surprisal values in the training data over the plotted range.

language models based on the transformer architecture (42): GPT-2(-small) (43), GPT-J (44), and GPT-3 (45).

We analyze these data using continuous-time deconvolutional regressive neural networks (32, 33), controlling for numerous perceptual, motor, and linguistic variables as well as participant and item effects in a mixed model design (CDRNNs recover expected effects of our word length and frequency controls; see **SI 3**). To shed light on the functional form of word predictability effects, we consider not only models that can find an unconstrained function of word surprisal ($f(\text{SURP})$), but also models that are constrained to be linear in either probability (PROB) or some fixed power of surprisal ($\text{SURP}^{1/2}$, $\text{SURP}^{3/4}$, SURP^1 , $\text{SURP}^{4/3}$, or SURP^2).

As in prior work (e.g., 1, 14, 25), part of our analysis rests on visualization of the model-estimated relationship between predictability and processing cost. However, we go beyond these visual impressions and compare model performance on a held-out portion of each dataset under different assumptions about the nature of predictability effects. All statistical comparisons are based on pre-trained CDRNNs' performance on data not seen in training, directly grounding hypothesis tests in how well models generalize.

For further details about the experimental tasks and materials, datasets, language models, regression analyses, and statistical testing protocols, see **Methods**. For simplicity, unless otherwise specified, we report comparisons that aggregate across all datasets considered in this study. Complete results tables for all statistical tests conducted in this study (including results on individual datasets) are given in **SI 4**.

What Is the Estimated Shape of Predictability Effects? We first establish qualitative impressions about the functional form of predictability effects by visualizing the estimates from the unconstrained $f(\text{SURP})$ CDRNN models. Estimates for the effect of word surprisal on fixations to that word (i.e., at no delay) are plotted across language models and datasets in **Figure 2** (for visualization of these effects over time following stimulus onset, see **SI 5**). With one exception (PCFG surprisal effects on GECO first pass reading times), all estimates show the expected positive relationship between surprisal and reading time (in fact, PCFG surprisal in GECO first pass reading times also shows a positive surprisal effect, albeit at longer latencies; see **SI 5** for visualizations and **SI 6** for additional discussion). Furthermore, estimates are primarily consistent with a logarithmic predictability (linear surprisal) effect. They are inconsistent with a linear predictability effect, according to which processing cost should essentially not vary beyond about 4 nats of surprisal (around 2%

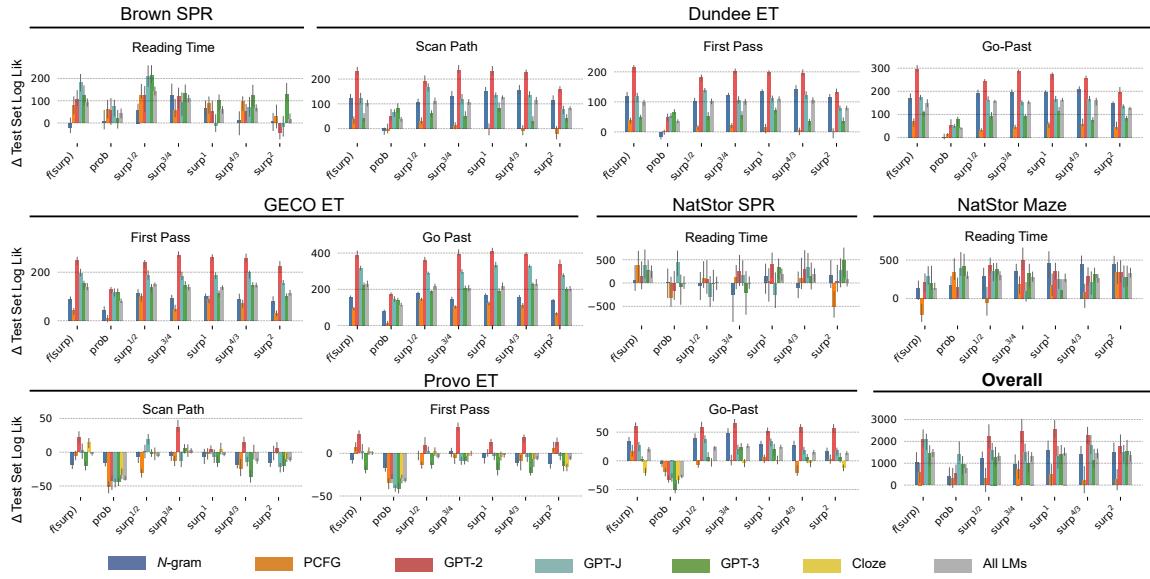


Figure 3: Change in test set log likelihood as a function of (i) language model and (ii) predictability-cost function, relative to a baseline model containing no predictability measure. Predictability-cost functions include the main CDRNN model that enforces no constraints on functional form ($f(\text{SURP})$), along with models assuming a linear effect of word probability (PROB) and models assuming a linear effect on some exponent of surprisal (from $\text{SURP}^{1/2}$ to SURP^2). Bars represent the median pairwise likelihood difference between the models of the critical and baseline ensembles (10 models each, resulting in 100 likelihood differences per bar). Error bars show 95% bootstrapped confidence intervals of the median pairwise likelihood difference.

predictability). Although there are hints of superlogarithmicity (superlinear surprisal effects) in some configurations (e.g., n -gram effects on Dundee scan path durations) and of sublogarithmicity (sublinear surprisal effects) in others (e.g., GPT-2 effects on GECO first pass durations), the uncertainty interval covers a logarithmic effect in nearly all cases. In SI 7, we also show that CDRNN models tend to recover a logarithmic predictability effect when provided with predictability measures on a linear or superlogarithmic scale. This outcome is at odds with some recent reports of superlogarithmic effects in a subset of this data (e.g., 13, 14). They are likewise at odds with recent claims that better language models find more strongly superlogarithmic effects (14)—in our results, estimates using a much larger model (GPT-3) are not systematically more superlogarithmic than estimates using smaller models with worse perplexity like GPT-2 (see also SI 8). We return to these divergences from prior work in the **Discussion**.

Are Predictability Effects Robust in Naturalistic Reading? We now confirm that our analyses replicate numerous prior findings of predictability effects in reading (e.g., 1, 4, 25, 46, *inter alia*). To this end, patterns of fit of pre-trained CDRNN models to unseen data are visualized in **Figure 3**, which shows the median change in out-of-sample test-set likelihood relative to a baseline containing no predictability variable. The primary models of interest— $f(\text{SURP})$ —use unconstrained (possibly nonlinear) functions of surprisal. The $f(\text{SURP})$ model for each statistical language model is significant over a baseline model with no predictability effect, as is the $f(\text{SURP})$ model for all language models in aggregate, supporting a generalizable effect of word predictability. Moreover, the more constrained models PROB, $\text{SURP}^{1/2}$, $\text{SURP}^{3/4}$, SURP^1 , $\text{SURP}^{4/3}$, and SURP^2 are also significant over the baseline, indicating that this finding does not critically depend on assumptions about functional form. We thus find strong evidence that reading behavior is modulated by predictability in context, consistent with much prior work.

Which Language Model Best Estimates Human Subjective Surprisal? We next evaluate differences in psychometric quality (predictive fit to reading times) across language models. The numerically

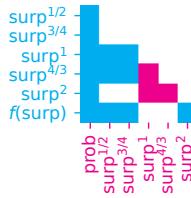


Figure 4: Results of statistical comparisons across all datasets and language models between pairs of assumed forms for the effect of word predictability. For a given pair, cyan indicates that the model on the row has significantly better test set performance than the model on the column, magenta indicates that the model on the column significantly outperforms the model on the row, and white indicates no significant difference. Only the lower triangle is shown. Tests use false discovery rate correction for multiple comparisons (49) across all tests. See **Figure S8** for results by dataset and language model.

best performing language model overall is GPT-2(-small), which significantly outperforms all other language models in the $f(\text{SURP})$ configuration except GPT-J (**Figure 3**), and which shows especially pronounced performance gains over other models in the more constrained configurations $\text{SURP}^{3/4}$ and SURP^1 . The finding that GPT-2-small substantially outperforms GPT-3 is striking given that GPT-3 has over 1,000 times more parameters than GPT-2-small, is trained on much more data, and has better overall perplexity (see the surprisal density plots in **Figure 2**; perplexities by language model and dataset are provided in **SI 9**). This result suggests that previously reported correlations between the linguistic and psychometric performance of language models (25, 47) may not hold for more recent large transformer language models, and instead suggests limitations on the benefits of language model perplexity for modeling human subjective word probabilities (see also 48). Given these performance differences, although we consider results across language models in the remainder of this article, we place special emphasis on results derived from GPT-2, since these most reliably characterize reading behavior overall.

Main Question: Is Processing Difficulty Linear, Logarithmic, or Superlogarithmic on Word Predictability? We now turn to the statistical analyses that bear on our core question, using out-of-sample model performance to assess hypothesized functional forms of predictability effects. As shown in **Figure 3**, we compare the performance of the unconstrained $f(\text{SURP})$ models to that of models constrained to respect some fixed predictability-cost function, namely models that are linear on raw probability (PROB, as predicted by the FACILITATION view) and on powers of surprisal ($\text{SURP}^{1/2}$, $\text{SURP}^{3/4}$, SURP^1 , $\text{SURP}^{4/3}$, and SURP^2), where the SURP^1 model instantiates the logarithmic pattern predicted by the COST view and the $\text{SURP}^{4/3}$ and SURP^2 models instantiate superlogarithmic patterns consistent with the UID view. The $\text{SURP}^{1/2}$ and $\text{SURP}^{3/4}$ models instantiate sublogarithmic effects and are included for completeness, even though no existing theory predicts these functional forms.

Overall results across language models and datasets (**Figure 4**; see **SI 4** for full testing results by model and dataset) indicate (i) that PROB significantly under-performs all surprisal-based models, (ii) that SURP^1 is the best performing constrained model overall, significantly outperforming both sublogarithmic models ($\text{SURP}^{1/2}$ and $\text{SURP}^{3/4}$) and superlogarithmic models ($\text{SURP}^{4/3}$ and SURP^2), and (iii) that there is no significant advantage of unconstrained $f(\text{SURP})$ models over SURP^1 models constrained to have a logarithmic predictability effect. There is thus no systematic evidence in our study that predictability effects are anything other than logarithmic, and, of the constrained models, the logarithmic effect fits the data better than either the linear effect predicted by the COST view or the superlogarithmic effect predicted by the UID view. Results from this large-scale investigation therefore favor a logarithmic predictability effect.

One logical possibility is that both kinds of processes (anticipatory FACILITATION and inferential COST) act simultaneously, giving rise to a superposition of linear and logarithmic effects on predictability. This view has been advocated by a prior study of predictability effects on event-related potentials in electrophysiology experiments (50), which supported additive linear and logarithmic predictability effects. This position predicts a sharper fall-off in processing demand in the low-surprisal interval due to additional linear facilitation

at highly predictable words. Might a similar pattern hold in reading data? Visual estimates in **Figure 2** do not appear to support this hypothesis. To address this hypothesis directly, we focus on GPT-2 (the language model with the strongest overall psychometric performance) and fit models that contain strictly linear predictors for one or both of GPT-2 probability and GPT-2 surprisal. We then compare the generalization performance of the model containing both GPT-2 probability and GPT-2 surprisal to that of the models containing only one or the other (see *Prob vs. Surp of Tables S5–S8*). We only find significant contributions of GPT-2 probability (linear effect) above and beyond GPT-2 surprisal (logarithmic effect) in two datasets (Natural Stories SPR and Natural Stories Maze). However, in the largest of these (Natural Stories SPR), the GPT-2 probability effect does not go in the predicted direction: more probable words are associated with longer reading times (**Figure S11**). Thus, although the overall contribution of GPT-2 probability over GPT-2 surprisal alone across datasets is significant (**Table S9**), this significance is driven largely by the opposite pattern from that predicted by the FACILITATION view. We therefore do not find evidence to support the additive linear and logarithmic effects of word predictability reported by ref. (50), a difference that could be due to modality differences (reading in our study vs. electrophysiology in theirs). Instead, overall results are primarily consistent with logarithmic predictability effects alone.

Nonetheless, there are potentially important differences in testing outcomes between individual datasets and language models, as visualized in **Figure S8**. For example, superlogarithmic models tend to show stronger performance in the Natural Stories SPR dataset: aggregating across language models, $\text{SURP}^{4/3}$ outperforms SURP^1 . In addition, across all datasets, the larger transformer language models (GPT-J and GPT-3) favor a superlogarithmic model over a logarithmic one: $\text{SURP}^{4/3}$ outperforms SURP^1 using GPT-J predictability estimates, and SURP^2 outperforms SURP^1 using GPT-3 predictability estimates. Both of the outcomes above (superlogarithmic effects in Natural Stories SPR and a bias toward superlogarithmicity in larger language models) are consistent with recent findings from ref. (14). However, they should be interpreted with caution, since (i) we find no evidence that these dataset-specific superlogarithmicities are characteristic of reading in general (across our entire sample; see **SI 8** for in-depth discussion of this point), and (ii) the GPT-J and GPT-3 language models perform worse in overall psychometric comparisons than GPT-2 (especially in the critical SURP^1 condition; **Figure 3**), which does not exhibit a bias toward superlogarithmicity. The GPT-J and GPT-3 patterns are therefore a questionable basis for claims about predictability effects in humans, in the absence of similar patterns in better-performing GPT-2 models. In addition, GPT-3 in particular was trained on a large web corpus that is not publicly available. Given that all the reading stimuli in these experiments are available online, it is plausible that GPT-3 was trained on some or all of these texts, which could artificially reduce its surprisal estimates for them, especially for highly surprising words that contribute large training gradients. This could give rise to artifactual superlogarithmicities when using GPT-3 surprisal estimates to predict human reading times, since compression in the high-surprisal regime will lead to steeper increases in processing cost if the underlying cost function is logarithmic on human subjective probabilities. Therefore, although these exceptions are noteworthy and warrant future research, the overall pattern emerging from our study is most favorable to logarithmic predictability effects.

Do Results Change under Cloze Estimates of Word Predictability? The results reported here derive from statistical language models that perform next-word prediction. However, the current experimental gold standard measure of word predictability is the cloze task (26) in which predicted next-word continuations given a context are collected from human participants (e.g., 3, 6, 27, 46, 51–53). Because cloze estimates are human-derived, they avoid potential confounds due to mismatch between statistically estimated and human subjective next-word probabilities. Indeed, the use of statistical rather than cloze predictability estimates has been cited as a criticism of prior work on the functional form of word predictability effects (3, see **SI 1** for extended discussion). However, some have argued that the cloze task may measure different cognitive processes than those that underlie real-time language comprehension (27, 52), and there is currently debate as to whether cloze estimates perform better (27, 51, 53) or worse (54, 55) than statistical language models as estimators of human processing difficulty (see also **SI 1**).

Thoroughly investigating this question in our current experimental setup is prohibitive, since it would require word-by-word cloze distributions for all the large naturalistic texts in this study (including an entire

novel in the case of the GECO dataset). However, the Provo dataset was fully cloze-normed as part of its design (6). We therefore use Provo to address two questions: (i) do results depend critically on the use of statistical predictability estimates, and (ii) how does our best statistical language model (GPT-2) perform relative to cloze?

Regarding (i), estimates using cloze surprisal for the Provo dataset are plotted in **Figure 2**. As shown, estimates are if anything more strongly superlogarithmic than estimates using any of the statistical language models, and the $f(\text{SURP})$ model significantly outperforms the PROB model for all duration types. Despite the visually superlogarithmic estimates in **Figure 2**, the performance profile for cloze is similar to that of other predictability estimates (**Figure 3**), with peak performance from logarithmic (SURP^1) or slightly sublogarithmic ($\text{SURP}^{3/4}$) models, and worse performance from either superlogarithmic model ($\text{SURP}^{4/3}$ and SURP^2). Thus, results under cloze remain most consistent with logarithmic predictability effects.

Regarding (ii), GPT-2 surprisal numerically outperforms cloze surprisal in all comparisons, significantly so for first pass and go-past durations (**Figure 3**). This outcome suggests that at the scales of training corpus, language model architecture, and cloze norm dataset size investigated here, the benefits of artificial model-based surprisal estimation (e.g., differentiating among degrees of low probability, or capturing variability that may be under-represented by cloze distributions, see **SI 1**) may now outweigh whatever disadvantages model-based estimates might have in principle relative to cloze norms, at least for naturalistic-text datasets (see also 54, 55).

Discussion

In this study, we revisited a longstanding question about predictive processing during language comprehension, namely, what is the functional form of predictability effects on measures of incremental comprehension difficulty? We evaluated five statistical language models (n -gram, PCFG, GPT-2, GPT-J, and GPT-3 models) on six large-scale reading datasets using recent advances in nonlinear regression modeling for naturalistic language processing data (CDRNNs, 32, 33). Unlike most prior work on this question (cf., 13), our statistical tests are based exclusively on out-of-sample model fit, thus grounding the outcomes of tests in the generalizability of effects.

Results favor a logarithmic effect of word predictability (linear effect of word surprisal, 1) compared to a linear (3) or superlogarithmic (13, 14) effect. Nonlinear CDRNN models of human reading emergently discover estimates consistent with a logarithmic predictability effect, improve upon models constrained to have a linear or superlogarithmic predictability effect, and generally do not improve upon models constrained to have a logarithmic predictability effect. Similarly, models constrained to have a logarithmic effect generally outperform models constrained to have a linear effect, as well as models constrained to have slightly sublogarithmic or superlogarithmic effects. Supplementary analyses (**SI 7**) suggest that this logarithmic effect of word predictability is not due to an inductive bias of the CDRNN model. Moreover, when we reanalyze data from Brothers and Kuperberg (3)—the strongest current counterevidence supporting linear rather than logarithmic predictability effects—we find (**SI 1**) that GPT-2 predictability estimates instead favor a logarithmic over a linear effect and fit Brothers and Kuperberg’s self-paced reading data as well as the cloze estimates used in the original study.

Our findings have implications for current understanding of the cognitive processes that give rise to predictability effects, favoring the view that predictability effects primarily reflect the cost of probabilistic inference (17) over the view that predictability effects primarily reflect anticipatory facilitation (3). Furthermore, our results do not support the hypothesis that processing demand is superlogarithmic in predictability, which might give rise to uniform information density pressures (13, 14, 22).

In making this claim, we stress that we have used the term *facilitation* more narrowly than it is sometimes used in the field: by “FACILITATION view,” we are referring specifically to theories of a linear form for the predictability–cost relationship whereby predictability effects are driven primarily by highly predictable words, rather than the more general idea that contextually preactivated words are read more quickly. Our findings agree with the construal of predictable words as “facilitated” in this more general sense: when surprisal is low, the COST view predicts fast reading (because the inferential update is small).

Implications for Theories of Language Comprehension. A common stance among psycholinguists is that prediction serves a largely facilitatory (3)—and possibly optional (56)—role in a comprehension process dominated by the demands of incrementally assembling an ever-richer representation of sentence structure and meaning. These hypothesized demands include lexical retrieval (e.g., 57) and syntactic integration (e.g., 58), and successful prediction might allow the processor to discharge these demands early and thus use computational resources more efficiently. As word predictability nears zero, the processor gets little of this anticipatory benefit and converges to a wait-and-see mode. This view predicts little difference for processing between next-word probabilities of e.g., $p = 0.001$ vs. $p = 0.0001$: in both cases, the full processing burden will fall on the word itself. Our results challenge this FACILITATION view, instead showing large changes in processing cost due to small absolute differences within the low probability regime.

Rather, our results support an information-theoretic view (17, 18) in which a major driver of processing cost is probabilistic inference over a (possibly vast) space of interpretations of the unfolding sentence (possibly including syntactic parses, predicate logic, and any other cognitively-relevant form of sentence representation). Under this view, an interpretation is a probability distribution over this representation space, and words with small absolute differences in probability can have large differences in the size of the update they require to the interpretation distribution, due to the logarithmic form of the KL divergence between the interpreter states before and after observing a word. Our results bear out this prediction by supporting a linear increase in reading latencies as a function of this logarithmic divergence (surprisal), thereby supporting the COST view that prediction is not merely an aide to comprehension, but an inherent consequence of what it means to comprehend.

The importance of probabilistic inference draws support from computational parsing algorithms, the design of which is dominated by the problem of *finding* (rather than assembling) the correct analysis of a sentence (e.g., 59–67). Computationally implemented approaches thus suggest that the problem of local ambiguity in sentence interpretation goes well beyond the garden-path constructions and attachment ambiguities that have largely preoccupied psycholinguistic treatments of this problem (e.g., 68–78), and may instead be the primary obstacle to successful comprehension (see also e.g., 79–81). It is therefore not surprising to find evidence that probabilistic inference may also be a major preoccupation of the human language comprehension system.

That said, two points of clarification must be emphasized. *First*, our claims are not at odds with the notion of preactivation *per se*, but only with a facilitatory construal of its influence on processing cost. Diverse experimental evidence supports the hypothesis that predictable linguistic units are represented in the mind and brain before they are encountered (82–87). Probabilistic inference is perfectly compatible with this evidence, since the candidate interpretations among which the processor allocates probability mass might contain information about as-yet unobserved material. Our study simply constrains the hypothesis space around how these representations influence incremental processing demand.

Second, our claims are compatible with the existence of other, surprisal-independent determinants of incremental processing demand. In other words, our claims do not entail commitment to a strong view of surprisal as the sole causal bottleneck between representations and processing demand (c.f., 18). Experiments have identified diverse surprisal-independent influences on processing demand, including lexical (88, 89) and repetition (90) priming, word frequency (91, 92), dependency locality (93, 94), and garden path constructions (95). Whether all such influences can be reconciled with surprisal theory is currently unclear (for recent attempts to address some of them, see 96, 97). But the results of our study are orthogonal to this issue: we are not claiming that surprisal is the only determinant of processing difficulty, only that it is an important one, and that predictability effects in natural reading cannot be reduced to mere facilitation at highly predictable words. As a result, we argue that mechanisms of probabilistic inference should feature prominently in theories of language comprehension, regardless of any other constraints on constructing sentence representations in memory.

One potential challenge for the COST view that we have advocated is a well-replicated finding that invalid parafoveal preview (i.e., replacing words near the current fixation with other words or random characters) eliminates predictability effects in early eye movement measures (first fixation duration and first pass duration, e.g., 98–100). This finding has been taken to indicate that, at least in early measures, predictability

primarily affects only the earliest stages of lexical processing, when visual cues to word identity are poorly resolved in the paravofea and must be supplemented by top-down predictive signals (100). This interpretation is hard to reconcile with our construal of predictability effects as primarily reflecting high-level structural and semantic inference. Although we cannot address this concern empirically since all of our data used valid preview, we offer three comments. *First*, three of our six datasets used self-paced designs that have no preview (but still show strong predictability effects), and the same experimental studies above found that predictability effects were preserved under invalid preview in later measures (go-past durations and N400 amplitudes, 99, 100). Thus, predictability effects register consistently in later measures that plausibly reflect high-level inferential processing. *Second*, our finding of predictability effects (under valid preview) in early eye movement measures like scan path and first past durations may reflect inferential processing that began during parafoveal preview and continues after fixation (some models of surprisal effects, e.g., 1, assume that inferences are continuously updated, rather than being strictly post-lexical, which is consistent with inference during preview). Invalid preview would delay the start of such processing, potentially pushing predictability effects outside the time window within which they would normally be captured by earlier measures (but preserving them in later ones). *Third*, one interpretational challenge for studies that manipulate preview validity is that the parafovea provides incorrect information about the future realization of the text. Although participants are usually not conscious of the preview validity manipulation, invalid preview could still send signals to the language processing system that predictions are incorrect with unusual frequency (when in fact they are not). This could result in a strategic adaptation in which the processing system relies less on prediction (or, put information-theoretically, generates more entropic predictions), thereby attenuating predictability effects. The high-cloze (i.e., very predictable) items typically used in these experimental studies may be especially susceptible to such an attenuation, since they encourage strong predictions that are temporarily disconfirmed parafoveally. Current evidence about preview validity may therefore be compatible with the view of predictability effects we have advocated, although the discussion above offers many opportunities for follow-up study.

Our results also discriminate between extant information-theoretic models of language comprehension by favoring the logarithmic effect of word predictability predicted by standard surprisal theory (17, 18) over the superlogarithmic effect that has been hypothesized to give rise to pressures toward uniform information density (13, 14); although there are visually apparent superlogarithmicities in some model estimates (**Figures 2** and **S12**), superlogarithmic models generally underperform logarithmic or sublogarithmic ones (**Figure 3**). Our results nonetheless highlight the challenge of discriminating between fine differences in hypothesized functional form on the basis of reading data, even at scale. Despite some statistically significant advantages of a logarithmic effect shape, we tend to find a broad range of near-equivalence in model performance across the sublogarithmic-superlogarithmic spectrum, with variation across language models and datasets as to the precise peak of this continuum (**Figure 3**). Since UID does not make precise claims about how strong superlogarithmicity should be (and is thus consistent with an arbitrarily diminishing exponent on log probability), it may not be possible to rule out UID pressures on the basis of this kind of data. Our evidence is simply more consistent with a logarithmic than a superlogarithmic effect of word predictability on reading times, while placing some constraints on the strength of superlogarithmicity (e.g., squared surprisal is likely too strongly superlogarithmic).

Why do our results differ from those reported in other recent studies using partially overlapping data (13, 14)? With respect to Meister et al. (13), the strongest evidence for superlogarithmicity came from offline acceptability judgments; the evidence from online reading measures was more equivocal. The relationship between online and offline measures of comprehension difficulty is currently poorly understood, and we leave this discrepancy to future investigation. With respect to Hoover et al. (14), their claims of superlogarithmicity are based on visual estimates (and descriptive statistics derived from those estimates) from models fitted only to the Natural Stories SPR dataset. Our results in fact partially replicate theirs, since estimates tend to be visually superlogarithmic in Natural Stories SPR (especially over the long right tail of surprisal values, see **Figure S13**), and a slightly superlogarithmic model (SURP^{4/3}) outperforms a logarithmic one on that dataset, aggregating over all language models. However, this outcome appears to be largely restricted to Natural Stories SPR and does not generalize to a broader sample of reading data. Furthermore, a recent

study of predictability effects across languages (15) obtained strongly logarithmic estimates (with little hint of superlogarithmicity) in ten non-English languages. In the absence of reasons to think that Natural Stories SPR is an especially reliable source of evidence on this question (see SI 8 for counterarguments), our results suggest that the pattern reported in Hoover et al. may not be characteristic of reading in general.

Implications for Statistical Modeling of Human Subjective Word Probabilities. Our results additionally differentiate computational models of human next-word prediction. Surprisal estimates from GPT-2(-small) (43) substantially outperform surprisal estimates from n -gram, PCFG, GPT-J, and GPT-3 models. GPT-2 therefore appears to reside in a “Goldilocks” region of psychometric performance between language models that are too constrained on the one hand (n -gram and PCFG models) and too powerful on the other (GPT-J and GPT-3). This outcome challenges the notion that previously reported correlations between the linguistic and psychometric performance of language models (e.g., 25, 47, 101) will extrapolate to models of ever-increasing size, complexity, and quantity of training data (see also 48). Instead, the task of using language model predictions to estimate human reading times may be akin to tasks in natural language processing that show an “inverse scaling” property, whereby task performance is inversely related to model size (102–104). This result has both methodological and scientific implications. From a methodological standpoint, bigger is not always better; the selection of a language model for psycholinguistic research may need to consider additional dimensions (beyond perplexity). From a scientific standpoint, homing in on classes of models that best mimic human processing patterns offers the opportunity for new insights into the learning and processing mechanisms that underlie human language abilities (9, 105), a direction that we leave to future work.

In addition, our results also bear on the widespread perception of cloze norms as the gold standard method for estimating human next word predictability. Prior work has raised theoretical concerns about this perception, arguing that cloze predictions may reflect distinct cognitive processes from those recruited during real-time language comprehension (27, 106). Relatedly, some recent studies have found cloze estimates to underperform model-based predictability estimates in predicting human language processing measures (54, 55). Our results accord with these prior findings by showing that, when used as estimators of human reading effort, surprisal values from GPT-2 are, on average, at or beyond parity with cloze norms (based on the Provo dataset). Although additional research is needed to characterize the relative strengths of statistical vs. cloze predictability estimates in specific cases, our results suggest that the use of statistical predictability estimates, especially those from incremental transformer language models like GPT-2, should not generally be viewed as a design weakness relative to cloze norms in studies of language processing (see SI 1 for extended discussion).

Although this comparison between GPT-2 and cloze may seem purely methodological, it is in fact bound up in our core theoretical question about the cognitive sources of word predictability effects. This is because of the asymmetric importance assigned by the FACILITATION vs. COST views to low-probability events, for which the cloze task (under realistic sample sizes) provides poor quality estimates. Under a FACILITATION (linear predictability) view, the main drivers of predictability effects are high-probability words. If this view is correct, then accurately estimating degrees of low probability is of little consequence, and cloze is the preferred estimator. Under a COST (logarithmic predictability, i.e., surprisal) view, the main drivers of predictability effects are low-probability words, since small absolute differences in low predictability can correspond to large differences in surprisal. If this view is correct, then accurately estimating degrees of low probability is essential, and cloze is not the preferred estimator. Therefore, one consequence of the COST view is that accurately estimating fine-grained differences in low probability (via e.g., GPT-2) should be more important than accurately estimating human subjective probabilities within the high-probability regime (via cloze). Our results support this position.

Conclusion. In conclusion, using recent advances in computational language modeling and time series analysis, and using diverse large-scale naturalistic reading datasets, our results support a logarithmic effect of word predictability on processing difficulty (1), and therefore support probabilistic inference as a core component of human language comprehension.

Data. The datasets considered in this study span three modalities: self-paced reading, the Maze task, and eye-tracking during reading. In a self-paced reading task, participants are presented with texts in which words or characters are occluded until the participant reveals them one-by-one in left-to-right order by pressing a button. In a Maze task (107), like in a self-paced reading task, participants press buttons to progress word-by-word through a text. However, at each word position in the text, participants are presented with a forced choice between the true next word and a distractor, and they are tasked with selecting the correct continuation. In an eye-tracking during reading task, texts are presented on a screen to participants who read naturally, and their sequence of fixations to words in the text are recorded by an eye tracker.

The self-paced reading and Maze tasks yield a single word-by-word dependent variable: *reading time* (or *reaction time*, RT), that is, the time elapsed between stimulus presentation (a word in self-paced reading or a forced-choice decision in Maze) and pressing a button to indicate a decision (to reveal the next word in self-paced reading or to choose the continuation in Maze). Modeling eye movements during free reading is more challenging because the eyes do not progress linearly through the textual sequence of words. Studies of eye-tracking during reading have used a variety of measures derived from the reading record, each with a somewhat different cognitive interpretation (see e.g., 108, for review). In this study, we consider three different measures of fixation duration:

- **Scan path duration (e.g., 109).** Time elapsed from when the eyes enter any word region from either the left or the right to when they next enter a different word region (either to the left or to the right), regardless of whether the fixation is a part of a regressive eye movement. This definition of scan path duration sums across all consecutive fixations to the same word region, since we do not wish to treat consecutive fixations to the same word as distinct events (a word should likely not influence our analyses three times more for having been viewed by three consecutive fixations rather than one). Under this definition (and unlike the first pass and go-past durations discussed below), a given experimental participant can have more than one observation associated with a given word token in the text (when words are refixated). For example, if a word sequence *ABC* is fixated in the order *ACBBC*, the scan path record will contain a sequence of four events: the duration of the fixation to *A*, followed by the duration of the first fixation to *C*, followed by the summed durations of the fixations to *B*, followed by the duration of the second fixation to *C*. Scan path durations thus encode the entire sequence of word fixations in time rather than textual order, including fixations that are part of regressive eye movements (e.g., refixations and fixations to words that were skipped in the initial pass). Regressive and non-regressive scan path events are distinguished in our analyses by a binary indicator variable (see SI 10).
- **First pass duration (e.g., 108).** Time elapsed from when the eyes first enter a word region from the left to when they enter a different word region (either to the left or to the right). The sequence of first pass durations excludes all regressive eye movements, such that refixations or fixations to words that were skipped in the initial pass are not modeled.
- **Go-past duration (e.g., 108).** Time elapsed from when the eyes first enter a word region from the left to when they enter a different word region to its right (including all intervening regressive fixations). Like first pass durations, the sequence of go-past durations excludes all regressive eye movements, such that refixations or fixations to words that were skipped in the initial pass are not modeled (except indirectly via their influence on go-past durations for words that were fixated in the initial pass).

Scan path and first pass durations are both early measures, restricted to the fixation duration of a single word (108). They differ only in whether regressive eye movements are included (scan path) or discarded (first pass). Go-past duration is a late measure designed to capture all processing (including regressive eye movements) involved in moving beyond the current “frontier” in progressing through the text.

In all eye tracking datasets except the GECO dataset (see below), a stimulus “event” is considered to be any fixation to a word region in the text. Thus, the full sequence of fixations before entering a target word region, regressive or non-regressive, is used to predict all three types of fixation duration at that region. Note that this differs from standard regression analyses of first-pass and go-past durations in eye-tracking data,

which typically discard the full sequence of fixations and only consider the linear sequence of words. The ability to recruit the full scan path record to predict all response variables is an advantage of the deconvolutional regression approach described below.

In all datasets, following prior analyses of the Dundee and Natural Stories SPR datasets (109), we partition the data into training, validation, and test splits (approximately 50%, 25%, and 25%, respectively) using modular arithmetic on a split variable i , defined as a function of participant index p and sentence index s :

$$i = (s + p) \mod 4 \quad (1)$$

where datapoints are cycled into training if $i \in \{0, 1\}$, validation if $i = 2$, and test if $i = 3$. Models are only fitted to data from the training set. Validation data is used for tuning and early stopping, following ref. (33). Test data is only used for statistical comparisons between models. Per ref. (109), to enable valid deconvolution, all data partitioning and filtering (see below) are applied only to the response vectors (the modeled reading times). The entire predictor matrix (sequence of word fixation features) is retained in all models.

The preprocessed datasets are available at <https://osf.io/6wvqe/>. For instructions on reproducing our preprocessing pipeline for the reading data, see <https://github.com/coryshain/cdr>.

Brown SPR. The Brown SPR dataset (1) contains self-paced reading data from 35 participants reading short (292-902 word) passages from the Brown dataset of American English (110). The data can be accessed online at <https://github.com/wilcoxeg/neural-networks-read-times>.

The dataset contains a total of 450 sentences, 7,188 words, and 136,907 responses. Following established protocol for Natural Stories SPR (another self-paced reading dataset, described below), we remove sentence boundaries and RTs that were less than 100ms or greater than 3,000ms.

Dundee ET. The Dundee ET dataset (111) contains eye-tracking data from 10 participants who read newspaper articles from *The Independent* on a computer monitor. The data can be accessed online at <https://github.com/wilcoxeg/neural-networks-read-times>.

The dataset contains a total of 2,388 sentences, 51,501 words, and 408,439 distinct fixations to word regions on the screen. The responses in the Dundee dataset are filtered to exclude fixations following large outlier saccades (>20 words in either direction), based on the assumption that such outliers reflect track loss or inattention, rather than language processing. Following prior work (e.g., 109), we also remove fixations to words adjacent to a screen, line, or sentence boundary, as well as fixations interrupted by blinks.

GECO ET. The GECO ET dataset (36) contains eye-tracking data from participants who read *The Mysterious Affair at Styles* by Agatha Christie on a computer monitor. The full dataset contains data from 19 Dutch-English bilinguals who read the first half of the novel in either Dutch or English and the second half in the other language, along with data from 14 English monolinguals who read the entire novel in English. Because the computational language models used in this study are English-specific, here we only used the data from the 14 monolingual English readers. Unlike the other ET datasets analyzed in this study, the GECO dataset does not provide the full scan path record, but only a distilled format that contains first pass and go-past times by word. Thus, in the case of GECO, we do not analyze scan path durations, and we treat each fixated word in textual order as a stimulus “event” (rather than individual fixations) for the purposes of deconvolution. The data can be accessed online at <https://expsy.ugent.be/downloads/geco/>.

The portion of the dataset that we analyzed contains a total of 5,300 sentences, 56,440 words, and 374,179 events. Following the Dundee protocol (above), the responses in the GECO dataset are filtered to exclude fixations following large outlier saccades (>20 words in either direction) and fixations to sentence boundaries (screen and line boundaries were not annotated).

Natural Stories SPR. The Natural Stories SPR dataset (37) contains crowd-sourced self-paced reading responses from 178 participants to 10 naturally-occurring narrative or non-fiction pieces modified in order to

over-represent rare words and syntactic constructions without compromising perceived naturalness. The stimuli are thus designed to reflect the typical conditions of story comprehension, while subtly taxing the language processing system. The data can be accessed online at <https://github.com/languageMIT/naturalstories>.

The dataset contains a total of 485 sentences, 10,256 words, and 1,013,377 responses. Following previous work (e.g., 109), RTs are removed if they are less than 100ms or greater than 3,000ms, if they are to words adjacent to a sentence boundary, if participants answered less than 5/8 comprehension questions correctly, or if, subject to the aforementioned constraints, participants have fewer than 100 RTs.

Natural Stories Maze. The Natural Stories Maze dataset (38) contains crowd-sourced Maze task responses from 95 participants to the same materials as in the Natural Stories SPR dataset above, using a recently developed technique (A-Maze) to generate high quality forced-choice alternatives for long naturalistic passages (112). The data can be accessed online at <https://github.com/vboyce/amaze-natural-stories>.

The dataset contains a total of 97,527 responses (the textual statistics are the same as Natural Stories SPR above). Following ref. (38), RTs are removed if they are less than 100ms or greater than 5,000ms, if they are to words adjacent to a sentence boundary, or if the subject responded incorrectly (i.e., selected the wrong continuation). Inattentive subjects (defined as subjects with lower than 80% accuracy) are also removed.

Provo ET. The Provo ET dataset (39) contains eye-tracking data from 84 participants who read 55 short (39-62 word) passages from various online sources on a computer monitor. The data can be accessed online at <https://osf.io/sjefs/>.

The dataset contains a total of 134 sentences, 2,745 words, and 213,224 distinct fixations to word regions on the screen. Following the Dundee protocol (above), responses are filtered to exclude fixations following large outlier saccades (>20 words in either direction), fixations to words adjacent to a sentence boundary (screen and line boundaries were not annotated), and fixations interrupted by blinks.

Surprisal Estimates. We obtain the surprisal estimates used in our experiments from three different families of language models. *First*, we consider surprisal estimates derived from an n -gram model, a simple count-based method that estimates word probabilities by interpolating over prefix counts up to a fixed length, estimated from large text corpora. Many prior studies have reported n -gram effects in human language processing (e.g., 1, 12, 113, *inter alia*). We compute n -gram surprisal values using a 5-gram model estimated on the WikiText-103 dataset (114)—a large, popular language modeling dataset extracted from Wikipedia—with Kneser–Essen–Ney smoothing (115). Model parameters are estimated using the KenLM (116) library with default hyperparameter settings.

Second, we consider surprisal estimates from a probabilistic context-free grammar (PCFG) parser, which conditions its next-word predictions on hypotheses about the syntactic structure of the sentence, rather than on the preceding word sequence. Although incremental generative parsers generally perform poorly as language models due to their highly constrained representation of context, recent work has shown that they perform unexpectedly well as models of measures of sentence processing (48). Our PCFG (41) is trained on a generalized categorial grammar reannotation (117) of the Penn Treebank (118).

Third, we consider surprisal estimates from large autoregressive language models based on the transformer architecture (42), namely GPT-2(-small) (43), GPT-J (44), and GPT-3 (45). These models generate next-word predictions via a deep neural network transform of the linguistic context (preceding word sequence). Recent work has shown strong alignment between autoregressive transformer representations and measures of human sentence processing, both behavioral (101) and neural (119). GPT-2 is a 1.5B parameter model that has been open-sourced through the Hugging Face library (120). We generate GPT-2 surprisals using the default Hugging Face implementation of GPT-2 (GPT-2-small). At the time we conducted this study, GPT-J was among the largest fully open-source transformer language models, with 6B parameters. Open source models are a critical asset to repeatable science since their weights and training data are available for direct inspection, and the inclusion of GPT-J therefore allows us to incorporate more recent advances in language modeling since the release of GPT-2 without compromising replicability. GPT-3 is a very large (175B

parameter) proprietary commercial language model trained on proprietary data, and its weights have not been publicly released. At the time we conducted this study, access to GPT-3 surprisal estimates was only available through a paid service. Considering GPT-3 surprisal allows us to explore more recent advances in language modeling, at the expense of full replicability given the reliance on a proprietary model. In this study, we use GPT-3-davinci-002.

Before computing the GPT-2 and GPT-J surprisal estimates, text from all corpora is pre-processed using the Moses decoder (<http://www.statmt.org/moses/>) tokenizer and punctuation normalizer. Capitalization is kept intact. No text preprocessing is used for GPT-3. Note that additional tokenization is performed internally by the tokenizers associated with each of the neural models (likewise provided either by the Hugging Face library for GPT-2 and GPT-J and by the OpenAI API for GPT-3). Because of these tokenization protocols, transformer language models sometimes predict at the level of subwords. To align surprisal values from transformers to word tokens, we therefore sum surprisal values across tokens within each word to generate a word-level value. This procedure is licensed by the chain rule. Texts were entered to each model in their entirety when possible (except in the case of the PCFG, which requires sentence-tokenized text). In cases where text length exceeded the maximum allowed by the model, we used a sliding window approach guaranteeing at least 200 words of context per prediction. Code for reproducing our n -gram, GPT-2, and GPT-J estimates is available at <https://github.com/rycolab/revisiting-uid>. Code for reproducing the PCFG and GPT-3 estimates is available at <https://osf.io/6wvqe/>.

Models also include a number of control predictors described in **SI 10**; see **SI 11** for detailed model formulae. The preprocessed datasets, including all control and surprisal predictors, are available at <https://osf.io/6wvqe/>.

Analysis.

Continuous-Time Deconvolutional Regression. All analyses use continuous-time deconvolutional regressive neural networks (CDRNNs; 32, 33); see **SI 12** for a formal definition of the regression model. In brief, CDRNNs convolve the recent history of predictors (word features) in the experiment with continuous-time filters generated by deep neural networks in order to parameterize the distribution over the response (e.g., scan path duration) at a point in time. CDRNNs thus implicitly estimate continuous-time impulse response functions (IRFs) representing the effect of an impulse (a word) on the response (comprehension difficulty) at some delay. The properties of these IRFs can be queried using a combination of perturbation analysis (121) and Monte Carlo dropout (122), enabling interpretation of a black box deep neural model. Unlike standard approaches to time series regression like linear mixed-effects models (LMEs; 123) and generalized additive models (GAMs; 34), CDRNNs simultaneously relax assumptions that the IRF is discrete-time, linear, and stationary (time-invariant), all within a *distributional regression* framework (e.g., 124) that captures stimulus-driven effects on all parameters of the distribution over the dependent measure, not just its expected value. Critically, CDRNNs can be constrained to *enforce* linearity for certain predictors, permitting statistical evaluation of nonlinearity by comparing the fit of models that relax or enforce it. Full description of the CDRNN approach can be found in ref. (33). CDRNN implementation details used in this study are described in **SI 13**. Code for reproducing all analyses in this study can be found at <https://github.com/coryshain/cdr>. See **SI 14** for evidence that more commonly-used generalized additive models (GAMs) yield similar results to our own.

Response Distribution. Because the distribution of reading times is known to be heavily right skewed (e.g., 8), we assume an exGaussian response distribution (see e.g., 5, 125, for evidence that the exGaussian provides a strong distributional fit to human sentence reading). The exGaussian has three parameters: location (μ), dispersion (σ), and skewness (τ), where location, dispersion, and skewness all *increase* on their respective parameters. The quantity of interest targeted in this study is the influence of word probability estimates on the *mean* of this predictive distribution, where the mean depends linearly on the location and skewness parameters:

$$E_{F(\mu, \sigma, \tau)}(X) = \mu + \tau \quad (2)$$

Thus, a linear influence of surprisal on either μ or τ will yield a linear influence of surprisal on the mean of the response distribution. See [SI 15](#) for evidence both that assuming an exGaussian response substantially improves model fit over assuming a normal response and that similar findings to our main results still obtain when assuming normally distributed reading times.

Baseline Models. The main CDRNN models in this study are fully nonlinear on surprisal and can thus find any functional form ($f(\text{SURP})$) to a range of control models. The Baseline model contains no predictability effect of any kind, and thus provides a reference for the overall effect of including a predictability measure. The PROB model is constrained to be linear on probability, rather than surprisal, as predicted by some theories (e.g., [3](#)). The $\text{SURP}^{1/2}$, $\text{SURP}^{3/4}$, SURP^1 , $\text{SURP}^{4/3}$, and SURP^2 are constrained to be linear on some power of surprisal (denoted in superscript) and thus represent a cline of functional forms for the predictability effect, from sublogarithmic ($\text{SURP}^{1/2}$) to logarithmic (SURP^1) to superlogarithmic (SURP^2).

Statistical Procedure. Statistical testing within our continuous-time deconvolutional framework relies on out-of-sample model comparison: models instantiating the null vs. alternative hypotheses are trained on a portion of the data (training set), and conditional likelihoods from these models over an unseen portion of the data (test set) are statistically compared in order to determine whether the model instantiating the alternative hypothesis generalizes better than the model instantiating the null hypothesis ([109](#)). All results reported in this study are based in *ensembles* of 10 models, which reduces variability in effect estimation and predictive performance due to stochastic initialization and optimization. Following ref. ([33](#)), ensembles are compared using paired permutation tests of out-of-sample conditional likelihood. Full details of the testing protocol are described in [SI 13](#).

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Supplementary Information for *Large-scale evidence for logarithmic effects of word predictability on reading time*

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1. Revisiting the Linear Predictability Effects of Brothers & Kuperberg (2021)

The strongest current empirical support for linear (rather than our claimed logarithmic) predictability effects on processing difficulty comes from Brothers and Kuperberg (3), who report a striking convergence of evidence from three independent experiments. Brothers and Kuperberg's study is carefully implemented, with two key design features differentiating their approach from most other work on this question (including ours, but also 1, 13, 14, 25): (i) they used a normed predictability manipulation in a critical region of constructed stimulus sentences (i.e., an *experimental* or *controlled* study) rather than word-by-word modeling of responses to uncontrolled text (i.e., *naturalistic*, *corpus*, *observational*, or *correlational* studies, such as ours), and (ii) they used large-sample ($n \approx 90$) cloze predictability estimates in these critical regions, rather than model-based predictability estimates. These design features naturally travel together: word-by-word cloze-norming of free text is a large and rarely-attempted undertaking (though see 6). Thus, barring an unusually large investment of resources, cloze-norming is for practical purposes out of reach for naturalistic studies, but feasible for experimental studies, which have a relatively small number of critical regions to norm.

Citing (126–129), Brothers and Kuperberg argue that these design features are an advantage over studies that use (i) naturalistic stimuli and (ii) model-based statistical predictability estimates (including our study, by extension). They place special emphasis on (i), arguing in the article (and at length in an appendix) that naturalistic studies of language processing are especially susceptible to the correlation-causation problem: uncontrolled texts exhibit extensive covariation in linguistic features like length, frequency, predictability, syntactic complexity, semantic properties, etc., the underlying causes of which are still poorly understood. Naturalistic studies by definition lack experimental control for these confounds, and controlling for all plausible confounds statistically is difficult (or even impossible) given the complexity of the system. Furthermore, implemented statistical models require many assumptions about the set of relevant control variables and about the relationships among both the control variables and the response; these assumptions may lack strong theoretical or empirical support. These concerns are argued to make causal inference inherently less reliable from correlational studies than from experimental studies. Importantly in light of these arguments, across three experiments (self-paced reading, picture naming, and a meta-analysis of controlled studies of predictability effects) Brothers and Kuperberg found a linear rather than a logarithmic form for the predictability–cost relationship. Is the logarithmic effect found by our study (and others) merely an artifact of uncontrolled confounds in naturalistic reading, as suggested by Brothers and Kuperberg?

Here we address this concern by reanalyzing data from Brothers and Kuperberg's two key experiments (self-paced reading and picture naming) and attempting to nuance the arguments outlined above. In brief, (1) we show that Brothers and Kuperberg's experimental data are more equivocal than the impression given by the original study, supporting a logarithmic (rather than linear) predictability effect when modeled using GPT-2 (the language model with the best psychometric performance in our own study), and (2) we argue that Brothers and Kuperberg's advocacy for their design is somewhat selective: considering the full range of relative strengths and weaknesses for e.g., experimental vs. naturalistic studies and cloze vs. model-based predictability estimates results in a more ambivalent picture, without one unambiguously more reliable design.

1.1. Background. Brothers and Kuperberg constructed 648 stimulus sentences (9 to 16 words long, mean 13 words) representing low-cloze (less predictable), moderate-cloze, and high-cloze (more predictable) contexts for each of 216 critical words. The critical word was always followed by at least two additional words. An example item set is given below, with the critical word **glasses** in bold:

- High-cloze:** Her vision is terrible and she has to wear **glasses** in class.
- Moderate-cloze:** She looks very different when she has to wear **glasses** in class.
- Low-cloze:** Her mother was adamant that she has to wear **glasses** in class.

As the example makes clear, stimuli were constructed to read like natural sentences while carefully matching on length as well as left and right context of the critical word. Participants in all experiments saw each target

word in at most one condition. Cloze probabilities for the critical word of each stimulus were acquired in a crowd-sourced cloze completion task with about 90 completions per context, yielding an average resolution of $p = 1/90 \approx 0.01$ for the resulting predictability estimates. These stimuli were then used in a crowd-sourced self-paced reading (SPR) experiment (**Experiment 1**, $n = 216$) in which the dependent variable was summed reading time over a 3-word critical region starting at the critical word (the two additional words were included to capture spillover effects). A subset of 84 of these items (all with nouns as the critical word) were also used in an in-person cross-modal picture naming experiment (**Experiment 2**, $n = 36$) in which participants listened to an audio recording of the sentence prefix (up to but not including the critical word) being read aloud and were tasked with naming a picture of the critical word presented at a delay of 250ms (here the dependent variable was naming response time). As Brothers and Kuperberg acknowledge (p. 6), the naming experiment differs more than the SPR experiment from the normal conditions of word-by-word reading: it is an offline, explicit task that plausibly engages distinct cognitive processes (e.g., conscious reflection, visual object recognition, articulation) from those engaged in incremental language comprehension. It was included in the original study as a conceptual replication of the SPR experiment. Data from both experiments were analyzed using generalized additive models (GAMs; 34) to infer the form of the predictability–cost relationship from data and using linear mixed-effects models (LMEs; 123) to test prior hypotheses about this form (namely, linear vs. logarithmic). Brothers and Kuperberg also conducted a meta-analysis (**Study 3**) of 8 experiments across 5 prior studies in which they made inferences about the functional form of the predictability–cost relationship from condition-wise effect sizes (i.e., for high-, moderate-, and low-cloze conditions). We refer readers to Brothers and Kuperberg’s study for full methodological details.

Brothers and Kuperberg’s findings converged across experiments to support a linear over a logarithmic predictability effect. In both the SPR and naming experiments, GAM-estimated smoothing splines for the relationship between predictability and response time were approximately linear in cloze predictability and sublogarithmic in cloze surprisal (our analyses reproduce this finding, see the top row of **Figure S5A**), and LME models showed better fit to the data when cloze was represented linearly rather than logarithmically (our analyses also reproduce this finding, see rows “Cloze_{SURP1} vs. Cloze_{PROB}” and “Cloze_{PROB}+Cloze_{SURP1} vs. Cloze_{SURP1}” of **Table S1**). And in the meta-analysis, they found that the relative effect sizes reported for high-, moderate-, and low-cloze conditions in prior studies of predictability effects were more consistent with a linear than a logarithmic predictability effect. The authors made the data from these experiments and many of the analysis scripts publicly available: <https://osf.io/b9kns/>. We refer readers to Brothers and Kuperberg’s study for full results.

1.2. Methods. Here we use Brothers and Kuperberg’s public data release to revisit findings from the SPR and naming experiments. We leave the meta-analysis aside since the data release includes only aggregate measures, not the item-level stimulus and response data needed to e.g., reanalyze the data using model-based predictability estimates. Because the SPR task is more similar to incremental language comprehension than the cross-modal picture naming task (see above), we focus primarily on the SPR data, with the naming data included in our reanalysis for completeness. We follow Brothers and Kuperberg in using GAMs implemented by the `mgcv` package in R (34) to estimate the functional form of predictability effects and LMEs implemented by the `lme4` package in R (123) for statistical comparisons between prior hypotheses (e.g., linear vs. logarithmic predictability effects). We chose these methods over the CDRNNs used in our main study both to improve comparability to Brothers and Kuperberg’s results and because the public data release lacks word-level timing information; as a result, continuous-time deconvolution is not possible.

However, we go beyond the original study in two key ways. *First*, we use GAMs to additionally estimate predictability effects under other predictability models, not just cloze:

1. The **trigram** predictability estimates used as a control in Brothers and Kuperberg’s original study.
2. **GPT-2** predictability estimates computed by applying the same procedures from our main study to the stimuli used by Brothers and Kuperberg.
3. **GPT-2-region** predictability, i.e., the product of probabilities (sum of surprisals) assigned by GPT-2 to

the entire 3-word critical region analyzed in the SPR experiment. This variant is motivated by concern that responses in the critical region may be driven not only by spillover from the first (critical) word in the region, but also by the surprisals of the remaining words in the region (the critical region is lexically matched within item sets but not across them). Note that this measure only applies to the SPR experiment, since participants in the naming experiment did not hear the critical word or any words that followed it.

Because Brothers and Kuperberg have not as of this writing released code for their GAM models, we approximate their implementation based on the description in their article (our implementation can be found in our public repository: <https://github.com/coryshain/cdr>). We closely reproduce their visualizations (**Figure S5A**).

Second, we perform statistical comparisons using out-of-sample likelihood, as in our main analyses. By contrast, the key comparison in Brothers and Kuperberg (linear vs. logarithmic cloze effects) is based on numerical differences in in-sample likelihood. We use 5-fold cross-validation to estimate an LME on 4 folds of data and use the estimated model to assign a likelihood to the held-out fold. Roughly equally-sized folds were created by cycling stimulus sentences into folds based on modular arithmetic (mod 5) applied to their numerical IDs (these IDs are unrelated to order of presentation, which was randomized). Differences in cross-validated out-of-sample likelihood are permutation-tested for significance, allowing us to perform both non-nested comparisons (e.g., linear vs. logarithmic effects of some predictability estimate) and nested comparisons (e.g., the unique contribution of adding a logarithmic effect to a model containing a linear one).

Following Brothers and Kuperberg, we only include the key predictability variable(s) in each model. Follow-up analyses with additional controls (critical word length, summed critical region word length, critical word unigram surprisal, summed critical region unigram surprisal, critical word position in the sentence, and mean semantic distance of the critical word from content words in the preceding context) had little impact either on visualizations or measures of model fit, and we do not report on them further. All models include random intercepts by participant and item (richer random effects structures—e.g., with random slopes by participant for each predictor—led to frequent convergence errors).

About 15% of the time, the critical word in an item was not produced by any participant in the norming experiment, resulting in a cloze probability of 0. Because the logarithm (and thus, the surprisal) of cloze predictability for these items is undefined, we follow Brothers and Kuperberg in assigning half a response (out of an average of 90 responses per item) to these items, resulting in a minimum cloze predictability of about 0.01 and a maximum cloze surprisal of about 5.2.

1.3. Results.

1.3.1. What Is the Estimated Shape of Predictability Effects? Results from the GAM models are visualized in **Figure S5A**. Estimates using cloze predictability/surprisal (top row of the figure) closely match those reported by Brothers and Kuperberg and support a linear predictability effect: despite the flexibility of GAM regression, models find a clear straight-line effect of cloze probability on response times in both experiments, and, when given surprisal-scale cloze values instead, models find the characteristic “plateau” at high surprisal values, as expected when a linear predictability effect is projected onto a logarithmic scale (**Figure 1**). However, results using model-based predictability estimates (bottom three rows of the figure) show a strikingly different pattern. In the SPR experiment, GPT-2-based surprisal effects are strongly linear (trigram effects are too uncertain to be informative). In the naming experiment, model-based surprisal effects (including from Brothers and Kuperberg’s own trigram measures) are mostly consistent with a linear effect, except with a downward curve at high surprisal values where there are few datapoints (see the superposed density plots of **Figure S5A**); GAM models sometimes show a similar downward curve at high surprisal values in our own data (**Figure S14**). Despite this downward curve, model-based estimates for the naming experiment are inconsistent with a linear predictability effect. For example, the estimated GPT-2 effect is essentially logarithmic (linear in surprisal) over almost 10 orders of magnitude on a natural log scale (from

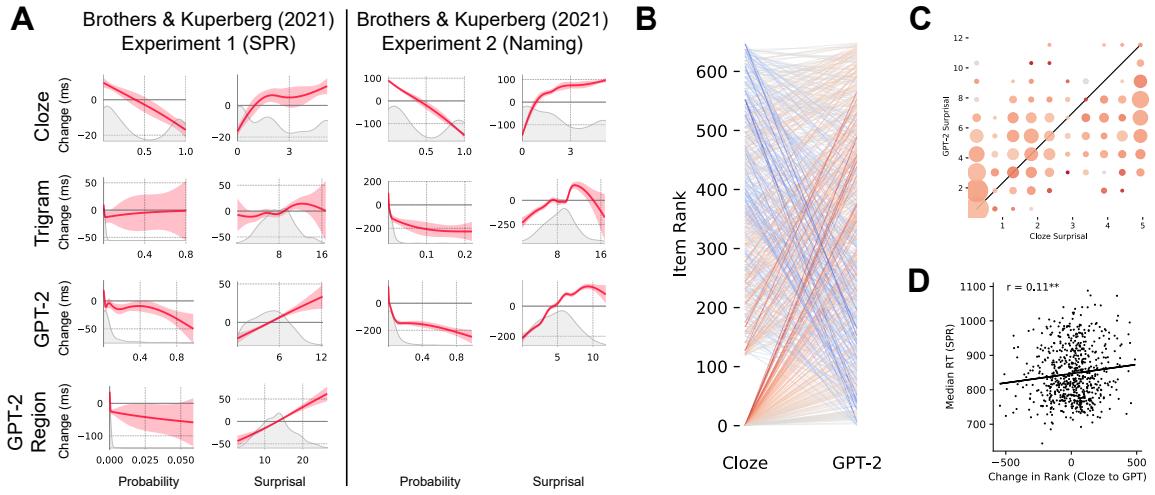


Figure S5: A. GAM-estimated functional form of effects in Brothers' & Kuperberg's (3) Experiment 1 (SPR) and Experiment 2 (cross-modal picture naming) across language model types (human cloze and trigram from the original study, GPT-2 predictability of the critical word, and GPT-2 predictability of the whole 3-word critical region). Plots cover the full empirical range of predictor values in each training dataset. Kernel density plots show the distribution of predictor values in the training data over the plotted range. Uncertainty intervals show the ± 2 standard errors used as a default in plots from `mgcv` (unlike CDRNN plots, which show 95% credible intervals). **B.** Change in rank for Brothers' and Kuperberg's 648 stimuli as a function of cloze probability (left) vs. GPT-2 probability (right). Color reflects the slope of the change in rank (red for positive, blue for negative). **C.** The distribution of items and reading latencies in Brothers and Kuperberg's SPR experiment as a function of cloze surprisal vs. GPT-2 surprisal. Point size represents the number of items that fall into the corresponding region of the cloze surprisal vs. GPT-2 surprisal coordinate space. Point color represents the median RT of those items in the SPR experiment (darker red indexes longer RTs). **D.** Median RT by item in Brothers and Kuperberg's SPR experiment as a function of the item's rank change in surprisal as estimated by GPT-2, relative to cloze.

$p = 1$ to approximately $p = 0.0005$), an interval that contains most of the data. Furthermore, when we examine trigram- and GPT-2-based effects estimated on a probability scale (left column of **Figure S5A**), both experiments show the characteristic spike in estimated processing cost as probability approaches zero from the right, as expected when a logarithmic predictability effect is projected onto a linear scale (**Figure 1**). The upshot of these visualizations is that the disagreement between Brothers and Kuperberg’s findings of linearity and the logarithmic findings of other studies (such as ours) is not due primarily to misalignment between experimental and naturalistic approaches, but instead due primarily to the choice of predictability model: Brothers and Kuperberg’s conclusions from these experiments depend critically on using cloze to represent predictability, whereas model-based analyses of the same data support a logarithmic pattern.

This could be because models underestimate the degree to which high-cloze words are predictable to humans, as Brothers and Kuperberg argue (pp. 5–6). We agree that this is true in some cases, but it is not the whole story, at least for the (full-context) GPT-2 model. *First*, GPT-2 does capture some high-cloze items (as shown by the fact that some GPT-2 probabilities fall near 1). *Second*, cloze may *overestimate* humans’ certainty about future words during real-time comprehension due to task-specific strategic effects (27), such as a prototypicality bias (130), that may suppress variance in (explicit) cloze responses relative to (implicit) expectations during comprehension. *Third*, as shown in **Figure S5B**, GPT-2 does not merely suppress cloze probabilities (which would result in stable item ranks between cloze and GPT-2), nor does it fail to capture human cloze intuitions altogether (Spearman rank correlation between cloze and GPT-2 probabilities for these stimuli is 0.55). Instead, the relationship is complex, with much shared structure between GPT-2 and human cloze, but also qualitative differences *in both directions*, with both (a) many high-cloze items given low probabilities by GPT-2 (consistent with Brothers and Kuperberg’s framing), but also (b) many low-cloze items given high probabilities by GPT-2, as shown by the many red lines representing items with cloze near zero that GPT-2 predicts with high probability. GPT-2’s rank changes confer a modeling benefit relative to cloze, as shown in **Figure S5D**, which plots the median RT by item in Brothers and Kuperberg’s SPR experiment as a function of the item’s rank change in surprisal from cloze to GPT-2. As shown, there is a weak but significant trend whereby items that are ranked as more surprising by GPT-2 relative to cloze also take longer to read. Furthermore, when we bin items by their cloze vs. GPT-2 surprisal values **Figure S5C**, we find that some items in the middle of the cloze surprisal range have both long reading times (dark red color) and high GPT-2 surprisal. This is important because moderate-surprisal items are critical to function estimation: if the predictability model overestimates the predictability of some items, this could push items with longer reading times leftward along the surprisal scale, potentially giving rise to a more plateau-like effect as predicted by the FACILITATION view (**Figure 1**).

Thus, we find evidence that GPT-2 contains potentially important signal about human subjective predictability during incremental language comprehension that may not be fully captured by cloze estimates. In **1.3.3** we present more detailed qualitative analyses of these differences.

1.3.2. Is Processing Difficulty Linear or Logarithmic on Word Predictability in Brothers and Kuperberg’s Experiments? We now evaluate how the patterns visualized in **Figure S5A** are reflected in model fit to unseen response time data, as summarized in **Table S1**. Predictability effects are generally significant over baseline (\emptyset) across experiments and predictability measures, with the key exception in the SPR study of the trigram estimates provided in the data release. In other words, predictability effects are indeed robust in these experiments, but Brothers and Kuperberg’s trigram estimates are a poor proxy for response times (especially in SPR), which perhaps contributed to their failure to explain variance over cloze in Brothers and Kuperberg’s SPR analyses. This limitation is addressed by GPT-2, which provides a strong fit to response times in both experiments. Regarding comparisons between linear and logarithmic variants of the same predictability measures, results replicate Brothers and Kuperberg’s key findings even using our more stringent held-out tests: in both experiments, cloze probability outperforms cloze surprisal, and cloze probability contributes significantly when added to a model containing cloze surprisal alone, whereas cloze surprisal does not contribute significantly when added to a model containing cloze probability alone. Thus, as in the visualizations above, cloze-based results support a linear predictability effect. However, model-based results again show the opposite pattern: across experiments and predictability models, a surprisal scale numerically outper-

Comparison		SPR Experiment		Naming Experiment	
		ΔLL	p	ΔLL	p
Overall	Cloze _{PROB} vs. \emptyset	83	0.0012	1477	0.0006
	Cloze _{SURP¹} vs. \emptyset	62	0.0012	1077	0.0006
	Trigram _{PROB} vs. \emptyset	-4	—	44	0.0006
	Trigram _{SURP¹} vs. \emptyset	2	1.0000	46	0.0038
	GPT-2 _{PROB} vs. \emptyset	37	0.0007	402	0.0005
	GPT-2 _{SURP¹} vs. \emptyset	74	0.0007	836	0.0005
	GPT-2-Region _{PROB} vs. \emptyset	-1	—		
	GPT-2-Region _{SURP¹} vs. \emptyset	73	0.0007		
Probability vs. Surprisal	Cloze _{SURP¹} vs. Cloze _{PROB}	-21	0.0107	-400	0.0011
	Trigram _{SURP¹} vs. Trigram _{PROB}	6	0.1506	2	1.0000
	GPT-2 _{SURP¹} vs. GPT-2 _{PROB}	37	0.0012	434	0.0007
	GPT-2-Region _{SURP¹} vs. GPT-2-Region _{PROB}	74	0.0012		
	Cloze _{PROB} -Cloze _{SURP¹} vs. Cloze _{SURP¹}	20	0.0107	395	0.0007
	Cloze _{PROB} -Cloze _{SURP¹} vs. Cloze _{PROB}	-1	—	-5	—
	Trigram _{PROB} -Trigram _{SURP¹} vs. Trigram _{SURP¹}	-2	—	5	1.0000
	Trigram _{PROB} -Trigram _{SURP¹} vs. Trigram _{PROB}	5	0.6184	7	1.0000
Cloze vs. Other	GPT-2 _{PROB} -GPT-2 _{SURP¹} vs. GPT-2 _{SURP¹}	-2	—	18	0.0600
	GPT-2 _{PROB} -GPT-2 _{SURP¹} vs. GPT-2 _{PROB}	35	0.0012	452	0.0007
	GPT-2-Region _{PROB} +GPT-2-Region _{SURP¹} vs. GPT-2-Region _{SURP¹}	-24	—		
	GPT-2-Region _{PROB} +GPT-2-Region _{SURP¹} vs. GPT-2-Region _{PROB}	50	0.0107		
	Trigram _{SURP¹} vs. Cloze _{PROB}	-81	0.0015	-1431	0.0005
	Cloze _{PROB} +Trigram _{SURP¹} vs. Cloze _{PROB}	-4	—	4	0.2979
	GPT-2 _{SURP¹} vs. Cloze _{PROB}	-9	0.9841	-641	0.0005
	Cloze _{PROB} +GPT-2 _{SURP¹} vs. Cloze _{PROB}	4	0.6076	16	0.0276
	GPT-2-Region _{SURP¹} vs. Cloze _{PROB}	-10	1.0000		
	Cloze _{PROB} +GPT-2-Region _{SURP¹} vs. Cloze _{PROB}	0	1.0000		

Table S1: Testing Results on Data from Brothers' & Kuperberg's (3) Experiment 1 (SPR) and Experiment 2 (cross-modal picture naming). Results of key statistical comparisons using permutation tests of the difference in average cross-validated log likelihood (ΔLL) between linear mixed effects (LME) models that assume some predictability estimate and functional form (PROB is linear in probability and SURP¹ is linear in surprisal). **Boldface** indicates statistical significance. In directional tests where one hypothesis subsumes the other (e.g., PROB vs. \emptyset , since a nonlinear surprisal effect subsumes a linear one), dashes (—) indicate failure of the alternative hypothesis (left) to improve over the null hypothesis (right). Comparisons involving GPT-2-Region are left empty in the column for the naming experiment, where this measure is not relevant. Otherwise, cells are color coded using **cyan** to indicate that the hypothesis on the left outperforms the one on the right, and using **magenta** to indicate that the hypothesis on the right outperforms the one on the left. All p -values are corrected for false discovery rate (49) across all tests within family of tests (delimited by single horizontal lines) for each experiment. Key comparisons are highlighted.

Rank change	Critical word	Stimulus
-545	leaves	Bill raked up all the leaves in front of his condo.
-511	spider	The web had been spun by the large spider on our porch.
-500	ring	Before proposing to his girlfriend he would need a ring with a big diamond.
-493	match	The gentleman lit the candle using a single match which was surprising.
-443	star	The hopeful girl wished upon a star up in the sky.
-441	beak	My son saw the bird pecking at the soil using its beak and claws.
-437	yearbook	To record some high school memories, he had everyone sign his yearbook after graduation.
-425	grace	Before eating dinner, the family held hands to say grace around the table.
-418	elves	In Santa's workshop, the toys are made by the elves on Christmas Eve.
-413	frame	To display the beautiful photo, she ordered an elegant frame made of silver.
486	bloom	We started watering the yard because the plants had begun to bloom out back.
456	wood	At the ancient burial site they found weapons made of wood with smooth edges.
449	narrow	They decided not to go rafting because the river was too narrow and cramped.
446	island	The rescue workers just learned that the captain had died on the island in Bermuda.
437	skin	The factory produces noxious chemicals that can damage your skin quite badly.
437	leather	For her project, she bought lots of black leather and tough fabric.
422	bed	There was a large pile of dirty towels on the bed in the guestroom.
384	quiet	The neighborhood at the top of the tall hill was very quiet and peaceful.
375	attic	The accountant put all of the old documents in the attic at home.
373	chess	They were learning another important rule of chess at Victor's house.

Table S2: Top 10 items from Brothers and Kuperberg (3) with the largest rank decrease in probability from cloze to GPT-2 (top) and top 10 items with the largest rank increase in probability from cloze to GPT-2 (bottom).

forms a probability scale, significantly so in GPT-2-based comparisons. Moreover, in GPT-2-based models for both experiments, surprisal contributes significantly when added to a model containing probability alone, but probability does not contribute significantly when added to a model containing surprisal alone.

Which of these patterns is to be believed? Perhaps Brothers and Kuperberg are right that cloze is a better approximation of human subjective predictability than corpus estimates, and therefore that results based on cloze should be trusted more. The SPR results undermine this position: although cloze probability outperforms trigram surprisal as in Brothers and Kuperberg’s own analyses, there is no significant difference in fit between cloze probability and either GPT-2 or GPT-2-Region. By contrast, cloze probability significantly outperforms GPT-2 surprisal on the naming data. However, picture naming is an offline task that is considerably more similar to a cloze task than it is to word-by-word reading as reflected in SPR and the experiments analyzed in our main study. It is thus perhaps not surprising that cloze provides a better fit to such data, and the relevance of picture naming latencies to online language processing is less direct than SPR or eye-tracking. In light of other findings (including our own, but see also 54, 55) that cloze sometimes provides a poorer fit to human data than recent language models like GPT-2, as well as arguments that there may be important differences in the cognitive mechanisms that underlie the cloze task vs. moment-by-moment language processing (52), the ambivalent results shown in **Table S1** do not convincingly support the notion that cloze is a systematically more reliable proxy (vs. statistical models) for human subjective predictability during real-time language comprehension.

1.3.3. How Do Cloze-Based and Model-Based Predictability Estimates Differ? Given these divergent results using cloze-based vs. model-based predictability, we now ask whether the two predictability estimates differ in systematic ways. The kernel densities shown in **Figure S5A** shed some light on this question. As shown, the cloze probability/surprisal values for items in these studies tend to fall (by design) into a roughly trimodal distribution corresponding to the high-, moderate-, and low-cloze conditions (the moderate- and low-cloze conditions collapse in probability space to a single large mode in these visualizations), with substantial spread of empirical support across the full spectrum of attested cloze values. Thus, according to the cloze estimates, Brothers and Kuperberg’s experimental manipulation successfully varies predictability relatively evenly. The model-based probability/surprisal values are distributed differently: they are strongly concentrated toward low values in probability space and moderate values in surprisal space, with little representation of the extremes of the surprisal continuum.

Critical word: leaves Cloze probability: 0.93333 GPT-2 probability: 0.00002 Prefix: Bill raked up all the	Critical word: break Cloze probability: 0.98876 GPT-2 probability: 0.00241 Prefix: My son saw the bird pecking at the soil using its
Top-10 GPT-2 predictions: Token p fuss 0.0373 money 0.0231 headlines 0.0208 talk 0.0131 right 0.0121 wrong 0.0093 hype 0.0093 way 0.0085 controversy 0.0082 media 0.0080	Top-10 GPT-2 predictions: Token p tail 0.0543 claws 0.0389 wings 0.0365 tal 0.0279 nose 0.0277 long 0.0277 mouth 0.0234 own 0.0184 tiny 0.0178 eyes 0.0143
Critical word: spider Cloze probability: 0.93333 GPT-2 probability: 0.00020 Prefix: The web had been spun by the large	Critical word: yearbook Cloze probability: 0.63516 GPT-2 probability: 0.00014 Prefix: To record some high school memories, he had everyone sign his
Top-10 GPT-2 predictions: Token p - 0.0686 media 0.0462 corporations 0.0352 multinational 0.0268 - 0.0238 and 0.0222 companies 0.0194 tech 0.0179 number 0.0128 internet 0.0108	Top-10 GPT-2 predictions: Token p name 0.3214 own 0.0264 aut 0.0246 letter 0.0176 first 0.0134 papers 0.0107 diploma 0.0099 book 0.0077 daughter 0.0070
Critical word: ring Cloze probability: 0.93333 GPT-2 probability: 0.00026 Prefix: Bill proposing to his girlfriend he would need a	Critical word: grace Cloze probability: 0.68539 GPT-2 probability: 0.00004 Prefix: Before eating dinner, the family held hands to say
Top-10 GPT-2 predictions: Token p permit 0.0300 driver 0.0197 lawyer 0.0178 passport 0.0158 doctor 0.0145 - 0.0142 car 0.0121 job 0.0115 lot 0.0109 divorce 0.0105	Top-10 GPT-2 predictions: Token p goodbye 0.3330 hello 0.1649 - 0.0809 - 0.0710 thank 0.0512 farewell 0.0404 thanks 0.0368 good 0.0353 - 0.0146 a 0.0116
Critical word: match Cloze probability: 1.00000 GPT-2 probability: 0.00139 Prefix: The gentleman lit the candle using a single	Critical word: elves Cloze probability: 1.00000 GPT-2 probability: 0.00326 Prefix: In Santa's workshop, the toys are made by the
Top-10 GPT-2 predictions: Token p candle 0.0740 hand 0.0556 - 0.0316 finger 0.0281 piece 0.0228 strand 0.0217 screw 0.0166 - 0.0149 brush 0.0137 stick 0.0132	Top-10 GPT-2 predictions: Token p same 0.0699 kids 0.0187 children 0.0179 company 0.0164 team 0.0159 Santa 0.0131 arts 0.0125 local 0.0122 crafts 0.0122 people 0.0101
Critical word: star Cloze probability: 0.96000 GPT-2 probability: 0.00164 Prefix: The hopeful girl wished upon a	Critical word: frame Cloze probability: 0.93333 GPT-2 probability: 0.00118 Prefix: To display the beautiful photo, she ordered an elegant
Top-10 GPT-2 predictions: Token p man 0.0452 young 0.0186 stranger 0.0161 friend 0.0142 boy 0.0107 bright 0.0105 white 0.0092 god 0.0090 father 0.0089 girl 0.0080	Top-10 GPT-2 predictions: Token p - 0.0438 dress 0.0297 black 0.0294 and 0.0280 white 0.0267 set 0.0153 red 0.0149 silver 0.0111 pair 0.0088 gown 0.0084

Table S3: Top-10 next-token predictions from GPT-2 for Brothers and Kuperberg’s (3) items with the largest rank decrease from cloze to GPT-2 probability. GPT-2 uses subword tokenization, which means that candidate outputs include punctuation, white space, control characters, and partial words.

To build qualitative intuitions about these differences, in **Table S2** we present the top 10 items with the largest rank decrease in probability from cloze to GPT-2 as well as the top 10 items with the largest rank increase in probability from cloze to GPT-2. The rank decrease items (high-cloze items that were assigned low probability by GPT-2) indeed seem like failure cases, i.e., items with a strong next-word candidate that the model should have anticipated but did not, perhaps due to failures of idiomatic knowledge (e.g., “say grace”) or world knowledge (e.g., that “proposing” in this context refers to a romantic gesture involving a ring) that humans generally have. To better understand what might be happening in these cases, in **Table S3** we present the top 10 highest probability continuations for each of these items according to GPT-2, with their associated probabilities. As shown, the model assigns implausibly low probability to the high-cloze continuation, and many of the top 10 predictions are nonsensical (e.g., *Bill raked up all the headlines*) or demonstrate misunderstanding (e.g., *To record some high school memories, he had everyone sign his name*, which suggests failure to understand the fact that one does not normally sign someone else’s name).

Nevertheless, even in these failure cases, the model shows sensitivity to phenomena that are unlikely to be produced in a cloze task but are plausible in ordinary texts and thus might register in human expectations during reading. These include:

- **Coordination.** E.g.:

- *The web had been spun by the large **and***
- *To display the beautiful photo, she ordered an elegant,* (here, the generated comma plausibly delimits coordinated adjectives, suggesting that the model may be entertaining this possibility)
- **Modifiers.** E.g.:
 - *My son saw the bird pecking at the soil using its **own***
 - *In Santa’s workshop, the toys are made by the **same***
- **Metaphors.** E.g.:
 - *The web had been spun by the large **multinational***
- **Effects of extrasentential context.** GPT-2 often receives truncated text in training and must contend with the possibility that relevant context is missing to an extent that human cloze participants may not (since they are aware that they are study participants and the sentences have no context). E.g.:
 - *Before proposing to his girlfriend he would need a **passport*** (e.g., perhaps the partners live in different countries)

These examples illustrate the diversity of linguistic inputs that a general-purpose sentence comprehension system must contend with during typical reading, diversity that models must also contend with in training but that cloze productions may not represent well. In light of this diversity, distributing probability mass across many such continuations, even for “high cloze” experimental items, may often be optimal, both for machines and plausibly human readers. Greater tolerance of diverse outcomes could even underlie the cases of rank *increase* (i.e., prescience) in **Table S2**, where GPT-2 assigns unexpectedly high probability to low-cloze but semantically plausible continuations (although these cases could also of course arise by chance). In any case, the results above suggest that model-based estimates are not only higher resolution than cloze (as discussed in the main article), but also qualitatively different due not only to their weaknesses but also plausibly to their strengths relative to cloze norms as models of human prediction during language comprehension.

1.4. Discussion. Here we revisited the data and arguments presented in Brothers and Kuperberg (3), the strongest extant evidence favoring linear over logarithmic predictability effects during incremental language comprehension. We showed that both of their key experimental findings are contingent on analysis choices, namely, the use of cloze norms rather than statistical language models to represent predictability. When their experimental data are reanalyzed using statistical language models, results favor logarithmic over linear predictability effects. Thus, even Brothers and Kuperberg’s controlled experiment replicates our own claimed logarithmic predictability effect when analyzed using methods more similar to those we have advocated. Although direct support is infeasible due to limitations of the public data release, the same discrepancy might also hold in Brothers and Kuperberg’s meta-analysis, which is based exclusively on cloze estimates. We further showed no statistical difference in fit to reading times between cloze probability and GPT-2 surprisal in Brothers and Kuperberg’s SPR experiment, which is the experimental task that most closely mimics incremental sentence comprehension (Brothers and Kuperberg’s other experiment is picture naming, which is an explicit, offline, cloze-like task that differs in potentially critical ways from incremental sentence comprehension). The ambivalence of this statistical outcome undermines a key pillar supporting Brothers and Kuperberg’s interpretation of their data, which turns on the use of cloze. This concern is reinforced by our main finding from the Provo corpus that cloze estimates are neither linear in predictability nor strong psychometrically—they are significantly outperformed as reading models by GPT-2. Other recent studies have reached a similar conclusion (54, 55). Finally, we qualitatively analyzed the items with the strongest discrepancy between cloze and GPT-2 predictability and identified not only clear failure cases in which GPT-2 assigns implausibly low predictability to high-cloze items, but also plausible advantages of model-based estimates in representing the full diversity of continuations in ordinary texts, diversity which is likely also relevant to human expectations during sentence comprehension. In light of these considerations, we do not

believe there is strong reason to favor linear cloze-based over logarithmic model-based interpretations of Brothers and Kuperberg’s data, and thus we do not consider their study to be strong counterevidence to our claims that predictability effects are more logarithmic than linear.

Although we have argued that the linear predictability effects reported in Brothers and Kuperberg’s study are due primarily to analysis choices rather than the use of experimental vs. observational data (and that under different, similarly justifiable analyses our two studies in fact agree), we would additionally like to address their argument (elaborated in their Appendix B) that correlational studies like ours are inherently less reliable than experimental ones like theirs. Their argument (and those they cite in its defense, namely 126, 127) invokes an uncontroversial principle of scientific inference: correlation does not imply causation, and thus well-designed experimental studies (e.g., randomized controlled trials) are strictly more informative about causation than comparable correlational studies, assuming that a pure manipulation is feasible. They demonstrated the practical significance of this issue for the study of predictability effects with an elegant proof-of-concept analysis: they replaced the dependent variable in their SPR experiment (reading time) with mean lexical decision times of the critical words in an independent experiment (131). These lexical decision times can have no causal relation to the trigram predictability of critical words in Brothers and Kuperberg’s experiment, since the lexical decision experiment used different stimuli in a different task; nonetheless, they found a significant effect of trigram log probability (over and above unigram log probability, a more straightforwardly causal influence on lexical decision times with which trigram probability covaries) on lexical decision times in a linear model. They attribute this to *residual confounding*, i.e., systematic covariation between a non-causal variable and *measurement error* of a causal variable, leading to a spurious effect. Mitigating such issues can require deep domain knowledge, but even with such knowledge, it is often impossible to completely eliminate the possibility of systematic uncontrolled confounds that could bias inferences. In Brothers and Kuperberg’s view, naturalistic studies’ systematic tendency to find logarithmic predictability effects is driven by residual confounding with word frequency, which is known to have a logarithmic effect on response times.

However, there is a less nefarious issue that can produce a similar result: correlation between the non-causal variable and random noise in a specific experimental sample. Although standard tests account for variance in the effect estimate, the fact remains that the degree of overfitting of an arbitrary model to an arbitrary finite dataset cannot be known in advance. Fortunately, this issue can be straightforwardly addressed using out-of-sample testing procedures like those we have used throughout this study. And it turns out that these procedures fix the spurious trigram finding above: when we replace Brothers and Kuperberg’s in-sample test with the 5-fold cross-validation procedure used in our analyses of their SPR and naming experiments, unigram log probability remains significant over trigram log probability as a predictor of lexical decision times ($p < 0.001$) but trigram log probability is not significant over unigram log probability ($p = 0.309$). This outcome does not invalidate Brothers and Kuperberg’s general concern: we agree that correlation does not imply causation and that confounds can influence results, including in studies of word predictability. Our point is that the practical significance of this concern can be reduced by careful analysis design.

In addition, we see little external support for Brothers and Kuperberg’s position that apparent logarithmicity in the predictability effects found by other studies is driven by poor control of frequency effects. Not only do the same logarithmic patterns emerge in their own data when reanalyzed with model-based predictability estimates as shown above (Brothers and Kuperberg’s design holds critical words—and thus, frequencies—constant while varying predictability within each item set, leaving no frequency-related variance within a set for GPT-2 surprisal to correlate with), but naturalistic studies regularly include statistical frequency controls with diverse (and increasingly accurate) implementations. For example, our own study estimates frequency from the 3.5-billion word Gigaword 3 corpus (132), and a related study (92) used frequency estimates from the Open Web Text corpus (133), which is about an order of magnitude larger than Gigaword, to directly compare log frequency and GPT-2 surprisal effects, finding additive effects of similar magnitude for both variables across diverse naturalistic datasets (see also 106, for review of convergent findings from experimental studies). Other studies reporting logarithmic effects in English have also used strong frequency controls derived from other text corpora (14, 25), and a recent study showed strongly logarithmic predictability effects in naturalistic reading data from English and 10 other languages under large-scale frequency controls (15).

Although Brothers and Kuperberg's claim cannot be directly falsified, we find it implausible that errors in estimating human subjective frequency are so systematically and similarly correlated with surprisal across such diverse training corpora, languages, and statistical language models that they spuriously drive the large and growing body of findings that favor logarithmic over linear predictability effects.

Moreover, it is not obvious that the correlation-causation problem applies uniquely or even especially to naturalistic studies relative to experimental studies for the domain of language research, because a pure manipulation of the theoretically relevant variable (e.g., predictability) is usually not possible. The manipulation usually must be carried out as a manipulation of *language* (e.g., words, morphemes, or syntactic structures), and linguistic units are complex objects with myriad poorly understood relationships not only to diverse and often collinear linguistic variables (frequency, predictability, syntactic category, age-of-acquisition, orthographic neighborhood, lexical semantics, etc.) but also to sensory systems by which words and their referents are perceived, to conceptual systems by which words and sentences interface with knowledge representations, and to executive systems that use these knowledge representations in order to control behavior. A linguistic manipulation therefore plucks at a potentially vast web of interrelated cognitive phenomena. Brothers and Kuperberg's own lexical decision demonstration illustrates the generality of this issue: even controlled studies of lexical decision times must contend with the possibility that the critical variable may covary with errors in any attempts at experimental or statistical control, potentially giving rise to spurious results. These potential confounds must be anticipated and addressed in experimental studies, just as they must be in naturalistic ones. In psycholinguistics, the type of a study (experimental or correlational) may be less informative about its reliability for causal inference than the quality of its experimental and statistical design.

More broadly, Brothers and Kuperberg's discussion of the relative strengths of experimental and naturalistic studies in psycholinguistics may be somewhat selective. Their discussion does not acknowledge known inferential challenges for experimental studies in cognitive science and psycholinguistics that have been treated at length elsewhere (29–31, 134–137), especially item and task effects. In brief, although an experimental study licenses narrow causal inferences about a specific set of items presented in a specific task (e.g., specific high and low cloze items in a picture naming task), it does not necessarily license broad causal inferences about the theoretical construct of interest (e.g., prediction during incremental sentence processing), in part because of the complexity of language itself (item effects, see the preceding paragraph) and in part because humans adapt flexibly to the experimental setting, and in so doing may draw on novel or heuristic task-specific processing strategies (task effects). In other words, it may matter how closely a given language processing experiment resembles things that people do with language in the real world. People regularly read connected texts for content, but they rarely read dozens of isolated sentences for a comprehension assessment, and the cognitive strategies they use in the latter setting may differ qualitatively from those they use in the former. This concern is reinforced by neuroscientific evidence that domain-general executive regions of the brain, which are largely dormant during passive language comprehension, come online strongly when language comprehension is layered with other tasks (138). Systematic differences with respect to which regions of the brain are engaged between an experiment (a task) and an inferential target (the human language comprehension system) could give rise to systematic differences in measurable responses (e.g., reading behavior). There is thus reason to suspect that experimental studies may be prone to task effects in ways that naturalistic studies plausibly are not (31). This is not to say that experimental studies are uninformative (members of our author team also do experimental work), but rather to establish that experimental studies of language processing also have inferential limitations that are distinct from but perhaps equally serious to those faced by naturalistic studies.

In other words, experimental and naturalistic studies of human language processing have largely complementary strengths and weaknesses. Experimental studies support strong (narrow) causal inferences but may have poor ecological validity (relevance to language processing in general), whereas naturalistic studies have strong ecological validity but only support causal inferences indirectly and in proportion to the degree to which their statistical design rules out other causal factors. Moreover, each approach is more naturally suited to some questions than others. Many experiments' scientific value lies precisely in their *unnaturalness*: deliberately implausible designs can be used to test theoretical predictions about events that rarely or never occur in the real world (139). Experimental studies are well suited to address such questions, and naturalistic

studies are not. However, for questions about the normal response of a system to events that regularly occur (e.g., word predictability effects in reading), naturalistic studies can offer substantial advantages in terms of power and ecological validity, and the causality gap with experimental studies can be narrowed using careful statistical design (31). We therefore agree with the spirit of Brothers and Kuperberg’s position (citing 126) that “claims made from regression analysis techniques should not be accepted until confirmed via controlled experimental techniques” (p. 9). But we also think that a balanced perspective acknowledges the converse, at least for questions that are amenable to naturalistic investigation. More generally, we think psycholinguistic science would benefit from an emphasis on both establishing consensus around convergent findings and understanding divergent findings across well-designed studies that differ in methodological details, including along the experimental-naturalistic axis. We believe such a consensus is emerging around the functional form of predictability effects on incremental processing demand.

2. Analysis of Word Skipping Behavior

Our study targets the theoretical construct of language *processing difficulty*, for which we (following much prior work) take reading times to be a reliable experimental proxy. Our main analyses therefore focus exclusively on measures of reading time. We have thus far not considered a related phenomenon from the literature on eye-tracking during reading: *word skipping*, when the eyes entirely skip (do not land on) a word during the first pass through a text (108, 140–143).

There are three reasons why word skipping behavior is not included in our main analyses. *First*, skipping words is not possible in self-paced experimental paradigms such as those used for the Brown, Natural Stories SPR, and Natural Stories Maze datasets. Thus, skipping is only relevant to a subset of the data we wish to analyze. *Second*, the predictions of theories of processing difficulty are less straightforward *a priori* for skipping probabilities than they are for reading times. For example, under all theories considered here, predictability effects are thought to derive from the degree of match between predicted and observed words. But what effect might predictability have on the decision to observe (i.e., fixate) a word in the first place? How to extend theories of processing difficulty to such questions is not immediately clear (see below for discussion). *Third*, and perhaps consequently, prior work on the functional form of predictability effects in reading has focused heavily on measures of reading time (1, 3, 14, 25), and we have chosen to follow this precedent. Nonetheless, motivated by prior work that found predictability effects on word skipping (e.g., 46, 51, 142, 144), here we report supplementary analyses of the effect of predictability on word skipping in our three eye-tracking datasets (the Dundee, GECO, and Provo datasets).

2.1. Methods. Our analyses of word skipping use the same CDRNN design for each dataset as our analyses of fixation duration, with the following changes:

- Words that were skipped during first-pass reading are added as events to the predictor and response matrices of the CDRNN models (by contrast, our main analyses only consider fixated words).
- The response variable is a boolean indicator for whether a word was skipped during first-pass reading (words that were initially skipped and subsequently fixated during a regressive eye movement are treated as skipped). The response is modeled as binomially distributed rather than exGaussian. The IRF thus describes the expected change in the logit (log-odds) of first-pass skipping as a function of continuous delay in time.
- The *saccade length* predictor (i.e., the length in words of the incoming saccade) is removed from all models. This is because *saccade length* is not well defined for skipped words, which (by virtue of not having been fixated) have no incoming saccade. Although in principle this could be addressed by setting *saccade length* to 0 for skipped words, *saccade length* = 0 would then be a near-perfect decision rule for whether a word was skipped (the only other fixations with this value are the first fixation in a text). This decision rule is non-linear but learnable by a neural network (e.g., a CDRNN). Thus, including *saccade length* in the model could trivialize the regression problem by providing a

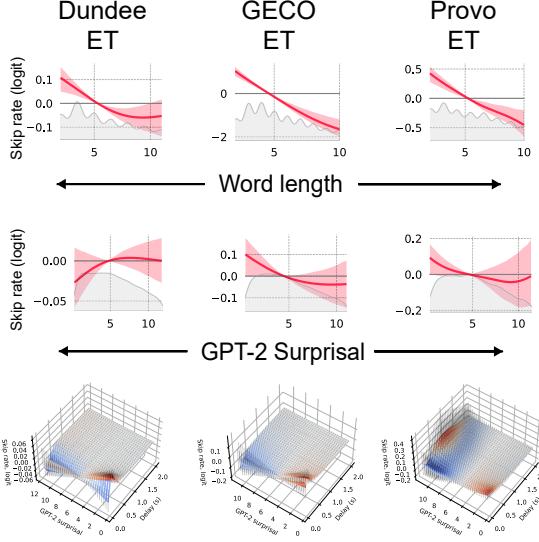


Figure S6: CDRNN-based analysis of word skipping in eye-tracking datasets (Dundee, GECO, and Provo datasets). **Top:** Functional form of word length (x -axis) effects on the log-odds of skipping (y -axis) with no delay (i.e. at the current word). **Middle:** Functional form of GPT-2 surprisal (x -axis) effects on the log-odds of skipping (y -axis) with no delay (i.e. at the surprising word). **Bottom:** Surface plots showing the estimated change in the log-odds of skipping (vertical axis) from observing a word with a given surprisal value (left axis) at a given delay (in seconds) from word onset (right axis). All plots cover the interdecile range of predictor values in each training dataset. Uncertainty intervals represent 95% variational Bayesian credible intervals. Kernel density plots show the distribution of predictor values in the training data over the plotted range. Word length densities appear to oscillate because word length is integer-valued (number of characters).

single covariate with near-perfect separation of the target classes (skipped vs. fixated), with potentially harmful downstream consequences for estimates of our effect of interest (predictability).

One challenge for modeling word skipping in our continuous-time deconvolutional framework is the necessity of timestamping skipped words, which are in a sense non-events since they are never fixated. Whereas fixations are naturally timestamped by their onsets (as in our main analyses), what timestamps should be assigned to unfixated words? Here we adopt the approach of assigning the timestamp of the *immediately following* first-pass fixation to all intervening skipped words, since this is the time point at which the skipped words first *could* have been fixated (but in fact were not). This timestamp is then used to determine the delay between responses (fixated or skipped words) and predictors (preceding fixated or skipped words up to and including the current response), which in turn parameterizes the IRF in the CDRNN model. For simplicity, we only consider predictability according to the best-performing language model from the reading time analyses (GPT-2-small) in three functional forms: $f(\text{SURP})$, PROB, and SURP¹.

2.2. Results. CDRNN-estimated predictability effects on word skipping are visualized in **Figure S6** (middle and bottom rows), along with word length effects for reference (top row) given the well-established tendency for shorter words to be skipped more frequently (141, 142, 145, 146). As shown, CDRNN analyses find the expected decrease in skipping probability for longer words, suggesting that our logistic CDRNN models are sensitive enough to recover a known effect. Estimated predictability effects on word skipping are substantially weaker. As shown in the middle row of **Figure S6**, the Dundee dataset shows no visible relationship between a word’s predictability and its probability of being skipped, whereas the GECO and Provo datasets show small estimated increases in skipping rate for less surprising words. The surface plots on the bottom row of **Figure S6** suggest that the peak predictability effect may in fact be delayed in time: although the predictability of a word has a weak association with its own skipping probability (at delay 0s), it has a stronger association with the skipping probability of *subsequent* words within a short time window (delays

Comparison	Dundee		GECO		Provo		
	ΔLL	p	ΔLL	p	ΔLL	p	
GPT-2	GPT-2 _{$f(SURP)$} vs. \emptyset	18	0.0005	49	0.0004	496	0.0002
	GPT-2 _{PROB} vs. \emptyset	17	0.0005	-7	—	586	0.0002
	GPT-2 _{SURP¹} vs. \emptyset	12	0.0005	35	0.0004	301	0.0002
	GPT-2 _{$f(SURP)$} vs. GPT-2 _{PROB}	0	1.0000	56	0.0004	-90	—
	GPT-2 _{$f(SURP)$} vs. GPT-2 _{SURP¹}	6	0.0979	14	0.0247	195	0.0002
	GPT-2 _{SURP¹} vs. GPT-2 _{PROB}	-5	0.0979	42	0.0004	-285	0.0002

Table S4: Testing Results for Word Skipping on Eye-Tracking Datasets. Results of key statistical comparisons between models of word skipping based on permutation tests of the difference in average test set likelihood between the alternative and null CDRNN model in each comparison (ΔLL). Subscripts indicate the assumed functional form of predictability effects: $f(SURP)$ (nonlinear in surprisal), and PROB (linear in probability). **Boldface** indicates statistical significance. In directional tests where one hypothesis subsumes the other (e.g., $f(SURP)$ vs. SURP¹, since a nonlinear surprisal effect subsumes a linear one), dashes (—) indicate failure of the alternative hypothesis (left) to improve over the null hypothesis (right). Otherwise, cells are color coded using **cyan** to indicate that the hypothesis on the left outperforms the one on the right, and using **magenta** to indicate that the hypothesis on the right outperforms the one on the left. All p -values are corrected for false discovery rate (49) across all tests within each dataset.

between 0s and about 0.3s).

Model comparisons on the held-out test set (Table S4) support the generalizability of these effects. In 8 of 9 comparisons, models of word skipping that contain a predictability effect ($f(SURP)$, PROB, and SURP¹) significantly outperform those that do not (\emptyset); the only exception is the PROB model on the GECO dataset. Thus, our analyses support prior claims that predictability is associated with word skipping (46). However, as indicated by the delayed effects shown in Figure S6, the bulk of these effects appear to be effects of the predictability of *prior* words on the skipping probability of the current word, with considerably weaker evidence that a word’s predictability affects its own skipping probability. We leave further investigation of this finding to future research.

Results regarding the functional form relating predictability to skipping probability are less clear. The $f(SURP)$ model finds a roughly linear relationship between surprisal and the log-odds of skipping, especially at the peak predictability effect (bottom row of SI S4), but the $f(SURP)$ model outperforms the more constrained PROB and SURP¹ models in half of the comparisons, suggesting that the optimal form for the predictability-cost relationship may be other than strictly linear (PROB) or strictly logarithmic (SURP¹). Direct comparisons between the PROB and SURP¹ models are also equivocal: neither model is significant over the other in the Dundee dataset, the SURP¹ model is significant over the PROB model in the GECO dataset, and the PROB model is significant over the SURP¹ model in the Provo dataset. Thus, results do little to clarify the optimal form of the function relating word predictability to the log-odds of skipping, and we leave further investigation of this question to future research.

2.3. Discussion. Using CDRNN analyses of word skipping in our three eye-tracking datasets, we showed a significant effect of predictability on skipping that is relatively weak on the (un)predictable word itself but stronger on subsequent words that follow closely in time. Statistical comparisons between linear and logarithmic predictability effects were inconclusive, with some evidence that neither of these assumed forms may be optimal for word skipping (since the unconstrained $f(SURP)$ model often outperformed them). What do these word skipping results reveal about our core scientific question: the influence of predictability on processing demand? The answer depends on why there is a relationship between word predictability and word skipping behavior in the first place. Prior theorizing about this relationship has focused on one of two potential causes. The first potential cause is parafoveal preview (46). Words that are not yet in foveal attention are still processed to some extent (147), as revealed e.g., by studies that show effects of preview validity (99, 148–150), by studies that show indirect effects of skipped words (151), and by studies that show effects of the content of words in the parafovea on reading behavior (152, though see 129). Parafoveal preview may provide enough information to partially (dis)confirm predictions, which may in turn allow predictability to influence

whether readers skip parafoveally accessed words. Assuming a preview-based mechanism, both the FACILITATION and COST views are consistent with an effect of predictability on skipping probability: when the parafovea provides enough information to determine that a subsequent word is strongly activated by context (FACILITATION view) or contributes little information (COST view), the reader may choose to skip it. With respect to the finer-grained question about the quantitative form of the relationship between predictability and skipping probability, additional theoretical commitments are needed as to precisely how the processing costs of (parafoveally accessed) words affect the skipping decision. In particular, although processing costs are in principle unbounded, skipping probability is constrained to the interval $[0, 1]$ and therefore cannot scale logarithmically on predictability as a matter of mathematical definition. The expected functional form will therefore depend on the choice of nonlinearity (e.g., log, logit) used to map between processing costs and skipping probabilities. This choice in turn depends on principles of elegance or mechanism that must be expounded by theorists. To our knowledge, this is uncharted theoretical territory, and developing such a theory is beyond the scope of our study.

The second potential cause is partial collinearity between predictability and *entropy*, an information-theoretic measure of the degree of spread of the predictive distribution over words given context (143). Lower entropy indexes greater concentration of probability mass on a smaller number of outcomes. Entropy is thus a formalization of the experimental construct of *contextual constraint* and plausibly influences the skipping decision: skipping may be more likely when there is greater certainty (lower entropy) in the distribution over the upcoming word. Entropy in natural texts is positively correlated with surprisal (153); that is, when language models are more certain about the identity of a word (prior to observing it), they tend to be less surprised by the word (after observing it). Thus, an association between predictability and skipping rate could emerge indirectly by correlation with entropy. Indeed, one recent study found no effect of surprisal on skipping rates once entropy was taken into account (87). Assuming an entropy-based mechanism (i.e., disregarding parafoveal preview), neither the FACILITATION nor the COST views predicts an entropy-independent predictability effect: without access to the identity of the predicted word, the degree of prediction (mis)match cannot be computed, nor used to guide the skipping decision.

In summary, word skipping behavior does not to our knowledge differentiate the classes of theories at issue in our study. Both the FACILITATION and COST views make broadly similar predictions about skipping behavior (predictability effects on skipping under a preview-based mechanism, no unique predictability effects on skipping under an entropy-based mechanism) without clear differences in their commitments as to the functional form relating predictability to skipping probability (under a preview-based mechanisms). Our findings support the existence of a predictability effect on skipping probability but locate this predictability effect primarily on the skipping rate of *subsequent words*, an outcome that does not appear to be anticipated by either theory. Thus, these word skipping analyses are largely orthogonal to our core claims about predictability and processing demand. They primarily serve to explore a dimension (skipping rate) of the predictability-cost relationship which is under-studied in naturalistic settings, and which warrants further theoretical analysis.

3. Visualization of Word Length and Frequency Effects

Although the primary construct of interest in our study is word predictability, reading times are also known to be influenced by other factors, especially word length (154) and frequency (155). For this reason, our own statistical models include length and unigram surprisal (negative normalized log frequency) as control covariates (see SI 10). To establish continuity with prior work, in this section we visualize the estimates for these length and frequency predictors in models that simultaneously estimate effects of GPT-2 surprisal (along with other control predictors). Results are shown in Figure S7. As expected, reading times increase across datasets for both longer words and less frequent words (higher unigram surprisal). These effects are similar in size to those obtained for our critical surprisal measures (Figure S13). Thus, our CDRNN-based analyses recover other known signatures of processing difficulty, beyond predictability effects.

Word length, frequency, and predictability covary in language, and the relationship (and degree of separability) between these variables is of central theoretical concern; for example, the degree to which frequency

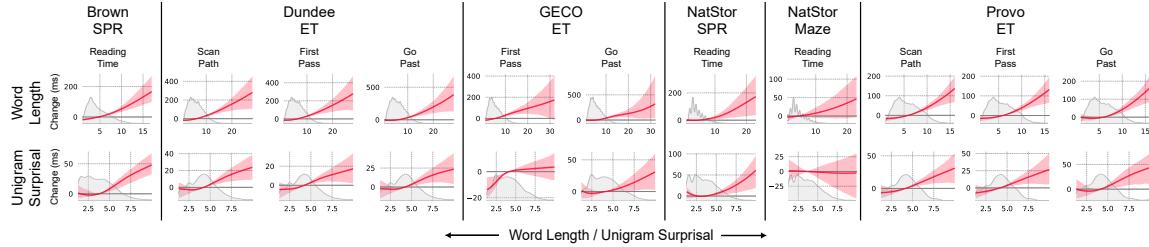


Figure S7: CDRNN-estimated functional form of word length and unigram surprisal (negative log frequency) effects with no delay (i.e. at the current word). Plots cover the full empirical range of predictor values in each training dataset. Kernel density plots show the distribution of predictor values in the training data over the plotted range.

and predictability effects are dissociable has been tied to theoretical debates about the nature of sentence comprehension that are closely related to the FACILITATION and COST views targeted by our study (106). Full investigation of these relationships is beyond the scope of this work, but a recent study used similar methods (CDRNN analyses of naturalistic reading) to investigate the relationship between frequency and predictability effects and found evidence supporting an additive dissociation between frequency and predictability in naturalistic reading (92), as has been claimed based on experimental designs (106, 156). This outcome suggests bounds on the role of probabilistic inference in sentence interpretation by pointing to the existence of inference-independent lexical retrieval difficulty (106), although inference-based explanations for this finding may exist (see 92).

4. Full Significance Testing Results

Tables S5–S9 provide the full results of all statistical tests conducted in this study. Figure S8 visualizes the results of key comparisons between hypothesized forms for the predictability-cost relationship, broken out by individual models and datasets. The bottom right subplot is identical to Figure 4 of the main article, which shows results of tests that aggregate across all datasets and language models.



Figure S8: Performance comparison across datasets and language models between pairs of assumed forms for the effect of word predictability. For a given pair, cyan indicates that the model on the row has significantly better test set performance than the model on the column, magenta indicates that the model on the column significantly outperforms the model on the row, and white indicates no significant difference. Only the lower triangle is shown. Note that comparisons to the unconstrained $f(\text{SURP})$ model are treated as directional (since $f(\text{SURP})$ subsumes the hypothesis space of all other models), and thus comparisons in which another model numerically outperforms $f(\text{SURP})$ on the test set are treated as non-significant. Tests use false discovery rate correction for multiple comparisons (49) within the set of comparisons covered by each cell of the figure. Overall results across language models and datasets (lower right subplot) indicate that PROB significantly underperforms surprisal-based models, that SURP^1 significantly outperforms superlogarithmic models ($\text{SURP}^{3/4}$ and SURP^2), and that SURP^1 is the best performing constrained model overall.

5. Full IRF Surface Plots

Figures S9–S10 provide detailed IRF estimates across language models and datasets. In these three-dimensional surface plots, the change in the dependent variable (e.g., reading time) is shown on the z -axis as a function of both surprisal (x -axis) and delay (in seconds) from stimulus onset (y -axis). These surface plots provide timecourse details that are important for interpretation of some results. For example, Figure 2 of the main article shows a null effect (with a puzzling negative slope) of PCFG surprisal on first pass reading times of the GECO corpus. This finding becomes less puzzling in the context of Figure S9, which makes clear that the estimated effect of PCFG surprisal on first pass reading times in GECO is still positive overall, but simply late-onset, peaking around 250ms after the word is first fixated. This estimated timecourse is an outlier: for most datasets, language models, and response types, the instantaneous surprisal effect (at delay 0) is positive. We avoid further interpretation of this finding in light of the fact that the PCFG is a weaker language model than the others considered in this study, both in terms of perplexity (Table S10) and psychometric performance (Figure 3 of the main article). Related work has explored better-performing incremental parsers as language models (48), a research question that was not a central focus of this study.

Figure S11 plots the estimated IRFs for GPT-2 probability (Figure S11a) vs. GPT-2 surprisal (Figure S11b) in a model containing strictly linear terms for both of these predictors. This configuration addresses the hypothesis that linear and logarithmic effects (deriving from distinct cognitive processes) may be superposed (50) by allowing the regression model to find a mixture between the linear (probability) and logarithmic (surprisal) terms. As shown, the estimated mixture heavily favors surprisal over probability. GPT-2 surprisal effect estimates (Figure S11b) have relatively large magnitude and show the expected positive association

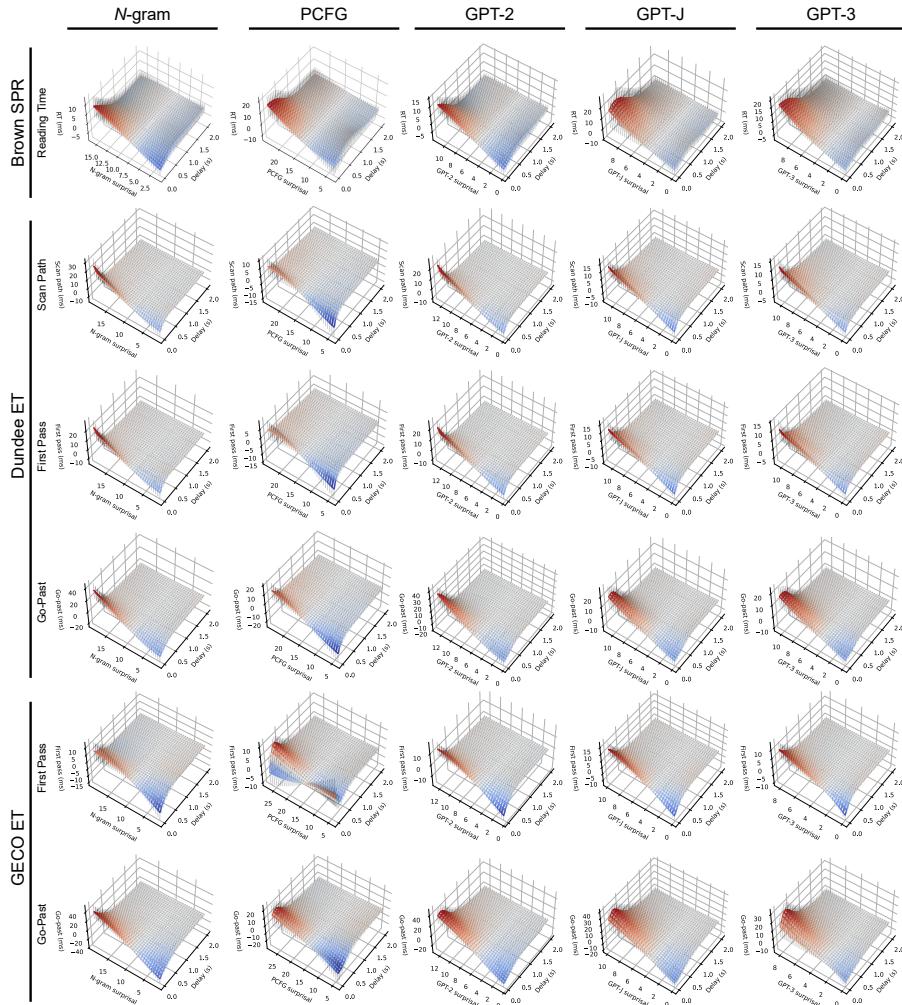


Figure S9: CDRNN-estimated functional form of effect estimates in the Brown, Dundee, and GECO datasets across language models over a 2s interval following initial fixation. Each subplot shows the estimated change in the dependent variable (vertical axis) from observing a word with a given surprisal value (left axis) at a given delay (in seconds) from word onset (right axis). Gray error bars indicate 95% variational Bayesian credible intervals.

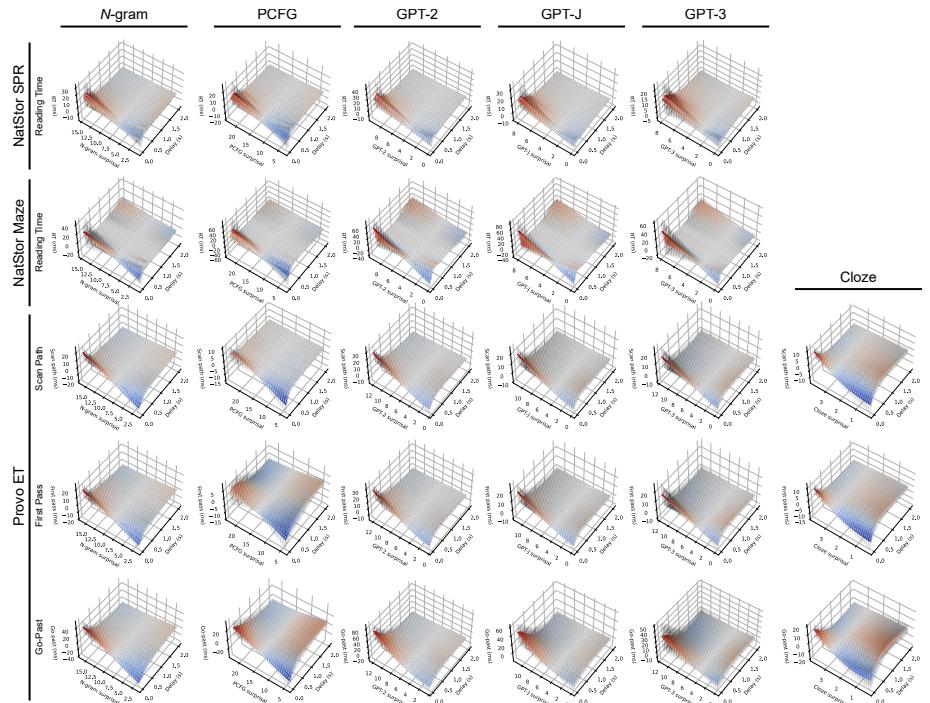


Figure S10: CDRNN-estimated functional form of effect estimates in the Natural Stories SPR, Natural Stories Maze, and Provo datasets across language models over a 2s interval following initial fixation. Each subplot shows the estimated change in the dependent variable (vertical axis) from observing a word with a given surprisal value (left axis) at a given delay (in seconds) from word onset (right axis). Gray error bars indicate 95% variational Bayesian credible intervals.

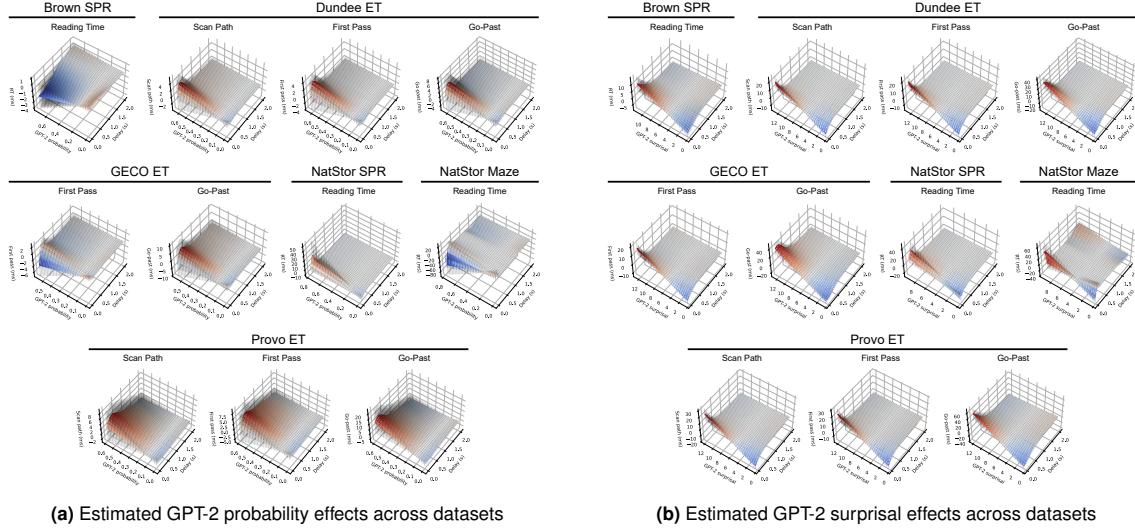


Figure S11: CDRNN-estimated impulse responses to GPT-2 probability (a) vs. GPT-2 surprisal (b) across datasets, under a model containing strictly linear terms for both GPT-2 probability and GPT-2 surprisal. Each subplot shows the estimated change in the dependent variable (vertical axis) from observing a word with a given probability/surprisal value (left axis) at a given delay (in seconds) from word onset (right axis). Gray error bars indicate 95% variational Bayesian credible intervals. Note that, whereas reading times consistently increase on word surprisal near word onset in panel (b), they do not consistently decrease on word predictability in panel (a). Instead, in some datasets (e.g., Natural Stories SPR), more probable words are estimated to lead to an *increase* in reading time.

predicted by the COST view. By comparison, GPT-2 probability effect estimates (Figure S11a) have small magnitude and high uncertainty, and they do not consistently show the expected negative association predicted by the FACILITATION view; indeed more often than not effects trend numerically in the wrong direction, with higher probability words eliciting longer reading times. We thus find no systematic support for superposed linear and logarithmic predictability effects.

6. Direct Comparison between GPT-2 and PCFG Language Models

We show in the main article that GPT-2 substantially outperforms PCFG language models as an estimator of human processing difficulty. Nonetheless, recent evidence indicates that surprisal derived from incremental generative parsers like our PCFG may contain signal about human language processing that is not fully captured by GPT-2 surprisal (48), despite the capacity of large neural network language models to acquire some syntactic information (e.g., 157, 158). To test this possibility, we compared our GPT-2 $f(\text{SURP})$ CDRNN models to comparable models augmented with PCFG surprisal, which allows us to quantify the contribution of PCFG surprisal to model fit over GPT-2 surprisal alone. Results do not clearly support a generalized contribution of PCFGs over GPT-2. Adding PCFG surprisal to a model containing GPT-2 surprisal does not provide a significant performance boost across datasets (Table S9) and is only significant in 2/11 comparisons on individual datasets. Notably, one of these is the Natural Stories SPR dataset, which has previously been used to argue for complementary PCFG effects over GPT-2 (48). Although we do not find evidence in these analyses of unique contributions of PCFGs over transformer networks as psychometric models, more sophisticated generative incremental parsers are known to afford substantial psychometric gains over GPT-2 models (48). Our lack of findings in favor of unique PCFG effects should therefore not be construed as evidence against the sensitivity of human surprisal to syntactic factors (see also e.g., 12, 159).

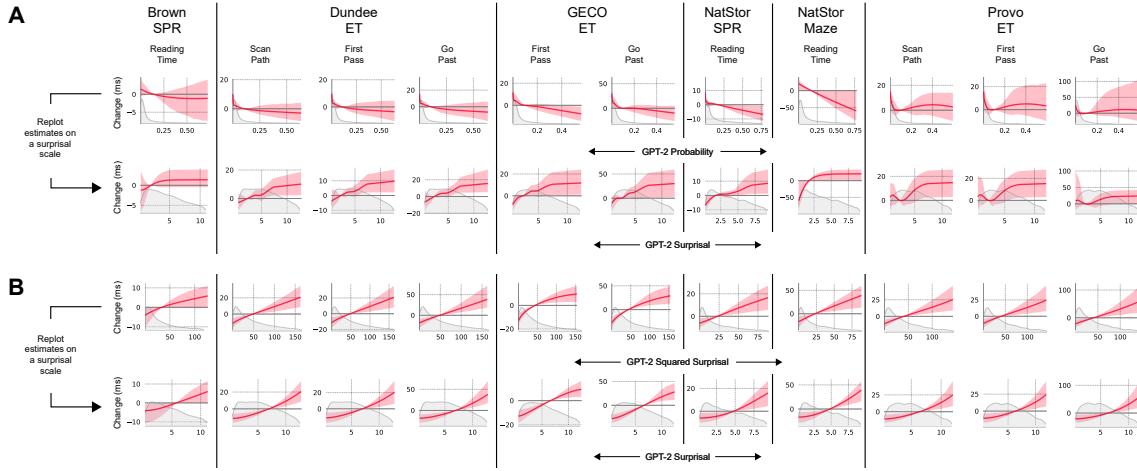


Figure S12: Inversion of predictability effects toward surprisal. The top row of each panel shows CDRNN estimated predictability effects on a (non-logarithmic) source scale (probability or squared surprisal) from which the CDRNN model estimates the shape and scale of a continuous function relating word predictability to processing cost (reading time). The bottom row of each panel shows the same estimated function (from the same models) replotted on a surprisal scale. Effects are shown with no delay (i.e. at the surprising word). **A.** Estimates from models using GPT-2 probability to represent word predictability. A logarithmic effect should be realized as a sharp increase in reading time as probability drops toward zero in the top row of the panel. When the resulting estimated function is replotted on a surprisal scale (bottom row of the panel), it should resemble a straight line. This prediction is largely borne out visually, especially in the largest datasets (Dundee ET, GECO ET, and Natural Stories SPR). With the exception of Natural Stories Maze, models do not find the linear effect (straight line with a negative slope in the top row of the panel) predicted by the facilitation view. **B.** Estimates from models using squared GPT-2 surprisal to represent word predictability. A logarithmic effect should be realized as decreasing slope the reading time effect as squared surprisal increases in the top row of the panel. When the resulting estimated function is replotted on a surprisal scale (bottom row of the panel), it should resemble a straight line. This prediction is borne out in some cases (e.g., Brown SPR and GECO ET), but not in others. In other words, a logarithmic effect is generally not (fully) recovered from a superlogarithmic source scale. Nonetheless, direct statistical comparisons favor a logarithmic effect over a superlogarithmic one overall (Figure 3).

7. Are Logarithmic Predictability Effects Recovered from a Non-Logarithmic Scale?

If CDRNN models learn to *invert* a predictability measure defined on a non-logarithmic scale into one that is logarithmic, this would provide convergent evidence for logarithmic predictability effects. To investigate this, we fit nonlinear CDRNNs equipped with either GPT-2 probability (linear in predictability) or squared GPT-2 surprisal (superlogarithmic in predictability) and evaluate the degree to which the model pushes the estimated predictability effect toward a logarithmic shape (i.e., linear in surprisal). We refer to these models respectively as $f(\text{PROB})$ and $f(\text{SURP}^2)$. Note that recovering a logarithmic surprisal scale from a linear probability scale is a challenging estimation problem for neural networks (like CDRNNs), which repeatedly modify their weights in small increments during estimation, subject to penalties on large-magnitude weights. This is because the model must find a probability-cost curve that rises sharply (to asymptotic infinity) over a small numerical range (probabilities approximately in the range $0 < p < 0.05$). The converse is not true: recovering a linear probability scale from a logarithmic surprisal scale is straightforward, since the model must simply find a “plateau” whereby cost increases little at large values of surprisal (Figure 1). To address this asymmetry, we substantially reduce the regularizer strength in these analyses (by a factor of 500), which harms generalization performance but permits more flexible estimation of nonlinearities. Results are plotted in Figure S12. As shown, CDRNNs tend to estimate a highly nonlinear effect of GPT-2 probability that suggests an inordinate cost associated with low-probability items across datasets, contrary to the FACILITATION view. When these estimates are replotted on a surprisal scale, they look roughly linear in the Dundee, GECO, and Natural Stories SPR datasets, which are substantially larger than the others and plausibly provide more reliable estimates.

Results using squared surprisal are less clear. In some cases, the squaring function is inverted to an effect that is linear on surprisal (Brown and GECO), and in others not. Note however that this is a similar distribution of models that found visually linear vs. superlinear estimates for GPT-2 surprisal in Figure 2, and, as discussed above, these estimated superlinearities do not lead systematically to improved fit. In sum, models tend to find a logarithmic predictability effect, even when starting from predictability measures that do not represent predictability logarithmically.

8. Comparison to Hoover et al. (2023)

Here we provide an extended discussion of Hoover et al. (14), a closely-related study that reached qualitatively different conclusions about the functional form of predictability effects. Hoover et al. make two key contributions, one theoretical and one empirical. Their key theoretical contribution is to derive superlogarithmic predictability effects from sampling algorithms as models of language comprehension, thus providing a mechanistic account for how UID pressures might arise from processing constraints. Their key empirical contribution is to show superlogarithmic estimates of word predictability effects (consistent with UID) in the Natural Stories SPR dataset (also analyzed in this study), especially for large language models like GPT-3.

The methods in Hoover et al. are rigorous, using generalized additive mixed models for location and scale (34, 160, 161) with spillover regressors from preceding words (162) to estimate and control for delayed, nonlinear, and heteroscedastic patterns of influence of word predictability on reading times, and using 6-fold cross-validation to insure against high-leverage outliers. Nevertheless, our approach offers some advantages. *First*, we consider a broader range of datasets (six) and reading modalities (three), which allows us to avoid drawing conclusions from patterns that may be idiosyncratic to a given dataset or modality. *Second*, our exGaussian CDRNN models are less assumption-laden, allowing for continuous-time effect delays (rather than indexical delays over word positions, which are implausible for naturalistic reading; 33, 109), arbitrary predictor interactions, nonstationarity in the response function, and control over skewness in the distribution over reading times, which plays a major role in the Natural Stories SPR dataset (our exGaussian model improves over a comparable Normal model on Natural Stories SPR by over 120,000 log likelihood points; Table S9). *Third*, our claims are based on the generalization performance of pretrained regression models, rather than on visual estimates (or descriptive statistics derived from visual estimates, as in Hoover et al.). This aspect of our design offers critical insurance against non-replication, since in-sample tests from highly expressive models applied to large datasets may be prone to finding small but significant effects that may not hold in general, especially when analyses are concentrated on estimates from the empirical tail (a small number of high-surprisal words). Although Hoover et al.’s cross-validation procedure offers some protection against this concern, generalization performance is not evaluated, and 5/6 of the training data (used to estimate the critical surprisal effects) is shared across any pair of folds.

As discussed in the main article, our procedures nonetheless replicate key aspects of the Hoover et al. findings on Natural Stories SPR. *First*, the effect estimates tend to be superlogarithmic across language models (Figure 2 of the main article). This impression is even more pronounced when considering the full empirical range of surprisal values in the training data (Figure S13), as reported in Hoover et al., rather than the interdecile range reported in the main article. Natural Stories SPR shows a visually pronounced superlogarithmic pattern at the highest levels of surprisal. *Second*, these superlogarithmic estimates in Natural Stories SPR confer performance advantages: in aggregate, $f(\text{SURP})$ outperforms SURP^1 , and $\text{SURP}^{4/3}$ (but not SURP^2) also outperforms SURP^1 . This outcome is reassuring, since different research teams using different methods converge in large part on the same result when analyzing the same dataset.

However, our approach allows us to zoom out to a broader picture that is considerably less favorable to superlogarithmic effects, and which suggests that the Natural Stories SPR dataset may be an outlier. The other datasets show little evidence of superlogarithmic patterns, even at the highest values of surprisal (Figure S13). In addition, as reported in the main article, over all datasets, SURP^1 outperforms $\text{SURP}^{4/3}$, SURP^2 , and $\text{SURP}^{>1}$ (the ensemble of $\text{SURP}^{4/3}$ and SURP^2), and $\text{SURP}^{>1}$ does not outperform $\text{SURP}^{\leq 1}$ (the ensemble of $\text{SURP}^{1/2}$ and $\text{SURP}^{3/4}$, and SURP^1). This pattern holds both when we consider all language models in aggregate and when we focus on the best-performing model overall (GPT-2). Furthermore, this

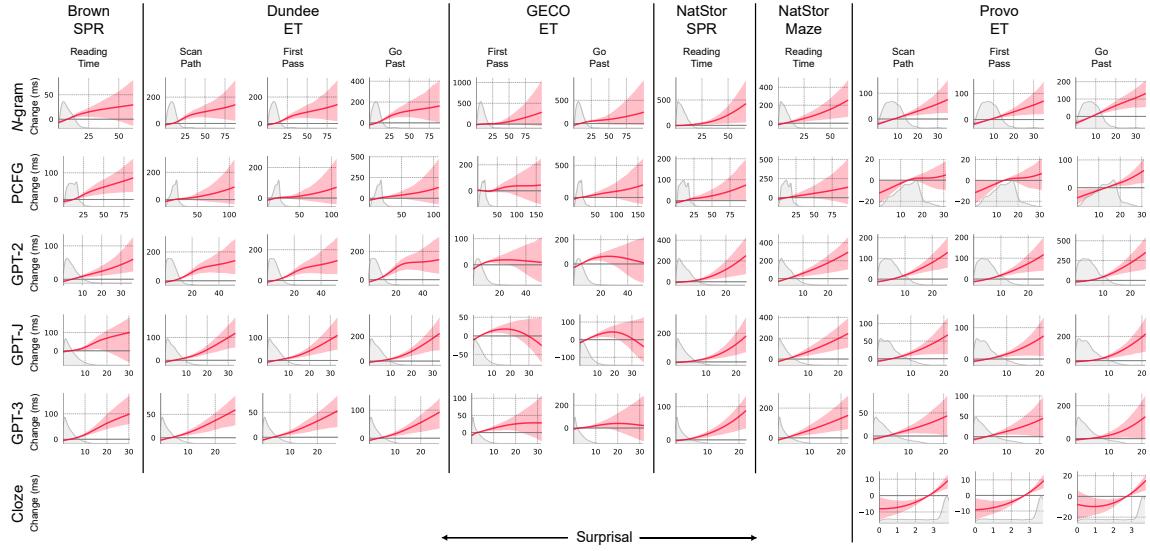


Figure S13: CDRNN-estimated functional form of effects across language model types (*n*-gram, PCFG, GPT-2, GPT-J, GPT-3, and human cloze) with no delay (i.e. at the surprising word). Plots cover the full empirical range of surprisal values in each training dataset. Kernel density plots show the distribution of surprisal values in the training data over the plotted range.

general picture is unchanged when we revisit our key questions using GAMs, as in Hoover et al., instead of CDRNNs (SI 14).

One could potentially argue that Natural Stories SPR has a privileged status relative to the other datasets, and that results using it should be given greater weight. In particular, it is the largest of the datasets considered here (>1M datapoints), and it was specifically designed to tax the language processing system using rare words and syntactic constructions that plausibly increase overall surprisal (37). Perhaps this makes it a better testbed for questions about predictability effects relative to the five other datasets we considered (which used naturally occurring written texts, and which may therefore under-represent effects in the high surprisal regime). We find the following problems with this argument. *First*, the texts in Natural Stories are not in fact more surprising overall than those in the other datasets. Across language models, surprisal values in Natural Stories are among the lowest in our sample, whether considering overall model perplexity (i.e., exponentiated average surprisal, Table S10), the interdecile range (Figure 2 of the main article), the total range (Figure S13), or the surprisal densities within either of these intervals. There is thus little empirical support for the notion that these materials are better for investigating the extremes of the surprisal continuum. *Second*, the Natural Stories Maze dataset used the same materials in a different sample of participants under a different reading modality, but Natural Stories Maze shows no evidence of superlogarithmicity, either in terms of estimates (Figure S13) or model performance (if anything, the best-performing models are sublogarithmic; Figure 3 of the main article). Thus, the Natural Stories SPR patterns do not appear to derive from the textual materials, but are instead specific to the particular sample and/or modality (self-paced reading). *Third*, it is known that responses in Natural Stories SPR show strong effects of continuous reading rate that are much larger in magnitude than linguistic effects like word frequency or surprisal (33, 109). These rate effects are plausibly driven by motor habituation, rather than language processing demand (33, 109), and they cannot be estimated using discrete-time methods like those in Hoover et al. Studies that control for rate effects in Natural Stories SPR show that linguistic effects can be still detected (33, 109), but that they are faint relative to more ecologically valid modalities like eye-tracking. *Fourth*, unlike the other datasets (except Natural Stories Maze), Natural Stories SPR was crowdsourced on Amazon Mechanical Turk and may consequently have issues with quality control that may not be present in the other data. For example, one participant fixated a word for over an hour, and another participant had an average reaction time of 35ms, far faster than the time

Dataset	Perplexity				
	<i>n</i> -gram	PCFG	GPT-2	GPT-J	GPT-3
Brown	1560	6379	98	32	23
Dundee	1142	4028	74	28	20
GECO	1702	6444	86	23	10
Natural Stories	645	4216	43	18	12
Provo	796	3172	72	32	23

Table S10: Language model perplexity (exponentiated average surprisal) by model type and dataset.

generally needed to develop and execute a motor plan (for button pressing), let alone process the meanings of words. Although these extreme cases are removed through outlier filtering, it is unclear how indicative they may be about broader quality issues in crowd-sourced data that may not always be caught by outlier filters. For example, the participant with the 35ms average reading time still managed to get an average of 6/8 comprehension questions correct, enough to pass quality checks for any of their reading times that happen to exceed the 100ms minimum filter. These issues are unlikely to arise in an in-person laboratory setting.

Consistent with the third and fourth concerns above, our results show large variation in model performance on Natural Stories SPR relative to the other datasets, despite its size (**Figure 3** of the main article), which suggests weak signal. This is not to say Natural Stories SPR is bad data; as we said above, it has major advantages, and prior work has supported the existence of linguistic effects in that dataset. For these reasons, we have chosen to include it in our own analyses. Our point is only that, like all datasets, Natural Stories SPR has both strengths and weaknesses, and we do not think that it should be privileged relative to the other datasets in our sample, especially given the clearer signals about predictive processing that emerge using other datasets. In summary, we believe the full pattern of results in our study suggests that the superlogarithmic effects reported by Hoover et al. may be idiosyncratic to the dataset they analyzed, rather than being characteristic of reading in general.

Up to this point, we have focused on points of disagreement between our study and Hoover et al., but these points should be considered against a larger context of similarity. Both studies concur that predictability effects in natural reading are *at least* logarithmic and thus agree with the COST (cf., FACILITATION) view that probabilistic inference is a major concern of the human language processing system. Where they differ is with respect to the presence or absence of additional pressures favoring uniform information density, over and above a set of shared assumptions about the underlying inferential processes. Furthermore, as discussed in the main article, our design cannot falsify the UID view, only constrain the strength of superlogarithmic patterns. For example, our results indicate that SURP² may be too strongly superlogarithmic. Nonetheless, there remains an infinite space of superlogarithmic functions that are consistent with our pattern of results. We hope this work will encourage the development of experimental and analytical innovations that could shed additional light on this question.

9. Language Model Perplexity

Table S10 presents language model perplexities by dataset.

10. Statistical Controls

We include the following control predictors in all models:

- **Rate.** A “deconvolutional intercept” (109) describing the average response to a word, independent of its properties. *Rate* is so named because its influence on the response depends solely on stimulus timing.
- **Word length.** The length of the word (in characters).

- **Unigram surprisal.** A “context-free” surprisal driven by the relative frequency of the word. Unigram surprisal was obtained from a KenLM unigram model (40) trained on the Gigaword 3 corpus (132).
- **End of sentence.** Whether a word ends a sentence (binary indicator), designed to capture diffuse effects of sentence boundaries (e.g., 163, 164), even though the final words of sentences themselves are excluded from analysis (see above).

Models of eye-tracking datasets additionally contained the following control predictors that are specifically relevant to the eye-tracking modality:

- **Saccade length.** Length in words of incoming saccade (eye movement).
- **Regression.** Whether the fixation is part of a regressive (backward) eye movement (binary indicator).

Since the Dundee corpus additionally provides annotations for screen and line boundaries, we included these as regressors in Dundee models only:

- **End of line.** Whether a word ends a line of text on the display (binary indicator).
- **End of screen.** Whether a word ends a screen on the display (binary indicator).

Finally, the Maze task used in Natural Stories Maze involves a potentially errorful word-by-word forced choice task. Therefore, for this dataset alone, we modeled the possibility of effects from task errors using the following regressor:

- **Incorrect.** Whether the incorrect continuation was chosen in the A-Maze task (binary indicator).

The CDRNN models used here flexibly capture interactions between any combinations of these variables (33). Thus, by including e.g., the *regression* predictor, the models can learn not only overall differences in response to regressive vs. non-regressive fixations, but also e.g., differences in surprisal or word length effects between regressive vs. non-regressive fixations. This ability is important because fixations during regressive eye movements plausibly differ in their processing demands, since they involve material that was likely already viewed either foveally or parafoveally.

11. Model Formulae

For ease of reference, here we present the CDRNN model formulae used throughout this study. Variable names are changed from those used in our codebase for readability. For full software implementation details, see <https://github.com/coryshain/cdr>. Because the sets of control predictors differ across datasets, we abbreviate “control₁ + … + control_n” (e.g., “WordLength + SaccadeLength + EndOfSentence + …” as “CONTROLS”. Control variables used for each dataset are described in SI 10. The keywords “Surprisal” and “Probability” respectively represent surprisal- and probability-scale estimates from a given language model.

- **\emptyset :** $y \sim C(\text{CONTROLS}, \text{NN}()) + (C(\text{CONTROLS}, \text{NN(ran=T)}) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **$f(\text{SURP})$:** $y \sim C(\text{CONTROLS} + \text{Surprisal}, \text{NN}()) + (C(\text{CONTROLS} + \text{Surprisal}, \text{NN(ran=T)}) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **$f(\text{PROB})$:** $y \sim C(\text{CONTROLS} + \text{Probability}, \text{NN}()) + (C(\text{CONTROLS} + \text{Probability}, \text{NN(ran=T)}) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **$f(\text{SURP}^2)$:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^2, \text{NN}()) + (C(\text{CONTROLS} + \text{Surprisal}^2, \text{NN(ran=T)}) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$

- **PROB:** $y \sim C(\text{CONTROLS} + \text{Probability}, \text{NN}(\text{inputs_to_drop}=[\text{Probability}])) + (C(\text{CONTROLS} + \text{Probability}, \text{NN}(\text{inputs_to_drop}=[\text{Probability}], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **SURP^{1/2}:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^{1/2}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{1/2}])) + (C(\text{CONTROLS} + \text{Surprisal}^{1/2}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{1/2}], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **SURP^{3/4}:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^{3/4}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{3/4}])) + (C(\text{CONTROLS} + \text{Surprisal}^{3/4}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{3/4}], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **SURP¹:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^1, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^1])) + (C(\text{CONTROLS} + \text{Surprisal}^1, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^1], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **SURP^{4/3}:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^{4/3}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{4/3}])) + (C(\text{CONTROLS} + \text{Surprisal}^{4/3}, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^{4/3}], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **SURP²:** $y \sim C(\text{CONTROLS} + \text{Surprisal}^2, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^2])) + (C(\text{CONTROLS} + \text{Surprisal}^2, \text{NN}(\text{inputs_to_drop}=[\text{Surprisal}^2], \text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$
- **GPT-2+PCFG_{f(SURP)}:** $C(\text{CONTROLS} + \text{GPT2Surprisal} + \text{PCFGSurprisal}, \text{NN}()) + (C(\text{CONTROLS} + \text{GPT2Surprisal} + \text{PCFGSurprisal}, \text{NN}(\text{ran}=T)) | \text{Participant}) + (1 | \text{DocumentID: SentenceID: WordPositionInSentence})$

12. CDRNN Model Definition

For ease of reference, here we reproduce the formal definition of the CDRNN model from ref. (33). Let $\mathbf{y} \in \mathbb{R}^Y$ be a Y -dimensional random variable that we seek to model (the response). Let \mathcal{F} be a probability distribution with S -dimensional parameter vector $\mathbf{s} \in \mathbb{R}^S$ such that $\mathbf{y} \sim \mathcal{F}(\mathbf{s})$. Let $\mathbf{X} \in \mathbb{R}^{N \times K}$ be a matrix of N K -dimensional predictor vectors $\mathbf{x}_n, 1 \leq n \leq N$. Let $t_y \in \mathbb{R}$ be the timestamp of \mathbf{y} , and let $\mathbf{t} \in \mathbb{R}^N$ be the vector of predictor timestamps $t_{\mathbf{x}_1}, \dots, t_{\mathbf{x}_n}$ such that $t_{\mathbf{x}_n}$ is the timestamp of \mathbf{x}_n . Let $\mathbf{d} \in \mathbb{R}^N$ then be the vector of temporal offsets $d_{\mathbf{x}_1}, \dots, d_{\mathbf{x}_n}$ such that $d_{\mathbf{x}_n} = t_y - t_{\mathbf{x}_n}$, i.e. the signed distance in time between \mathbf{y} and \mathbf{x}_n .¹

The timestamps \mathbf{t} are horizontally concatenated with \mathbf{X} to yield the inputs to $f_{\text{in}} \in \mathbb{R}^{N \times (K+1)} \rightarrow \mathbb{R}^{N \times J}$ with parameters \mathbf{u}_{in} . f_{in} is an input processing function that yields $\mathbf{X}' \in \mathbb{R}^{N \times J}$, a matrix of J -dimensional impulse vectors $\mathbf{x}'_n, 1 \leq n \leq N$:

$$\mathbf{X}' \stackrel{\text{def}}{=} f_{\text{in}}([\mathbf{t} \quad \mathbf{X}] ; \mathbf{u}_{\text{in}}) \quad (3)$$

\mathbf{X}' is horizontally concatenated with \mathbf{d} and \mathbf{t} to yield the inputs to $\text{IRF} f_{\text{IRF}} \in \mathbb{R}^{N \times (J+2)} \rightarrow \mathbb{R}^{N \times S \times (J+1)}$ with parameters \mathbf{u}_{IRF} . The output of the IRF is a sequence of convolution weight matrices $\mathbf{G}_n \in \mathbb{R}^{S \times (J+1)}, 1 \leq n \leq N$:

$$\mathbf{G}_1, \dots, \mathbf{G}_N \stackrel{\text{def}}{=} f_{\text{IRF}}([\mathbf{d} \quad \mathbf{t} \quad \mathbf{X}'] ; \mathbf{u}_{\text{IRF}}) \quad (4)$$

The final outputs of the model—parameters \mathbf{s} of \mathcal{F} —are computed as the sum of (i) the temporal convolution of \mathbf{X}' with $\mathbf{G}_1, \dots, \mathbf{G}_N$ and (ii) learned bias vector \mathbf{s}_0 , where each transposed row $\mathbf{x}'_n, 1 \leq n \leq N$ of \mathbf{X}' is

¹We note that, in this work, we never use future stimuli for predicting \mathbf{y} , so in practice $t_y \geq t_{\mathbf{x}_n}$ and all $d_{\mathbf{x}_n}$ will be non-negative.

vertically concatenated with a bias² and weighted by learned coefficient vector $\mathbf{b} \in \mathbb{R}^{J+1}$:

$$\mathbf{s} \stackrel{\text{def}}{=} \mathbf{s}_0 + \sum_{n=1}^N \mathbf{G}_n \text{diag}(\mathbf{b}) \begin{bmatrix} 1 \\ \mathbf{x}'_n \end{bmatrix} \quad (5)$$

Letting $\mathbf{v} \in \mathbb{R}^V$ represent the concatenation of \mathbf{u}_{in} , \mathbf{u}_{IRF} , \mathbf{b} , and \mathbf{s}_0

$$\mathbf{v} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{u}_{\text{in}} \\ \mathbf{u}_{\text{IRF}} \\ \mathbf{b} \\ \mathbf{s}_0 \end{bmatrix} \quad (6)$$

mixed effects models can be defined by letting \mathbf{v} be the sum of a fixed part $\mathbf{v}_0 \in \mathbb{R}^V$ and random part, where $\mathbf{V} \in \mathbb{R}^{V \times Z}$ is a random effects matrix whose rows sum to 0 and $\mathbf{z} \in \{0, 1\}^Z$ indicates which of Z random effects levels apply to \mathbf{y} :

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{V}\mathbf{z} \quad (7)$$

The parameters of the model are therefore \mathbf{v}_0 and \mathbf{V} , which may be fitted via maximum likelihood or (given priors) Bayesian inference. This definition assumes a singleton dataset $\mathcal{D} = \{\langle \mathbf{X}, \mathbf{t}, \mathbf{y}, t_y \rangle\}$, but it extends without loss of generality to any finite dataset by applying eq. 5 independently to each of M elements in $\mathcal{D} = \{\langle \mathbf{X}_m, \mathbf{t}_m, \mathbf{y}_m, t_{y_m} \rangle \mid 1 \leq m \leq M\}$.

In this study, following ref. (33), f_{in} is identity, and f_{IRF} is a feedforward neural network; i.e., a network consisting solely of linear transformations followed by a nonlinearity—in our case, the GELU function (165). To enforce a linear effect for the k^{th} predictor dimension, f_{IRF} is preceded by a mask $\mathbf{f}^{(k)} \in \{0, 1\}^K$ such that each of its rows is defined as:

$$\mathbf{f}_i^{(k)} \stackrel{\text{def}}{=} \begin{cases} 1 & i \neq k \\ 0 & \text{otherwise} \end{cases}$$

This removes the k^{th} predictor from the inputs to the IRF, but retains it in the convolution defined in eq. 5, thereby preventing the IRF from conditioning on the predictor's value and enforcing a linear effect.

13. CDRNN Implementation and Statistical Procedure

Unless otherwise indicated, all CDRNNs were implemented as described in ref. (33) with the following parameter settings:

- Feedforward IRF with two hidden layers of 32 units each.
- Full random effects (zero-centered deviations in the model intercepts, linear coefficients, and layerwise bias terms permitting variation in IRF shape) by subject and random intercepts by token.
- Dropout rate (166) of 0.1 on (a) all hidden layers and (b) random grouping factor variables.
- L2 weight regularization constant of 5.
- L2 regularization constant of 10 on random IRF bias terms.
- To speed convergence, prior to fitting, response variables are z-scored and predictor variables are rescaled by their standard deviations.
- Fixed intercepts and coefficients assume a standard normal prior, and, following ref. (109), random intercepts and coefficients assume a normal prior with mean 0 and standard deviation 0.1. Variational posteriors over these parameters are estimated using variational expectation maximization.

²The bias term, referred to here and in ref. (109) as *rate*, serves as a deconvolutional “intercept” capturing general effects of event timing.

- For computational efficiency, histories are truncated at 32 words or 60s into the past, whichever is shorter.
- Convergence is diagnosed based on a time-loss correlation criterion, where the loss is the validation set likelihood evaluated every 10 epochs and the correlation is computed over a window of 250 consecutive epochs. Thus, convergence is declared whenever the validation set likelihood is statistically non-increasing at $\alpha = 0.5$ for at least 13 of the preceding 25 evaluations. For full details about this procedure, see ref. (109). Following convergence, the model state with the best validation set performance is used for all evaluation and visualization.

Full code and model configuration files needed for reproduction are provided at <https://github.com/coryshain/cdr>. Hypotheses are statistically evaluated on the test set, with separate tests for each response variable (scan path, first pass, and go-past durations in Dundee and reading time in Natural Stories).

In this study, null hypotheses assume a linear effect of some fixed function of word predictability (e.g., surprisal), and the alternative hypothesis is a non-linear effect. To enforce linearity, dependencies to predictors are removed from impulse response functions, which prevents the network from adapting its convolution weights to the value of the predictor, resulting in a strictly linear effect. We present the specific formulae used to define our CRDNN models in **SI 11**. For formal definition of the CDRNN model, see **SI 12**.

Following ref. (33), in order to account for optimization noise in the statistical tests, we statistically compare *ensembles* of 10 model replicates per hypothesis, using a hierarchical paired permutation test inspired by ref. (167). In some cases, we combine ensembles to test *composite* hypotheses that encompass multiple simple hypotheses in our design. For example, given our design spanning five different exponents on surprisal (1/2, 3/4, 1, 4/3, and 2) for each language model, the hypothesis that predictability effects are superlogarithmic is a composite test in which exponents 1/2, 3/4, and 1 together define the composite null hypothesis and exponents 4/3 and 2 together define the composite alternative hypothesis. For such questions, we ensemble all models fitted using each composite hypothesis, resulting (in the example above) in a null model with 30 replicates (3×10) and an alternative model with 20 replicates (2×10).

Tests use the following procedure, in which A is the ensemble size for hypothesis \mathcal{A} and B is the ensemble size for hypothesis \mathcal{B} :

1. For each of the N evaluation items $1 \leq n \leq N$, repartition the $A + B$ log-likelihood statistics into two random sets of likelihoods $\hat{\mathcal{A}}_n \in \mathbb{R}^A$, $\hat{\mathcal{B}}_n \in \mathbb{R}^B$.
 2. Compute the resampled dataset likelihood as the median of summed likelihoods within the resampled partition:
- $$\mathcal{L}_{\hat{\mathcal{A}}} = \underset{1 \leq a \leq A}{\text{med}} \left[\sum_{n=1}^N \hat{\mathcal{A}}_{n,a} \right], \mathcal{L}_{\hat{\mathcal{B}}} = \underset{1 \leq b \leq B}{\text{med}} \left[\sum_{n=1}^N \hat{\mathcal{B}}_{n,b} \right]$$
3. Compute and store the absolute difference $|\mathcal{L}_{\hat{\mathcal{A}}} - \mathcal{L}_{\hat{\mathcal{B}}}|$.

This process is repeated many (10,000) times to construct an empirical null distribution over the likelihood differences between ensembles, which is then compared to the observed difference in mean likelihood between ensembles in order to compute a p value. Because tests are based on out-of-sample performance, alternative models are not guaranteed to outperform null models. Thus, for directional hypotheses in which the alternative subsumes the null (e.g., testing $f(\text{SURP})$ against SURP, since the former's solution space includes the latter), cases where test set likelihood *degrades* in the alternative model relative to the null model are assigned a default p value of 1.

In some cases, we combine all response variables from all datasets in order to test comparisons across the entire set. To do so, given D dataset-response pairs (in this study, $D = 11$) with M total datapoints between them, we vertically concatenate the item-wise likelihood matrices into joint likelihood matrices

$\mathcal{A}^{(\text{all})} \in \mathbb{R}^{M \times A}$, $\mathcal{B}^{(\text{all})} \in \mathbb{R}^{M \times B}$ as follows:

$$\mathcal{A}^{(\text{all})} = \begin{bmatrix} \mathcal{A}^{(1)} \\ \vdots \\ \mathcal{A}^{(M)} \end{bmatrix}, \mathcal{B}^{(\text{all})} = \begin{bmatrix} \mathcal{B}^{(1)} \\ \vdots \\ \mathcal{B}^{(M)} \end{bmatrix}$$

These combined likelihood matrices serve as inputs to the testing procedure outlined above.

All visualizations aggregate across the entire ensemble using 1,000 bootstrap resampling iterations. In each iteration, an ensemble component (i.e., a CDRNN fit) is sampled uniformly, then a model is sampled from that component's variational posterior, then the sampled model is queried with respect to the estimate of interest. This procedure jointly takes into account uncertainty in the posterior of each CDRNN fit as well as uncertainty across the ensemble.

14. Revisiting Our Main Findings Using Generalized Additive Models (GAMs)

The generalized additive model (GAM; 34, 160) is the statistical method of choice in related work on the functional form of predictability effects in reading (e.g., 1, 3, 14, 25) because it permits inference about the functional form of the predictability-cost relationship, rather than requiring an assumed form. However, for reasons discussed in the **Introduction** to the main article and elaborated in ref. (33), GAMs as standardly implemented (e.g., in the `mgcv` package, 34) retain a number of problematic simplifying assumptions for the reading domain, including discrete-time (spillover) rather than continuous effect delays, constant error, and (barring multidimensional interaction smooths that are difficult to estimate in practice) additive effects. For these reasons, we have chosen to use continuous-time deconvolutional regressive neural network (CDRNN) models for our key analyses, since they allow us to relax all of these assumptions in a data-driven manner (33), thereby reducing the likelihood that results depend critically on poor correspondence between aspects of model design and aspects of the underlying cognitive process. However, to clarify the extent to which results depend on this modeling choice, here we revisit our key analyses using GAMs instead of CDRNNs.

In so doing, we attempt to match the design of the GAM models to that of our main CDRNN models as closely as possible. With some exceptions noted below, all predictors and random effects from a given CDRNN model were included in the corresponding GAM model. In addition, although GAMs cannot estimate continuous-time impulse response functions, we provide them with some ability to detect effect delays by including two additional spillover positions for each predictor. For example, to model word length effects, GAM models include as predictors both the length of the current word and the lengths of the two preceding words. Given that predictability effects are represented by three distinct predictors in GAM models, all three of these predictors are removed from the baseline (\emptyset) GAM model (since this corresponds most closely to the ablation used in the main CDRNN analyses, where predictability effects are not modeled in the baseline at any delay). All predictors are modeled using thin-plate splines with default bases except for boolean indicators like *End of sentence* (which only take two values and thus do not support nonlinear regression) and constrained terms like $GPT-2_{\text{PROB}}$, both of which are modeled as linear. We apply the same hypothesis testing paradigm to GAM models as we do to CDRNNs (permutation testing of the likelihood difference statistic on the held-out test set). We also apply the same exclusion criteria to the training and evaluation data for GAMs as we did for CDRNNs.

That said, practical considerations led to the following deviations from the CDRNN design:

- Both by-token random intercepts and by-participant random splines led to out-of-memory errors on our compute resource, and, as a result, models with these terms could not be fitted. Therefore, GAM models only contain by-participant random intercepts and by-participant random slopes for each fixed effect in a given model, allowing the magnitude (but not the shape) of the response to each variable to vary by participant.
- Although CDRNNs implicitly estimate interactions between all subsets of variables, estimating such rich multidimensional smooths using GAMs (via tensor-product smooths) is intractable for these datasets.

Therefore, predictors are assumed to combine additively in all GAM models.

- Whenever sentence, line, and/or screen starts and ends are excluded from analysis, the *End of sentence*, *End of line*, and/or *End of screen* predictors have no variance both *in situ* and in spillover position 1 (which corresponds to the start of the next sentence, line, or screen, hence also excluded). Therefore, only spillover position 2 is considered for these boundary predictors.
- Because items with incorrect responses are excluded from analysis of the Natural Stories Maze dataset, the *Incorrect* predictor has no variance *in situ*. Therefore, only spilled over variants of *Incorrect* are considered.
- Because first pass and go-past durations by definition exclude words fixated as part of a regressive eye movement, the *Regression* predictor has no variance (in any spillover position) for first pass and go-past durations and is therefore excluded from all models.
- Because GAMs lack the capacity for continuous-time deconvolution, they cannot estimate the *Rate* predictor (33, 109). Therefore, *Rate* is excluded from all models.
- Because (unlike our CDRNN models) GAMs do not rely on early stopping for convergence, no validation set is needed. Therefore, GAMs are fitted to the training and validation sets together. The test set remains the same as that used for CDRNNs.
- In rare cases, numerical imprecision led to NaN likelihoods for some datapoints during test-set prediction from GAMs (this never occurred with CDRNN models). Any datapoint assigned such a likelihood by either model in a given statistical comparison was excluded prior to performing the permutation test. This led to exclusion of at most two datapoints in any given test.
- Unlike CDRNNs, GAM fitting is deterministic. Therefore, only a single GAM is fitted to each model configuration (rather than the ensemble of 10 CDRNN models used in the main analyses).

The GAM-estimated predictability-cost functions at no delay (i.e., at the surprising word) are plotted in **Figure S14**. Although estimates differ those produced by CDRNNs (**Figure S13**), this is to be expected given important design differences between the two modeling approaches. Nonetheless, the overall visual impression remains similar to that supported by our main finding: estimates are generally consistent with a logarithmic predictability (linear surprisal) effect (especially at lower surprisal values where the datapoints are concentrated) without a systematic trend either toward the plateau predicted by the FACILITATION view or the superlogarithmic pattern predicted by the UID view: some estimates look more sublogarithmic (e.g., the GPT-2 surprisal effect on Dundee scan path durations), others look more superlogarithmic (e.g., the GPT-2 surprisal effect on Provo scan path durations), and others simply look logarithmic (e.g., the GPT-J surprisal effect on GECO go-past durations). Thus, replacing CDRNNs with GAMs does not lead to systematic differences in the theoretical conclusions suggested visually by model estimates.

Full statistical testing results for our main comparisons using GAM models are given in **Tables S11–S15**. In general, comparisons using GAM models reject the null hypothesis less frequently than comparisons using CDRNN models, suggesting lower sensitivity. For example, no comparison is significant in the Provo dataset (**Table S14**). However, in aggregate, comparisons using GAM models on the held-out test set largely accord with those of our main CDRNN-based findings (**Table S15**): across all datasets and language models, (i) models containing predictability effects significantly outperform the baseline containing no predictability effect, (ii) the strictly logarithmic SURP¹ model is one of the best performing models, (iii) the logarithmic SURP¹ model significantly outperforms the linear PROB model, and (iv) the superlogarithmic SURP^{4/3} and SURP² models do not significantly outperform the SURP¹ model. The key difference from our main findings is that GAM models do not show a significant improvement of the SURP¹ model over the SURP^{3/4} and SURP² models, whereas CDRNN models show significant improvements over both. Thus, the GAM results decide less clearly between the predictions of the COST and UID views of predictability effects. This difference aside, these reanalyses show that, despite their limitations, GAMs largely reproduce our key findings, suggesting that the conclusions we advocate are not critically dependent on the use of CDRNNs rather than GAMs.

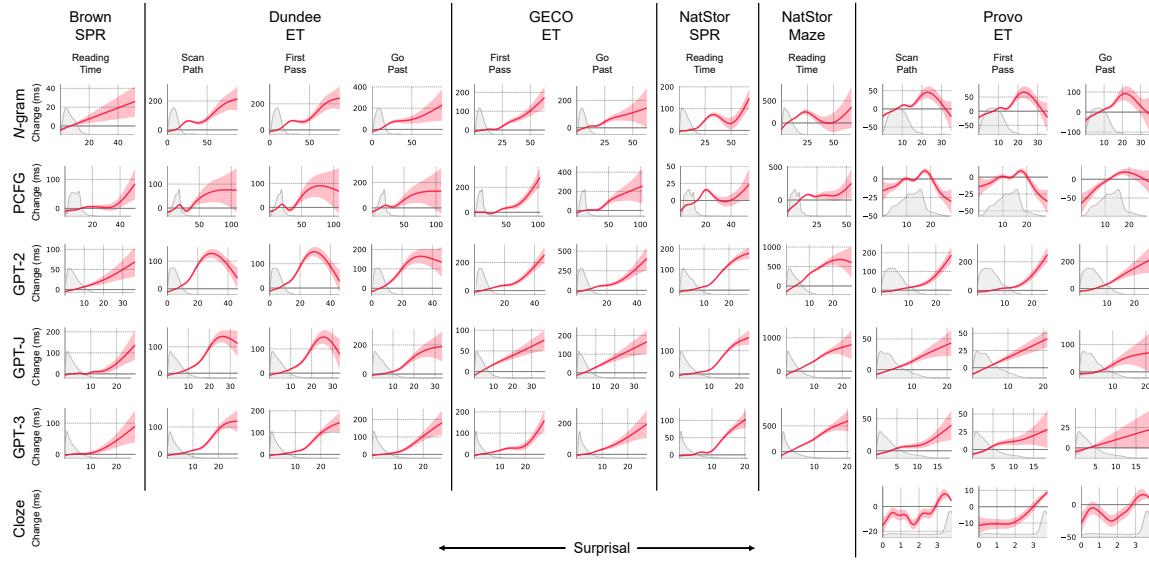


Figure S14: GAM-estimated functional form of effects across language model types (*n*-gram, PCFG, GPT-2, GPT-J, GPT-3, and human cloze) with no delay (i.e. at the surprising word). Plots cover the full empirical range of surprisal values in each training dataset (and are thus most comparable to the CDRNN plots in **Figure S13**, rather than **Figure 2** of the main article, which uses the interdecile range). Kernel density plots show the distribution of surprisal values in the training data over the plotted range. Uncertainty intervals show the ± 2 standard errors used as a default in plots from `mgcv` (unlike CDRNN plots, which show 95% credible intervals).

15. Reanalysis Using Normal Error

To better understand the potential impact on results of our assumed exGaussian distribution over reading times—which is motivated by prior considerations from psycholinguistic research (e.g., 5, 125) but deviates from the more widely assumed normal distribution, we re-ran the GPT-2 subset of our main analyses assuming a normal rather than exGaussian distribution. The estimated form of GPT-2 surprisal effects under both distributions are plotted in **Figure S15**. As shown, estimates are highly stable across distributions.

Tables S5–S9 report key results of testing GPT-2 models under a normal distribution. In general, a similar qualitative pattern of results holds across both distributions, and most of the key results survive (e.g., $f(\text{SURP})$ outperforms both \emptyset and PROB, and linear or sublogarithmic models outperform superlinear ones). Nonetheless, fewer of these results are significant, suggesting that the exGaussian analyses may be more sensitive. In any case, the exGaussian results are more credible on the basis of generalization performance: as shown in the final row of each table, the exGaussian GPT-2 $f(\text{SURP})$ model obtains a test set log likelihood

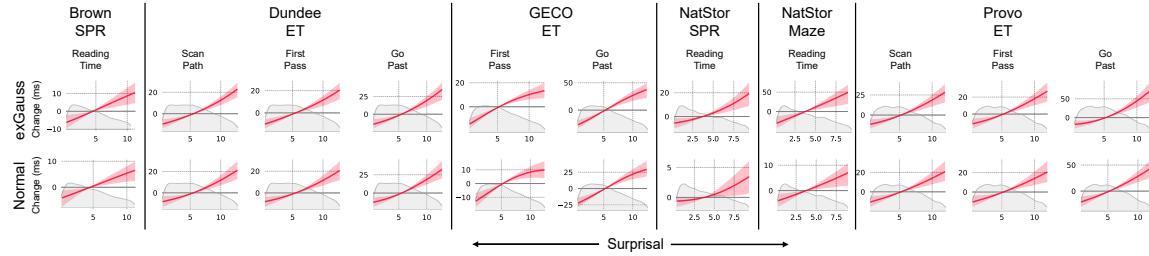


Figure S15: CDRNN-estimated functional form of GPT-2 surprisal effects using the ExGaussian predictive distribution assumed in our main analyses (top row, reproduced from **Figure 2** for convenience) vs. the more widely used Normal predictive distribution (bottom row). Plots cover the interdecile range of surprisal values in each training dataset. Estimates are highly similar across distributions.

improvement of several thousand points over the normal GPT-2 f (SURP) model in every comparison (all $p < 0.0001$), accumulating to an improvement of over 280,000 log likelihood points in the combined dataset (**Table S9**). Thus, our results suggest that better models of the underlying data distribution not only improve fit, but also the statistical power of critical tests. This outcome could be of relevance to future analysis design for similarly skewed response time data.