

A new analytical method of estimation of rigid wing characteristics during its non-stationary movement was worked out. General mathematical expressions were obtained for thrust, power and efficiency for rigid wing during linear and angular oscillations of high amplitude. The phase angle between them is considered to be arbitrary. These expressions can be useful for estimation of thrust and power produced by the wing with the help of simple calculation. Moreover general mathematical expressions were converted into simple design formulas, with the help of which it is possible to estimate thrust, power and efficiency using a simple engineering calculator. The results of the calculations of hydrodynamic characteristics of some wings were compared with numerical calculations mentioned in some publications. An agreement was achieved in a wide interval of Strouhal number. The received design formulas were used for estimation thrust characteristics of dolphin's fluke. Two possible models of dolphin's fluke flexibility were analyzed. This analytical method can be used for calculations of ship swimming movers. The book can be useful for engineers in ship constructing.

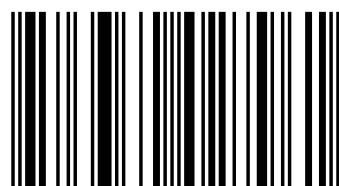
Oscillating Wing Theory



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Introduction

In the unsteady-state of the wing hydrodynamics the problem of the hydrodynamic forces determination as a rule reduces to the integral equations yet to be solved by means of numerical (more often than not) or analytical (in specific cases) methods (Nekrasov, 1947; Sedov, 1966; Belotserkovsky, 1958; Lighthill, 1969,1970; Wu, 1971a;1971b,1971c, Garric, 1936).

The books of Belotserkovsky, (1958); Belotserkovsky, S.M., V.K., Scripatch, V.G. Tabatchnikov. (1971) and Gorelov, D.N. (2000) describe the numerical methods for solving of singular integral equations.

Analytical methods are of limited usefulness.

They were laid down in detail for infinite foils in linear approximation exclusively.

In the case of the limit span the analytical solution of the problem has been existed only for low aspect-ratio wing for very small and very large Strouhal number exclusively. The analytical solutions are nonexistent for large aspect-ratio flapping foils.

It is significant that even the nonlinear theory has met with only limited success (or is not universally true) in studying of unsteady-state of the wing hydrodynamics and the wake production.

At the same time the practical tasks of swimming and flight of the different vehicles invite further investigations to obtain the simple formulas to calculate the foils hydrodynamic forces. Such investigations have been carried out first and foremost to study fish and dolphin swimming, assessment of the propulsive characteristics of the wing movers.

The basis for investigations are models of a splitting apart of hydrodynamic forces into the components depending on the media inertia and circulation component as well as linear relationship among hydrodynamic characteristics of a wing with the use of hydrodynamic derivative coefficients (Belotserkovsky, 1958). This method is very perspective because the hydrodynamic derivative coefficients have been much studied for wings of different shapes by numerical methods for the linear setting up a problem. By this means there is a good reason to believe that relatively simple formulas are derivable for estimations of hydrodynamic forces and efficiency of flat and rigid foils.

Some fundamental tenets and results are shown up in this book.

Chapter 1 gives setting up a problem and design formulas have been arrived at to calculate the flat rigid wing thrust, power, suction force and inductive reactance. The wing oscillates in non viscous infinite

medium. Oscillation amplitudes are sufficiently large. Pitch-axes positions and phase angle between heave and pitch oscillations are arbitrary. Three variants of the wing movement cinematic parameters have been given adequate consideration.

1. Heave and pitch oscillates harmonically. Phase angle is arbitrary. The design formulas have been obtained for the case when phase angle is equal to 90^0 .
2. Heave and angle of attack oscillates harmonically. The design formulas have been obtained for the case when phase angle is equal to 90^0 .
3. Pitch and angle of attack oscillates harmonically. As this takes place, the heaving oscillation is non harmonic.

The second and the third cinematic parameter variants are less common than the first one practically. Because of this, the design formulas were obtained for phase angle 90^0 only.

Pure heaving and pure pitching wing oscillations were investigated in all variants.

Chapter 2 is devoted to the obtaining of the design formulas for deformable wing as a model of flexible wing which is found naturally (fishes, cetaceans, insects, birds). Two sectional wings and a wing of composite profile were investigated. Such wings are capable of developing a larger thrust and efficiency than rigid wing when Strouhal number and Reynolds number are not so large.

Chapter 3 shows the comparison of thrust and efficiency calculated by using design formulas and published experimental data as well as theoretical models. In most cases a good agreement is achieved between the results when Strouhal number and Reynolds number are not so large. By it meant that design formulas can be useful in the design of the flapping-wing propulsors.

The book may be useful for specialists working in the design of the flapping-wing propulsors.

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Chapter 1. Hydrodynamic forces on oscillating rigid wing undergoing large amplitude heaving and pitching oscillations

1.1 Setting up a problem

Plane transient movement of a thin wing problem was discussed by L.I. Sedov (1966) and A.I. Nekrasov (1948). In case of low amplitude oscillations of the profile in relation to some main motion the authors obtained the expressions for hydrodynamic forces, which allow a simple physical interpretation.

A given thin wing is moving in boundless volume of liquid, which rests on the infinity. The movement of the wing can be represented as the main movement at the speed of U_0 and some additional movement with small drift and low speed. When the wing movement is described by the coordinates XOY , which moves of the speed of U_0 , we consider that vortex wake comes off the back edge of the wing, and the Chaplygin–Jouckovsky condition for finite speed is true. There were obtained the following expressions for lifting force Y normal to the wing line and for suction force X oriented along the wing line, (1.1.1)

$$Y_L = -\lambda_{22} \frac{dv_n}{dt} - \rho \pi b U_0 (v_n - b \omega_z / 4) - \rho \frac{b}{2} U_0 \int_{b/2}^{\infty} \frac{\gamma(\xi, t) d\xi}{\sqrt{\xi^2 - (b/2)^2}},$$

(1.1.1)

$$X_s = \rho\pi b \left(v_n + (1/2\pi) \int_{b/2}^{\infty} \frac{\gamma(\xi, t) d\xi}{\sqrt{\xi^2 - (b/2)^2}} \right)^2.$$

where $\lambda_{22} = \rho\pi(b/2)^2$ is virtual mass of the wing, $b/2$ is half of chord, v_n is normal speed in the centre of the wing, $\omega = \frac{\partial \vartheta}{\partial t}$ is angular velocity, $\gamma(\xi, t)$ is vortex intensity in the trace at the distance of ξ from the centre of the wing.

With the help of simple transformations equation (1.1.1) can be transformed into notation

$$\begin{aligned} Y_L &= -\lambda_{22} \frac{dv_n}{dt} - \rho U_0 \Gamma \\ X_s &= \lambda_{22} v_n \omega_z + \rho v_n \Gamma - \rho\pi b u_* (v_n - u_*). \end{aligned} \quad (1.1.2)$$

Here the value $\Gamma = 2\pi\alpha \left(v_n - \frac{b\omega_z}{4} - u_* \right)$ may be considered as added circulation and the value $u_* = -\frac{1}{2\pi} \int_{b/2}^{\infty} \frac{\gamma(\xi, t) d\xi}{\sqrt{\xi^2 - (b/2)^2}}$ as some effective velocity induced by presence of vortex wake behind the wing.

Now let's analyze the task concerning non-stationary finite-span wing movement (fig. 1.1) analogical to the case of the infinite-span wings.

Let the wing shape be symmetrical to the wing central line OZ_1 .

Let's assume that in case of unsteady wing of finite span movement the wake influence on the wing hydrodynamic characteristics can be assumed as well as in case with infinite-span wings using some induced speed. In this case the method of flat sections can be used as it is shown in formulas (1.1.1).

$$\begin{aligned} Y_L &= -\lambda_{22} \frac{dv_n}{dt} - \rho U_0 \int_{-l}^l \Gamma(z) dz, \\ X_S &= \lambda_{22} v_n \omega_z + \rho v_n \int_{-l}^l \Gamma(z) dz - X_i, \end{aligned} \quad (1.1.3)$$

Here λ_{22} - virtual mass of wing, $X_i = \rho\pi \int_{-l}^l b(z)f_*(z)(v_n - f_*(z))dz$ -

induced drag, f_* - some effective speed induced by the wake, v_n - normal wing speed in the OZ points, $b(z)$ - wing chord in points $z=\text{const.}$, l - semi-span wing.

As v_n does not depend on z then it is possible to make an upper estimation for X_i :

$$X_i \leq \rho\pi S \frac{v_n^2}{4}, \quad (1.1.4)$$

where S is the square of the wing. The last expression shows that in case of non-steady-state motion of a finite span wing the coefficient of inductive resistance can not exceed $C_{xi} \leq \pi\alpha^2/2$, where $\alpha = v_n/U_0$ - is the instant angle of attack, U_0 is the velocity of the main movement.

Up to this point we have considered the case of low amplitude oscillations of the profile and the finite span wing. Now let us consider the case of large oscillations of the wing.

Let us consider the motion of the finite span wing into the unlimited volume of the liquid. Let the plane projection of the wing be of symmetric shape relative to OZ -axis. Let the motion of the wing be defined as periodic oscillation $y = y(t)$ and $\vartheta = \vartheta(t)$ in the coordinate system of $OXYZ$, which moves with the constant speed of U_0 in the direction of OX , see fig. 1.1. Here ϑ is the angle of a pitch.

Let us assume that under the transverse and angle oscillations at high amplitude the instant values of the angle of attack are of small values and that there is no breakaway.

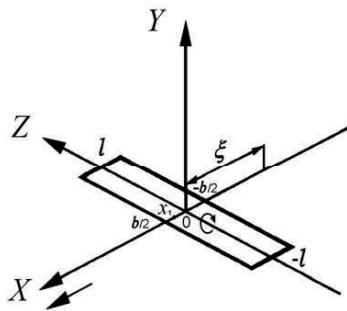


Fig..1.1 – The co-ordinate system in the wing frame of reference.

In this case due to the laws of physics the equations like (1.1.3) for the components of hydrodynamic forces are valid:

$$\begin{aligned} Y &= -\lambda_{22} \frac{dv_n}{dt} - \rho U \cos \alpha \int_{-l}^l \Gamma(z) dz, \\ X &= \lambda_{22} v_n \omega_z + \rho v_n \int_{-l}^l \Gamma(z) dz - X_i \end{aligned} \quad (1.1.5)$$

The lifting force Y , which is normal to the plane of the wing, has two components: the component depending on the media inertia and the circulation component. The vector of suction force X is perpendicular to OZ -axis in the plane of the wing. The value of X is determined by the

values of inertia component $\lambda_{22} v_n \omega_z$, of circulation component $\rho v_n \int_{-l}^l \Gamma(z) dz$ and of inductive resistance X_i . The circulation components in the expressions of the lifting force and the suction force are the corresponding projections of the Jouckovsky force $\rho U \int_{-l}^l \Gamma(z) dz$ which is normal to the vector of the instant velocity of the wing motion U . In expression (5): U is the absolute velocity of the wing movement respecting to motionless liquid, v_n is normal to the wing plane component of the velocity U ; m^* is the virtual mass, Γ is the circulation in the section of the wing Z , ρ is density of the liquid, $\omega = d\vartheta / dt$. The values of U , v_n are determined in the points of the wing symmetry axis OZ

$$v_n = V_y \cos \vartheta - U_0 \sin \vartheta = U \sin \alpha, \quad (1.1.6)$$

here $V_y = dy/dt$, α is an instant angle of attack of the wing.

1.2 Wing thrust

The OX -component of the hydrodynamic forces are

$$F_x = X \cos \vartheta - Y \sin \vartheta - \frac{\rho S U^2}{2} C_p \cos \vartheta. \quad (1.2.1)$$

On the basis of (1.1.5) and (1.2.1) we can take the expression for F_x into another expression:

$$F_x = \lambda_{22} \frac{d(v_n \sin \vartheta)}{dt} + \rho V_y \int_{-l}^l \Gamma(z) dz - X_i \cos \vartheta - \frac{\rho S U^2}{2} C_p \cos \vartheta. \quad (1.2.2)$$

Therefore under periodic oscillations of the wing, period average F_x depends mainly on a circulation member and on a inductive resistance.

Under linear approximation like in case of small oscillations of the wing the lifting force Y can be estimated with equation

$$Y = -\lambda_{22} \frac{dv_n}{dt} - \rho U \cos \alpha \int_{-l}^l \Gamma(z) dz = \frac{\rho U^2}{2} S \begin{pmatrix} -C_y^\alpha \frac{v_n}{U} - C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} + \\ + C_y^{\omega_z} \frac{\omega_z}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \end{pmatrix}. \quad (1.2.3)$$

Let us consider that the coefficients of the hydrodynamic derivatives keep constant during the period of oscillation and depend on the Strouhal number, which looks like:

$$Sh_0 = \frac{\omega b}{U_0} \quad (1.2.4)$$

From (1.1.5) and ((1.2.3) we have

$$\int_{-l}^l F(z) dz = -\frac{\lambda_{22}\dot{v}_n}{\rho U \cos \alpha} + \frac{US}{2 \cos \alpha} \begin{pmatrix} C_y^\alpha \frac{v_n}{U} + C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} - \\ -C_y^{\omega_z} \frac{\omega_z b}{U} - C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \end{pmatrix}. \quad (1.2.5)$$

Taking into account (1.2.5) we get

$$F_x = \lambda_{22} \frac{d(v_n \sin \theta)}{dt} - \frac{\lambda_{22} \dot{v}_n V_y}{U \cos \alpha} + \frac{\rho V_y US}{2 \cos \alpha} \begin{pmatrix} C_y^\alpha \frac{v_n}{U} + C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} - \\ -C_y^{\omega_z} \frac{\omega_z b}{U} - C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \end{pmatrix} - X_i \cos \theta - \frac{\rho S U^2}{2} C \cos \theta. \quad (1.2.6)$$

Considering $U \sin \theta = V_y$ we get

$$F_x = \left\{ \begin{array}{l} \lambda_{22} \frac{d(v_n \sin \theta)}{dt} + \frac{\rho S}{2 \cos \alpha} \begin{pmatrix} C_y^\alpha v_n V_y + b \left(C_y^{\dot{\alpha}} - \frac{2m^*}{\rho S b} \right) \dot{v}_n \sin \theta - \\ -C_y^{\omega_z} b \omega_z V_y - C_y^{\dot{\omega}_z} b^2 \dot{\omega}_z \sin \theta \end{pmatrix} - \\ -X_i \cos \theta - \frac{\rho S U^2}{2} C \cos \theta \end{array} \right\} \quad (1.2.7)$$

Here $\theta = \alpha + \vartheta$ is the angle of the slope of the wing motion trajectory.

The formula (1.2.7) stems from the assumption that the wing cinematic parameters are given with respect to the wing center. But, more important is when the wing cinematic parameters are given with respect to the arbitrary point of the longitudinal wing axis. This is of particular value for fish and dolphin swimming. There is a reason to think that the dolphin rotational wing axes should be arranged as near as possible to the back end of the dolphins fluke (Lighthill, 1969, 1970, Wu, 1971). The existing estimates of the dolphin rotational wing axes position on a basis of experimental data reinforce this assumptions (Romanenko, 2001; Romanenko, 2002).

Let's assume that the coordinate system $OXYZ$ movement is directed in OX axes and at a steady rate of U_0 . The wing motion is given by the periodic law of the point x_1 (Fig. 1.1).

To make an estimate of the wing hydrodynamic forces in this case we can use the formula (1.2.5) but all the wing kinematic equations must be written relatively of the wing center.

The wing movement relatively of the wing center is determined by equations:

$$V_{xc} = V_0 - \omega_z x \sin \vartheta , \quad (1.2.8)$$

$$V_{yc} = V_{y1} + \omega_z x \cos \theta, \quad (1.2.9)$$

where $V_{y1} = \dot{y}(t)$, $\omega_z = \dot{\theta}(t)$, $y(t)$ - heaving, x - the distance of the wing center from the point x_1 . The point which is placed directly above of symbols (here and next) denotes the time derivative.

The formula (1.2.7) may be written as

$$F_{xc} = \lambda_{22} \frac{d(v_{nc} \sin \theta)}{dt} + \frac{\rho S}{2 \cos \alpha_c} \left(C_{yc}^{\alpha} v_{nc} V_{yc} + b \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \dot{v}_{nc} \sin \theta_c - \right. \\ \left. - C_{yc}^{\omega_z} b \omega_z V_{yc} - C_{yc}^{\dot{\omega}_z} b^2 \dot{\omega}_z \sin \theta_c \right) - \\ - X_i \cos \theta - \frac{\rho S U_c^2}{2} C \cos \theta. \quad (1.2.10)$$

Here and next F_{xc} - the thrust, λ_{22} - virtual mass of the wing, v_{nc} - the normal velocity, ρ - the water density, θ_c - the angle between the flow of water and horizontal axes, C - the double sum of the drag coefficient and the shape drag coefficient, U_c - the instantaneous flow velocity, X_i - the wing induced drag, b - the wing chord, $C_{yc}^{\alpha}, C_{yc}^{\alpha'}, C_{yc}^{\omega_z}, C_{yc}^{\dot{\omega}_z'}$ - aerodynamic (rotary) derivatives coefficients (Belotserkovskii, 1958; Belotserkovskii, Scripatch, Tabatchnicov, 1971). The formula (1.2.10) differs from formula (1.2.7) by the availability of index “ c ” by

symbols, which was written relatively 4o 4he wing center. In a similar manner design equations may be derived for other parameters

$$v_{yc} = V_{y1} \cos \vartheta - U_0 \sin \vartheta + \omega_z x = U_c \sin \alpha_c, \quad (1.2.11)$$

$$\theta_c = \alpha_c + \vartheta = \arctg(V_{yc} / V_{xc}), \quad (1.2.12)$$

$$U_c^2 = V_{yc}^2 + V_{xc}^2, \quad (1.2.13)$$

here α_c - the angle of attack which was written relatively of the wing center. The angle of the slope of the wing is free from index “c” because it is alike on the wing surface. Parameters $\sin \vartheta$ and $\cos \vartheta$ can be expressible as (with the supposition of the small in size of angle of attack)

$$\sin \vartheta \approx \sin \theta_i - \alpha_i \cos \theta_i, \quad (1.2.14)$$

$$\cos \vartheta \approx \cos \theta_i + \alpha_i \sin \theta_i. \quad (1.2.15)$$

here

$$\cos \theta_i = \frac{U_0}{U_i}, \quad (1.2.16)$$

$$\sin \theta_i = \frac{\dot{y}}{U_i}, \quad (1.2.17)$$

$$U_i = \sqrt{U_0^2 + (\dot{y})^2}. \quad (1.2.18)$$

1.3 Wing efficiency

The wing efficiency is equal to useful and common energy ratio

$$\eta = \frac{\bar{A}_c}{\bar{P}_c}, \quad (1.3.1)$$

where

$$\bar{A}_c = \bar{F}_{sc} U_0 \quad (1.3.2)$$

and

$$\bar{P}_c = -\overline{F_{yc}V_{yc}} - \overline{M_{zc}\omega_z}. \quad (1.3.3)$$

Here F_{xc} and F_{yc} are horizontal and vertical forces, V_{yc} is the wing vertical velocity, M_{zc} is the forces moment relatively of the wing pitch-axes. This moment is usually reported as

$$M_{zc} = \frac{\rho S b U_c^2}{2} \left[-m_z^\alpha \frac{v_n}{U_c} - m_z^{\dot{\alpha}} \frac{\dot{v}_n b}{U_c^2} + m_z^{\omega_z} \frac{\omega_z b}{U_c} + m_z^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U_c^2} \right]. \quad (1.3.4)$$

Here $m_z^\alpha, m_z^{\dot{\alpha}}, m_z^{\omega_z}, m_z^{\dot{\omega}_z}$ - aerodynamic (rotary) derivative coefficients of the moment which were written relatively of the wing center (Belotserkovskii, 1958; Belotserkovskii, Scripatch, Tabatchnicov, 1971).

Projection of the hydrodynamic forces onto the OY axes can be expressed as

$$F_y = X_S \sin \vartheta + Y_L \cos \vartheta - \frac{\rho S U^2}{2} C \sin \vartheta. \quad (1.3.5)$$

This formula (with regard to formula 1.2.5) can be expressed as

$$F_y = \begin{bmatrix} \lambda_{22}v_n\omega_z \sin \vartheta - \frac{\lambda_{22}v_n\dot{v}_n \sin \vartheta}{U \cos \alpha} + \\ + \frac{\rho v_n S U \sin \vartheta}{2 \cos \alpha} \left(C_y^\alpha \frac{v_n}{U} + C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} - \right. \\ \left. - C_y^{\omega_z} \frac{\omega_z b}{U} - \frac{C_y^{\dot{\omega}_z} \dot{\omega}_z b^2}{U^2} \right) - \\ - \frac{\rho U^2 S \cos \vartheta}{2} \left(C_y^\alpha \frac{v_n}{U} + C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} - \right. \\ \left. - C_y^{\omega_z} \frac{\omega_z b}{U} - C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right) - \\ - X_i \sin \vartheta - \frac{\rho S U^2}{2} C \sin \vartheta \end{bmatrix}. \quad (1.3.6)$$

In the right-hand side of the formula (1.3.6) the expression $\lambda_{22}\dot{v}_n \cos \vartheta$ can be added to and subtracted from. Then

$$F_y = \begin{cases} -\lambda_{22} \frac{d(v_n \cos \theta)}{dt} + \\ + \left(\cos \theta - \frac{v_n \sin \theta}{U \cos \alpha} \right) \left[\frac{2\lambda_{22}\dot{v}_n}{\rho S U^2} - \right. \\ \left. - \left(C_y^\alpha \frac{v_n}{U} + C_y^\dot{\alpha} \frac{\dot{v}_n b}{U^2} - \right. \right. \\ \left. \left. - C_y^{\omega_z} \frac{\omega_z b}{U} - \frac{C_y^{\dot{\omega}_z} \dot{\omega}_z b^2}{U^2} \right) \right] \frac{\rho S U^2}{2} - \\ - X_i \sin \theta - \frac{\rho S U^2}{2} C \sin \theta. \end{cases} \quad (1.3.7)$$

Taking into account that $\frac{v_n}{U} = \sin \alpha$ we can derivate the equation

$$F_y = \begin{cases} -\lambda_{22} \frac{d(v_n \cos \theta)}{dt} + \\ + \frac{\cos \theta}{\cos \alpha} \frac{\rho S}{2} \left[-C_y^\alpha v_n U - b \left(C_y^\dot{\alpha} - \frac{2\lambda_{22}}{\rho S b} \right) \dot{v}_n + \right. \\ \left. + C_y^{\omega_z} \omega_z b U + C_y^{\dot{\omega}_z} \dot{\omega}_z b^2 \right] - \\ - X_i \sin \theta - \frac{\rho S U^2}{2} C \sin \theta. \end{cases} \quad (1.3.8)$$

The formula (1.3.8) stems from the assumption that the wing cinematic parameters are given with respect to the wing center. But when the wing cinematic parameters are given with respect to the arbitrary point of the longitudinal wing axis and aerodynamic derivatives

coefficients are given with respect to the wing pressure center (Belotserkovskii, 1958) this formula can be represented as

$$F_{yc} = \left\{ \begin{array}{l} -\lambda_{22} \frac{d(v_{nc} \cos \theta)}{dt} + \\ + \frac{\rho S}{2 \cos \alpha_c} \left[-C_{ye}^\alpha v_{nc} U_c \cos \theta_c - \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) b \dot{v}_{nc} \cos \theta_c + \right. \\ \left. + C_{ye}^{\omega_z} \omega_z b U_c \cos \theta_c + C_{yc}^{\dot{\omega}_z} \dot{\omega}_z b^2 \cos \theta_c \right] - \\ - X_{ic} \sin \theta - \frac{\rho S U_c^2}{2} C \sin \theta. \end{array} \right\} \quad (1.3.9)$$

In this formula index «c» denotes that parameters where written relatively of the wing center. When it is considered that $U_c \cos \theta_c = V_{xc}$ and $\cos \alpha \approx 1$ then

$$F_{yc} = \left\{ \begin{array}{l} -\lambda_{22} \frac{d(v_{nc} \cos \theta)}{dt} + \\ + \frac{\rho S}{2} \left[-C_{ye}^\alpha v_{nc} V_{xc} - \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) b \dot{v}_{nc} \cos \theta_c + \right. \\ \left. + C_{ye}^{\omega_z} \omega_z b V_{xc} + C_{yc}^{\dot{\omega}_z} \dot{\omega}_z b^2 \cos \theta_c \right] - \\ - X_{ic} \sin \theta - \frac{\rho S U_c^2}{2} C \sin \theta. \end{array} \right\} \quad (1.3.10)$$

1.4. Suction force

Let's estimate the contribution of suction force (X_{sc}) in the total thrust. Then

$$X_{sc} = \lambda_{22} v_{nc} \omega_z \cos \theta - \lambda_{22} \dot{v}_{nc} \alpha_c \cos \theta + \\ + \frac{\rho S}{2} \left(C_{y\bar{n}}^{\alpha} v_{nc}^2 + C_{y\bar{n}}^{\dot{\alpha}} \dot{v}_{nc} b \alpha_c - C_{y\bar{n}}^{\omega_z} \omega_z b v_{nc} - C_{y\bar{n}}^{\dot{\omega}_z} \dot{\omega}_z b^2 \alpha_c \right) \cos \theta. \quad (1.4.1)$$

1.5. (Aerodynamic (rotary) derivative coefficients conversation to the wing center.

Tabulated the rotary derivatives coefficients are consistent with standart coordinate system (x,y,z) at the origin placed at a range of $\frac{1}{4}$ wing chord from a wing leading edge.

Since the cinematic parameters are converted to the wing center the aerodynamic (rotary) derivatives coefficients must be converted as well.

By way of example let us convert aerodynamic (rotary) derivatives coefficients from the two-dimensional wing pressure centre (of $\frac{1}{4}$ wing chord from a wing leading edge) to the wing center. Here we examine the case when Strouhal number is equal to 1. The conversion results are given in the Table.

The first and the second rows content the initial values of the aerodynamic (rotary) derivatives coefficients from Table number 13 of the paper (Belotserkovskii, 1958). The third and the fourth rows content the results of the conversions from the wing pressure centre to the wing centre.

Table. Aerodynamic (rotary) derivative coefficients: initial values and results of the conversions from the wing pressure centre to the wing centre.

C_y^α	$C_y^{\dot{\alpha}}$	$C_y^{\omega_z}$	$C_y^{\dot{\omega}_z}$	m_z^α	$m_z^{\dot{\alpha}}$	$m_z^{\omega_z}$	$m_z^{\dot{\omega}_z}$
3.757	0.6239	1.878	-0.0807	0	$-\pi/8$	$-\pi/8$	$-3\pi/64$
C_{yc}^α	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{\omega_z}$	$C_{yc}^{\dot{\omega}_z}$	m_{zc}^α	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{\omega_z}$	$m_{zc}^{\dot{\omega}_z}$
3.757	0.6239	0.9388	-0.2367	0.9393	-0.2365	-0.1578	-0.1082

Conversion formulas would be expressible as: $C_{yc}^\alpha = C_y^\alpha$, $C_{yc}^{\dot{\alpha}} = C_y^{\dot{\alpha}}$,

$$C_{yc}^{\omega_z} = C_y^{\omega_z} + C_y^\alpha \xi_0, \quad C_{yc}^{\dot{\omega}_z} = C_y^{\dot{\omega}_z} + C_y^{\dot{\alpha}} \xi_0, \quad m_{zc}^\alpha = m_z^\alpha - C_y^\alpha \xi_0, \quad m_{zc}^{\dot{\alpha}} = m_z^{\dot{\alpha}} - C_y^{\dot{\alpha}} \xi_0,$$

$$m_{zc}^{\omega_z} = m_z^{\omega_z} - (C_y^{\omega_z} - m_z^\alpha) \xi_0 - C_y^\alpha \xi_0^2,$$

Here: $\xi_0 = -0.25$.

The formulas (1.2.10), (1.3.1)-(1.3.10) и (1.4.1) are true for all cinematic parameters of the symmetric wings.

In the special case that oscillating wings are used as a mover it is essential to calculate the time-average wing hydrodynamic characteristics. To do this formulas (1.2.10), (1.3.1)-(1.3.10) и (1.4.1) evolved into calculating formulas for hydrodynamic characteristics by three variants of heaving and pitching motion.

a) The wing executes heaving and pitching periodic motion. The phase angle by which the pitch motion leads the heave motion is arbitrary.

$$y = y_0 \sin \omega t, \quad (1.5.1)$$

$$\vartheta = \vartheta_0 \sin(\omega t + \varphi). \quad (1.5.2)$$

Here φ - the phase angle.

6) The wing executes heaving and angle of attack periodic motion. The phase angle by which the pitch motion leads the heave motion is 90° .

$$y = y_0 \sin \omega t, \quad (1.5.3)$$

$$\alpha = \alpha_0 \cos \omega t. \quad (1.5.4)$$

b) The wing executes pitching and angle of attack periodic motion. The phase angle between the pitching and angle of attack is absent.

$$\theta = \theta_0 \cos \omega t, \quad (1.5.5)$$

$$\alpha = \alpha_0 \cos \omega t. \quad (1.5.6)$$

It was mentioned above that the wing heaving is not harmonic (Prempraneerach P, 2003) and it shows up as: $\dot{y} = U_0 \operatorname{tg}(\alpha + \theta)$. When angles α and θ are very small the wing heaving is near harmonic. The

derived calculating formulas can be found in the published literature (Pushkov и др., 2000,2006; Romanenko, 2001; Romanenko, 2002; Romanenko и др., 2005,2007,2008,2009).

The formulas (1.2.10), (1.3.10) and (1.4.1) are useful for pure heaving and pure pitching. In the former case should be $\vartheta_0 = 0$. In another case should be $\lambda_p = \frac{U_0}{y_0 \omega} = \infty$.

1.6. The thrust design formulas.

Let us analyse a common case when the point x_1 cinematic is presented as (see above 1.5.1 and 1.5.2)

$$y = y_0 \sin \omega t, \quad (1.6.1)$$

$$\vartheta = \vartheta_0 \sin(\omega t + \varphi). \quad (1.6.2)$$

The time-average rate of the thrust has been obtained formerly (see above 1.2.10) and is shown up as (on the assumption that the angle of attack is small)

$$\overline{F_{xc}} = \frac{\rho S}{2} \left\{ \begin{aligned} & C_y^\alpha \overline{v_{nc} V_{yc}} + b \left(C_y^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \overline{\dot{v}_{nc} \sin \theta_c} - \\ & - b C_y^{\omega_z} \overline{\omega_z V_{yc}} - C_y^{\dot{\omega}_z} b^2 \overline{\dot{\omega}_z \sin \theta_c} - \\ & - \frac{\pi}{2} \overline{v_{nc}^2 \cos \vartheta} - C \overline{U_c^2 \cos \vartheta} \end{aligned} \right\}. \quad (1.6.3)$$

The formula (1.6.3) can be shown up as

$$C_T = \frac{2\overline{F_{xc}}}{\rho S U_0^2} = C_{T1} + C_{T2} + C_{T3} + C_{T4} + C_{T5} + C_{T6}. \quad (1.6.4)$$

The thrust coefficients can be shown up as

$$C_{T1} = \frac{C_{yc}^\alpha \overline{v_{nc} V_{yc}}}{U_0^2} = C_{yc}^\alpha \left\{ \begin{aligned} & \frac{1}{2\lambda_p^2} \left[1 - 0.125 g_0^2 (2 - \cos 2\varphi) \right] - \\ & - \frac{g_0 \sin \varphi}{2\lambda_p} (1 - 0.125 g_0^2) \\ & + \frac{(Sh_0) g_0 X_b \cos \varphi}{2\lambda_p} (1 - 0.25 g_0^2) + \\ & + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125 g_0^2) \end{aligned} \right\}, \quad (1.6.5)$$

Here and further $X_b = \frac{x}{b}$.

$$C_{T2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{\overline{b \dot{v}_{nc} \sin \theta_c}}{U_0^2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) G. \quad (1.6.6)$$

Here $G = \sum_{n=1}^{n=13} G_n$

$$G_1 = (Sh_0)g_0 \begin{Bmatrix} -\frac{\cos \varphi}{2\lambda_p} (1 - 0.5g_0^2) - \\ -\frac{g_0^2 \cos \varphi}{8\lambda_p} (\cos^2 \varphi - 0.1667g_0^2) - \\ -\frac{(Sh_0)g_0 X_b}{2} (1 - 0.125g_0^2) \end{Bmatrix}, \quad (1.6.7)$$

$$G_2 = \frac{\sqrt{2}(Sh_0)g_0^2 \sin 2\varphi}{\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \begin{Bmatrix} 0.75 \left(0.25 + \frac{0.0233}{(2\lambda_p^2 + 1)^2} \right) - \\ 0.25 - \frac{0.0313}{(2\lambda_p^2 + 1)} + \\ + \left(\frac{0.0238}{(2\lambda_p^2 + 1)^2} - \frac{0.0194}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi - \\ - \left(\frac{0.0468}{(2\lambda_p^2 + 1)^2} + \frac{0.0342}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \end{Bmatrix}, \quad (1.6.8)$$

$$G_3 = \frac{\sqrt{2} (Sh_0) g_0^2}{\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[\begin{aligned}
& -0.5 \left(\begin{aligned}
& 0.25 - \frac{0.125}{(2\lambda_p^2 + 1)} + \\
& + \frac{0.07}{(2\lambda_p^2 + 1)^2} - \frac{0.041}{(2\lambda_p^2 + 1)^3} \end{aligned} \right) \sin 2\varphi + \\
& \left[\begin{aligned}
& 0.125 - \frac{0.0586}{(2\lambda_p^2 + 1)^2} + \\
& + \frac{0.1234}{(2\lambda_p^2 + 1)^3} \end{aligned} \right] \sin \varphi \cos^3 \varphi + \\
& + \left(\begin{aligned}
& -\frac{0.0391}{(2\lambda_p^2 + 1)} + \\
& + \frac{0.0587}{(2\lambda_p^2 + 1)^2} - \frac{0.0733}{(2\lambda_p^2 + 1)^3} \end{aligned} \right) \cos^4 \varphi + \\
& + \left(\begin{aligned}
& 0.125 - \frac{0.0625}{(2\lambda_p^2 + 1)} + \\
& + \frac{0.0469}{(2\lambda_p^2 + 1)^2} - \frac{0.1016}{(2\lambda_p^2 + 1)^3} \end{aligned} \right) \sin^3 \varphi \cos \varphi + \\
& + \left(\begin{aligned}
& -\frac{0.07}{(2\lambda_p^2 + 1)} + \\
& + \frac{0.1055}{(2\lambda_p^2 + 1)^2} - \frac{0.1318}{(2\lambda_p^2 + 1)^3} \end{aligned} \right) \sin^2 \varphi \cos^2 \varphi
\end{aligned} \right], \quad (1.6.9)$$

$$G_4 = G_{4-0} \begin{cases} \left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) - \\ \quad \left[\left(0.125 + \frac{0.0118}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \right. \\ \quad \left. - 1.5g_0^2 \left[\left(0.125 - \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \right. \right. \\ \quad \left. \left. \left. + \frac{0.0586}{(2\lambda_p^2 + 1)^2} - \frac{0.041}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \right] \right], \end{cases}, \quad (1.6.10)$$

Here $G_{4-0} = -\frac{\sqrt{2}(Sh_0)g_0}{\sqrt{(2\lambda_p^2 + 1)}} \cos \varphi.$

$$G_5 = G_{5-0} \begin{cases} \left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) - \\ \quad \left[\left(0.375 + \frac{0.0116}{(2\lambda_p^2 + 1)^2} + \frac{0.0589}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi + \right. \\ \quad \left. - g_0^2 \left[\left(0.375 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0819}{(2\lambda_p^2 + 1)^2} - \frac{0.0409}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \right] \right], \end{cases}, \quad (1.6.11)$$

here $G_{5-0} = -\frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi$

$$G_6 = \frac{\sqrt{2}(Sh_0)g_0 \cos \varphi}{\sqrt{(2\lambda_p^2 + 1)}} \left\{ \begin{array}{l} 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} + \frac{0.0585}{(2\lambda_p^2 + 1)^3} - \\ - 1.67g_0^2 \left[\begin{array}{l} \left(0.375 + \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ \left. + \frac{0.0819}{(2\lambda_p^2 + 1)^2} + \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi + \\ + \left(0.375 + \frac{0.0411}{(2\lambda_p^2 + 1)} \right) \sin^2 \varphi \end{array} \right] \end{array} \right\} \quad (1.6.12)$$

$$G_7 = G_{7-0} \left\{ \begin{array}{l} \left(0.125 - \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \left(\begin{array}{l} 0.125 - \\ - \frac{0.125}{(2\lambda_p^2 + 1)} + \\ + \frac{0.0586}{(2\lambda_p^2 + 1)^2} \end{array} \right) \sin^2 \varphi - \\ - 0.83g_0^2 \left[\begin{array}{l} \left(0.0625 + \right. \\ \left. + \frac{0.2167}{(2\lambda_p^2 + 1)^2} \right) \cos^4 \varphi + \\ \left(0.031 - \right. \\ \left. - \frac{0.02}{(2\lambda_p^2 + 1)} + \right) \\ + \frac{0.019}{(2\lambda_p^2 + 1)^2} + \sin^2 2\varphi + \\ + \frac{0.0625}{(2\lambda_p^2 + 1)^3} \\ + \left(0.0625 - \frac{0.086}{(2\lambda_p^2 + 1)} + \frac{0.2322}{(2\lambda_p^2 + 1)^2} \right) \sin^4 \varphi \end{array} \right] \end{array} \right\}, \quad (1.6.13)$$

$$\text{Here } G_{7-0} = \frac{\sqrt{2}(Sh_0)g_0^3 \cos \varphi}{\sqrt{(2\lambda_p^2 + 1)}}.$$

$$G_8 = G_{8-0} \begin{cases} \left(\frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1172}{(2\lambda_p^2 + 1)^3} \right) - \\ -1.17g_0^2 \left[\left(\frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \right. \\ \left. + \left(\frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right) \sin^2 \varphi \right] \end{cases}, \quad (1.6.14)$$

$$\text{Here } G_{8-0} = -\frac{\sqrt{2}(Sh_0)g_0^2 \lambda_p \sin \varphi \cos \varphi}{\sqrt{(2\lambda_p^2 + 1)}}.$$

$$G_9 = \frac{\sqrt{2}(Sh_0)^2 g_0^2 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \left\{ \begin{aligned} & \left[\begin{aligned} & \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \right. \\ & \left. + \left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} \right) \sin^2 \varphi \end{aligned} \right] - \\ & - 0.67g_0^2 \left[\begin{aligned} & \left(0.75 + \frac{0.0235}{(2\lambda_p^2 + 1)^2} \right) \sin^2 \varphi \cos^2 \varphi + \\ & + \left(0.375 - \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ & \left. + \frac{0.0819}{(2\lambda_p^2 + 1)^2} - \frac{0.0409}{(2\lambda_p^2 + 1)^3} \right) \sin^4 \varphi + \\ & + \left(0.375 + \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ & \left. + \frac{0.0813}{(2\lambda_p^2 + 1)^2} + \frac{0.06}{(2\lambda_p^2 + 1)^3} \right) \cos^4 \varphi \end{aligned} \right] \end{aligned} \right\}, \quad (1.6.15)$$

$$G_{10} = G_{10-0} \left\{ \begin{aligned} & \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} + \frac{0.0589}{(2\lambda_p^2 + 1)^3} \right) - \\ & - g_0^2 \left[\begin{aligned} & \left(0.125 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0586}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \\ & + \left(0.125 + \frac{0.3632}{(2\lambda_p^2 + 1)^2} - \frac{0.8788}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \end{aligned} \right] \end{aligned} \right\}, \quad (1.6.16)$$

$$\text{here } G_{10-0} = \frac{\sqrt{2}(Sh_0)^2 g_0 X_b \sin \varphi}{\sqrt{(2\lambda_p^2 + 1)}}.$$

$$G_{11} = G_{11-0} \left\{ \begin{aligned} & \left[0.125 - \frac{0.0625}{(2\lambda_p^2 + 1)} + \frac{0.0469}{(2\lambda_p^2 + 1)^2} - \right. \\ & \left. - \left(\frac{0.0625}{(2\lambda_p^2 + 1)} + \frac{0.0234}{(2\lambda_p^2 + 1)^3} \right) \cos 2\varphi + \right. \\ & \left. + \left(-\frac{0.0821}{(2\lambda_p^2 + 1)^2} + \frac{0.1153}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi + \right. \\ & \left. + \left(\frac{0.0587}{(2\lambda_p^2 + 1)^2} - \frac{0.0786}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \right] - \\ & \left[0.25 - \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ & \left. + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.1478}{(2\lambda_p^2 + 1)^3} \right] \cos^4 \varphi + \\ & -0.67g_0^2 + \left[-0.0625 + \frac{0.1173}{(2\lambda_p^2 + 1)} - \right. \\ & \left. - \frac{0.0856}{(2\lambda_p^2 + 1)^2} \right] \sin^2 \varphi \cos^2 \varphi + \\ & + \left(0.0625 - \frac{0.0154}{(2\lambda_p^2 + 1)^3} \right) \sin^4 \varphi \end{aligned} \right\} \quad (1.6.17)$$

$$\text{here } G_{11-0} = -\frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b \sin \varphi}{\sqrt{(2\lambda_p^2 + 1)}}.$$

$$G_{12} = G_{12-0} \left\{ \begin{aligned} & \left[\left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi + \right. \\ & \left. + \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.3163}{(2\lambda_p^2 + 1)^2} + \frac{0.0589}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \right] - \\ & - g_0^2 \left[\begin{aligned} & \left(0.125 + \frac{0.0118}{(2\lambda_p^2 + 1)^2} \right) (\cos^4 \varphi + \sin^4 \varphi) + \\ & + \left(0.25 + \frac{0.1171}{(2\lambda_p^2 + 1)^2} + \frac{0.018}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \cos^2 \varphi \end{aligned} \right] \end{aligned} \right\}, \quad (1.6.18)$$

$$\text{here } G_{12-0} = -\frac{\sqrt{2}(Sh_0)^2 g_0^2 \lambda_p X_b}{\sqrt{(2\lambda_p^2 + 1)}}$$

$$G_{13} = G_{13-0} \left\{ \begin{aligned} & \left[\left(-\frac{0.25}{(2\lambda_p^2 + 1)} - \frac{1}{(2\lambda_p^2 + 1)^3} \right) - \right. \\ & \left. \left(-0.25 - \frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.07}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \right] \\ & -0.5g_0^2 \left[\left(\frac{0.25 +}{(2\lambda_p^2 + 1)^2} + \frac{0.0235}{(2\lambda_p^2 + 1)^2} \right) \cos 2\varphi + \right. \\ & \left. \left(\frac{0.25 -}{(2\lambda_p^2 + 1)} + \frac{0.07}{(2\lambda_p^2 + 1)^2} - \frac{0.5875}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \right] \end{aligned} \right\}, \quad (1.6.19)$$

here $G_{13-0} = -\frac{(Sh_0)^3 g_0^2 \lambda_p X_b^2 \sin \varphi \cos \varphi}{\sqrt{(2\lambda_p^2 + 1)}}$

$$C_{T3} = -C_{yc}^{m_z} \frac{(Sh_0) g_0}{2} \left\{ \frac{1}{\lambda_p} \cos \varphi + (Sh_0) g_0 X_b (1 - 0.125 g_0^2) \right\}, \quad (1.6.20)$$

$$C_{T4} = -C_{yc}^{\dot{\omega}_z} \left(\sum_{n=1}^{n=3} K_n \right). \quad (1.6.21)$$

here

$$K_1 = (Sh_0)^2 g_0^2 \left\{ \begin{aligned} & -\frac{1}{2}(1 - 0.125g_0^2) + \\ & + \frac{\sqrt{2}g_0}{2\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \cos^2 \varphi \left[0.5 + \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right] - \\ & - \frac{\sqrt{2}}{2g_0\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} \right. - \\ & \quad \left. - \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] - \\ & - g_0^2 \left[\begin{aligned} & \left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} \right) + \\ & - \frac{0.0586}{(2\lambda_p^2 + 1)^3} + \frac{0.0854}{(2\lambda_p^2 + 1)^4} \end{aligned} \right] \\ & + \left[\begin{aligned} & \left(-0.25 + \frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0703}{(2\lambda_p^2 + 1)^2} \right) \cos 2\varphi \\ & - \frac{0.0586}{(2\lambda_p^2 + 1)^3} - \frac{0.0427}{(2\lambda_p^2 + 1)^4} \end{aligned} \right] \end{aligned} \right\} \quad (1.6.22)$$

$$K_2 = K_{2-0} \left\{ \begin{aligned} & \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.3517}{(2\lambda_p^2 + 1)^3} \right) - \\ & - 0.67g_0^2 \left[\begin{aligned} & \left(0.375 + \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ & \quad \left. + \frac{0.082}{(2\lambda_p^2 + 1)^2} + \frac{0.1758}{(2\lambda_p^2 + 1)^3} \right) \cos^2 \varphi + \\ & + \left(0.375 + \frac{0.0352}{(2\lambda_p^2 + 1)^2} \right) \sin^2 \varphi \end{aligned} \right] \end{aligned} \right\}, \quad (1.6.23)$$

$$\text{here } K_{2-0} = \frac{\sqrt{2}(Sh_0)^2 g_0^2 \lambda_p}{\sqrt{(2\lambda_p^2 + 1)}} \cos^2 \varphi.$$

$$K_3 = K_{3-0} \begin{cases} \left(0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) - \\ - 0.67g_0^2 \left[\begin{array}{l} \left(0.375 + \frac{0.0352}{(2\lambda_p^2 + 1)^2} \right) \cos^2 \varphi + \\ + \left(\frac{0.375 - 0.1055}{(2\lambda_p^2 + 1)^2} + \frac{0.1935}{(2\lambda_p^2 + 1)^3} \right) \sin^2 \varphi \end{array} \right] \end{cases}. \quad (1.6.24)$$

$$\text{here } K_{3-0} = \frac{\sqrt{2}(Sh_0)^2 g_0^2 \lambda_p}{\sqrt{(2\lambda_p^2 + 1)}} \sin^2 \varphi$$

$$C_{T5} = -\frac{\pi}{2} \begin{cases} \left(\frac{1}{\lambda_p^2} \left[\begin{array}{l} 0.5 - 0.1875g_0^2(2 - \cos 2\varphi) + \\ 0.0469g_0^4(1 + 4\sin^2 \varphi) \end{array} \right] - \right. \\ \left. - \frac{g_0}{\lambda_p} \sin \varphi (1 - 0.875g_0^2 + 0.256g_0^4) + \right. \\ \left. + g_0^2 (0.5 - 0.3125g_0^2 + 0.0608g_0^4) + \right. \\ \left. + \frac{2(Sh_0)g_0 X_b}{\lambda_p} \cos \varphi (0.5 - 0.125g_0^2) + \right. \\ \left. + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125g_0^2) \right] \end{cases}, \quad (1.6.25)$$

$$C_{T6} = -C \left\{ \begin{aligned} & \left(1 - 0.25g_0^2 \right) + \frac{1}{2\lambda_p^2} \left[1 - 0.125g_0^2 (2 - \cos 2\varphi) \right] + \\ & \frac{2(Sh_0)g_0X_b}{\lambda_p} \cos \varphi (0.5 - 0.125g_0^2) + \\ & + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125g_0^2) \end{aligned} \right\}. \quad (1.6.26)$$

The terms in the right-hand side of the formula (1.6.2) are of different values. The first, the fifth and the sixth members are considerably more than the second, the third and the fifth. The first approximation of the small members can be eliminated from consideration. In this case the error incurred in shall be no more than 10%.

Inductive reactance (formula C_{T5}) was estimated as maximum («upper» estimation):

$$X_i \leq \frac{\rho\pi S v_n^2}{4}, \quad (1.6.27)$$

It was shown before (paragraph 1.1) that this formula is a reasonable approximation to the exact equation to calculate the wing propulsive characteristics when the wing aspect ratio is in the range from 2 to 5 or when inductive reactance is very small. In a general case when the wing aspect ratio is unbounded on the interval from 2 to 5 a closer approximation to the exact equation is derivable from common equations (paragraph 1.7.1).

It is easy to convert the formulas to pure heaving and to pure pitching oscillation. In the first case it was reasoned that $\vartheta = 0$, in the second case $\lambda_p = \infty$.

To pure heaving

$$C_{T1} = C_y^\alpha \left(0.5 \frac{1}{\lambda_p^2} \right), \quad (1.6.28)$$

$$C_{T2} = C_{T3} = C_{T4} = 0, \quad (1.6.29)$$

$$C_{T5} = -\frac{\pi}{2} \left(0.5 \frac{1}{\lambda_p^2} \right), \quad (1.6.30)$$

$$C_{T6} = -C \left(1 + \frac{0.5}{\lambda_p^2} \right). \quad (1.6.31)$$

To pure pitching oscillations

$$C_{T1} = C_{yc}^\alpha \left[\frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left(1 - 0.125 g_0^2 \right) \right], \quad (1.6.32)$$

$$C_{T2} = -\left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) (G_1 + G_{12}), \quad (1.6.33)$$

where

$$G_1 = -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125 g_0^2), \quad (1.6.34)$$

$$G_{12} = -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.25 g_0^2), \quad (1.6.35)$$

$$C_{T3} = -C_{yc}^{\omega_z} \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125 g_0^2), \quad (1.6.36)$$

$$C_{T4} = 0.1875 C_{yc}^{\dot{\omega}_z} (Sh_0)^2 g_0^4, \quad (1.6.37)$$

$$C_{T5} = -\frac{\pi}{2} \left\{ g_0^2 (0.5 - 0.3125 g_0^2) + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125 g_0^2) \right\}, \quad (1.6.38)$$

$$C_{T6} = -C \left\{ (1 - 0.25 g_0^2) + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125 g_0^2) \right\}. \quad (1.6.39)$$

Let us $\varphi = \frac{\pi}{2}$ then it can be shown that

$$C_{T_1} = \frac{C_{yc}^{\alpha} v_{nc} V_{yc}}{U_0^2} = C_{yc}^{\alpha} \left\{ \begin{array}{l} \frac{1}{2\lambda_p^2} \left[1 - 0.375g_0^2 \right] - \frac{g_0}{2\lambda_p} \left(1 - 0.125g_0^2 \right) + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} \left(1 - 0.125g_0^2 \right) \end{array} \right\}, \quad (1.6.40)$$

$$C_{T_2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\overline{b\dot{v}_{nc} \sin \theta_c}}{U_0^2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) G. \quad (1.6.41)$$

here

$$G = \sum_{n=1}^{n=13} G_n. \quad (1.6.42)$$

and

$$G_1 = -\frac{(Sh_0)^2 g_0^2 X_b}{2} \left(1 - 0.125g_0^2 \right), \quad (1.6.43)$$

$$G_2 = G_3 = G_4 = G_6 = G_7 = G_8 = G_{13} = 0, \quad (1.6.44)$$

$$G_5 = -\frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \left\{ \begin{array}{l} 0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \\ - g_0^2 \left[\begin{array}{l} 0.375 - \frac{0.125}{(2\lambda_p^2 + 1)} + \\ + \frac{0.0819}{(2\lambda_p^2 + 1)^2} - \frac{0.0409}{(2\lambda_p^2 + 1)^3} \end{array} \right] + \\ + 0.25g_0^4 \left[0.3125 - \frac{0.1172}{(2\lambda_p^2 + 1)} \right] \end{array} \right\}, \quad (1.6.45)$$

$$G_9 = \frac{\sqrt{2}(Sh_0)^2 g_0^2 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \begin{cases} 0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} - \\ -0.6667g_0^2 \left[0.375 - \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\ \left. + \frac{0.0819}{(2\lambda_p^2 + 1)^2} - \frac{0.0409}{(2\lambda_p^2 + 1)^3} \right] \end{cases}, \quad (1.6.46)$$

$$G_{10} = \frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \begin{cases} 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} + \frac{0.0589}{(2\lambda_p^2 + 1)^3} - \\ -g_0^2 \left[0.125 + \frac{0.3632}{(2\lambda_p^2 + 1)^2} - \frac{0.8788}{(2\lambda_p^2 + 1)^3} \right] \end{cases}, \quad (1.6.47)$$

$$G_{11} = -\frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{\sqrt{(2\lambda_p^2 + 1)}} \begin{cases} 0.125 + \frac{0.1056}{(2\lambda_p^2 + 1)^2} - \frac{0.0943}{(2\lambda_p^2 + 1)^3} - \\ -0.6667g_0^2 \left[0.0625 - \frac{0.0154}{(2\lambda_p^2 + 1)^3} \right] \end{cases}, \quad (1.6.48)$$

$$G_{12} = -\frac{\sqrt{2}(Sh_0)^2 g_0^2 \lambda_p X_b}{\sqrt{(2\lambda_p^2 + 1)}} \begin{cases} 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} - \\ -\frac{0.3163}{(2\lambda_p^2 + 1)^2} + \frac{0.0589}{(2\lambda_p^2 + 1)^3} - \\ -g_0^2 \left[0.125 + \frac{0.0118}{(2\lambda_p^2 + 1)^2} \right] \end{cases}. \quad (1.6.49)$$

$$C_{T3} = -C_{yc}^{\omega_2} \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125 g_0^2), \quad (1.6.50)$$

$$C_{T4} = -C_{yc}^{\dot{\omega}_2} \left(\sum_{n=1}^{n=3} K_n \right), \quad (1.6.51)$$

where

$$K_1 = (Sh_0)^2 g_0^2 \left\{ \begin{array}{l} -\frac{1}{2} (1 - 0.125 g_0^2) - \\ -\frac{\sqrt{2}}{2g_0 \sqrt{2\lambda_p^2 + 1}} \left\{ \begin{array}{l} 1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} - \\ -\frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \end{array} \right\} - \\ -g_0^2 \left[\begin{array}{l} 0.75 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1641}{(2\lambda_p^2 + 1)^2} - \\ -\frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1281}{(2\lambda_p^2 + 1)^4} \end{array} \right] \end{array} \right\}, \quad (1.6.52)$$

$$K_2 = 0, \quad (1.6.53)$$

$$K_3 = \frac{\sqrt{2} (Sh_0)^2 g_0^2 \lambda_p}{\sqrt{(2\lambda_p^2 + 1)}} \begin{cases} 0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \\ -0.67g_0^2 \left[0.375 - \frac{0.1055}{(2\lambda_p^2 + 1)^2} \right] \end{cases}. \quad (1.6.54)$$

$$C_{T5} = -\frac{\pi}{2} \overline{v_{nc}^2 \cos \vartheta} = -\frac{\pi}{2} \begin{cases} \frac{1}{\lambda_p^2} \left[0.5 - 0.5625g_0^2 + 0.2345g_0^4 \right] - \\ -\frac{g_0}{\lambda_p} \left(1 - 0.875g_0^2 + 0.256g_0^4 \right) + \\ + g_0^2 \left(0.5 - 0.3125g_0^2 + 0.0608g_0^4 \right) + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} \left(1 - 0.125g_0^2 \right) \end{cases}, \quad (1.6.55)$$

$$C_{T6} = -C \begin{cases} \left(1 - 0.25g_0^2 \right) + \frac{1}{2\lambda_p^2} \left(1 - 0.375g_0^2 \right) + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} \left(1 - 0.125g_0^2 \right) \end{cases}. \quad (1.6.56)$$

Let us consider the second case of the wing cinematic parameters. In this case the kinematic of the point x_1 (fig 1.1) may be specified by the expressions (1.5.3) and (1.5.4). In this case the time-average thrust is defined by the formulas (1.6.3 and 1.6.4) as before. The thrust coefficients can be shown up as

$$C_{T1} = C_{yc}^\alpha \frac{\overline{v_{nc} V_{yc}}}{U_0^2} = C_{yc}^\alpha \left(\frac{\overline{v_{nl} V_{yl}}}{U_0^2} + B_1 + B_2 + B_3 + B_4 \right). \quad (1.6.57)$$

here

$$\frac{\overline{v_{nl} V_{yl}}}{U_0^2} = \frac{\alpha_0 \sqrt{2\lambda_p^2 + 1}}{2\sqrt{2}\lambda_p^2} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \frac{0.0234}{(2\lambda_p^2 + 1)^3} - \right. \\ \left. - \frac{0.0146}{(2\lambda_p^2 + 1)^4} + \frac{0.0086}{(2\lambda_p^2 + 1)^5} \dots \right], \quad (1.6.58)$$

$$B_1 = B_0 \left\{ \begin{array}{l} \frac{\alpha_0 \lambda_p^2}{\sqrt{2}(2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{1.094}{(2\lambda_p^2 + 1)^2} \right] - \\ - \frac{\alpha_0^2 \lambda_p}{\sqrt{2}(2\lambda_p^2 + 1) \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{0.469}{(2\lambda_p^2 + 1)^2} \right] \end{array} \right\}, \quad (1.6.59)$$

$$B_2 = B_0 \left\{ \begin{array}{l} \frac{2\sqrt{2}\lambda_p^3}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} \left[1 + \frac{1.25}{(2\lambda_p^2 + 1)} + \frac{2.188}{(2\lambda_p^2 + 1)^2} + \right. \\ \left. + \frac{2.461}{(2\lambda_p^2 + 1)^3} + \frac{3.384}{(2\lambda_p^2 + 1)^4} \right] \end{array} \right\}, \quad (1.6.60)$$

$$B_3 = B_0 \left\{ \begin{array}{l} \frac{2\sqrt{2}\alpha_0 \lambda_p^2}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \left[1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} + \right. \\ \left. + \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.923}{(2\lambda_p^2 + 1)^4} \right] \end{array} \right\}, \quad (1.6.61)$$

$$B_4 = B_0 \left\{ \frac{\alpha_0^2 \lambda_p}{\sqrt{2} \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.188}{(2\lambda_p^2 + 1)^2} + \frac{0.117}{(2\lambda_p^2 + 1)^3} \right] \right\}, \quad (1.6.62)$$

$$B_0 = (Sh_0)^2 X_b^2. \quad (1.6.63)$$

$$C_{T2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) (1/X_b) (B_1 - B_2 - B_3 - B_4), \quad (1.6.64)$$

$$C_{T3} = -C_{yc}^{\omega_y} (1/X_b) (B_1 + B_2 + B_3 + B_4), \quad (1.6.65)$$

$$\begin{aligned}
C_{T4} = A_{T4} \left\{ \right. & \left. \begin{aligned}
& 1 - \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.9375}{(2\lambda_p^2 + 1)^2} - \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.9229}{(2\lambda_p^2 + 1)^4} - \\
& - \frac{0.4565}{(2\lambda_p^2 + 1)^5} + \frac{0.3}{(2\lambda_p^2 + 1)^6} - \frac{0.16}{(2\lambda_p^2 + 1)^7} + \frac{0.075}{(2\lambda_p^2 + 1)^8} + \\
& + \frac{2}{(2\lambda_p^2 + 1)} \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \frac{0.3494}{(2\lambda_p^2 + 1)^6} \right] - \\
& - \frac{\alpha_0 (2\lambda_p^2 + 1)}{2\lambda_p} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} - \frac{0.1172}{(2\lambda_p^2 + 1)^3} \right] + \\
& + \frac{\alpha_0^2}{2} \left[1.5 - \frac{1.5}{(2\lambda_p^2 + 1)} + \frac{1.6406}{(2\lambda_p^2 + 1)^2} - \right. \\
& \left. - \frac{1.6406}{(2\lambda_p^2 + 1)^3} + \frac{1.6919}{(2\lambda_p^2 + 1)^4} - \frac{0.9131}{(2\lambda_p^2 + 1)^5} \right] + \\
& + \frac{\alpha_0^2}{(2\lambda_p^2 + 1)} \left[0.5 - \frac{0.3125}{(2\lambda_p^2 + 1)} + \frac{0.5469}{(2\lambda_p^2 + 1)^2} - \right. \\
& \left. - \frac{0.4102}{(2\lambda_p^2 + 1)^3} + \frac{0.564}{(2\lambda_p^2 + 1)^4} - \frac{0.3705}{(2\lambda_p^2 + 1)^5} \right] - \\
& \left. - \frac{\alpha_0^3 (2\lambda_p^2 + 1)}{4\lambda_p} \left[1.5 - \frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.3281}{(2\lambda_p^2 + 1)^2} - \frac{0.2344}{(2\lambda_p^2 + 1)^3} \right] \right\} , \quad (1.6.66)
\end{aligned}
\right.$$

here

$$A_{T4} = \frac{C_{yc}^{\omega_z} \sqrt{2} (Sh_0)^2 \lambda_p}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.6.67)$$

$$\begin{aligned}
C_{T5} = & A_{T5-1} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \frac{0.0234}{(2\lambda_p^2 + 1)^3} \right] + \\
& + A_{T5-2} \left\{ \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} \right] - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} \left[0.5 + \frac{0.2344}{(2\lambda_p^2 + 1)^2} \right] \right\} + \\
& + A_{T5-3} \left\{ \left[1.5 + \frac{5.5}{(2\lambda_p^2 + 1)} + \frac{15.641}{(2\lambda_p^2 + 1)^2} + \frac{33.516}{(2\lambda_p^2 + 1)^3} \right] - \right. \\
& \left. - \frac{2\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} \left[1.5 + \frac{4.5}{(2\lambda_p^2 + 1)} + \frac{10.83}{(2\lambda_p^2 + 1)^2} \right] + \right\} + \\
& + A_{T5-4} \left\{ \left[0.5 - \frac{0.688}{(2\lambda_p^2 + 1)} + \frac{2.234}{(2\lambda_p^2 + 1)^2} - \frac{2.793}{(2\lambda_p^2 + 1)^3} \right] - \right. \\
& \left. - \frac{2\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} \left[0.5 - \frac{0.563}{(2\lambda_p^2 + 1)} + \frac{1.547}{(2\lambda_p^2 + 1)^2} \right] \right\} + \\
& + A_{T5-5} \left\{ \left[1 + \frac{1.25}{(2\lambda_p^2 + 1)} + \frac{2.188}{(2\lambda_p^2 + 1)^2} + \frac{2.461}{(2\lambda_p^2 + 1)^3} \right] - \right. \\
& \left. - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} \left[1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} \right] + \right\} + \\
& + A_{T5-6} \left\{ \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.188}{(2\lambda_p^2 + 1)^2} \right] \right. \\
& \left. + 0.5 + \frac{0.547}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \frac{0.349}{(2\lambda_p^2 + 1)^6} \right]. \quad (1.6.68)
\end{aligned}$$

here

$$A_{T5-1} = -\frac{\pi}{2} \frac{\alpha_0^2 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p}, \quad (1.6.69)$$

$$A_{T5-2} = -\frac{\pi}{2} \frac{\sqrt{2}(Sh_0)^2 \alpha_0 \lambda_p^2 X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}, \quad (1.6.70)$$

$$A_{T5-3} = -\frac{\pi}{2} \frac{8\sqrt{2}(Sh_0)^4 \lambda_p^7 X_b^4}{(2\lambda_p^2 + 1)^5 \sqrt{(2\lambda_p^2 + 1)}}, \quad (1.6.71)$$

$$A_{T5-4} = -\frac{\pi}{2} \frac{4\sqrt{2}(Sh_0)^4 \alpha_0 \lambda_p^6 X_b^4}{(2\lambda_p^2 + 1)^5 \sqrt{(2\lambda_p^2 + 1)}}, \quad (1.6.72)$$

$$A_{T5-5} = -\frac{\pi}{2} \frac{2\sqrt{2}(Sh_0)^2 \lambda_p^3 X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}, \quad (1.6.73)$$

$$A_{T5-6} = -\frac{\pi}{2} \frac{5\sqrt{2}(Sh_0)^2 \alpha_0^2 \lambda_p^3 X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.6.74)$$

$$C_{T_6} = -\frac{C\sqrt{(2\lambda_p^2 + 1)}}{\sqrt{2}\lambda_p} \left\{ \begin{aligned} & \left(1 + \frac{\alpha_0}{2\lambda_p} \right) \left[1 - \frac{0.0625}{(2\lambda_p^2 + 1)^2} \right] + \\ & + \frac{\alpha_0}{2\lambda_p} \left[\frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.0234}{(2\lambda_p^2 + 1)^3} \right] \end{aligned} \right\}. \quad (1.6.75)$$

It is an easy matter to convert the formulas to pure heaving and to pure pitching oscillations (as before). In the former case it was reasoned that $\vartheta = 0$, in the other case $\lambda_p = \infty$.

To pure heaving

$$C_{T_1} = C_{ye}^\alpha \left(\frac{1}{2\lambda_p^2} \right), \quad (1.6.76)$$

$$C_{T_2} = C_{T_3} = C_{T_4} = 0, \quad (1.6.77)$$

$$C_{T_5} = -\frac{\pi}{2} \left(\frac{1}{2\lambda_p^2} \right), \quad (1.6.78)$$

$$C_{T_6} = -C \left(\frac{1}{2\lambda_p^2} + 1 \right). \quad (1.6.79)$$

To pure pitching oscillations

$$C_{T1} = C_{yc}^{\alpha} \frac{\overline{V_{yc}}}{U_0^2} = C_{yc}^{\alpha} \left(\frac{\overline{V_{yc}}}{U_0^2} + B_1 + B_2 + B_3 + B_4 \right), \quad (1.6.80)$$

where

$$\frac{\overline{V_{yc}}}{U_0^2} = B_1 = B_2 = B_3 = 0, \quad (1.6.81)$$

$$B_4 = B_0 \left\{ \frac{\alpha_0^2}{2} \right\}, \quad (1.6.82)$$

$$B_0 = (Sh_0)^2 X_b^2. \quad (1.6.83)$$

$$C_{T2} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\overline{b\dot{V}_{yc} \sin \theta_c}}{U_0^2} = - \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) (1/X_b) B_4, \quad (1.6.84)$$

$$C_{T3} = -C_{yc}^{\omega_s} \frac{\overline{b\omega_z V_{yc}}}{U_0^2} = -C_{yc}^{\omega_s} (1/X_b) B_4, \quad (1.6.85)$$

$$C_{T4} = 0, \quad (1.6.86)$$

$$C_{T5} = -\frac{\pi \alpha_0^2}{4}, \quad (1.6.87)$$

$$C_{T6} = -C. \quad (1.6.88)$$

And at last let us consider the third case of the cinematic parameters (formulas 1.5.5 and 1.5.6). In this case the heaving is different from harmonic and is defined as (Prempraneerach et al., 2003):

$$\dot{y} = U_0 \operatorname{tg}(\alpha + \vartheta) = U_0 \operatorname{tg}(\theta_0 \cos \omega t). \quad (1.6.89)$$

Here $\theta_0 = \alpha_0 + \vartheta_0$.

The wing heaving is near to harmonic when the angles are small.

The formula (1.6.3) can be simplified to (as before). It may be conceived as the sum of the thrust coefficients

$$C_T = \frac{2\bar{F}_{xc}}{\rho S U_0^2} = C_{T1} + C_{T2} + C_{T3} + C_{T4} + C_{T5} + C_{T6}, \quad (1.6.90)$$

where

$$C_{T1} = C_{yc}^\alpha \left\{ \frac{\alpha_0}{2} \left(\begin{array}{l} \theta_0 + 0.62\theta_0^3 + 0.32\theta_0^5 + 0.15\theta_0^7 + \\ 0.07\theta_0^9 + 0.014\theta_0^{11} - 0.027\theta_0^{13} \end{array} \right) + C_0 \right\}, \quad (1.6.91)$$

$$C_0 = \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \begin{cases} 1 + 0.25\alpha_0\theta_0 - 0.125\theta_0^2 - 0.021\alpha_0\theta_0^3 + \\ + 0.005\theta_0^4 - 0.001\alpha_0\theta_0^5 - \\ - 0.001\alpha_0\theta_0^7 + 0.01\theta_0^{10} + 0.008\alpha_0\theta_0^{11} + \\ + 0.019\theta_0^{12} + 0.018\alpha_0\theta_0^{13} + \\ + 0.023\theta_0^{14} + 0.025\alpha_0\theta_0^{15} \end{cases}, \quad (1.6.92)$$

$$C_{T2} = -\frac{g_0^2 (Sh_0)^2 X_b}{2} \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \begin{bmatrix} 1 - 0.125g_0^2 - \frac{\alpha_0 g_0}{4} (1 - 0.083g_0^2) - \\ - \frac{\alpha_0 \theta_0^2}{4g_0} \left(1 + 0.17\theta_0^2 + 0.0416\theta_0^4 - \right. \\ \left. - 0.25g_0^2 - 0.05g_0^2\theta_0^2 + \right. \\ \left. + 0.013g_0^4 + 0.003g_0^4\theta_0^2 \right) \end{bmatrix}, \quad (1.6.93)$$

$$C_{T3} = -C_{yc}^{\omega_2} \frac{C_0}{X_b}, \quad (1.6.94)$$

$$C_{T4} = -C_{yc}^{\dot{\alpha}_2} \left\{ - (Sh_0)^2 g_0^2 \left[0.5 + 0.0625g_0^2 + \frac{\alpha_0}{g_0} (0.5 - 0.1875g_0^2) \right] \right\}, \quad (1.6.95)$$

$$C_{T5} = -\frac{\pi}{2} \left[\frac{\alpha_0^2}{2} \left(1 + 0.375\theta_0^2 + 0.13\theta_0^4 + \right. \right. \\ \left. \left. + 0.046\theta_0^6 + 0.017\theta_0^8 - 0.006\theta_0^{10} \right) + C_0 \right], \quad (1.6.96)$$

$$C_{T6} = -C \left[\left(1 + 0.5\alpha_0\theta_0 + 0.25\theta_0^2 + 0.31\alpha_0\theta_0^3 + \right. \right. \\ \left. \left. + 0.078\theta_0^4 + 0.16\alpha_0\theta_0^5 + 0.026\theta_0^6 \right) + C_0 \right]. \quad (1.6.97)$$

As it was made before let us obtain the formulas to pure heaving and to pure pitching oscillations

To pure heaving ($\vartheta = 0$)

$$C_{T1} = C_{yc}^{\alpha} \left\{ \frac{\alpha_0^2}{2} \left(1 + 0.62\alpha_0^2 + 0.32\alpha_0^4 + 0.15\alpha_0^6 \right) \right\}, \quad (1.6.98)$$

$$C_0 = C_{T2} = C_{T3} = C_{T4} = 0, \quad (1.6.99)$$

$$C_{T5} = -\frac{\pi}{2} \left[\frac{\alpha_0^2}{2} \left(1 + 0.375\alpha_0^2 + 0.13\alpha_0^4 + 0.046\alpha_0^6 \right) \right], \quad (1.6.100)$$

$$C_{T6} = -C \left(1 + 0.75\alpha_0^2 + 0.388\alpha_0^4 + 0.186\alpha_0^6 \right). \quad (1.6.101)$$

To pure pitching ($y = 0, \dot{y} = 0, \theta = 0, \vartheta = -\alpha$)

$$C_{T1} = C_{yc}^{\alpha} \{C_0\}, \quad (1.6.102)$$

$$C_0 = \frac{\vartheta_0^2 (Sh_0)^2 X_b^2}{2}, \quad (1.6.103)$$

$$C_{T2} = -\frac{\vartheta_0^2 (Sh_0)^2 X_b}{2} \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \left[\begin{aligned} & 1 - 0.125\vartheta_0^2 + 0.0052\vartheta_0^4 - \\ & - \frac{\alpha_0 \vartheta_0}{4} (1 - 0.0833\vartheta_0^2) \end{aligned} \right], \quad (1.6.104)$$

$$C_{T3} = -C_{yc}^{\omega_2} \frac{C_0}{X_b}, \quad (1.6.105)$$

$$C_{T4} = -C_{yc}^{\dot{\omega}_2} \left\{ -\left(Sh_0\right)^2 g_0^2 \left[0.5 + 0.0625g_0^2 + 0.0026g_0^4 + \right. \right. \\ \left. \left. + \frac{\alpha_0}{g_0} (0.5 - 0.1875g_0^2 + 0.0586g_0^4) \right] \right\}, \quad (1.6.106)$$

$$C_{T5} = -\frac{\pi}{2} \left[\frac{\alpha_0^2}{2} + C_0 \right], \quad (1.6.107)$$

$$C_{T6} = -C \left[1 + \frac{\left(Sh_0\right)^2 g_0^2 X_b^2}{2} \right]. \quad (1.6.108)$$

The formulas (1.6.35 – 1.6.41) contain the angles α_0, g_0 and the sum of angles θ_0 . Let us expand the right-hand side of the formula (1.6.89) as power series in θ_0 and will be bound by four members. After the term-by-term integrating with respect to θ_0 we obtain

$$Sh_0 = \frac{\theta_0}{y_0} \left(1 + 0.222\theta_0^2 + 0.071\theta_0^4 + 0.01\theta_0^6 \right). \quad (1.6.109)$$

If the Strouhal number, the angle of attack and the relative oscillation amplitude are known we can calculate the angles θ_0 and g_0 .

1.7. The suction force design formulas.

Let us estimate the suction force contribution to the common thrust (X_{sc}) if the wing cinematic parameters are represented as (1.5.1) and (1.5.2). In this case we derive the formula

$$\overline{X_{sc}} = \frac{\rho S}{2} \left[\begin{aligned} & \left(C_{yc}^{\alpha} \overline{v_{nc}^2 \cos \vartheta} - C_{yc}^{\dot{\omega}_z} \overline{\dot{\omega}_z b^2 \alpha_c \cos \vartheta} \right) + \\ & + b \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \overline{\dot{v}_{nc} \alpha_c \cos \vartheta} + \\ & + b \left(\frac{2\lambda_{22}}{\rho S b} - C_{yc}^{\omega_z} \right) \overline{v_{nc} \omega_z \cos \vartheta} \end{aligned} \right]. \quad (1.7.1)$$

Or, alternatively, in the form of the suction force coefficients if

$$\varphi = \frac{\pi}{2}$$

$$\frac{2\overline{X_{sc}}}{\rho S U_0^2} = \left[\begin{aligned} & \left(C_{yc}^{\alpha} \frac{\overline{v_{nc}^2 \cos \vartheta}}{U_0^2} - C_{yc}^{\dot{\omega}_z} \frac{\overline{\dot{\omega}_z b^2 \alpha_c \cos \vartheta}}{U_0^2} \right) + \\ & + b \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{\overline{\dot{v}_{nc} \alpha_c \cos \vartheta}}{U_0^2} + \\ & + b \left(\frac{2\lambda_{22}}{\rho S b} - C_{yc}^{\omega_z} \right) \frac{\overline{v_{nc} \omega_z \cos \vartheta}}{U_0^2} \end{aligned} \right]. \quad (1.7.2)$$

This formula can be conceived of as

$$C_S = \frac{2\bar{X}_{SC}}{\rho S U_0^2} = C_{S1} + C_{S2} + C_{S3} + C_{S4}, \quad (1.7.3)$$

where

$$C_{S1} = C_{yc}^\alpha \frac{\frac{v_{nc}^2 \cos \theta}{U_0^2}}{U_0^2} = 0.5 C_{yc}^\alpha \left[\begin{array}{l} \left(\frac{1}{\lambda_p} - \theta_0 \right)^2 - \\ -1.125 \theta_0^2 \left(\frac{1}{\lambda_p^2} - 1.556 \frac{\theta_0}{\lambda_p} + 0.556 \theta_0^2 \right) + \\ +0.547 \theta_0^4 \left(\frac{1}{\lambda_p^2} - 1.162 \frac{\theta_0}{\lambda_p} + 0.289 \theta_0^2 \right) - \\ -0.137 \theta_0^6 \left(\frac{1}{\lambda_p^2} - 0.844 \frac{\theta_0}{\lambda_p} + 0.144 \theta_0^2 \right) \end{array} \right], \quad (1.7.4)$$

$$C_{S2} = C_{yc}^{\dot{\theta}_0} (Sh_0)^2 \frac{\theta_0}{2} \left[\begin{array}{l} \left(\frac{1}{\lambda_p} - \theta_0 \right) \left(1 - \frac{3}{8} \theta_0^2 + \frac{5}{192} \theta_0^4 \right) - \\ \frac{1}{4\lambda_p^3} \left(1 - \frac{1}{2\lambda_p^2} \right) + \frac{5}{48} \frac{\theta_0^2}{\lambda_p^3} \left(1 - \frac{21}{40\lambda_p^2} \right) \end{array} \right], \quad (1.7.5)$$

$$C_{S3} = C_{S3-0} \left[\begin{aligned} & \left[1 + 0.375g_0^2 + 0.078g_0^4 - \frac{0.125g_0^2}{(2\lambda_p^2 + 1)} + \right. \\ & + \frac{0.1876}{(2\lambda_p^2 + 1)^2} + \frac{0.0817g_0^2}{(2\lambda_p^2 + 1)^2} + \frac{0.0269g_0^4}{(2\lambda_p^2 + 1)^2} - \\ & \left. - \frac{0.0188g_0^4}{(2\lambda_p^2 + 1)^3} + \frac{0.1026}{(2\lambda_p^2 + 1)^4} + \frac{0.047g_0^2}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \lambda_p \left[\frac{0.25g_0}{(2\lambda_p^2 + 1)} - \frac{0.0555g_0^3}{(2\lambda_p^2 + 1)} + \frac{0.0091g_0^3}{(2\lambda_p^2 + 1)^2} + \right. \\ & \left. + \frac{0.1172g_0}{(2\lambda_p^2 + 1)^3} - \frac{0.026g_0^3}{(2\lambda_p^2 + 1)^3} + \frac{0.0114g_0^3}{(2\lambda_p^2 + 1)^4} \right] \end{aligned} \right], \quad (1.7.6)$$

here

$$C_{S3-0} = - \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{\sqrt{2\lambda_p^2 + 1}}$$

$$C_{S4} = \left(\frac{2\lambda_{22}}{\rho Sb} - C_{yc}^{\omega_c} \right) (Sh_0)^2 g_0^2 X_b (0.5 - 0.0625g_0^2). \quad (1.7.7)$$

To pure heaving

$$C_{S1} = C_{yc} \left(\frac{1}{2\lambda_p^2} \right) \quad (1.7.8)$$

$$C_{S2} = C_{S3} = C_{S4} = 0. \quad (1.7.9)$$

To pure pitching

$$C_{S1} = C_{yc}^\alpha \left[\frac{1}{2} \left(g_0^2 - 0.6255g_0^4 + 0.1581g_0^6 - 0.0197g_0^8 \right) \right], \quad (1.7.10)$$

$$C_{S2} = C_{yc}^{\dot{\omega}_z} \frac{(Sh_0)^2 g_0}{2} \left(-g_0 + 0.375g_0^3 - 0.026g_0^5 \right), \quad (1.7.11)$$

$$C_{S3} = 0, \quad (1.7.12)$$

$$C_{S4} = \left(\frac{2\lambda_{22}}{\rho S b} - C_{yc}^{\omega_z} \right) (Sh_0)^2 g_0^2 X_b (0.5 - 0.0625g_0^2). \quad (1.7.13)$$

By way of example let us estimate the suction force contribution to the value of the thrust (X_{sc}) if the wing cinematic parameters are represented as an arbitrary values. Figure 1.7.1 gives the relative suction force contribution to the value of the wing thrust if the wing cinematic parameters are represented as (1.5.1) and (1.5.2) with the proviso that $\varphi = \frac{\pi}{2}$ and two values of Strouhal number.

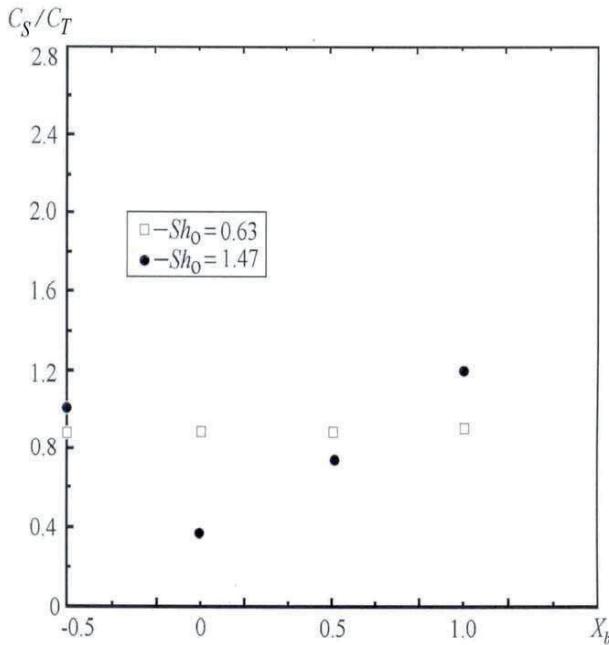


Fig.1.7.1. The relative suction force contribution to the value of the rectangular rigid wing common thrust if the wing aspect ratio equal 4 and harmonic heaving and pitching with pitch-axes position. The wing cinematic parameters were taken from the work (Anderson at all, 1998):

the relative heaving is $y_0/b = 0.75$, the angle of the wing slope is $\vartheta_0 = 0.188$, $Sh_0 = 0.63$, $Sh_0 = 1.47$ and $\vartheta_0 = 0.63$.

The suction force contribution to the wing thrust value is practically constant (near 0.9) when $Sh_0 = 0.63$ in the manner indicated in Fig. 1.7.1. By this is meant that the thrust is due to the suction force (near 90%). When the $Sh_0 = 1.47$ the suction force contribution to the wing thrust value depends on the pitch-axes location. The suction force is minimum when the pitch-axes locates at the wing centre. When the pitch-axes locates outside the wing ($X_b = 1$) the thrust is due to the suction force. This result is in agreement with the linear Lighthill wing theory (1969).

Let us use the second wing cinematic parameters (1.7.1)-(1.7.3) which after the time-average gives

$$C_{s1} = C_{yc}^{\alpha} \left(-\frac{2}{\pi} C_{Ts} \right), \quad (1.7.14)$$

$$C_{s2} = -C_{yc}^{\dot{\omega}_z} \frac{A}{X_b} \left\{ \begin{array}{l} -\frac{\sqrt{2}(\lambda_p + \alpha_0)}{\lambda_p(2\lambda_p^2 + 1)\sqrt{2\lambda_p^2 + 1}} \left[1 - \frac{0.75}{(2\lambda_p^2 + 1)} + \right. \right. \\ \left. \left. + \frac{0.938}{(2\lambda_p^2 + 1)^2} - \right. \right. \\ \left. \left. - \frac{0.82}{(2\lambda_p^2 + 1)^3} \right] \right. \\ -\frac{\sqrt{2}}{(2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{1.094}{(2\lambda_s^2 + 1)^2} + \right. \\ \left. \left. + \frac{1.128}{(2\lambda_s^2 + 1)^4} \right] \right. \\ \left. + \frac{\alpha_0}{\sqrt{2}\lambda_p \sqrt{2\lambda_p^2 + 1}} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \right. \right. \\ \left. \left. + \frac{0.188}{(2\lambda_p^2 + 1)^2} - \frac{0.117}{(2\lambda_p^2 + 1)^3} \right] \right. \end{array} \right\}, \quad (1.7.15)$$

$$C_{S3} = A_1 \left\{ \begin{aligned}
& -\frac{\sqrt{2}}{(2\lambda_p^2 + 1)\sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l} (1 + 0.75\alpha_0) - \\ -\frac{0.75(1 + \alpha_0)}{(2\lambda_p^2 + 1)} + \\ + \frac{1.094 + 0.817\alpha_0}{(2\lambda_p^2 + 1)^2} - \\ - \frac{0.82(1 + \alpha_0)}{(2\lambda_p^2 + 1)^3} \end{array} \right] - \\
& -\frac{\sqrt{2}}{(2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l} (1 + 0.5\alpha_0) - \frac{0.313\alpha_0}{(2\lambda_p^2 + 1)} + \\ + \frac{(1.094 + 0.547\alpha_0)}{(2\lambda_p^2 + 1)^2} \end{array} \right] + \\
& + \frac{\alpha_0}{\sqrt{2}\lambda_p \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l} 1 + \frac{0.75\alpha_0}{\lambda_p} - \frac{0.25(\lambda_p + \alpha_0)}{\lambda_p(2\lambda_p^2 + 1)} + \\ + \frac{0.188(\lambda_p + \alpha_0)}{\lambda_p(2\lambda_p^2 + 1)^2} - \\ - \frac{0.117\lambda_p + 0.195\alpha_0}{\lambda_p(2\lambda_p^2 + 1)^3} \end{array} \right] + \\
& + \frac{\sqrt{2}}{(2\lambda_p^2 + 1)\sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l} 1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} + \\ + \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.923}{(2\lambda_p^2 + 1)^4} \end{array} \right] - \\
& - \frac{\alpha_0}{\sqrt{2}\lambda_p \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.188}{(2\lambda_p^2 + 1)^2} \right] + \\
& + \frac{1}{\sqrt{2}(2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[1 + \frac{1.094}{(2\lambda_p^2 + 1)^2} + \frac{1.128}{(2\lambda_p^2 + 1)^4} \right], \quad (1.7.16)
\end{aligned} \right.$$

$$C_{s4} = A_2 \left\{ \begin{array}{l}
\frac{\alpha_0}{\sqrt{2}\lambda_p \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l}
1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.188}{(2\lambda_p^2 + 1)^2} + \\
+ \frac{0.117}{(2\lambda_p^2 + 1)^3} + \frac{0.103}{(2\lambda_p^2 + 1)^4}
\end{array} \right] - \\
- \frac{2\sqrt{2}}{(2\lambda_p^2 + 1)\sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l}
1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} + \\
+ \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.923}{(2\lambda_p^2 + 1)^4}
\end{array} \right] + \\
+ \frac{2\sqrt{2}\lambda_p}{\alpha_0 (2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l}
1 + \frac{1.25}{(2\lambda_p^2 + 1)} + \\
+ \frac{2.188}{(2\lambda_p^2 + 1)^2} + \\
+ \frac{2.461}{(2\lambda_p^2 + 1)^3}
\end{array} \right] - \\
- \frac{\alpha_0}{\sqrt{2}\lambda_p (2\lambda_p^2 + 1) \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l}
1 + \frac{0.461}{(2\lambda_p^2 + 1)^2}
\end{array} \right] + \\
+ \frac{1}{\sqrt{2}(2\lambda_p^2 + 1)^2 \sqrt{2\lambda_p^2 + 1}} \left[\begin{array}{l}
1 + \frac{1.094}{(2\lambda_p^2 + 1)^2}
\end{array} \right]
\end{array} \right\}. \quad (1.7.17)$$

In the formulas above we have

$$\begin{aligned}
 A &= \alpha_0 (Sh_0)^2 \lambda_p^2 X_b, \\
 A_1 &= -\left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho S b} \right) A, \\
 A_2 &= \left(\frac{2\lambda_{22}}{\rho S b} - C_{yc}^{\omega_z} \right) A.
 \end{aligned} \tag{1.7.18}$$

The first term in the round brackets of the right-hand side of the formula (1.7.2) is many times higher than the other terms. The neglect of the small terms and the time-average gives

$$\overline{X_{sc}} = \frac{\rho S}{2} \left(C_{yc}^{\alpha} \overline{v_{nc}^2 \cos \theta} \right). \tag{1.7.19}$$

To pure heaving and pure pitching we shall have:

To pure heaving

$$C_{s1} = C_{yc}^{\alpha} \left(\frac{1}{2\lambda_p^2} \right), \tag{1.7.20}$$

$$C_{s2} = C_{s3} = C_{s4} = 0. \tag{1.7.21}$$

To pure pitching

$$C_{S1} = C_{yc}^{\alpha} \left(\frac{\alpha_0^2}{2} \right), \quad (1.7.22)$$

$$C_{S2} = -C_{yc}^{\dot{\alpha}_z} \frac{(Sh_0)^2 \alpha_0^2}{2}, \quad (1.7.23)$$

$$C_{S3} = 0, \quad (1.7.24)$$

$$C_{S4} = \left(\frac{2\lambda_{22}}{\rho S b} - C_{yc}^{\alpha_z} \right) \frac{(Sh_0)^2 \alpha_0^2 X_b}{2}. \quad (1.7.25)$$

As it was make before let us assess the relative suction force contribution to the value of the rigid wing common thrust. The wing cinematic parameters were taken from the work (Anderson et all, 1998). Let us use the second variant of the wing cinematic parameters (1.7.1)-(1.7.3).

Figure 1.7.2 gives the relative suction force contribution to the value of the wing thrust if the wing cinematic parameters are represented as (1.7.1)-(1.7.3). with the proviso that $\varphi = \frac{\pi}{2}$ and two values of Strouhal number.

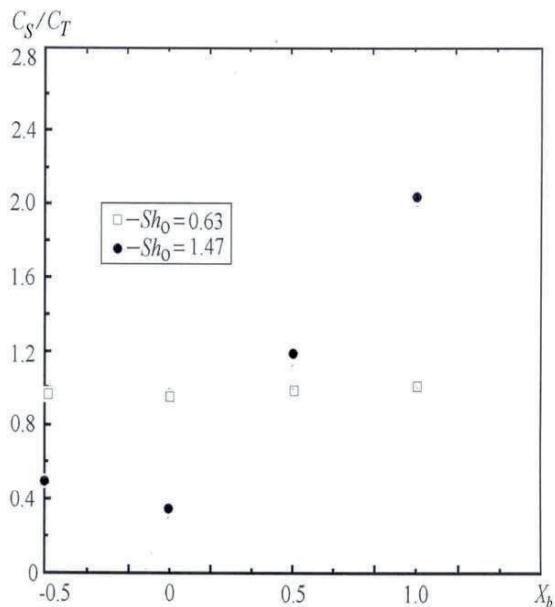


Fig.1.7.2. The relative suction force contribution to the value of the rectangular rigid wing common thrust if the wing aspect ratio equal 4 and harmonic heaving and angle of attack with pitch-axes position. The wing cinematic parameters were taken from the work (Anderson at all, 1998): the relative heaving is $y_0/b = 0.75$, the angle of attack is $\alpha_0 = 15^\circ = 0.262$, $Sh_0 = 0.63$, $Sh_0 = 1.47$ and $\vartheta_0 = 0.63$.

From this figure we notice that the suction force contribution to the wing thrust value is practically constant (near 0.9) when $Sh_0 = 0.63$. By this it is meant that the thrust is due to the suction force (near 90%). If $Sh_0 = 1.47$ relationship between C_s/C_T and X_b is different from those shown before (Fig. 1.7.1.).

Let us use the third variant of the wing cinematic parameters (1.5.5)-(1.5.6). In this case the coefficient of the relative suction force contribution to the value of the rectangular rigid wing common thrust takes the form which after the time-average gives $C_s = C_{s1} + C_{s2} + C_{s3} + C_{s4}$ where

$$C_{s1} = C_{yc}^{\alpha} \left\{ (Sh_0)^2 g_0^2 \begin{bmatrix} \frac{\alpha_0^2}{(Sh_0)^2 g_0^2} (0.5 + 0.1875\theta_0^2) + \\ + X_b^2 \left(0.5 - 0.0625\theta_0^2 + \right. \\ \left. + 0.125\alpha_0\theta_0 - 0.0104\alpha_0\theta_0^3 \right) \end{bmatrix} \right\}, \quad (1.7.26)$$

$$C_{s2} = C_{yc}^{\dot{\alpha}_0} (Sh_0)^2 \alpha_0 g_0 \begin{pmatrix} 0.5 - 0.1875\theta_0^2 + \\ + 0.375\alpha_0\theta_0 - 0.052\alpha_0\theta_0^3 \end{pmatrix}, \quad (1.7.27)$$

$$C_{s3} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) (Sh_0)^2 \alpha_0 g_0 X_b \begin{pmatrix} 1 - 0.125\theta_0^2 - 0.0573\theta_0^4 + \\ + 0.0179\theta_0^6 - 0.25\alpha_0\theta_0 + \\ + 0.1042\alpha_0\theta_0^3 - 0.0349\alpha_0\theta_0^5 \end{pmatrix}, \quad (1.7.28)$$

$$C_{S4} = \left(\frac{2\lambda_{22}}{\rho Sb} - C_{yc}^{\alpha_2} \right) \left[(Sh_0)^2 \alpha_0^2 X_b \begin{pmatrix} 0.5 - 0.0625\theta_0^2 + 0.0026\theta_0^4 + \\ + 0.125\alpha_0\theta_0 - 0.0104\alpha_0\theta_0^3 \end{pmatrix} \right]. \quad (1.7.29)$$

To pure heaving and pure pitching we shall have:

To pure heaving ($\vartheta = 0$)

$$C_{S1} = C_{yc}^{\alpha} \left[\frac{\alpha_0^2}{2} (1 + 0.375\alpha_0^2) \right], \quad (1.7.30)$$

$$C_{S2} = C_{S3} = C_{S4} = 0. \quad (1.7.31)$$

To pure pitching ($y = 0, \theta = 0$)

$$C_{S1} = C_{yc}^{\alpha} \frac{\alpha_0^2}{2} \left[1 + (Sh_0)^2 X_b^2 \right], \quad (1.7.32)$$

$$C_{S2} = -C_{yc}^{\alpha_2} (Sh_0)^2 \frac{\alpha_0^2}{2}, \quad (1.7.33)$$

$$C_{S3} = - \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) (Sh_0)^2 \alpha_0^2 X_b, \quad (1.7.34)$$

$$C_{s4} = \left(\frac{2\lambda_{22}}{\rho Sb} - C_{yc}^{\omega_z} \right) \left[(Sh_0)^2 \frac{\alpha_0^2}{2} X_b \right]. \quad (1.7.35)$$

If $Sh_0 = 0.63$, $\alpha_0 = 15^\circ = 0.262$, $\theta_0 = 0.188$, and $Sh_0 = 1.47$, $\alpha_0 = 15^\circ = 0.262$, $\theta_0 = 0.63$ a relationship between C_s/C_T and X_b like shown in Fig. 1.7.1.

Having the thrust and suction force formulas we can calculate the drag force as

$$\frac{2(F_{xc} - X_{xc})}{\rho S U_0^2} = \begin{cases} C_{yc}^{\alpha} \frac{\overline{v_{nc} V_{yc}}}{U_0^2} + b \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\dot{v}_{nc} \sin \theta}{U_0^2} - \\ -b C_{yc}^{\omega_z} \frac{\overline{\omega_z V_{yc}}}{U_0^2} - C_{yc}^{\omega_z} b^2 \frac{\dot{\omega}_z \sin \theta}{U_0^2} - \\ - \left(\frac{\pi}{2} + C_{yc}^{\alpha} \right) \frac{\overline{v_{nc}^2 \cos \theta}}{U_0^2} - C \frac{\overline{U_c^2 \cos \theta}}{U_0^2} \end{cases}. \quad (1.7.36)$$

or

$$C_{TC} = C_{TC1} + C_{TC2} + C_{TC3} + C_{TC4} + C_{TC5} + C_{TC6}, \quad (1.7.37)$$

For the first cinematic variant the drag force coefficients are obtainable from

$$C_{TC1} = C_{T1}, \quad (1.7.38)$$

$$C_{TC2} = C_{T2}, \quad (1.7.39)$$

$$C_{TC3} = C_{T3}, \quad (1.7.40)$$

$$C_{TC4} = -C_{yc}^{\phi_2} \left[-\left(Sh_0 \right)^2 g_0^2 \left(0.5 - 0.0625 g_0^2 \right) \right], \quad (1.7.41)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^\alpha}{\pi} \right) C_{T5}, \quad (1.7.42)$$

$$C_{TC6} = C_{T6}. \quad (1.7.43)$$

To pure heaving

$$C_{TC1} = C_{yc}^\alpha \left(0.5 \frac{1}{\lambda_p^2} \right), \quad (1.7.44)$$

$$C_{TC2} = C_{TC3} = C_{TC4} = 0, \quad (1.7.45)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^\alpha}{\pi} \right) \left(0.5 \frac{1}{\lambda_p^2} \right), \quad (1.7.46)$$

$$C_{TC6} = C_{T6}. \quad (1.7.47)$$

To pure pitching

$$C_{TC1} = C_{yc}^a \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.48)$$

$$C_{TC2} = - \left(C_{yc}^{\dot{a}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{g_0^2 (Sh_0)^2 X_b}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.49)$$

$$C_{TC3} = -C_{yc}^{\omega_z} \frac{g_0^2 (Sh_0)^2 X_b}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.50)$$

$$C_{TC4} = -C_{yc}^{\dot{\omega}_z} \left[- (Sh_0)^2 g_0^2 (0.5 - 0.0625 g_0^2) \right], \quad (1.7.51)$$

$$C_{TCS} = \left(1 + \frac{2C_{yc}^a}{\pi} \right) \left\{ \frac{\overline{v_n^2 \cos \theta}}{U_0^2} + \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right] \right\}, \quad (1.7.52)$$

where

$$\frac{\overline{v_n^2 \cos \theta}}{U_0^2} = 0.5 \left[g_0^2 - 0.6255 g_0^4 + 0.1581 g_0^6 - 0.0197 g_0^8 \right].$$

$$C_{TC6} = -C \frac{\overline{U_c^2 \cos \theta}}{U_0^2} = -C \left\{ \frac{\overline{U_1^2 \cos \theta}}{U_0^2} + \left[\frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right] \right] \right\}, \quad (1.7.53)$$

where

$$\frac{\overline{U_1^2 \cos \theta}}{U_0^2} = (1 - 0.25 g_0^2)$$

For the second cinematic variant the drag force coefficients are obtainable from

$$C_{TC1} = C_{T1}, \quad (1.7.54)$$

$$C_{TC2} = C_{TC2-0} \left\{ \begin{aligned} & \left(1 + \lambda_p \right) \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} - \right. \right. \\ & \left. \left. - \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] + \right. \\ & \left. + \frac{\lambda_p^2}{(2\lambda_p^2 + 1)} \left(1 - \frac{2}{\alpha_0 \lambda_p} \right) \left[1 - \frac{0.75}{(2\lambda_p^2 + 1)} + \right. \right. \\ & \left. \left. + \frac{0.9375}{(2\lambda_p^2 + 1)^2} - \right. \right. \\ & \left. \left. + \frac{0.82}{(2\lambda_p^2 + 1)^3} + \right. \right. \\ & \left. \left. + \frac{0.9229}{(2\lambda_p^2 + 1)^4} \right] \right. \\ & \left. + \frac{4\lambda_p^2}{(2\lambda_p^2 + 1)^2} \left(1 - \frac{1}{\alpha_0 \lambda_p} \right) \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} \right] \right], \end{aligned} \right. \quad (1.7.55)$$

here $C_{TC2-0} = \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\sqrt{2} (Sh_0)^2 \alpha_0 X_b}{2\sqrt{(2\lambda_p^2 + 1)}}$

$$C_{TC3} = C_{T3}, \quad (1.7.56)$$

$$C_{TC4} = C_{TC4-0} \left\{ \begin{aligned} & \left(1 - \alpha_0\right) \left[1 - \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.9375}{(2\lambda_p^2 + 1)^2} - \right. \\ & \quad \left. - \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.9229}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \frac{1}{(2\lambda_p^2 + 1)} \left(1 - \alpha_0 \lambda_p\right) \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \right. \\ & \quad \left. + \frac{0.564}{(2\lambda_p^2 + 1)^4} \right] - \end{aligned} \right\}, \quad (1.7.57)$$

here $C_{TC4-0} = -\frac{\sqrt{2}(Sh_0)^2 \lambda_p}{(2\lambda_p^2 + 1)\sqrt{(2\lambda_p^2 + 1)}}$

$$C_{TCS} = \left(1 + \frac{2C_{yc}^\alpha}{\pi}\right) C_{T5}, \quad (1.7.58)$$

$$C_{TC6} = C_{T6}. \quad (1.7.59)$$

To pure heaving

$$C_{TC1} = C_{yc}^\alpha \left(0.5 \frac{1}{\lambda_p^2}\right), \quad (1.7.60)$$

$$C_{TC2} = C_{TC3} = C_{TC4} = 0, \quad (1.7.61)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^{\alpha}}{\pi} \right) \left(0.5 \frac{1}{\lambda_p^2} \right), \quad (1.7.62)$$

$$C_{TC6} = C_{T6}. \quad (1.7.63)$$

To pure pitching

$$C_{TC1} = C_{yc}^{\alpha} \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.64)$$

$$C_{TC2} = - \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{g_0^2 (Sh_0)^2 X_b}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.65)$$

$$C_{TC3} = -C_{yc}^{\omega_c} \frac{g_0^2 (Sh_0)^2 X_b}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.7.66)$$

$$C_{TC4} = -C_{yc}^{\dot{\omega}_c} \left[- (Sh_0)^2 g_0^2 (0.5 - 0.0625 g_0^2) \right], \quad (1.7.67)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^\alpha}{\pi} \right) \left\{ \frac{\overline{v_n^2 \cos \theta}}{U_0^2} + \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right] \right\}, \quad (1.7.68)$$

where

$$\frac{\overline{v_n^2 \cos \theta}}{U_0^2} = 0.5 \left[g_0^2 - 0.6255g_0^4 + 0.1581g_0^6 - 0.0197g_0^8 \right].$$

$$C_{TC6} = -C \frac{\overline{U_e^2 \cos \theta}}{U_0^2} = -C \left\{ \frac{\overline{U_1^2 \cos \theta}}{U_0^2} + \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right] \right\}, \quad (1.7.69)$$

where

$$\frac{\overline{U_1^2 \cos \theta}}{U_0^2} = \left(1 - 0.25g_0^2 \right).$$

For the third cinematic variant the drag force coefficients are obtainable from

$$C_{TC1} = C_{T1}, \quad (1.7.70)$$

$$C_{TC2} = -\left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) \left[(Sh_0)^2 g_0 X_b (0.5g_0 - 0.0625g_0^3) \right], \quad (1.7.71)$$

$$C_{TC3} = C_{T3}, \quad (1.7.72)$$

$$C_{TC4} = C_{yc}^{\dot{\alpha}_2} \left[(Sh_0)^2 g_0 (0.5g_0 - 0.0625g_0^3) \right], \quad (1.7.73)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^{\alpha}}{\pi} \right) C_{T5}, \quad (1.7.74)$$

$$C_{TC6} = C_{T6}. \quad (1.7.75)$$

To pure heaving

$$C_{TC1} = C_{yc}^{\alpha} \left\{ \frac{\alpha_0}{2} (\alpha_0 + 0.62\alpha_0^3 + 0.32\alpha_0^5) \right\}, \quad (1.7.76)$$

$$C_{TC2} = C_{TC3} = C_{TC4} = 0, \quad (1.7.77)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^{\alpha}}{\pi} \right) \left[\frac{\alpha_0^2}{2} (1 + 0.375\alpha_0^2 + 0.13\alpha_0^4) \right], \quad (1.7.78)$$

$$C_{TC6} = -C \left(1 + 0.5\alpha_0^2 + 0.25\alpha_0^4 \right). \quad (1.7.79)$$

To pure pitching

$$C_{TC1} = C_{yc}^\alpha \frac{(Sh_0)^2 g_0^2 X_b^2}{2}, \quad (1.7.80)$$

$$C_{TC2} = -\frac{g_0^2 (Sh_0)^2 X_b}{2} \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho Sb} \right) \begin{bmatrix} 1 - 0.125g_0^2 + 0.0052g_0^4 & \\ -\frac{\alpha_0 g_0}{4} (1 - 0.0833g_0^2) & \end{bmatrix}, \quad (1.7.81)$$

$$C_{TC3} = -C_{yc}^{\omega_z} \frac{(Sh_0)^2 g_0^2 X_b}{2}, \quad (1.7.82)$$

$$C_{TC4} = C_{yc}^{\phi_z} \left[(Sh_0)^2 g_0 (0.5g_0 - 0.0625g_0^3) \right], \quad (1.7.83)$$

$$C_{TC5} = \left(1 + \frac{2C_{yc}^\alpha}{\pi} \right) \left[\frac{\alpha_0^2}{2} + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} \right], \quad (1.7.84)$$

$$C_{TC6} = -C. \quad (1.7.85)$$

1.8. The power and efficiency design formulas.

Let's recall that common formulas to wing efficiency calculate has the general form as equations (1.3.1) – (1.3.3):

$$\eta = \frac{\bar{A}_c}{\bar{P}_c}. \quad (1.3.1)$$

Here

$$\bar{A}_c = \overline{F_{xc}} U_0 = \frac{\rho S U_0^3}{2} \left(\frac{2 \overline{F}_{xc}}{\rho S U_0^2} \right) \quad (1.3.2)$$

and

$$\bar{P}_c = -\overline{F_{yc} V_{yc}} - \overline{M_{zc} \omega_z} = \frac{\rho S U_0^3}{2} \left(-\frac{\overline{2 F_{yc} V_{yc}}}{\rho S U_0^3} - \frac{\overline{2 M_{zc} \omega_z}}{\rho S U_0^3} \right). \quad (1.3.3)$$

Formula (1.3.2) is the wing useful power and equal to the product of the thrust by the flow speed. The wing thrust is determined by the formula (1.4.1). The parameters in the right-hand side of the formula (1.3.2) are determined by the expressions (1.2.7), (1.3.4.) and (1.3.5). With

allowance made for $V_{yc} \cos \theta_c = V_{xc} \sin \theta_c$ the first term in the right-hand side of the formula (1.3.3) can be obtained as

$$\left. -\frac{2F_{yc}V_{yc}}{\rho SU_0^3} = \left[+ \begin{array}{l} \left. \frac{2}{\rho SU_0^3} \lambda_{22} \overline{V_{yc} \frac{d(v_{nc} \cos \theta)}{dt}} + \right. \\ \left. \left[C_y^\alpha v_{nc} V_{yc} V_{xc} \frac{1}{U_0^3 \cos \alpha_c} + \right. \right. \\ \left. \left. + b \left(C_y^\alpha - \frac{2\lambda_{22}}{\rho Sb} \right) \overline{\dot{v}_{nc} V_{xc} \frac{\sin \theta_c}{U_0^3 \cos \alpha_c}} - \right. \right. \\ \left. \left. - C_y^{\omega_z} \omega_z V_{yc} V_{xc} b \frac{1}{U_0^3 \cos \alpha_c} - C_y^{\omega_z} \dot{\omega}_z V_{xc} b^2 \frac{\sin \theta_c}{U_0^3 \cos \alpha_c} \right] \right. \\ \left. + \frac{\pi}{2U_0^3} \overline{v_{nc}^2 V_{yc} \sin \theta} + \overline{\frac{U_c^2}{U_0^3} CV_{yc} \sin \theta} \end{array} \right] \right\}. \quad (1.8.1)$$

If angle of attack is small it can be conceived of as $\cos \alpha_c = 1$.

The second term in the formula (1.3.3) can be conceived of as

$$\left. -\frac{2M_{zc}\omega_z}{\rho SU_0^3} = \left[\begin{array}{l} \left. \frac{m_z^\alpha b \overline{v_{nc} U_c \omega_z}}{U_0^3} + m_{zc}^{\dot{\alpha}} \frac{b^2 \overline{\dot{v}_{nc} \omega_z}}{U_0^3} - \right. \\ \left. - m_{zc}^{\omega_z} \frac{b^2 \overline{U_c \omega_z^2}}{U_0^3} - m_z^{\dot{\omega}_z} \frac{b^3 \overline{\omega_z \dot{\omega}_z}}{U_0^3} \right] \end{array} \right]. \quad (1.8.2)$$

Let us obtain the design formulas for the first cinematic regime (1.5.1) and (1.5.2) when the phase angle by which the pitch motion leads the heave motion is arbitrary. Formula (1.3.3) can be envisioned as the sum of the power coefficients

$$-\frac{2F_{yc}V_{yc}}{\rho SU_0^3} - \frac{2M_{zc}\omega_z}{\rho SU_0^3} = \left(C_{p1} + C_{p2} + C_{p3} + C_{p4} + C_{p5} + C_{p6} + \right. \\ \left. + C_{p7} + C_{p8} + C_{p9} + C_{p10} + C_{p11} \right). \quad (1.8.3)$$

The power coefficients in the right-hand side of the formula (1.8.3) take the form

$$C_{p1} = \frac{2\lambda_{22}}{\rho Sb} \left\{ \begin{aligned} & -\frac{(Sh_0)\vartheta_0}{2\lambda_p} \cos\varphi (1 - 0.5\vartheta_0^2) - \\ & -\frac{(Sh_0)^2}{2} \vartheta_0^2 X_b (1 - 0.625\vartheta_0^2 + 0.104\vartheta_0^4) - \\ & -\frac{(Sh_0)^2}{4\lambda_p^2} \vartheta_0^2 \sin\varphi \cos\varphi (1 - 0.3333\vartheta_0^2) - \\ & -\frac{(Sh_0)^2}{8\lambda_p} \vartheta_0^3 X_b \sin\varphi \left(3 - 1.25\vartheta_0^2 + \right. \\ & \left. + 0.33\vartheta_0^2 \sin^2 2\varphi \right) + \\ & + \frac{0.33(Sh_0)\vartheta_0^4}{4\lambda_p^2} \sin\varphi \cos\varphi + \\ & + \frac{(Sh_0)^3}{8} \vartheta_0^6 X_b^2 \sin\varphi \cos^5 \varphi \end{aligned} \right\}. \quad (1.8.4)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P1} = \frac{2\lambda_{22}}{\rho Sb} \begin{cases} -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.625g_0^2 + 0.104g_0^4) - \\ -\frac{(Sh_0)^2 g_0^3 X_b}{8\lambda_p} (3 - 1.25g_0^2) \end{cases}. \quad (1.8.5)$$

$$C_{P2} = C_{yc}^\alpha \begin{cases} \frac{1}{2\lambda_p^2} [1 - 0.125g_0^2 (1 + 2\sin^2 \varphi)] - \\ -\frac{g_0}{2\lambda_p} \sin \varphi (1 - 0.125g_0^2) + \frac{(Sh_0) g_0 X_b}{\lambda_p} \cos \varphi + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} \left[1 + 0.125g_0^2 - \right. \\ \left. - 0.1g_0^4 \begin{cases} \cos^6 \varphi + 4\sin \varphi \cos^5 \varphi + \\ + 11\sin^2 \varphi \cos^4 \varphi + \\ + 20\sin^3 \varphi \cos^3 \varphi + \\ + 3\sin^4 \varphi \cos^2 \varphi + \sin^6 \varphi \end{cases} \right] - \\ - \frac{(Sh_0)^2 g_0^3 X_b^2}{4\lambda_p} \sin \varphi (1 - 0.33g_0^2) + \\ + \frac{(Sh_0)^3 g_0^6 X_b^3}{8} \sin \varphi \cos^5 \varphi \end{cases}. \quad (1.8.6)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P2} = C_{yc}^\alpha \begin{cases} \frac{1}{2\lambda_p^2} (1 - 0.375g_0^2) - \frac{g_0}{2\lambda_p} (1 - 0.125g_0^2) + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 + 0.125g_0^2 - 0.104g_0^4) - \\ - \frac{(Sh_0)^2 g_0^3 X_b^2}{4\lambda_p} (1 - 0.3333g_0^2) \end{cases}. \quad (1.8.7)$$

$$C_{P3} = \left(C_{yc}^a - \frac{2\lambda_{22}}{\rho Sb} \right) \begin{cases} -\frac{(Sh_0)g_0}{2\lambda_p} \cos \varphi (1 - 0.25g_0^2) - \\ -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 1.125g_0^2 + 0.625g_0^4) - \\ -\frac{(Sh_0)^2 g_0^3 X_b}{8\lambda_p} \sin \varphi (1 - 0.917g_0^2) + \\ +\frac{(Sh_0)g_0^4}{2} \sin^3 \varphi \cos \varphi \end{cases}. \quad (1.8.8)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P3} = \left(C_{yc}^a - \frac{2\lambda_{22}}{\rho Sb} \right) \begin{cases} -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 1.125g_0^2 + 0.625g_0^4) - \\ -\frac{(Sh_0)^2 g_0^3 X_b}{8\lambda_p} (1 - 0.917g_0^2) \end{cases}. \quad (1.8.9)$$

$$C_{P4} = -C_{yc}^a \begin{cases} \frac{(Sh_0)g_0}{2\lambda_p} \cos \varphi + \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125g_0^2) - \\ -\frac{(Sh_0)^2 g_0^3 X_b}{4\lambda_p} \sin \varphi \cos^2 \varphi (1 - 0.0833g_0^2) + \\ +\frac{(Sh_0)g_0^3 X_b}{8\lambda_p} \cos 2\varphi (1 - 0.0833g_0^2) + \\ +\frac{(Sh_0)^3 g_0^6 X_b^2}{8} \sin^3 \varphi \cos^3 \varphi \end{cases}. \quad (1.8.10)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{p4} = -C_{yc}^{\omega_c} \left\{ \begin{array}{l} \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125 g_0^2) - \\ - \frac{(Sh_0) g_0^3 X_b}{8\lambda_p} (1 - 0.0833 g_0^2) \end{array} \right\}. \quad (1.8.11)$$

$$C_{p5} = C_{yc}^{\dot{\omega}_c} \left[\frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125 g_0^2) \right]. \quad (1.8.12)$$

In the case when $\varphi = \frac{\pi}{2}$ this coefficient is the same

$$\begin{aligned}
C_{P_6} = \frac{\pi}{2} & \left\{ \begin{array}{l}
\frac{3g_0}{8\lambda_p^3} \left[1 - 0.9722g_0^2 (1 - 0.4 \cos^2 \varphi) \right] - \\
-\frac{g_0^2}{4\lambda_p^2} \left[1 + 2 \sin^2 \varphi - 0.417g_0^2 (1 + 4 \sin^2 \varphi) \right] + \\
+\frac{3g_0^3}{8\lambda_p} \sin \varphi (1 - 0.417g_0^2) + \\
+\frac{3(Sh_0)g_0^2 X_b}{8\lambda_p^2} \sin 2\varphi \left[1 - 0.2778 \cos 2\varphi - 0.2222g_0^2 \right] + \\
+\frac{(Sh_0)g_0^3 X_b}{2\lambda_p} \sin^2 \varphi \cos \varphi - \\
-\frac{(Sh_0)g_0^3 X_b}{4\lambda_p} \cos \varphi \left[1 + 3 \sin^2 \varphi - \right. \\
\left. - 0.67g_0^2 (1 + 2 \sin^4 \varphi) \right] + \\
+\frac{(Sh_0)^2 g_0^3 X_b^2}{4\lambda_p} \sin \varphi \left[1 - \right. \\
\left. - 0.5834g_0^2 (\cos^2 2\varphi - \right. \\
\left. - 0.5 \sin^2 2\varphi) \right] - \\
-\frac{(Sh_0)^2 g_0^4 X_b^2}{4} + 0.375(Sh_0)^3 g_0^4 X_b^3 \sin^3 \varphi \cos^3 \varphi
\end{array} \right\} . \quad (1.8.13)
\end{aligned}$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$\begin{aligned}
C_{P_6} = \frac{\pi}{2} & \left\{ \begin{array}{l}
\frac{3g_0}{8\lambda_p^3} (1 - 0.9722g_0^2) - \frac{g_0^2}{4\lambda_p^2} (3 - 2.085g_0^2) + \\
+\frac{3g_0^3}{8\lambda_p} (1 - 0.417g_0^2) + \\
+\frac{(Sh_0)^2 g_0^3 X_b^2}{4\lambda_p} (1 - 0.5834g_0^2) - \frac{(Sh_0)^2 g_0^4 X_b^2}{4}
\end{array} \right\} . \quad (1.8.14)
\end{aligned}$$

$$C_{P7} = C \left\{ \frac{g_0(2\lambda_p^2 + 1)}{4\lambda_p^3} \sin \varphi \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - \right. \right. \\ \left. \left. - 0.125g_0^2 \left(1 + \frac{0.6667}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right) \right] + \right. \\ \left. + \frac{(Sh_0)g_0^2 X_b}{8\lambda_p^2} \sin 2\varphi (1 - 0.0833g_0^2) \right\}. \quad (1.8.15)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P7} = C \left\{ \frac{g_0(2\lambda_p^2 + 1)}{4\lambda_p^3} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.125g_0^2 \left(1 + \frac{0.6667}{(2\lambda_p^2 + 1)} \right) \right] \right\}. \quad (1.8.16)$$

$$C_{P8} = m_x^\alpha \left\{ \frac{(Sh_0)\sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p^2} \cos \varphi \left[g_0 \left(1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right) - \right. \right. \\ \left. \left. - 0.1g_0^3 \left(1 + \frac{4\lambda_p^2 + 3}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right) \right] + \right. \\ \left. + \frac{(Sh_0)g_0^3 \sqrt{(2\lambda_p^2 + 1)}}{8\sqrt{2}\lambda_p^2} \sin^2 \varphi \cos \varphi - \right. \\ \left. - \frac{(Sh_0)g_0^2}{8\sqrt{2}\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi + \right. \\ \left. + \frac{(Sh_0)^2 g_0^2 X_b \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left(1 + \frac{0.25}{(2\lambda_p^2 + 1)} \cos 2\varphi \right) \right\}. \quad (1.8.17)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P8} = m_{zc}^{\alpha} \left\{ \frac{(Sh_0)^2 g_0^2 X_b \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right\}. \quad (1.8.18)$$

$$C_{P9} = m_{zc}^{\alpha} \left[\begin{array}{l} \frac{(Sh_0)^2 g_0}{2\lambda_p} \sin \varphi (1 - 0.125g_0^2) - \frac{(Sh_0)^2 g_0^3}{8\lambda_p} \sin \varphi - \\ - \frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125g_0^2) \end{array} \right]. \quad (1.8.19)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P9} = m_{zc}^{\alpha} \left[\begin{array}{l} \frac{(Sh_0)^2 g_0}{2\lambda_p} (1 - 0.125g_0^2) - \frac{(Sh_0)^2 g_0^3}{8\lambda_p} - \\ - \frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125g_0^2) \end{array} \right]. \quad (1.8.20)$$

$$C_{P10} = -m_{zc}^{\alpha} \frac{(Sh_0)^2 g_0^2 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[1 + \frac{0.125}{(2\lambda_p^2 + 1)} \cos 2\varphi \right]. \quad (1.8.21)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P10} = -m_{zc}^{\omega_2} \frac{(Sh_0)^2 g_0^2 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right]. \quad (1.8.22)$$

$$C_{P11} = -m_{zc}^{\dot{\omega}_2} \frac{(Sh_0)^3 g_0^2}{4\lambda_p^2} \sin \varphi \cos \varphi. \quad (1.8.23)$$

In the case when $\varphi = \frac{\pi}{2}$ we have

$$C_{P11} = 0. \quad (1.8.24)$$

All formulas (1.8.4) – (1.8.24) can be transformed to formulas for pure heaving and pure pitching oscillations.

In the case of pure heaving oscillations

$$C_{P1} = C_{P3} = C_{P4} = C_{P5} = C_{P6} = C_{P7} = C_{P8} = C_{P9} = C_{P10} = C_{P11} = 0, \quad (1.8.25)$$

$$C_{P2} = C_y^\alpha \frac{0.5}{\lambda_p^2}. \quad (1.8.26)$$

In the case of pure pitching oscillations

$$C_{p1} = \frac{2\lambda_{22}}{\rho Sb} \left\{ \begin{aligned} & -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.625g_0^2 + 0.104g_0^4) - \\ & -\frac{(Sh_0)^3 g_0^6 X_b^2}{8} \sin \varphi \cos^5 \varphi \end{aligned} \right\}. \quad (1.8.27)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{p1} = \frac{2\lambda_{22}}{\rho Sb} \left\{ -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.625g_0^2 + 0.104g_0^4) \right\}. \quad (1.8.28)$$

$$C_{p2} = C_{yc}^\alpha \left\{ \begin{aligned} & +\frac{(Sh_0)^2 g_0^2 X_b^2}{2} \left[1 + 0.125g_0^2 - \right. \\ & \left. \left(\cos^6 \varphi + \right. \right. \\ & \left. \left. + 4 \sin \varphi \cos^5 \varphi + \right. \right. \\ & \left. \left. + 11 \sin^2 \varphi \cos^4 \varphi + \right. \right. \\ & \left. \left. + 20 \sin^3 \varphi \cos^3 \varphi + \right. \right. \\ & \left. \left. + 3 \sin^4 \varphi \cos^2 \varphi + \right. \right. \\ & \left. \left. + \sin^6 \varphi \right) \right] + \\ & +\frac{(Sh_0)^3 g_0^6 X_b^3}{8} \sin \varphi \cos^5 \varphi \end{aligned} \right\}. \quad (1.8.29)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{P2} = C_{yc}^{\alpha} \left\{ \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 + 0.125g_0^2 - 0.104g_0^4) \right\}. \quad (1.8.30)$$

$$C_{P3} = \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) \left\{ \begin{aligned} & -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 1.125g_0^2 + 0.625g_0^4) + \\ & + \frac{(Sh_0) g_0^4}{2} \sin^3 \varphi \cos \varphi \end{aligned} \right\}. \quad (1.8.31)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{P3} = \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) \left\{ -\frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 1.125g_0^2 + 0.625g_0^4) \right\}. \quad (1.8.32)$$

$$C_{P4} = -C_{yc}^{\omega_z} \left\{ \begin{aligned} & \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125g_0^2) + \\ & + \frac{(Sh_0)^3 g_0^6 X_b^2}{8} \sin^3 \varphi \cos^3 \varphi \end{aligned} \right\}. \quad (1.8.33)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{P4} = -C_{yc}^{\omega_z} \left\{ \frac{(Sh_0)^2 g_0^2 X_b}{2} (1 - 0.125g_0^2) \right\}. \quad (1.8.34)$$

$$C_{P5} = C_{yc}^{\dot{\phi}_c} \left[\frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125 g_0^2) \right]. \quad (1.8.35)$$

When $\varphi = \frac{\pi}{2}$ this coefficient is invariant

$$C_{P6} = \frac{\pi}{2} \left\{ -\frac{(Sh_0)^2 g_0^4 X_b^2}{4} + 0.375 (Sh_0)^3 g_0^4 X_b^3 \sin^3 \varphi \cos^3 \varphi \right\}. \quad (1.8.36)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{P6} = \frac{\pi}{2} \left\{ -\frac{(Sh_0)^2 g_0^4 X_b^2}{4} \right\}. \quad (1.8.37)$$

$$C_{P7} = C \left\{ \begin{aligned} & \frac{g_0(2\lambda_p^2 + 1)}{4\lambda_p^3} \sin \varphi \left[\begin{aligned} & 1 + \frac{0.5}{(2\lambda_p^2 + 1)} - \\ & -0.125 g_0^2 \left(1 + \frac{0.67}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right) \end{aligned} \right]^{+} \\ & + \frac{(Sh_0) g_0^2 X_b}{8\lambda_p^2} \sin 2\varphi (1 - 0.083 g_0^2) \end{aligned} \right\}. \quad (1.8.38)$$

When $\varphi = \frac{\pi}{2}$

$$C_{P7} = C \left\{ \frac{g_0(2\lambda_p^2 + 1)}{4\lambda_p^3} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.125g_0^2 \left(1 + \frac{0.6667}{(2\lambda_p^2 + 1)} \right) \right] \right\}. \quad (1.8.39)$$

$$C_{P8} = m_{zc}^\alpha \left\{ \begin{array}{l} \frac{(Sh_0)\sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p^2} \cos \varphi \left[g_0 \left(1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right) - \right. \\ \left. - 0.1g_0^3 \left(1 + \frac{4\lambda_p^2 + 3}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right) \right] + \\ + \frac{(Sh_0)g_0^3\sqrt{(2\lambda_p^2 + 1)}}{8\sqrt{2}\lambda_p^2} \sin^2 \varphi \cos \varphi - \\ - \frac{(Sh_0)g_0^2}{8\sqrt{2}\lambda_p\sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi + \\ + \frac{(Sh_0)^2 g_0^2 X_b \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left(1 + \frac{0.25}{(2\lambda_p^2 + 1)} \cos 2\varphi \right) \end{array} \right\} \quad (1.8.40)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{P8} = m_{zc}^\alpha \left\{ \frac{(Sh_0)^2 g_0^2 X_b \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right\}. \quad (1.8.41)$$

$$C_{p9} = m_{zc}^{\dot{\alpha}} \begin{bmatrix} \frac{(Sh_0)^2 g_0}{2\lambda_p} \sin \varphi (1 - 0.125g_0^2) - \frac{(Sh_0)^2 g_0^3}{8\lambda_p} \sin \varphi - \\ - \frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125g_0^2) \end{bmatrix}. \quad (1.8.42)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{p9} = m_{zc}^{\dot{\alpha}} \begin{bmatrix} \frac{(Sh_0)^2 g_0}{2\lambda_p} (1 - 0.125g_0^2) - \frac{(Sh_0)^2 g_0^3}{8\lambda_p} - \\ - \frac{(Sh_0)^2 g_0^2}{2} (1 - 0.125g_0^2) \end{bmatrix}. \quad (1.8.43)$$

$$C_{p10} = -m_{zc}^{\omega_z} \frac{(Sh_0)^2 g_0^2 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[1 + \frac{0.125}{(2\lambda_p^2 + 1)} \cos 2\varphi \right]. \quad (1.8.44)$$

When $\varphi = \frac{\pi}{2}$ we shall have

$$C_{p10} = -m_{zc}^{\omega_z} \frac{(Sh_0)^2 g_0^2 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right]. \quad (1.8.45)$$

$$C_{p11} = 0. \quad (1.8.46)$$

We are coming to the second variant of the cinematic parameters (1.5.3) and (1.5.4). The design formulas (the power coefficients when $y = y_0 \sin \omega t$ и $\alpha = \alpha_0 \cos \omega t$) are derivable from common formulas (1.3.1) – (1.3.3). This variant is rarely used in the wing investigations. Because of this, we shall restrict our consideration to the case when $\varphi = \frac{\pi}{2}$.

$$\begin{aligned}
C_{Pl} = A_{Pl} \left\{ \right. & 2\alpha_0 \lambda_p^2 \left[0.5 + \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} + \right. \\
& \left. + \frac{0.4102}{(2\lambda_p^2 + 1)^3} + \frac{0.4615}{(2\lambda_p^2 + 1)^4} + \right. \\
& \left. + \frac{0.2283}{(2\lambda_p^2 + 1)^5} + \frac{0.15}{(2\lambda_p^2 + 1)^6} \right] - \\
& -2\lambda_p \left[0.5 - \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} - \frac{0.4102}{(2\lambda_p^2 + 1)^3} + \right. \\
& \left. + \frac{0.4615}{(2\lambda_p^2 + 1)^4} - \frac{0.2283}{(2\lambda_p^2 + 1)^5} + \frac{0.15}{(2\lambda_p^2 + 1)^6} \right] - \\
& - \frac{3\lambda_p + 2\alpha_0 - \alpha_0\lambda_p^2}{(2\lambda_p^2 + 1)} \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \right. \\
& \left. + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \frac{0.3494}{(2\lambda_p^2 + 1)^6} \right] + \\
& + \alpha_0 (2\lambda_p^2 + 1) \left[0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \right. \\
& \left. + \frac{0.0938}{(2\lambda_p^2 + 1)^2} - \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right] - \\
& - \alpha_0 \left[0.25 - \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.586}{(2\lambda_p^2 + 1)^2} - \frac{0.8204}{(2\lambda_p^2 + 1)^3} \right] - \\
& - \frac{\alpha_0^2 \lambda_p (2\lambda_p^2 + 1)}{2} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \right. \\
& \left. + \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] \left. \right\}, \quad (1.8.47)
\end{aligned}$$

where

$$A_{P1} = \frac{2\lambda_{22}}{\rho Sb} \frac{\sqrt{2}(Sh_0)^2 X_b}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.48)$$

$$C_{P2} = A_{P2} \left\{ \begin{aligned} & \left[0.5 + \frac{0.625}{(2\lambda_p^2 + 1)} + \frac{1.094}{(2\lambda_p^2 + 1)^2} + \frac{1.23}{(2\lambda_p^2 + 1)^3} + \right. \\ & \left. + \frac{1.692}{(2\lambda_p^2 + 1)^4} + \frac{1.482}{(2\lambda_p^2 + 1)^5} + \frac{1.4}{(2\lambda_p^2 + 1)^6} \right] + \\ & + \frac{\alpha_0 (2\lambda_p^2 + 1)^3}{8\lambda_p^5 (Sh_0)^2 X_b^2} \left[\begin{aligned} & 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} - \\ & - \frac{0.0313}{(2\lambda_p^2 + 1)^2} + \frac{0.0117}{(2\lambda_p^2 + 1)^3} \end{aligned} \right] - \\ & - \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} \left[\begin{aligned} & 0.5 + \frac{0.378}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} + \\ & + \frac{0.41}{(2\lambda_p^2 + 1)^3} + \frac{0.461}{(2\lambda_p^2 + 1)^4} \end{aligned} \right] - \\ & - \frac{1}{4\lambda_p^2} \left[\begin{aligned} & 0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \\ & + \frac{0.3494}{(2\lambda_p^2 + 1)^6} + \frac{0.157}{(2\lambda_p^2 + 1)^8} \end{aligned} \right] + \\ & + \frac{\alpha_0 (2\lambda_p^2 + 1)}{4\lambda_p^3} \left[0.5 + \frac{0.2344}{(2\lambda_p^2 + 1)^2} + \frac{0.1538}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \frac{3\alpha_0}{4\lambda_p} \left[\begin{aligned} & 0.5 - \frac{0.833}{(2\lambda_p^2 + 1)} + \frac{2.052}{(2\lambda_p^2 + 1)^2} - \\ & - \frac{1.64}{(2\lambda_p^2 + 1)^3} + \frac{2.068}{(2\lambda_p^2 + 1)^4} - \frac{1.98}{(2\lambda_p^2 + 1)^5} \end{aligned} \right] \end{aligned} \right\}, \quad (1.8.49)$$

where

$$A_{P_2} = C_{yc}^\alpha \frac{4\sqrt{2}(Sh_0)^2 \lambda_p^3 X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.50)$$

$$C_{P_3} = A_{P_3} \left\{ \begin{aligned} & \frac{\alpha_0 \lambda_p - 2}{(2\lambda_p^2 + 1)} \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} \right] + \\ & + 2\alpha_0 \lambda_p \left[0.5 + \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} + \right. \\ & \quad \left. + \frac{0.41}{(2\lambda_p^2 + 1)^3} + \frac{0.4615}{(2\lambda_p^2 + 1)^4} \right] - \\ & - 2 \left[0.5 - \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} - \frac{0.42}{(2\lambda_p^2 + 1)^3} + \right. \\ & \quad \left. + \frac{0.4615}{(2\lambda_p^2 + 1)^4} - \frac{0.2283}{(2\lambda_p^2 + 1)^5} + \frac{0.15}{(2\lambda_p^2 + 1)^6} \right] + \\ & + \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} \left[0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} \right] - \\ & - \frac{1.5\alpha_0}{\lambda_p} \left[0.5 - \frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.5468}{(2\lambda_p^2 + 1)^2} - \right. \\ & \quad \left. - \frac{0.5468}{(2\lambda_p^2 + 1)^3} + \frac{0.564}{(2\lambda_p^2 + 1)^4} \right] - \\ & - \frac{\alpha_0}{\lambda_p (2\lambda_p^2 + 1)} \left[0.5 - \frac{0.3125}{(2\lambda_p^2 + 1)} + \frac{0.5469}{(2\lambda_p^2 + 1)^2} - \right. \\ & \quad \left. - \frac{0.41}{(2\lambda_p^2 + 1)^3} + \frac{0.564}{(2\lambda_p^2 + 1)^4} \right] \end{aligned} \right\}, \quad (1.8.51)$$

where

$$A_{P3} = \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) \frac{\sqrt{2} (Sh_0)^2 \lambda_p X_b}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.52)$$

$$C_{P4} = A_{P4} \left\{ \begin{array}{l} \frac{\alpha_0^2 (2\lambda_p^2 + 1)^2}{2} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \right. \\ \left. + \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] + \\ + (2\alpha_0 \lambda_p - 1) \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} \right] + \\ + \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} \left[0.5 + \frac{0.234}{(2\lambda_p^2 + 1)^2} + \frac{0.1538}{(2\lambda_p^2 + 1)^4} \right] + \\ + 4\alpha_0 \lambda_p (2\lambda_p^2 + 1) \left[0.5 + \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} + \right. \\ \left. + \frac{0.41}{(2\lambda_p^2 + 1)^3} + \frac{0.4615}{(2\lambda_p^2 + 1)^4} \right], \end{array} \right\}, \quad (1.8.53)$$

where

$$A_{p4} = -C_{yc}^{\omega_2} \frac{\sqrt{2}(Sh_0)^2 \lambda_p X_b}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.54)$$

$$C_{p5} = A_{p5} \left\{ \begin{aligned} & \left[0.5 - \frac{0.375}{(2\lambda_p^2 + 1)} + \frac{0.4688}{(2\lambda_p^2 + 1)^2} - \frac{0.41}{(2\lambda_p^2 + 1)^3} + \right. \\ & \left. + \frac{0.4615}{(2\lambda_p^2 + 1)^4} - \frac{0.2283}{(2\lambda_p^2 + 1)^5} + \frac{0.15}{(2\lambda_p^2 + 1)^6} \right] + \\ & + \frac{1}{(2\lambda_p^2 + 1)} \left[0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \right. \\ & \left. + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \frac{0.3494}{(2\lambda_p^2 + 1)^6} \right] - \\ & - \frac{\alpha_0 (2\lambda_p^2 + 1)}{4\lambda_p} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} - \right. \\ & \left. - \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \frac{\alpha_0^2 \lambda_p}{4} \left[1.5 - \frac{1.5}{(2\lambda_p^2 + 1)} + \frac{1.64}{(2\lambda_p^2 + 1)^2} - \right. \\ & \left. - \frac{1.64}{(2\lambda_p^2 + 1)^3} + \frac{1.6919}{(2\lambda_p^2 + 1)^4} - \frac{0.9131}{(2\lambda_p^2 + 1)^5} + \right. \\ & \left. + \frac{0.5628}{(2\lambda_p^2 + 1)^6} - \frac{0.32}{(2\lambda_p^2 + 1)^7} + \frac{0.142}{(2\lambda_p^2 + 1)^8} \right], \end{aligned} \right\}, \quad (1.8.55)$$

where

$$A_{P5} = C_{yc}^{\phi_2} \frac{2\sqrt{2}(Sh_0)^2 \lambda_p}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.56)$$

$$C_{P6} = A_{P6} \left\{ -\frac{(2\lambda_p^2 + 1)}{\lambda_0^2} \left[\begin{array}{l} 0.5 + \frac{0.5469}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \\ + \frac{0.3494}{(2\lambda_p^2 + 1)^6} + \frac{0.157}{(2\lambda_p^2 + 1)^8} \end{array} \right] + \right. \\ \left. + \frac{\alpha_0 (2\lambda_p^2 + 1)^3}{8(Sh_0)^2 \lambda_p^5 X_b^2} \left[1.5 + \frac{0.5}{(2\lambda_p^2 + 1)} - \frac{0.1094}{(2\lambda_p^2 + 1)^2} \right] \right\}, \quad (1.8.57)$$

where

$$A_{P6} = \frac{\pi}{2} \frac{\sqrt{2}(Sh_0)^2 \alpha_0 \lambda_p^2 X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}}. \quad (1.8.58)$$

$$C_{P7} = C \frac{\sqrt{(2\lambda_p^2 + 1)}}{\sqrt{2}\lambda_p^2} \left(\frac{1}{\lambda_p} - \alpha_0 \right) \left[0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0316}{(2\lambda_p^2 + 1)^2} \right], \quad (1.8.59)$$

$$C_{p_8} = C_{p_8-0} \left\{ \begin{aligned} & -\alpha_0 \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \right. \\ & \left. + \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \frac{\lambda_p}{(2\lambda_p^2 + 1)} \left[1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.9375}{(2\lambda_p^2 + 1)^2} + \right. \\ & \left. + \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.9223}{(2\lambda_p^2 + 1)^4} \right] + \\ & + \frac{\alpha^2 (2\lambda_p^2 + 1)}{4\lambda_p} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^2} - \right. \\ & \left. - \frac{0.0234}{(2\lambda_p^2 + 1)^3} - \frac{0.0146}{(2\lambda_p^2 + 1)^4} \right] \end{aligned} \right\}, \quad (1.8.60)$$

here

$$C_{p_8-0} = m_{zc}^\alpha \frac{\sqrt{2} (Sh_0)^2 X_b}{\sqrt{(2\lambda_p^2 + 1)}}$$

$$C_{p_9} = m_{zc}^{\dot{\alpha}} \frac{\alpha_0 (Sh_0)^2}{2\lambda_p} \left\{ 1 - \frac{\alpha_0 (2\lambda_p^2 + 1)}{2} \left[1 - \frac{0.5}{(2\lambda_p^2 + 1)} \right] \right\}, \quad (1.8.61)$$

$$C_{P10} = -m_{zc}^{\omega_c} (Sh_0)^2 \left\{ \frac{1}{(2\lambda_p^2 + 1)} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.5}{(2\lambda_p^2 + 1)^2} + \right. \right. \\ \left. \left. + \frac{0.375}{(2\lambda_p^2 + 1)^3} + \frac{0.375}{(2\lambda_p^2 + 1)^4} + \right. \right. \\ \left. \left. + \frac{0.3125}{(2\lambda_p^2 + 1)^5} + \frac{0.3125}{(2\lambda_p^2 + 1)^6} \right] - \right. \\ \left. - \frac{\alpha_0}{\lambda_p} + \frac{\alpha_0^2}{4\lambda_p^2} \left[1 - \frac{0.5}{(2\lambda_p^2 + 1)} \right] \right\}, \quad (1.8.62)$$

$$C_{P11} = 0. \quad (1.8.63)$$

The design formulas are derivable from common formulas
 (1.8.47) – (1.8.63)

To pure heaving oscillations this formulas are invariant because
 foil pitch angle is absent.

To pure pitching oscillations

$$C_{P1} = \left\{ -\frac{\lambda_{22}}{\rho S b} (Sh_0)^2 X_b \alpha_0^2 \right\}, \quad (1.8.64)$$

$$C_{P2} = C_{P3} = C_{P5} = C_{P6} = C_{P7} = C_{P10} = C_{P11} = 0, \quad (1.8.65)$$

$$C_{P4} = \left\{ -C_{yc}^{\omega_2} (Sh_0)^2 X_b \frac{\alpha_0^2}{2} \right\}, \quad (1.8.66)$$

$$C_{P8} = \left\{ m_{zc}^{\alpha} (Sh_0)^2 X_b \frac{\alpha^2}{2} \right\}, \quad (1.8.67)$$

$$C_{P9} = -m_{zc}^{\dot{\alpha}} \frac{\alpha_0^2 (Sh_0)^2}{2}. \quad (1.8.68)$$

We are coming to the third variant of the cinematic parameters (1.5.5) and (1.5.6). The design formulas are derivable from common formulas (1.6.3). In this case the wing pitch angle and the angle of attack are harmonic. Heaving oscillations are non harmonious. We are confined to the consideration of the case when $\varphi = \frac{\pi}{2}$ (as in the previous variant).

The time derivative of the heaving oscillations is (Prempraneerach at all, 2003)

$$\frac{dy}{dt} = U_0 t g(\alpha + \vartheta), \quad (1.8.69)$$

The power coefficients in the formula (1.8.3) will look like

$$C_{P1} = C_{P1-0} \begin{bmatrix} -0.5 + 0.3125g_0^2 - 0.1g_0^4 + 0.0125g_0^6 - \\ -\frac{1}{g_0} \left(0.5\theta_0 + 0.125\theta_0^3 + 0.0417\theta_0^5 - \right. \\ \left. -0.1875g_0^2\theta_0 - 0.0521g_0^2\theta_0^3 - 0.0182g_0^2\theta_0^5 \right) - \\ -0.375g_0\theta_0 - 0.0624g_0\theta_0^3 - 0.0156g_0\theta_0^5 + \\ +0.1562g_0^3\theta_0 + 0.0326g_0^3\theta_0^3 + 0.01g_0^3\theta_0^5 - \\ -0.0326g_0^5\theta_0 - 0.0076g_0^5\theta_0^3 + \\ +\frac{\theta_0}{g_0} \left(0.5 - 0.1875g_0^2 + 0.0469g_0^4 + \right. \\ \left. +0.125\theta_0^2 - 0.0938g_0^2\theta_0^2 + \right. \\ \left. +0.0293g_0^4\theta_0^2 - 0.0156\theta_0^4 + \right. \\ \left. +0.0146g_0^2\theta_0^4 - 0.0051g_0^4\theta_0^4 \right) \end{bmatrix}, \quad (1.8.70)$$

here

$$C_{P1-0} = \lambda_{22} \frac{2(Sh_o)^2 g_0^2 X_b}{\rho Sb}.$$

$$C_{P2} = C_{jyc}^a \theta_0 \begin{bmatrix} 0.5\theta_0 + 0.25\theta_0^3 - 0.1875g_0^2\theta_0 - 0.1042g_0^2\theta_0^3 - \\ -0.0516g_0^2\theta_0^5 - 0.5g_0 - 0.125g_0\theta_0^2 - 0.0417g_0\theta_0^4 - \\ +0.0625g_0^3 + 0.0174g_0^3\theta_0^2 - \\ -\frac{(Sh_0)^2 g_0^2 X_b^2}{\theta_0} \begin{bmatrix} -0.5 + 0.0625g_0^2 + 0.25g_0\theta_0 + \\ +0.0417g_0\theta_0^3 + 0.0104g_0\theta_0^5 - \\ -0.0833g_0^3\theta_0 - 0.0174g_0^3\theta_0^3 - \\ -0.0049g_0^5\theta_0 + 0.0163g_0^5\theta_0 + \\ +0.0038g_0^5\theta_0^3 - 0.125g_0^2 + \\ +0.0521g_0^4 \end{bmatrix} \end{bmatrix}, \quad (1.8.71)$$

$$C_{P3} = C_{P3-0} \begin{bmatrix} -0.5 + 0.1875g_0^2 - 0.0521g_0^4 + 0.0076g_0^6 - \\ -0.125g_0\theta_0 + 0.0521g_0^3\theta_0 - 0.0195g_0^5\theta_0 + \\ +0.013g_0^3\theta_0^3 - 0.0046g_0^5\theta_0^3 + 0.0091g_0^3\theta_0^5 \end{bmatrix}, \quad (1.8.72)$$

here

$$C_{P_3-0} = C_{yc}^{\omega_z} \left(C_{yc}^{\alpha} - \frac{2\lambda_{22}}{\rho Sb} \right) (Sh_0)^2 g_0^2 X_b \\ C_{P_4} = -C_{yc}^{\omega_z} \begin{bmatrix} -0.5 + 0.0625g_0^2 + 0.125g_0\theta_0 + \\ +(Sh_0)^2 g_0^2 X_b \\ +0.0208g_0\theta_0^3 + 0.0052g_0\theta_0^5 - \\ -0.0104g_0^3\theta_0 - 0.0022g_0^3\theta_0^3 \end{bmatrix}, \quad (1.8.73)$$

$$C_{P_5} = -C_{yc}^{\omega_z} \begin{bmatrix} g_0(-0.5 + 0.0625g_0^2) + \\ +(Sh_0)^2 g_0 \begin{bmatrix} +\alpha_0(-0.5 + 0.1875g_0^2) + \\ +(Sh_0)^2 g_0^3 X_b^2 (0.125 - 0.0287g_0^2) \end{bmatrix} \end{bmatrix}, \quad (1.8.74)$$

$$C_{P_6} = \frac{\pi}{2} \begin{bmatrix} 0.375g_0\theta_0^3 + 0.313g_0\theta_0^5 + 0.2005g_0\theta_0^7 + 0.375g_0^3 + \\ +(Sh_0)^2 g_0^2 X_b^2 \begin{bmatrix} 0.375g_0\theta_0 + 0.0625g_0\theta_0^3 - \\ -0.1562g_0^3\theta_0 - 0.0326g_0^3\theta_0^3 + \\ +0.0326g_0^5\theta_0 + 0.0076g_0^5\theta_0^3 - \\ -0.25g_0^2 + 0.1042g_0^4 - 0.0152g_0^6 \end{bmatrix} - \\ -0.1563g_0^5\theta_0 + 0.0683g_0^5\theta_0^3 + 0.0861g_0^5\theta_0^5 - \\ -0.26g_0^3\theta_0^3 - 0.75g_0^2\theta_0^2 - 0.4166g_0^2\theta_0^4 + \\ +0.5208g_0^4\theta_0^2 + 0.3038g_0^4\theta_0^4 + 0.1548g_0^4\theta_0^6 - \\ -0.1064g_0^6\theta_0^2 - 0.0638g_0^6\theta_0^4 - 0.0332g_0^6\theta_0^6 \end{bmatrix}, \quad (1.8.75)$$

$$C_{P_7} = C \begin{pmatrix} 0.5g_0\theta_0 + 0.5g_0\theta_0^3 + 0.3542g_0\theta_0^5 + 0.2066g_0\theta_0^7 - \\ -0.0625g_0^3\theta_0 - 0.0695g_0^3\theta_0^3 - 0.0517g_0^3\theta_0^5 \end{pmatrix}, \quad (1.8.76)$$

$$C_{P8} = m_{zc}^{\alpha} (Sh_0)^2 g_0^2 X_b \begin{cases} 0.5 + 0.0625\theta_0^2 + \\ + 0.013\theta_0^4 - 0.0016\theta_0^6 \end{cases}, \quad (1.8.77)$$

$$C_{P9} = m_{zc}^{\alpha} (Sh_0)^2 \alpha_0 g_0 \begin{cases} 0.5 + 0.125\theta_0^2 + 0.0417\theta_0^4 + \\ + 0.0148\theta_0^6 + 0.0036\theta_0^8 \end{cases}, \quad (1.8.78)$$

$$C_{P10} = -m_{zc}^{\alpha} (Sh_0)^2 g_0^2 \begin{cases} 0.5 + 0.125\theta_0^2 + 0.0417\theta_0^4 + \\ + 0.0148\theta_0^6 + 0.0036\theta_0^8 \end{cases}, \quad (1.8.79)$$

$$C_{P11} = -m_{zc}^{\alpha} \frac{(Sh_0)^4 g_0^4 X_b}{4} \begin{cases} 1 - 0.0833g_0^2 - \frac{\theta_0}{g_0} - \\ - 0.1668 \frac{\theta_0^3}{g_0} - 0.0417 \frac{\theta_0^5}{g_0} + \\ + 0.25g_0\theta_0 + 0.052g_0\theta_0^3 \end{cases}. \quad (1.8.80)$$

To pure heaving

$$C_{P1} = C_{P3} = C_{P4} = C_{P5} = C_{P6} = C_{P7} = C_{P8} = C_{P9} = C_{P10} = C_{P11} = 0, \quad (1.8.81)$$

$$C_{P2} = C_{yc}^{\alpha} \alpha_0^2 \{0.5 + 0.25\alpha_0^2\}. \quad (1.8.82)$$

To pitching ($y = 0, \dot{y} = 0, \theta = 0, \vartheta = -\alpha$) we shall have

$$C_{P1} = \lambda_{22} \frac{2(Sh_o)^2 g_0^2 X_b}{\rho Sb} \left[-0.5 + 0.3125 g_0^2 - 0.1 g_0^4 \right], \quad (1.8.83)$$

$$C_{P2} = C_{yc}^\alpha \left\{ - (Sh_0)^2 g_0^2 X_b^2 \left[-0.5 + 0.0625 g_0^2 \right] \right\}, \quad (1.8.84)$$

$$C_{P3} = \left(C_{yc}^\alpha - \frac{2\lambda_{22}}{\rho Sb} \right) (Sh_0)^2 g_0^2 X_b \begin{pmatrix} -0.5 + 0.1875 g_0^2 - \\ -0.0521 g_0^4 + 0.0076 g_0^6 \end{pmatrix}, \quad (1.8.85)$$

$$C_{P4} = C_{yc}^{\dot{\alpha}_z} \left[(Sh_0)^2 g_0^2 X_b (-0.5 + 0.0625 g_0^2) \right], \quad (1.8.86)$$

$$C_{P5} = -C_{yc}^{\dot{\alpha}_z} \left\{ (Sh_0)^2 g_0 \begin{bmatrix} -0.125 g_0 + \\ + (Sh_0)^2 g_0^3 X_b^2 \begin{bmatrix} 0.125 - 0.0287 g_0^2 + \\ + 0.0033 g_0^4 \end{bmatrix} \end{bmatrix} \right\}, \quad (1.8.87)$$

$$C_{P6} = \frac{\pi}{2} \left\{ (Sh_0)^2 g_0^2 X_b^2 \left(-0.25 g_0^2 + 0.1042 g_0^4 - 0.0152 g_0^6 \right) \right\}, \quad (1.8.88)$$

$$C_{P7} = 0, \quad (1.8.89)$$

$$C_{p8} = \frac{m_{zc}^{\alpha} (Sh_0)^2 g_0^2 X_b}{2}, \quad (1.8.90)$$

$$C_{p9} = \frac{m_{zc}^{\dot{\alpha}} (Sh_0)^2 \alpha_0 g_0}{2}, \quad (1.8.91)$$

$$C_{p10} = -\frac{m_{zc}^{o_z} (Sh_0)^2 g_0^2}{2}, \quad (1.8.92)$$

$$C_{p11} = -m_{zc}^{\dot{o}_z} \frac{(Sh_0)^4 g_0^4 X_b}{4} (1 - 0.0833 g_0^2). \quad (1.8.93)$$

1.9. The rigid wing inductive reactance.

Before we evolved the mathematical model of a flat and rigid wing with different form and aspect ratio when pitch-axes location varies and heaving and pitching amplitudes are sufficiently large. A peculiarity of this model is the usage of the first order aerodynamic derivatives coefficients and cinematic parameters. To calculate the thrust and efficiency the design formulas were derived. These design formulas involve the induced drag which will look like (upper estimation)

$$X_i \leq \frac{\rho \pi S v_n^2}{4}, \quad (1.9.1)$$

The designations in this and other formulas are given in parts 1.1. and 1.2. this book.

Induced drag coefficient looks like

$$C_{x_i} = \frac{\pi}{2U_0^2} v_n^2. \quad (1.9.2)$$

In this part we shall obtain the design formulas for more precised evaluation of the rigid wing induced drag. The general formula for the rigid finite-span wings induced drag will look like (Romanenko E.V. Pushkov S.G., Lopatin V.N. 2009)

$$X_i = \rho\pi \int_{-l}^l b(z)u_*(z)(v_n - u_*(z))dz. \quad (1.9.3)$$

Here $b(z)$ - the wing chord, $u_*(z)$ - the velocity induced by the wake, l - the wing semispan. It is believed that $b(z)$, $u_*(z)$ and v_n do not depend on z when high-aspect-ratio (or infinite in size) wing oscillates. Upon integrating with respect to the span (z) formula (1.9.3) will look like

$$X_i = 2\rho\pi l b u_* (v_n - u_*). \quad (1.9.4)$$

Induced drag coefficient will look like

$$C_i = \frac{2X_i}{\rho S U_0^2} = \frac{2\pi}{U_0^2} u_* (v_n - u_*). \quad (1.9.5)$$

In this formula u_* - the unknown term which can be obtain from the lifting force expression

$$Y_L = -\lambda_{22} \dot{v}_n - \rho U T = -\lambda_{22} \dot{v}_n - \rho U \pi b \left(v_n - \frac{\omega_z b}{4} - u_* \right) \cos \alpha, \quad (1.9.6)$$

On the other hand the lifting force expression in the linear approximation will look like

$$Y_L = \frac{\rho U^2 b}{2} \left(-C_y^\alpha \frac{v_n}{U} - C_y^\alpha \frac{\dot{v}_n b}{U^2} + C_y^{\omega_z} \frac{\omega_z b}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right). \quad (1.9.7)$$

We equate two right-hand parts of the expressions (1.9.6) and (1.9.7)

$$\begin{aligned} \frac{\rho U^2 b}{2} \left(-C_y^\alpha \frac{v_n}{U} - C_y^\dot{\alpha} \frac{\dot{v}_n b}{U^2} + C_y^{\omega_z} \frac{\omega_z b}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right) = \\ = -\lambda_{22} \dot{v}_n - \rho U \pi b \left(v_n - \frac{\omega_z b}{4} - u_* \right) \cos \alpha. \end{aligned} \quad (1.9.8)$$

The left-hand side formula (1.9.8) includes four members containing four variables: $v_n, \dot{v}_n, \omega_z, \dot{\omega}_z$. Let us suppose that u_* also includes four members $u_* = u_1 + u_2 + u_3 + u_4$ and each member includes one of the four variables. In this case formula (1.9.8) will look like

$$\begin{aligned} \frac{\rho U^2 b}{2} \left(-C_y^\alpha \frac{v_n}{U} - C_y^\dot{\alpha} \frac{\dot{v}_n b}{U^2} + C_y^{\omega_z} \frac{\omega_z b}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right) = \\ = -\lambda_{22} \dot{v}_n - \rho U \pi b \left(v_n - \frac{\omega_z b}{4} - u_1 - u_2 - u_3 - u_4 \right) \cos \alpha. \end{aligned} \quad (1.9.9)$$

Further we can write down

$$\pi b (v_n - u_1) \cos \alpha = \frac{b}{2} v_n C_y^\alpha, \quad (1.9.10)$$

$$\pi b \left(\frac{\omega_z b}{4} + u_2 \right) \cos \alpha = \frac{Ub}{2} C_y^{\omega_z} \frac{\omega_z b}{U}, \quad (1.9.11)$$

$$-\lambda_{22}\dot{v}_n + \rho U \pi b u_3 \cos \alpha = -\frac{\rho U^2}{2} b C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2}, \quad (1.9.12)$$

$$\rho U \pi b u_4 \cos \alpha = \frac{\rho U^2 b}{2} C_y^{\dot{\alpha}} \frac{\dot{\phi}_z b^2}{U^2}. \quad (1.9.13)$$

From the formulas (1.9.10) – (1.9.13) we can write down

$$u_1 = \left(1 - \frac{1}{2\pi \cos \alpha} C_y^{\alpha} \right) v_n, \quad (1.9.14)$$

$$u_2 = \frac{\omega_z b}{4} \left(\frac{2}{\pi \cos \alpha} C_y^{\omega_z} - 1 \right), \quad (1.9.15)$$

$$u_3 = \frac{\dot{v}_n}{U \pi \cos \alpha} \left(\frac{\lambda_{22}}{\rho b} - \frac{b}{2} C_y^{\dot{\alpha}} \right), \quad (1.9.16)$$

$$u_4 = \frac{\dot{\phi}_z b^2}{2\pi U \cos \alpha} C_y^{\dot{\phi}_z}. \quad (1.9.17)$$

Further we can write down

$$u_* = v_n - \frac{v_n}{2\pi} C_y^\alpha + \frac{\omega_z b}{2\pi} C_y^{\omega_z} - \frac{\omega_z b}{4} + \frac{\lambda_{22} \dot{v}_n}{\rho \pi b U} - \frac{\dot{v}_n b}{2\pi U} C_y^\alpha + \frac{\dot{\omega}_z b^2}{2\pi U} C_y^{\dot{\omega}_z}. \quad (1.9.18)$$

After conversion of kinematic parameters from the arbitrary point to the wing center this formula will look like

$$u_{*c} = v_{nc} - \frac{v_{nc}}{2\pi} C_{yc}^\alpha + \frac{\omega_z b}{2\pi} C_{yc}^{\omega_z} - \frac{\omega_z b}{4} + \frac{\lambda_{22} \dot{v}_{nc}}{\rho \pi b U_c} - \frac{\dot{v}_{nc} b}{2\pi U_c} C_{yc}^\alpha + \frac{\dot{\omega}_z b^2}{2\pi U_c} C_{yc}^{\dot{\omega}_z}$$

Previously we obtained the formula for the wing thrust (1.2.10)

$$\overline{F_{xc}} = \frac{\rho S}{2} \left\{ \begin{aligned} & C_{yc}^\alpha \overline{v_{nc} V_{yc}} + b \left(C_{yc}^\alpha - \frac{2\lambda_{22}}{\rho S b} \right) \overline{\dot{v}_{nc} \sin \theta_c} - C_{yc}^{\dot{\omega}_z} b^2 \overline{\dot{\omega}_z \sin \theta_c} - \\ & - b C_{yc}^{\omega_z} \overline{\omega_z V_{yc}} - \overline{X_{ic} \cos \vartheta} - C \overline{U_c^2 \cos \vartheta} \end{aligned} \right\}. \quad (1.9.19)$$

This formula can be expressible as a thrust coefficients sum

$$C_T = C_{T1} + C_{T2} + C_{T3} + C_{T4} + C_{T5} + C_{T6}. \quad (1.9.20)$$

One of the thrust coefficients which includes the induced drag will look like

$$C_{T5} = \frac{\overline{2X_{ic} \cos \vartheta}}{\rho S U_0^2}. \quad (1.9.21)$$

This formula (1.9.21) with formulas (1.9.4), (1.9.5), (1.9.18) would be expressible as

$$C_{T5} = -\frac{1}{U_0^2} \left(\begin{array}{l} D_1 \overline{v_{nc}^2 \cos \theta} + D_2 \overline{v_{nc} \omega_z \cos \theta} + D_3 \overline{\frac{v_{nc} \dot{\phi}_z}{U_c} \cos \theta} + \\ + D_4 \overline{\frac{v_{nc} \dot{v}_{nc}}{U_c} \cos \theta} + D_5 \overline{\frac{\dot{v}_{nc} \omega_z}{U_c} \cos \theta} + D_6 \overline{\omega_z^2 \cos \theta} + \\ D_7 \overline{\frac{\omega_z \dot{\phi}_z}{U_c} \cos \theta} + D_8 \overline{\frac{\dot{v}_{nc}^2}{U_c^2} \cos \theta} + D_9 \overline{\frac{\dot{v}_{nc} \dot{\phi}_z}{U_c^2} \cos \theta} + \\ + D_{10} \overline{\frac{\dot{\phi}_z^2}{U_c^2} \cos \theta} \end{array} \right). \quad (1.9.22)$$

1.9.1. The harmonically oscillating wing induced drag

Let us investigate the common case when the infinite wing executes a periodic heaving and pitching motion when phase angle by which the pitch motion leads the heave motion is arbitrary. In this case $y = y_0 \sin \omega t$ and $\theta = \theta_0 \sin(\omega t + \phi)$.

The variables in formula (1.9.22) are shown in part 1.2.

$$v_{nc} = V_{y1} \cos \theta - U_0 \sin \theta + \omega_z x, \quad (1.9.23)$$

$$U_c^2 = V_{yc}^2 + V_{xc}^2, \quad (1.9.24)$$

$$V_{xc} = U_0 - \omega_z x \sin \vartheta, \quad (1.9.25)$$

$$V_{yc} = V_{y1} + \omega_z x \cos \vartheta, \quad (1.9.26)$$

where $V_{y1} = \dot{y}(t)$, $\omega_z = \dot{\vartheta}(t)$, $y(t)$ - wing linear oscillations.

The formula (1.9.22) can be expressible as the coefficients sum

$$C_{T5} = \left(C_{T5-1} + C_{T5-2} + C_{T5-3} + C_{T5-4} + C_{T5-5} + \right. \\ \left. + C_{T5-6} + C_{T5-7} + C_{T5-8} + C_{T5-9} + C_{T5-10} \right). \quad (1.9.27)$$

The terms in the right-hand of the formula (1.9.27) will look like
(here it can be assumed that $U_c^2 \approx U_0^2 + V_{y1}^2$)

$$C_{T5-1} = -D_1 \left\{ \begin{array}{l} \frac{1}{2\lambda_p^2} \left[1 - 0.375g_0^2(1 + 2\sin^2 \varphi) + \right. \\ \left. + 0.0938g_0^4(1 + 4\sin^2 \varphi) \right] - \\ - \frac{g_0}{\lambda_p} \sin \varphi (1 - 0.875g_0^2 + 0.26g_0^4) + \\ + \frac{g_0^2}{2} (1 - 0.625g_0^2 + 0.158g_0^4) + \\ + \frac{(Sh_0)g_0 X_b}{\lambda_p} \cos \varphi (1 - 0.25g_0^2 + 0.0312g_0^4) + \\ + \frac{(Sh_0)^2 g_0^2 X_b^2}{2} (1 - 0.125g_0^2) \end{array} \right\}, \quad (1.9.28)$$

where

$$D_1 = \left[C_{yc}^\alpha \left(1 - \frac{1}{2\pi} C_{yc}^\alpha \right) \right]. \quad (1.9.29)$$

$$C_{T5-2} = -D_2 \begin{cases} \frac{(Sh_0)g_0}{2b\lambda_p} \cos \varphi \left[1 - 0.25g_0^2(1 + 2\sin^2 \varphi) \right] + \\ + \frac{(Sh_0)^2 g_0^2 X_b}{2b} \left(1 - 0.125g_0^2 \right) + \frac{(Sh_0)g_0^3}{4b\lambda_p} \sin^2 \varphi \cos \varphi \end{cases}, \quad (1.9.30)$$

where

$$D_2 = b \left(\frac{1}{\pi} C_{yc}^\alpha C_{yc}^{\omega_z} - \frac{1}{2} C_{yc}^\alpha - C_{yc}^{\omega_z} + \frac{\pi}{2} \right). \quad (1.9.31)$$

$$\begin{aligned}
C_{T3-3} = -D_3 & \left\{ \frac{\sqrt{2}(Sh_0)^2 g_0^3}{4b \sqrt{2\lambda_3^2 + 1}} \sin \varphi \cos^2 \varphi + \right. \\
& + \frac{\sqrt{2}(Sh_0)^2 \lambda_p}{2b^2 \sqrt{2\lambda_3^2 + 1}} \cos^2 \varphi \left[g_0^2 \left(1 + \frac{0.25}{(2\lambda_3^2 + 1)} \right) - \right. \\
& \left. \left. - 0.6g_0^4 \right] + \right. \\
& \left. \frac{\sqrt{2}(Sh_0)^3 \lambda_p X_b}{4b^2 \sqrt{2\lambda_3^2 + 1}} \sin 2\varphi \left[g_0^3 \left(1 + \frac{0.25}{(2\lambda_3^2 + 1)} \right) \right] - \right. \\
& \left. - \frac{\sqrt{2}(Sh_0)^2 \sin \varphi}{2b^2 \sqrt{2\lambda_3^2 + 1}} \left[g_0 \left(1 - \frac{0.25}{(2\lambda_3^2 + 1)} \right) - \right. \right. \\
& \left. \left. - 0.3g_0^3 \left(1 + \frac{4\lambda_p^2 + 1.75}{(2\lambda_3^2 + 1)} \sin^2 \varphi \right) \right] + \right. \\
& \left. + \frac{\sqrt{2}(Sh_0)^2 \lambda_p}{2b^2 \sqrt{2\lambda_3^2 + 1}} \sin^2 \varphi \left[g_0^2 \left(1 - \frac{0.25}{(2\lambda_3^2 + 1)} \right) - \right. \right. \\
& \left. \left. - 0.5g_0^4 \right] - \right. \\
& \left. - \frac{\sqrt{2}(Sh_0)^3 \lambda_p X_b}{2b^2 \sqrt{2\lambda_3^2 + 1}} \sin 2\varphi \left[g_0^3 \left(1 - \frac{0.25}{(2\lambda_3^2 + 1)} \right) \right] \right\}, \quad (1.9.32)
\end{aligned}$$

where

$$D_3 = b^2 \left(\frac{1}{\pi} C_{yc}^\alpha C_{yc}^{\omega_z} - C_{yc}^{\omega_z} \right). \quad (1.9.33)$$

$$\begin{aligned}
C_{T5-4} = -D_4 \left\{ \begin{array}{l}
\frac{3\sqrt{2}(Sh_0)}{16\lambda_p b \sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi - \\
-\frac{\sqrt{2}(Sh_0)g_0^2}{16\lambda_p b \sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi \left[2 - \frac{1}{(2\lambda_p^2 + 1)} - 1.17g_0^2 \right] - \\
-\frac{\sqrt{2}(Sh_0)g_0^2}{2b\sqrt{(2\lambda_p^2 + 1)}} \cos \varphi \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} - 0.375g_0^2 \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{2b\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} - 0.25g_0^2 \right] + \\
+\frac{\sqrt{2}(Sh_0)g_0}{2b\sqrt{(2\lambda_p^2 + 1)}} \cos \varphi \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - 0.875g_0^2 \right] + \\
+\frac{\sqrt{2}(Sh_0)g_0^3}{8b\sqrt{(2\lambda_p^2 + 1)}} \cos \varphi \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) + \\
+\frac{\sqrt{2}(Sh_0)^3 g_0^2 \lambda_p X_b^2}{8b\sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi (1 - 0.25g_0^2) - \\
-\frac{\sqrt{2}(Sh_0)g_0^2 \lambda_p \sin 2\varphi}{8b(2\lambda_p^2 + 1)\sqrt{(2\lambda_p^2 + 1)}} + \\
+\frac{\sqrt{2}(Sh_0)^2 \lambda_p X_b}{2b\sqrt{(2\lambda_p^2 + 1)}} \left[g_0^2 + \frac{0.25g_0^2}{(2\lambda_p^2 + 1)} \cos 2\varphi - 0.5g_0^4 \right] + \\
+\frac{\sqrt{2}(Sh_0)^2 g_0 X_b}{2b\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - 0.25g_0^2 \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{8b\sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \left[1 - \frac{1}{(2\lambda_p^2 + 1)} \cos^2 \varphi \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 \lambda_p X_b}{2b\sqrt{(2\lambda_p^2 + 1)}} \left[g_0^2 - \frac{0.25g_0^2}{(2\lambda_p^2 + 1)} \cos 2\varphi - 0.25g_0^4 \right]
\end{array} \right\}, \quad (1.9.34)
\end{aligned}$$

where

$$D_4 = b \left(\frac{2\lambda_{22}}{\rho\pi b^2} C_{ye}^\alpha - \frac{1}{\pi} C_{ye}^\alpha C_{ye}^{\dot{\alpha}} + C_{ye}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho b^2} \right). \quad (1.9.35)$$

$$C_{T5-5} = -D_5 \left\{ \begin{array}{l} \frac{\sqrt{2}(Sh_0)^2 g_0^3}{4b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \cos \varphi + \\ + \frac{\sqrt{2}(Sh_0)^2 g_0}{2b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right] - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^3 \sin \varphi \cos^2 \varphi}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[2 - \frac{1}{(2\lambda_p^2 + 1)} \right] + \\ + \frac{\sqrt{2}(Sh_0)^2 g_0^3}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \cos 2\varphi - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^2}{2b^2 \sqrt{(2\lambda_p^2 + 1)}} \cos^2 \varphi \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{-0.25g_0^2}{-0.25g_0^2} \right] - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^2 \lambda_p}{2b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin^2 \varphi \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{-0.25g_0^2}{-0.25g_0^2} \right] + \\ + \frac{\sqrt{2}(Sh_0)^3 g_0^2 \lambda_p X_b}{8b^2 (2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi \end{array} \right\}, \quad (1.9.36)$$

where

$$D_5 = b^2 \left(-\frac{1}{2} C_{ye}^{\omega_z} + \frac{1}{\pi} C_{ye}^{\dot{\alpha}} C_{ye}^{\omega_z} + \frac{\pi}{4} - \frac{1}{2} C_{ye}^{\dot{\alpha}} \right). \quad (1.9.37)$$

$$C_{T5-6} = -D_6 \frac{(Sh_0)^2 g_0^2}{2b^2} (1 - 0.125g_0^2), \quad (1.9.38)$$

where

$$D_6 = b^2 \left[-\frac{1}{2\pi} (C_{yc}^{\omega_z})^2 + \frac{1}{2} C_{yc}^{\omega_z} - \frac{\pi}{8} \right]. \quad (1.9.39)$$

$$C_{T5-7} = -D_7 \left\{ \begin{array}{l} \frac{\sqrt{2}(Sh_0)^3 g_0^4 \lambda_p}{16\sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi \cos 2\varphi + \\ + \frac{\sqrt{2}(Sh_0)^3 g_0 \lambda_p}{4\sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi \left[\begin{array}{l} \frac{0.5}{(2\lambda_p^2 + 1)} - \\ - 0.25g_0^2 \left(\cos 2\varphi + \frac{1}{(2\lambda_p^2 + 1)} \right) \end{array} \right] \end{array} \right\}, \quad (1.9.40)$$

where

$$D_7 = b^3 \left(-\frac{1}{\pi} C_{yc}^{\omega_z} C_{yc}^{\dot{\omega}_z} + \frac{1}{2} C_{yc}^{\dot{\omega}_z} \right). \quad (1.9.41)$$

$$\begin{aligned}
C_{T5-8} = -D_8 \left\{ \right. & \left. \frac{\left(Sh_0\right)^2 g_0^2 \cos 2\varphi}{2b^2 (2\lambda_p^2 + 1)} (1 - 0.833g_0^2) + \right. \\
& + \frac{2\left(Sh_0\right)^2 g_0^4}{16b^2 (2\lambda_p^2 + 1)} \left[1 - \frac{5}{4(2\lambda_p^2 + 1)} \sin^2 2\varphi \right] - \\
& - \frac{2\left(Sh_0\right)^2 g_0 \lambda_p}{b^2 (2\lambda_p^2 + 1)} \sin \varphi \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.375g_0^2 \right] + \\
& + \frac{\left(Sh_0\right)^2 g_0^3 \lambda_p \sin \varphi \cos^2 \varphi}{b^2 (2\lambda_p^2 + 1)} \left[1 - \frac{1.25}{(2\lambda_p^2 + 1)} - 1.17g_0^2 \right] + \\
& + \frac{2\left(Sh_0\right)^3 g_0 \lambda_p X_b}{b^2 (2\lambda_p^2 + 1)} \cos \varphi \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.75g_0^2 \right] - \\
& - \frac{\left(Sh_0\right)^3 g_0^3 \lambda_p X_b}{2b^2 (2\lambda_p^2 + 1)} \cos \varphi \left[1 - \frac{3}{(2\lambda_p^2 + 1)} \sin^2 \varphi - \right. \\
& \left. \left. - 0.(3)g_0^2 \right] + \right. \\
& + \frac{\left(Sh_0\right)^2 g_0^2 \lambda_0^2}{b^2 (2\lambda_p^2 + 1)} \left(1 - \frac{0.5 \cos 2\varphi}{(2\lambda_p^2 + 1)} - 0.375g_0^2 \right) + \\
& + \frac{\left(Sh_0\right)^2}{b^2 (2\lambda_p^2 + 1)} \left(1 + \frac{0.5}{(2\lambda_p^2 + 1)} \right) - \\
& - \frac{2\left(Sh_0\right)^3 g_0^2 \lambda_0^2 X_b}{b^2 (2\lambda_p^2 + 1)^2} \sin \varphi \cos \varphi (1 - 0.5g_0^2) + \\
& + \frac{\left(Sh_0\right)^4 g_0^2 \lambda_0^2 X_b^2}{b^2 (2\lambda_p^2 + 1)} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} \cos 2\varphi - 0.5g_0^2 \right] \quad (1.9.42)
\end{aligned}$$

where

$$D_8 = b^2 \left[\frac{2\lambda_{22}}{\rho\pi b^2} C_{yc}^{\dot{\alpha}} - \frac{2(\lambda_{22})^2}{\rho^2\pi b^4} - \frac{1}{2\pi} (C_{yc}^{\dot{\alpha}})^2 \right]. \quad (1.9.43)$$

$$C_{T5-9} = -D_9 \left\{ \begin{aligned} & \frac{(Sh_0)^3 g_0 \lambda_p \cos \varphi}{b^3 (2\lambda_p^2 + 1)} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.75 g_0^2 \right] - \\ & - \frac{(Sh_0)^3 g_0^3 \lambda_p}{2b^3 (2\lambda_p^2 + 1)} \sin^2 \varphi \cos \varphi + \\ & + \frac{(Sh_0)^3 g_0 \lambda_p}{4b^3 (2\lambda_p^2 + 1)} \cos \varphi \left[\cos 2\varphi - 0.33 g_0^2 \left(\frac{\cos 2\varphi + 0.25 \cos 4\varphi}{(2\lambda_p^2 + 1)} \right) \right] + \\ & + \frac{(Sh_0)^3 g_0^3 \lambda_p}{2b^3 (2\lambda_p^2 + 1)} \sin^2 \varphi \cos \varphi \left[1 - \frac{1}{(2\lambda_p^2 + 1)} - \right. \\ & \left. - 0.3 g_0^2 \left(1 - \frac{0.5(1 + \sin^2 \varphi)}{(2\lambda_p^2 + 1)} \right) \right] - \\ & - \frac{(Sh_0)^3 g_0^2 \lambda_p^2}{2b^3 (2\lambda_p^2 + 1)} \sin 2\varphi \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} \right] + \\ & + \frac{(Sh_0)^3 g_0^3 \lambda_p^2}{2b^3 (2\lambda_p^2 + 1)} \sin 2\varphi \left[1 - \frac{0.5}{(2\lambda_p^2 + 1)} \right] + \\ & + \frac{(Sh_0)^4 g_0^2 \lambda_p^2 X_b}{b^3 (2\lambda_p^2 + 1)} \cos^2 \varphi \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} - 0.375 g_0^2 \right] + \\ & + \frac{(Sh_0)^4 g_0^2 \lambda_p^2 X_b}{b^3 (2\lambda_p^2 + 1)} \sin^2 \varphi \left[1 - \frac{0.5}{(2\lambda_p^2 + 1)} - 0.375 g_0^2 \right] \end{aligned} \right\}, \quad (1.9.44)$$

where

$$D_9 = b^3 \left(-\frac{2\lambda_{22}}{\rho\pi b^2} C_{yc}^{\dot{\phi}_2} + \frac{1}{\pi} C_{yc}^{\dot{\alpha}} C_{yc}^{\dot{\phi}_2} \right). \quad (1.9.45)$$

$$C_{T5-10} = -D_{10} \frac{(Sh_0)^4 g_0^2 \lambda_p^2}{b^4 (2\lambda_p^2 + 1)} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} \cos 2\varphi \right], \quad (1.9.46)$$

where

$$D_{10} = -\frac{b^4}{2\pi} \left(C_{yc}^{\dot{\phi}_2} \right)^2. \quad (1.9.47)$$

If $\varphi = \frac{\pi}{2}$ the formula can be written

$$C_{T5-1} = -D_1 \frac{\overline{v_n^2 \cos \theta}}{U_0^2} = -D_1 \left(\frac{\overline{v_n^2 \cos \theta}}{U_0^2} + A_1 \right), \quad (1.9.48)$$

where

$$\frac{v_n^2 \cos \theta}{U_0^2} = 0.5 \left[\begin{aligned} & \left(\frac{1}{\lambda_p} - g_0 \right)^2 - 1.1g_0^2 \left(\frac{1}{\lambda_p^2} - 1.556 \frac{g_0}{\lambda_p} + \right. \\ & \left. \left. + 0.56g_0^2 \right) \right] + \\ & + 0.547g_0^4 \left(\frac{1}{\lambda_p^2} - 1.162 \frac{g_0}{\lambda_p} + 0.289g_0^2 \right) - \\ & - 0.137g_0^6 \left(\frac{1}{\lambda_p^2} - 0.844 \frac{g_0}{\lambda_p} + 0.144g_0^2 \right) \end{aligned} \right]. \quad (1.9.49)$$

Here and further

$$A_i = \frac{g_0^2 (Sh_0)^2 X_b^2}{2} \left[1 - \frac{g_0^2}{8} \left(1 - \frac{g_0^2}{24} \right) \right], \quad (1.9.50)$$

$$Sh_0 = \frac{\omega b}{U_0}, \quad (1.9.51)$$

$$C_{T5-2} = -D_2 \frac{A_i}{X_b}, \quad (1.9.52)$$

$$C_{T5-3} = -D_3 \left[-\frac{\sqrt{2} (Sh_0)^2 g_0}{\sqrt{2\lambda_p^2 + 1}} \right] (J_{3-1} + J_{3-2}), \quad (1.9.53)$$

where

$$J_{3-1} = \left\{ \begin{aligned} & \left(0.5 - 0.25g_0^2 + 0.0417g_0^4 \right) \left[1 + \frac{0.1875}{(2\lambda_p^2 + 1)^2} \right] + \\ & + \left(0.5 - 0.5g_0^2 + 0.125g_0^4 \right) \left[-\frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.1172}{(2\lambda_p^2 + 1)^3} \right] + \\ & + \left(-0.25g_0^2 + 0.125g_0^4 \right) \left[0.5 + \frac{0.1406}{(2\lambda_p^2 + 1)^2} \right] \end{aligned} \right\}, \quad (1.9.54)$$

$$J_{3-2} = -\lambda_p \left\{ \begin{aligned} & \left(0.5g_0 - 0.1667g_0^3 + 0.0167g_0^5 \right) \left[1 + \frac{0.1875}{(2\lambda_p^2 + 1)^2} \right] + \\ & + \left(0.5g_0 - 0.3333g_0^3 + 0.05g_0^5 \right) \left[-\frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.1172}{(2\lambda_p^2 + 1)^3} \right] + \\ & + \left(-0.1667g_0^3 + 0.05g_0^5 \right) \left[0.5 + \frac{0.1406}{(2\lambda_p^2 + 1)^2} \right] \end{aligned} \right\}. \quad (1.9.55)$$

Next

$$C_{T5-4} = -D_4 \frac{\overline{v_{nc} \dot{v}_{nc}}}{U_0^2 U_c} \cos \theta = 0, \quad (1.9.56)$$

$$C_{T^5-5} = -D_3 \left[-\frac{\sqrt{2}(Sh_0)^2 g_0^2}{\sqrt{2\lambda_p^2 + 1}} \right] \{J_{5-1} + J_{5-2} + J_{5-3}\}, \quad (1.9.57)$$

where

$$J_{5-1} = - \left\{ \begin{aligned} & \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} + \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) + \\ & + (-0.25g_0^2 + 0.0417g_0^4) \left[0.5 + \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right] + \\ & 0.0417g_0^4 \left[-\frac{0.0625}{(2\lambda_p^2 + 1)} - \frac{0.0195}{(2\lambda_p^2 + 1)^3} \right] \end{aligned} \right\}, \quad (1.9.58)$$

$$J_{5-2} = g_0 \left\{ \begin{aligned} & (0.25g_0 - 0.083g_0^3 + 0.0083g_0^5) \left[0.5 + \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right] + \\ & + (-0.083g_0^3 + 0.0167g_0^5) \left[-\frac{0.0625}{(2\lambda_p^2 + 1)} - \frac{0.0196}{(2\lambda_p^2 + 1)^3} \right] \end{aligned} \right\}, \quad (1.9.59)$$

$$J_{5-3} = -g_0 \lambda_p \left\{ \begin{aligned} & \left(0.5 + \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.0938}{(2\lambda_p^2 + 1)^2} + \frac{0.0586}{(2\lambda_p^2 + 1)^3} \right) + \\ & + \left(-0.25g_0^2 + 0.0417g_0^4 \right) \left[0.5 + \frac{0.0469}{(2\lambda_p^2 + 1)^2} \right] + \\ & 0.0417g_0^4 \left[-\frac{0.0625}{(2\lambda_p^2 + 1)} - \frac{0.0195}{(2\lambda_p^2 + 1)^3} \right] \end{aligned} \right\}. \quad (1.9.60)$$

$$C_{T5-6} = -D_6 \left[(Sh_0)^2 g_0^2 \right] \{ J_{6-1} \}, \quad (1.9.61)$$

where

$$J_{6-1} = (0.5 - 0.0625g_0^2 + 0.0026g_0^4). \quad (1.9.62)$$

$$C_{T5-7} = -D_7 \overline{\frac{\omega_z \dot{\omega}_z}{U_0^2 U_c} \cos \theta} = 0, \quad (1.9.63)$$

$$C_{T5-8} = -D_8 \left[\frac{2(Sh_0)^2}{(2\lambda_p^2 + 1)} \right] \left\{ \sum_1^7 J_{8-n} \right\} \quad (1.9.64)$$

where

$$J_{8-1} = \left\{ \begin{array}{l} \left\{ 0.5 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.25}{(2\lambda_p^2 + 1)^2} + \frac{0.1875}{(2\lambda_p^2 + 1)^3} + \frac{0.1875}{(2\lambda_p^2 + 1)^4} \right\} \\ + (-0.375g_0^2 + 0.1093g_0^4) \left[\begin{array}{l} \left\{ 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)^2} + \right. \\ \left. + \frac{0.0625}{(2\lambda_p^2 + 1)^4} \right] + \\ + (0.1093g_0^4 - 0.0313g_0^6) \left[\begin{array}{l} -\frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^3} \end{array} \right] + \\ + (-0.0156g_0^6) \left[\begin{array}{l} 0.125 + \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \frac{0.0391}{(2\lambda_p^2 + 1)^4} \end{array} \right] \end{array} \right\}, \quad (1.9.65) \end{array}$$

$$J_{8-2} = -2g_0 \left\{ \begin{array}{l} \left(0.25g_0 - 0.1458g_0^3 \right) \left[0.5 + \frac{0.125}{(2\lambda_p^2 + 1)^2} \right] - \\ - 0.1458g_0^3 \left[\begin{array}{l} -\frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^3} \end{array} \right] + \\ + 0.054g_0^4 \left[\begin{array}{l} 0.125 + \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \\ + \frac{0.039}{(2\lambda_p^2 + 1)^4} \end{array} \right] \end{array} \right\}, \quad (1.9.66)$$

$$J_{8-3} = g_0^2 \left\{ \begin{aligned} & \left(0.125g_0^2 - 0.052g_0^4 \right) \left[\begin{aligned} & 0.5 + \frac{0.125}{(2\lambda_p^2 + 1)^2} + \\ & + \frac{0.0625}{(2\lambda_p^2 + 1)^4} + \frac{0.0091}{(2\lambda_p^2 + 1)^6} \end{aligned} \right] + \\ & + \left(0.125g_0^2 - 0.104g_0^4 \right) \left[\begin{aligned} & - \frac{0.125}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^3} - \\ & - \frac{0.0198}{(2\lambda_p^2 + 1)^5} \end{aligned} \right] + \\ & + \left(-0.052g_0^4 + 0.0213g_0^6 \right) \left[\begin{aligned} & 0.125 + \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \\ & + \frac{0.0391}{(2\lambda_p^2 + 1)^4} + \frac{0.0064}{(2\lambda_p^2 + 1)^6} \end{aligned} \right] \end{aligned} \right\}, \quad (1.9.67)$$

$$J_{8-4} = -2g_0\lambda_p J_{8-1}, \quad (1.9.68)$$

$$J_{8-5} = -g_0\lambda_p J_{8-2}, \quad (1.9.69)$$

$$J_{8-6} = g_0^2 \lambda_p^2 J_{8-1}, \quad (1.9.70)$$

$$J_{8-7} = (Sh_0)^2 g_0^2 \lambda_p^2 X_b^2 \left[\begin{aligned} & 0.5 - \frac{0.125}{(2\lambda_p^2 + 1)} + \frac{0.125}{(2\lambda_p^2 + 1)^2} - \\ & - \frac{0.1875}{(2\lambda_p^2 + 1)^3} + \frac{0.1875}{(2\lambda_p^2 + 1)^4} \end{aligned} \right]. \quad (1.9.71)$$

$$C_{T5-9} = -D_9 \left[\frac{2(Sh_0)^4 g_0^2 \lambda_p^2 X_b}{(2\lambda_p^2 + 1)} \right] \{J_{9-1}\}, \quad (1.9.72)$$

here

$$J_{9-1} = \begin{cases} \left(0.5 - 0.125g_0^2\right) \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)^2} + \frac{0.1875}{(2\lambda_p^2 + 1)^4} \right] + \\ + \left(0.5 - 0.25g_0^2\right) \left[-\frac{0.5}{(2\lambda_p^2 + 1)} - \frac{0.375}{(2\lambda_p^2 + 1)^3} \right] - \\ - 0.125g_0^2 \left[0.5 + \frac{0.375}{(2\lambda_p^2 + 1)^2} + \frac{0.3125}{(2\lambda_p^2 + 1)^4} \right] \end{cases}. \quad (1.9.73)$$

$$C_{T5-10} = -D_{10} \left[\frac{2(Sh_0)^4 g_0^2 \lambda_p^2}{(2\lambda_p^2 + 1)} \right] \{J_{10-1}\}, \quad (1.9.74)$$

where

$$J_{10-1} = J_{9-1}. \quad (1.9.75)$$

One component of the propulsive coefficient, which includes the inductive reactance, has the following form (1.1.4), (1.8.1) and (1.8.3)

$$C_{P6} = \frac{2V_{yc}X_i \sin \vartheta}{\rho S U_0^3}. \quad (1.9.76)$$

Having expanded expression (1.9.76), we obtain the equation

$$C_{P6} = \frac{1}{U_0^3} \left(\begin{array}{l} D_1 \overline{V_{yc} v_{nc}^2 \sin \vartheta} + D_2 \overline{V_{yc} v_{nc} \omega_z \sin \vartheta} + D_3 \overline{\frac{V_{yc} v_{nc} \dot{\omega}_z}{U_c} \sin \vartheta} + \\ + D_4 \overline{\frac{V_{yc} v_{nc} \dot{v}_{nc}}{U_c} \sin \vartheta} + D_5 \overline{\frac{V_{yc} \dot{v}_{nc} \omega_z}{U_c} \sin \vartheta} + D_6 \overline{V_{yc} \omega_z^2 \sin \vartheta} + \\ + D_7 \overline{\frac{V_{yc} \omega_z \dot{\omega}_z}{U_c} \sin \vartheta} + D_8 \overline{\frac{V_{yc} \dot{v}_{nc}^2}{U_c^2} \sin \vartheta} + \\ + D_9 \overline{\frac{V_{yc} \dot{v}_{nc} \dot{\omega}_z}{U_c^2} \sin \vartheta} + D_{10} \overline{\frac{V_{yc} \dot{\omega}_z^2}{U_c^2} \sin \vartheta} \end{array} \right). \quad (1.9.77)$$

Formula (1.9.77) can be expressible as

$$C_{P6} = \sum_{n=1}^{n=10} \{C_{P6-n}\}, \quad (1.9.78)$$

where the terms in the brackets look like

$$C_{P6-1} = D_1 \left\{ \begin{aligned} & \left[\frac{g_0}{8\lambda_p^3} \sin \varphi \left[3 - 0.(3) g_0^2 (3 + 2 \sin^2 \varphi) \right] - \right. \\ & - \frac{g_0^2}{4\lambda_p^2} (1 + 2 \sin^2 \varphi) + \frac{3g_0^3}{8\lambda_p} \sin \varphi + \\ & + \frac{(Sh_0) g_0^2 X_b}{4\lambda_p^2} \sin 2\varphi \left[1 - 0.(3) g_0^2 \right] - \\ & - \frac{(Sh_0) g_0^3 X_b}{2\lambda_p} \cos \varphi + \\ & + \frac{(Sh_0)^2 g_0^3 X_b^2}{8\lambda_p} \sin \varphi (1 - 0.083 g_0^2) + \\ & + \frac{(Sh_0) g_0^2 X_b}{8\lambda_p^2} \sin 2\varphi (1 - 0.83 g_0^2) + \\ & + \frac{(Sh_0)^2 g_0^3 X_b^2}{4\lambda_p} \sin \varphi (1 - 0.83 g_0^2) - \\ & \left. - \frac{(Sh_0)^2 g_0^4 X_b^2}{4} \right] \end{aligned} \right\}, \quad (1.9.79)$$

$$C_{P6-2} = D_2 \left\{ \begin{aligned} & \left[\frac{(Sh_0) g_0^2}{8b\lambda_p^2} \sin 2\varphi \left[1 - 1.(3) g_0^2 \sin^2 \varphi \right] - \right. \\ & - \frac{(Sh_0) g_0^3}{8b\lambda_p} \cos \varphi + \\ & + \frac{(Sh_0)^2 g_0^3 X_b}{8b\lambda_p} \sin \varphi (1 - 0.083 g_0^2) + \\ & + \frac{(Sh_0)^2 g_0^3 X_b}{8b\lambda_p} \sin \varphi (1 - 0.58 g_0^2) - \\ & \left. - \frac{(Sh_0)^2 g_0^4 X_b}{8b} \right] \end{aligned} \right\}, \quad (1.9.80)$$

$$C_{P6-3} = D_3 \left\{ -\frac{\sqrt{2}(Sh_0)^2 g_0^2}{8b^2 \lambda_p \sqrt{(2\lambda_p^2 + 1)}} \begin{bmatrix} 1 + \left(2 - \frac{1}{(2\lambda_p^2 + 1)} \right) \sin^2 \varphi - \\ -0.(3) g_0^2 \begin{pmatrix} \cos^4 \varphi + \\ +1.5 \sin^2 2\varphi + \\ +5 \sin^4 \varphi \end{pmatrix} \end{bmatrix} + \right. \\ \left. + \frac{\sqrt{2}(Sh_0)^2 g_0^3}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[3 - \frac{1}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right] - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{4b^2 \sqrt{(2\lambda_p^2 + 1)}} \cos \varphi \left[\cos 2\varphi + 4 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \sin^2 \varphi \right] - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^3 g_0^4 \lambda_p}{16b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin 2\varphi - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^4 g_0^4 \lambda_p X_b^2}{16b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[\begin{array}{l} 2 \cos^2 2\varphi + \\ \left(0.5 + \frac{0.125}{\sqrt{(2\lambda_p^2 + 1)}} \right) \sin^2 2\varphi \end{array} \right] \right\}, \quad (1.9.81)$$

$$C_{P6-4} = D_4 (C_{P6-4-1} + C_{P6-4-2} + C_{P6-4-3}), \quad (1.9.82)$$

where

$$C_{P6-4-1} = \left\{ \begin{array}{l}
-\frac{\sqrt{2}(Sh_0)g_0}{8b\lambda_p^2\sqrt{(2\lambda_p^2+1)}}\cos\varphi\left[1-0.6g_0^2(1+2\sin^2\varphi)\right] + \\
+\frac{\sqrt{2}(Sh_0)g_0^2}{8b\lambda_p\sqrt{(2\lambda_p^2+1)}}\sin 2\varphi - \\
-\frac{\sqrt{2}(Sh_0)^2g_0^2X_b}{8b\lambda_p\sqrt{(2\lambda_p^2+1)}}\left[\cos 2\varphi - 0.(3)g_0^2\cos^4\varphi\right] - \\
-\frac{\sqrt{2}(Sh_0)g_0^3}{16b\lambda_p^2\sqrt{(2\lambda_p^2+1)}}\cos\varphi\left[1+\left(2-\frac{1.9375}{(2\lambda_p^2+1)}\right)\sin^2\varphi\right] + \\
+\frac{\sqrt{2}(Sh_0)g_0^4\sin 2\varphi}{16b\lambda_p\sqrt{(2\lambda_p^2+1)}}\left[1-\frac{0.44}{(2\lambda_p^2+1)}\cos^2\varphi - \right. \\
\left. -\frac{0.75}{(2\lambda_p^2+1)}\sin^2\varphi\right] - \\
-\frac{\sqrt{2}(Sh_0)^2g_0^4X_b}{16b\lambda_p\sqrt{(2\lambda_p^2+1)}}\left[1-\frac{0.312}{(2\lambda_p^2+1)}\cos^4\varphi - \right. \\
\left. -\frac{0.266}{(2\lambda_p^2+1)}\sin^2 2\varphi\right] - \\
-\frac{\sqrt{2}(Sh_0)g_0^2}{8b\lambda_p\sqrt{(2\lambda_p^2+1)}}\sin 2\varphi\left[1-\frac{0.5}{(2\lambda_p^2+1)}-0.58g_0^2\right]
\end{array} \right\}, \quad (1.9.83)$$

$$C_{P6-4-2} = \left\{ \begin{array}{l} \frac{\sqrt{2}(Sh_0)g_0^3}{8b\sqrt{(2\lambda_p^2+1)}} \cos \varphi \left[1 - \frac{1}{(2\lambda_p^2+1)} \sin^2 \varphi \right] - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{8b\sqrt{(2\lambda_p^2+1)}} \sin \varphi \left[\frac{1}{(2\lambda_p^2+1)} \cos^2 \varphi \right] + \\ + \frac{\sqrt{2}(Sh_0)^2 g_0^2 X_b}{8b\sqrt{(2\lambda_p^2+1)}} \left(\frac{1}{\lambda_p} + g_0 \right) \left[\cos 2\varphi - 2 \sin^2 \varphi \right] - \\ - \frac{\sqrt{2}(Sh_0)^3 g_0^3 X_b^2}{8b\sqrt{(2\lambda_p^2+1)}} \cos \varphi \left[\cos 2\varphi + 2 \sin^2 \varphi \right] - \\ - \frac{\sqrt{2}(Sh_0)^3 g_0^2 X_b}{8b\lambda_p\sqrt{(2\lambda_p^2+1)}} \cos 2\varphi \end{array} \right\}, \quad (1.9.84)$$

$$\begin{aligned}
C_{p6-4-3} = & \left\{ \begin{array}{l}
-\frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{8b\sqrt{(2\lambda_p^2+1)}} \sin \varphi \left[1 + \frac{1}{(2\lambda_p^2+1)} \cos^2 \varphi \right] - \\
-\frac{\sqrt{2}(Sh_0)^3 g_0^3 X_b^2}{8b\sqrt{(2\lambda_p^2+1)}} \cos \varphi \left[1 + \frac{\sin^2 \varphi}{(2\lambda_p^2+1)} \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 g_0^4 X_b}{16b\lambda_p\sqrt{(2\lambda_p^2+1)}} \left[\begin{array}{l} \left(1 - \frac{0.125}{(2\lambda_p^2+1)} \right) \cos^4 \varphi + \\ \left(1.25 - \frac{0.6875}{(2\lambda_p^2+1)} \right) \sin^2 2\varphi - \\ - \frac{0.125}{(2\lambda_p^2+1)} \sin^4 \varphi \end{array} \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 g_0^3 X_b}{8b\sqrt{(2\lambda_p^2+1)}} \sin \varphi \left[1 - \frac{1}{(2\lambda_p^2+1)} \cos^2 \varphi \right] + \\
\frac{\sqrt{2}(Sh_0)^2 g_0^4 \lambda_p X_b}{8b\sqrt{(2\lambda_p^2+1)}} + \\
+\frac{\sqrt{2}(Sh_0)^3 g_0^4 \lambda_p X_b^2}{16b(2\lambda_p^2+1)\sqrt{(2\lambda_p^2+1)}} \sin 2\varphi + \\
+\frac{\sqrt{2}(Sh_0)^3 g_0^3 X_b^2}{8b\sqrt{(2\lambda_p^2+1)}} \cos \varphi \left[\begin{array}{l} \cos 2\varphi + \\ \left(2 - \frac{1}{(2\lambda_p^2+1)} \right) \sin^2 \varphi \end{array} \right] - \\
-\frac{\sqrt{2}(Sh_0)^3 g_0^4 \lambda_p X_b^2}{16b(2\lambda_p^2+1)\sqrt{(2\lambda_p^2+1)}} \sin 2\varphi + \frac{\sqrt{2}(Sh_0)^4 g_0^4 \lambda_p X_b^3}{8b\sqrt{(2\lambda_p^2+1)}} \end{array} \right\} \quad (1.9.85)
\end{aligned}$$

$$C_{P6-5} = D_5 \left\{ -\frac{\sqrt{2}(Sh_0)^2 g_0^2}{8b\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \begin{bmatrix} \cos 2\varphi - \\ -0.3 g_0^2 \left(\cos 2\varphi + \frac{0.125}{(2\lambda_p^2 + 1)} \cos 4\varphi \right) \end{bmatrix} - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^2 g_0^4}{16b^2 \lambda_p \sqrt{(2\lambda_p^2 + 1)}} (1 + 0.5 \sin^2 2\varphi) - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^2 g_0^3}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \sin \varphi \begin{bmatrix} 1 - \\ -\frac{1}{(2\lambda_p^2 + 1)} \cos^2 \varphi \end{bmatrix} \right\}, \quad (1.9.86)$$

$$C_{P6-6} = D_6 \frac{(Sh_0)^2 g_0^3}{8b^2 \lambda_p} \sin \varphi, \quad (1.9.87)$$

$$C_{P6-7} = D_7 \left\{ -\frac{\sqrt{2}(Sh_0)^3 g_0^3}{8b^3 \sqrt{(2\lambda_p^2 + 1)}} \cos \varphi \begin{bmatrix} \cos 2\varphi + \\ + \sin^2 \varphi \left[2 - \frac{1}{(2\lambda_p^2 + 1)} \right] \end{bmatrix} - \right. \\ \left. - \frac{\sqrt{2}(Sh_0)^4 g_0^4 \lambda_p X_b}{8b^3 \sqrt{(2\lambda_p^2 + 1)}} \begin{bmatrix} \cos^2 2\varphi \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right] + \\ + \sin^2 2\varphi \end{bmatrix} \right\}, \quad (1.9.88)$$

$$C_{P6-8} = D_8 \{ C_{P6-8-1} + C_{P6-8-2} + C_{P6-8-3} + C_{P6-8-4} \}, \quad (1.9.89)$$

where

$$\begin{aligned}
C_{P6-8-1} = & \left\{ \begin{array}{l}
\frac{(Sh_0)^2 g_0 \sin \varphi}{4b^2 \lambda_p (2\lambda_p^2 + 1)} \left[1 - 0.6g_0^2 \left(\begin{array}{l} 1 + 2 \cos^2 \varphi + \\ \frac{0.25}{(2\lambda_p^2 + 1)} (4 \cos^2 \varphi - \sin^2 \varphi) \end{array} \right) \right] + \\
+ \frac{(Sh_0)^2 g_0^3 \sin \varphi}{4b^2 \lambda_p (2\lambda_p^2 + 1)} \left[\cos 2\varphi - \frac{0.25}{(2\lambda_p^2 + 1)} (3 \cos^2 \varphi + \cos 2\varphi) \right] + \\
+ \frac{(Sh_0)^2 g_0^3 \lambda_p}{4b^2 (2\lambda_p^2 + 1)} \sin \varphi \left[1 - \frac{2}{(2\lambda_p^2 + 1)} \cos^2 \varphi \right] + \\
+ \frac{(Sh_0)^4 g_0^3 \lambda_p X_b^2}{4b^2 (2\lambda_p^2 + 1)} \sin \varphi \left[3 - \frac{2}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right] \\
+ \frac{(Sh_0)^3 g_0^3 \lambda_p X_b}{2b^2 (2\lambda_p^2 + 1)} \cos \varphi \left[\cos 2\varphi + 2 \left(1 - \frac{1}{(2\lambda_p^2 + 1)} \right) \sin^2 \varphi \right] +
\end{array} \right\}, \quad (1.9.90)
\end{aligned}$$

$$C_{p6-8-2} = \left\{ \begin{array}{l}
\frac{(Sh_0)^2 g_0^2}{2b^2(2\lambda_p^2+1)} \left[\begin{array}{l} \cos 2\varphi - \\ -0.58g_0^2 \left(\cos 2\varphi + \frac{0.25}{(2\lambda_p^2+1)} \cos 4\varphi \right) \end{array} \right] + \\
+ \frac{(Sh_0)^3 g_0^2 X_b}{2b^2(2\lambda_p^2+1)} \left[\begin{array}{l} \cos^2 \varphi + \sin \varphi \cos \varphi - \\ \left(\begin{array}{l} 0.6g_0^2 \sin 2\varphi - \\ -\frac{0.8g_0^2}{(2\lambda_p^2+1)} \left(\begin{array}{l} 9 \cos^4 \varphi + \\ +8 \sin^3 \varphi \cos \varphi + \\ +17 \sin \varphi \cos^3 \varphi \end{array} \right) \end{array} \right) \end{array} \right] + \\
+ \frac{(Sh_0)^2 g_0^4}{4b^2(2\lambda_p^2+1)} \left[1 - \frac{0.25}{(2\lambda_p^2+1)} (1 + 2 \sin^2 2\varphi) \right] + \\
+ \frac{(Sh_0)^3 g_0^4 X_b}{8b^2(2\lambda_p^2+1)} \sin 2\varphi \left[\begin{array}{l} 2 \cos^2 \varphi - \\ -\frac{0.25 \sin 2\varphi}{(2\lambda_p^2+1)} \left(\cos 2\varphi + \cos^2 \varphi \right) - \\ -\frac{3.5}{(2\lambda_p^2+1)} \sin 2\varphi \sin^2 \varphi \end{array} \right], \quad (1.9.91)
\end{array} \right.$$

$$C_{P6-8-3} = \left\{ \begin{array}{l}
\left[\begin{array}{c}
1 + \frac{1}{(2\lambda_p^2 + 1)} - \\
-0.8g_0^2 \left(\begin{array}{c}
1 + \\
+ \frac{1.5 \cos^2 \varphi}{(2\lambda_p^2 + 1)} + \\
+ \frac{0.5 \sin^2 \varphi}{(2\lambda_p^2 + 1)}
\end{array} \right) - \\
- \frac{(Sh_0)^3 g_0^2 X_b}{4b^2 (2\lambda_p^2 + 1)} \sin 2\varphi \left(\cos 2\varphi + 0.75 \sin^2 \varphi \right) - \\
- \frac{(Sh_0)^3 g_0^4 \lambda_p^2 X_b}{8b^2 (2\lambda_p^2 + 1)^2} \sin 2\varphi (1 + \cos^2 \varphi) + \\
+ \frac{(Sh_0)^4 g_0^4 X_b^2}{2b^2 (2\lambda_p^2 + 1)} - \frac{(Sh_0)^5 g_0^4 \lambda_p^2 X_b^3}{4b^2 (2\lambda_p^2 + 1)^2} \sin 2\varphi + \\
+ \frac{(Sh_0)^3 g_0^3 \lambda_p X_b}{2b^2 (2\lambda_p^2 + 1)} \cos \varphi \left(1 + \frac{2}{(2\lambda_p^2 + 1)} \right)
\end{array} \right] , (1.9.92) \\
\end{array} \right\}$$

$$C_{P6-8-4} = \left\{ \begin{array}{l} \left(\frac{(Sh_0)^4 g_0^3 \lambda_p X_b^2}{2b^2 (2\lambda_p^2 + 1)} \cos \varphi \left[\begin{array}{l} \cos 2\varphi - \\ - \left(1 + \frac{1}{(2\lambda_p^2 + 1)} \right) \sin 2\varphi \end{array} \right] + \right. \\ \left. + \frac{(Sh_0)^3 g_0^5 \lambda_p X_b}{4b^2 (2\lambda_p^2 + 1)} \cos \varphi \left[\begin{array}{l} 1 - \frac{0.25}{(2\lambda_p^2 + 1)} \cos^4 \varphi - \\ - \frac{0.75}{(2\lambda_p^2 + 1)} \sin^2 \varphi \cos^2 \varphi - \\ - \frac{1.25}{(2\lambda_p^2 + 1)} \sin^4 \varphi \end{array} \right] + \right. \\ \left. + \frac{(Sh_0)^4 g_0^5 \lambda_p X_b^2}{4b^2 (2\lambda_p^2 + 1)} \left[\begin{array}{l} \cos^5 \varphi + \sin^3 \varphi (1 + \cos^2 \varphi) - \\ - \frac{0.25}{(2\lambda_p^2 + 1)} \sin \varphi (1 + 4 \cos^2 \varphi) \end{array} \right] \right] \end{array} \right\}. \quad (1.9.93)$$

$$C_{P6-9} = D_9 \{ C_{P6-9-1} + C_{P6-9-2} + C_{P6-9-3} \}, \quad (1.9.94)$$

where

$$C_{P6-9-1} = \left\{ \begin{array}{l} \frac{(Sh_0)^3 g_0^2}{8b^3(2\lambda_p^2+1)} \sin 2\varphi \left[1 - \frac{0.083}{(2\lambda_p^2+1)} (3 \cos^2 \varphi + 0.75) \right] + \\ + \frac{(Sh_0)^3 g_0^4}{16b^3(2\lambda_p^2+1)} \sin 2\varphi \left[\frac{\cos 2\varphi -}{-\frac{0.25}{(2\lambda_p^2+1)} (2 \cos^2 \varphi + \cos 2\varphi)} \right] + \\ + \frac{(Sh_0)^3 g_0^3 \lambda_p}{4b^3(2\lambda_p^2+1)} \cos 2\varphi + \frac{(Sh_0)^4 g_0^3 \lambda_p X_b}{2b^3(2\lambda_p^2+1)} \sin \varphi \cos^2 \varphi + \\ + \frac{(Sh_0)^3 g_0^2 \sin 2\varphi}{8b^3(2\lambda_p^2+1)} \left\{ \begin{array}{l} 1 - \\ -0.3g_0^2 \left[\begin{array}{l} 1 + 2 \sin^2 \varphi + \\ + \frac{0.125}{(2\lambda_p^2+1)} (2 \cos^2 \varphi - 9 \sin^2 \varphi) \end{array} \right] \end{array} \right\} \end{array} \right\}, \quad (1.9.95)$$

$$C_{P6-9-2} = \left\{ \begin{array}{l} \frac{(Sh_0)^3 g_0^4}{16b^3(2\lambda_p^2+1)} \sin 2\varphi (1 + 2 \sin^2 \varphi) \left(1 + \frac{0.375}{(2\lambda_p^2+1)} \right) + \\ + \frac{(Sh_0)^3 g_0^3 \lambda_p}{2b^3(2\lambda_p^2+1)} \sin^2 \varphi \cos \varphi \left(1 - \frac{1}{(2\lambda_p^2+1)} \right) - \\ - \frac{(Sh_0)^4 g_0^3 \lambda_p}{2b^3(2\lambda_p^2+1)} \sin \varphi \cos^2 \varphi \left(1 + \frac{1}{(2\lambda_p^2+1)} \right) - \\ - \frac{(Sh_0)^4 g_0^5 \lambda_p X_b}{8b^3(2\lambda_p^2+1)} \sin \varphi \cos^2 \varphi \cos 2\varphi + \\ + \frac{(Sh_0)^4 g_0^4 \lambda_p^2 X_b}{4b^3(2\lambda_p^2+1)} \cos^2 \varphi \left[1 - \frac{0.5}{(2\lambda_p^2+1)} \sin^2 \varphi \right] + \\ + \frac{(Sh_0)^5 g_0^4 \lambda_p^2 X_b^2}{8b^3(2\lambda_p^2+1)} \sin 2\varphi \left[\begin{array}{l} \left(1 - \frac{0.5}{(2\lambda_p^2+1)} \right) \cos 2\varphi - \\ - 2 \left(1 + \frac{0.75}{(2\lambda_p^2+1)} \right) \cos^2 \varphi \end{array} \right], \end{array} \right\}, \quad (1.9.96)$$

$$C_{P6-9-3} = \left\{ \begin{array}{l} \frac{(Sh_0)^4 g_0^3 \lambda_p X_b}{4b^3(2\lambda_p^2+1)} \sin \varphi \{ \cos 2\varphi - 0.583 g_0^2 \cos 2\varphi \} + \\ + \frac{(Sh_0)^4 g_0^5 \lambda_p X_b}{8b^3(2\lambda_p^2+1)} \sin \varphi \left[1 - \frac{0.25}{(2\lambda_p^2+1)} (\cos 2\varphi - -2.5 \sin^2 2\varphi) \right] + \\ + \frac{(Sh_0)^4 g_0^4 \lambda_p^2 X_b}{4b^3(2\lambda_p^2+1)} \sin^2 \varphi \left[1 - \frac{2}{(2\lambda_p^2+1)} \cos^2 \varphi \right] + \\ + \frac{(Sh_0)^5 g_0^4 \lambda_p^2 X_b^2}{8b^3(2\lambda_p^2+1)} \sin 2\varphi \left[\begin{array}{l} \cos 2\varphi + \\ 2 \sin^2 \varphi \left(1 - \frac{2}{(2\lambda_p^2+1)} \right) \end{array} \right] \end{array} \right\}. \quad (1.9.97)$$

$$C_{P6-10} = D_{10} \left\{ \begin{array}{l} \frac{(Sh_0)^4 g_0^3 \lambda_p}{4b^4 (2\lambda_p^2 + 1)} \sin \varphi \left(3 - \frac{2}{(2\lambda_p^2 + 1)} \sin^2 \varphi \right) - \\ - \frac{(Sh_0)^5 g_0^4 \lambda_p^2 X_b}{4b^4 (2\lambda_p^2 + 1)^2} \sin 2\varphi (1 - 0.5 \sin^2 \varphi) \end{array} \right\} \quad (1.9.98)$$

If $\varphi = \frac{\pi}{2}$

$$C_{P6-1} = D_1 \left\{ \begin{array}{l} \frac{3g_0}{8\lambda_p^3} (1 - 0.9723g_0^2 + 0.36g_0^4) - \\ - \frac{3g_0^2}{4\lambda_p^2} (1 - 0.694g_0^2) + \\ + \frac{3g_0^3}{8\lambda_p} (1 - 0.417g_0^2) + \\ + \frac{(Sh_0)^2 g_0^3 X_b^2}{8\lambda_p} (3 - 1.25g_0^2 + 0.312g_0^4) - \\ - \frac{(Sh_0)^2 g_0^4 X_b^2}{8} (1 - 0.417g_0^2 + 0.066g_0^4) \end{array} \right\}, \quad (1.9.99)$$

$$C_{P6-2} = D_2 \left\{ \begin{array}{l} \frac{(Sh_0)^2 g_0^3 X_b}{4b\lambda_p} (1 - 0.333g_0^2 + 0.078g_0^4) - \\ - \frac{(Sh_0)^2 g_0^4 X_b}{8b} (1 - 0.417g_0^2 + 0.066g_0^4) \end{array} \right\}, \quad (1.9.100)$$

$$C_{p6-3} = D_3 \left\{ \begin{array}{l} -\frac{3\sqrt{2}(Sh_0)^2 g_0^2}{8b^2 \lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.3}{(2\lambda_p^2 + 1)} - \right. \\ \left. -0.5 g_0^2 \left(1 - \frac{0.375}{(2\lambda_p^2 + 1)} \right) \right]_+ \\ + \frac{3\sqrt{2}(Sh_0)^2 g_0^3}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.3}{(2\lambda_p^2 + 1)} - \left(\frac{0.25g_0^2 -}{(2\lambda_p^2 + 1)} \right. \right. \\ \left. \left. - \frac{0.094g_0^2}{(2\lambda_p^2 + 1)} \right)_- \\ - \frac{\sqrt{2}(Sh_0)^4 g_0^4 \lambda_p X_b^2}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.33g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] \end{array} \right\}, \quad (1.9.101)$$

$$C_{p6-4} = D_4 \left\{ \begin{array}{l}
\frac{\sqrt{2}(Sh_0)^2 g_0^2 X_b}{8b\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3)g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] - \\
-\frac{\sqrt{2}(Sh_0)^2 g_0^4 X_b}{16b\lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.125}{(2\lambda_p^2 + 1)} - \right. \\
\left. - 0.208g_0^2 \left(1 - \frac{0.11}{(2\lambda_p^2 + 1)} \right) \right] - \\
-\frac{3\sqrt{2}(Sh_0)^2 g_0^3 X_b}{8b\sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.333}{(2\lambda_p^2 + 1)} - \right. \\
\left. - 0.25g_0^2 \left(1 - \frac{0.375}{(2\lambda_p^2 + 1)} \right) \right] - \\
-\frac{3\sqrt{2}(Sh_0)^2 g_0^2 X_b}{8b\sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.(3)}{(2\lambda_p^2 + 1)} - \right. \\
\left. - 0.(5)g_0^2 \left(1 - \frac{0.75}{(2\lambda_p^2 + 1)} \right) \right] - \\
-\frac{\sqrt{2}(Sh_0)^3 g_0^3 X_b}{b\sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3)g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right]
\end{array} \right\}, \quad (1.9.102)$$

$$C_{P6-5} = D_5 \left\{ \begin{array}{l} \frac{\sqrt{2}(Sh_0)^2 g_0^2}{8b^2 \lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3) g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^4}{16b^2 \lambda_p \sqrt{(2\lambda_p^2 + 1)}} \left[1 - \frac{0.125}{(2\lambda_p^2 + 1)} - \right. \\ \left. - 0.208 g_0^2 \left(1 - \frac{0.0688}{(2\lambda_p^2 + 1)} \right) \right] - \\ - \frac{\sqrt{2}(Sh_0)^2 g_0^3}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3) g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] - \\ - \frac{\sqrt{2}(Sh_0)^4 \lambda_p X_b^2}{8b^2 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3) g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] \end{array} \right\}, \quad (1.9.103)$$

$$C_{P6-6} = D_6 \frac{(Sh_0)^2 g_0^3}{8b^2 \lambda_p} \left(1 - 0.0833 g_0^2 \right), \quad (1.9.104)$$

$$C_{P6-7} = D_7 \left\{ - \frac{\sqrt{2}(Sh_0)^4 g_0^4 \lambda_p X_b}{8b^3 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3) g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] \right\}, \quad (1.9.105)$$

$$\begin{aligned}
C_{p6-8} = D_8 & \left\{ \begin{array}{l}
\frac{(Sh_0)^2 g_0}{4b^2 \lambda_p (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] - \\
- \frac{(Sh_0)^2 g_0^3}{4b^2 \lambda_p (2\lambda_p^2 + 1)} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right] + \\
+ \frac{3(Sh_0)^4 g_0^3 \lambda_p X_b^2}{4b^2 (2\lambda_p^2 + 1)} \left[1 - \frac{0.6(6)}{(2\lambda_p^2 + 1)} \right] + \\
+ \frac{(Sh_0)^4 g_0^5 \lambda_p X_b^2}{4b^2 (2\lambda_p^2 + 1)} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right] - \\
- \frac{(Sh_0)^2 g_0^2}{2b^2 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] + \\
+ \frac{(Sh_0)^2 g_0^3 \lambda_p}{4b^2 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] + \\
+ \frac{(Sh_0)^4 g_0^4 \lambda_p^2 X_b^2}{2b^2 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] - \\
- \frac{(Sh_0)^4 g_0^3 \lambda_p X_b^2}{4b^2 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] + \\
+ \frac{5(Sh_0)^2 g_0^5}{64b^2 \lambda_p (2\lambda_p^2 + 1)} \left[1 - \frac{0.22}{(2\lambda_p^2 + 1)} \right]
\end{array} \right\}, \quad (1.9.106)
\end{aligned}$$

$$C_{P6-9} = D_9 \left\{ \begin{aligned} & - \frac{(Sh_0)^4 g_0^3 \lambda_p X_b}{4b^3 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] + \\ & + \frac{3(Sh_0)^4 g_0^3 \lambda_p X_b}{4b^3 (2\lambda_p^2 + 1)} \left[1 - \frac{0.833}{(2\lambda_p^2 + 1)} - \right. \\ & \quad \left. - 0.139 g_0^2 \left(1 - \frac{0.75}{(2\lambda_p^2 + 1)} \right) \right] + \\ & + \frac{(Sh_0)^4 g_0^5 \lambda_p X_b}{8b^3 (2\lambda_p^2 + 1)} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} - \right. \\ & \quad \left. - 0.521 g_0^2 \left(1 - \frac{0.22}{(2\lambda_p^2 + 1)} \right) \right] + \\ & + \frac{(Sh_0)^4 g_0^4 \lambda_p^2 X_b}{4b^3 (2\lambda_p^2 + 1)} \left[1 - 0.583 g_0^2 \left(1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right) \right] \end{aligned} \right\}, \quad (1.9.107)$$

$$C_{P6-10} = D_{10} \left\{ \frac{3(Sh_0)^4 g_0^3 \lambda_p}{4b^4 (2\lambda_p^2 + 1)} \left[1 - \frac{0.6}{(2\lambda_p^2 + 1)} - \right. \right. \\ \left. \left. - 0.139 g_0^2 \left(1 - \frac{0.75}{(2\lambda_p^2 + 1)} \right) \right] \right\}. \quad (1.9.108)$$

To pure heaving

$$C_{p6} = 0. \quad (1.9.109)$$

To pure pitching

$$C_{p6-1} = -D_1 \frac{(Sh_0)^2 g_0^4 X_b^2}{8} \left(1 - 0.417 g_0^2 + 0.066 g_0^4 \right) \quad (1.9.110)$$

$$C_{p6-2} = -D_2 \frac{(Sh_0)^2 g_0^4 X_b}{8b} \left(1 - 0.417 g_0^2 + 0.066 g_0^4 \right), \quad (1.9.111)$$

$$C_{p6-3} = -D_3 \frac{(Sh_0)^4 g_0^4 X_b^2}{8b^2} \left[1 - 0.(3) g_0^2 \right], \quad (1.9.112)$$

$$C_{p6-4} = 0, \quad (1.9.113)$$

$$C_{p6-5} = -D_5 \frac{(Sh_0)^4 X_b^2}{8b^2} \left[1 - 0.(3) g_0^2 \right], \quad (1.9.114)$$

$$C_{p6-6} = 0, \quad (1.9.115)$$

$$C_{p6-7} = -D_7 \frac{(Sh_0)^4 g_0^4 X_b}{8b^3} \left[1 - 0.(3) g_0^2 \right], \quad (1.9.116)$$

$$C_{p6-8} = D_8 \left\{ \frac{(Sh_0)^4 g_0^4 X_b^2}{4b^2} \left(1 - 0.583 g_0^2 \right) \right\}, \quad (1.9.117)$$

$$C_{p6-9} = D_9 \frac{(Sh_0)^4 g_0^4 X_b}{8b^3} (1 - 0.583g_0^2), \quad (1.9.118)$$

$$C_{p6-10} = 0. \quad (1.9.119)$$

As an example figure 1.9.1 illustrates the calculation results of thrust coefficients C_{Ts} and power C_p

The mathematical treatments are of the infinite wing with the pitch axes distance equal 1/3 of chord from the wing leading edge ($X_b = -0.1667$), angles of attack 5° and 15° , relative heaving equal 0.75 (kinematic parameters were taken from Anderson, 1998). Other parameters are taken from Table 1 and 2.

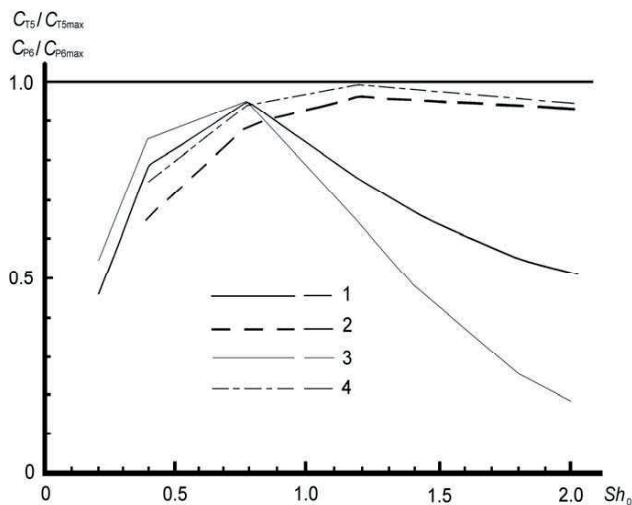


Fig. 1.9.1 Relative thrust and power coefficients versus the Strouhal number and angles of attack. 1 - $C_{T5}/C_{T5\max}$ with an angle of attack equal to 5° , 2 - $C_{T5}/C_{T5\max}$ with an angle of attack equal to 15° , 3 - $C_{P6}/C_{P6\max}$ with an angle of attack equal to 5° , 4 - $C_{P6}/C_{P6\max}$ with an angle of attack equal to 15° . $C_{T5\max}$ and $C_{P6\max}$ equal maximum associated values.

Fig. 1.9.1 shows that the thrust and power coefficients can differ a lot from maximum values (especially with very small angle of attack).

The aerodynamic (rotary) kinematic derivative coefficients measured in the arbitrary point can be converted to the wing center using the method given in the work (Belotserkovskii, 1958).

Nevertheless one additional remark is necessary.

The point is that in the above-mentioned work the aerodynamic (rotary) kinematic derivatives coefficients measured with the proviso that wing amplitude oscillation is small. In these conditions wing and vortex wake is reasoned that lying in the same plane.

When the wing amplitude oscillation is large the wing plane and vortex wake plane do not coincide.

In this case the Strouhal number is more correctly reported in form $Sh = \omega b/U$ where $U = \sqrt{U_0^2 + \omega^2 y_0^2}$. This Strouhal number form was used to obtain the aerodynamic (rotary) kinematic derivative coefficients from the tabulated data (Belotserkovskii, 1958). In the course of calculations it is necessary to use the Strouhal number in the form $Sh_0 = \omega b/U_0$.

Table 1. Aerodynamic (rotary) kinematic derivatives coefficients and other parameters (with the proviso that $\alpha_0 = 5^\circ$)

№ III	1	2	3	4	5	6	7
Sh_0	0.2	0.4	0.8	1.2	1.4	1.8	2.0
Sh	0.198	0.383	0.686	0.892	0.965	1.072	1.109

g_0	0.062	0.204	0.4531	0.646	0.723	0.846	0.895
λ_P	6.666	3.3333	1.667	1.111	0.952	0.741	0.666
C_{yc}^{α}	5.228	4.6162	4.065	3.852	3.782	3.715	3.697
$C_{yc}^{\dot{\alpha}}$	-3.842	-1.533	-0.035	0.4341	0.5748	0.7028	0.7366
$C_{yc}^{w_z}$	1.307	1.1541	1.016	0.963	0.945	0.928	0.924
$C_{yc}^{\dot{w}_z}$	-1.354	-0.776	-0.402	-0.284	-0.249	-0.217	-0.209

Table 2. Aerodynamic (rotary) kinematic derivatives coefficients and other parameters (with the proviso that $\alpha_0 = 15^\circ$)

№пп	1	2	3	4
Sh_0	0.4	0.8	1.2	2.0
Sh	0.383	0.686	0.892	1.109
g_0	0.03	0.279	0.471	0.721
λ_P	3.3333	1.667	1.111	0.666
C_{yc}^{α}	4.6162	4.065	3.852	3.691
$C_{yc}^{\dot{\alpha}}$	-1.533	-0.035	0.4341	0.7479
$C_{yc}^{w_z}$	1.1541	1.016	0.963	0.9223
$C_{yc}^{\dot{w}_z}$	-0.776	-0.402	-0.284	-0.2058

1.9.2. The wing inductive reactance in the course of the harmonic angle of attack.

Using formulas (1.9.22) and (1.9.85) we get inductive reactance coefficients for the second variant of kinematic parameters ($y = y_0 \sin \omega t, \alpha = \alpha_0 \cos \omega t$)

$$C_{T5-1} = -2\pi D_1 \frac{\overline{v_{nc}^2 \cos \theta}}{U_0^2} = -D_1 \frac{2\sqrt{2}(Sh_0)^2 \lambda_p^2 X_b^2}{(2\lambda_p^2 + 1)\sqrt{2\lambda_p^2 + 1}} J_{5-1}, \quad (1.9.120)$$

where

$$J_{5-1} = \left[\begin{aligned} & \frac{\lambda_p}{(2\lambda_p^2 + 1)} J_{5-1-1} - \alpha_0 J_{5-1-2} + \frac{\alpha_0}{2(2\lambda_p^2 + 1)} J_{5-1-3} + \\ & + \frac{\alpha_0^2 (2\lambda_p^2 + 1)^2}{8\lambda_p^3 (Sh_0)^2 X_b^2} J_{5-1-4} + \frac{\alpha_0^2 (2\lambda_p^2 + 1)}{4\lambda_p} J_{5-1-5} + \\ & + \frac{\alpha_0^3 (2\lambda_p^2 + 1)^2}{16(Sh_0)^2 \lambda_p^4 X_b^2} J_{5-1-6} - \frac{\alpha_0^2}{2\lambda_p} J_{5-1-7} + \\ & + \frac{\alpha_0^3 (2\lambda_p^2 + 1)}{8\lambda_p^2} J_{5-1-8} \end{aligned} \right] \quad (1.9.121)$$

and

$$J_{5-1-1} = \left[1 + \frac{1.25}{(2\lambda_p^2 + 1)} + \frac{2.188}{(2\lambda_p^2 + 1)^2} + \frac{2.461}{(2\lambda_p^2 + 1)^3} + \frac{3.384}{(2\lambda_p^2 + 1)^4} + \right. \\ \left. + \frac{2.964}{(2\lambda_p^2 + 1)^5} + \frac{2.795}{(2\lambda_p^2 + 1)^6} + \frac{2.094}{(2\lambda_p^2 + 1)^7} + \frac{1.57}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.122)$$

$$J_{5-1-2} = \left[1 + \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} + \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.923}{(2\lambda_p^2 + 1)^4} + \right. \\ \left. + \frac{0.457}{(2\lambda_p^2 + 1)^5} + \frac{0.3}{(2\lambda_p^2 + 1)^6} + \frac{0.16}{(2\lambda_p^2 + 1)^7} + \frac{0.075}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.123)$$

$$J_{5-1-3} = \left[0.5 + \frac{0.547}{(2\lambda_p^2 + 1)^2} + \frac{0.564}{(2\lambda_p^2 + 1)^4} + \frac{0.349}{(2\lambda_p^2 + 1)^6} + \frac{0.157}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.124)$$

$$J_{5-1-4} = \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.0625}{(2\lambda_p^2 + 1)^2} + \frac{0.0234}{(2\lambda_p^2 + 1)^3} - \frac{0.0146}{(2\lambda_p^2 + 1)^4} \right], \quad (1.9.125)$$

$$J_{5-1-5} = \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right], \quad (1.9.126)$$

$$J_{5-1-6} = \left[1.5 + \frac{0.5}{(2\lambda_p^2 + 1)} - \frac{0.1094}{(2\lambda_p^2 + 1)^2} + \frac{0.0469}{(2\lambda_p^2 + 1)^3} - \frac{0.0286}{(2\lambda_p^2 + 1)^4} \right], \quad (1.9.127)$$

$$J_{s-1-7} = \left[0.5 + \frac{0.2344}{(2\lambda_p^2 + 1)^2} + \frac{0.1538}{(2\lambda_p^2 + 1)^4} + \frac{0.0375}{(2\lambda_p^2 + 1)^6} + \frac{0.0075}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.128)$$

$$J_{s-1-8} = \left[0.5 + \frac{0.0469}{(2\lambda_p^2 + 1)^2} + \frac{0.0171}{(2\lambda_p^2 + 1)^4} \right]. \quad (1.9.129)$$

$$C_{Ts-2} = -D_2 \left(\frac{v_{nc}\omega_z \cos \vartheta}{U_0^2} \right) = -D_2 \frac{2\sqrt{2}(Sh_0)^2 \lambda_p^3 X_b}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} J_{s-2}, \quad (1.9.130)$$

where

$$J_{s-2} = \begin{pmatrix} J_{s-1-1} - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} J_{s-1-2} + \frac{\alpha_0}{2\lambda_p} J_{s-1-3} - \\ - \frac{\alpha_0^2(2\lambda_p^2 + 1)}{2\lambda_p^2} J_{s-2-4} + \frac{\alpha_0^3(2\lambda_p^2 + 1)^2}{2\lambda_p^3} J_{s-1-8} \end{pmatrix} \quad (1.9.131)$$

and

$$J_{s-2-4} = \left[0.5 + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \frac{0.1538}{(2\lambda_p^2 + 1)^4} \right]. \quad (1.9.132)$$

$$C_{T_{5-3}} = -D_3 \frac{\overline{v_{nc} \dot{\omega}_z \cos \theta}}{U_0^2 U_c} = -D_3 \left[-\frac{\sqrt{2} (Sh_0)^2 \alpha_0 \lambda_p^2}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-3}, \quad (1.9.133)$$

where

$$J_{5-3} = \begin{bmatrix} J_{5-3-1} + \frac{2}{(2\lambda_p^2 + 1)} J_{5-3-2} + \frac{\alpha_0}{2\lambda_p} J_{5-3-3} + \\ + \frac{\alpha_0}{\lambda_p (2\lambda_p^2 + 1)} J_{5-3-4} + \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} J_{5-3-5} + \\ + \frac{\alpha_0^2 (2\lambda_p^2 + 1)}{4\lambda_p^2} J_{5-3-6} \end{bmatrix}, \quad (1.9.134)$$

and

$$J_{5-3-1} = \begin{bmatrix} 1 - \frac{0.75}{(2\lambda_p^2 + 1)} + \frac{0.938}{(2\lambda_p^2 + 1)^2} - \frac{0.82}{(2\lambda_p^2 + 1)^3} + \frac{0.923}{(2\lambda_p^2 + 1)^4} - \\ - \frac{0.457}{(2\lambda_p^2 + 1)^5} + \frac{0.3}{(2\lambda_p^2 + 1)^6} - \frac{0.16}{(2\lambda_p^2 + 1)^7} + \frac{0.075}{(2\lambda_p^2 + 1)^8} \end{bmatrix}, \quad (1.9.135)$$

$$J_{5-3-2} = \begin{bmatrix} 1.5 - \frac{1.5}{(2\lambda_p^2 + 1)} + \frac{1.6406}{(2\lambda_p^2 + 1)^2} - \frac{1.6406}{(2\lambda_p^2 + 1)^3} + \\ + \frac{1.6869}{(2\lambda_p^2 + 1)^4} - \frac{0.9131}{(2\lambda_p^2 + 1)^5} + \frac{0.5628}{(2\lambda_p^2 + 1)^6} \end{bmatrix}, \quad (1.9.136)$$

$$J_{5-3-3} = \left[0.5 - \frac{0.3125}{(2\lambda_p^2 + 1)} + \frac{0.5469}{(2\lambda_p^2 + 1)^2} - \frac{0.41}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{0.564}{(2\lambda_p^2 + 1)^4} - \frac{0.37}{(2\lambda_p^2 + 1)^5} + \frac{0.3494}{(2\lambda_p^2 + 1)^6} \right], \quad (1.9.137)$$

$$J_{5-3-4} = \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} - \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \right], \quad (1.9.138)$$

$$J_{5-3-5} = \left[0.5 - \frac{0.3125}{(2\lambda_p^2 + 1)} + \frac{0.5469}{(2\lambda_p^2 + 1)^2} - \frac{0.41}{(2\lambda_p^2 + 1)^3} + \frac{0.564}{(2\lambda_p^2 + 1)^4} - \right. \\ \left. - \frac{0.37}{(2\lambda_p^2 + 1)^5} + \frac{0.3494}{(2\lambda_p^2 + 1)^6} - \frac{0.209}{(2\lambda_p^2 + 1)^7} + \frac{0.157}{(2\lambda_p^2 + 1)^8} \right] \quad (1.9.139)$$

$$J_{5-3-6} = \left[1.5 - \frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.3281}{(2\lambda_p^2 + 1)^2} - \frac{0.2344}{(2\lambda_p^2 + 1)^3} + \frac{0.1879}{(2\lambda_p^2 + 1)^4} \right]. \quad (1.9.140)$$

$$C_{TS-4} = -D_4 \left(\frac{v_{nc} \dot{v}_{nc} \cos \theta}{U_0^2 U_c} \right) = -D_4 \left[-\frac{\sqrt{2} (Sh_0)^2 \alpha_0 \lambda_p^2 X_b}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-4}, \quad (1.9.141)$$

where

$$J_{5-4} = \left[J_{5-4-1} + \frac{\alpha_0 (2\lambda_p^2 + 1)}{2\lambda_p} J_{5-4-2} + \frac{\alpha_0}{2\lambda_p} J_{5-4-3} + \right. \\ \left. + \frac{\alpha_0^2 (2\lambda_p^2 + 1)}{4\lambda_p^2} J_{5-4-4} + \frac{\alpha_0^2}{4\lambda_p} J_{5-4-5} \right], \quad (1.9.142)$$

and

$$J_{5-4-1} = \left[-\frac{1}{(2\lambda_p^2 + 1)} - \frac{1.093}{(2\lambda_p^2 + 1)^3} - \frac{0.349}{(2\lambda_p^2 + 1)^5} + \frac{0.029}{(2\lambda_p^2 + 1)^7} \right], \quad (1.9.143)$$

$$J_{5-4-2} = \left[\frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.2344}{(2\lambda_p^2 + 1)^3} \right], \quad (1.9.144)$$

$$J_{5-4-3} = \left[0.5 + \frac{0.2344}{(2\lambda_p^2 + 1)^2} + \frac{0.1538}{(2\lambda_p^2 + 1)^4} + \frac{0.0375}{(2\lambda_p^2 + 1)^6} \right], \quad (1.9.145)$$

$$J_{5-4-4} = \left[-1.5 + \frac{0.5}{(2\lambda_p^2 + 1)} - \frac{0.2812}{(2\lambda_p^2 + 1)^2} + \frac{0.2344}{(2\lambda_p^2 + 1)^3} - \frac{0.1704}{(2\lambda_p^2 + 1)^4} \right], \quad (1.9.146)$$

$$J_{5-4-5} = \left[0.5 - \frac{0.1875}{(2\lambda_p^2 + 1)} + \frac{0.2344}{(2\lambda_p^2 + 1)^2} - \frac{0.1367}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{0.1538}{(2\lambda_p^2 + 1)^4} - \frac{0.0571}{(2\lambda_p^2 + 1)^5} + \frac{0.0375}{(2\lambda_p^2 + 1)^6} \right]. \quad (1.9.147)$$

$$C_{T5-5} = -D_s \frac{\bar{v}_{nc} \omega_z \cos \vartheta}{U_0^2 U_c} = -D_s \left[\frac{\sqrt{2} (Sh_0)^2 \alpha_0 \lambda_p}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-5}, \quad (1.9.148)$$

where

$$J_{5-5} = \begin{bmatrix} J_{5-1-1} + \frac{\lambda_p}{(2\lambda_p^2 + 1)} J_{5-1-3} + \frac{\alpha_0}{2\lambda_p} J_{5-1-7} - \\ - \frac{\alpha_0(2\lambda_p^2 + 1)}{2} J_{5-1-5} - \frac{\alpha_0^2(2\lambda_p^2 + 1)}{4\lambda_p} J_{5-1-8} - \\ - \frac{\alpha_0}{2} J_{5-1-7} - \frac{\alpha_0^2}{4\lambda_p} J_{5-4-5} \end{bmatrix}. \quad (1.9.149)$$

$$C_{T5-6} = -D_6 \overline{(\omega_z^2 \cos \theta)} = -D_6 \left[\frac{2\sqrt{2}(Sh_0)^2 \lambda_p^3}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-6}, \quad (1.9.150)$$

where

$$J_{5-6} = \begin{bmatrix} J_{5-1-1} - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} J_{5-1-2} + \frac{\alpha_0}{2\lambda_p} J_{5-1-3} - \\ - \frac{\alpha_0^2(2\lambda_p^2 + 1)}{2\lambda_p^2} J_{5-1-7} + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{4\lambda_p^2} J_{5-3-5} + \\ + \frac{\alpha_0^3(2\lambda_p^2 + 1)^2}{8\lambda_p^3} J_{5-1-8} \end{bmatrix}, \quad (1.9.151)$$

$$C_{T5-7} = 0, \quad (1.9.152)$$

$$C_{T5-8} = -D_8 \left[\frac{\dot{v}_{nc}^2 \cos \theta}{U_0^2 U_1^2} \right] = -D_8 \left[\frac{4\sqrt{2}(Sh_0)^4 \lambda_p^5 X_b^2}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-8}, \quad (1.9.153)$$

where

$$J_{5-8} = \left[\begin{aligned} & J_{5-8-1} + \frac{4\lambda_p}{(2\lambda_p^2 + 1)} J_{5-8-2} + \\ & \frac{4}{(2\lambda_p^2 + 1)^2} J_{5-8-3} - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} J_{5-8-4} + \\ & + \frac{2\alpha_0}{\lambda_p} J_{5-8-5} + \frac{\alpha_0^2}{8(Sh_0)^2 \lambda_p^4 X_b^2} J_{5-8-6} + \\ & + \frac{\alpha_0}{2\lambda_p} J_{5-8-6} + \frac{2\alpha_0}{\lambda_p(2\lambda_p^2 + 1)} J_{5-8-7} + \\ & + \frac{2\alpha_0}{\lambda_p(2\lambda_p^2 + 1)^2} J_{5-8-8} + \\ & + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{4(Sh_0)^2 \lambda_p^4 X_b^2} J_{5-8-7} + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{16(Sh_0)^2 \lambda_p^6 X_b^2} J_{5-8-5} + \\ & + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{2(Sh_0)^2 \lambda_p^2 X_b^2} J_{5-8-1} + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{4\lambda_p^2} J_{5-8-3} + \\ & + \frac{\alpha_0^3(2\lambda_p^2 + 1)^3}{4(Sh_0)^2 \lambda_p^5 X_b^2} J_{5-8-7} + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{8(Sh_0)^2 \lambda_p^5 X_b^2} J_{5-8-5} + \\ & + \frac{\alpha_0^3(2\lambda_p^2 + 1)}{16(Sh_0)^2 \lambda_p^5 X_b^2} J_{5-8-9} + \frac{\alpha_0^3(2\lambda_p^2 + 1)^2}{4(Sh_0)^2 \lambda_p^3 X_b^2} J_{5-8-3} + \\ & + \frac{\alpha_0^3(2\lambda_p^2 + 1)^2}{8\lambda_p^3} J_{5-8-3} - \frac{\alpha_0^2(2\lambda_p^2 + 1)}{2\lambda_p^2} J_{5-8-10} - \frac{\alpha_0^2}{\lambda_p^2} J_{5-8-11} \end{aligned} \right] \quad (1.9.154)$$

and

$$J_{5-8-1} = \left[1 - \frac{1.75}{(2\lambda_p^2 + 1)} + \frac{3.9375}{(2\lambda_p^2 + 1)^2} - \frac{5.4143}{(2\lambda_p^2 + 1)^3} + \frac{8.798}{(2\lambda_p^2 + 1)^4} - \right. \\ \left. - \frac{10.139}{(2\lambda_p^2 + 1)^5} + \frac{12.48}{(2\lambda_p^2 + 1)^6} - \frac{12.25}{(2\lambda_p^2 + 1)^7} + \frac{12.58}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.155)$$

$$J_{5-8-2} = \left[0.5 + \frac{1.547}{(2\lambda_p^2 + 1)^2} + \frac{3.142}{(2\lambda_p^2 + 1)^4} + \right. \\ \left. + \frac{4.616}{(2\lambda_p^2 + 1)^6} + \frac{5.1}{(2\lambda_p^2 + 1)^8} + \frac{4.064}{(2\lambda_p^2 + 1)^{10}} \right], \quad (1.9.156)$$

$$J_{5-8-3} = \left[0.5 + \frac{0.688}{(2\lambda_p^2 + 1)} + \frac{2.234}{(2\lambda_p^2 + 1)^2} + \frac{2.793}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{5.935}{(2\lambda_p^2 + 1)^4} + \frac{6.889}{(2\lambda_p^2 + 1)^5} + \frac{11.403}{(2\lambda_p^2 + 1)^6} + \frac{12.027}{(2\lambda_p^2 + 1)^7} + \right. \\ \left. + \frac{16.68}{(2\lambda_p^2 + 1)^8} + \frac{15.17}{(2\lambda_p^2 + 1)^9} + \frac{16.64}{(2\lambda_p^2 + 1)^{10}} \right], \quad (1.9.157)$$

$$J_{5-8-4} = \left[1 - \frac{1.25}{(2\lambda_p^2 + 1)} + \frac{2.188}{(2\lambda_p^2 + 1)^2} - \frac{2.461}{(2\lambda_p^2 + 1)^3} + \frac{3.384}{(2\lambda_p^2 + 1)^4} - \right. \\ \left. - \frac{2.964}{(2\lambda_p^2 + 1)^5} + \frac{2.795}{(2\lambda_p^2 + 1)^6} - \frac{2.094}{(2\lambda_p^2 + 1)^7} + \frac{1.57}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.158)$$

$$J_{5-8-5} = \left[0.5 + \frac{0.984}{(2\lambda_p^2 + 1)^2} + \frac{1.466}{(2\lambda_p^2 + 1)^4} + \frac{1.561}{(2\lambda_p^2 + 1)^6} + \frac{1.258}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.159)$$

$$J_{5-8-6} = \left[1.5 - \frac{3.5}{(2\lambda_p^2 + 1)} + \frac{6.891}{(2\lambda_p^2 + 1)^2} - \frac{10.829}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{16.13}{(2\lambda_p^2 + 1)^4} - \frac{20.278}{(2\lambda_p^2 + 1)^5} + \frac{23.409}{(2\lambda_p^2 + 1)^6} - \right. \\ \left. - \frac{24.5}{(2\lambda_p^2 + 1)^7} + \frac{23.898}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.160)$$

$$J_{5-8-7} = \left[0.5 - \frac{0.563}{(2\lambda_p^2 + 1)} + \frac{1.547}{(2\lambda_p^2 + 1)^2} - \frac{1.676}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{3.142}{(2\lambda_p^2 + 1)^4} - \frac{3.192}{(2\lambda_p^2 + 1)^5} + \frac{4.616}{(2\lambda_p^2 + 1)^6} - \right. \\ \left. - \frac{4.241}{(2\lambda_p^2 + 1)^7} + \frac{5.103}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.161)$$

$$J_{5-8-8} = \left[0.375 + \frac{1.117}{(2\lambda_p^2 + 1)^2} + \frac{2.226}{(2\lambda_p^2 + 1)^4} + \right. \\ \left. + \frac{3.421}{(2\lambda_p^2 + 1)^6} + \frac{4.17}{(2\lambda_p^2 + 1)^8} \right]. \quad (1.9.162)$$

$$J_{5-8-9} = \left[\begin{array}{l} 0.625 - \frac{0.625}{(2\lambda_p^2 + 1)} + \frac{0.82}{(2\lambda_p^2 + 1)^2} - \frac{0.82}{(2\lambda_p^2 + 1)^3} + \\ + \frac{0.9164}{(2\lambda_p^2 + 1)^4} - \frac{0.741}{(2\lambda_p^2 + 1)^5} + \frac{0.594}{(2\lambda_p^2 + 1)^6} - \\ - \frac{0.4187}{(2\lambda_p^2 + 1)^7} + \frac{0.2748}{(2\lambda_p^2 + 1)^8} \end{array} \right]. \quad (1.9.163)$$

$$C_{T5-9} = -D_9 \left[\frac{\dot{v}_{nc} \dot{\phi}_c \cos \vartheta}{U_0^2 U_c^2} \right] = -D_9 \left[\frac{4\sqrt{2} (Sh_0)^4 \lambda_p^5 X_b}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-9}, \quad (1.9.164)$$

where

$$J_{5-9} = \left[\begin{array}{l} J_{5-9-1} + \frac{4}{(2\lambda_p^2 + 1)} J_{5-9-2} - \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} J_{5-9-3} + \\ + \frac{4}{(2\lambda_p^2 + 1)^2} J_{5-9-4} - \frac{\alpha_0}{2\lambda_p} J_{5-9-5} + \frac{\alpha_0}{2\lambda_p} J_{5-9-6} + \\ + \frac{16\alpha_0}{\lambda_p (2\lambda_p^2 + 1)} J_{5-9-7} + \frac{2\alpha_0}{\lambda_p (2\lambda_p^2 + 1)^2} J_{5-9-8} - \\ - \frac{\alpha_0^2 (2\lambda_p^2 + 1)}{4\lambda_p^2} J_{5-9-9} - \frac{\alpha_0^2}{2\lambda_p^2} J_{5-9-10} + \\ + \frac{\alpha_0^2 (2\lambda_p^2 + 1)^2}{4\lambda_p^2} J_{5-3-1} + \frac{\alpha_0^3 (2\lambda_p^2 + 1)^2}{8\lambda_p^3} J_{5-3-3} \end{array} \right], \quad (1.9.165)$$

and

$$J_{5-9-1} = J_{5-8-1}, \quad (1.9.166)$$

$$J_{5-9-2} = J_{5-8-2}, \quad (1.9.167)$$

$$J_{5-9-3} = J_{5-8-4}, \quad (1.9.168)$$

$$J_{5-9-4} = J_{5-8-3}, \quad (1.9.169)$$

$$J_{5-9-5} = J_{5-8-5}, \quad (1.9.170)$$

$$J_{5-9-6} = J_{5-8-6}, \quad (1.9.171)$$

$$J_{5-9-7} = J_{5-8-7}, \quad (1.9.172)$$

$$J_{5-9-8} = J_{5-8-8}, \quad (1.9.173)$$

$$J_{5-9-9} = \left[1.5 - \frac{2.5}{(2\lambda_p^2 + 1)} + \frac{3.8281}{(2\lambda_p^2 + 1)^2} - \frac{4.9219}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{6.2036}{(2\lambda_p^2 + 1)^4} - \frac{5.9277}{(2\lambda_p^2 + 1)^5} + \frac{5.2414}{(2\lambda_p^2 + 1)^6} - \right. \\ \left. - \frac{4.187}{(2\lambda_p^2 + 1)^7} + \frac{2.9831}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.174)$$

$$J_{5-9-10} = \left[\begin{array}{l} 0.5 - \frac{0.4375}{(2\lambda_p^2 + 1)} + \frac{0.9843}{(2\lambda_p^2 + 1)^2} - \frac{0.9024}{(2\lambda_p^2 + 1)^3} + \frac{1.4663}{(2\lambda_p^2 + 1)^4} - \\ - \frac{1.2674}{(2\lambda_p^2 + 1)^5} + \frac{1.5606}{(2\lambda_p^2 + 1)^6} - \frac{1.2252}{(2\lambda_p^2 + 1)^7} + \frac{1.2578}{(2\lambda_p^2 + 1)^8} \end{array} \right]. \quad (1.9.175)$$

$$C_{75-10} = -D_{10} \left[\frac{\bar{\omega}_z^2 \cos \theta}{U_0^2 U_c^2} \right] = -D_{10} \left[\frac{4\sqrt{2}(Sh_0)^4 \lambda_p^5}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] J_{5-10}, \quad (1.9.176)$$

where

$$J_{5-10} = \left[\begin{array}{l} J_{5-10-1} + \frac{4}{(2\lambda_p^2 + 1)} J_{5-10-2} + \frac{4}{(2\lambda_p^2 + 1)^2} J_{5-10-3} - \\ - \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} J_{5-10-4} - \frac{2\alpha_0}{\lambda_p} J_{5-10-5} + \frac{\alpha_0}{2\lambda_p} J_{5-10-6} + \\ + \frac{2\alpha_0}{\lambda_p(2\lambda_p^2 + 1)} J_{5-10-7} + \frac{2\alpha_0}{\lambda_p(2\lambda_p^2 + 1)^2} J_{5-10-8} - \\ - \frac{\alpha_0^2(2\lambda_p^2 + 1)}{2\lambda_p^2} J_{5-10-9} - \frac{\alpha_0^2}{\lambda_p^2} J_{5-10-10} + \\ + \frac{\alpha_0^2(2\lambda_p^2 + 1)^2}{4\lambda_p^2} J_{5-10-11} + \frac{\alpha_0^3(2\lambda_p^2 + 1)^2}{8\lambda_p^3} J_{5-10-12} \end{array} \right], \quad (1.9.177)$$

and

$$J_{5-10-1} = J_{5-9-1}, \quad (1.9.178)$$

$$J_{5-10-2} = J_{5-9-2}, \quad (1.9.179)$$

$$J_{5-10-3} = \left[0.5 + \frac{0.688}{(2\lambda_p^2 + 1)} + \frac{1.5469}{(2\lambda_p^2 + 1)^2} + \frac{2.793}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. \frac{5.1421}{(2\lambda_p^2 + 1)^4} + \frac{6.882}{(2\lambda_p^2 + 1)^5} + \frac{11.4}{(2\lambda_p^2 + 1)^6} + \right. \\ \left. + \frac{12.027}{(2\lambda_p^2 + 1)^7} + \frac{16.68}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.180)$$

$$J_{5-10-4} = J_{5-9-3}, \quad (1.9.181)$$

$$J_{5-10-5} = \left[0.5 + \frac{0.9844}{(2\lambda_p^2 + 1)^2} + \frac{1.4663}{(2\lambda_p^2 + 1)^4} + \frac{1.561}{(2\lambda_p^2 + 1)^6} + \frac{1.1845}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.182)$$

$$J_{5-10-6} = J_{5-9-6}, \quad (1.9.183)$$

$$J_{5-10-7} = J_{5-9-7}, \quad (1.9.184)$$

$$J_{5-10-8} = J_{5-9-8}, \quad (1.9.185)$$

$$J_{5-10-9} = \left[1.5 - \frac{2.5}{(2\lambda_p^2 + 1)} + \frac{3.8281}{(2\lambda_p^2 + 1)^2} - \frac{4.9219}{(2\lambda_p^2 + 1)^3} + \frac{6.2036}{(2\lambda_p^2 + 1)^4} - \right. \\ \left. - \frac{5.9277}{(2\lambda_p^2 + 1)^5} + \frac{5.2414}{(2\lambda_p^2 + 1)^6} - \frac{4.187}{(2\lambda_p^2 + 1)^7} + \frac{2.9831}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.186)$$

$$J_{5-10-10} = \left[0.5 - \frac{0.4375}{(2\lambda_p^2 + 1)} + \frac{0.9844}{(2\lambda_p^2 + 1)^2} - \frac{0.9023}{(2\lambda_p^2 + 1)^3} + \frac{1.4663}{(2\lambda_p^2 + 1)^4} - \right. \\ \left. - \frac{1.2677}{(2\lambda_p^2 + 1)^5} + \frac{1.561}{(2\lambda_p^2 + 1)^6} - \frac{1.2255}{(2\lambda_p^2 + 1)^7} + \frac{1.1845}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.187)$$

$$J_{5-10-11} = J_{5-3-1} \quad (1.9.188)$$

$$J_{5-10-12} = J_{5-10-9}. \quad (1.9.189)$$

$$C_{P6-1} = D_1 \left[\frac{V_{yc} v_{nc}^2 \sin \theta}{U_0^3} \right] = D_1 \left[\frac{\sqrt{2} (Sh_0)^2 \lambda_p X_b^2}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-1}, \quad (1.9.190)$$

where

$$I_{6-1} = \left[\begin{array}{l} I_{6-1-1} - \alpha_0 \lambda_p I_{6-1-2} + \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} I_{6-1-3} + 2\alpha_0 \lambda_p I_{6-1-4} + \\ + \alpha_0^2 (2\lambda_p^2 + 1) I_{6-1-5} + \frac{\alpha_0^2 (2\lambda_p^2 + 1)^2}{4\lambda_p^2} I_{6-1-6} + \alpha_0^2 I_{6-1-7} - \\ - 2\alpha_0^2 \lambda_p^2 I_{6-1-8} - \alpha_0^2 (2\lambda_p^2 + 1) I_{6-1-9} \end{array} \right], \quad (1.9.191)$$

and

$$I_{6-l-1} = J_{5-l-3}, \quad (1.9.192)$$

$$I_{6-l-2} = J_{5-l-3}, \quad (1.9.193)$$

$$I_{6-l-3} = \left[0.5 + \frac{0.234}{(2\lambda_p^2 + 1)^2} + \frac{0.154}{(2\lambda_p^2 + 1)^4} + \frac{0.038}{(2\lambda_p^2 + 1)^6} + \frac{0.008}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.194)$$

$$I_{6-l-4} = J_{5-l-3}, \quad (1.9.195)$$

$$I_{6-l-5} = I_{6-l-3}, \quad (1.9.196)$$

$$I_{6-l-6} = J_{5-l-8}, \quad (1.9.197)$$

$$I_{6-l-7} = J_{5-3-4}, \quad (1.9.198)$$

$$I_{6-l-8} = J_{5-l-3}, \quad (1.9.199)$$

$$I_{6-l-9} = J_{5-l-7}. \quad (1.9.200)$$

$$C_{P6-2} = D_2 \left[\frac{V_{yc} v_{nc} \omega_z \sin \vartheta}{U_0^3} \right] = D_2 \left[\frac{\sqrt{2} (Sh_0)^2 \alpha_0 \lambda_p^2 X_b}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-2}, \quad (1.9.201)$$

where

$$I_{6-2} = \left[\begin{array}{l} I_{6-2-1} + \frac{1}{\alpha_0 \lambda_p} I_{6-2-2} - \frac{2(2\lambda_p^2 + 1)}{\lambda_p^2} I_{6-2-3} - I_{6-2-4} + \\ + \frac{\alpha_0}{2\lambda_p} I_{6-2-5} - \alpha_0 \lambda_p I_{6-2-6} + \frac{\alpha_0 (2\lambda_p^2 + 1)^2}{4\lambda_p^3} I_{6-2-7} \end{array} \right], \quad (1.9.202)$$

and

$$I_{6-2-1} = I_{6-1-1}, \quad (1.9.203)$$

$$I_{6-2-2} = I_{6-1-1}, \quad (1.9.204)$$

$$I_{6-2-3} = I_{6-1-3}, \quad (1.9.205)$$

$$I_{6-2-4} = I_{6-1-1}, \quad (1.9.206)$$

$$I_{6-2-5} = J_{5-3-4}, \quad (1.9.207)$$

$$I_{6-2-6} = J_{5-1-3}, \quad (1.9.208)$$

$$I_{6-2-7} = J_{5-1-8}. \quad (1.9.209)$$

$$C_{p6-3} = D_3 \left[\frac{\bar{V}_{yc} V_{nc} \dot{\phi}_z \sin \theta}{U_0^3 U_c} \right] = D_3 \left[\frac{\sqrt{2} (Sh_0)^2 \alpha_0}{(2\lambda_p^2 + 1) \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-3}, \quad (1.9.210)$$

where

$$I_{6-3} = \left[\begin{aligned} & I_{6-3-1} - \frac{4(Sh_0)^2 \lambda_p^5 X_b^2}{\alpha_0 (2\lambda_p^2 + 1)^3} I_{6-3-2} + \frac{4(Sh_0)^2 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^2} I_{6-3-3} + \\ & + \frac{4(Sh_0)^2 \lambda_p^6 X_b^2}{(2\lambda_p^2 + 1)^3} I_{6-3-4} + \frac{2(Sh_0)^2 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^3} I_{6-3-5} - \\ & - \frac{1}{(2\lambda_p^2 + 1)} I_{6-3-6} - \frac{8(Sh_0)^2 \lambda_p^5 X_b^2}{\alpha_0 (2\lambda_p^2 + 1)^4} I_{6-3-7} + \\ & + \frac{8(Sh_0)^2 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^3} I_{6-3-8} + \frac{8(Sh_0)^2 \lambda_p^6 X_b^2}{(2\lambda_p^2 + 1)^4} I_{6-3-9} - \\ & - \frac{4(Sh_0)^2 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^4} I_{6-3-10} + \frac{2(Sh_0)^2 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^2} I_{6-3-11} \end{aligned} \right] \quad (1.9.211)$$

and

$$I_{6-3-1} = -0.5 J_{5-3-3}, \quad (1.9.212)$$

$$I_{6-3-2} = J_{5-8-2}, \quad (1.9.213)$$

$$I_{6-3-3} = J_{5-8-5}, \quad (1.9.214)$$

$$I_{6-3-4} = J_{5-8-2}, \quad (1.9.215)$$

$$I_{6-3-5} = J_{5-8-7}, \quad (1.9.216)$$

$$I_{6-3-6} = J_{5-3-4}, \quad (1.9.217)$$

$$I_{6-3-7} = J_{5-8-3}, \quad (1.9.218)$$

$$I_{6-3-8} = \left[0.5 + \frac{0.563}{(2\lambda_p^2 + 1)} + \frac{1.547}{(2\lambda_p^2 + 1)^2} + \frac{1.676}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{3.142}{(2\lambda_p^2 + 1)^4} + \frac{3.192}{(2\lambda_p^2 + 1)^5} + \frac{4.616}{(2\lambda_p^2 + 1)^6} + \right. \\ \left. + \frac{4.241}{(2\lambda_p^2 + 1)^7} + \frac{5.103}{(2\lambda_p^2 + 1)^8} + \right. \\ \left. + \frac{4.131}{(2\lambda_p^2 + 1)^9} + \frac{4.064}{(2\lambda_p^2 + 1)^{10}} \right], \quad (1.9.219)$$

$$I_{6-3-9} = J_{5-8-3}, \quad (1.9.220)$$

$$I_{6-3-10} = J_{5-8-8}, \quad (1.9.221)$$

$$I_{6-3-11} = J_{5-8-5}. \quad (1.9.222)$$

$$C_{P6-4} = D_4 \left[\frac{V_{yc} v_{nc} \dot{v}_{nc} \sin \theta}{U_0^3 U_c} \right] = D_4 \left[\frac{4\sqrt{2} (Sh_0)^4 \alpha_0 \lambda_p^6 X_b^3}{(2\lambda_p^2 + 1)^4 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-4}, \quad (1.9.223)$$

where

$$\begin{aligned}
I_{6-4} = & \left[\begin{array}{l}
\frac{(2\lambda_p^2 + 1)^3}{8(Sh_0)^2 \lambda_p^6 X_b^2} I_{6-4-1} - \frac{(2\lambda_p^2 + 1)^2}{8(Sh_0)^2 \lambda_p^6 X_b^2} I_{6-4-2} - \\
-\frac{1}{\alpha_0 \lambda_p} I_{6-4-3} + I_{6-4-4} - \frac{1}{2\lambda_p^2} I_{6-4-5} - \\
-\frac{2}{\lambda_p \alpha_0 (2\lambda_p^2 + 1)} I_{6-4-6} + \frac{2}{(2\lambda_p^2 + 1)} I_{6-4-7} - \\
-\frac{1}{\lambda_p^2 (2\lambda_p^2 + 1)} I_{6-4-8} + \\
+\frac{(2\lambda_p^2 + 1)}{2\lambda_p^2} I_{6-4-9} + \frac{(2\lambda_p^2 + 1)}{\lambda_p^2} I_{6-4-10} + \\
+\frac{2}{\lambda_p^2} I_{6-4-11} - \frac{\alpha_0 (2\lambda_p^2 + 1)^4}{16(Sh_0)^2 \lambda_p^7 X_b^2} I_{6-4-12} - \\
-\frac{\alpha_0 (2\lambda_p^2 + 1)^3}{16(Sh_0)^2 \lambda_p^7 X_b^2} I_{6-4-13} - \frac{(2\lambda_p^2 + 1)^3}{8(Sh_0)^2 \lambda_p^6 X_b^2} I_{6-4-14} + \\
+\frac{\alpha_0 (2\lambda_p^2 + 1)^4}{16(Sh_0)^2 \lambda_p^7 X_b^2} I_{6-4-15} + \\
+\frac{\alpha_0 (2\lambda_p^2 + 1)^3}{4(Sh_0)^2 \lambda_p^5 X_b^2} I_{6-4-16} + \frac{\alpha_0 (2\lambda_p^2 + 1)^2}{4(Sh_0)^2 \lambda_p^5 X_b^2} I_{6-4-17} - \\
-\frac{\alpha_0 (2\lambda_p^2 + 1)^2}{2\lambda_p^3 X_b^2} I_{6-4-18} - \\
-\frac{\alpha_0 (2\lambda_p^2 + 1)^2}{4\lambda_p^3} I_{6-4-19} - \frac{\alpha_0 (2\lambda_p^2 + 1)}{2\lambda_p^3} I_{6-4-20}
\end{array} \right] \quad (1.9.224)
\end{aligned}$$

and

$$I_{6-4-1} = I_{6-1-3}, \quad (1.9.225)$$

$$I_{6-4-2} = I_{6-3-6}, \quad (1.9.226)$$

$$I_{6-4-3} = J_{5-8-2}, \quad (1.9.227)$$

$$I_{6-4-4} = J_{5-8-2}, \quad (1.9.228)$$

$$I_{6-4-5} = J_{5-8-7}, \quad (1.9.229)$$

$$I_{6-4-6} = J_{5-8-3}, \quad (1.9.230)$$

$$I_{6-4-7} = J_{5-8-3}, \quad (1.9.231)$$

$$I_{6-4-8} = J_{5-8-8}, \quad (1.9.232)$$

$$I_{6-4-9} = J_{5-8-5}, \quad (1.9.233)$$

$$I_{6-4-10} = J_{5-8-5}, \quad (1.9.234)$$

$$I_{6-4-11} = I_{6-3-8}, \quad (1.9.235)$$

$$I_{6-4-12} = J_{5-1-8}, \quad (1.9.236)$$

$$I_{6-4-13} = J_{5-4-5}, \quad (1.9.237)$$

$$I_{6-4-14} = I_{6-3-1}, \quad (1.9.238)$$

$$I_{6-4-15} = J_{5-3-6}, \quad (1.9.239)$$

$$I_{6-4-16} = I_{6-3-1}, \quad (1.9.240)$$

$$I_{6-4-17} = I_{6-3-6}, \quad (1.9.241)$$

$$I_{6-4-18} = J_{5-1-3}, \quad (1.9.242)$$

$$I_{6-4-19} = J_{5-1-3}, \quad (1.9.243)$$

$$I_{6-4-20} = I_{6-8-6}. \quad (1.9.244)$$

$$C_{P6-5} = D_5 \left[\frac{V_{yc} \dot{v}_{nc} \omega_z \sin \theta}{U_0^3 U_c} \right] = D_5 \left[\frac{4\sqrt{2} (Sh_0)^4 \alpha_0 \lambda_p^6 X_b^2}{(2\lambda_p^2 + 1)^4 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-5}, \quad (1.9.245)$$

where

$$\begin{aligned}
I_{6-5} = & \left[\begin{array}{l}
\frac{(2\lambda_p^2 + 1)^3}{8(Sh_0)^2 \lambda_p^6 X_b^2} I_{6-5-1} + \frac{(2\lambda_p^2 + 1)^2}{8(Sh_0)^2 \lambda_p^6 X_b^2} I_{6-5-2} + \\
+ \frac{1}{\alpha_0 \lambda_p} I_{6-5-3} - \frac{2}{\lambda_p (2\lambda_p^2 + 1)} I_{6-5-4} + I_{6-5-5} + \\
+ \frac{2}{(2\lambda_p^2 + 1)} I_{6-5-6} + \frac{1}{2\lambda_p^2} I_{6-5-7} - \\
- \frac{1}{\lambda_p^2 (2\lambda_p^2 + 1)} I_{6-5-8} + \frac{(2\lambda_p^2 + 1)}{2\lambda_p^2} I_{6-5-9} + \frac{1}{\lambda_p^2} I_{6-5-10} - \\
- \frac{\alpha_0 (2\lambda_p^2 + 1)^3}{16(Sh_0)^2 \lambda_p^7 X_b^2} I_{6-5-11} - \\
- \frac{\alpha_0 (2\lambda_p^2 + 1)^2}{8(Sh_0)^2 \lambda_p^5 X_b^2} I_{6-5-12} - \frac{\alpha_0 (2\lambda_p^2 + 1)^3}{8(Sh_0)^2 \lambda_p^5 X_b^2} I_{6-5-13} - \\
- \frac{\alpha_0 (2\lambda_p^2 + 1)^4}{16(Sh_0)^2 \lambda_p^7 X_b^2} I_{6-5-14} - \frac{3\alpha_0 (2\lambda_p^2 + 1)^2}{4\lambda_p^3} I_{6-5-15}
\end{array} \right] \quad (1.9.246)
\end{aligned}$$

and

$$I_{6-5-1} = I_{6-1-3}, \quad (1.9.247)$$

$$I_{6-5-2} = I_{6-3-6}, \quad (1.9.248)$$

$$I_{6-5-3} = J_{5-8-2}, \quad (1.9.249)$$

$$I_{6-5-4} = J_{5-8-3}, \quad (1.9.250)$$

$$I_{6-5-5}=J_{5-8-2}, \quad (1.9.251)$$

$$I_{6-5-6}=J_{5-8-3}, \quad (1.9.252)$$

$$I_{6-5-7}=J_{5-8-7}, \quad (1.9.253)$$

$$I_{6-5-8}=J_{5-8-8}, \quad (1.9.254)$$

$$I_{6-5-9}=J_{5-8-5}, \quad (1.9.255)$$

$$I_{6-5-10}=I_{6-3-8}, \quad (1.9.256)$$

$$I_{6-5-11}=J_{5-4-5}, \quad (1.9.257)$$

$$I_{6-5-12}=J_{5-3-4}, \quad (1.9.258)$$

$$I_{6-5-13}=I_{6-1-3}, \quad (1.9.259)$$

$$I_{6-5-14}=J_{5-1-8}, \quad (1.9.260)$$

$$I_{6-5-15}=J_{5-1-3}. \quad (1.9.261)$$

$$C_{p6-6} = D_6 \left[\frac{V_{yc} \omega_z^2 \sin \vartheta}{U_0^3} \right] = D_6 \left[\frac{\sqrt{2} (Sh_0)^2 \lambda_p}{(2\lambda_p^2 + 1)^2 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-6}, \quad (1.9.262)$$

where

$$I_{6-6} = \left[I_{6-6-1} - \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} I_{6-6-2} - \alpha_0 \lambda_p I_{6-6-3} \right] \quad (1.9.263)$$

and

$$I_{6-6-1} = J_{5-1-3}, \quad (1.9.264)$$

$$I_{6-6-2} = I_{6-1-3}, \quad (1.9.265)$$

$$I_{6-6-3} = J_{5-1-3}. \quad (1.9.266)$$

$$C_{p6-7} = D_7 \left[\frac{V_{yc} \omega_z \dot{\omega}_z \sin \vartheta}{U_0^3 U_c} \right] = D_7 \left[\frac{4\sqrt{2} (Sh_0)^4 \lambda_p^5 X_b}{(2\lambda_p^2 + 1)^4 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-7}, \quad (1.9.267)$$

where

$$I_{6-7} = \left[-I_{6-7-1} + \frac{\alpha_0}{2\lambda_p} I_{6-7-2} + \frac{\alpha_0(2\lambda_p^2 + 1)}{\lambda_p} I_{6-7-3} - \right. \\ \left. - \frac{4}{(2\lambda_p^2 + 1)} I_{6-7-4} - \frac{\alpha_0}{\lambda_p(2\lambda_p^2 + 1)} I_{6-7-5} + \right. \\ \left. + \frac{2\alpha_0}{\lambda_p} I_{6-7-6} + \frac{\alpha_0(2\lambda_p^2 + 1)}{2\lambda_p} I_{6-7-7} + \right. \\ \left. + \alpha_0 \lambda_p I_{6-7-8} + \frac{2\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)} I_{6-7-9} \right] \quad (1.9.268)$$

and

$$I_{6-7-1} = J_{5-8-2}, \quad (1.9.269)$$

$$I_{6-7-2} = J_{5-8-7}, \quad (1.9.270)$$

$$I_{6-7-3} = J_{5-8-5}, \quad (1.9.271)$$

$$I_{6-7-4} = \left[0.5 + \frac{2.234}{(2\lambda_p^2 + 1)^2} + \frac{5.935}{(2\lambda_p^2 + 1)^4} + \frac{11.403}{(2\lambda_p^2 + 1)^6} + \frac{16.68}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.272)$$

$$I_{6-7-5} = J_{5-8-8}, \quad (1.9.273)$$

$$I_{6-7-6} = J_{5-8-3}, \quad (1.9.274)$$

$$I_{6-7-7} = J_{5-8-5}, \quad (1.9.275)$$

$$I_{6-7-8} = J_{5-8-2}, \quad (1.9.276)$$

$$I_{6-7-9} = J_{5-8-3}. \quad (1.9.277)$$

$$C_{P6-8} = D_8 \left[\frac{V_{yc} \dot{v}_{nc}^2 \sin \theta}{U_0^3 U_c^2} \right] = D_8 \left[\frac{4\sqrt{2} (Sh_0)^4 \alpha_0 \lambda_p^4 X_b^2}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-8}, \quad (1.9.278)$$

where

$$I_{6-8} = \begin{bmatrix} I_{6-8-1} + \frac{1}{(2\lambda_p^2 + 1)} I_{6-8-2} + \frac{8\lambda_p^2}{(2\lambda_p^2 + 1)^2} I_{6-8-3} - \\ - \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} I_{6-8-4} - \frac{2\alpha_0}{\lambda_p} I_{6-8-5} - \frac{4\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)^2} I_{6-8-6} - \\ - \alpha_0 \lambda_p I_{6-8-7} - \frac{4\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)} I_{6-8-8} \end{bmatrix} \quad (1.9.279)$$

and

$$I_{6-8-1} = J_{5-8-6}, \quad (1.9.280)$$

$$I_{6-8-2} = J_{5-8-7}, \quad (1.9.281)$$

$$I_{6-8-3} = J_{5-8-8}, \quad (1.9.282)$$

$$I_{6-8-4} = \left[1.5 - \frac{2.5}{(2\lambda_p^2 + 1)} + \frac{3.829}{(2\lambda_p^2 + 1)^2} - \frac{4.922}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{6.204}{(2\lambda_p^2 + 1)^4} - \frac{5.928}{(2\lambda_p^2 + 1)^5} + \frac{5.241}{(2\lambda_p^2 + 1)^6} - \right. \\ \left. - \frac{4.187}{(2\lambda_p^2 + 1)^7} + \frac{2.983}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.283)$$

$$I_{6-8-5} = \left[0.5 - \frac{0.438}{(2\lambda_p^2 + 1)} + \frac{0.984}{(2\lambda_p^2 + 1)^2} - \frac{0.902}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{1.466}{(2\lambda_p^2 + 1)^4} - \frac{1.267}{(2\lambda_p^2 + 1)^5} + \frac{1.561}{(2\lambda_p^2 + 1)^6} - \right. \\ \left. - \frac{1.225}{(2\lambda_p^2 + 1)^7} + \frac{1.258}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.284)$$

$$I_{6-8-6} = J_{5-8-8}, \quad (1.9.285)$$

$$I_{6-8-7} = J_{5-8-6}, \quad (1.9.286)$$

$$I_{6-8-8} = J_{5-8-7}. \quad (1.9.287)$$

$$C_{P6-9} = D_9 \left[\frac{V_{yc} \dot{V}_{nc} \dot{\phi}_z \sin \theta}{U_0^3 U_c^2} \right] = D_9 \left[\frac{2\sqrt{2} (Sh_0)^4 \lambda_p^3 X_b}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-9}, \quad (1.9.288)$$

where

$$I_{6-9} = \begin{bmatrix} I_{6-9-1} + \frac{4}{(2\lambda_p^2 + 1)} I_{6-9-2} + \frac{4}{(2\lambda_p^2 + 1)^2} I_{6-9-3} - \\ -\alpha_0 \lambda_p I_{6-9-4} - \frac{5\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)} I_{6-9-5} - \\ -\frac{6\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)^2} I_{6-9-6} - \alpha_0 \lambda_p I_{6-9-7} \end{bmatrix} \quad (1.9.289)$$

and

$$I_{6-9-1} = J_{5-8-6}, \quad I_{6-9-2} = J_{5-8-7}, \quad (1.9.290)$$

$$I_{6-9-3} = J_{5-8-8}, \quad I_{6-9-4} = J_{5-8-5}, \quad (1.9.291)$$

$$I_{6-9-5} = J_{5-8-7}, \quad I_{6-9-7} = J_{5-8-6}. \quad (1.9.292)$$

$$C_{P6-10} = D_{10} \left[\frac{V_{yc} \dot{\phi}_z^2 \sin \theta}{U_0^3 U_c^2} \right] = D_{10} \left[\frac{2\sqrt{2} (Sh_0)^4 \lambda_p^3}{(2\lambda_p^2 + 1)^3 \sqrt{(2\lambda_p^2 + 1)}} \right] I_{6-10}, \quad (1.9.293)$$

where

$$I_{6-10} = \left[\begin{array}{l} I_{6-10-1} + \frac{4}{(2\lambda_p^2 + 1)} I_{6-10-2} + \frac{4}{(2\lambda_p^2 + 1)^2} I_{6-10-3} - \\ - \frac{\alpha_0 (2\lambda_p^2 + 1)}{\lambda_p} I_{6-10-4} - \frac{2\alpha_0}{\lambda_p} I_{6-10-5} - \alpha_0 \lambda_p I_{6-10-6} - \\ - \frac{4\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)} I_{6-10-7} - \frac{4\alpha_0 \lambda_p}{(2\lambda_p^2 + 1)^2} I_{6-10-8} + \\ + \frac{\alpha_0^2 (2\lambda_p^2 + 1)^2}{4\lambda_p} I_{6-10-9} + \alpha_0^2 (2\lambda_p^2 + 1) I_{6-10-10} + \\ + 2\alpha_0^2 \lambda_p^4 I_{6-10-11} - \frac{\alpha_0^3 (2\lambda_p^2 + 1)^2}{4\lambda_p} I_{6-10-12} \end{array} \right] \quad (1.9.294)$$

and

$$I_{6-10-1} = J_{5-8-6}, I_{6-10-2} = J_{5-8-7}, I_{6-10-3} = J_{5-8-8}, \quad (1.9.295)$$

$$I_{6-10-4} = I_{6-8-5}, I_{6-10-5} = I_{6-8-6}, I_{6-10-6} = J_{5-8-6}, \quad (1.9.296)$$

$$I_{6-10-7} = J_{5-8-7}, I_{6-10-8} = J_{5-8-8}, I_{6-10-9} = I_{6-3-1}, \quad (1.9.297)$$

$$I_{6-10-10} = I_{6-8-5}, \quad (1.9.298)$$

$$I_{6-10-11} = \left[0.5 + \frac{0.4375}{(2\lambda_p^2 + 1)} + \frac{0.9844}{(2\lambda_p^2 + 1)^2} + \frac{0.9024}{(2\lambda_p^2 + 1)^3} + \right. \\ \left. + \frac{1.4663}{(2\lambda_p^2 + 1)^4} + \frac{1.2674}{(2\lambda_p^2 + 1)^5} + \frac{1.5606}{(2\lambda_p^2 + 1)^6} + \right. \\ \left. + \frac{1.2252}{(2\lambda_p^2 + 1)^7} + \frac{1.2578}{(2\lambda_p^2 + 1)^8} \right], \quad (1.9.299)$$

$$I_{6-10-12} = I_{6-3-1}. \quad (1.9.300)$$

For pure heaving ($\theta = 0$)

$$C_{T5-1} = D_1 \left(\frac{1}{2\lambda_p^2} \right), \quad (1.9.301)$$

$$C_{T5-2} = C_{T5-3} = C_{T5-4} = C_{T5-5} = C_{T5-6} = C_{T5-7} = C_{T5-9} = C_{T510} = 0, \quad (1.9.302)$$

$$C_{T5-8} = -D_8 \left\{ \frac{1}{(2\lambda_p^2 + 1)} \left[1 + \frac{0.5}{(2\lambda_p^2 + 1)} + \frac{0.5}{(2\lambda_p^2 + 1)^2} + \right. \right. \\ \left. \left. + \frac{0.375}{(2\lambda_p^2 + 1)^3} + \frac{0.17}{(2\lambda_p^2 + 1)^4} + \right. \right. \\ \left. \left. + \frac{0.073}{(2\lambda_p^2 + 1)^5} \right] \right\}, \quad (1.9.303)$$

$$C_{P6} = 0. \quad (1.9.304)$$

For pure pitching ($\lambda_p = \infty$)

$$C_{T5-1} = -D_1 \frac{\overline{v_{nc}^2 \cos \theta}}{U_0^2} = -D_1 \frac{\alpha_0^2}{2} \left[1 + (Sh_0)^2 X^2 \right], \quad (1.9.305)$$

$$C_{T5-2} = 0, \quad (1.9.306)$$

$$C_{T5-3} = -D_3 \left[-(Sh_0)^2 \alpha_0^2 \right], \quad (1.9.307)$$

$$C_{T5-4} = 0, \quad (1.9.308)$$

$$C_{T5-5} = -D_5 \left[-\frac{(Sh_0)^2 \alpha_0^2}{2} \right], \quad (1.9.309)$$

$$C_{T5-6} = -D_6 (Sh_0)^2 \frac{\alpha_0^2}{2} \left[1 + \frac{\alpha_0}{4} \right], \quad (1.9.310)$$

$$C_{T5-7} = 0. \quad (1.9.311)$$

$$C_{T5-8} = -D_8 (Sh_0)^2 \alpha_0^2 \left[2 + (Sh_0)^2 X_b^2 \right], \quad (1.9.312)$$

$$C_{T5-9} = -D_9 \left[(Sh_0)^4 \alpha_0^2 X_b \right], \quad (1.9.313)$$

$$C_{T5-10} = -D_{10} \left[(Sh_0)^4 \alpha_0^2 \right], \quad (1.9.314)$$

$$C_{P6-1} \div C_{P6-9} = 0 \quad (1.9.315)$$

$$C_{P6-10} = D_{10} \left[2\sqrt{2} (Sh_0)^4 \alpha_0^2 \right], \quad (1.9.316)$$

1.9.3. The wing inductive reactance in the course of the harmonic pitch angle and angle of attack.

In this case of the kinematic parameters $\theta = \theta_0 \cos \omega t, \alpha = \alpha_0 \cos \omega t$

The wing amplitude is changes according non-harmonically

$$\dot{y} = U_0 \operatorname{tg} \theta, \quad (1.9.317)$$

Let us obtain C_{T5} and C_{P6} . In each coefficient there are ten components.

$$C_{T5-1} = -D_1 \left[\frac{(Sh_0)^2 X_b^2}{2} \right] J_{5-1}, \quad (1.9.318)$$

where

$$J_{5-1} = \left[J_{5-1-1} + \frac{\alpha_0^2}{(Sh_0)^2 X_b^2} J_{5-1-2} - \right. \\ \left. - \frac{\alpha_0^2 g_0^2}{(Sh_0)^2 X_b^2} J_{5-1-3} + \frac{\alpha_0^2 g_0^4}{12(Sh_0)^2 X_b^2} J_{5-1-4} \right] \quad (1.9.319)$$

and

$$J_{5-1-1} = (g_0^2 - 0.125g_0^4 + 0.0052g_0^6), \quad (1.9.320)$$

$$J_{5-1-2} = (1 + 0.75\theta_0^2 - 0.1563\theta_0^4), \quad (1.9.321)$$

$$J_{5-1-3} = (0.375 + 0.3125\theta_0^2 - 0.0684\theta_0^4), \quad (1.9.322)$$

$$J_{5-1-4} = (0.3125 + 0.2733\theta_0^2 - 0.0615\theta_0^4), \quad (1.9.323)$$

$$C_{T5-2} = -D_2 \left[\frac{(Sh_0)^2 g_0^2 X_b}{2} \right] J_{5-2}, \quad (1.9.324)$$

where

$$J_{5-2} = (1 - 0.125g_0^2). \quad (1.9.325)$$

$$C_{T5-3} = -D_3 \left[-\frac{(Sh_0)^2 \alpha_0 g_0}{2} \right] J_{5-3}, \quad (1.9.326)$$

where

$$J_{5-3} = (1 - 0.375 g_0^2 + 0.026 g_0^4). \quad (1.9.327)$$

$$C_{T5-4} = -D_4 \left[\frac{(Sh_0)^2 \alpha_0 g_0 X_b}{8} \right] J_{5-4}, \quad (1.9.328)$$

where

$$J_{5-4} = [J_{5-4-1} + \theta_0^2 J_{5-4-2}] \quad (1.9.329)$$

and

$$J_{5-4-1} = g_0^2, \quad (1.9.330)$$

$$J_{5-4-2} = (1 - 0.25 g_0^2 + 0.1667 \theta_0^2 - 0.052 g_0^2 \theta_0^2). \quad (1.9.331)$$

$$C_{T5-5} = -D_5 \left[\frac{(Sh_0)^2 \alpha_0 g_0}{2} \right] J_{5-5}, \quad (1.9.332)$$

where

$$J_{5-5} = [J_{5-5-1} + J_{5-5-2} + \theta_0 J_{5-5-3}] \quad (1.9.333)$$

and

$$J_{5-5-1} = 1, \quad (1.9.334)$$

$$J_{5-5-2} = (-0.125\theta_0^2), \quad (1.9.335)$$

$$J_{5-5-3} = (0.1875\theta_0 + 0.0403\theta_0^3 + 0.0068\theta_0^5). \quad (1.9.336)$$

$$C_{T5-6} = -D_6 \left[\frac{(Sh_0)^2 \theta_0^2}{2} \right] [J_{5-6-1}], \quad (1.9.337)$$

where

$$J_{5-6-1} = (1 - 0.125\theta_0^2). \quad (1.9.338)$$

$$C_{T5-7} = 0. \quad (1.9.339)$$

$$C_{T5-8} = -D_8 \left[\frac{(Sh_0)^2 \alpha_0^2}{2} \right] J_{5-8}, \quad (1.9.340)$$

where

$$J_{5-8} = \left[\begin{array}{l} \frac{2(Sh_0)^2 g_0^2 X_b^2}{\alpha_0^2} J_{5-8-1} - \frac{(Sh_0)^2 g_0^4 X_b^2}{\alpha_0^2} J_{5-8-2} + J_{5-8-3} + \\ + 4\theta_0 J_{5-8-4} + 2\theta_0^2 J_{5-8-5} - 2g_0^2 \theta_0 J_{5-8-6} - g_0^2 \theta_0^2 J_{5-8-7} \end{array} \right] \quad (1.9.341)$$

and

$$J_{5-8-1} = (0.5 - 0.375\theta_0^2 + 0.078\theta_0^4), \quad (1.9.342)$$

$$J_{5-8-2} = (0.375 - 0.313\theta_0^2 + 0.068\theta_0^4), \quad (1.9.343)$$

$$J_{5-8-3} = (1 - 0.125g_0^2), \quad (1.9.344)$$

$$J_{5-8-4} = (0.125\theta_0 + 0.0208\theta_0^3 + 0.0052\theta_0^5), \quad (1.9.345)$$

$$J_{5-8-5} = (0.125\theta_0^2 + 0.0417\theta_0^4 + 0.0043\theta_0^6), \quad (1.9.346)$$

$$J_{5-8-6} = (0.0625\theta_0 + 0.013\theta_0^3 + 0.0036\theta_0^5), \quad (1.9.347)$$

$$J_{5-8-7} = \left(0.0391\theta_0^2 + 0.0182\theta_0^4 + 0.0023\theta_0^6 \right). \quad (1.9.348)$$

$$C_{T5-9} = -D_9 \left[\frac{(Sh_0)^4 g_0^2 X_b}{2} \right] J_{5-9}, \quad (1.9.349)$$

where

$$J_{5-9} = \begin{pmatrix} 1 - 0.75\theta_0^2 + 0.156\theta_0^4 + \\ 0.375g_0^2 - 0.313g_0^2\theta_0^2 + 0.068g_0^2\theta_0^4 \end{pmatrix}, \quad (1.9.350)$$

$$C_{T5-10} = -D_{10} \left[\frac{(Sh_0)^4 g_0^2}{2} \right] J_{5-10}, \quad (1.9.351)$$

where

$$J_{5-10} = \begin{pmatrix} 1 - 0.75\theta_0^2 + 0.156\theta_0^4 - \\ -0.375g_0^2 + 0.313g_0^2\theta_0^2 - 0.068g_0^2\theta_0^4 \end{pmatrix}. \quad (1.9.352)$$

$$C_{P6-1} = 2\pi D_1 \left[(Sh_0)^2 g_0^3 X_b^2 \right] \left[\begin{array}{l} I_{6-1-1} - \frac{g_0^2}{6} I_{6-1-2} + \\ + \frac{2\alpha_0}{g_0} I_{6-1-3} + \frac{\alpha_0\theta_0^2}{g_0} I_{6-1-4} + \\ + \frac{\alpha_0^2}{(Sh_0)^2 g_0^3 X_b^2} I_{6-1-5} \end{array} \right] \quad (1.9.353)$$

where

$$I_{6-1-1} = \left(0.125\theta_0 + 0.021\theta_0^3 + 0.005\theta_0^5 \right), \quad (1.9.354)$$

$$I_{6-1-2} = \left(0.0625\theta_0 + 0.013\theta_0^3 + 0.004\theta_0^5 \right), \quad (1.9.355)$$

$$I_{6-1-3} = \left(0.125\theta_0 - 0.042\theta_0^3 + 0.003\theta_0^5 \right), \quad (1.9.356)$$

$$I_{6-1-4} = \left(0.0625\theta_0 - 0.026\theta_0^3 + 0.002\theta_0^5 \right), \quad (1.9.357)$$

$$I_{6-1-5} = \begin{pmatrix} 0.375\theta_0\theta_0 + 0.4167\theta_0\theta_0^3 + 0.0591\theta_0\theta_0^5 + 0.0122\theta_0\theta_0^7 - \\ -0.0521\theta_0^3\theta_0 - 0.0608\theta_0^3\theta_0^3 - 0.0089\theta_0^3\theta_0^5 \end{pmatrix}, \quad (1.9.358)$$

$$C_{P6-2} = D_2 \left[(Sh_0)^2 \theta_0^2 X_b \right] \left[I_{6-2-1} - \frac{\theta_0^3}{6} I_{6-2-2} + \alpha_0 I_{6-2-3} \right], \quad (1.9.359)$$

where

$$I_{6-2-1} = \left(0.125\theta_0\theta_0 + 0.021\theta_0\theta_0^3 + 0.005\theta_0\theta_0^5 \right), \quad (1.9.360)$$

$$I_{6-2-2} = I_{6-1-2}, \quad (1.9.361)$$

$$I_{6-2-3} = \begin{pmatrix} 0.125g_0 - 0.042g_0^3 + 0.003g_0^5 + \\ + 0.031g_0\theta_0^2 - 0.013g_0^3\theta_0^2 + 0.001g_0^5\theta_0^2 \end{pmatrix}. \quad (1.9.362)$$

$$C_{P6-3} = D_3 \left[- (Sh_0)^2 \alpha_0 g_0 \right] \left[I_{6-3-1} + \frac{(Sh_0)^2 g_0^2 X_b^2}{\alpha_0} I_{6-3-2} \right], \quad (1.9.363)$$

$$I_{6-3-1} = \begin{pmatrix} 0.375g_0\theta_0 + 0.104g_0\theta_0^3 + 0.036g_0\theta_0^5 - \\ - 0.052g_0^3\theta_0 - 0.015g_0^3\theta_0^3 - 0.006g_0^3\theta_0^5 \end{pmatrix}, \quad (1.9.364)$$

$$I_{6-3-2} = \begin{pmatrix} 0.125g_0 - 0.042g_0^3 + 0.003g_0^5 - \\ - 0.031g_0\theta_0^2 + 0.013g_0^3\theta_0^2 - 0.001g_0^5\theta_0^2 \end{pmatrix}. \quad (1.9.365)$$

$$C_{P6-4} = D_4 \left[- (Sh_0)^2 \alpha_0 g_0 X_b \right] \left[I_{6-4-1} - \theta_0 I_{6-4-2} + \frac{(Sh_0)^2 g_0^2 X_b^2}{\alpha_0} I_{6-4-3} \right], \quad (1.9.366)$$

where

$$I_{6-4-1} = \begin{pmatrix} 0.25g_0\theta_0 + 0.083g_0\theta_0^3 + 0.031g_0\theta_0^5 - \\ - 0.042g_0^3\theta_0 - 0.013g_0^3\theta_0^3 - 0.005g_0^3\theta_0^5 \end{pmatrix}, \quad (1.9.367)$$

$$I_{6-4-2} = \begin{pmatrix} 0.063g_0\theta_0^2 + 0.026g_0\theta_0^4 + 0.019g_0\theta_0^6 + 0.002g_0\theta_0^8 - \\ - 0.007g_0^3\theta_0^2 - 0.003g_0^3\theta_0^4 - 0.001g_0^3\theta_0^6 \end{pmatrix}, \quad (1.9.368)$$

$$I_{6-4-3} = I_{6-3-2}. \quad (1.9.369)$$

$$C_{P6-5} = D_5 \left[(Sh_0)^2 \alpha_0 g_0 \right] \left[I_{6-5-1} + \theta_0 I_{6-5-2} - \frac{(Sh_0)^2 g_0^2 X_b^2}{\alpha_0} I_{6-5-3} \right], \quad (1.9.370)$$

where

$$I_{6-5-1} = \begin{pmatrix} 0.125g_0\theta_0 + 0.021g_0\theta_0^3 + 0.005g_0\theta_0^5 \\ -0.01g_0^3\theta_0 - 0.002g_0^3\theta_0^3 \end{pmatrix}, \quad (1.9.371)$$

$$I_{6-5-2} = I_{6-4-2}, \quad (1.9.372)$$

$$I_{6-5-3} = I_{6-4-3}. \quad (1.9.373)$$

$$C_{P6-6} = D_6 \left[(Sh_0)^2 g_0^2 \right] I_{6-6}, \quad (1.9.374)$$

where

$$I_{6-6} = I_{6-5-1}. \quad (1.9.375)$$

$$C_{P6-7} = D_7 \left[-(Sh_0)^4 g_0^3 X_b \right] I_{6-7}, \quad (1.9.376)$$

where

$$I_{6-7} = I_{6-3-2}, \quad (1.9.377)$$

$$C_{P6-8} = D_8 \left[(Sh_0)^4 g_0^2 X_b^2 \right] \left[I_{6-8-1} - 2\alpha_0 I_{6-8-2} - 2\alpha_0 \theta_0 I_{6-8-3} \right], \quad (1.9.378)$$

where

$$I_{6-8-1} = \begin{pmatrix} 0.375g_0\theta_0 - 0.209g_0\theta_0^3 + 0.013g_0\theta_0^5 - 0.012g_0\theta_0^7 - \\ -0.052g_0^3\theta_0 + 0.031g_0^3\theta_0^3 - 0.002g_0^3\theta_0^5 \end{pmatrix}, \quad (1.9.379)$$

$$I_{6-8-2} = I_{6-3-2}, \quad (1.9.380)$$

$$I_{6-8-3} = \begin{pmatrix} 0.063g_0\theta_0 - 0.007g_0\theta_0^3 - 0.026g_0^3\theta_0 + \\ +0.003g_0^3\theta_0^3 + 0.002g_0^5\theta_0 \end{pmatrix}. \quad (1.9.381)$$

$$C_{P6-9} = 2\pi D_9 \left[(Sh_0)^4 g_0^2 X_b \right] \left[I_{6-9-1} - \alpha_0 I_{6-9-2} - \alpha_0 \theta_0 I_{6-9-3} \right], \quad (1.9.382)$$

where

$$I_{6-9-1} = I_{6-8-1}, \quad (1.9.383)$$

$$I_{6-9-2} = I_{6-3-2}, \quad (1.9.384)$$

$$I_{6-9-3} = I_{6-8-3}. \quad (1.9.385)$$

$$C_{P6-10} = D_{10} \left[(Sh_0)^4 g_0^2 \right] I_{6-10-1}, \quad (1.9.386)$$

where

$$I_{6-10-1} = I_{6-8-1}. \quad (1.9.387)$$

For pure heaving ($\theta = 0$)

$$C_{T5-1} = -D_1 \left[\frac{\alpha_0^2}{2} \right] (1 + 0.75\alpha_0^2 - 0.1563\alpha_0^4), \quad (1.9.388)$$

$$C_{T5-2} \div C_{T5-7} = C_{T5-9} = C_{T5-10} = 0 \quad (1.9.389)$$

$$C_{T5-8} = -D_8 \left[\frac{(Sh_0)^2 \alpha_0^2}{2} \right] \left(1 + 0.5\alpha_0^2 + 0.3332\alpha_0^4 + \right. \\ \left. + 0.1041\alpha_0^6 + 0.0086\alpha_0^8 \right), \quad (1.9.390)$$

$$C_{P6} = 0 \quad (1.9.391)$$

For pure pitching ($\theta = 0$)

$$C_{T5-1} = -D_1 \left[\frac{(Sh_0)^2 X_b^2}{2} \left(g_0^2 - 0.125g_0^4 + 0.0052g_0^6 \right) + \right. \\ \left. + 0.5\alpha_0^2 - 0.1875\alpha_0^2 g_0^2 + 0.013\alpha_0^2 g_0^4 \right], \quad (1.9.392)$$

$$C_{T5-2} = -D_2 \left[\frac{(Sh_0)^2 g_0^2 X_b}{2} \right] \left(1 - 0.125 g_0^2 \right). \quad (1.9.393)$$

$$C_{T5-3} = -D_3 \left[-\frac{(Sh_0)^2 \alpha_0 g_0}{2} \right] \left(1 - 0.375 g_0^2 + 0.026 g_0^4 \right), \quad (1.9.394)$$

$$C_{T5-4} = -D_4 \left[\frac{(Sh_0)^2 \alpha_0 g_0^3 X_b}{8} \right], \quad (1.9.395)$$

$$C_{T5-5} = -D_5 \left[\frac{(Sh_0)^2 \alpha_0 g_0}{2} \right] \left(1 - 0.125 g_0^2 \right), \quad (1.9.396)$$

$$C_{T5-6} = -D_6 \left[\frac{(Sh_0)^2 g_0^2}{2} \right] \left(1 - 0.125 g_0^2 \right), \quad (1.9.397)$$

$$C_{T5-7} = 0. \quad (1.9.398)$$

$$C_{T5-8} = -D_8 \left[\frac{(Sh_0)^2}{2} \right] \left[\begin{aligned} & \left(\alpha_0^2 - 0.125 \alpha_0^2 g_0^2 \right) + \left(Sh_0 \right)^2 g_0^2 X_b^2 - \\ & - 0.375 \left(Sh_0 \right)^2 g_0^4 X_b^2 \end{aligned} \right], \quad (1.9.399)$$

$$C_{T5-9} = -D_9 \left[\frac{(Sh_0)^4 g_0^2 X_b}{2} \right], \quad (1.9.400)$$

$$C_{P6-10} = -D_{10} \left[\frac{(Sh_0)^4 g_0^2}{2} \right] (1 - 0.375g_0^2), \quad (1.9.401)$$

$$C_{P6-1} = D_1 \left[(Sh_0)^2 g_0^3 X_b^2 \right] \left[2\alpha_0 (0.125 - 0.042g_0^2 + 0.003g_0^4) \right], \quad (1.9.402)$$

$$C_{P6-2} = D_2 \left[(Sh_0)^2 \alpha_0 g_0^2 X_b \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.403)$$

$$C_{P6-3} = D_3 \left[-(Sh_0)^4 g_0^3 X_b^2 \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.404)$$

$$C_{P6-4} = D_4 \left[-(Sh_0)^4 g_0^3 X_b^3 \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.405)$$

$$C_{P6-5} = D_5 \left[-(Sh_0)^4 g_0^3 X_b^2 \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.406)$$

$$C_{P6-6} = 0, \quad (1.9.407)$$

$$C_{P6-7} = D_7 \left[-(Sh_0)^4 g_0^3 \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.408)$$

$$C_{P6-8} = D_8 \left[-2\alpha_0 (Sh_0)^4 g_0^2 X_b^2 \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.409)$$

$$C_{P6-9} = D_9 \left[-\alpha_0 (Sh_0)^4 g_0^2 X_b \right] (0.125g_0 - 0.042g_0^3 + 0.003g_0^5), \quad (1.9.410)$$

$$C_{P6-10} = 0, \quad (1.9.411)$$

Fig. 1.9.3 shows the relative inductive reactance $\left(\frac{C_{T5}}{C_{T5\max}} \right)$ versus the wing kinematic parameters. Here $C_{T5\max} = -\frac{\pi}{2U_0^2} \overline{v_{nc}^2 \cos \theta}$ (upper estimation).

The wing kinematic parameters are shown in the Table 1.3.

Table 1.3. The wing kinematic parameters

Sh_0	0.4	0.7	1.0	1.2	1.6	2.0
θ_0 , rad	0.39	0.64	0.85	0.98	1.14	1.3
g_0 , rad	0.09	0.34	0.55	0.68	0.84	1.0
ω , c^{-1}	4.0	7.0	10	12.0	16.0	20.0
C_{yc}^α	4.67	4.242	4.087	4.06	4.06	4.295
$C_{yc}^{\dot{\alpha}}$	-1.7	-0.47	-0.099	-0.067	-0.067	-0.061
$C_{yc}^{\omega_z}$	1.16	1.061	1.024	1.021	1.021	1.075
$C_{yc}^{\dot{\omega}_z}$	-0.82	-0.51	-0.4175	-0.41	-0.41	-0.545

The calculations were performed when wing maximum relative amplitude $\left(\frac{y_{\max}}{b_0} \right)$ equal 1 and angle of attack equal 0.3 rad. The magnitude of θ_0 is obtained from equation

$$\frac{y_{\max}}{b} Sh_0 = \theta_0 (1 + 0.222\theta_0^2 + 0.071\theta_0^4 + 0.016\theta_0^6) \quad (1.9.411)$$

This formula was published in the work (Romanenko, Pushkov, Lopatin, 2007).

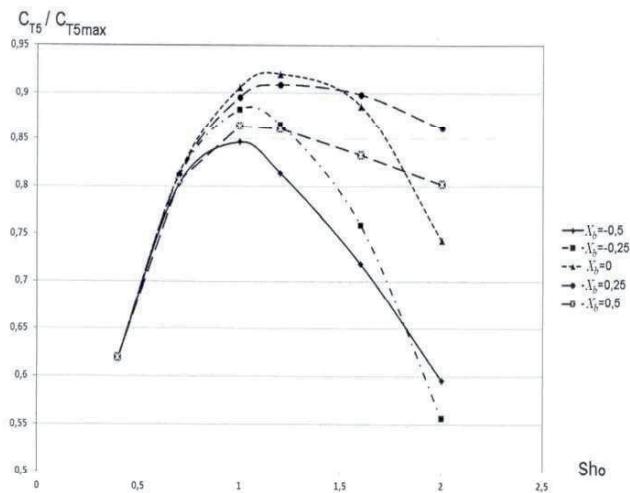


Fig. 1.9.3. The wing relative inductive reactance versus Strouhal number Sh_0 and pitch axes location X_b .

The analysis of the wing inductive reactance estimation using the formulas shows that inductive reactance depends a lot on the wing kinematic parameters.

This suggests the usage of the formulas but not upper estimation to obtain the rigid wing hydrodynamic characteristics especially with inductive reactance. The previous comment refers equally to the infinite- and finite span wings.

Chapter 2 Deformable wing

The great majority of the theoretical and experimental aero-hydrodynamic investigations are devoted to the research of the flat and rigid wings. In many cases the rigid wings are used to modeling of the animals wings: cetaceans, fishes, birds and insects. At the same time it is known that animals naturally use extremely adaptable wings. In particular the dolphins kinematic and hydrodynamic research (Romanenko, Pushkov, 1998; Romanenko, 2001; Romanenko, 2002) attests that during active swimming of a dolphin its fluke gets considerably deformed, especially in upper and lower positions (fig. 2.1.).



Fig.2.1. Form of a dolphin fluke during a dolphin active swimming when it moves from the top down

Traditionally, investigators estimate the performance of the dolphin's fluke as a mover as approximated by the flat rigid hydrodynamic wing. Such estimations give the rough results because the fluke bending is not taken into account.

The flexible flapping foils were investigated theoretically (Bose, 1995; Gordon, Ryzhov, 1996 and other).

Nevertheless, numerical methods described in the works are limited in use because the foils springiness systematic data are absent. But this problem can be solved by means using of the aerodynamic (rotary) derivatives. As this takes place there is no need to know data about the foils springiness.

2.1. Setting up a problem

The lifting force coefficient of such wing will look like

$$c_y = c_y^\alpha \alpha + c_y^\dot{\alpha} \dot{\alpha} + c_y^{\omega_z} \omega_z + c_y^{\dot{\omega}_z} \dot{\omega}_z + c_y^\delta \delta + c_y^{\dot{\delta}} \dot{\delta} \quad (2.1.1)$$

Two previous terms in the right-hand of the formula can be described as the lifting force part which is due to the wing deformation during its oscillations. The major parameter in this case is the deformation law (the time displacement of the wing every point in the coordinate system x, y, z). Let us have a look at the two variants of the wing deformation.

The first variant in the common case will look like

$$\eta(x, z, t) = \frac{y^*}{b} \left(\frac{x}{b} \right)^n \left(1 - \frac{z}{b} \right)^m \cos \omega t. \quad (2.1.2)$$

This formula would be expressible as

$$\eta = f_\delta(x, z) \delta(t), \quad (2.1.3)$$

Here the second factor of the right-hand is the time parameter. The first factor will look like

$$f_\delta = \left(\frac{x}{b} \right)^n \left(1 - \frac{z}{b} \right)^m \quad (2.1.4)$$

The time parameter will look like

$$\delta(t) = \frac{y^*}{b} \cos \omega t \quad (2.1.5)$$

Here y^* is the maximum displacement of the wing plane.

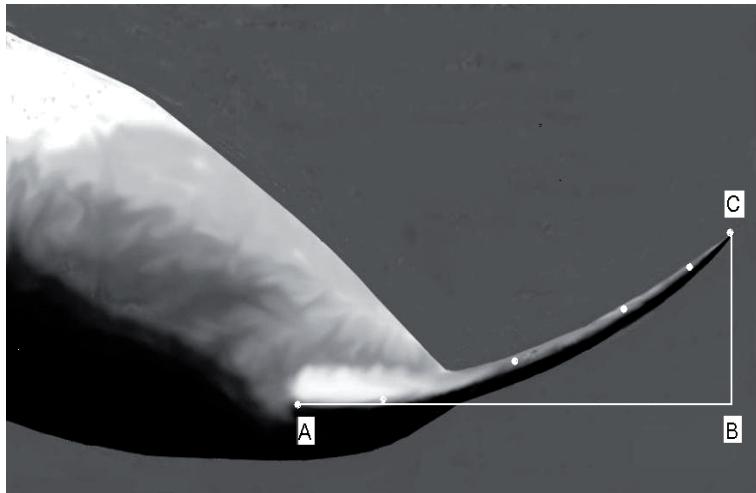


Fig. 2.2 The simulation of the dolphin fluke deformation using the function (2.1.2)

In the case of the Fig. 2.2 we can write

$$f_s = \xi^2, m = 0 \quad (2.1.6),$$

and

$$\delta(t) = -0.332 \cos \omega t \quad (2.1.7)$$

This value of δ_0 are derived from formula (2.1.5) if y^* measured as shown in the Fig. 2.2 (length of CB).

At the same time from fig. 3.33 we notice that part of the dolphin fluke is accounted for by the terminal areas (relatively small space). We shall not include these two parts into the estimation of the δ_0 value. We reason that the end of the fluke is its rear end (more exactly the end of the central chord). In this case estimation of y^* formula (2.1.7) can be brought into form

$$\delta(t) = -0.29 \cos \omega t \quad (2.1.8)$$

In the expression (2.1.6) condition $m = 0$ implies that deformation along the span is absent.

Judging from the photo (fig. 2.1) such deformation is either absent or very small.

In the formulas for the lifting force and power of the deformable wing (Belotserkovskii, Skripach, Tabatchnikov 1971; Belotserkovskii, Skripach, 1975) the unknown terms are the aerodynamic (rotary) derivatives $C_y^\delta, C_z^\delta, m_z^\delta$ and δ_0 .

When the function (2.1.2) simulates the dolphin fluke it's not so difficult to derive the hydrodynamic coefficients. We can use the reversibility (variational) theorem deductions. According to this theorem it is conceivable to gauge the characteristics of the wing when the flow

streaming past it from leading edge to the trailing edge («forward flow wing») using the characteristics of the wing when the flow streaming past it from the trailing edge to the leading edge («reverse flow wing»).

Let us suppose that the intensity of the rotational layer distribution along the wing chord of the «reverse flow wing» $\gamma^{\alpha}, \gamma^{\dot{\alpha}}, \gamma^{\omega_z}, \gamma^{\dot{\omega}_z}$ is known (with minus sign). Taking into account that heaving regard to circulatory flow over the wing the relationship $p^{q_i} = 2\gamma^{q_i}$ is true we can write for «forward flow wing» (with plus sign)

$$C_{y1+}^{\delta} = \frac{1}{S} \iint_S p_-^{\alpha} \left(\frac{\partial f_{\delta}}{\partial \xi} \right)_+ dS \quad (2.1.9)$$

$$C_{y1+}^{\dot{\delta}} = \frac{1}{S} \iint_S p_-^{\dot{\alpha}} \left(\frac{\partial f_{\dot{\delta}}}{\partial \xi} \right)_+ dS \quad (2.1.10)$$

$$C_{y2+}^{\delta} = -\frac{1}{S} \iint_S p_-^{\alpha} f_{\delta+} dS \quad (2.1.11)$$

$$C_{y2+}^{\dot{\delta}} = -\frac{1}{S} \iint_S p_-^{\dot{\alpha}} f_{\dot{\delta}+} dS \quad (2.1.12)$$

$$m_{z1+}^{\delta} = \frac{1}{S} \iint_S p_-^{\omega_z} \left(\frac{\partial f_{\delta}}{\partial \xi} \right)_+ dS \quad (2.1.13)$$

$$m_{z1+}^{\delta} = \frac{1}{S} \iint_S p_-^{\phi_z} \left(\frac{\partial f_{\delta}}{\partial \xi} \right)_+ dS \quad (2.1.14)$$

$$m_{z2+}^{\delta} = -\frac{1}{S} \iint_S p_-^{\phi_z} f_{\delta+} dS \quad (2.1.15)$$

$$m_{z2+}^{\ddot{\delta}} = -\frac{1}{S} \iint_S p_-^{\phi_z} f_{\ddot{\delta}+} dS. \quad (2.1.16)$$

To calculate the aerodynamic (rotary) derivatives of the deformable wing we can use the formulas

$$C_y^{\delta} = C_{y1}^{\delta} - (Sh_0)^2 C_{y2}^{\ddot{\delta}}, \quad (2.1.17)$$

$$C_y^{\dot{\delta}} = C_{y1}^{\dot{\delta}} + C_{y2}^{\dot{\delta}}, \quad (2.1.18)$$

$$m_z^{\delta} = m_{z1}^{\delta} - (Sh_0)^2 m_{z2}^{\ddot{\delta}}, \quad (2.1.19)$$

$$m_z^{\dot{\delta}} = m_{z1}^{\dot{\delta}} + m_{z2}^{\dot{\delta}}. \quad (2.1.20)$$

In the work (Belotserkovskii, Skripach, Tabatchnikov, 1971) values of the $\gamma_-^\alpha, \gamma_-^{\dot{\alpha}}, \gamma_-^{\omega_1}, \gamma_-^{\dot{\omega}_1}$ are illustrated for the infinite wing

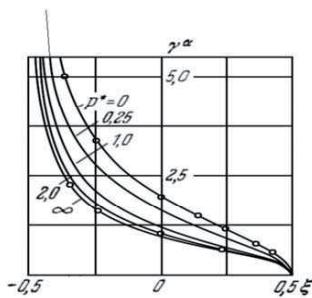


Fig. 2.3 The distribution of the circulation density γ^α along the infinite wing chord from the leading edge (-0.5) to the trailing edge (+0.5). Here $p^* = Sh_0$, ξ - coordinate along the wing chord.

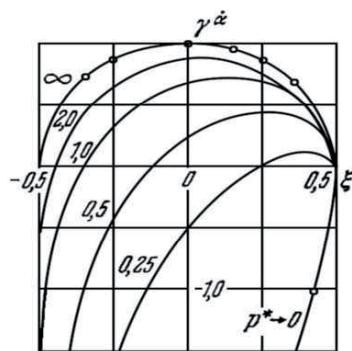


Fig. 2.4. The distribution of the circulation density γ^α along the infinite wing chord.

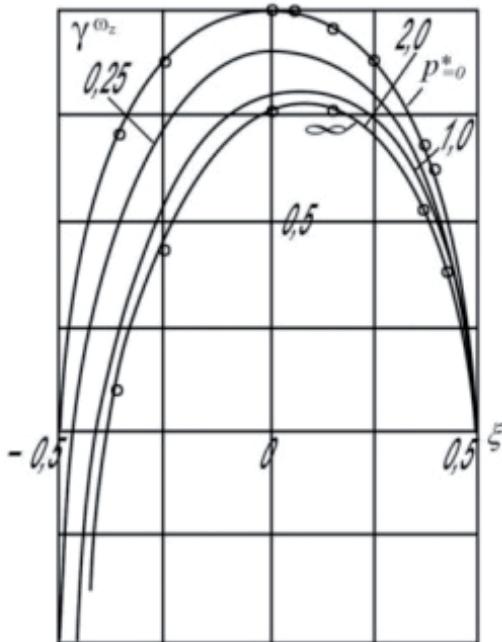


Fig. 2.5. The distribution of the circulation density γ^{oo} along the infinite wing chord. (The wing centre placed at $\bar{x}_r = 0.5$).

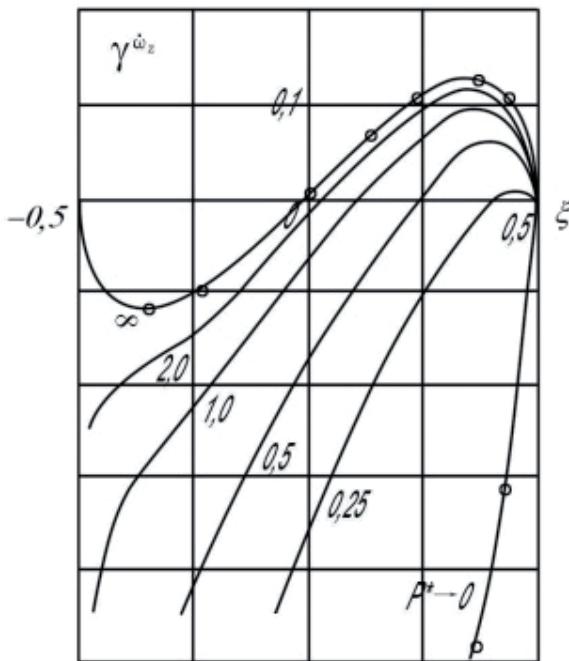


Fig. 2.6. The distribution of the circulation density γ^{ω_z} along the infinite wing chord. (The wing centre placed at $\bar{x}_T = 0.5$).

Using illustrations we can calculate the values of the $\gamma_-^\alpha, \gamma_-^{\dot{\alpha}}, \gamma_-^{o_z}, \gamma_-^{\dot{o}_z}$ then values $p_-^\alpha, p_-^{\dot{\alpha}}, p_-^{o_z}, p_-^{\dot{o}_z}, f_{\delta+}$ and $\left(\frac{\partial f_\delta}{\partial \xi}\right)_+$. Further using formulas (2.1.10) – (2.1.19) we can calculate the aerodynamic (rotary) derivatives of the deformable wing.

The second variant is a two-section wing which fits into the profile of the dolphins fluke (fig. 2.7.).

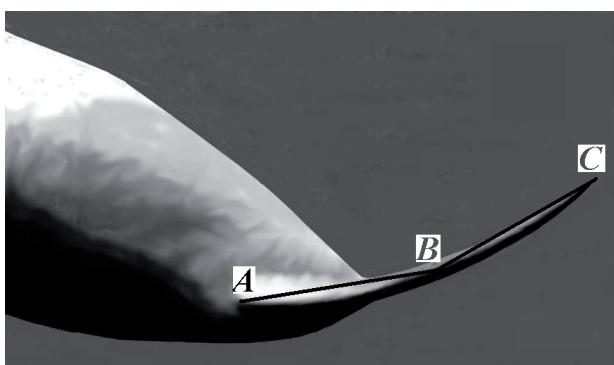


Fig. 2.7. Two-section wing (ABC) with swivel at the point B (BC – flowing part).

Fig. 2.8 illustrates the two-section wing schematic sketch

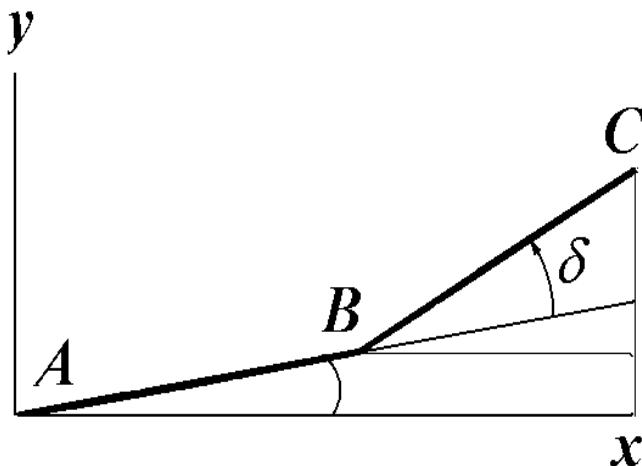


Fig. 2.8. The two-section wing schematic sketch.

We will now look at the rigid wing part of which adjacent to the trailing edge is liable to deflect through some one angle. This problem is the variant of an aerodynamic problem of the oscillation the wing which has the tail controls (Belotserkovskii, Skripach, 1975;).

Aside from the deflection angle the flowing part (in the wing chord) is very important. This parameter determines values of the aerodynamic (rotary) derivatives coefficients which are tabulated for the infinite wings and different sizes of the flowing part (Belotserkovskii, Skripach, 1975).

In the case of the infinite two sectional wing aerodynamic (rotary) derivatives coefficients can be determine using Theodorsen and Kussner functions. In this case we can write

$$C_{y1}^{\delta}(Sh_0) = 2\Phi_l F(Sh_0), \quad (2.1.21)$$

$$C_{y1}^{\delta}(Sh_0) = \frac{1}{2} \left[\Phi_3 + 4\Phi_l \frac{G(Sh_0)}{Sh_0} \right], \quad (2.1.22)$$

$$C_{y2}^{\delta}(Sh_0) = \frac{1}{2} \Phi_2 F(Sh_0), \quad (2.1.23)$$

$$C_{y2}^{\delta}(Sh_0) = \frac{1}{4} \left[\frac{1}{2} \Phi_4 + 2\Phi_2 \frac{G(Sh_0)}{Sh_0} \right], \quad (2.1.24)$$

$$m_{z1}^{\delta}(Sh_0) = -\frac{1}{2} [\Phi_5 + \Phi_l F(Sh_0)], \quad (2.1.25)$$

$$m_{z1}^{\delta}(Sh_0) = -\frac{1}{8} [\Phi_6 + \Phi_3 + 4\Phi_l \frac{G(Sh_0)}{Sh_0}], \quad (2.1.26)$$

$$m_{z2}^{\delta}(Sh_0) = -\frac{1}{8}\Phi_2 F(Sh_0), \quad (2.1.27)$$

$$m_{z2}^{\delta}(Sh_0) = -\frac{1}{8} \left[\Phi_7 + \frac{1}{4}\Phi_4 + \Phi_2 \frac{G(Sh_0)}{Sh_0} \right]. \quad (2.1.28)$$

In this case the aerodynamic (rotary) derivative coefficients can be determined using the same formulas (2.1.16) – (2.1.19) as in the previous variant. Here $F(Sh_0)$ and $G(Sh_0)$ are the real and imaginary part of the Theodorsen functions. The values of the Φ_1 – Φ_7 are the Kussner functions which are dictated by the values of the flowing part.

Particular attention has been given to the sign problem of the function $\delta(t) = \delta_0 \cos \omega t$. In the case which is illustrated in fig. 2.2 and fig 2.3 we chose preferred «minus» because:

1. When the wing moves down the points of the wing flowing part has been falling behind from the corresponding points of the non deformable wing. Consequently, the value $\frac{y^*}{b}$ must be negative.
2. Experimentally, when Strouhal number is small (for example, dolphins fluke) the thrust is larger than in the case of a rigid wing. It is possible only when the time function is negative.

Now we will study only infinite deformable wings which simulate dolphin's fluke although its more correctly to simulate by a three

dimensional deformable wing. However in the literature the data about the distribution of the circulation density $\gamma_-^\alpha, \gamma_-^{\alpha'}, \gamma_-^{\alpha''}, \gamma_-^{\alpha'''}$ (rectangular and triangular wings) are absent.

Special researches are likely to be done by means of numerical methods to obtain this data and then to use thereof. Nevertheless, the dolphin fluke simulation by the infinite deformable wing allows us to estimate the wing propulsive parameters.

2.2. The deformable wing thrust.

The thrust can be written (look at chapter 1)

$$F_{xc} = \left[\lambda_{22} \frac{d(v_{nc} \sin \vartheta)}{dt} - \frac{\lambda_{22} \dot{v}_{nc} V_{yc}}{U \cos \alpha_c} + \right. \\ \left. + \frac{\rho V_y U_c S}{2 \cos \alpha_c} \begin{pmatrix} C_{yc}^\alpha \frac{v_{nc}}{U_c} + C_{yc}^{\dot{\alpha}} \frac{\dot{v}_{nc} b}{U_c^2} - C_{yc}^{\omega_z} \frac{\omega_z b}{U_c} - \\ - C_{yc}^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U_c^2} - C_{yc}^\delta \delta - C_{yc}^{\dot{\delta}} \frac{\dot{\delta} b}{U_c} \end{pmatrix} - \right]. \quad (2.2.1)$$

After the simple rearranging and time average we can write

$$C_T = \frac{2}{\rho S U_0^2} \bar{F}_{xc} = \left[\begin{aligned} & \left(C_{yc}^{\alpha} \frac{\overline{V_{yc}}}{U_0^2} + \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{\overline{\dot{v}_{nc} b \sin \theta_c}}{U_0^2} - \right. \\ & \left. - C_{yc}^{\omega} \frac{\overline{\omega_z V_{yc} b}}{U_0^2} - C_{yc}^{\dot{\omega}} \frac{\overline{\dot{\omega}_z b^2 \sin \theta_c}}{U_0^2} - \right. \\ & \left. - C_{yc}^{\delta} \frac{\overline{\delta U_c V_{yc}}}{U_0^2} - C_{yc}^{\dot{\delta}} \frac{\overline{\dot{\delta} b V_{yc}}}{U_0^2} \right. \\ & \left. - \frac{\overline{2X_i}}{\rho S U_0^2} \cos \vartheta - C \frac{\overline{U_c^2}}{U_0^2} \cos \vartheta \right] . \end{aligned} \right] \quad (2.2.2)$$

This formula we can write

$$C_T = C_{T1} + C_{T2} + C_{T3} + C_{T4} + C_{T5} + C_{T6} + C_{T7} + C_{T8} . \quad (2.2.3)$$

Coefficients $C_{T1} - C_{T4}$, C_{T7} , C_{T8} determine the rigid wing work and were determined previously (chapter 1). Coefficients C_{T5} and C_{T6} determine the deformable wing work and must be calculated.

The change in the wing thrust after time average can be written as

$$\Delta C_T = \Delta \left(\frac{2}{\rho S U_0^2} \bar{F}_{xc} \right) = \left(-C_{yc}^{\delta} \frac{\overline{\delta U_c V_{yc}}}{U_0^2} - C_{yc}^{\dot{\delta}} \frac{\overline{\dot{\delta} b V_{yc}}}{U_0^2} \right) = C_{T5} + C_{T6} \quad (2.2.4)$$

2.3. The deformable wing power

The deformable wing power can be written as

$$\overline{P}_c = -\overline{F}_{yc} \overline{V}_{yc} - \overline{M}_{zc} \overline{\omega_z}. \quad (2.3.1)$$

The terms in the right side of this formula will look like

$$F_{yc} V_{yc} = \left\{ \begin{array}{l} \lambda_{22} V_{yc} \frac{d(v_{nc} \cos \theta)}{dt} + \\ + \frac{\rho S}{2} \left[C_{yc}^\alpha v_{nc} V_{xc} V_{yc} + \left(C_{yc}^\alpha - \frac{2\lambda_{22}}{\rho S b} \right) b \dot{v}_{nc} V_{yc} \cos \theta_c - \right. \\ \left. - C_{yc}^{\omega_z} \omega_z b V_{xc} V_{yc} - C_{yc}^{\dot{\omega}_z} \dot{\omega}_z V_{yc} b^2 \cos \theta_c - \right. \\ \left. - C_{yc}^\delta \delta V_{yc} U_c^2 \cos \theta_c - C_{yc}^{\dot{\delta}} \dot{\delta} b V_{yc} U_c \cos \theta_c \right] + \\ + X_u V_{yc} \sin \theta + \frac{\rho S U_c^2 V_{yc}}{2} C \sin \theta. \end{array} \right\} \quad (2.3.2)$$

and

$$-M_{zc} \omega_z = \frac{\rho S}{2} \left[m_{zc}^\alpha \alpha_c \omega_z b U_c^2 + m_{zc}^{\dot{\alpha}} \dot{\alpha}_c \omega_z b^2 U_c - m_{zc}^{\omega_z} \omega_z^2 b^2 U_c - \right. \\ \left. - m_{zc}^{\dot{\omega}_z} \dot{\omega}_z b^3 - m_{zc}^{\dot{\delta}} \dot{\delta} \omega_z b U_c^2 - m_z^{\dot{\delta}} \dot{\delta} b \omega_z U_c^2 \right]. \quad (2.3.3)$$

The total power coefficient we can write (chapter 1)

$$C_p = \left(C_{p1} + C_{p2} + C_{p3} + C_{p4} + C_{p5} + C_{p6} + C_{p7} + \right. \\ \left. C_{p8} + C_{p9} + C_{p10} + C_{p11} + C_{p12} + C_{p13} + C_{p14} + C_{p15} \right), \quad (2.3.4)$$

In this formula $C_{p1} - C_{p5}, C_{p8}, C_{p9}, C_{p10} - C_{p13}$ were determined in chapter 1 for the rigid wing. The terms $C_{p6}, C_{p7}, C_{p14}, C_{p15}$ are due to the wing deformation and must be calculated.

The change in the wing power after time average can be written as

$$\Delta \left(-\frac{2F_{yc}V_{yc}}{\rho SU_0^3} - \frac{2M_{zc}\omega_z}{\rho SU_0^3} \right) = \begin{pmatrix} -C_{yc}^\delta \frac{\overline{\delta V_{xc}V_{yc}U_c}}{U_0^3} - C_{yc}^\delta \frac{\overline{\dot{\delta}V_{xc}V_{yc}b}}{U_0^3} - \\ -m_{zc}^\delta \frac{\overline{\delta\omega_z b U_c^2}}{U_0^3} - m_{zc}^\delta \frac{\overline{\dot{\delta}b^2 \omega_z U_c}}{U_0^3} \end{pmatrix}. \quad (2.3.5)$$

2.4. The change in the wing thrust

Let us considerate the common case

$$y = y_0 \sin \omega t, \quad (2.4.1)$$

$$\vartheta = \vartheta_0 [\sin(\omega t + \varphi_1)], \quad (2.4.2)$$

$$\delta = \delta_0 [\sin(\omega t + \varphi_2)]. \quad (2.4.3)$$

Here φ_1 and φ_2 are the phase angles between heaving and pitching

Correspondent to the thrust coefficients will look like

$$C_{T5} = C_{T5-0} \left\{ \begin{aligned} & \frac{\sin \varphi_2}{\lambda_p} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right] + \\ & \left[\frac{0.25 g_0 (Sh_0) X_b}{(2\lambda_p^2 + 1)} \sin(\varphi_1 + \varphi_2) - \sin(\varphi_1 - \varphi_2) - \right. \\ & \left. + \left[- \left(\frac{0.06 g_0^3 (Sh_0) X_b}{(2\lambda_p^2 + 1)} \begin{pmatrix} \sin(\varphi_1 + \varphi_2) + \\ + \sin(\varphi_1 - \varphi_2) \left(\begin{pmatrix} \cos 2\varphi_1 - \\ - 0.5 \end{pmatrix} \right) \end{pmatrix} \right) \right] \end{aligned} \right\}, \quad (2.4.4)$$

$$\text{where } C_{T5-0} = -C_{yc}^\delta \frac{\delta_0 \sqrt{2\lambda_p^2 + 1}}{2\sqrt{2}\lambda_p}$$

$$C_{T6} = -C_{yc}^\delta \frac{\delta_0}{2} \left[\begin{aligned} & \frac{(Sh_0) \cos \varphi_2}{\lambda_p} + \\ & + (Sh_0)^2 X_b \cos \varphi_1 \cos \varphi_2 \begin{pmatrix} g_0 - \\ -0.1g_0^3 \left(1 + \right. \\ \left. + 2 \sin^2 \varphi_1 \right) \end{pmatrix} + \\ & + \frac{(Sh_0)^2 g_0^3 X_b \sin \varphi_1 \cos^2 \varphi_1 \sin \varphi_2}{8} + \\ & + \frac{(Sh_0)^2 g_0^3 X_b \sin^2 \varphi_1 \cos^2 \varphi_1}{8} + \\ & + (Sh_0)^2 X_b \sin \varphi_1 \sin \varphi_2 \begin{pmatrix} g_0 - \\ -0.1g_0^3 \left(1 + \right. \\ \left. + 2 \cos^2 \varphi_1 \right) \end{pmatrix} \end{aligned} \right]. \quad (2.4.5)$$

In the case when $\varphi_1 = \varphi_2 = \frac{\pi}{2}$ we can write

$$C_{T5} = -C_{yc}^{\delta} \frac{\delta_0 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p^2} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right], \quad (2.4.6)$$

$$C_{T6} = -C_{yc}^{\delta} \frac{(Sh_0)^2 \delta_0 g_0 X_b}{2} \left(1 - 0.125 g_0^2 \right). \quad (2.4.7)$$

To pure heaving ($g = \delta = 0$) we can write

$$C_{T5} = 0, \quad (2.4.8)$$

$$C_{T6} = 0. \quad (2.4.9)$$

To pure pitching $\lambda = \infty$ we can write

$$C_{T5} = C_{yc}^{\delta} \left[\frac{(Sh_0) g_0 \delta_0 X_b}{2} \right] \sin(\varphi_1 - \varphi_2) \begin{bmatrix} 0.25 g_0^2 \begin{pmatrix} \cos 2\varphi_1 & - \\ -0.5 & \end{pmatrix}^+ \\ +1 \end{bmatrix}, \quad (2.4.10)$$

$$C_{T6} = -C_{yc}^{\delta} \frac{(Sh_0)^2 \delta_0 g_0 X_b}{2} \left[\begin{aligned} & \cos \varphi_1 \cos \varphi_2 \left(\begin{aligned} & 1 - \\ & -0.1 g_0^2 \left(\begin{aligned} & 1 + \\ & +2 \sin^2 \varphi_1 \end{aligned} \right) \end{aligned} \right) + \\ & + \frac{g_0^2 \sin \varphi_1 \cos^2 \varphi_1 \sin \varphi_2}{8} + \\ & + \frac{g_0^2 \sin^2 \varphi_1 \cos^2 \varphi_1}{8} + \\ & + \sin \varphi_1 \sin \varphi_2 \left(\begin{aligned} & 1 - \\ & -0.1 g_0^2 \left(\begin{aligned} & 1 + \\ & +2 \cos^2 \varphi_1 \end{aligned} \right) \end{aligned} \right) \end{aligned} \right]. \quad (2.4.11)$$

2.5. Power coefficients

Correspondent power coefficients can be written (see (2.3.5))

$$C_{P6} = A(C_{P6-1} + C_{P6-2}) \quad (2.5.1)$$

where

$$A = -C_{yc}^{\delta} \frac{\delta_0 \sqrt{(2\lambda_p^2 + 1)}}{\sqrt{2}\lambda_p}, \quad (2.5.2)$$

$$\begin{aligned}
C_{P6-1} = & \left\{ \frac{\sin \varphi_2}{2\lambda_p} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right] - \right. \\
& - \frac{(Sh_0) \vartheta_0 \sin \varphi_1 \cos \varphi_2 X_b}{2} \left[\begin{array}{l} 1 - 0.25\vartheta_0 - \\ - 0.125\vartheta_0 \cos 2\varphi_1 + \\ + \frac{0.0625\vartheta_0 \cos 2\varphi_1}{(2\lambda_p^2 + 1)} - \\ - \frac{0.25}{(2\lambda_p^2 + 1)} + \\ + \frac{0.0625\vartheta_0}{(2\lambda_p^2 + 1)} \end{array} \right] + \\
& + \frac{(Sh_0) \vartheta_0 \sin \varphi_2 \cos \varphi_1 X_b}{2} \left[\begin{array}{l} 1 - 0.25\vartheta_0 + \\ + 0.125\vartheta_0 \cos 2\varphi_1 + \\ + \frac{0.0625\vartheta_0 \cos 2\varphi_1}{(2\lambda_p^2 + 1)} + \\ + \frac{0.25}{(2\lambda_p^2 + 1)} - \frac{0.0625\vartheta_0}{(2\lambda_p^2 + 1)} \end{array} \right] - \\
& - \frac{(Sh_0) \vartheta_0^2 \sin \varphi_2 \cos \varphi_1 X_b}{\lambda_p} \left[\begin{array}{l} 0.375 \sin \varphi_1 - \\ - 0.0313\vartheta_0^2 \sin \varphi_1 \cos^2 \varphi_1 - \\ - 0.0521\vartheta_0^2 \sin^3 \varphi_1 + \\ + \frac{0.125 \sin \varphi_1}{(2\lambda_p^2 + 1)} - \\ - \frac{0.0039\vartheta_0^2 \sin \varphi_1}{(2\lambda_p^2 + 1)} - \\ - \frac{0.0313\vartheta_0^2 \sin^3 \varphi_1}{(2\lambda_p^2 + 1)} \end{array} \right] \left. \right\}, \quad (2.5.3)
\end{aligned}$$

$$\begin{aligned}
C_{p6-2} = A_1 & \left\{ \begin{array}{l} \left[\begin{array}{l} 0.125 \cos^2 \varphi_1 + \\ + 0.125 \sin^2 \varphi_1 \left(1 - \frac{1}{(2\lambda_p^2 + 1)} \right) - \\ - 0.01 g_0^2 \cos^4 \varphi_1 \left(1 - \frac{0.5}{(2\lambda_p^2 + 1)} \right) - \\ \cos \varphi_1 \cos^2 \varphi_2 \left[- 0.083 g_0^2 \sin^2 \varphi_1 \cos^2 \varphi_1 - \right. \\ \left. - 0.25 \sin^2 \varphi_1 \cos^2 \varphi_1 + \right. \\ \left. + 0.042 g_0^2 \sin^4 \varphi_1 \left(1 - \frac{0.375}{(2\lambda_p^2 + 1)} \right) + \right. \\ \left. + \frac{0.031 g_0^2 \sin^2 \varphi_1}{(2\lambda_p^2 + 1)} \right] - \\ \left. \left[\begin{array}{l} 0.125 \left(1 + \cos^2 \varphi_1 \left(g_0^2 + \frac{1}{(2\lambda_p^2 + 1)} \right) \right) + \\ + 0.083 g_0^2 \cos^3 \varphi_1 - \\ - 0.042 g_0^2 \sin^2 \varphi_1 \cos^2 \varphi_1 \left(1 - \frac{0.729}{(2\lambda_p^2 + 1)} \right) + \\ + 0.0417 g_0^2 \sin^4 \varphi_1 \leq \left(1 + \frac{0.125}{(2\lambda_p^2 + 1)} \right) - \\ - \frac{0.01 g_0^2 \sin \varphi_1 \cos^3 \varphi_1}{(2\lambda_p^2 + 1)} + \frac{0.016 g_0^2 \cos^4 \varphi_1}{(2\lambda_p^2 + 1)} \end{array} \right] \right] \end{array} \right\} . \\
& \quad (2.5.4)
\end{aligned}$$

here

$$A_1 = -(Sh_0)^2 g_0^3 X_b^2$$

$$C_{P7} = C_{P7-0} \left\{ \begin{array}{l} \frac{\cos \varphi_2}{2\lambda_p} + \\ + \frac{(Sh_0)g_0 X_b}{2} \left[(1 - 0.125g_0^2) \cos(\varphi_1 - \varphi_2) \right] - \\ - \frac{(Sh_0)g_0^2 X_b}{\lambda_p} \left[0.125 \sin(2\varphi_1 - \varphi_2) \right] + \\ -0.19g_0^3 \sin(\varphi_1 - \varphi_2) + \\ \left. \begin{array}{l} 0.06 \sin^4 \varphi_1 \sin(\varphi_1 - \varphi_2) + \\ + 0.06 \sin \varphi_1 \cos^4 \varphi_1 \cos \varphi_2 - \\ - 0.06 \cos^5 \varphi_1 \cos \varphi_2 + \\ + 0.06 \sin^2 \varphi_1 \sin \varphi_2 \cos^3 \varphi_1 + \\ + 0.5 \sin^3 \varphi_1 \cos^2 \varphi_1 \cos \varphi_2 - \\ - 0.4 \sin^2 \varphi_1 \cos^2 \varphi_1 \cos \varphi_2 - \\ - 0.19 \sin^2 \varphi_1 \sin \varphi_2 \cos^3 \varphi_1 + \\ + 0.06 \sin^4 \varphi_1 \sin(\varphi_1 - \varphi_2) \end{array} \right] \end{array} \right\}, \quad (2.5.5)$$

where $C_{P7-0} = -C_{yc}^{\delta} (Sh_0) \delta_0$

$$C_{P14} = -m_{zc}^{\delta} \frac{(Sh_0)g_0 \delta_0 (2\lambda_p^2 + 1)}{4\lambda_p^2} \left[\sin(\varphi_1 + \varphi_2) - \frac{\sin(\varphi_2 - \varphi_1)}{2(2\lambda_p^2 + 1)} \right], \quad (2.5.6)$$

$$C_{P15} = -m_{zc}^{\delta} \left\{ \frac{(Sh_0)^2 g_0 \delta_0 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[\cos(\varphi_2 - \varphi_1) + \frac{\cos(\varphi_1 + \varphi_2)}{4(2\lambda_p^2 + 1)} \right] \right\}. \quad (2.5.7)$$

when $\varphi_1 = \varphi_2 = \frac{\pi}{2}$ we can write

$$C_{P6} = -C_{yc}^{\delta} \frac{\delta_0 \sqrt{(2\lambda_p^2 + 1)}}{\sqrt{2}\lambda_p} \begin{Bmatrix} \frac{1}{2\lambda_p} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} \right] - \\ - (Sh_0)^2 g_0^3 X_b^2 \begin{Bmatrix} 0.125 - \\ -0.04g_0^2 \end{Bmatrix} \end{Bmatrix}, \quad (2.5.8)$$

$$C_{P7} = -C_{yc}^{\delta} (Sh_0)^2 \delta_0 g_0 X_b \begin{Bmatrix} \frac{1}{2} (1 - 0.125g_0^2) - \\ - \frac{g_0}{\lambda_p} [0.125 - 0.01g_0^2] \end{Bmatrix}, \quad (2.5.9)$$

$$C_{P14} = 0, \quad (2.5.10)$$

$$C_{P15} = -m_{zc}^{\delta} \frac{(Sh_0)^2 \delta_0 g_0 \sqrt{(2\lambda_p^2 + 1)}}{2\sqrt{2}\lambda_p} \left[1 - \frac{0.25}{(2\lambda_p^2 + 1)} \right]. \quad (2.5.11)$$

To pure heaving we can write

$$C_{P6} = C_{P7} = C_{P14} = C_{P15} = 0. \quad (2.5.12)$$

To pure pitching

$$C_{p_6} = C_{yc}^{\delta} \delta \left\{ \begin{array}{l} \frac{(Sh_0)g_0 \sin \varphi_1 \cos \varphi_2 X_b}{2} \left[\begin{array}{c} 1 - 0.25g_0 \\ -0.125g_0 \cos 2\varphi_1 \end{array} \right] - \\ - \frac{(Sh_0)g_0 \sin \varphi_2 \cos \varphi_1 X_b}{2} \left[\begin{array}{c} 1 - 0.25g_0 \\ +0.125g_0 \cos 2\varphi_1 \end{array} \right] - \\ + (Sh_0)^2 g_0^3 X_b^2 \cos \varphi_1 \cos^2 \varphi_2 \left[\begin{array}{c} 0.125 \\ -0.01g_0^2 \cos^4 \varphi_1 \\ -0.1g_0^2 \sin^2 \varphi_1 \cos^2 \varphi_1 \\ -0.25 \sin^2 \varphi_1 \cos^2 \varphi_1 \\ +0.04g_0^2 \sin^4 \varphi_1 \end{array} \right] - \\ + (Sh_0)^2 g_0^3 X_b^2 \sin \varphi_1 \sin \varphi_2 \left[\begin{array}{c} 0.125(1 + \cos^2 \varphi_1(g_0^2)) \\ +0.1g_0^2 \cos^3 \varphi_1 \\ -0.04g_0^2 \sin^2 \varphi_1 \cos^2 \varphi_1 \\ +0.04g_0^2 \sin^4 \varphi_1 \end{array} \right] \end{array} \right\}, \quad (2.5.13)$$

$$C_{p_7} = A \left\{ \begin{array}{l} + \frac{(Sh_0)g_0 X_b}{2} \left[(1 - 0.125g_0^2) \cos(\varphi_1 - \varphi_2) \right] + - \\ + g_0^5 \left[\begin{array}{c} -3g_0^3 \sin(\varphi_1 - \varphi_2) \\ \left(\begin{array}{c} \sin^4 \varphi_1 \sin(\varphi_1 - \varphi_2) \\ + \sin \varphi_1 \cos^4 \varphi_1 \cos \varphi_2 \\ - \cos^5 \varphi_1 \cos \varphi_2 \\ + \sin^2 \varphi_1 \sin \varphi_2 \cos^3 \varphi_1 \\ + 8 \sin^3 \varphi_1 \cos^2 \varphi_1 \cos \varphi_2 \\ - 6 \sin^2 \varphi_1 \cos^2 \varphi_1 \cos \varphi_2 \\ - 3 \sin^2 \varphi_1 \sin \varphi_2 \cos^3 \varphi_1 \\ + \sin^4 \varphi_1 \sin(\varphi_1 - \varphi_2) \end{array} \right) \end{array} \right] \end{array} \right\}, \quad (2.5.14)$$

here

$$A = -C_{yc}^{\delta}(Sh_0)\delta_0$$

$$C_{P14} = -m_{zc}^{\delta} \frac{(Sh_0) \vartheta_0 \delta_0}{2} [\sin(\varphi_1 + \varphi_2)], \quad (2.5.15)$$

$$C_{P15} = -m_{zc}^{\delta} \left\{ \frac{(Sh_0)^2 \vartheta_0 \delta_0}{2} [\cos(\varphi_2 - \varphi_1)] \right\}. \quad (2.5.16)$$

2.6. Additional induced drag terms of the deformable wing

Earlier the velocity induced by the rotary wake of the non deformable wing was obtained in the form (1.9.18). A related expression can be obtained in the case of the deformable wing.

$$u_* = \begin{pmatrix} v_n - \frac{v_n}{2\pi} C_y^\alpha + \frac{\omega_z b}{2\pi} C_y^{\omega_z} - \frac{\omega_z b}{4} + \frac{\lambda_{22} \dot{v}_n}{\rho \pi b U} - \\ - \frac{\dot{v}_n b}{2\pi U} C_y^\alpha + \frac{\dot{\omega}_z b^2}{2\pi U} C_y^{\dot{\omega}_z} + \frac{U}{2\pi} C_y^\delta \delta + \frac{1}{2\pi} C_y^\delta \dot{\delta} b \end{pmatrix}, \quad (2.6.1)$$

This expression is distinct from obtained before by the availability of the two additional terms. These terms lead to the production of additional terms in the formula of the induced drag which will look like

$$C_{T5} = -\frac{1}{U_0^2} \left(\begin{array}{l} D_1 \overline{v_{nc}^2 \cos \vartheta} + D_2 \overline{v_{nc} \omega_z \cos \vartheta} + D_3 \overline{\frac{v_{nc} \dot{\phi}_z}{U_c} \cos \vartheta} + \\ + D_4 \overline{\frac{v_{nc} \dot{v}_{nc}}{U_c} \cos \vartheta} + D_5 \overline{\frac{\dot{v}_{nc} \omega_z}{U_c} \cos \vartheta} + D_6 \overline{\omega_z^2 \cos \vartheta} + \\ + D_7 \overline{\frac{\omega_z \dot{\phi}_z}{U_c} \cos \vartheta} + D_8 \overline{\frac{\dot{v}_{nc}^2}{U_c^2} \cos \vartheta} + D_9 \overline{\frac{\dot{v}_{nc} \dot{\phi}_z}{U_c^2} \cos \vartheta} + \\ + D_{10} \overline{\frac{\dot{\phi}_z^2}{U_c^2} \cos \vartheta} + D_{11} \overline{v_{nc} \delta U_c \cos \vartheta} + \\ + D_{12} \overline{v_{nc} \dot{\delta} b \cos \vartheta} + D_{13} \overline{\omega_z \delta U b \cos \vartheta} + \\ + D_{14} \overline{\omega_z \dot{\delta} b^2 \cos \vartheta} + D_{15} \overline{\dot{v}_{nc} \delta b \cos \vartheta} + \\ + D_{16} \overline{\frac{\dot{v}_{nc} \dot{\delta} b^2}{U_c} \cos \vartheta} + D_{17} \overline{\dot{\phi}_z \delta b^2 \cos \vartheta} + \\ + D_{18} \overline{\frac{\dot{\phi}_z \dot{\delta} b^3}{U_c} \cos \vartheta} + D_{19} \overline{\delta^2 U_c^2 \cos \vartheta} + \\ + D_{20} \overline{(\dot{\delta}^2) b^2 \cos \vartheta} + D_{21} \overline{\delta \dot{\delta} U_c b \cos \vartheta} \end{array} \right). \quad (2.6.2)$$

In this formula the first 10 terms correspond to the rigid wing (non deformable) and are depicted in chapter 1. The next 11 terms (from 11 to 21) are due to the wing deformation. When $\varphi_1 = \varphi_2 = \frac{\pi}{2}$ after time average we can write

$$C_{T5-11} = A \left\{ \begin{array}{l} \left[1 + \frac{0.25}{(2\lambda_p^2 + 1)} - 0.5g_0^2 \left(1.5 + \frac{0.5}{(2\lambda_p^2 + 1)} \right) + \right. \\ \left. + 0.156g_0^4 \left(1 + \frac{0.375}{(2\lambda_p^2 + 1)} \right) - \right. \\ \left. - g_0 \lambda_p \left(1 + \frac{0.25}{(2\lambda_p^2 + 1)} - 0.5g_0^2 \left(1 + \frac{0.33}{(2\lambda_p^2 + 1)} \right) + \right. \right. \\ \left. \left. + 0.05g_0^4 \left(1 + \frac{0.375}{(2\lambda_p^2 + 1)} \right) \right] \right\}, \quad (2.6.3)$$

where

$$A = -D_{11} \frac{\delta_0 \sqrt{2\lambda_p^2 + 1}}{2\sqrt{2}\lambda_p^2}$$

$$D_{11} = \left[\frac{1}{\pi} C_{yc}^\alpha C_{yc}^\delta - C_{yc}^\delta \right]. \quad (2.6.4)$$

$$C_{T5-12} = -D_{12} \left[\frac{(Sh_0)^2 \delta_0 g_0 X_b}{2} \left(1 - 0.125g_0^2 \right) \right], \quad (2.6.5)$$

where

$$D_{12} = b \left[\frac{1}{\pi} C_{yc}^\alpha C_{yc}^\delta - C_{yc}^\delta \right]. \quad (2.6.6)$$

$$C_{T5-13} = -D_{13} \frac{\omega_z \delta U_c b \cos \vartheta}{U_0^2} = 0, \quad (2.6.7)$$

where

$$D_{13} = b \left[-\frac{1}{\pi} C_{yc}^{\omega_z} C_{yc}^{\delta} + \frac{1}{2} C_{yc}^{\delta} \right]. \quad (2.6.8)$$

$$C_{T5-14} = -D_{14} \left[\frac{(Sh_0)^2 \delta_0 \vartheta_0}{2} (1 - 0.125 \vartheta_0^2) \right], \quad (2.6.9)$$

where

$$D_{14} = b^2 \left[-\frac{1}{\pi} C_{yc}^{\omega_z} C_{yc}^{\delta} + \frac{1}{2} C_{yc}^{\delta} \right]. \quad (2.6.10)$$

$$C_{T5-15} = -D_{15} \left[-\frac{(Sh_0)^2 \delta_0 \vartheta_0 X_b}{2} (1 - 0.375 \vartheta_0^2) \right], \quad (2.6.11)$$

where

$$D_{15} = b \left[\frac{1}{\pi} C_{yc}^{\dot{\alpha}} C_{yc}^{\delta} - \frac{2\lambda_{22}}{\rho\pi b^2} C_{yc}^{\delta} \right]. \quad (2.6.12)$$

$$C_{T5-16} = -D_{16} \left[\frac{\sqrt{2}(Sh_0)^2 \delta_0}{2\sqrt{(2\lambda_p^2 + 1)}} \left(\begin{array}{l} 1 + \frac{0.25}{(2\lambda_p^2 + 1)} + \frac{0.1875}{(2\lambda_p^2 + 1)^2} + \\ + \frac{0.1172}{(2\lambda_p^2 + 1)^3} + \frac{0.1025}{(2\lambda_p^2 + 1)^4} \end{array} \right) \right], \quad (2.6.13)$$

where

$$D_{16} = b^2 \left[\frac{1}{\pi} C_{yc}^{\dot{\alpha}} C_{yc}^{\delta} - \frac{2\lambda_{22}}{\rho\pi b^2} C_{yc}^{\delta} \right]. \quad (2.6.14)$$

$$C_{T5-17} = -D_{17} \left[-\frac{(Sh_0)^2 \delta_0 g_0}{2} (1 - 0.375 g_0^2) \right], \quad (2.6.15)$$

where

$$D_{17} = b^2 \left[-\frac{1}{\pi} C_{yc}^{\dot{\phi}_z} C_{yc}^{\delta} \right]. \quad (2.6.16)$$

$$C_{T5-18} = -D_{18} \frac{\dot{\phi}_z \dot{\delta} b^3}{U_0^2 U_c} = 0, \quad (2.6.17)$$

where

$$D_{18} = b^3 \left[-\frac{1}{\pi} C_{yc}^{\dot{\phi}_z} C_{yc}^{\delta} \right]. \quad (2.6.18)$$

$$C_{T5-19} = -D_{19} \left[\frac{\delta_0^2 (2\lambda_p^2 + 1)}{4\lambda_p^2} \right] \begin{bmatrix} 1 + \frac{0.625}{(2\lambda_p^2 + 1)} - \\ -0.375 g_0^2 \left(1 - \frac{0.(3)}{(2\lambda_p^2 + 1)} \right) \end{bmatrix}, \quad (2.6.19)$$

where

$$D_{19} = \left[-\frac{1}{2\pi} (C_{yc}^{\delta})^2 \right]. \quad (2.6.20)$$

$$C_{T5-20} = -D_{20} \left[\frac{(Sh_0)^2 \delta_0^2}{2} \right] [1 - 0.125 g_0^2], \quad (2.6.21)$$

where

$$D_{20} = b^2 \left[-\frac{1}{2\pi} \left(C_{yc}^\delta \right)^2 \right]. \quad (2.6.22)$$

$$C_{T5-21} = -D_{21} \frac{\overline{\delta \dot{\delta} U_c \sin \theta}}{U_0^2} = 0, \quad (2.6.23)$$

$$D_{21} = \frac{1}{\pi} C_{yc}^\delta C_{yc}^\delta. \quad (2.6.24)$$

To pure heaving ($\theta = 0, \delta = 0$)

$$C_{T5-11} - C_{T5-20} = 0. \quad (2.6.25)$$

To pure pitching

$$C_{T5-11} = 0, \quad (2.6.26)$$

$$C_{T5-12} = -D_{12} \left[\frac{(Sh_0)^2 \delta_0 \theta_0 X_b}{2} \left(1 - 0.125 \theta_0^2 \right) \right], \quad (2.6.27)$$

$$C_{T5-13} = 0, \quad (2.6.28)$$

$$C_{T5-14} = -D_{14} \left[\frac{(Sh_0)^2 \delta_0 g_0}{2} (1 - 0.125 g_0^2) \right], \quad (2.6.29)$$

$$C_{T5-15} = -D_{15} \left[-\frac{(Sh_0)^2 \delta_0 g_0 X_b}{2} (1 - 0.375 g_0^2) \right], \quad (2.6.30)$$

$$C_{T5-16} = 0, \quad (2.6.31)$$

$$C_{T5-17} = -D_{17} \left[-\frac{(Sh_0)^2 \delta_0 g_0}{2} (1 - 0.375 g_0^2) \right], \quad (2.6.32)$$

$$C_{T5-18} = 0, \quad (2.6.33)$$

$$C_{T5-19} = -D_{19} \left[\frac{\delta_0^2}{2} \right], \quad (2.6.34)$$

$$C_{T5-20} = -D_{20} \left[\frac{(Sh_0)^2 \delta_0^2}{2} \right], \quad (2.6.35)$$

$$C_{T5-21} = 0. \quad (2.6.36)$$

2.7. Additional power coefficients terms including the induced drag of deformable wing

One of the power coefficients terms including the induced drag of the deformable wing will look like

$$C_{p6} = \frac{\overline{2V_{yc}X_i \sin \theta}}{\rho S U_0^3} \quad (2.7.1)$$

The expression (2.7.1) can be expressed in terms of the components

$$C_{p6} = \frac{1}{U_0^3} \left\{ \begin{aligned} & D_1 \overline{V_{yc} v_{nc}^2 \sin \theta} + D_2 \overline{V_{yc} v_{nc} \omega_z \sin \theta} + D_3 \overline{\frac{V_{yc} v_{nc} \dot{\omega}_z}{U_c} \sin \theta} + \\ & + D_4 \overline{\frac{V_{yc} v_{nc} \dot{v}_{nc}}{U_c} \sin \theta} + D_5 \overline{\frac{V_{yc} \dot{v}_{nc} \omega_z}{U_c} \sin \theta} + D_6 \overline{V_{yc} \omega_z^2 \sin \theta} + \\ & + D_7 \overline{\frac{V_{yc} \omega_z \dot{\omega}_z}{U_c} \sin \theta} + D_8 \overline{\frac{V_{yc} \dot{v}_{nc}^2}{U_c^2} \sin \theta} + D_9 \overline{\frac{V_{yc} \dot{v}_{nc} \dot{\omega}_z}{U_c^2} \sin \theta} + \\ & + D_{10} \overline{\frac{V_{yc} \dot{\omega}_z^2}{U_c^2} \sin \theta} + D_{11} \overline{V_{yc} v_{nc} U_c \delta \sin \theta} + D_{12} \overline{V_{yc} v_{nc} \dot{\delta} \sin \theta} + \\ & + D_{13} \overline{V_{yc} \omega_z U_c \delta \sin \theta} + D_{14} \overline{V_{yc} \omega_z \dot{\delta} \sin \theta} + D_{15} \overline{V_{yc} \dot{v}_{nc} \delta \sin \theta} + \\ & + D_{16} \overline{\frac{V_{yc} \dot{v}_{nc} \dot{\delta} \sin \theta}{U_c}} + D_{17} \overline{V_{yc} \dot{\omega}_z \delta \sin \theta} + D_{18} \overline{\frac{V_{yc} \dot{\omega}_z \dot{\delta} \sin \theta}{U_c}} + \\ & + D_{19} \overline{V_{yc} \delta^2 U_c^2 \sin \theta} + D_{20} \overline{V_{yc} \dot{\delta}^2 \sin \theta} + D_{21} \overline{V_{yc} \delta \dot{\delta} U_c \sin \theta} \end{aligned} \right\}. \quad (2.7.2)$$

The first 10 terms are shown in chapter 1 (formulas 1.9.87 – 1.9.127) to the rigid wing. Here we will show the formulas for the deformable wing (formulas from 11 to 21). As the formulas to common case are too cumbersome we shall deal with the case when ($\varphi_1 = \varphi_2 = 90^\circ$)

$$C_{P6-11} = D_{11} \left\{ -\frac{3g_0\delta_0\sqrt{2\lambda_p^2+1}}{8\sqrt{2}\lambda_p^3} \left[1 + \frac{0.(3)}{(2\lambda_p^2+1)} - 0.(5)g_0^2 \left(1 + \frac{0.375}{(2\lambda_p^2+1)} \right) \right] - \right. \\ \left. - \frac{3g_0^2\delta_0\sqrt{2\lambda_p^2+1}}{8\sqrt{2}\lambda_p^2} \left[1 + \frac{0.(3)}{(2\lambda_p^2+1)} - 0.2778g_0^2 \left(1 + \frac{0.375}{(2\lambda_p^2+1)} \right) \right] + \right. \\ \left. + \frac{(Sh_0)^2 g_0^3 \delta_0 \sqrt{2\lambda_p^2+1} X_b^2}{8\sqrt{2}\lambda_p} \left[1 - 0.(3)g_0^2 \left(1 + \frac{0.125}{(2\lambda_p^2+1)} \right) \right] \right\}, \quad (2.7.3)$$

$$C_{P6-12} = D_{12} \left\{ \begin{aligned} & \frac{(Sh_0)^2 g_0^2 \delta_0 X_b}{8b} \left[1 - 0.0833g_0^2 \right] + \\ & + \frac{(Sh_0)^2 g_0^2 \delta_0 X_b}{8b\lambda_p} \left[1 - 0.5834g_0^2 \right] - \\ & - \frac{(Sh_0)^2 g_0^3 \delta_0 X_b}{8b} \left[1 - 0.417g_0^2 \right] \end{aligned} \right\}, \quad (2.7.4)$$

$$C_{P6-13} = D_{13} \left\{ \frac{(Sh_0)^2 g_0^3 \delta_0}{8b\sqrt{2}\lambda_p} \left[1 - 0.(3)g_0^2 \left(1 + \frac{0.125}{(2\lambda_p^2+1)} \right) \right] \right\}, \quad (2.7.5)$$

$$C_{P6-14} = D_{14} \left\{ \frac{(Sh_0)^2 g_0^2 \delta_0}{8b^2 \lambda_p} (1 - 0.0833 g_0^2) \right\}, \quad (2.7.6)$$

$$C_{P6-15} = D_{15} \left\{ \begin{aligned} & -\frac{3(Sh_0)^2 g_0^2 \delta_0 X_b}{8b \lambda_p} (1 - 0.139 g_0^2) + \\ & + \frac{(Sh_0)^2 g_0^2 \delta_0 X_b}{8b \lambda_p} (1 - 0.5834 g_0^2) - \\ & - \frac{(Sh_0)^2 g_0^4 \delta_0 X_b}{16b \lambda_p} (1 - 0.521 g_0^2) - \\ & - \frac{(Sh_0)^2 g_0^3 \delta_0 X_b}{8b} (1 - 0.583 g_0^2) \end{aligned} \right\}, \quad (2.7.7)$$

$$C_{P6-16} = D_{16} \left\{ \begin{aligned} & \frac{\sqrt{2}(Sh_0)^2 g_0^2 \delta_0}{8b^2 \lambda_p \sqrt{2\lambda_p^2 + 1}} \left[1 - 0.3 g_0^2 \left(1 - \frac{0.3125}{(2\lambda_p^2 + 1)} \right) \right] - \\ & - \frac{\sqrt{2}(Sh_0)^2 g_0^3 \delta_0}{16b^2 \lambda_p \sqrt{2\lambda_p^2 + 1}} \left[1 - \frac{0.125}{(2\lambda_p^2 + 1)} - \right. \\ & \left. - 0.208 g_0^2 \left(1 - \frac{0.11}{(2\lambda_p^2 + 1)} \right) \right] - \\ & - \frac{\sqrt{2}(Sh_0)^2 g_0^2 \delta_0}{8b^2 \sqrt{2\lambda_p^2 + 1}} \left[1 - 0.3 g_0^2 \left(1 - \frac{0.3125}{(2\lambda_p^2 + 1)} \right) \right] - \\ & - \frac{\sqrt{2}(Sh_0)^4 g_0^3 \delta_0 \lambda_p X_b^2}{8b^2 \sqrt{2\lambda_p^2 + 1}} \left[1 - 0.3 g_0^2 \left(1 - \frac{0.3125}{(2\lambda_p^2 + 1)} \right) \right] \end{aligned} \right\}, \quad (2.7.8)$$

$$C_{P6-17} = D_{17} \left\{ -\frac{3(Sh_0)^2 g_0^2 \delta_0}{8b^2 \lambda_p} \left(1 - 0.139 g_0^2 \right) \right\}, \quad (2.7.9)$$

$$C_{P6-18} = D_{18} \left\{ -\frac{\sqrt{2}(Sh_0)^4 g_0^3 \delta_0 \lambda_p X_b}{8b^3 \sqrt{(2\lambda_p^2 + 1)}} \left[1 - 0.(3) g_0^2 \left(1 - \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] \right\} \quad (2.7.10)$$

$$C_{P6-19} = D_{19} \left\{ \frac{3(Sh_0)g_0\delta_0^2}{16b\lambda_p^3} \left[1 + \frac{0.(6)}{(2\lambda_p^2 + 1)} - 0.139 g_0^2 \left(1 + \frac{0.75}{(2\lambda_p^2 + 1)} \right) \right] \right\}, \quad (2.7.11)$$

$$C_{P6-20} = D_{20} \left\{ \frac{(Sh_0)^2 g_0 \delta_0^2}{8b^2 \lambda_p} \left(1 - 0.0833 g_0^2 \right) \right\}, \quad (2.7.12)$$

$$C_{P6-21} = D_{21} \left\{ \frac{(Sh_0)^2 g_0^2 \delta_0^2 \sqrt{(2\lambda_p^2 + 1)} X_b}{8b\sqrt{2}\lambda_p} \left[1 - 0.(3) g_0^2 \left(1 + \frac{0.125}{(2\lambda_p^2 + 1)} \right) \right] \right\}. \quad (2.7.13)$$

Chapter 3. A comparison with experimental results and theoretical models

A comparison with experimental and theoretical simulations is the best check of the design formulas feasibility.

Prior to the comparison let us describe the methods estimation of some parameters inherent in the design formulas. First and foremost we will consider the aerodynamic (rotary) derivatives.

3.1. Aerodynamic (rotary) derivatives

In the works (Belotserkovskii, 1958; Belotserkovskii, Skripach, Tabachnikov, 1971) the aerodynamic (rotary) derivatives coefficients are tabulated. These data are obtained by the application of the numerical method to the different wings and Strouhal numbers.

But derivatives values are given for the wing pressure center which are positioned in the $\frac{1}{4}$ wing chord from the wing leading edge.

In the design formulas all parameters are given relatively to the wing center. Therefore tabulated derivatives values were calculated relatively to the wing centre. The table 3.1 lists the converted values of the aerodynamic (rotary) derivative coefficients.

Table. 3.1. The aerodynamic (rotary) derivatives coefficients of the infinite wing (relatively to the wing centre)

Sh	C_{yc}^{α}	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{o_z}$	$C_{yc}^{\dot{o}_z}$	m_{zc}^{α}	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{o_z}$	$m_{zc}^{\dot{o}_z}$
0.04	6.055	-10.232	1.514	-2.956	1.514	-2.950	-0.014	-0.788
0.08	5.823	-7.540	1.455	-2.278	1.456	-2.278	-0.029	-0.618
0.12	5.593	-5.896	1.399	-1.867	1.399	-1.867	-0.043	-0.516
0.16	5.406	-4.728	1.352	-1.575	1.352	-1.575	-0.055	-0.443
0.20	5.228	-3.842	1.304	-1.354	1.307	-1.353	-0.066	-0.387
0.24	5.066	-3.144	1.267	-1.179	1.267	-1.179	-0.076	-0.344
0.28	4.922	-2.578	1.231	-1.038	1.231	-1.037	-0.085	-0.308
0.32	4.793	-2.111	1.198	-0.920	1.198	-0.920	-0.093	-0.279
0.36	4.676	-1.723	1.169	-0.823	1.169	-0.823	-0.100	-0.255
0.40	4.572	-1.392	1.143	-0.741	1.143	-0.741	-0.107	-0.234
0.44	4.477	-1.110	1.119	-0.6700	1.119	-0.670	-0.113	-0.216
0.48	4.391	-0.8666	1.098	-0.6094	1.098	-0.609	-0.118	-0.201
0.52	4.313	-0.6549	1.079	-0.5565	1.078	-0.556	-0.123	-0.188
0.56	4.242	-0.4701	1.061	-0.5103	1.061	-0.510	-0.127	-0.176
0.60	4.178	-0.3068	1.045	-0.4694	1.045	-0.469	-0.131	-0.166
0.64	4.119	-0.1630	1.030	-0.4335	1.030	-0.4335	-0.150	-0.157
0.68	4.055	-0.0351	1.018	-0.4015	1.018	-0.4015	-0.138	-0.150
0.72	4.015	+0.0794	1.004	-0.373	1.004	-0.373	-0.141	-0.142
0.76	3.969	0.1827	0.993	-0.3470	0.993	-0.347	-0.145	-0.136
0.80	3.927	0.2749	0.982	-0.324	0.982	-0.324	-0.147	-0.130
0.84	3.887	0.3583	0.972	-0.3032	0.972	-0.3032	-0.149	-0.125
0.88	3.852	0.4341	0.963	-0.2841	0.963	-0.2841	-0.151	-0.120
0.92	3.818	0.5033	0.955	-0.2668	0.955	-0.2668	-0.154	-0.116
0.96	3.786	0.5661	0.947	-0.2511	0.947	-0.2511	-0.156	-0.112
1.00	3.757	0.6239	0.939	-0.2367	0.939	-0.2367	-0.158	-0.108
1.20	3.637	0.8493	0.909	-0.1804	0.909	-0.1804	-0.165	-0.094
1.40	3.549	1.004	0.887	-0.1419	0.887	-0.1419	-0.171	-0.085
1.60	3.482	1.113	0.871	-0.1144	0.871	-0.1144	-0.175	-0.078
1.80	3.430	1.195	0.858	-0.0942	0.858	-0.0942	-0.178	-0.073

2.00	3.389	1.256	0.848	-0.0788	0.848	-0.0788	-0.181	-0.069
3.00	3.274	1.417	0.819	-0.0387	0.819	-0.0387	-0.188	-0.059
4.00	3.223	1.480	0.806	-0.0226	0.806	-0.0226	-0.191	-0.055
5.00	3.196	1.511	0.799	-0.0128	0.799	-0.0128	-0.192	-0.052
10.0	3.157	1.555	0.789	-0.0038	0.789	-0.0038	-0.195	-0.050
∞	3.142	1.571	0.786	-0.0001	0.786	-0.0001	-0.196	-0.049

Table 3.2 The aerodynamic (rotary) derivative coefficients to the rectangular wings (relatively to the wing centre)

λ	Sh	C_{yc}^α	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{\omega_z}$	$C_{yc}^{\dot{\omega}_z}$	m_{zc}^α	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{\omega_z}$	$m_{zc}^{\dot{\omega}_z}$
0.25	0.50	0.392	0.339	0.162	0.0003	0.171	-0.001	-0.077	-0.022
0.25	1.0	0.391	0.344	0.170	-0.001	0.171	0	-0.075	-0.023
0.25	2.0	0.389	0.342	0.171	-0.001	0.17	0	-0.074	-0.023
0.50	0.25	0.772	0.576	0.300	-0.008	0.30	-0.006	-0.114	-0.035
0.50	0.50	0.771	0.584	0.299	-0.003	0.3	-0.004	-0.114	-0.033
0.50	1.0	0.770	0.589	0.299	-0.002	0.3	-0.002	-0.114	-0.033
0.50	2.0	0.766	0.590	0.298	-0.001	0.3	0	-0.114	-0.033
1.00	0.25	1.45	0.832	0.486	-0.026	0.483	-0.021	-0.141	-0.047
1.00	0.50	1.44	0.850	0.485	-0.019	0.480	-0.019	-0.141	-0.046
1.00	1.0	1.43	0.865	0.475	-0.006	0.477	-0.012	-0.143	-0.043
1.00	2.0	1.40	0.884	0.467	-0.006	0.467	-0.007	-0.144	-0.042
2.0	0.25	2.43	0.806	0.713	-0.112	0.706	-0.11	-0.166	-0.074
2.0	0.50	2.38	0.880	0.685	-0.087	0.693	-0.085	-0.144	-0.069
2.0	1.0	2.27	0.989	0.663	-0.056	0.661	-0.054	-0.133	-0.060
2.0	2.0	2.15	1.10	0.623	-0.026	0.628	-0.023	-0.105	-0.052
4.0	0.25	3.47	0.100	0.933	-0.492	0.933	-0.336	-0.116	-0.112
4.0	0.50	3.26	0.473	0.875	-0.243	0.877	-0.238	-0.130	-0.106
4.0	1.0	2.96	0.895	0.800	-0.128	0.797	-0.119	-0.151	-0.083
4.0	2.0	2.72	1.18	0.730	-0.046	0.734	-0.047	-0.171	-0.068

Table 3.3 The aerodynamic (rotary) derivatives coefficients to the triangular wings (relatively to the wing centre)

λ	Sh	C_{yc}^α	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{\omega_z}$	$C_{yc}^{\dot{\omega}_z}$	m_{zc}^α	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{\omega_z}$	$m_{zc}^{\dot{\omega}_z}$
0.50	0.5	0.698	0.210	0.269	0.045	-0.096	-0.047	-0.107	-0.017
0.50	1.0	0.697	0.212	0.269	0.044	-0.092	-0.046	-0.107	-0.017
0.50	2.0	0.698	0.214	0.268	0.045	-0.091	-0.046	-0.07	-0.017

1.0	0.25	1.26	0.322	0.544	0.052	-0.132	-0.072	-0.148	-0.026
1.0	0.50	1.26	0.336	0.543	0.054	-0.133	-0.073	-0.144	-0.024
1.0	1.0	1.26	0.344	0.537	0.058	-0.131	-0.07	-0.146	-0.024
1.0	2.0	1.24	0.347	0.535	0.060	-0.133	-0.069	-0.146	-0.025
2.0	0.25	2.15	0.383	0.863	0.017	-0.141	-0.117	-0.144	-0.041
2.0	0.50	2.13	0.414	0.858	0.030	-0.147	-0.112	-0.145	-0.039
2.0	1.0	2.10	0.438	0.835	0.038	-0.149	-0.111	-0.149	-0.036
2.0	2.0	2.02	0.476	0.805	0.054	-0.153	-0.104	-0.150	-0.033
4.0	0.25	3.33	0.077	1.248	-0.163	-0.065	-0.261	-0.216	-0.096
4.0	0.50	3.24	0.182	1.22	-0.121	-0.091	-0.221	-0.224	-0.082
4.0	1.0	3.07	0.352	1.153	-0.061	-0.126	-0.166	-0.239	-0.062
4.0	2.0	2.81	0.522	1.058	+0.005	-0.160	-0.130	-0.247	-0.049

Table 3.4 The aerodynamic (rotary) derivatives coefficients to the ring wings (relatively to the wing centre)

λ	C_{yc}^α	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{\omega_z}$	$C_{yc}^{\dot{\omega}_z}$	m_{zc}^α	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{\omega_z}$	$m_{zc}^{\dot{\omega}_z}$
0.5	1.54	1.12	0.586	-0.001	0.586	-0.001	-0.209	-0.019
1.0	2.90	1.53	0.935	-0.044	0.933	-0.043	-0.247	-0.084
1.5	3.99	1.46	1.173	-0.133	1.171	-0.133	-0.237	-0.111
2.0	4.83	1.12	1.343	-0.256	0.34	-0.256	-0.217	-0.144
2.5	5.47	0.632	1.473	-0.98	0.471	-0.398	-0.198	-0.18
3.0	5.99	0.086	1.583	-0.44	0.581	-0.545	-0.190	-0.214

The formulas to the convert will look like

$$C_{yc}^\alpha = C_y^\alpha, \quad C_{yc}^{\dot{\alpha}} = C_y^{\dot{\alpha}}, \quad C_{yc}^{\omega_z} = C_y^{\omega_z} + C_y^\alpha \xi_0, \quad C_{yc}^{\dot{\omega}_z} = C_y^{\dot{\omega}_z} + C_y^{\dot{\alpha}} \xi_0,$$

$$m_{zc}^\alpha = m_z^\alpha - C_y^\alpha \xi_0, \quad m_{zc}^{\dot{\alpha}} = m_z^{\dot{\alpha}} - C_y^{\dot{\alpha}} \xi_0, \quad m_{zc}^{\omega_z} = m_z^{\omega_z} - (C_y^{\omega_z} - m_z^\alpha) \xi_0 - C_y^\alpha \xi_0^2,$$

$$m_{zc}^{\dot{\omega}_z} = m_z^{\dot{\omega}_z} - (C_y^{\dot{\omega}_z} - m_z^{\dot{\alpha}}) \xi_0 - C_y^{\dot{\alpha}} \xi_0^2. \quad \text{Здесь } \xi_0 = -0.25.$$

3.2. The wing virtual mass

Other parameter which we must determine is the virtual mass. In the book (Belotserkovskii, Skripach, Tabatchnikov, 1971) the virtual mass is defined as

$$\lambda_{22} = k_{22} \rho S b, \text{ где } k_{22} = \left(\frac{b_a}{b} \right) (k_{22})_a \quad (3.2.1)$$

where $(k_{22})_a$ is the virtual mass coefficient tabulated to the wings of the different forms and Strouhal numbers.

The virtual mass coefficients were defined by means of the methodic described in the book (Belotserkovskii, Skripach, Tabachnikov, 1971) in the origin of the coordinates placed in the leading edge of the median aerodynamic chord.

To convert the virtual mass coefficients values to the wing root chord b we can use formula

$$k_{22} = \frac{b_a}{b} (k_{22})_a, \quad (3.2.2)$$

where

$$\frac{b_a}{b} = \frac{2(\eta^2 + \eta + 1)}{3\eta(\eta + 1)}. \quad (3.2.3)$$

Here $\eta = \frac{b}{b_k}$, b – the chord at the wing centre, b_k - the chord at the wing end.

In particular for the rectangular wing we can write $k_{22} = (k_{22})_a$. For the triangular wing $k_{22} = \frac{2}{3}(k_{22})_a$. The table illustrates values of the $(k_{22})_a$

Table 3.5 The virtual mass coefficients

χ_0	η	λ	$(k_{22})_a$	χ_0	η	λ	$(k_{22})_a$	χ_0	η	λ	$(k_{22})_a$
0	1	0.25	0.1605	30°	5	0.25	0.1206	60°	1	0.25	0.1514
		0.5	0.2797			0.5	0.2241			0.5	0.2446
		1.0	0.4283			1.0	0.3819			1.0	0.3165
		1.5	0.5098			1.5	0.4882			1.5	0.3367
		2.5	0.5905			2.5	0.6024			2.5	0.3487
		5.0	0.6575			5.0	0.6764			5.0	0.3545
		10.0	0.6893			10.0	0.6867			10.0	0.3545

		0.25	0.1462		0.25	0.0923		0.25	0.1526		
		0.5	0.2628		0.5	0.1738		0.5	0.2750		
		1.0	0.4211		1.0	0.3070		1.0	0.3959		
0	2	1.5	0.5157	30°	1.5	0.4078	60°	2	1.5	0.4220	
		2.5	0.6142		2.5	0.5391		2.5	0.4189		
		5.0	0.6944		5.0	0.6600		5.0	0.3919		
		10.0	0.7278		10.0	0.6939		10.0	0.3722		
		0.25	0.1173		0.25	0.1571		0.25	0.1248		
		0.5	0.2140		0.5	0.2656		0.5	0.2338		
		1.0	0.3571		1.0	0.3812		1.0	0.3902		
0	5	1.5	0.4536	45°	1	1.5	0.4297	60°	5	1.5	0.4575
		2.5	0.5696		2.5	0.4668		2.5	0.4689		
		5.0	0.6822		5.0	0.4904		5.0	0.4382		
		10.0	0.7364		10.0	0.4983		10.0	0.4120		
		0.25	0.0892		0.25	0.1511		0.25	0.0961		
		0.5	0.1656		0.5	0.2753		0.5	0.1830		
		1.0	0.2861		1.0	0.4343		1.0	0.3241		

0	∞	1.5 2.5 5.0 10.0	0.3757 0.4966 0.6378 0.7202	45°	2	1.5 2.5 5.0 10.0	0.5069 0.5505 0.5559 0.5454	60°	∞	1.5 2.5 5.0 10.0	0.4185 0.4816 0.4625 0.4291
30°	1	0.25 0.5 1.0 1.5 2.5 5.0 10.0	0.1593 0.2746 0.4109 0.4791 0.5401 0.5851 0.6040	45°	5	0.25 0.5 1.0 1.5 2.5 5.0 10.0	0.1226 0.2291 0.3909 0.4925 0.5770 0.5966 0.5787	86°25'> 82°52'> 75°57'> 69°28'> 58°00'> 38°40'> 21°48'>	∞	0.25 0.5 1.0 1.5 2.5 5.0 10.0	0.0892 0.1656 0.2861 0.3757 0.4966 0.6378 0.7201
		0.25 0.5 1.0 1.5 2.5 5.0	0.1495 0.2719 0.4384 0.5306 0.6103 0.6518			0.25 0.5 1.0 1.5 2.5 5.0	0.0940 0.1783 0.3168 0.4197 0.5398 0.6063				

		10.0	0.6566			10.0	0.5962				
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The table contains the parameters which need to be explained. The values of the parameters are shown in fig. 3.1.

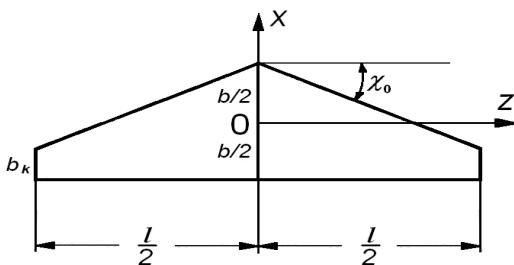


Fig. 3.1 The wing diagrammatic sketch

The wing aspect ratio

$$\lambda = \frac{l^2}{S}. \quad (3.2.4)$$

Parameter χ_0 is clearly visible at the wing diagrammatic sketch.

3.3. A comparison with Russian experimental and theoretical works (rigid wing)

In Russian scientific literature there are few experiments and theoretical simulations.

One of the first experimental works is: (Grebeschov, Sagoyan, 1976) In this paper hydrodynamic investigation of two wings is reported. The wings execute the small amplitude heaving and pitching oscillations (the first cinematic regime) with the constraint

$$\varphi = \frac{\pi}{2}. \quad (3.2.5)$$

The first wing has the profile NACA-0015 (relative thickness 15%) and the second wing has the profile TSAGI KV-1-7 (relative thickness 7%).

But we will compare the theoretical results (calculated using design formulas) with the results of the experimental research of the wing NACA-0015 only. The point is that in studies of the wing TSAGI KV-1-7 the separation flow is observed on the wing leading edge when angle of attack is more than 8 degrees.

The wing kinematic in the work (Grebeschov, Sagoyan, 1976) has the following form

$$y = y_0 \cos \omega t, \quad (3.3.1)$$

$$\vartheta = \alpha_0 - \beta \sin \omega t. \quad (3.3.2)$$

Here $y_0/b = 0.285$, $\alpha_0 = 3.7^\circ = 0.065$ rad, $\beta = 3^\circ = 0.052$ rad. We suppose that $\cos \vartheta \approx 1$ and $\sin \vartheta \approx \vartheta$.

In the work (Grebeschov, Sagoyan, 1976) the thrust coefficient has the following form

$$\bar{k}_T = \frac{2\bar{F}_{xc}}{\rho S (U_0^2 + y_0^2 \omega^2)} = \frac{2\bar{F}_{xc}}{\rho S U_0^2 (\lambda_p^2 + 1)}, \quad (3.3.3)$$

The thrust coefficient can be shown as:

$$\bar{k}_T = \bar{k}_{T1} + \bar{k}_{T2} + \bar{k}_{T3} + \bar{k}_{T4} + \bar{k}_{T5} + \bar{k}_{T6} \quad (3.3.4)$$

The terms in the right hand of the formula can be written as:

$$\bar{k}_{T1} = C_{yc}^{\alpha} \frac{\lambda_p^2}{\lambda_p^2 + 1} \left[\frac{1}{2\lambda_p^2} \left(\frac{1}{\lambda_p} - \beta \right) + A \right], \quad (3.3.5)$$

$$\bar{k}_{T2} = -\frac{\lambda_p^2}{\lambda_p^2 + 1} \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) \frac{1}{X_b} A, \quad (3.3.6)$$

$$\bar{k}_{T3} = -C_{yc}^{\omega_2} \frac{\lambda_p^2}{\lambda_p^2 + 1} \frac{1}{X_b} A, \quad (3.3.7)$$

$$\bar{k}_{T4} = C_{yc}^{\dot{\omega}_2} \frac{\lambda_p^2}{\lambda_p^2 + 1} \frac{(Sh_0)^2 \beta}{\sqrt{2(2\lambda_p^2 + 1)}}, \quad (3.3.8)$$

$$\bar{k}_{T5} = -\frac{\pi}{2} \frac{\lambda_p^2}{\lambda_p^2 + 1} \left[\frac{1}{2} \left(\frac{1}{\lambda_p} - \beta \right)^2 + \alpha_0^2 + A \right], \quad (3.3.9)$$

$$\bar{k}_{T6} = -C \frac{\lambda_p^2}{\lambda_p^2 + 1} \left(1 + \frac{1}{2\lambda_p^2} \right) ..(3.3.10)$$

here

$$A = \frac{(Sh_b)^2 \beta^2 X_b^2}{2}.(3.3.11)$$

Fig. 3.2 shows experimental data from the work (Grebeschov, Sagoyan, 1976) and calculation results using the formulas for the case of harmonic heaving and pitching wing oscillations.

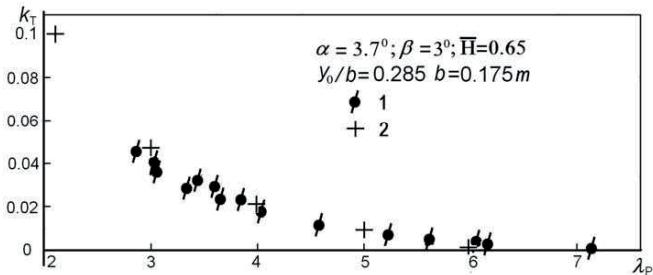


Fig. 3.2 Experimental thrust coefficient (1) and calculated using the formulas (2) for the case of harmonic heaving and pitching wing oscillations. The wing submergence into water during experiments was equal $\bar{H} = 0.65$ (wing chords).

Fig. 3.3 illustrates the results of the thrust coefficient measuring and calculation using formulas to pure heaving wing oscillations.

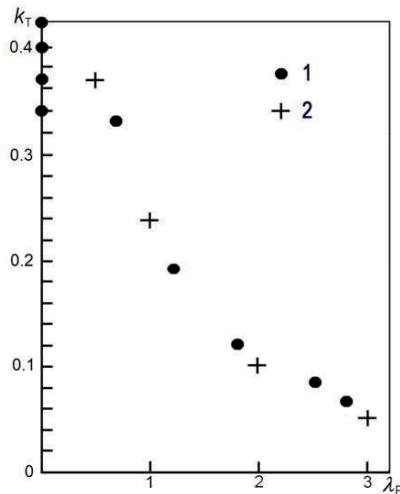


Fig. 3.3 The thrust coefficient measured (1) and calculated using formulas (2) to pure heaving wing oscillations

Some interesting experimental results were obtained in the work (Grebeschov, Sagoyan, 1976) to pure pitching wing oscillations.

In the work (Grebeschov, Sagoyan, 1976) the pitching amplitudes were equal 10^0 , 15^0 and 20^0 . Such angles are not to be supposed small. In this case the wing wake is not to be supposed plane.

The wake nonlinearity can be accounted by the next method. As the wing pitch-axes is situated at $\frac{1}{4}$ chord from the wing leading edge we can write the oscillation law of the wing trailing edge

$$y = 0.75b \sin \theta, \quad (3.3.12)$$

where

$$\theta = \beta \sin \omega t. \quad (3.3.13)$$

The velocity of the transverse moving of the wing trailing edge will look like

$$\dot{y} = 0.75b\dot{\theta} \cos \theta = 0.75b\omega\beta \cos(\beta \sin \omega t) \cos \omega t. \quad (3.3.14)$$

Suppose that

$$\cos(\beta \sin \omega t) \approx 1. \quad (3.3.15)$$

Next

$$\dot{y} = 0.75b\dot{\beta} \cos \theta = 0.75b\omega\beta \cos \omega t. \quad (3.3.16)$$

Maximum speed of the flow around the wing trailing edge can be written down

$$U = U_0 \sqrt{1 + (Sh_0)^2 (0.75\beta)^2}. \quad (3.3.17)$$

Then Strouhal number to obtain the aerodynamic (rotary) derivatives using tables 1-4 can be written down

$$Sh = \frac{Sh_0}{\sqrt{1 + (Sh_0)^2 (0.75\beta)^2}}. \quad (3.3.18)$$

Fig.3.4 shows measured thrust coefficients and calculations using design formulas to pure pitching oscillations

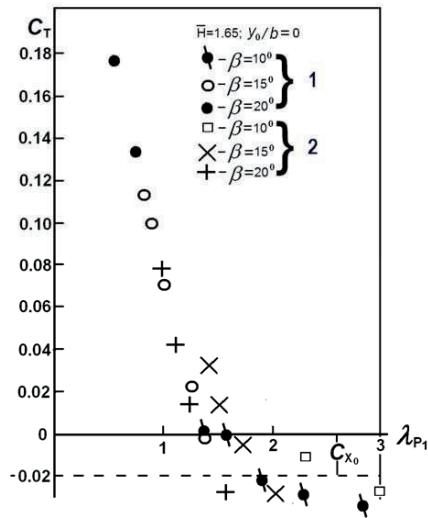


Fig.3.4 The experimental thrust coefficients (1) and calculated using design formulas (2) to pure pitching oscillations

In the works (Zaitsev, Fedotov, 1986; Fedotov, 1987; Tchekhovtsov, 1995, 1999) the mathematical model was devised. This model describes rigid wing oscillations when linear moving and angle of attack are varied harmonically.

The numerical method is used for hydrodynamic forces and efficiency estimation of the infinite wing (Tchekhovtsov, 1999) and of the finite aspect ratio complex-shaped wing (Zaitsev, Fedotov, 1986).

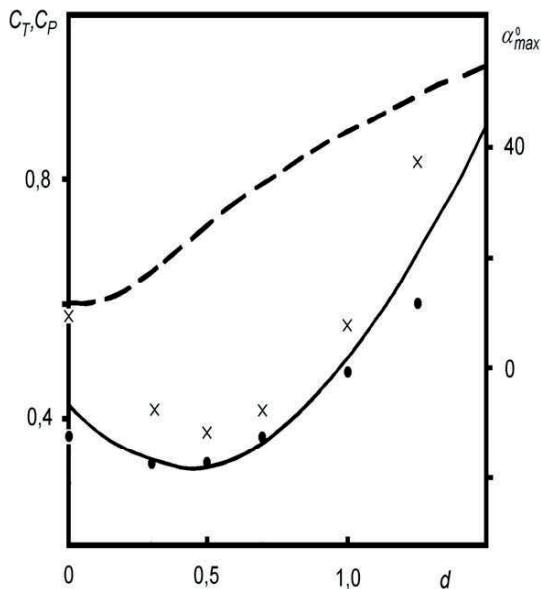


Fig.3.5 Comparison of the trust coefficients of the infinite wing obtained using numerical method (Tchekhovtsov, 1999) (line) and calculated using design formulas (black dots). Fig.3.5 shows also the power coefficients (crosses) obtained using design formulas too and

instantaneous angle of attack using data of the work (Tchekhovtsov, 1999) (dotting). Axis of abscissa shows the distance between the pitch-axes and the wing leading edge.

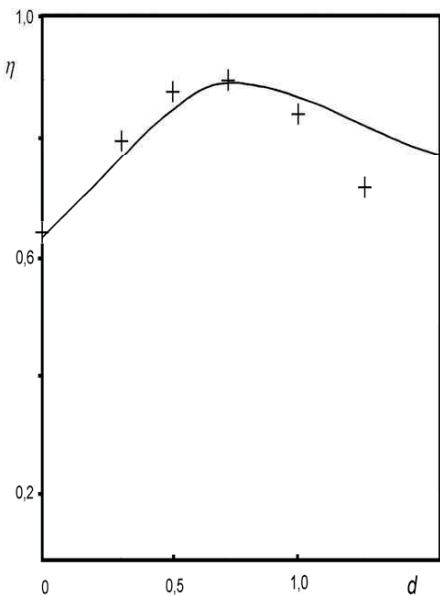


Fig.3.6 Comparison of the efficiency coefficients of the infinite wing obtained using numerical method (Tchekhovtsov, 1999) (line) and calculated using design formulas (crosses).

We can conclude that accord between the results obtained by the numerical methods and calculated using design formulas is satisfactory. It is true when the wing pitch-axes is placed within the wing chord. The theoretical results have failed to agree with calculated using design formulas when pitch-axes is placed outside the chord ($d \geq 1$). The reason is that instantaneous angle of attack is very large. (Fig. 3.5).

3.4. A Comparison with English experimental and theoretical works (rigid wing).

The results obtained by using the inviscid numerical methods are presented in the works (Pedro at al.,2003, Zhou and C. Shu, 2011). The thrust coefficients were obtained as a function of the phase angle between the heaving and pitching oscillations of the rigid infinite wing. The kinematic parameters (in the our designations) of the wing are: $Sh_0 = 1.57$, $\theta_0 = 30^0$, $\frac{y_0}{b} = 0.5$, $\lambda_p = 1.27$, $X_b = -1/4$.

Fig.3.7 shows the results of the cited works and our results obtained by using the design formulas. The thrust coefficients were obtained as a function of the phase angle between the heaving and pitching oscillations of the rigid infinite wing. The aerodynamic (rotary)

derivatives were obtained using table 1 are: $C_{yc}^{\alpha} = 3,6216$, $C_{yc}^{\dot{\alpha}} = 0.8764$, $C_{yc}^{\omega_z} = 0.9052$, $C_{yc}^{\dot{\omega}_z} = -0.1737$.

Fig. 3.8 shows the numerical estimation results of the thrust coefficients from the work (Pedro at al., 2003) and our results obtained by using the design formulas to pure pitching oscillations versus the Strouhal number. The aerodynamic (rotary) derivatives obtained using table 1 for Strouhal number equal 10 are: $C_{yc}^{\alpha} = 3.157$, $C_{yc}^{\dot{\alpha}} = 1.555$, $C_{yc}^{\omega_z} = 0.789$, $C_{yc}^{\dot{\omega}_z} = -0.0038$. For Strouhal numbers equal 20, 30 and 40 derivatives coefficients are of the same value and equal $C_{yc}^{\alpha} = 3.142$, $C_{yc}^{\dot{\alpha}} = 1.571$, $C_{yc}^{\omega_z} = 0.786$, $C_{yc}^{\dot{\omega}_z} = -0.0001$. The pitching amplitude $\theta_0 = 5^0$.

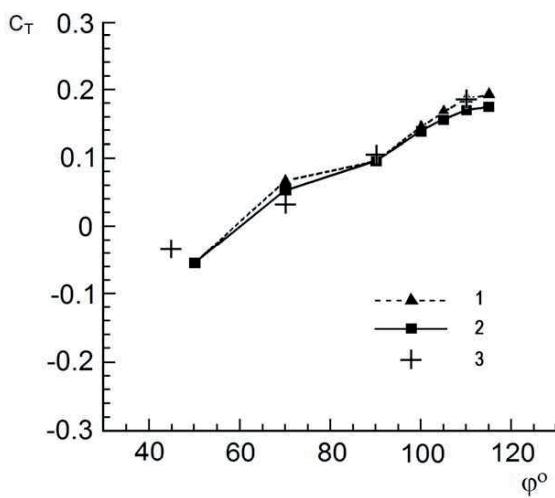


Fig.3.7. Comparison of the results presented in the works (Pedro et al., 2003, Zhou and C. Shu, 2011) (1 and 2) and our results obtained using the design formulas (3) versus phase between heaving and pitching oscillations.

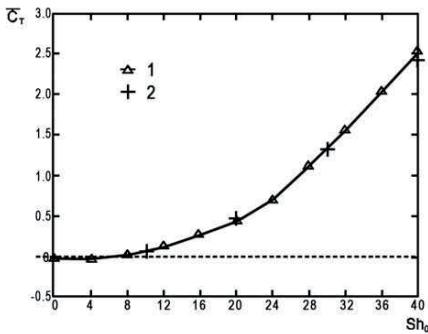


Fig. 3.8. Comparison of the results presented in the work (Pedro et al., 2003,) (1) and our results obtained by using the design formulas (2) versus Strouhal number to pure pitching oscillations.

Fig. 3.9-3.17 show the comparison of the results obtained by using numerical methods, experimental results and results obtained by using the design formulas. The thrust coefficients, the power coefficients and efficiency of the rigid infinite wing were estimated using linear and nonlinear theory, experiment and calculations using the design formulas. Phase angle between the heaving and pitching oscillations equal 90^0 .

Horizontal axes shows Strouhal numbers $Sh_0 = \frac{\omega b}{U_0}$ (lower scale)

and $St_{TE} = \frac{fA_{TE}}{U_0}$. Here A_{TE} is the double heave amplitude of the wing trailing edge. To estimate this value it is necessary to set up the equation of the wing trailing edge moving. The solution of the equation will give us the oscillation amplitude.

The simulation of the ideal two-dimensional flow around the Joukowski profile was used in the work (Anderson et. al., 1998) to obtain non-linear numerical solution. The author of this work supposed that the wing wake consists of an array of alternating vortices.

In the work (Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005) to investigate the mechanism of thrust generation by flapping wing, the inviscid version of a three-dimensional unsteady compressible Euler/Navier Stokes flow solver is used to simulate the flow field around flapping wing NACA 0012 at low speeds. Sinusoidally plunging or/and pitching oscillations are studied.

In the work (Young J., Lai J.C.S., Kaya M., Tuncer I.H. 2004) a two-dimensional unsteady Navier Stokes solver was used too. In addition, large-amplitude unsteady panel method (UPM) was used.

In the experiment (Anderson et. al., 1998) the wing NACA 0012 was used with chord equal to 10 cm and span 60 cm, fitted with circular end plates of radius $r = 30$ cm to ensure a two-dimensional flow along the major part of the wing and reduce end effects. As a result, the effective aspect ratio is larger than the normal value of 6. The one-third-chord point was used as the pivot point. A single harmonic heave and pitch motion was imposed with various combinations of heave amplitude, pitch amplitude, frequency, and relative phase angle. Most tests were conducted with a phase angle between heave and pitch equal to 90^0 , while a limited investigation was also made for the effect of a phase angle, which was varied between 75^0 and 105^0 . Force and power data are measured at Reynolds number 40000.

We should focus our attention at the two contingencies: first, different simulations give the distinguishable results, second, in specific cases experimental results differ from theoretical. This problems arise from ignoring the friction and form drag.

The results obtained by using design formulas correlate well with the results of the numerical solutions and experimental results.

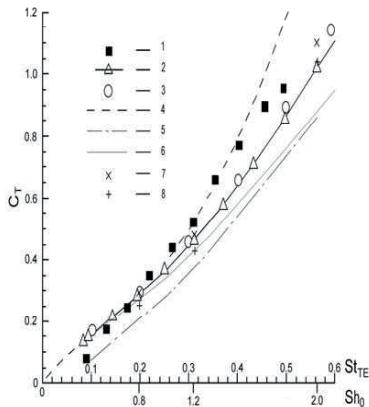


Fig.3.9: The thrust coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3,5,6 – nonlinear numerical solutions (Anderson et. al., 1998, Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005, Young J.,Lai J.C.S., Kaya M., Tuncer I.H. 2004), 4 – linear theory, 7,8 – calculation results using design formulas. Calculations (7) obtained without considering the wing drag, (8) – the results with consideration the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 15^\circ$.

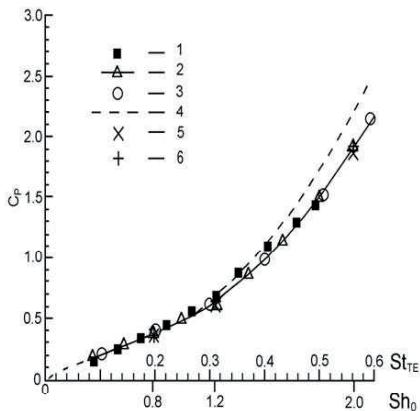


Fig. 3.10: The power coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3 - nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005), 4 - linear theory (Garrick, 1936), 5 - calculation results by using design formulas obtained without considering the wing drag, 6 – the same with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 15^\circ$.

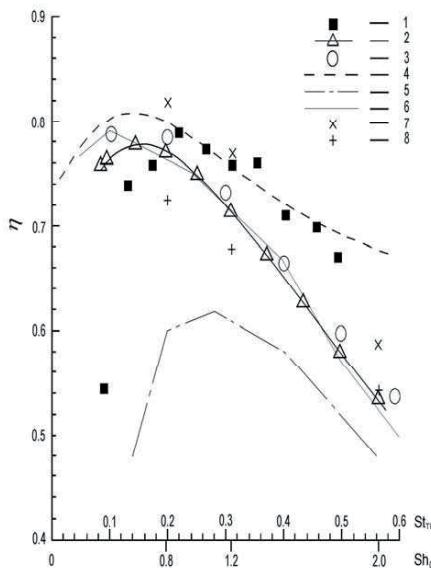


Fig.3.11. The efficiency coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3,5,6- nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005 Young J.,Lai J.C.S., Kaya M., Tuncer I.H. 2004), 4 - linear theory (Garrick, 1936), 7 - calculation results using design formulas obtained without considering the wing drag, 8 – the same with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 15^\circ$.

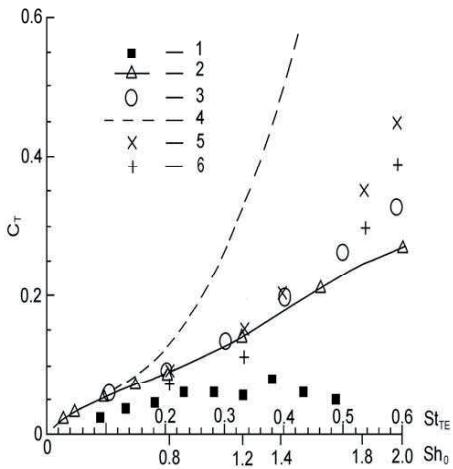


Fig.3.12. The thrust coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3 – nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005, 4 – linear theory, 7,8 – calculation results using design formulas. Calculations (7) obtained without considering the wing drag, (8) – the results with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 5^\circ$.

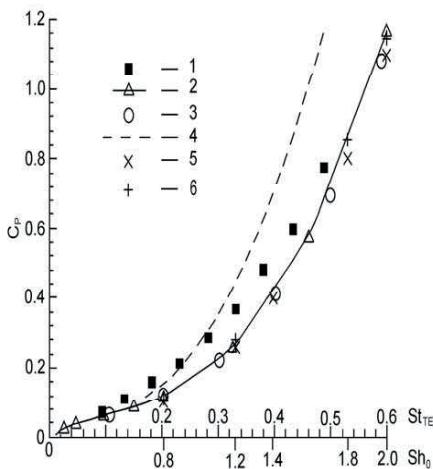


Fig.3.13. The power coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3 - nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005), 4 - linear theory (Garrick, 1936), 5 - calculation results using design formulas obtained without considering the wing drag, 6 – the same with consideration the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 5^\circ$.

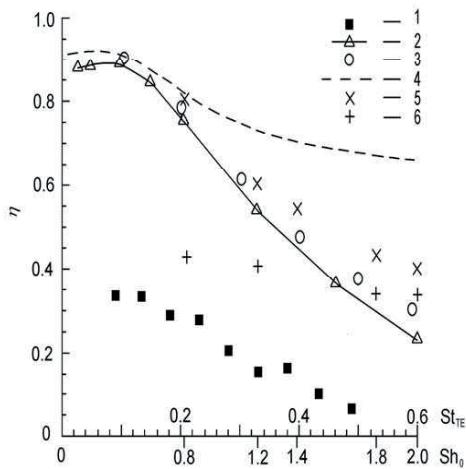


Fig.3.14. The efficiency coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3 - nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005),, 4 - linear theory (Garrick, 1936), 5 - calculation results using design formulas obtained without considering the wing drag, 6 – the same with consideration the wing drag. The wing kinematic parameters are: $y_0/b = 0.75$, $\alpha_0 = 5^\circ$.

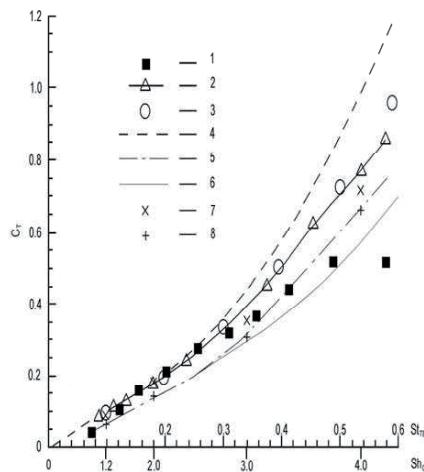


Fig.3.15. The thrust coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3,5,6 – nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005; Young J., Lai J.C.S., Kaya M., Tuncer I.H. 2004), 4 – linear theory (Garrick, 1936), 7,8 – calculation results using design formulas. Calculations (7) obtained without considering the

wing drag, (8) – the results with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.25, \alpha_0 = 15^\circ$.

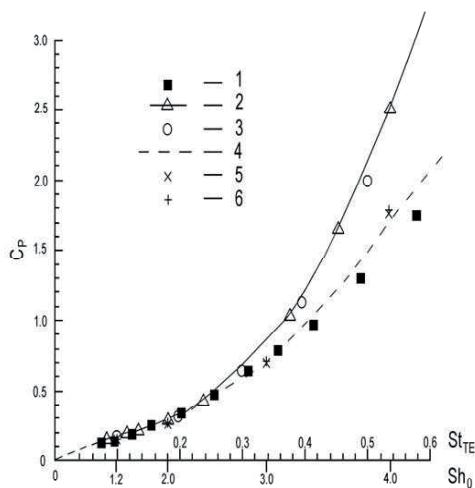


Fig.3.16. The power coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3 - nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005), 4 - linear theory (Garrick, 1936), 5 - calculation results using design formulas obtained without considering the wing

drag, 6 – the same with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.25, \alpha_0 = 15^\circ$.

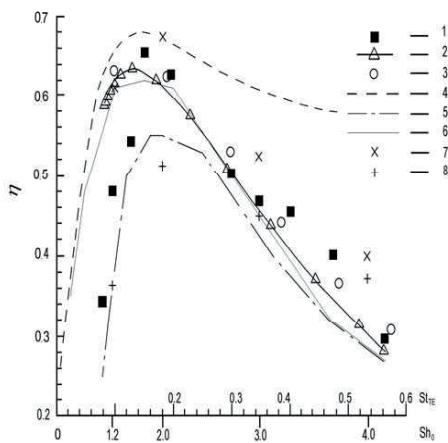


Fig.3.17. The efficiency coefficients versus Strouhal number: 1 – experimental results (Anderson et. al., 1998), 2,3,5,6 - nonlinear numerical solutions (Anderson et. al., 1998; Shuchi Yang, Shijun Luo, Feng Liu, Her-Mann Tsai, 2005; Young J.,Lai J.C.S., Kaya M., Tuncer I.H. 2004), 4 - linear theory (Garrick, 1936), 7 - calculation results using

design formulas obtained without considering of the wing drag, 8 – the same with consideration of the wing drag. The wing kinematic parameters are: $y_0/b = 0.25$, $\alpha_0 = 15^\circ$.

Let us use design formulas to estimate the rectangular rigid wing thrust coefficient. Such wing was used in the experimental research (Prempraneerach et al., 2003). The wing aspect ratio equal 5. The wing kinematic parameters correspond to the third variant (formulas (1.5.6)-(1.5.7)) and characterized the following values: pitch-axes is at a distance of 1/3 chord from the leading edge (in this case relative coordinate is $X_b = -1/6$), interval of the max angle of attack is from 10° to 30° . $\frac{y_0}{b} = 0.75$, $U_0 = 0.4 \text{ m/c}$, $St = 0.15-0.45$ ($Sh_0 = 0.628-1.884$),

Strouhal number in the work (Prempraneerach et al., 2003) will look like:

$$St = \frac{2y_0 f}{U_0}. \quad (3.4.1)$$

This expression and Sh_0 are in the ratio

$$St = \frac{y_0 (Sh_0)}{b\pi}. \quad (3.4.2)$$

The design formulas (2.4.32) – (2.4.39) contain angels α_0, ϑ_0 and thereof sum θ_0 .

In the experiment onely the angle of attack is known. Other two angles can be obtained from the formula (1.8.50), which will look like:
 $\dot{y} = U_0 \operatorname{tg} \theta$.

Let us expand the right-hand of this function in series and limit only by the four terms. Upon integrating the series with respect to the time we obtain following expression which links Sh_0 and θ_0 :

$$\frac{y_0}{b} Sh_0 = \theta_0 \left(1 + 0.222\theta_0^2 + 0.071\theta_0^4 + 0.01\theta_0^6 \right). \quad (3.4.3)$$

The Tabl.3.6 illustrates calculated values of θ_0 when Sh_0 is varied through a range 0.4-2.0 and $\frac{y_0}{b} = 1$ (as an example).

Tabl.3.6 Computed values θ_0 vs. Sh_0 when $\frac{y_0}{b} = 1$.

Sh_0	θ_0 , rad
0.4	0.39
0.7	0.64
1.0	0.85
1.2	0.98
1.6	1.14
2.0	1.27

Tabl.3.7 shows aerodynamic (rotary) derivatives coefficients (converted relatively to the wing centre) for Strouhal numbers used in the experiments (Prempraneerach et al., 2003)

Table.3.7. Aerodynamic (rotary) derivatives coefficients (converted relatively to the wing centre) for the wing aspect-ratio equal 5.

St	Sh_0	Sh	C_{yc}^α	$C_{yc}^{\dot{\alpha}}$	$C_{yc}^{\omega_z}$	$C_{yc}^{\dot{\omega}_z}$	m_{zc}^α	$m_{zc}^{\dot{\alpha}}$	$m_{zc}^{\omega_z}$	$m_{zc}^{\dot{\omega}_z}$
0.15	0.63	0.568	3.348	0.45	0.873	-0.26	0.901	-0.24	-0.13	-0.11

0.25	1.05	0.8	3.202	0.665	0.843	-0.2	0.863	-0.18	-0.14	-0.1
0.35	1.46	0.921	3.082	0.777	0.828	-0.17	0.844	-0.15	-0.14	-0.1
0.45	1.88	0.949	2.985	0.803	0.824	-0.17	0.837	-0.15	-0.14	-0.1

Fig.3.18 and Fig.3.19 show the thrust coefficients and efficiency from the work (Prempraneerach et. al. 2003) and values calculated with the help of the design formulas (third kinematic variant) versus angle of attack

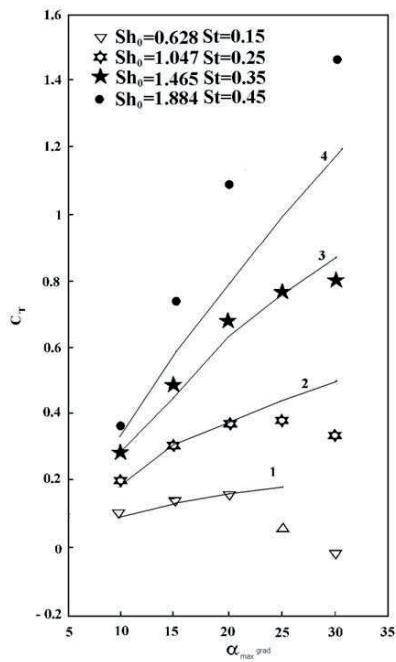


Fig.3.18. Comparison of the experimental (different points) and calculated (solid lines) data.

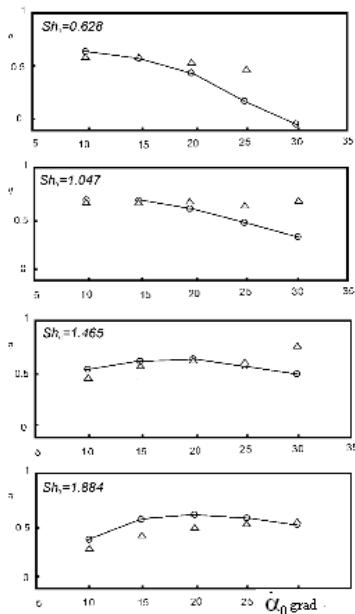


Fig.3.19. Comparison of the experimental (circles) (Prempraneerach et. al., 2003) and calculated (triangular) data.

We notice that calculated values agree closely with experimental when angles of attack and Strouhal number are small.

Fig.3.20 shows the results of the theoretical estimations of the rigid wing thrust coefficients. The wing executes only heaving movement (Platzer et al., 2008). Fig.3.20 illustrates also the results calculated using design formulas.

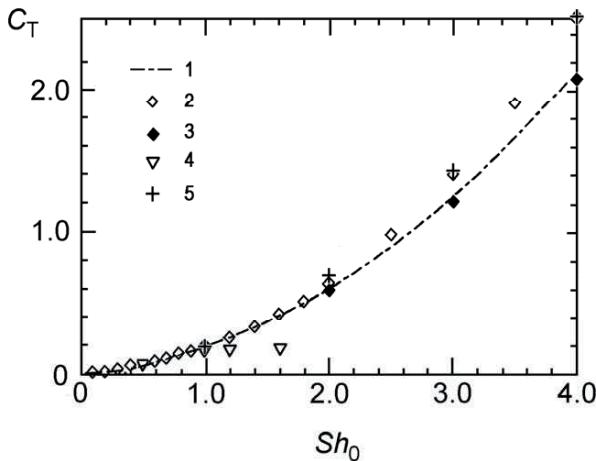


Fig.3.20 Thrust coefficients vs. Strouhal number when $\frac{y_0}{b} = 0.4$. The wing executes only heaving movement: 1 – Garrick linear theory, 2 – panel method (deforming wake), 3 – panel method (flat wake) 4 – Navier Stokes theory, 5 – calculation using design formulas

We notice that calculated values agree closely with results obtained using nonlinear theory for the deformed wake.

Fig.3.21 and Fig.3.22 show calculated results of the rigid wing thrust coefficients to the pure heaving oscillations versus $\frac{y_0}{b}$ and Sh_0 (Tunzer at all., 1998).

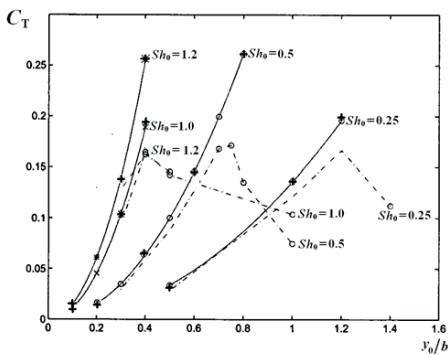


Fig.3.21. The rigid wing thrust coefficient vs. relative amplitude and relatively small Strouhal numbers calculated to the pure heaving oscillations (Tunzer at all., 1998) (different points: nonlinear theory). Figure shows also results calculated using design formulas (crosses)

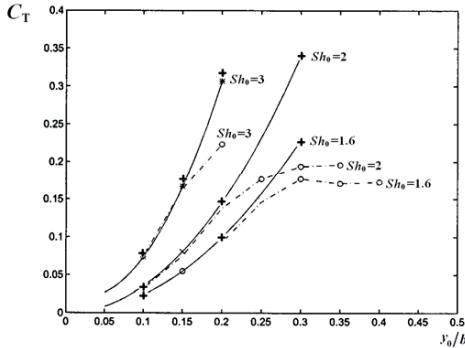


Fig.3.22. The rigid wing thrust coefficient vs. relative amplitude and relatively large Strouhal numbers calculated to the pure heaving oscillations (Tunzer at all., 1998) (different points: nonlinear theory). The Figure also shows results calculated using design formulas (crosses)

Fig.3.23 shows rigid wing efficiency (Tunzer at all., 1998) for different flow over mode.

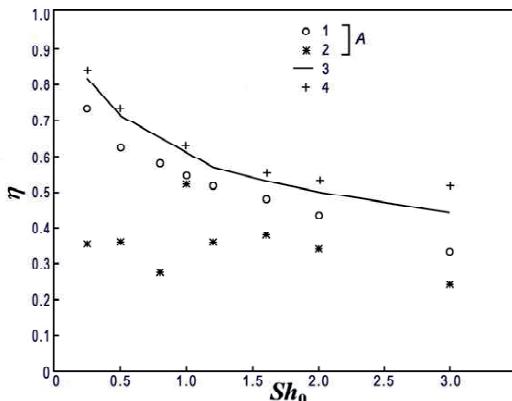


Fig.3.23. The rigid wing efficiency (Tunzer et all., 1998) vs. Strouhal number for different flow over mode: 1 – flow over mode without flow separation (Navier Stoks theory), 2. - flow over mode with flow separation (Navier Stoks theory), 3 – panel method, 4 – calculation using design formulas. The wing executes the pure heaving oscillations.

Analysis results shows (Fig. 3.21 - 3.22) remarkable accord between nonlinear theory and calculation using design formulas when wing flow over mode is without flow separation.

3.5. The estimation of the application limit of the design formulas

Let us estimate the values interval of the kinematic parameters characterized application of the design formulas.

The work (Jones, Platzer, 1997) illustrates the wake classification of the pure heaving wing oscillations versus the Strouhal number and relative amplitude when $Sh_0 y_0 / b = 1$. In the cited work $k = Sh_0$, $h = y_0 / b$.

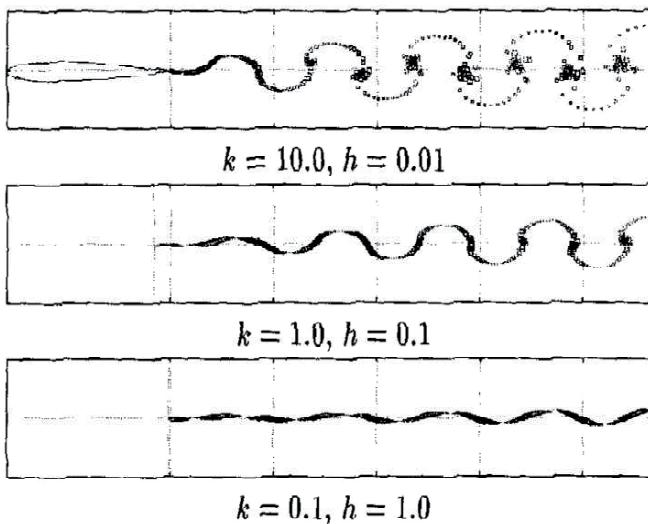


Fig.3.24. A wake classification to pure heaving wing oscillations.

Here $k=Sh_0$, $h=\frac{y_0}{b}$.

When $k = Sh_0 = 0.1$ and $h = \frac{y_0}{b} = 1.0$ the wake is almost linear.

When $k = Sh_0 = 1$ and $h = \frac{y_0}{b} = 0.1$ the wake is non linear and non deformed.

In this case the wake parts move down and shape the sinusoidal wake with constant wave length and amplitude. In this case it is possible to use the design formulas. As this takes place, aerodynamic (rotary) derivatives coefficients can be calculated using Strouhal number as (1.5.7).

Fig.3.25 shows the thrust, power and efficiency coefficients versus Strouhal number to pure heaving oscillations (Jones, Platzer, 1997).

We can see that numerical methods results considerably differ from linear theory results and calculated using the design formulas when $Sh_0 \geq 4-5$ because the wake is very deformed.

Fig.3.26 shows analogical results for combined plunging and pitching (Jones, Platzer, 1997). We can see that efficiency calculated using the design formulas differs from numerical methods results when $Sh_0 \gg 5-10$.

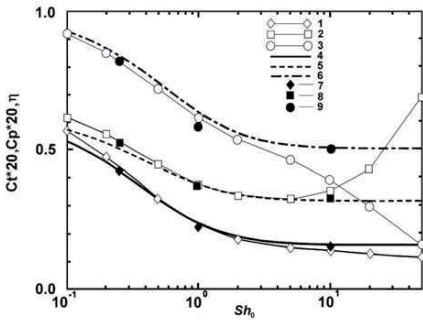


Fig.3.25. The thrust, power and efficiency coefficients versus Strouhal number to pure heaving oscillations $Sh_0 y_0 / b = 0.1$: '1, 2, 3 - from work (Jones, Platzer, 1997), 4, 5, 6 - from work (Garrick, 1936), 7, 8, 9 - calculated results using design formulas.

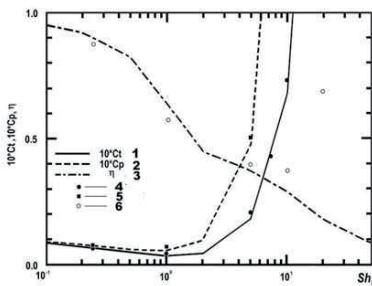


Fig.3.26 The thrust, power and efficiency coefficients versus Strouhal number for combined plunging and pitching when $Sh_0 y_0 / b = 0.1$: 1, 2, 3 - numerical methods results (Jones, Platzer, 1997), 4, 5, 6 - calculated results using design formulas.

We can see that the design formulas allowed investigations to be made of linear and non linear problems when Strouhal numbers are no more than 5.

3.6. A comparison with Russian and English experimental and theoretical papers (deformable wings)

In Russian literature some investigations of the deformable wings are known (Belotsercovski, 1972; Glusckho at.al.;1986; Gordon, Rizchov, 1995; Rizchov, Gordon, 1998; Grundfest, 1995; Khramuschin, 2005).

In English literature such investigations are known much more. The problem of deformable wings in these works is solved by using the numerical methods.

There are some good experimental investigations (Prempraneerach et al., 2003; Heathcote, Gursul, 2007). In some theoretical papers the

authors conclude that deformable wing thrust is smaller than of a rigid wing. In this case in contrast the efficiency is bigger (Katz, Weihs, 1978; Rizchov, Gordon, 1998).

The results of the experimental investigations are not uniquely and dependent on the wing kinematic parameters.

The deformable wing thrust is bigger than thrust of the rigid wing when Strouhal number is small. When Strouhal number is large the deformable wing thrust becomes smaller than one of the rigid wing. The efficiency of the deformable wing is bigger than one of the rigid wing under all values of the Strouhal number.

Fig.3.27 shows measurement results of the deformable wing thrust coefficient versus Strouhal number and wing flexibility to pure heaving oscillations. The different relative thicknesses steel plates were used as flexible (deformable) wings (S. Heathcote and I. Gursul, 2007).

It is seen that the most flexible (deformable) wing ($b/c = 0.00056$) thrust coefficient is significantly higher than one of the stiffest, essentially rigid wing ($b/c = 0.00423$) when Strouhal number is in the range from 0.1 to 0.5. When Strouhal number is bigger than 0.5 the effect is reversed. Here b – the plate thickness.

Fig.3.28 shows the efficiency of the same flexible wings versus Strouhal number and relative wing thickness. It is seen that the

efficiency of the flexible wings is significantly higher than of one of the stiffest, essentially rigid wing (S. Heathcote and I. Gursul, 2007).

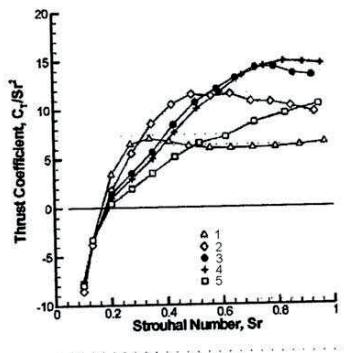


Fig.3.27. Thrust coefficient versus Strouhal number and relative wing thickness: 1 – $b/c = 0.56 \cdot 10^{-3}$, 2 – $b/c = 0.85 \cdot 10^{-3}$, 3 – $b/c = 1.13 \cdot 10^{-3}$, 4 – $b/c = 1.41 \cdot 10^{-3}$, 5 – $b/c = 4.23 \cdot 10^{-3}$. Reynolds number is equal 9000. Strouhal number is $Sr = \frac{2fa_{LE}}{U_0}$ (here and next), a_{LE} – the wing leading edge amplitude.

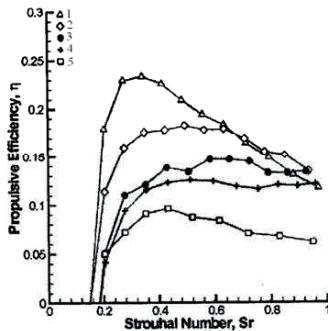


Fig.3.28. Efficiency versus Strouhal number and relative wing thickness: 1 – $b/c = 0.56 \cdot 10^{-3}$, 2 – $b/c = 0.85 \cdot 10^{-3}$, 3 – $b/c = 1.13 \cdot 10^{-3}$, 4 – $b/c = 1.41 \cdot 10^{-3}$, 5 – $b/c = 4.23 \cdot 10^{-3}$. Reynolds number is equal 9000.

Let us estimate of the two section wing thrust coefficient using design formulas versus Strouhal number to pure heaving oscillations. We will use kinematic parameters: $\frac{y_0}{b} = 0.175$, $\delta_0 = -9^\circ = -0.157 \text{ rad}$, Reynolds number equal 9000.

Fig.3.29 illustrates calculation results. It is seen that results in qualitative agreement with data shown on the fig. 3.27.

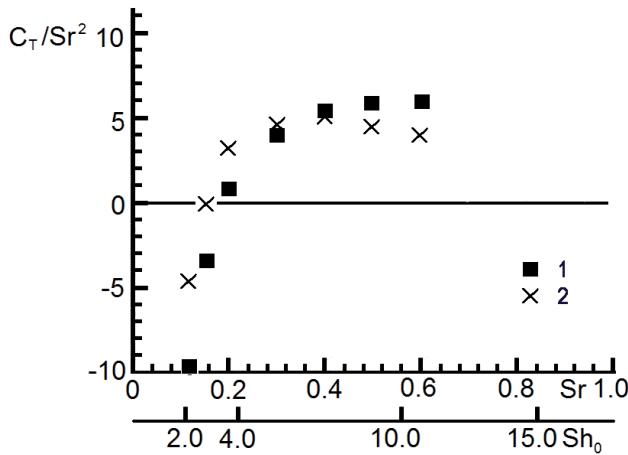


Fig. 3.29. Two section wing thrust coefficient versus Strouhal number. 1 – rigid wing, 2 - two section wing.

The paper (Prempraneerach et al., 2003) described the results of the flexible rubber wing experimental researches. Some rubber types of different flexibility were studied in the experiment. A more appropriate type was identified as A60. In the paper (Prempraneerach et al., 2003) the results are presented in the tabular form and were evolved into graphical form (fig. 3.30 and fig. 3.31).

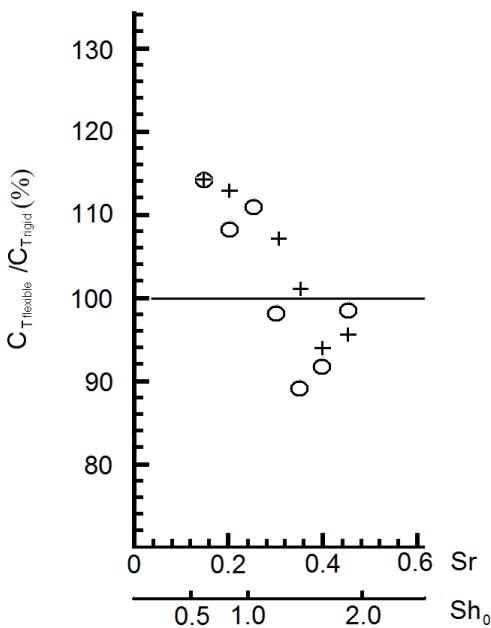


Fig. 3.30. Flexible wing thrust coefficient is normalized to the rigid wing thrust coefficient versus Strouhal number (sort A60): 1 – harmonic heave and pitch, 2 - harmonic heave and angle of attack.

It is seen that a flexible wing thrust coefficient is bigger than rigid wing thrust coefficient when Strouhal number is small. When Strouhal number is bigger than 0.4 the effect is reversed.

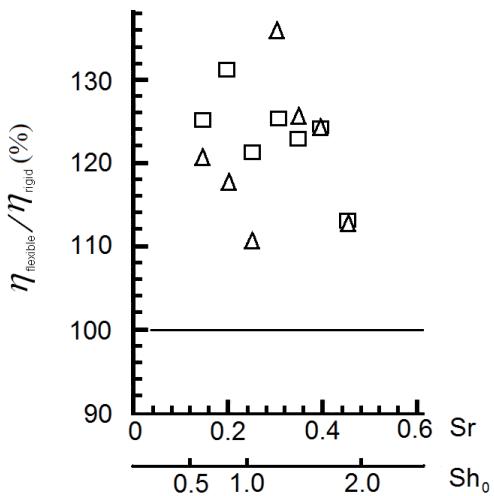


Fig. 3.31. Flexible wing efficiency is normalized to the rigid wing versus Strouhal number (sort *A60*): 1 – harmonic heave and pitch, 2 - harmonic heave and angle of attack.

It is seen that a flexible wing efficiency is higher than efficiency of rigid wing under all values of the Strouhal number.

3.7. Dolphin's fluke propulsion

Most of the wing theory studies have been performed for the special case when pitch-axes placed at a range of 1/3 (or 1/4) chord from the leading-edge and phase shift between heave and pitch near to 90°. Systematic wing propulsion studies to different pitch-axes positions and phase shift are absent. This makes mathematical simulation and dolphin's fluke propulsion correct estimation difficult.

3.7.1. Dolphin's fluke simulation using flat rigid wing.

Let us estimate hydrodynamic forces of the rigid rectangular wing on wide interval pitch-axes and phase shifts under experimental dolphin's fluke kinematic parameters. Solving this problem holds significance in the flapping-wing propulsor's design.

Experimental studies of the dolphin's fluke kinematic are few in number (Romanenko, Pushkov, 1998; Romanenko, 2001; Romanenko, 2002). Nevertheless principal motion parameters were measured: swimming velocity, oscillation amplitude and frequency, shape and geometrical parameters of the body and fluke, harmonic angular (pitch) motion.

Kinematic parameters were measured for two moving conditions: constant velocity swimming and an accelerated swimming. Above mentioned moving condition is most important because dolphin may need to have either maximum thrust (for example, under accelerated swimming) or maximum efficiency (under constant velocity swimming).

The experimental results allow definite conclusions that heaving and pitching oscillations, as a rule, are well described by harmonic functions. The phase angle between transverse oscillation and angular motion is the critical parameter affecting the interaction of leading-edge and trailing-edge vorticity. It is variable and may be different from 90°.

The position of the pitch-axes is variable too. Experimental data (Romanenko, 2001; Romanenko. 2002) show that the pitch-axes position was placed between 0.88 and 1.31 of chord when dolphin swimming velocity was in the range from 2.2 to 4.3 m/c.

Thus estimation of the rigid wing propulsion in the wide range kinematic parameters (pitch-axes position and phase angle between heave and pitch) is very important to study dolphin's fluke hydrodynamic.

Estimations of the wing propulsion are though comparatively uncommon when phase angle is arbitrary. The papers (Pedro at al, 2003; Mittal, 2004; Zhou and Shu 2011) illustrate numerical solution of the flat problem (infinite wing) when phase angle is arbitrary and in one pitch-axis position.

Fig.3.32 shows dolphin's fluke photo. The outline (ABCD) is symbolized as a rigid rectangular wing which model dolphin's fluke.

The wing (ABCD) area and span are equal to dolphin's fluke area and span. (It is necessary to stress that on fig. 3.32 axis OX is directed rightward as distinct from fig. 1.1. The reason is that in the published papers axis OX directed rightward. As of this, in the design formulas it is need to change X_b by $(-X_b)$). Table. 3.8 and Table. 3.9 gave basic dolphin's kinematic parameters when he swims with constant speed and with acceleration.

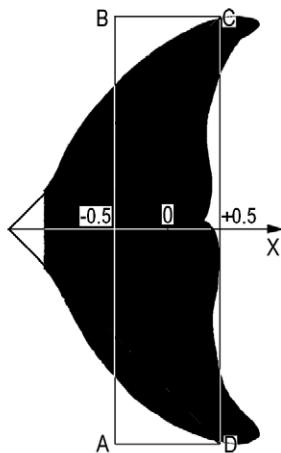


Fig.3.32. Dolphin's fluke photograph. The outline (ABCD) is rigid wing model. The explanations are in the text.

Wing chord (ABCD) equal 0.126m (recall that real dolphins fluke shape is near triangular and its chord equal 0.19m). On this assumption the relative oscillations amplitude of the simulating wing $\frac{y_0}{b} = 2.135$

when swimming speed is constant. And $\frac{y_0}{b} = 3.175$ when dolphin accelerated the rate of his swimming. The virtual mass of the simulating rectangular wing equal $\lambda_{22} = 5\text{kg}$ (Belotserkovsky at. al. 1971), wing area equal $S = 0.063\text{m}^2$.

Table 3.8 A Dolphin and dolphin fluke kinematic parameters when swimming speed is constant.

U_0, MC^{-1}	y_0/L	f, c^{-1}	$\theta_0, \text{рад}$	C
1 4.3	0.12	2.22	0.665	0.02

Table 3.9 A dolphin and dolphin fluke kinematic parameters when dolphin accelerated the rate of his swimming.

$U_0, \text{ мс}^{-1}$	$a, \text{ мс}^{-2}$	y_0/L	$f, \text{ с}^{-1}$	$\beta_0, \text{ рад}$	C
1.5	2.6	0.18	1.46	0.766	0.02

Here U_0 - flow velocity, a – acceleration, y_0/L - the ratio of dolphin fluke oscillation amplitude to the body length, f – oscillation frequency, β_0 - the angle of the wing slope, C – the dolphin fluke drag coefficient.

The thrust, efficiency and inductive reactance estimations of the simulating rectangular wing were made by using design formulas (chapter 1). As this takes place experimental dolphin fluke kinematic parameters were used. Particular emphasis has been placed on the phase angles when efficiency maximum was attained (in the case of the different pitch-axes). The efficiency maximum was found attains when the phase angle is near 90° and swimming speed is constant and accelerated (X_b is in the range from -0.5 to 0.5). This result conforms with popular opinion about profitable wing mower work.

Fig 3.33 gives efficiency values of the rectangular wing versus of the pitch-axes position.

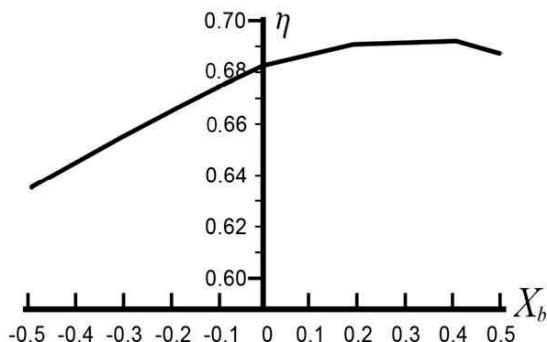


Fig 3.33 Efficiency values of the rectangular wing versus of the pitch-axes position when phase angle equal 90^0 and flow velocity equal 4.3 m s^{-1} .

Figure shows that rectangular wing efficiency has a peak when the pitch-axes are positioned on the near trailing edge of the wing and almost equal 70% (it is noteworthy that efficiency variation is no more than $\pm 4\%$. In this case we can say that efficiency depends only slightly on the pitch-axes position). The calculation results conform with experimental data and allow definite conclusion that dolphins use in this case the most effective kinematic.

Fig 3.34 shows inductive reactance and thrust of the rectangular wing simulating dolphin fluke versus phase angle and pitch-axes position when flow speed is constant.

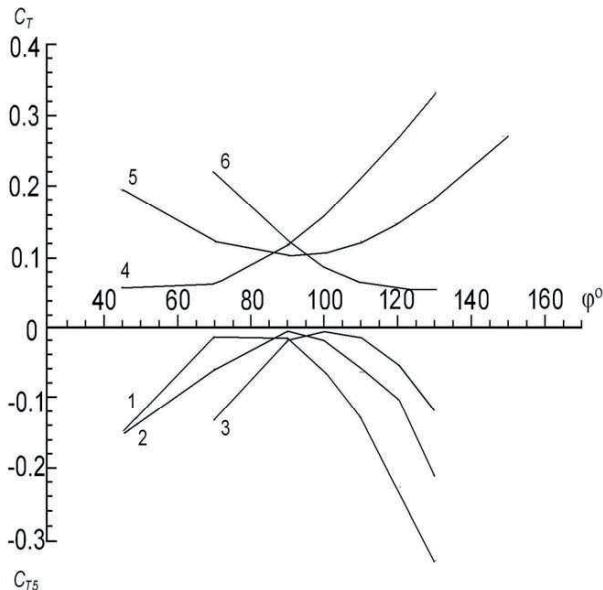


Fig 3.34 Inductive reactance (1-3) and thrust (4-6) of the rectangular wing simulating dolphin fluke versus phase angle and pitch-axes position (1 and 4 when $X_b=-0.5$, 2 and 5 when $X_b=0.3$, 3 and 6 when $X_b=0.5$) when flow speed is constant and equal $4.3mc^{-1}$.

It is seen that inductive reactance maximum positions and thrust character curve depend on a pitch-axes position.

Fig 3.35 shows analogical data of an acceleration regime.

Strictly speaking, these data are unique to events tentative because the design formulas were derived when flow velocity is constant. Nevertheless, curves common character allows making conclusions about the tendency for wing propulsion forces variations when swimming regime varies from constant to accelerated.

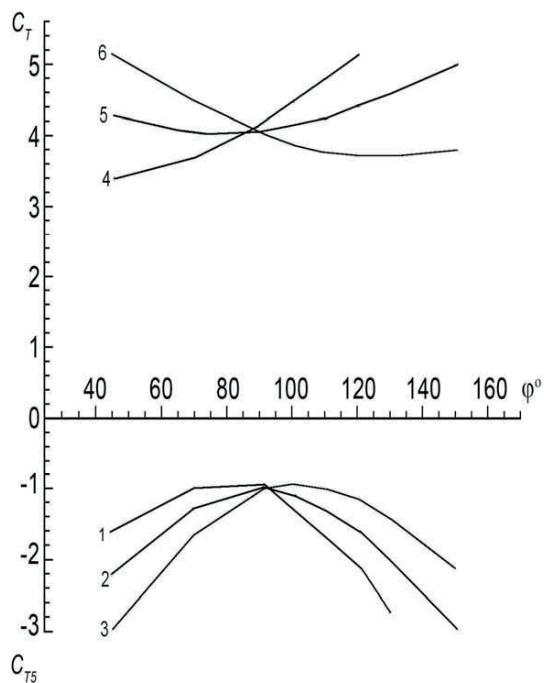


Fig 3.35 Inductive reactance (1-3) and thrust (4-6) of the rectangular wing when regime is accelerated (1 and 4 when $X_b=-0.5$, 2 and 5 when $X_b=0.4$ and 3 and 6 when $Xb = 0.5$).

As it is seen from the figure the curves character in acceleration regime is analogical to those in constant regime. But absolute values differed widely. These values are bigger on the order when acceleration regime exists.

Maximum efficiency in the acceleration regime is smaller than in the constant regime and equal near 60%.

It's necessary to notice that efficiency evaluation of the high-aspect-ratio wing (in this case wing aspect-ratio equal 4 and equal dolphin fluke aspect-ratio) is smaller than one of infinite wing (Romanenko at.al., 2009).

In addition, it is necessary to stress that efficiency evaluation reported in this part of the work no account has been taken of the wing deformation which holds for dolphin fluke. The efficiency evaluation becomes even greater when dolphin fluke deformations are also considered (Heathcote and Gursul, 2007).

3.7.2. Dolphin fluke simulation using deformable wing (the first variant)

In Chapter 2 two variants of the dolphin fluke simulation using infinite deformable wing are described. In the first variant simulating wing is assumed to have deformation character which coincides very closely with the dolphin fluke deformation character (Fig 2.2).

The second variant is more simple: the dolphin fluke simulation is carried out using rigid two-section wing.

The theoretical substantiation of these variants was presented in the book (Belotserkovskii et al. 1971). The design formulas are presented in Chapter 2 to propulsion estimation of the deformable wings.

The estimation was made by using dolphin fluke kinematic parameters (Chapter 3, Table 3.8). The aerodynamic (rotary) derivatives coefficients were obtained by using method described in part 2.1 using figs 2.2-2.6 and is presented in Table 3.10.

Let us consider in detail the process of aerodynamic (rotary) derivatives coefficients calculation of the infinite deformed wing (first variant).

Let us consider Fig 2.3 and obtain values circulation density γ^a in the chord points (from leading edge) spaced 0.05 of chord apart.

Obtained values γ_-^α for $Sh_0 = 0.62$ which is due to dolphin fluke kinematic parameters ($U_0 = 4.3 \text{ m}^{-1}$, $\omega = 13.95\text{c}^{-1}$, $b = 0.19m$) is given in Table 3.10 (the second column). The third column shows values $p_-^\alpha = 2\gamma_-^\alpha$. In the forth – values $f_{\delta+} = \xi^2$, in the fifth – value $\left(\frac{\partial f_\delta}{\partial \xi}\right)_+ = 2\xi$, in the sixth and seventh – product of the values involved into the integrant in formulas (2.1.8) and (2.1.10).

Here we emphasize that some graphs (for example, Fig 2.3) give no way to calculate values γ^α in the wing leading edge (when $\xi = 0$). In this case we can use the known expression for infinite wing derivative C_y^α which will look like

$$C_y^\alpha = \frac{1}{S} \iint_S p^\alpha dS, \quad (3.7.1)$$

Here S is integration area.

This formula can be written down

$$C_y^\alpha \approx \left(2\gamma_-^\alpha\right)_0 (\Delta\xi)_0 + \sum_{\xi_i}^1 2\gamma^\alpha \Delta\xi. \quad (3.7.2)$$

In this formula $(\Delta\xi)_0$ is the wing part near the leading edge. At this place value of $(2\gamma^\alpha)_0$ is not defined on the graph.

Taking into account that $2\gamma^\alpha = p^\alpha$ we can insert p^α into the right-hand side of the equation (3.7.2) from the third column of Table 3.10 (beginning with value ξ when values γ^α can be defined on graph). On the other hand value C_y^α can be obtained in Table 3.1 for the particular value of the Strouhal number. We equate the right-hand side of equation (3.7.2) to value C_y^α which were obtained in Table 3.1 we can obtain $(2\gamma^\alpha)_0$.

Table 3.10. Determination of the aerodynamic (rotary) derivatives coefficients using graphs.

1	2	3	4	5	6	7
ξ	γ_-^α	p_-^α	$f_{\delta+}$	$\left(\frac{\partial f_\delta}{\partial \xi}\right)_+$	$p_-^\alpha f_{\delta+} \Delta \xi$	$p_-^\alpha \left(\frac{\partial f_\delta}{\partial \xi}\right)_+ \Delta \xi$
0	7.3	14.6	0	0	0	0
0.05	5.74	11.1	0.005	0.1	0.0019	0.056
0.1	4.18	7.56	0.01	0.2	0.0038	0.0756
0.15	3.38	6.22	0.0225	0.3	0.007	0.0933
0.2	2.84	5.28	0.04	0.4	0.0106	0.1056

0.25	2.44	4.6	0.0625	0.5	0.0144	0.115
0.3	2.16	4.08	0.09	0.6	0.0184	0.1224
0.35	1.92	3.7	0.1225	0.7	0.0227	0.1295
0.4	1.78	3.33	0.16	0.8	0.0266	0.1332
0.45	1.55	2.95	0.2025	0.9	0.0299	0.1328
0.5	1.4	2.69	0.25	1.0	0.0336	0.1345
0.55	1.29	2.43	0.3025	1.1	0.0368	0.1337
0.6	1.14	2.15	0.36	1.2	0.0387	0.129
0.65	1.01	1.91	0.4225	1.3	0.0403	0.1242
0.7	0.9	1.7	0.49	1.4	0.0417	0.119
0.75	0.8	1.5	0.5625	1.5	0.0422	0.1125
0.8	0.7	1.29	0.64	1.6	0.0413	0.1032
0.85	0.59	1.07	0.7225	1.7	0.0387	0.091
0.9	0.48	0.8	0.81	1.8	0.0324	0.072
0.95	0.32	0.32	0.9025	1.9	0.0144	0.0304
1.0	0	0	1	2	0	0

Graphs (Fig. 2.4 – Fig. 2.6) are completed analogically. Table 3.11 contains parameters which are necessary to calculate the aerodynamic (rotary) derivatives coefficients using graphs.

Table 3.11 Parameters obtained after graphs handling (Fig. 2.3 – Fig. 2.6).

C_{y1+}^{δ}	$C_{y1+}^{\dot{\delta}}$	C_{y2+}^{δ}	$C_{y2+}^{\dot{\delta}}$	m_{z1+}^{δ}	$m_{z1+}^{\dot{\delta}}$	m_{z2+}^{δ}	$m_{z2+}^{\dot{\delta}}$
2.03	0.5442	-0.4952	-0.2162	1.2594	-0.1285	-0.3882	0.0038

We return once more to Fig. 2.3 – Fig. 2.6. As it was specially intimated in the book (Belotserkovsky et al. 1971) the graphs were designed relatively to the wing centre. By this is meant that data in the Table 3.11 have no need of convert

These data we will be used to calculate (using formulas (2.1.16) – (2.1.19)) the aerodynamic (rotary) derivatives coefficients of the deformed infinite wing which simulates the dolphin fluke. The calculated derivatives coefficients are shown in Table 3.12.

Table 3.12. The aerodynamic (rotary) derivatives coefficients of the deformed infinite wing

C_{yc}^δ	$C_{yc}^{\dot{\delta}}$	m_{zc}^δ	$m_{zc}^{\dot{\delta}}$
2.113	0.049	1.2579	-0.5167

3.7.3. Dolphin fluke simulation using deformable wing (the second variant)

Let us consider in more detail the process of calculations of the aerodynamic (rotary) derivatives coefficients of the deformed infinite wing. In this case we need design formulas (2.1.16) – (2.1.27) which contain Theodorsen and Kussner functions. These functions are tabulated in the book (Nekrasov 1947). Let us show it below.

Table 3.13. Real and imaginary parts of the Theodorsen function.

k	$F(k)$	$-G(k)$	k	$F(k)$	$-G(k)$
∞	0.5000	0.0000	0.56	0.5857	0.1428

10	0.5006	0.0124	0.54	0.5895	0.1453
5.0	0.5024	0.0246	0.52	0.5936	0.1480
4.0	0.5037	0.0305	0.50	0.5979	0.1507
3.0	0.5063	0.0400	0.48	0.6026	0.1535
2.5	0.5087	0.0473	0.46	0.6076	0.1563
2.0	0.5130	0.0577	0.44	0.6130	0.1592
1.5	0.5210	0.0736	0.42	0.6187	0.1621
1.2	0.5300	0.0877	0.40	0.6250	0.1650
1.1	0.5342	0.0936	0.38	0.6317	0.1679
1.0	0.5394	0.1003	0.36	0.6390	0.1709
0.98	0.5406	0.1017	0.34	0.6469	0.1738
0.94	0.5431	0.1047	0.32	0.6556	0.1766
0.90	0.5459	0.1078	0.30	0.6650	0.1793
0.86	0.5490	0.1112	0.28	0.6752	0.1819
0.82	0.5523	0.1147	0.26	0.6865	0.1842
0.80	0.5541	0.1165	0.24	0.6989	0.1862

0.78	0.5560	0.1184	0.22	0.7125	0.1877
0.76	0.5581	0.1203	0.20	0.7276	0.1886
0.74	0.5602	0.1223	0.18	0.7442	0.1887
0.72	0.5624	0.1243	0.16	0.7628	0.1875
0.70	0.5648	0.1264	0.14	0.7834	0.1849
0.68	0.5673	0.1286	0.12	0.8063	0.1801
0.66	0.5699	0.1308	0.10	0.8319	0.1723
0.64	0.5727	0.1330	0.08	0.8604	0.1604
0.62	0.5756	0.1354	0.06	0.8902	0.1426
0.60	0.5788	0.1378	0.04	0.9267	0.1160
0.58	0.5822	0.1402	0.02	0.9637	0.0752

In Table 3.13 $k = Sh_0$ in ouer designations.

In Table 3.14 $\cos \varphi = 1 - 2\bar{b}$ here \bar{b} is the ratio of wing moving part to the chord.

Table 3.14. Values of the Kussner function $\Phi(\varphi)$.

$\cos \varphi$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7

0.0	2.5708	3.5708	1.5708	1.3333	1.0000	4.4749	2.1187
0.2	2.3492	2.5853	1.1735	0.7847	1.1758	3.6011	1.3088
0.4	2.0758	1.6983	0.7927	0.3924	1.2831	2.6118	0.6860
0.6	1.7273	0.9345	0.4473	0.1459	1.2800	1.5773	0.2672
0.8	1.2435	0.3339	0.1635	0.0264	1.0800	0.6151	0.0505
0.84	1.1161	0.2394	0.1177	0.0152	0.9984	0.4485	0.0293
0.88	0.9699	0.1558	0.0770	0.0074	0.8929	0.2968	0.0145
0.92	0.7946	0.0850	0.0421	0.0027	0.7525	0.1646	0.0053
0.96	0.5638	0.0301	0.0150	0.0005	0.5488	0.0593	0.0009
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

In Table 3.14 $\cos\varphi=1-2\bar{b}$ here \bar{b} is the ratio of wing moving part to the chord.

Let us calculate the aerodynamic (rotary) derivatives coefficients of the deformed infinite wing using formulas (2.1.16) – (2.1.27). We will use dolphin fluke kinematic parameters ($Sh_0 = 0.62$, $b = 0.19$) for two

cases: wing moving parts equal 0.5 and 0.3 chord ($\cos\varphi = 0$ and 0.4 accordingly). Calculation results are given in Table 3.15.

In this case (formulas (2.1.16) – (2.1.27)) forces and moments centre is in the wing leading edge. Because of this, derivatives values which were calculated using these formulas must be converted to the wing centre.

Table 3.15. Calculation results using formulas (2.1.20) – (2.1.27).

$\cos\varphi$	C_{y1+}^δ	$C_{y1+}^{\dot{\delta}}$	C_{y2+}^δ	$C_{y2+}^{\dot{\delta}}$	m_{z1+}^δ	$m_{z1+}^{\dot{\delta}}$	m_{z2+}^δ	$m_{z2+}^{\dot{\delta}}$
0	3.1112	-0.5	1.08	-0.279	-1.278	-0.435	-0.27	-0.2
0.4	2.5119	-0.64	0.5138	-0.163	-1.269	-0.166	-0.13	-0.05

Derivative values calculated using these data are shown in Table 3.16.

Table 3.16. The aerodynamic (rotary) derivative coefficients (second variant)

$\cos\varphi$	C_{yc}^δ	$C_{yc}^{\dot{\delta}}$	m_{zc}^δ	$m_{zc}^{\dot{\delta}}$
0	3.219	0.581	- 1.203	- 0.705
0.4	2.575	- 0.127	- 1.252	- 0.295

Let us convert derivative coefficients to the wing centre using formulas

$$C_{yc}^{\delta} = C_y^{\delta}, \quad C_{yc}^{\dot{\delta}} = C_y^{\dot{\delta}}, \quad m_{zc}^{\delta} = m_z^{\delta} - \xi_0 C_y^{\delta}, \quad m_{zc}^{\dot{\delta}} = m_z^{\dot{\delta}} - \xi_0 C_y^{\dot{\delta}}.$$

Here $\xi_0 = -0.5$ wing chord.

Conversation results are shown in Table 3.17.

Table 3.17. Derivative values which are converted to the wing centre.

$\cos \varphi$	C_{yc}^{δ}	$C_{yc}^{\dot{\delta}}$	m_{zc}^{δ}	$m_{zc}^{\dot{\delta}}$
0	3.219	0.581	0.407	-0.995
0.4	2.575	-0.127	0.035	-0.358.

Let us estimate value δ_0 at the different deformation variants.

Two-section wing is illustrated by Fig. 2.8 with the proviso that $\cos \varphi = 0$. This illustration also shows angle δ_0 . This angle equal - 0.233 radian (averaged over 5 measurements). Analogically we can make measurement with the proviso that $\cos \varphi = 0.4$. In this case $\delta_0 = -0.268$ radian.

We shall use values of the aerodynamic (rotary) derivative coefficients of the deformed infinite wing from Table 3.12 and Table 3.17 and measured values δ_0 to calculate the wing propulsion efficiency.

We shall use the formulas: the thrust of non deformed wing (1.6.38) – (1.6.54), power - (1.8.5) – (1.8.23), inductive reactance - (1.9.22), (1.9.48) – (1.9.83), (1.9.85), (1.9.107) – (1.9.116). An added terms for deformed wing: thrust - ((2.4.6) и (2.4.7)), power - (2.5.6) – (2.5.9), inductive reactance - (2.6.3) – (2.6.35). Calculation results are shown in Table 3.18.

Table 3.18. Efficiency estimation results of the non deformed and deformed wings

Deformation variants	Non deformed wing efficiency	Deformed wing efficiency
First variant deformation (function)	0.57	0.81
Two-section wing (the second variant with the proviso that $\cos\varphi = 0$).	0.57	0.84
Two-section wing (the second variant with the proviso that $\cos\varphi = 0.4$).	0.57	0.81

3.7.4. Dolphin fluke simulation using deformable wing of finite span

As far as it is known the information about Strouhal number and aspect ratio dependence of the aerodynamic (rotary) derivative coefficients of the wing with rudder is absent in literature. Let us recall that we can consider rudder as the wing deformation.

The book (Belotserkovskii et al. 1975) contains data about Strouhal number dependence of the aerodynamic (rotary) derivative coefficients of the infinite wing with rudder (when $\frac{b_p}{b} = \bar{b}_p = 0.1, 0.2$, и 0.3). Here $\frac{b_p}{b} = \bar{b}_p$ is the rudder relative size. On the other hand the same book contains data about aspect ratio dependence of the aerodynamic (rotary) derivative coefficients of the finite span wing with rudder when $Sh_0 \rightarrow 0$.

It may be suggested that Strouhal number dependence of the aerodynamic (rotary) derivative coefficients of the finite span wing with rudder will be analogical to the same dependence of the infinite wing.

Of course, such suggestion is not sufficiently justified but some probability exists that the wing derivatives estimations with allowance made for this suggestion will be more correct than by dolphin fluke simulation with the infinite wing.

With allowance made for this remark we shall estimate deformed wing aerodynamic (rotary) derivatives coefficients. The wing aspect ratio equal 4 and the relative size of the wing second (movable) part equal $\frac{b_p}{b} = \bar{b}_p = 0.3$.

Let us use Table 9.4 and graphs Fig. 11.16 – Fig. 11.35 from the work (Белоцерковский и др., 1975), and considering that forces centre of the whole wing is placed at the leading edge. As a result we have two-section finite span wing aerodynamic (rotary) derivative coefficients shown in Table 3.19.

Table 3.19 Two-section the finite span wing aerodynamic (rotary) derivative coefficients.

Sh_0	Sh	C_{yc}^δ	$C_{yc}^{\dot{\delta}}$	m_{zc}^δ	$m_{zc}^{\dot{\delta}}$
0.41	0.31	2.028	-0.068	-0.049	-0.041

Let's recall that $Sh_0 = 0.41$ is Strouhal number of the dolphin fluke.

Formula $Sh = \frac{(Sh_0)\lambda_p}{\sqrt{\lambda_p^2 + 1}}$ accounts for the wing wake nonlinearity (chapter 1).

Using these data we can calculate the wing efficiency. The Results are shown in Table 3.20.

Table 3.20 Two-section rectangular wing simulating dolphin fluke.

Aspect ratio $\lambda = 4$, chord $b = 0.126\text{m}$, span $l = 0.5\text{ m}$, area $S = 0.063\text{ m}^2$.	Nondeformed wing efficiency	Deformed wing efficiency
$Sh_0 = 0.41$.	0.62	0.89

Comparison of the results in Table 3.19 and Table 3.20 shows that deformed wing efficiency is considerably higher than those of the non deformed wing. The finite aspect ratio wing efficiency and infinite wing efficiency using different deformation variants are sufficiently closely related (within the scatter in the data about 10%).

Conclusions

A new analytical method was developed to propulsive efficiency estimation of the non deformable and deformable wings at the arbitrary amplitudes of linear and angle oscillations and phase angles.

The design formulas have been derived to propulsive efficiency estimation by using a simple calculator without using special programs and computers. Comparisons have been carried out of the calculations using design formulas, experiments and theoretical studies. Comparison results made it possible to estimate the application limits of the method and make a conclusion that the method can be used in solving of very important problems.

The estimations of the dolphin fluke hydrodynamic forces and power were made using the new analytical method. The dolphin fluke was simulated by rigid finite span wing.

The simulating wing oscillates in non viscous infinite medium. Oscillation amplitudes are sufficiently large. Pitch-axes positions and phase angle between heave and pitch oscillations are arbitrary. It is shown that the efficiency of such wing is practically (in the dispersion limit near $\pm 4\%$) independent of pitch-axes positions in the chord limit and close to 70%. It turns out that efficiency is maximum when the flow velocity is constant or variable and phase angle close to 90°.

The dolphin fluke cinematic may be thought of as close to optimum. The maximum value of the inductive reactance and curve character thrust coefficient depend on pitch axes position.

Analogical estimated data have been arrived at acceleration regime. The common curve character offers a clearer view of the change tendency of the wing propulsive forces when flow velocity changes to

acceleration regime. This tendency appears as the appreciable thrust magnification.

The dolphin fluke mathematical simulation as flexible wing is poorly known. The literature data (Prempraneerach P., et al., 2003) allow account that wing chord-wise flexibility gives the appreciable thrust and power magnification.

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