Traffic Flow Models

Team 60243

Contents

0 Letter to the Governor

Dear Mr. Governor,

Here at the Motoring Club of Mathematicians (MCM), we take pride in our dedication to combining mathematics and motoring in new and interesting ways that can help shape the future. It is with this in mind that we present to you our latest findings regarding self-driving cars and the positive effect they can have on the heavy congestion that we see almost every day in the Greater Seattle area on Interstates 5, 90, 405, and State Route 520. Although self-driving cars can neither shorten congestion nor remove it altogether, they are capable of both breaking up traffic to prevent excessive braking and increasing the bulk flow of cars in traffic. In the midst of this exciting transition, there are several policy initiatives that we wish to present to you in order to maximize the positive impact of these self-driving cars.

Before we get into the details, you must surely be asking: "Why should I trust this very attractive conglomerate of mathematicians?" Firstly, thank you! As for credibility, our rigorous mathematical process consists of two cooperative models on both the small and large scales of this issue. On the small scale, we implemented a simulation of individual cars in order to mimic the behavior of humans and autonomous vehicles, as well as the cooperation (or lack thereof) between them. From this, we found a relationship between traffic density and the flow of traffic. After finding this relationship, we tested it on real-world data in order to ensure that our calibrations were accurate. Using this information, we modeled the flow of traffic as a function of traffic density at several locations along the previously mentioned highways.

For our large-scale model, we simulated the flow of traffic between different segments of roadways. Using real-world data, we determined where traffic is most likely to travel and its effect on large-scale congestion across the Greater Seattle area.

Our modeling led us to several interesting results about the impact of self-driving cars. First, self-driving cars do not become distracted and slow down randomly, nor do they slow down when merging lanes, so they are less likely to cause congestion and can keep the flow of traffic moving smoothly. Second, self-driving cars are capable of determining the optimal speed needed to eliminate stop-and-go waves of traffic. Although this can slow down the cars as a whole, it also prevents excessive acceleration and deceleration in traffic, which causes unnecessary emotional stress on people as well as uses excess fuel. Not much change will be seen when self-driving cars are only 10% integrated into current roads, especially on congested roads, where speeds may actually decrease, but traffic could flow at rates up to almost twice the rate it does today on a road made up of 90% self-driving cars.

However, self-driving cars do have limitations. The results of our model suggest that traffic will slow if the density of cars on the road is too high, no matter what. Depending upon the roadway (not the type of cars driving), the overall flow of vehicles does begin to decrease after the density of the road surpasses approximately 60-70% capacity.

It is with these issues in mind that we at the MCM humbly present a few suggestions in order to facilitate a friendlier road system for both humans and automated drivers. To begin, we believe that dynamic speed limits based on current traffic density should be put in place to determine the optimal speed necessary to prevent stop-and-go traffic. This will create the same effect in human drivers as the ones observed in self-driving cars, as long as those human drivers follow the rules! In addition, in order to cut down on the density of the roads, we believe that self-driving buses and other types of smart transportation should be invested in as well as self-driving cars. The flow of traffic will unavoidably suffer as the density of cars increases, so perhaps the best option to combat traffic is to cut down on the number of vehicles on the road in the first place.

We at the MCM are excited for the future of self-driving cars and we are glad that you are joining us in shepherding in a better tomorrow. Thank you for your time.

- The Motoring Club of Mathematicians (MCM)

1 Introduction

In this paper, we present a critical analysis of how autonomous vehicles could affect traffic using computational and mathematical models based on data for the greater Seattle area in Washington state. We specifically focus on major Interstates 5, 90 (a state route until its intersection with 405), and 405, in addition to State Route 520 in Thurston, Pierce, King, and Snohomish counties. Much like many urban areas in America, these regions are prone to heavy congestion during peak traffic hours because the roads do not have enough capacity to handle such a high vehicle occupancy. Autonomous vehicles, therefore, potentially offer an interesting solution to this problem by being able to increase traffic flow without the need to build new roads or widen existing ones.

Our analysis focuses on how autonomous vehicles could cooperatively interact with each other and human-driven vehicles to optimize traffic conditions. Furthermore, we analyze different percentages of autonomous vehicles (10%, 50%, and 90%) on the roads to determine whether there is a potential equilibrium or tipping point at which autonomous vehicles can optimize traffic without completely eliminating human drivers. This also includes an analysis of whether dedicating entire lanes of highways to autonomous vehicles increases the efficiency of traffic for the aforementioned highways in Washington and, if so, how many autonomous vehicles need to be on the road for this to improve efficiency. Finally, we propose suggested policy changes to existing traffic laws to accommodate autonomous vehicles most optimally based on the results of our model.

2 Assumptions

To keep our models within the realm of possibility in such a short time period, we made the following assumptions to simplify the models' implementations. These are not the only assumptions we made, however, as mentioned in section 6.1.1 where we discuss specific behaviors of autonomous vehicles in our models.

- **Traffic blocks result only from vehicles.** That is to say, there are no extraneous circumstances (such as a family of deer crossing the highway, inclement weather, etc.) that would otherwise cause traffic in the real world.
- **Changing lanes/merging does not significantly affect traffic flow.** Building off the previous assumption, we assume that cars do not get significantly slowed down by merging into lanes in a way that differs from a normal traffic jam.
- **Autonomous vehicles are advanced enough to never require human intervention.** While autonomous vehicles in the real world currently still tend to require human intervention at points in their testing, [15] we assume an idealistic scenario in which system failures are so rare they are negligible.
- **All autonomous vehicles on the road have the same communication platform.** It is reasonable to assume that once self-driving cars become commonplace, there will be government regulation on how they communicate with each other and, therefore, all of them will be able to communicate with each other.
- **Autonomous vehicles are able to identify human drivers.** Following the previous assumption, self-driving cars not only communicate with each other, but also use mobile GPS data/road sensors to track current traffic information about all cars in the system at a given time. This is based off how Google Maps currently delivers real-time traffic information. [10]
- **Human drivers behave relatively uniformly.** While a major component that factors into real traffic is human psychology (e.g., aggressive drivers can potentially cause more accidents by

merging erratically and/or following other cars too closely), we assume that, overall, people have similar driving patterns.

- **Human drivers act to optimize their own interest, while autonomous vehicles act to optimize the whole system.** When people drive, they have a limited view of traffic; they can only see the car(s) in front of them, so they act to maximize their speed based on this simple knowledge. On the other hand, since autonomous vehicles can communicate with each other and track human drivers, they are able to maximize overall traffic efficiency.
- **Aggregate traffic is the same in both directions at a given mile marker on each road.** Each driver is assumed to return from where they came after driving to some destination.
- **The total number of drivers in each mile block stays constant based on traffic density at the time.** This means that when a driver exits a stretch of road, a new driver will simultaneously enter that stretch of road from the other side.
- **The only accidents in our system are rear-end collisions.** Since we are only modeling traffic on highways, there are no accidents that could be caused by driving through a stop sign/red light, backing out in a parking lot without checking mirrors, etc.
- **People are willing to drive at a maximum speed about 10 mph above the speed limit, but autonomous vehicles always follow traffic laws explicitly.** To remain relevant to the example of S
- **All vehicles are approximately 15 feet long.** In particular, each car is the distance that an object traveling at 10 miles per hour goes in 1 second ($\approx$14.6667 feet).

3 Terms and Definitions

All variables and terms that we use in this paper are defined below.

- **Traffic flow** q - in cars per second.
- **Density** ρ - the density of cars on a particular road in cars per road segment.
- **Velocity** v - in distance per time. This takes on discrete values in our models. v_{cur} is a vehicle's current velocity and v_{max} is a vehicle's maximum velocity.
- **Length** L - the length of a road segment in number of car lengths ($\approx$ 15 ft.).
- **Number of lanes** - N

4 Existing Models

Current approaches for modeling traffic fall into two categories: microscopic models and macroscopic models. Microscopic models focus on individual vehicles and their interactions in terms of Newtonian mechanics while macroscopic models focus on overall traffic flow rates and are usually based off partial differential equations. In this section, we discuss existing microscopic and macroscopic traffic flow models.

4.1 Microscopic models

The most common approach to modeling traffic on a microscopic level is the Nagel–Schreckenberg model (1992), which uses cellular automata to represent a single-lane, circular road. [11] A cellular automaton is a collection of cells in a discrete grid that are in one of any given number of states. These cells update their states over time by following given rules and each cell in the entire system is updated simultaneously. [7]

In the Nagel–Schreckenberg model, cells either contain a value between 0-5 (meaning the cell is occupied by a car going at that velocity in cells/sec) or -1 to represent an empty piece of road. At each step in time, all cars simultaneously perform the following four actions in order:

1. Accelerate so that $v_{cur} = min(v_{cur} + 1, v_{max})$
2. Check the distance (d) between themselves and the nearest car in front and update their velocity so that $v_{cur} = min(v_{cur}, d)$.
3. Randomly decelerate at a rate of 1 cell/sec with a given probability p so that $v_{cur} = min(v_{cur} - 1, 0)$.
4. Move into a new position based on v_{cur} (e.g., if $v_{cur} = 5$, the car moves forward 5 cells).

This model does not represent accidents or lane merging/overtaking. It does, however, accurately represent how traffic jams propagate similarly to waves in space-time. [11]

Knospe et al. (including Schreckenberg) later extended the model to include a second lane in 2002 and found that traffic jams in one lane extend to the other lane, but the behavior of traffic jams caused by merging is the same as traffic jams in the original, single-lane model. [12] Lee et al. (once again including Schreckenberg) extended the model as well in 2004 so that in step 2, cars can only decelerate at a rate of 1 cell/sec instead of immediately updating their speed to the distance between themselves and a nearby car in front. Furthermore, they added an element of human psychology to make the model more realistic in which people speed up more rapidly if the car in front of them begins speeding up. [13]

4.2 Macroscopic models

The basic relation that macroscopic traffic flow models follow is

$$q = \rho v \tag{1}$$

[3] [4]. Two common approaches for modeling traffic based on this relation are Greenshield's model (1935) and Greenberg's model (1959). Greenshield's model assumes a linear relationship between traffic speed and density and is written

$$q = v_0(1 - \frac{\rho}{\rho_0}) \tag{2}$$

[5] where v_0 is the speed limit of traffic (because it occurs at free-flow, or when the $\rho = 0$) and ρ_0 is the density of traffic when there's a jam. Greenberg's model, on the other hand, assumes a nonlinear relationship and assumes traffic behaves like a continuous fluid. The model, which we will primarily focus on in this paper, is based off the partial differential equation of motion for a one-dimensional fluid:

$$\frac{dv}{dt} = -\frac{c^2}{\rho}\frac{\partial \rho}{\partial x} \tag{3}$$

where c is a given speed related to the medium and x represents the distance the car has traveled. The relationship between flow and density can be found by solving the prior equation and the conservation of fluid flow equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{4}$$

for v. This results in the relation

$$v = v_0 \ln(\frac{\rho_0}{\rho}) \tag{5}$$

with c replaced by v_0, which allows us to find the relationship between flow and density by plugging Eq. 5 into Eq. 1

$$q = v_0 \rho \ln(\frac{\rho_0}{\rho}) \tag{6}$$

Since a free flow velocity occurs at $\rho = 0$, as $\rho \to 0$, $q \to \infty$ for Greenberg's model. To avoid this, v_0 in Greenberg's model instead represents the velocity when $\frac{\rho_0}{\rho} = e$ (i.e., when $\rho = \frac{\rho_0}{e}$).

5 Analysis of Given Data

For this problem, we used not only existing models to develop our own model, but also a given data set about traffic on Washington highways. The data gives information on the average daily traffic counts for stretches of road between mileposts on the 5, 405, and 90 interstates and the 520 state route throughout the year 2015. It also includes the number of lanes in both the increasing and decreasing direction for each road stretch and information on where intersections between roads occur.

Further using the given data and the fact that 8% of cars are on the road during peak hours, we were able to determine more information about the highways of interest. Namely, we were able to find the average number of cars that drive through each road stretch per peak hour in one direction to be

$$\text{cars per peak hour in one direction} = \frac{\text{average daily traffic count} * 0.08}{2} \tag{7}$$

We also found the length of each stretch of road in miles and used this to find the number of vehicles that could fit on each road segment in a given direction (S) where

$$S = \text{number of lanes in given direction} * \text{number of vehicles that fit in length } L \tag{8}$$

based off our assumption that each vehicle is approximately 15 feet long. These results are plotted below as well. Average values (rounded so they more accurately reflect reality with total spaces and lanes averaged across both directions) and their standard deviations for these results are shown below.

Route	Daily Traffic Count	Lanes	Total Spaces	Cars per Peak Hour
I-5	156281 ± 42975.90	4 ± 0.63	1108 ± 909.83	6294 ± 1699.24
I-90	95556 ± 40697.01	3 ± 0.39	941 ± 631.30	3702 ± 1627.88
I-405	140438 ± 37235.12	3 ± 0.55	645 ± 484.53	5618 ± 1489.40
SR-520	72000 ± 18365.34	2 ± 0.00	641 ± 603.09	2880 ± 734.61

An interesting feature of the data to observe is the high standard deviations, which most likely relates to the nature of the roads themselves. For example, I-5 stretches across the entirety of the west coast of Washington, so there will be parts near more densely populated areas with a higher traffic count and parts in sparsely populated areas with fewer cars driving through. Furthermore, there are many points where these highways intersect each other/other roads and this causes large spikes or drops in the amount of traffic on the highway at a given mile marker, which would further influence the standard deviation.

The high standard deviations do lead to an interesting conclusion, however. Since Washington's four major highways do not have consistent layouts or population demographics, traffic will inevitably build up because people are frequently going from wider to smaller stretches of road and people in densely populated areas occupy more spaces on the road that can cause backups in less dense areas. The above also verifies the conclusion that on our four highways of interest, there are more cars driving on the road during peak hours than can actually fit in the amount of available space, which results in traffic jams.

6 Our Models

In order to analyze both the behavior of individual vehicles and traffic as a whole, we chose to use both a microscopic model, the Nagel–Schreckenberg model, and a macroscopic model, Greenberg's model applied to a Markov chain. Using the microscopic model, we simulated the behavior of traffic in different traffic densities and autonomous vehicle ratios. We then fitted the simulated data to the Greenberg model, which we used in the macroscopic model to determine traffic density given a flow rate.

6.1 Microscopic: Nagel–Schreckenberg model extension

For our microscopic model, we began with an implementation of the Nagel–Schreckenberg (Na–Sch) model in its most basic form, rather than its extensions with added lane merging and human psychology/less rapid deceleration. While this was done mainly in the interest of time, we found this to be a justifiable and accurate implementation for our purposes nonetheless. As described in section 4, adding lane merging to the model did not fundamentally alter the nature of traffic jams aside from creating new pockets of higher density (e.g., when one road intersects another), so (as mentioned in our assumptions) we assume that this behavior will already be modeled by areas with high vehicle density.

Furthermore, we decided that the probability p of randomly decelerating in the original model accounted for all types of human behavior (including lane merging) and also chose to assume that the random deceleration in the Na–Sch model is representative enough of human psychology to not require the additional acceleration as defined by Lee et al. This was done because, as reported by the U.S. Federal Highway Administration [2], there are three main causes of breakdowns in traffic:

- visual effects on drivers (e.g., people slowing to look at accidents, people slowing in areas where barriers are too close to the road, and people slowing to look at sights of interest while driving)
- controlled breakdowns (e.g., lane closures due to construction)
- vehicle merging (e.g., people slowing slightly to check their blind spots while changing lanes and people entering/exiting highways)

We decided that the probability of randomly decelerating models all of the above behaviors. People in the real world have some probability p of decelerating and creating breakdowns in traffic because their eyes are not fully focused on the road and/or because they need to check their blind spots when merging.

Finally, rather than modeling people decelerating at a non-immediate rate like in the Lee et al. extension to the second step of the Na–Sch model, we chose to represent situations where people need to brake many cells at a time as what would result in a rear-end collision in the real world.

6.1.1 Assumptions in incorporating autonomous vehicles

Modeling autonomous vehicles realistically is a difficult problem due to the fact that currently there are just not enough of these cars driving in non-simulated environments to give us a concrete view of their behavior. We, therefore, chose to implement autonomous vehicles in our Na–Sch model similarly to human drivers with a few modifications based on the following assumptions:

- **Autonomous vehicles have no probability of randomly slowing down.** Unlike people, self-driving cars do not slow down to look at a beautiful sunset in the horizon nor do they need to slow down when checking their blind spots because they are always aware of their blind spots.
- **Autonomous vehicles never cause accidents.** Since we only model rear-end collisions and autonomous vehicles use computer vision technology to be aware of their surroundings at all times, we assume that even in a sharp braking scenario, an autonomous vehicle would know exactly how much time it needs to brake without running into another car. This assumption is backed by a study done by Subaru, which found that cars with automatic braking technology were 60% less likely to get into accidents (even with human drivers) [8], and a report by Bloomberg that shows how purely self-driving vehicles currently really only get into accidents when other people crash into them during road tests [14].
- **Autonomous vehicles communicate with each other and track traffic to optimize the flow of traffic.** As mentioned above in our assumptions section, autonomous vehicles calculate the optimal speed to go based on traffic density and all of them will set their velocity to this speed rather than always trying to accelerate to the maximum possible velocity as people do.

6.2 Macroscopic: A Markovian Model

6.2.1 Determining transition probabilities

Once the flow rate as a function of density is determined for each segment of road in the Seattle area, the interaction between these segments still needs to be analyzed. In order to address this problem, we represent each segment of road (as defined in the given data) as a node in a directed graph.

It is possible to determine the total flow of cars from the i^{th} road segment to the $(i+1)^{st}$ segment, q_i from the Greenberg fluid model-

$$q_i = l_i \rho_i v_0 \ln(\frac{\rho_0}{\rho_i}) \tag{9}$$

where l_i is the number of lanes in the segment of road represented by node i. Therefore, the total density change across one time step Δt for node i is:

$$\rho_i^{(t+1)} = \rho_i^{(t)} + \frac{(q_{i-1}^{(t)} - q_i^{(t)})\Delta t}{N_i L_i} \tag{10}$$

This model assumes that the density in each road segment remains constant throughout a given time step, and the transfer of cars between nodes happens instantaneously. If the time step is small enough, this assumption is not a terrible one. However, this model alone fails in two other respects:

- It fails to account for backups on node i from road segments in front of it.
- It is possible for ρ to grow above 1, which only makes sense if cars can drive on top of one another.

Therefore, we must include the density of node $i+1$ in the flow of cars from node i. For the purposes of this model, we assume that node $i+1$ only has enough room to accept $(1-\rho_{i+1}^{(t)}) * 100\%$ of the cars flowing into it. As a result, we have:

$$\rho_i^{(t+1)} = \rho_i^{(t)} + \frac{((1-\rho_i^{(t)})q_{i-1}^{(t)} - (1-\rho_{i+1}^{(t)})q_i^{(t)})\Delta t}{N_i L_i} \tag{11}$$

This system defines a Markov chain where the transition probabilities are:

$$p_{i,i+1} = \frac{q_i^{(t)}\Delta t}{\rho_i^{(t)} N_i L_i} \tag{12}$$

$$p_{i,i} = 1 - p_{i,i+1} \tag{13}$$

Therefore, we must have $q_i^{(t)}\Delta t$ (total flow from node i to node $i+1$) less than $\rho_i^{(t)} N_i L_i$ (the number of cars in node i). This is a result of representing a continuous system discretely, and as $\Delta t \to 0$, this is not a problem.

6.2.2 What about cars entering and leaving the highway?

Two major issues are left in this model:

- cars will pile up at the end of the chain
- cars cannot get on or off the highway

In order to fix the first of these problems, we use an assumption from earlier in this paper: the total number of cars going one direction on a roadway remains constant throughout a simulation. Keeping this in mind, we can wrap our Markov chain around so that the final node feeds into the first node going one direction and vice versa for the other direction. By doing this, we retrieve a number of useful results. Mainly, the chain is now irreducible and aperiodic, so there exists no node that is transient or absorbing.

The second problem is more difficult to fix. If cars cannot get on or off the highway, then we are left with the following property:

$$\forall i \; E(q_i) = C \tag{14}$$

where C is some constant. This is because the overall flow into a state must equal the overall flow out of that state in order to prevent accumulation, and therefore $q_1 = q_2$, $q_2 = q_3$, etc. This is a significant problem because it is known that some parts of the freeway see more traffic flow than others. In order to rectify this issue, we start with the following observation:

$$E(q_i) = \frac{\int_0^{24} q_i(t)dt}{24} = \frac{\text{average daily traffic count}}{24} \tag{15}$$

where q_i is measured in cars per hour here. Total daily traffic count is measurable and we can, therefore, solve for the expected flow rate out of each node. However, since $E(q_i)$ is not the same for each node, there must be some way for cars to join or escape each node. As a result, we must include a source flow term for node i which is proportional to q_i. Let this constant of proportionality be α_i. Using conservation of mass, we have the following:

$$E(q_{i-1}) = E(q_i + \alpha_i q_i) \tag{16}$$

$$E(q_{i-1}) = (1 + \alpha_i)E(q_i) \tag{17}$$

$$\alpha_i = \frac{E(q_{i-1})}{E(q_i)} - 1 \tag{18}$$

So we know how may cars are getting on and off a particular section of highway as a function of flow. Note that this α parameter encompasses *all* traffic leaving and entering the highway, so intersections between highways need not be considered in this model. As as result, we will look at each highway individually in this analysis.

Also, we can use the equations above to retrieve information on the steady state of the model, $q^*_{1:n}$ (if one exists):

$$q^*_{i-1} = (1 + \alpha_i)q^*_i \tag{19}$$

However, we do not know if the chain approaches this steady state since the process is not homogeneous in time. In order to determine if it converges to this equilibrium state, we must run simulations with varying road densities to see if the solution converges (See section 7.2).

7 Results

7.1 Microscopic Model: Sensitivity Analysis

To ensure accurate and stable results from our cellular automaton model, we used Monte Carlo simulations to verify it against real data. Our input parameters are the overall density of traffic, the ratio of autonomous vehicles on the road, the number of lanes, the length of the road, the time to run the model, and a list of all cars' positions and respective speeds. Every time we run a simulation, we sample positions of cars from $Unif(0, L)$, and sample speeds from $Unif(1, v_{max})$. This way, we can be certain that the behavior of the model is not tied to a specific initialization. The road is "circular" meaning that traffic that exits it enters back into it from the other end of the road.

Our first task was to verify that the model accurately reflects real traffic behavior on the road. We first ran the simulation with no autonomous vehicles and varied the density of traffic on the road to obtain the flow of traffic as an output of the simulation. Below we compare our model's behavior to the "fundamental diagrams" [6] (Flow vs. Density) of real traffic behavior tracked on the Autobahn in Germany. These specific data plots are taken from a course on traffic flow theory at Virginia Tech. [16]

In our plots, we present a minimal L^2-norm fit of our simulation to Greenberg's traffic flow formulation while in the plots of the fundamental diagrams, the red line is an averaging of the data points. We can see that the fit of Greenberg's model approximates the general trend of traffic well, but it misses small kinks that form and are captured by the Na–Sch model.

To find the optimal fit, we minimize the L^2 distance between each known data point x_i and the predicted Greenberg's data point y_i by varying the v_0 and ρ_0 in Eq. 6. L^2 distance is defined as

$$\sum_{i=1}^{i=n} \sqrt{(x_i - y_i)^2} \tag{20}$$

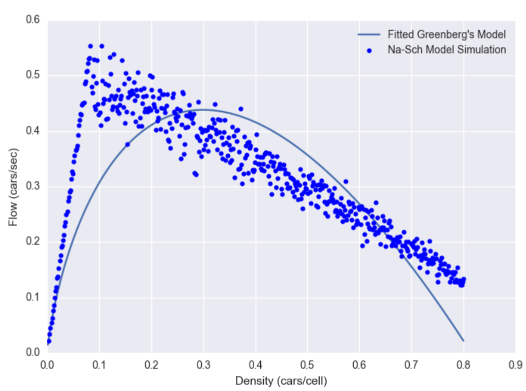

Figure 1: Flow vs. Density, 500 simulations

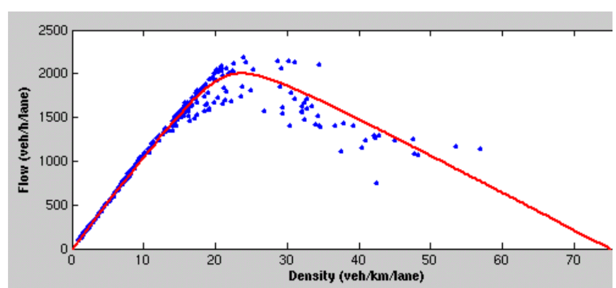

Figure 2: Real life traffic behavior [16]

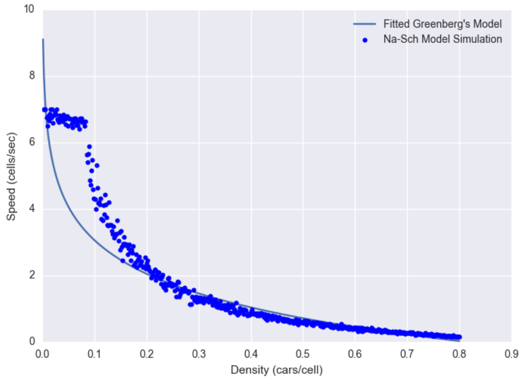

Figure 3: Speed vs. Density, 500 simulations

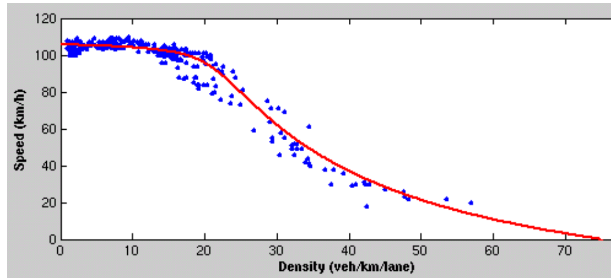

Figure 4: Real life traffic behavior [16]

7.2 Microscopic model: Waves

One key phenomenon of traffic to notice is its resemblance to water waves, if we plot the movement of traffic over time, we can see a resemblance to water waves.

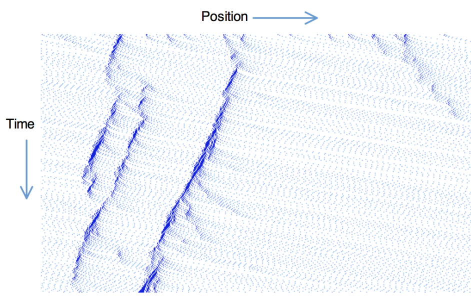

Figure 5: Traffic density=0.1, AV_{ratio}=0.0

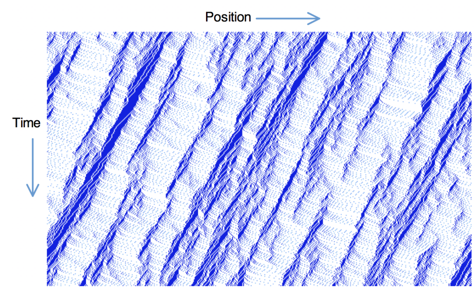

Figure 6: Traffic density=0.3, AV_{ratio}=0.0

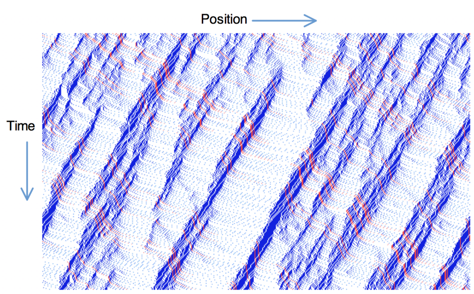

Figure 7: Traffic density=0.3, AV_{ratio}=0.1

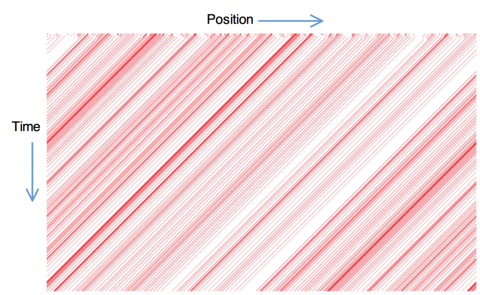

Figure 8: Traffic density=0.3, AV_{ratio}=1.0

Regular cars (blue) slow down with probability p. That creates a chain reaction in which cars behind the car that just slowed down also need to slow down in order to avoid the collision. In such a way, the wave of traffic propagates backwards through the traffic. We can clearly see the blue "waves" on the images. Note that by our assumption autonomous vehicles (red) do not slow down randomly like human-driven vehicles; therefore, setting the ratio of them to be 1.0 just results in them traveling at constant speed.

7.3 Microscopic Model: Stop-and-Go waves

Observe how once the wave has started, it keeps propagating backwards. The downside of this behavior is that cars have to slow down and accelerate back to their desired speed (stop-and-go movement) every time they encounter such a wave. Even though the average velocity of all cars stays about the same, the velocity of each car individually fluctuates a lot, causing unnecessary fuel waste and potentially dangerous situations. Theoretically, if all cars just moved at the expected average speed, the individual fluctuation should decrease.

Our objective was to see if we can mitigate such behavior with the use of self-driving cars. A simple idea of what we are trying to do is to break down the waves, while maintaining approximately the same flow of traffic and reducing the amount of rapid deceleration that human-driven cars have to do as shown in Figure 9.

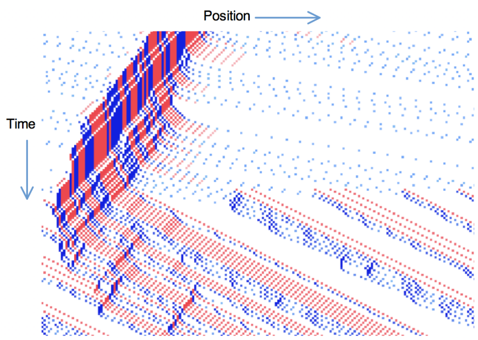

Figure 9: Traffic density=0.21, AV_{ratio}=0.5

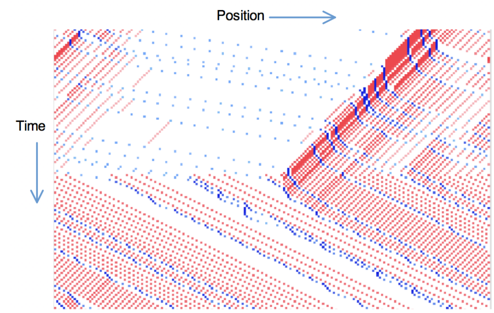

Figure 10: Traffic density=0.21, AV_{ratio}=0.9

To achieve this goal, we adjusted the v_{max} of self-driving cars to the expected speed for the current given density of cars. By limiting the speed of self driving cars, we are also implicitly forcing regular cars to follow that speed limit. Therefore, we encourage all of the vehicles on the road to reach an expected speed for a current density of traffic.

Since our cellular automaton model is not dynamic at the moment, we simply took an average of the current cars' speeds to get the expected speed for a given density. In more dynamic scenarios, we could use Greenberg's model as an estimate or live data from cellphones and other self-driving cars.

First, we verified that by adjusting the flow of self-driving cars to the optimal flow, we did not lose any amount of the flow of traffic. We verify that with a simple Monte Carlo simulation in Figure 12, which shows almost perfect alignment between adjusted flow simulation and non-adjusted simulation.

Next, we computed the average deceleration of cars during the simulation with and without adjusting the speed of self-driving cars. In Figure 11, we see how the adjusted simulation consistently has its mean below that of the non-adjusted simulation. $AV_{ratio} = 0.0$ is obviously an exception, since it has no self-driving cars, so there was nothing to adjust and the two plots overlap. With more optimization and verification, we believe this concept could be employed even further to break down expected traffic jams or ones already formed on the road. With our current optimization, for $AV_{ratio} = 0.5$, the average strong deceleration (more than 2 $\frac{cells}{sec^2}$) was reduced by 12.02% of its original value.

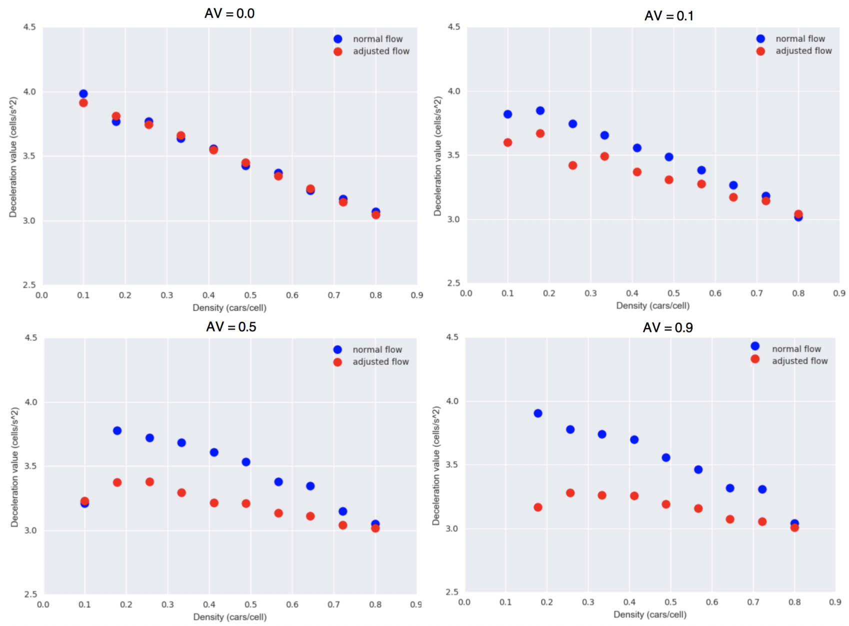

Figure 11: Average deceleration above 2 $\frac{cell}{sec^2}$ vs. Density for different AV_{ratio}

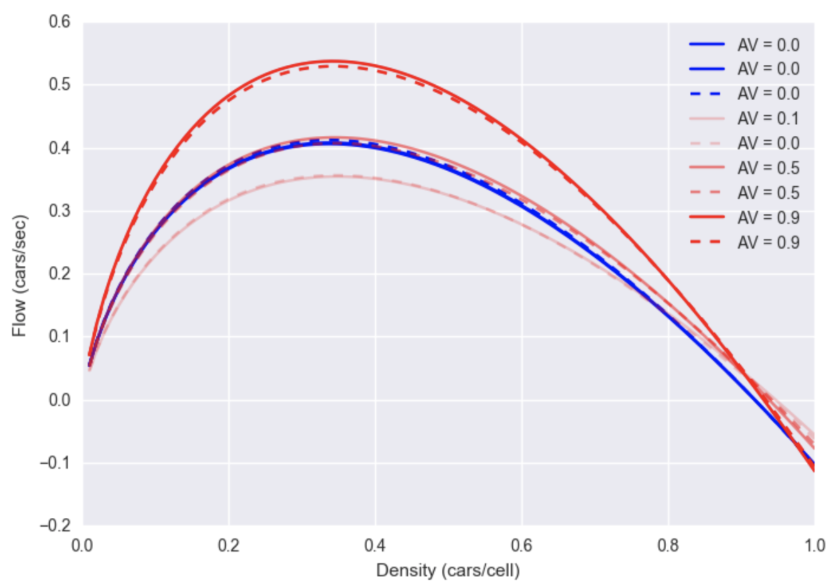

Figure 12: Adjusted (solid line) vs regular (dashed line), fitted Greenberg's model

7.4 Microscopic Model: Dedicated Self-Driving Car Only Lanes

We experimentally found that dedicating lanes to self driving cars is only effective when you have enough autonomous cars to fill an entire lane. Otherwise, the lane is not running at its full capacity. We found that dedicating a lane to only self driving cars allows them to avoid the unpredictable interaction with regular cars. In the images below, we modeled roads after the four Washington highways of interest in this problem: the 4 lane road represents I-5, the 3 lane road represents I-90 and I-405, and the 2 lane road represents SR-520. We chose these representations based on the average number of lanes we calculated for each highway from our given data.

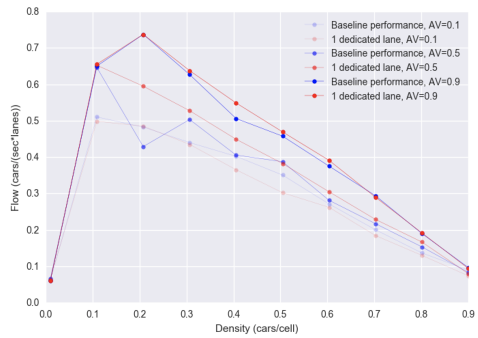

Figure 13: 4 lane road, 1 AV dedicated

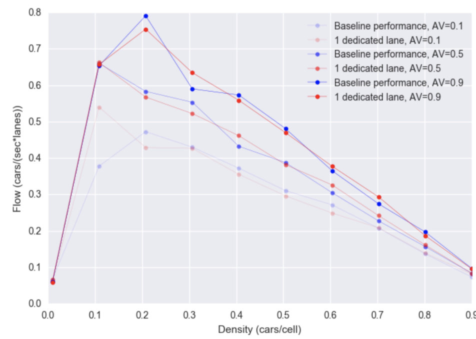

Figure 14: 3 lane road, 1 dedicated AV lane

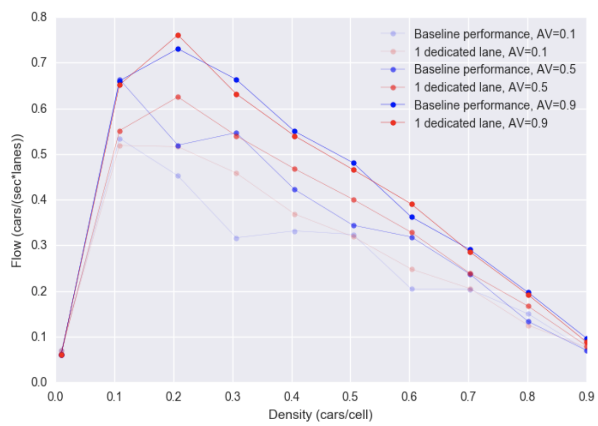

Figure 15: 2 lane road, 1 dedicated AV lane

7.5 Macroscopic Model

To analyze the results of our model, we set the density at every node to be either 0.01, 0.2, 0.5, or 0.8, and then run the simulation. The following group of rather busy plots shows the flow rates of each road section (from the excel file provided in the problem statement) as a function of time when there are no self driving cars, as well as a time average of several parameters as a function of mile marker on Interstate 5.

7.5.1 Interstate 5

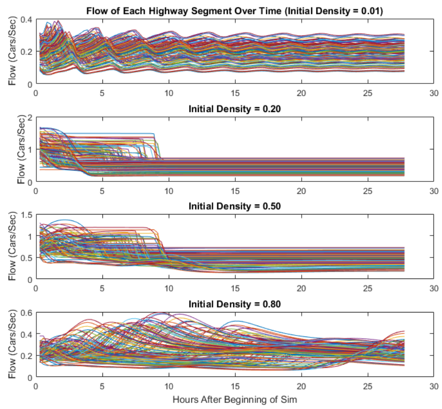

Figure 16: Flow rate on I-5 vs time

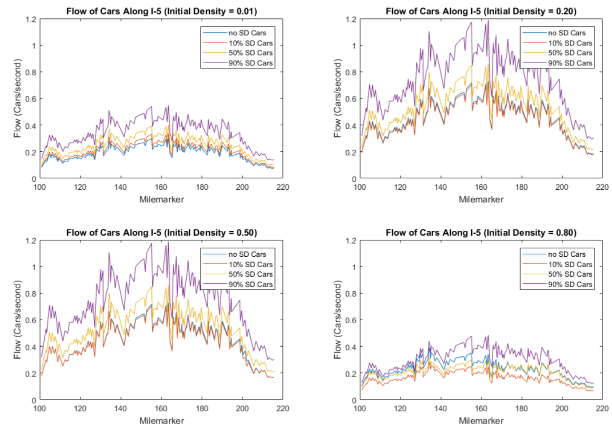

Figure 17: Flow rate on I-5 vs distance

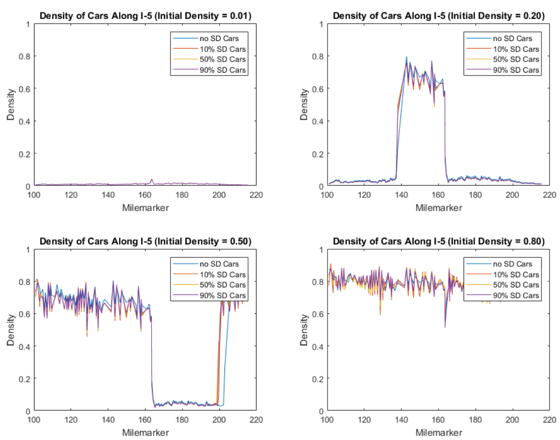

Figure 18: density rate on I-5 vs distance

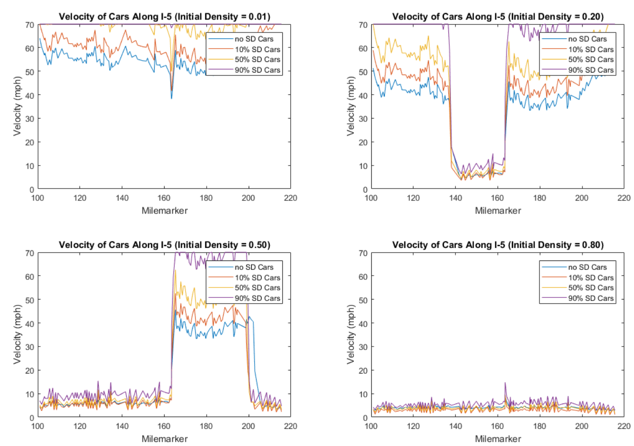

Figure 19: Velocity on I-5 vs distance

For Interstate-5 in particular, it is easy to distinguish between areas on the road that are and are not jammed. We can see that as road density goes up, there are three stages of congestion. First, the overall flow of the road goes up, as extra traffic does not cause much more congestion. Then, bulk flow stays constant, but more sections of road become congested as the extra density of cars must be accounted for. Finally, the entire roadway is congested and bulk flow must decrease as we add more density, as the Greenberg model tells us.

It is in the second stage of this progression that the model achieves equilibrium, as flow remains constant between each of the road segments. If density is too low, then flow oscillates quickly around a steady state, and if density is too high, then flow oscillates slowly around the steady state.

Although self-driving cars do not decrease the size of the traffic jams, they do increase the speed at which those traffic jams move and, therefore, the bulk flow of the roadways as a whole. As a result, it is not wise to drive on a road when the entire roadway is congested, with or without a self-driving car, as the bulk flow will only go down with extra congestion. However, no matter what, driving goes a bit faster with self-driving cars in the mix.

7.5.2 Interstate 90

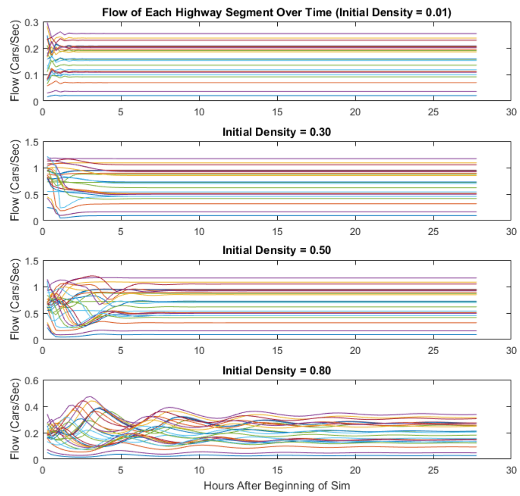

Figure 20: Flow rate on I-90 vs time

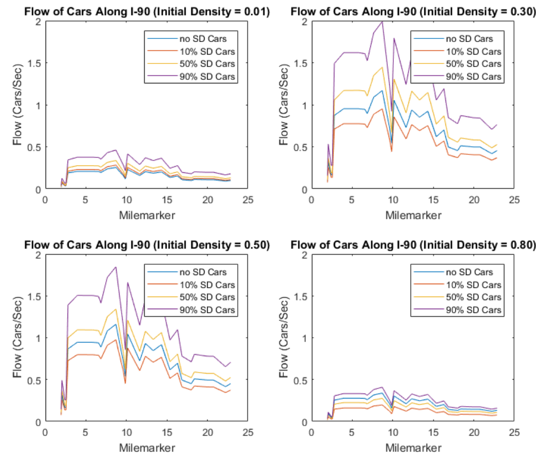

Figure 21: Flow rate on I-90 vs distance

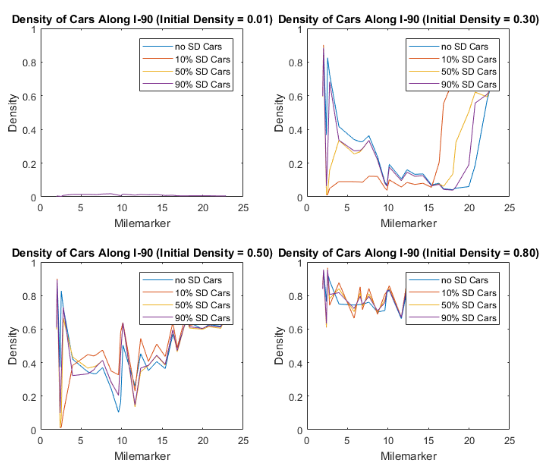

Figure 22: Density on I-90 vs distance

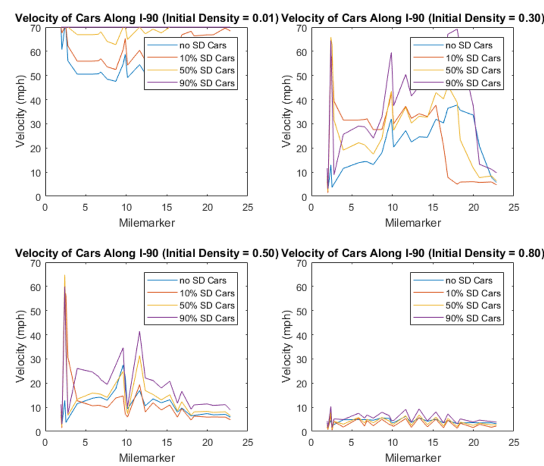

Figure 23: Velocity on I-90 vs distance

The primary distinction between Interstate 90 and Interstate 5 is that the locations of traffic jams are less predictable on I-90 than they are on I-5. This is likely due to both numerical instabilities in the model as well as the fact that this system is quite chaotic.

In addition, it can be seen that the simulation with 10% self-driving cars has a slower bulk velocity on average than the simulation with no self-driving cars when density is above 0.5. This is likely due to the fact that self-driving cars are hindered by and compete with traditional drivers when traffic is high and there are not enough self-driving cars to make an appreciable difference in the aggregate.

Finally, Interstate 90 reaches a steady state much more quickly than interstate 5 at both low and high densities. The frequency of oscillation is much quicker for interstate 90 than it is for interstate 5. This could be due to the fact that it is much shorter in length, so instabilities travel around its imposed periodic domain more quickly.

7.5.3 Interstate 405

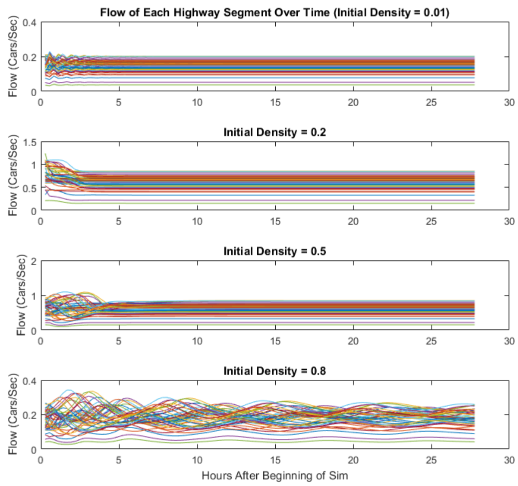

Figure 24: Flow rate on I-405 vs time

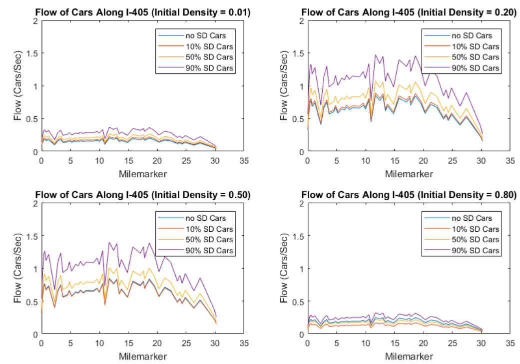

Figure 25: Flow rate on I-405 vs distance

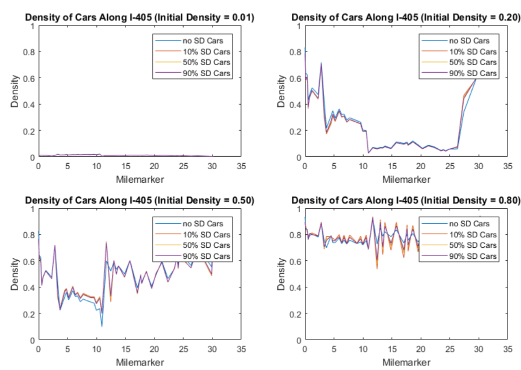

Figure 26: Density on I-405 vs distance

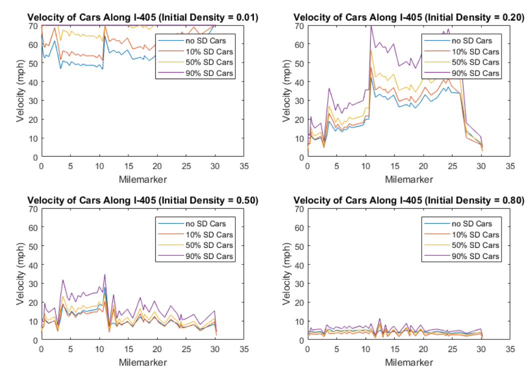

Figure 27: Velocity on I-405 vs distance

Interstate 405 fails to converge to a steady state for an initial density of 0.8. It also yields slower results for the 10% self-driving case than the 0% case in traffic densities higher than 0.5. the 90% case yields significantly faster speeds than any other simulation.

One interesting feature of Interstate 405 is that as traffic density increases, congestion first gets worse close to mile marker 5, but after this congestion gets worse close to mile marker 15 before returning to mile marker 5 again.

7.5.4 State Road 520

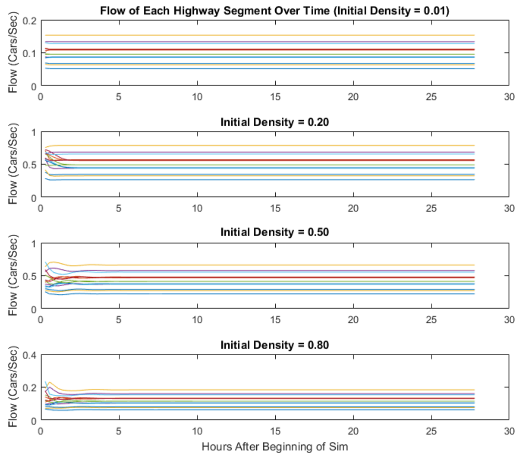

Figure 28: Flow rate on SR-520 vs time

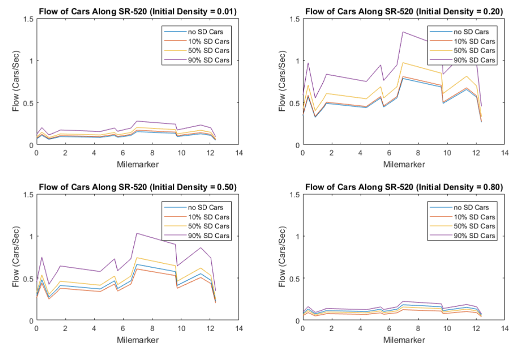

Figure 29: Flow rate on SR-520 vs distance

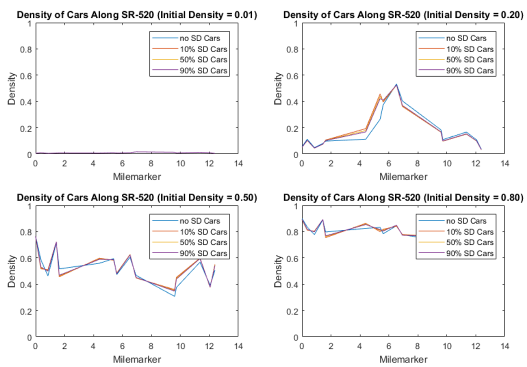

Figure 30: Density on SR-520 vs distance

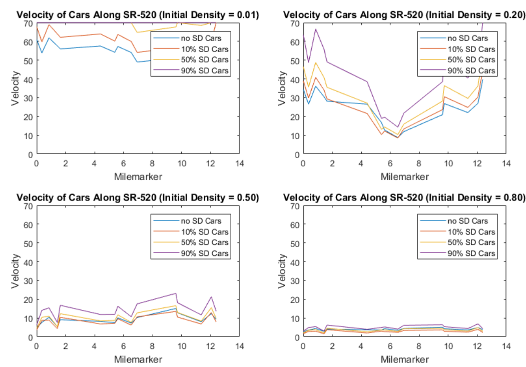

Figure 31: Velocity on SR-520 vs distance

The primary difference between State Road 520 and the Interstates of this model is that State road 520 has a much lower capacity for density on its road. While the interstates are able to get up to a density of 0.5 and maintain peak bulk flow, state road 520 becomes completely congested and losses some of its bulk flow at a density of 0.5. This can be seen both by the plot of bulk flow and by the fact that density spikes around mile marker 6 when initial density equals 0.2, but density is equally distributed when initial density is set at 0.5. This is likely due to the fact that this is a smaller state road with a more uniform average flow of cars.

The self-driving cars, like in previous models, cannot effect the location and size of congestion. However, in this case they do increase bulk flow and make traffic go faster for all initial densities.

7.5.5 Sensitivity Analysis

In order to test the sensitivity of each equilibrium solution to different input parameters, the model was run 10 different times (without autonomous vehicles). The initial input density, $\rho^{(0)}$, was set to be i.i.d $U(0, 0.5)$ to introduce randomness. Below are plots of the final distribution of density for each of the 10 simulations for both I-5 and state road 520.

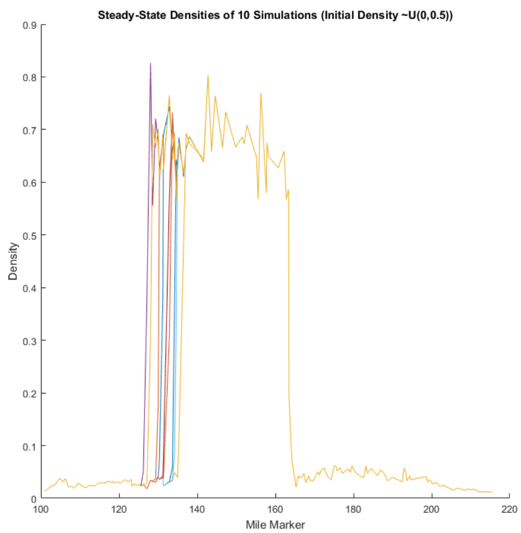

Figure 32: Density on SR-520 vs distance

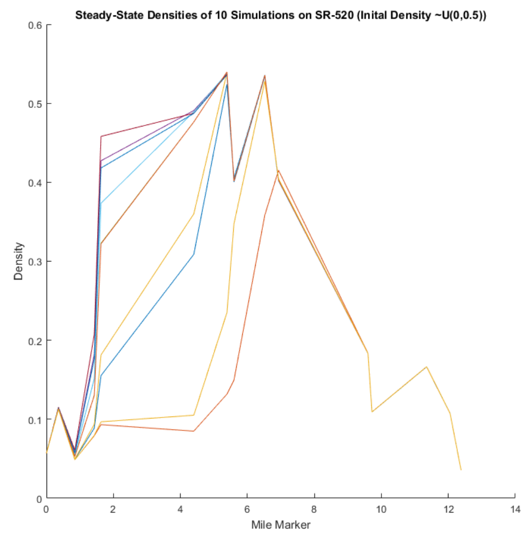

Figure 33: Velocity on SR-520 vs distance

The sensitivity of the model depends upon the highway in question. Initial conditions do not effect the final distribution of density much for I-5, but the final distribution for SR-520 is highly sensitive to the initial conditions. This is likely because the variability of the inputs randomly effect the number of cars in the system, and each of these steady state solutions correspond to different overall levels of congestion.

8 Strengths and weaknesses

8.1 Strengths

- **Our models incorporate both small-scale and large-scale phenomena.** The cellular automata model on the small scale takes into account individual drivers while the Markov model looks at the effects of traffic as a whole backing up and funneling into other sections of the roadway.
- **The cellular automata model is supported by data.** We compared the cellular automata model to real-world data of traffic flow from Virginia Tech and found it to be consistent. In addition, the excess flow rates of the Markov model are determined so that the relative flow rates into and out of sections of road match the data given in the problem statement; therefore, our Markov model is able to accurately reflect traffic conditions on Washington highways.
- **The cellular automata model is flexible.** Not much is known now about how self-driving cars will behave in the future. As this behavior becomes clear, it can easily be implemented as additional rules in the cellular automata model to refine the results we have obtained here.

8.2 Weaknesses

- **The model is discrete in a continuous world.** In the case of the cellular automata model, each car takes up only one space, which is approximately 15 ft. As a result, each car can only move at speeds that are multiples of 15 ft/sec, which is around 10 mph. Therefore, self driving cars can only tune their velocity by large increments, even though the optimal speed for them to travel may be more precise.

- **The Markov model assumes constant mass on the road.** Although this might not be a problem if the Markov model converged to steady state very quickly, it takes several hours in our model for the Markov model to converge to a steady state, at which time the volume of cars on the road certainly will have changed.
- **The cellular automata model is a simplification of behavior.** The model assumes that all humans have similar driving patterns, which is not true. Human psychology is a significant factor of traffic flow in the real world, but is much more difficult to model with simple rules. In addition, because not much is known about how self-driving cars will behave, our model is rather speculative about how these cars will act on the road.

9 Future Work

Another potential aspect of autonomous vehicle behavior that we did not have a chance to model in this report is their ability to redirect themselves using a different route to their chosen destination. This would remove some of the congestion on highways by redistributing it to other roads and would almost certainly further improve overall traffic efficiency. It is also a reasonable assumption that these vehicles will behave this way because they already use GPS data to determine which routes they take, so by knowing in advance that they are coming close to an area of high congestion, they could avoid congestion entirely and would actually ease the congestion by doing so. We believe this could be modeled using fluid mass conservation laws by modeling traffic flow as a fluid flowing in a pipe that forks off into different segments and then comes back together again later.

10 Conclusions

Our most significant finding is that autonomous vehicles improve the flow of traffic by removing stop-and-go waves due to a combination of what we assumed about their behavior: that they do not cause accidents, that they do not randomly slow down, and that they are able to dynamically adjust their speeds. None of these assumptions about their behavior seem unrealistic, however, and some are even backed by current data (specifically that they do not get into accidents often and that humans impede their ability to be as optimal as possible). Because of this, we concluded that there really is no equilibrium or tipping point ratio where autonomous vehicles are able to coexist with human drivers and maintain optimal efficiency; optimal efficiency in traffic flow involves completely removing human behavior from traffic.

Something to note is that, although autonomous vehicles do improve overall traffic flow, eventually they fall victim to too much occupancy on a road at a given time just like normal cars. While this seems like a slightly trivial conclusion (replacing cars with other cars would obviously not decrease the number of cars on the road), it is an important one. Autonomous vehicles should not be used to solve overcrowding on roads, but rather to soothe the nature of traffic because they do not cause jams nor do they get into accidents.

Another improvement autonomous vehicles will have on traffic is one that is very difficult to measure using a mathematical model so we did not focus on it throughout this paper, but is non-negligible nonetheless: the quality of life and productivity improvements society will experience when people do not have to sit in a car doing nothing for so much of their day. In fact, the quality of life improvements might be incredibly significant. In a sample of 900 women from Texas, Princeton economists found that the women rated their morning commute as the activity that makes them the least happy during their every day lives, below cleaning the house and other chores. [17]

10.1 Policy suggestions

Because of our findings above and the conclusions we came to, we finish this report with what we see as feasible policy changes to improve traffic flow in Washington. These policies could be applied to anywhere with heavy traffic as well.

- **Implementing dynamic speed limits.** We found that our autonomous vehicles were able to effectively eliminate stop-and-go traffic by dynamically adjusting their speed as a function of traffic conditions. Since governments already use highway sensors to track traffic, it would be an easy change to add new speed limit signs on Washington highways that display a different speed limit based on traffic conditions. While this does not make use of self-driving cars, it uses the same cooperative abilities we assume they will have to optimize traffic in a way that is easy to implement in the very short term future and does not require building new roads.
- **Dedicating lanes to self-driving cars.** We found that, as the total proportion of autonomous vehicles increases, efficiency improves by giving them their own lanes. This is primarily because, as mentioned before, humans are the main impediment to self-driving car efficiency. This is another easy to implement policy that requires no new road building, but is not necessarily a short term implementation as we really only saw efficiency improvements in this case starting at 50% autonomous vehicles on the road.
- **Higher speed limits for self-driving cars.** Since self-driving cars will always be aware of how long it takes them to stop before hitting a car in front of them and do not cause traffic jams, it makes sense to allow them to have higher maximum speed limits on roads to optimize overall efficiency. This is a moderately short term goal; the total number of self-driving cars on the road does not change the fact that they essentially eliminate jams or accidents.
- **Investing into autonomous buses/public transportation.** At the end of the day, self-driving cars can only do so much for improving congestion on roads. They can stop jams and accidents, but cars in general are not the most efficient use of space, since they can only transport so few people. We think investing into autonomous public transportation would include all the benefits of self-driving technology we found in this report while also using road space more optimally. This could potentially also influence more people to take public transportation, since currently buses tend to have rigid routes and schedules, whereas having a fleet of them that do not need drivers would allow them to be more flexible in their routes. This is a long term goal, but its benefits are easily seen in the image below that was on display as a poster in the Muenster, Germany city planning office. [18]

Amount of space required to transport the same number of passengers by car, bus or bicycle.
(Poster in city of Muenster Planning Office, August 2001)

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