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Introduction

When I first started to think seriously about reform in mathematics education, some 45 years ago, I asked advice from my then Mathematics Dept. Chair at Cornell, Alex Rosenberg. I remember him telling me that he believed that every 20 years or so it is necessary to "stir the pot" so that we are forced to re-think how we teach and re-imagine how children learn.

Well, 40 years ago, I began to work with applications and modeling; 20 years ago, I was immersed in creating materials that exemplified the NCTM standards; and today, we are all dealing with implementation of the Common Core. The pot sure as hell is getting stirred.

But there is so much noise that it's hard to discern what's going on and what it all means. So here's my take.

The NCTM Standards and Reactions

In 1989, I believe that NCTM basically got it right. The standards that they wrote emphasized the need to improve the mathematics education of all students. There was no laundry list of topics to be mastered by a certain age, but rather some key notions of what some have called "mathematical habits of mind." The National Science Foundation (NSF) quickly funded a number of projects designed to produce "standards-based" curricula at the elementary, middle, and high school levels. It is interesting to note that the progeny of those curricula hold roughly 50% of the elementary market, 25% of the middle school market and 5% of the secondary market.

The UMAP Journal 35 (2–3) (2014) 93–96. ©Copyright 2014 by COMAP, Inc. All rights reserved. Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice. Abstracting with credit is permitted, but copyrights for components of this work owned by others than COMAP must be honored. To copy otherwise, to republish, to post on servers, or to redistribute to lists requires prior permission from COMAP.

So what went wrong? Enter the "math wars." Basically, a small—but extremely vocal—group of research mathematicians, right-wing politicians, the wealthy, and religious fundamentalists got scared, for a number of different reasons.

- The mathematicians were afraid that in creating curricula designed to serve all students, we would short-change the best and the brightest, potentially losing future Ph.D. candidates.
- For the political right, this smacked of federal interference in local control of education, i.e., a federally-mandated national curriculum.
- For the wealthy, we were potentially changing the rules in a game that they and their children were already winning—so why take the risk?
- For the fundamentalists, the emphasis on applications and modeling left the mathematics classroom open to influencing the hearts and minds of their children in a way that solving quadratic equations didn't.

And so words like "fuzzy math" were thrown around to make the new curricula look silly and non-rigorous, and members of this alliance turned out at every local and statewide school board meeting—and won.

There were many consequences of this turmoil. NCTM rewrote their standards in 2000 hoping in vain to placate their critics. NSF redefined its mission as one primarily—if not solely—devoted to research, backing away from 50 years of curriculum reform and implementation. But without the NCTM standards to provide direction, we were left without guidance on how to proceed with reform of math education.

NCLB and High-Stakes Testing

Enter No Child Left Behind (NCLB) and the mandated state-wide high-stakes tests from grade 3 onward in mathematics and language arts. Effectively, NCLB created 50 different sets of standards and the tests to assess them. As you can imagine, the tests are widely disparate in quality, and a satisfactory performance in one state may be woefully inadequate in another. But NCLB says nothing about what should be on the tests, how hard the tests should be, or what should be used as cutoff scores. Those are left up to the individual states to determine. And to be fair, it's a mess.

CCSSM: Higher Expectations—and Risks

Enter the Common Core State Standards for Mathematics (CCSSM). The logic of the Common Core rationale is simple and admittedly compelling: We have a standards/testing Tower of Babel; it makes no sense. We need one common set of standards to teach to and to hold students (and teachers) accountable.

The devil, however, is in the details. The current version of the CCSSM is, I believe, destined to disadvantage the already-disadvantaged. For the most part, it does obeisance to those who worry most about where our next generation of STEM researchers and workers will come from. In the name of raising expectations, it runs the risk of being irrelevant to the vast majority of students taking math today. The promise of the NCTM standards was "math for all." *CCSSM threatens to deliver math for the few.*

Ostensibly, the Common Core is a creature of the states; but its standards were certainly created in concert with administration education policy and strongly influenced by large private foundations, as well as publishers and test-makers.

And these standards, too, have their set opponents:

- educators who worry that the topic lists grade by grade are simply designed as an escalator to calculus;
- teachers and teacher unions who see the coming assessment of teacher performance—based in part on student performance—as an attack on tenure and a weapon to be used against teachers;
- the same right-wing politicians who fear federal control of education; and
- those who fear the undue influence of the aforementioned large corporate interests (publishers, test-makers), as well as that of private and public foundations.

All of this is confounded by the fact that, *to the general public, the Common Core is synonymous with the high-stakes tests being designed to assess them*. These tests will roll out this year and next. The results will likely be horrific, with predictions of failure rates in excess of 70%. This fact, the general cost of the assessments, and the factors mentioned above put the broader issue of acceptance of the CCSSM at great risk.

We Are Not in a Race

In some ways, this is a shame. There is promise here. CCSSM needs to be viewed as a living document, one that can be adapted as needed to serve the broad student population. If we can back away from our obsession with high-stakes testing, then we could use common standards to produce common assessments which we could then use to diagnose student needs in order to improve learning—not to punish students, teachers, or schools.

Forty years ago, we feared the Russians, 20 years ago the Japanese, and today the Chinese. Mathematics education is not a horse race. It is not about ensuring that our best are better than their best. It is about ensuring that every student has the opportunity to learn as much mathematics—and as much of how they can use that mathematics—as possible. That is a pot worth stirring.

About the Author

Solomon Garfunkel is the founder and Executive Director of COMAP and Executive Publisher of this *Journal*.

He served on the mathematics faculties of Cornell University and the University of Connecticut at Storrs, but he has dedicated the last 35 years to research and development efforts in mathematics education. He was project director for the Undergraduate Mathematics and Its Applications (UMAP) and the High School Mathematics and Its Applications (HiMAP) Projects funded by NSF, and directed three telecourse projects, including *Against All Odds: Inside Statistics* and *In Simplest Terms: College Algebra*, for the Annenberg/CPB Project. He has been the Executive Director of COMAP, Inc. since its inception in 1980.

Dr. Garfunkel was the project director and host for the video series *For All Practical Purposes: Introduction to Contemporary Mathematics*. He was the Co-Principal Investigator on the ARISE Project, and Co-Principal Investigator of the CourseMap, ResourceMap, and WorkMap projects. In 2003, Dr. Garfunkel was Chair of the National Academy of Sciences and Mathematical Sciences Education Board Committee on the Preparation of High School Teachers.

Editor's Note

About This Issue

This year we had almost 8,000 teams in the MCM and ICM contests combined; the 19 Outstanding papers ran to more than 500 manuscript pages. Editing and publishing all the Outstanding papers, which we once did, is simply not possible any more.

Hence, as in the past few years, we present in the pages of this *Journal* only one Outstanding paper for each of the MCM and ICM problems. The selection of which papers to publish reflected editorial considerations and was done blind to the affiliations of the teams.

All 19 Outstanding papers appear in their original form on the 2014 MCM-ICM CD-ROM, which also has the press releases for the two contests, the results, the problems, and some commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772-6627.

MCM Modeling Forum

Results of the 2014 Mathematical Contest in Modeling

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Introduction

A total of 6,755 teams of undergraduates from hundreds of institutions and departments in 18 countries spent a weekend in February working on applied mathematics problems in the 30th Mathematical Contest in Modeling (MCM)®.

The 2014 MCM began at 8:00 P.M. EST on Thursday, February 6, and ended at 8:00 P.M. EST on Monday, February 10. During that time, teams of up to three undergraduates researched, modeled, and submitted a solution to one of two open-ended modeling problems. Students registered, obtained contest materials, downloaded the problems and data, and entered completion data through COMAP's MCM Website. After a weekend of hard work, solution papers were sent to COMAP on Monday. Two of the top papers appear in this issue of *The UMAP Journal*, together with commentaries.

In addition to this special issue of *The UMAP Journal*, COMAP offers a supplementary *2014 MCM-ICM CD-ROM* containing the press releases for the two contests, the results, the problems, unabridged versions of the Outstanding papers, and judges' commentaries. Information about ordering is at

http://www.comap.com/product/?idx=1418

or at (800) 772–6627.

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Results and winning papers from the first 29 contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2013). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains the 20 problems used in the first 10 years of the contest and an Outstanding paper for each year. That volume and the special MCM issues of the *Journal* for the last few years are available from COMAP. The 1994 volume is also available on COMAP's special *Modeling Resource* CD-ROM. Also available is *The MCM at 21* CD-ROM, which contains the 20 problems from the second 10 years of the contest, an Outstanding paper from each year, and advice from advisors of Outstanding teams. These CD-ROMs can be ordered from COMAP at

`http://www.comap.com/product/cdrom/index.html`.

This year, the two MCM problems represented significant challenges:

- Problem A, "The Keep-Right-Except-To-Pass-Rule," asked teams to build and analyze a mathematical model to analyze the performance of this rule in light and in heavy traffic. Is this rule effective in promoting greater throughput? If not, teams were to suggest and analyze alternatives that might promote greater throughput, safety, and / or other factors that they deemed important.
- Problem B, "College Coaching Legends," asked teams to build a mathematical model to identify the "best all-time college coach," male or female, in any sport, over the past century. Teams were to clearly articulate their metrics for assessment, and to discuss how their model can be applied across both genders and across all sports. Teams also had to prepare a 1–2-page article intended for *Sports Illustrated* explaining their reasoning and results, including a nontechnical explanation of their model that sports fans will understand.

COMAP also sponsors:

- The MCM / ICM Media Contest (see p. 109).
- The Interdisciplinary Contest in Modeling (ICM)®, which runs concurrently with the MCM and next year again will offer a modeling problem involving network science, together with the choice of a second problem involving human-environment interactons. Results of this year's ICM are on the COMAP Website at

  `http://www.comap.com/undergraduate/contests`.

  The contest report, an Outstanding paper, and commentaries appear in this issue.
- The High School Mathematical Contest in Modeling (HiMCM)®, which offers high school students a modeling opportunity similar to the MCM. Further details are at

  `http://www.comap.com/highschool/contests`.

2014 MCM Statistics

- 6,755 teams participated (with 1,028 more in the ICM)
- 12 high school teams (0.2%)
- 391 U.S. teams (6%)
- 6,364 foreign teams (93%), from Canada, China, Finland, Hong Kong, India, Indonesia, Japan, Macao, Mexico, New Zealand, Scotland, Singapore, South Africa, South Korea, Spain, Sweden, and the United Kingdom
- 13 Outstanding Winners (0.2%)
- 12 Finalist Winners (0.2%)
- 656 Meritorious Winners (9%)
- 2,168 Honorable Mentions (31%)
- 3,891 Successful Participants (57%)

Problem A:
The Keep-Right-Except-To-Pass Rule

In countries where driving automobiles on the right is the rule (that is, the U.S.A., China, and most other countries except for Great Britain, Australia, and some former British colonies), multi-lane freeways often employ **a rule that requires drivers to drive in the right-most lane unless they are passing another vehicle, in which case they move one lane to the left, pass, and return to their former travel lane**.

Build and analyze a mathematical model to analyze the performance of this rule in light and heavy traffic. You may wish to examine tradeoffs between traffic flow and safety, the role of under- or over-posted speed limits (that is, speed limits that are too low or too high), and/or other factors that may not be explicitly called out in this problem statement. Is this rule effective in promoting better traffic flow? If not, suggest and analyze alternatives (to include possibly no rule of this kind at all) that might promote greater traffic flow, safety, and/or other factors that you deem important.

In countries where driving automobiles on the left is the norm, argue whether or not your solution can be carried over with a simple change of orientation, or would additional requirements be needed.

Lastly, the rule as stated above relies upon human judgment for compliance. If vehicle transportation on the same roadway was fully under the control of an intelligent system—either part of the road network or imbedded in the design of all vehicles using the roadway—to what extent would this change the results of your earlier analysis?

Problem B: College Coaching Legends

Sports Illustrated, a magazine for sports enthusiasts, is looking for the "best all-time college coach," male or female, for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer.

Does it make a difference which time line horizon that you use in your analysis, that is, does coaching in 1913 differ from coaching in 2013?

Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model's top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1–2-page article for *Sports Illustrated* that explains your results and includes a non-technical explanation of your mathematical model that sports fans will understand.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at either Appalachian State University (Keep Right Problem), Carroll College (Coach Problem), or by a panel in China. At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree, a third judge evaluated the paper.

Additional Regional Judging sites were created at the U.S. Military Academy and at the Naval Postgraduate School, to support the growing number of contest submissions.

Final judging took place at the Naval Postgraduate School, Monterey, CA. The judges classified the papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participation	Total
Keep Right Problem	6	6	453	957	2,453	3,885
Coach Problem	7	6	203	1,211	1,439	2,871
	13	12	656	2,168	3,892	6,756

We list here the 13 teams that the judges designated as Outstanding; the list of all participating schools, advisors, and results is at the COMAP Website.

Outstanding Teams

Institution and Advisor	Team Members

Keep Right Problem

"Keep Right to Keep 'Right'"	
Tsinghua University	Yaofeng Zhong
Beijing, China	Yunyi Zhang
Zhiming Hu	Xiao Zhao
"Rules of the Road"	
Tufts University	Michael Bird
Medford, MA	Kathleen Cachel
Scott MacLachlan	Charlie Colley
"Freeway Traffic Model Based on Cellular Automata and Monte-Carlo Method"	
Shanghai Jiaotong University	Dongyu Jia
Shanghai, China	Zhaoyang Shi
Jinliang Yue	Yanping Xie
"Simulating and Scoring the Performance of Traffic Driving Rules"	
Beijing Normal University	Yihan Sun
Beijing, China	Xiang Xu
Haigang Li	Junwei Zhang
"A New Traffic Rule"	
Nanjing University	Luowei Zho
Nanjing, China	Xiuyu Wang
Meilin Zhu	Wanjia Zhu
"The Keep-Right-Except-to-Pass Rule"	
Zhejiang University	Yuan Gong
Hangzhou, China	Shu Liu
Jianxin Zhu	Yandi Shen

Coach Problem

"Grey Correlation and Fuzzy Models for Best Coach"
Chongqing University
Chongqing, China
Xiaobing Hu

Yue Wang
Bo Hou
Qiang Zhang

"Finding Out the Best All-Time College Coach"
Southwest University for Nationalities
Chengdu, China
Gaoping Li

Yiping Liu
Yongyi Xie
Yao Zhang

"Who Is the Centennial Best Coach?"
Southeast University
Nanjing, China
Zhizhong Sun

Yatao Fu
Yuan Dong
Yuyang Wang

"College Coaches' Mount Rushmore"
Northeastern University
Shenyang, China
Dali Chen

Yantao Shen
Bingzhu Xie
Yingyi Ma

"An Evaluation Model of College Coaches"
University of International Business and Economics
Beijing, China
Shuyu Zhang

Zhuoyi Chen
Mengru Wang
Jie Hang

"A Networks and Machine Learning Approach to Determine the Best College Coaches of the 20th–21st Centuries"
NC School of Science and Mathematics
Durham, NC
Christine Belledin

Christopher Qian Yuan
Tian-Shun Allan Jiang
Zachary T. Polizzi

"Evaluation System for College Coaching Legends"
Huazhong Univ. of Science and Technology
Wuhan, China
Zhibin Han

Feng Xiong
Wenchao Ding
Jingling Li

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, recognized as INFORMS Outstanding teams two teams: the teams from Tsinghua University (Keep Right Problem) and from NC School of Science and Mathematics (Coach Problem) and provided the following recognition:

- a letter of congratulations from the current president of INFORMS to each team member and to the faculty advisor;
- a check in the amount of $300 to each team member;
- a bronze plaque for display at the team's institution, commemorating team members' achievement;
- individual certificates for team members and faculty advisor as a personal commemoration of this achievement; and
- a one-year student membership in INFORMS for each team member, which includes their choice of a professional journal plus the *OR/MS Today* periodical and the INFORMS newsletter.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The SIAM Award teams were from Zhejiang University (Keep Right Problem) and Southwest University for Nationalities (Coach Problem). Each team member was awarded a $300 cash prize. The teams were offered partial expenses to present their results in a special Minisymposium at the SIAM Annual Meeting in Chicago, IL in July, and the team from Southwest University for Nationalities was able to come. Their schools were given framed hand-lettered certificates in gold leaf.

The Mathematical Association of America (MAA) designated one North American team from each problem as an MAA Winner. The MAA Winners were from Tufts University (Keep Right Problem) and the NC School of Science and Mathematics (Coach Problem). With partial travel support from the MAA, the teams presented their solutions at a special session of the MAA Mathfest in Portland, OR in August. Each team member was presented a certificate by an official of the MAA Committee on Undergraduate Student Activities and Chapters.

Ben Fusaro Award

One Meritorious, Finalist, or Outstanding paper is selected for the Ben Fusaro Award, named for the Founding Director of the MCM and awarded for the 11th time this year. It recognizes an especially creative approach; details concerning the award, its judging, and Ben Fusaro are in Vol. 25 (3) (2004): 195–196. The Ben Fusaro Award Winner was the Outstanding team from Tsinghua University (Keep Right Problem). A commentary about it appears in this issue.

Frank Giordano Award

For the third time, the MCM is designating a paper with the Frank Giordano Award. This award goes to a paper that demonstrates a very good example of the modeling process in a problem featuring discrete mathematics—this year, the Coach Problem. Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. The Frank Giordano Award for 2014 went to the Outstanding team from Huazhong University of Science and Technology. A commentary about it appears in this issue.

Judging

Director
William P. Fox, Dept. of Defense Analysis, Naval Postgraduate School,
  Monterey, CA
Associate Director
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy,
  West Point, NY

Keep Right Problem

Head Judge
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy,
  West Point, NY
Associate Judges
William C. Bauldry, Chair-Emeritus, Dept. of Mathematical Sciences,
  Appalachian State University, Boone, NC (Head Triage Judge)
Kelly Black, Mathematics Dept., Clarkson University, Potsdam, NY
Karen Bolinger, Dept. of Mathematics, Clarion University, Clarion, PA
Tim Elkins, Dept. of Systems Engineering, U.S. Military Academy,
  West Point, NY
Thomas Fitzkee, Mathematics Dept., Francis Marion University,
  Florence, SC
Ben Fusaro, Dept. of Mathematics, Florida State University, Tallahassee, FL
  (SIAM Judge)
Jerry Griggs, Mathematics Dept., University of South Carolina,
  Columbia, SC

Marvin Keener, Mathematics Dept., Oklahoma State University, Stillwater, OK
Yongji Tan, Dept. of Mathematics, Fudan University, Shanghai, China
Michael Tortorella, Dept. of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ

Regional Judging Session at the U.S. Military Academy

Head Judge
Patrick J. Driscoll, Dept. of Systems Engineering
Associate Judges
Dave Chennault, Tim Elkins, James Enos, Daniel McCarthy,
Kenny McDonald, Elizabeth Schott, and Russell Schott,
Dept. of Systems Engineering
Steve Horton, Dept. of Mathematical Sciences
—all from the United States Military Academy at West Point, NY

Paul Heiney, Dept of Mathematics, U.S. Military Academy Preparatory School, West Point, NY

Ed Pohl, Dept. of Industrial Engineering
Tish Pohl, Dept. of Civil Engineering
—both from University of Arkansas, Fayetteville, AR

Triage Session at Appalachian State University

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Associate Judges
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—all from the Dept. of Mathematical Sciences, Appalachian State University, Boone, NC
Amy H. Erickson and Keith Erickson
—Dept. of Mathematics, Georgia Gwinnett College, Lawrenceville, GA

Steven Kaczkowski
—Dept. of Mathematics, University of South Carolina, Columbia, SC
Douglas Meade
—Governor's School for Science and Mathematics, Hartsville, SC
Harrison Schramm
—Office of the Chief of Naval Operations, Washington, DC
Rich West
—Francis Marion University, Florence, SC

Coach Problem

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Jack Picciuto, Director of Operations Analysis and Planning at IT Cadre, Ashburn, VA
Kathleen M. Shannon, Dept. of Mathematics and Computer Science, Salisbury University, Salisbury, MD (MAA Judge)
Dan Solow, Case Western Reserve University, Cleveland, OH (INFORMS Judge)
Maynard Thompson, Mathematics Dept., University of Indiana, Bloomington, IN
Marie Vanisko, Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT (Giordano Award Judge)

Regional Judging Session at the Naval Postgraduate School

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Associate Judges
Michael Jaye, Dept. of Defense Analysis
Robert Burks, Dept. of Defense Analysis
David Olwell, Dept. of Systems Engineering
—all from the Naval Postgraduate School, Monterey, CA
Richard West, Emeritus Professor
Thomas Fitzkee, Mathematics Dept.
—both from Francis Marion University, Florence, SC
Jay Belanger, Truman State University, Kirksville, MO
Jack Picciuto, Director of Operations Analysis and Planning at IT Cadre, Ashburn, VA

Triage Session at Carroll College
Head Judge
Marie Vanisko
Associate Judges
Kelly Cline, Terry Mullen, John Scharf, Eric Sullivan, and Theodore Wendt
—all from Dept. of Mathematics, Engineering, and Computer Science,
Carroll College, Helena, MT

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Laifu Liu, Beijing Normal University, Beijing
Liqiang Lu, Fudan University, Shanghai
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Sources of the Problems

The Keep Right Problem was contributed by Michael Tortorella (Dept. of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ).

The Coach Problem was contributed by William P. Fox (Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA).

Acknowledgments

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We also thank for their involvement and unflagging support the MCM judges and MCM Board members, as well as

- **Two Sigma Investments.** "This group of experienced, analytical, and technical financial professionals based in New York builds and operates sophisticated quantitative trading strategies for domestic and international markets. The firm is successfully managing several billion dollars using highly-automated trading technologies. For more information about Two Sigma, please visit `http://www.twosigma.com`."

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each paper here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential MCM advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are the only international modeling contests in which students work in teams. Centering its educational philosophy on mathematical modeling, COMAP serves the educational community as well as the world of work by preparing students to become better-informed and better-prepared citizens.

About the Author

Dr. William P. Fox is a professor in the Department of Defense Analysis at the Naval Postgraduate School and teaches a three-course sequence in mathematical modeling for decision making. He received his B.S. degree from the United States Military Academy at West Point, New York, his M.S. at the Naval Postgraduate School, and his Ph.D. at Clemson University. Previously he has taught at the United States Military Academy and at Francis Marion University, where he was the Chair of Mathematics for eight years. He has many publications and scholarly activities including books, chapters of books, journal articles, conference presentations, and workshops. He directs several mathematical modeling contests through COMAP: HiMCM and MCM. His interests include applied mathematics, optimization (linear and nonlinear), mathematical modeling, statistical models for medical research, and computer simulations. He is President-Emeritus of the NPS Faculty Council and President of the Military Application Society of INFORMS.

Editor's Note

The complete roster of participating teams and results is too long to reproduce in the *Journal*. It can be found at the COMAP Website, in separate files for each problem:

```
http://www.comap.com/undergraduate/contests/
  mcm/contests/2014/results/2014_MCM_Problem_A_Results.pdf
http://www.comap.com/undergraduate/contests/
  mcm/contests/2014/results/2014_MCM_Problem_B_Results.pdf
```

Media Contest

This year, COMAP again organized an MCM/ICM Media Contest.

Over the years, contest teams have increasingly taken to various forms of documentation of their activities over the grueling 96 hours—frequently in video, slide, or presentation form. This material has been produced to provide comic relief and let off steam, as well as to provide some memories days, weeks, and years after the contest. We *love* it, and we want to encourage teams (outside help is allowed) to create media pieces and share them with us and the MCM/ICM community.

The media contest is *completely separate* from MCM and ICM. No matter how creative and inventive the media presentation, it has *no* effect on the judging of the team's paper for MCM or ICM. We do not want work on the media project to detract or distract from work on the contest problems in any way. This is a separate competition, one that we hope is fun for all.

Further information about the contest is at

`http://www.comap.com/undergraduate/contests/mcm/media.html.`

There were 34 entries—31 of them from Dalian Maritime University! (Come on, you other teams!)

Outstanding Winner:

- Dalian Maritime University, Dalian, China
  (Xiaonan Wang, Hang Li, Yang Cui)

Finalists:

- Dalian Maritime University
  (Lei Zheng, Xufan Liu, Yuhan Weng)
- Dalian Maritime University
  (Tianzi Yang, Guirong Zhang, Yuyan Qiao)

The remaining entries were judged Meritorious Winners.

Complete results, including links to the Outstanding videos, are at

`http://www.comap.com/undergraduate/contests/mcm/contests/2014/solutions/index.html.`

Keep Right to Keep "Right"

Yaofeng Zhong
Yunyi Zhang
Xiao Zhao

Tsinghua University
Beijing, China

Advisor: Zhiming Hu

Abstract

Our goal is a model to evaluate the performance of the keep-right-except-to-pass (KRETP) rule and other alternatives, by simulating the traffic flow on a freeway. We construct models to analyze five influencing factors. Then we integrate multiple criteria to judge the performance of nine rules using a fuzzy synthetic evaluation (FSE).

Our basic model focuses on lane-changing behavior, an essential component of overtaking (passing).

We extend our model with a cellular automaton-based approach. We assume that the drivers will change the lane with a specific probability if trigger and safety conditions are satisfied. We simulate traffic flow on a long section of a freeway, controlling occupancy, varying the *number of lanes*, *maximum speed limit*, *minimum speed limit*, and *signaling behavior*.

In addition to KRETP, we examine four other rules by revising the laws governing the cells in the cellular automaton. Then we design five improved rules.

We choose flow rate and average speed as traffic flow criteria, sharp braking frequency as a safety criterion, and satisfaction and standard deviation of speed as experience criteria. Then we use a fuzzy synthetic evaluation technique to integrate these criteria to determine the performance of each rule. We find that in a light traffic, *a partial-assigned-lane-and-keep-right rule* performs the best, while in heavy traffic, *a different-speed-limit-on-each-lane rule* is preferred.

We change the probability of lane-changing to adjust our model to a country such as Great Britain. Moreover, we also simulate a freeway fully controlled by an intelligent system.

Additionally, we refine our extended model by considering the on- and off-ramps. We adopt open boundary conditions and assume that the vehicles flowing in are Poisson-distributed.

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Introduction

A freeway is a controlled-access highway designed for high-speed vehicles. It provides an unhindered flow of traffic with no traffic lights or intersections. The Keep-Right-Except-To-Pass (KRETP) rule, also known as "Slower Traffic Keep Right," is often employed in right-hand traffic to raise traffic flow. In this paper, we simulate different rules for overtaking and compare them so as to attempt to find an optimal rule.

Restatement of the Problem

We are required to build a mathematical model to analyze the performance of KRETP and alternative rules. We have two subproblems:

- Build a model that can simulate the overtaking process.
- Propose mathematical criteria to determine the performance of a rule.

In the first step, we build a model with inputs such as the speed limit plus other factors. In the second step, we consider the tradeoff between traffic flow, safety, and other factors.

Literature Review

Nagel and Schreckenberg [1992] built a model to simulate freeway traffic, a simple cellular automaton model known as the "N-S model." They defined a one-dimensional lane. In their model, each site may be occupied by one vehicle or else be empty. Each vehicle has an integer velocity between 0 and $v_{\max}$. At each time step, four sub-steps are performed: acceleration, slowing down, random speeding or slowing, and car motion.

Rickert et al. [1996] introduced a model with two parallel lanes. Several conditions have to be fulfilled before a vehicle changes lanes:

- no other vehicle is in the way,
- other lanes are better, and
- no collision will occur.

They too simulated using a cellular automaton, with reasonable results.

A multi-lane model does not have to be lane-symmetric. Differences may include different speed limits on each lane, different kinds of vehicles, etc. Chowdhury et al. [1997] created a model with different kinds of vehicles with different maximum speeds. They showed that even if the share of "slow cars" is relatively low, "fast cars" can move only at a low speed. However, Knospe and Santen [1999] suggested that the influence of "slow cars" might have been overestimated.

Assumptions and Justifications

- **No pedestrian can affect the vehicles on freeways.** Usually, pedestrians have no access to freeways, let alone crossing a freeway.
- **We ignore crosswinds during overtaking.** This impact is negligible compared with that of the headwind.
- **Drivers cannot drive in the emergency lane or on the shoulder.**
- **The freeway is completely flat and straight, with no curves or slopes.** This assumption allows us to focus on the nature of overtaking.
- **We assume that all drivers act based on the same set of rules.** Drivers may be aggressive or not, but both groups follow the same rules.

Model Overview

Most research for traffic flow can be classified as either microscopic and macroscopic. Since macroscopic methods are difficult to apply to our problem, we approach the problem with microscopic techniques.

We focus on the incentive for changing lanes and the conditions for a successful lane-change. We treat changing-to-the-left-lane behavior and changing-to-the-right-lane behavior differently. This model gives us some intuition about the rule and serves as a stepping stone to our later study.

Our extended model views the problem from a wider perspective. We consider a section of freeway and divide it into lattice cells. Then we run a cellular automaton to simulate the behavior of vehicles. We derive the laws governing the cells according to the analysis of our basic model. Moreover, using periodic boundary conditions, we treat the freeway as a "ring road" so as to accurately control the density. Thus, we call it a "Ring Road" model.

Our refined model adds an entrance ramp and an exit ramp to our cellular automaton, with laws for entering and exiting vehicles. We use a Poisson distribution to simulate vehicles moving in from the start point.

We use the extended "Ring Road" model as a standard model to analyze the problem and all results have this model at their cores.

The Keep-Right-Except-To-Pass Model

The Basic Lane-changing Model

The basic model is a microscopic approach. A typical overtaking behavior consists of five actions:

- signal for three seconds,

Table 1.
Symbol Table.

Symbol	Definition	Units
	Constants	
λ	Mean of Poisson distribution	unitless
p_{slow}	Probability that a vehicle slows down randomly	unitless
p_{left}	Probability that a vehicle shifts to the left lane when possible	unitless
p_{right}	Probability that a vehicle shifts to the right lane when possible	unitless
p_{exit}	Probability that a vehicle wants to move off through the exit ramp	unitless
	Variables	
v_s	Speed of vehicle s	cell/time step
v_{expect}	Expected speed of vehicle s	cell/time step
v_{lf}	Speed of the vehicle in the left lane in front	cell/time step
v_{lb}	Speed of the vehicle in the left lane behind	cell/time step
v_{rf}	Speed of the vehicle in the right lane in front	cell/time step
v_{rb}	Speed of the vehicle in the right lane behind	cell/time step
t	Time	time step
$D_{l,f,gap}$	Left front gap	cell
$D_{l,b,gap}$	Left back gap	cell
$D_{r,f,gap}$	Right front gap	cell
$D_{r,b,gap}$	Right back gap	cell
vehicle_j^i	The j^{th} vehicle in the i^{th} lane	unitless
$v_j^i\,(t)$	Speed of vehicle i_j at t^{th} time step	cell/time step
$v_{j,\text{expect}}^i$	Expected speed of vehicle i_j	cell/time step
$gap_j^i\,(t)$	Front gap of vehicle i_j at t^{th} time step	cell
$x_j^i\,(t)$	Location of vehicle i_j at t^{th} time step	cell
$lfgap_j^i$	Left front gap of vehicle i_j	cell
$lbgap_j^i$	Left back gap of vehicle i_j	cell
$rfgap_j^i$	Right front gap of vehicle i_j	cell
$rbgap_j^i$	Right back gap of vehicle i_j	cell
lbv_j^i	Speed of the vehicle behind vehicle i_j in the left lane	cell/time step
rbv_j^i	Speed of the vehicle behind vehicle i_j in the right lane	cell/time step
$\overline{v}\,(t)$	Average speed at t^{th} time step	cell/time step
N	Number of vehicles passing a certain point on the highway	unitless
$N(t)$	Number of vehicles on the highway at t^{th} time step	unitless
$N_j(t)$	Number of vehicles on the j^{th} lane at t^{th} time step	unitless
$N_{\text{shift}}(t)$	Number of vehicles changing lanes at t^{th} time step	unitless
t_{expect}	Expected time	time step
t_{actual}	Actual time	time step
a_{ij}	Value of the j^{th} criterion of the i^{th} rule	uncertain
u_j^0	Value of the j^{th} criterion of the ideal scheme	uncertain
r_{ij}	Relative deviation of the j^{th} criterion of the i^{th} rule	unitless
v_j	coefficient of variation of the j^{th} criterion	unitless
w_j	Weight of the j^{th} criterion	unitless
F_i	Relative deviation of the i^{th} rule	unitless

- change lane,
- accelerate,
- signal back, and
- change back to the former lane.

Among these actions, lane-changing is the most crucial part.

Changing to the Left Lane

There are two main considerations [Chowdhury et al. 1997]:

- a reason or a trigger consideration, and
- a safety consideration.

The former means that the vehicle ahead moves slowly enough, triggering the driver to overtake it. The latter indicates that the driver will take safety into account. In other words, if there is a high-speed vehicle driving on the left lane, the driver will choose to stay in the current lane to avoid collision. Based on these considerations, we can introduce some mathematical intuition into the problem. **Figure 1** illustrates the situation that the red (dark) car intends to change to the left lane.

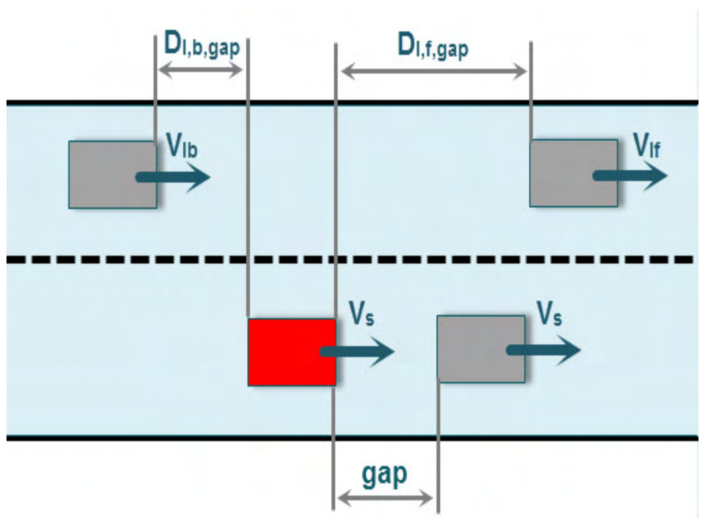


Figure 1. Change to the left lane.

We give mathematical expressions for basic conditions for a change to the left lane:

- **Trigger:** The speed of the cars satisfies $v_{\text{expect}} > v_s$.
- **Safety:** The left back gap should satisfy $D_{l,b,gap} > (v_{lb} - v_s)t$.

Additionally, the car attempts to accelerate (it is unreasonable to change to the passing lane while slowing down). So the left front gap should satisfy

$$D_{l,f,gap} > (v_s - v_{lf})t.$$

Changing Back to the Right Lane

After accelerating and passing the slow vehicle ahead, the driver tends to change back to the former lane due to KRETP. However, this lane-changing behavior is subject to the following constraints:

- **There is no incentive to pass the car in the current passing lane.** Otherwise, on a multi-lane freeway, the driver might prefer to pass the car ahead and hence change again left rather than return to the former lane.
- **It is safe to change back.** The driver should pass the slow car and ensure that there is no collision when changing back.
- **After changing back to the former lane, the driver can maintain a relatively high speed.** Otherwise, the driver will want to pass more than one car in the overtaking process.

The first constraint can be treated as in the changing-to-the-left-lane situation as mentioned above and the other conditions can be stated as the following mathematical expressions:

- **Safety:** The right back gap should satisfy $D_{r,b,gap} > (v_{rb} - v_s)t$.
- **To pass more cars:** We need $D_{r,b,gap} > (v_s - v_{rf})t$.

Figure 2 illustrates this change-back-to-the-right-lane situation.

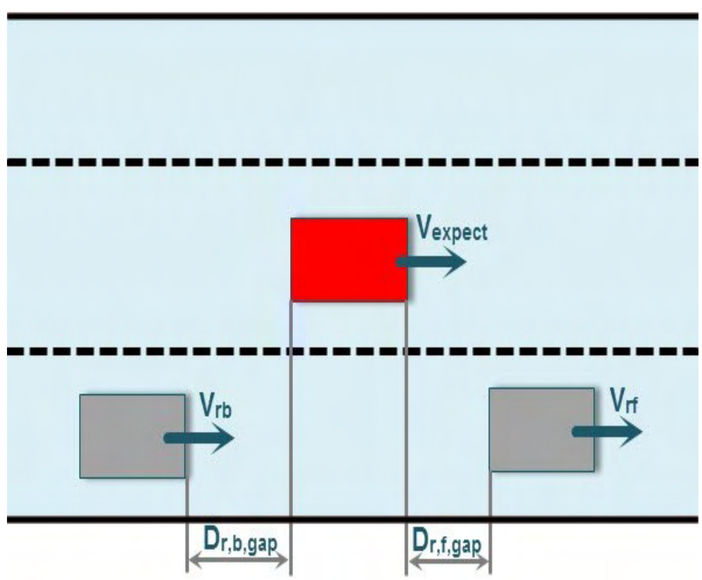


Figure 2. Change back to the right lane.

The Extended "Ring Road" Model

To understand how the rule works in a traffic flow, we have to analyze the behavior of vehicles over a period of time. One intuition for modeling the problem is to think of it as a stochastic process. Therefore, we use a cellular automaton to simulate the behavior of vehicles on a freeway.

A cellular automaton is a discrete model that describes the time development of a system; it treats time as a discrete variable. The model requires an initial configuration and a set of laws that determine how the system develops. At every time step, the cellular automaton advances incrementally and all the laws are implemented.

Assumptions of the Model

- **Drivers follow the rules with a specified probability.** A driver might not want to change lanes even if all the conditions are satisfied. We assume that a driver changes to the left lane or to the right lane with probabilities p_{left} or p_{right}, when possible.
- **All drivers tend to drive as fast as possible while keeping a safe following distance.** Nearly every driver wants to drive faster on the freeway as long as there is enough time for him to react if the vehicle ahead decelerates. Also, the maximum speed is limited by the vehicle type and the speed limit of freeway.
- **Drivers are "myopic": They can see only one vehicle in front, one behind, and several in the neighboring lanes.** A driver can see the vehicle in front directly and the vehicle behind with the help of rearview mirrors. Moreover, a driver can turn the head to left or right to see cars in the immediately neighboring lanes but not in lanes beyond those.
- **Drivers make decisions only according to their own interest.** Because drivers are "myopic," they cannot know the conditions of the whole freeway. Consequently, they make greedy decisions so as to traverse the freeway in the shortest time.

Additionally, to implement a cellular automaton , we propose the following assumptions:

- **Lane-changing does not cost additional time.** Although the distance traveled in lane-changing is longer compared with staying in one lane, drivers tend to accelerate to change lanes. Thus, we suppose that lane-changing costs the same amount of time as staying in the current lane.
- **Each cell represents a 4 m $\times$ 6 m area. The road's length is 2,000 cells. A lane's width is 1 cell.** We divide a multilane freeway into equally-partitioned lanes. We choose a length of 12 km to simulate because of the trade-off between time complexity and the completeness of the model. Each array of cells represents a lane.
- **A time step represents 1 second.** Such an assumption is made by nearly all cellular automaton techniques.
- **We run 20,000 time steps and analyze the last 1,000 steps.** This procedure ensures that we obtain steady-state conditions.

- **While all vehicles trend toward the maximum speed, every single vehicle randomly slows down with probability p_{slow}.** This randomized slowing is characteristic of traffic flow.
- **Acceleration is done steadily while any kind of deceleration can be done in one time step.** Steady acceleration is an energy-saving behavior, and drivers will decelerate to avoid possible collisions.

Characteristics of Vehicles

We classify the vehicles into three groups:

- **Cars**: Cars are small vehicles, which can have high speeds.
- **Buses**: Buses are large vehicles, and their speeds can be relatively high.
- **Trucks**: Trucks are large vehicles that can have only lower speeds.

Then we define the characteristics of the three types of vehicles:

- **Occupancy**: Each car occupies one cell. Since a typical car's length is 3.6 m–4.6 m, a car cannot fully occupy a cell (which has length 6 m). We place the car in the middle of the cell and treat the space in front of and behind as *safe distance*. Accordingly, each bus and each truck occupies two cells with safe distance preserved.
- **Maximum speed**: Cars: 6 cells per time step (130 km/h); buses: 5 cells per time step (108 km/h); trucks: 3 cells per time step (65 km/h).
- **Percentage**: We assume that cars account for 60% of the traffic flow, buses 30%, and trucks 10%, based on the data collected by Anhui [2013].

Laws Governing the Cellular Automaton

Our cellular automaton is implemented by laws sequentially implemented in every time step. These laws are based on the previous analysis and are expressed from a computational perspective.

1. **Moving:** These laws are set based on the assumptions of the model and the characteristics of the vehicles. We denote the j^{th} vehicle in lane i by vehicle$_j^i$. We point out some notation in **Figure 3**, and we diagram the algorithm in **Figure 4**.

1. **Determine the speed:**

   Our three laws below were implemented sequentially, so we use $\left(t+\frac{1}{3}\right)$, $\left(t+\frac{2}{3}\right)$ to denote intermediate times between t and $t+1$.

   (a) **Acceleration:** All drivers tend to drive as fast as possible:

   $$\text{If } v_j^i(t) < v_{j,\text{expect}}^i, \quad \text{then} \quad v_j^i\left(t+\tfrac{1}{3}\right) = v_j^i(t)+1,$$

   where $v_j^i(t)$ is the speed of vehicle$_j^i$ at time t, and $v_{j,\text{expect}}^i$ means the expected speed of vehicle$_j^i$.

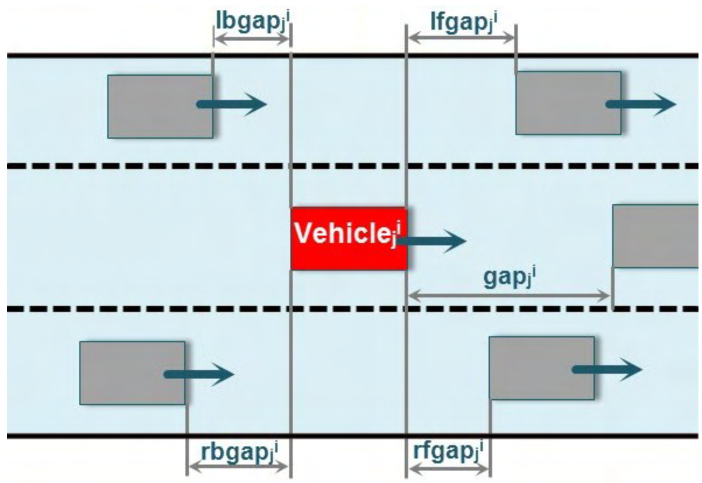


Figure 3. Clarification of some notation.

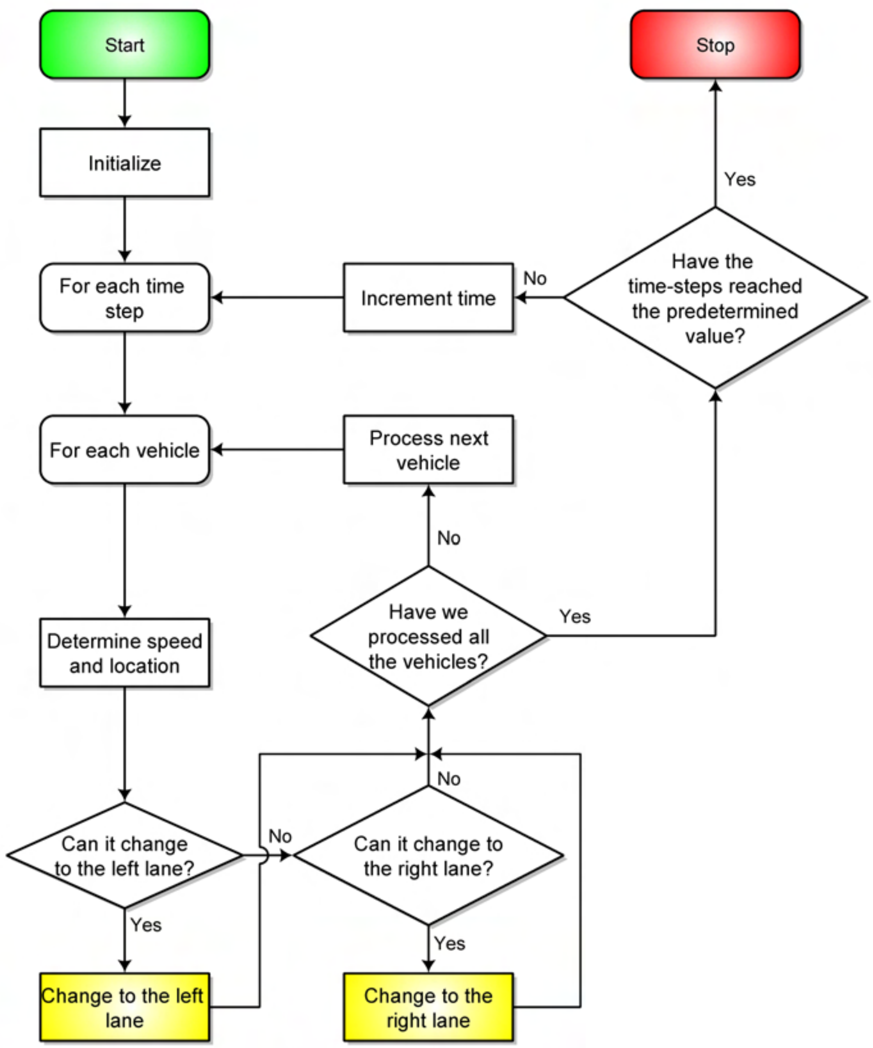


Figure 4. Flow chart of the cellular automaton.

(b) **Randomized slowing:** Every vehicle randomly slows down by 1 cell/sec with probability p_{slow}. Mathematically,

$$v_j^i\left(t+\tfrac{2}{3}\right)=v_j^i\left(t+\tfrac{1}{3}\right)-1.$$

(c) **Deceleration(because of other vehicles):** To maintain a safe following distance, a cell cannot be occupied by more than one vehicle at the same time step.

$$\text{If } v_j^i\left(t+\tfrac{2}{3}\right)>gap_j^i(t), \quad \text{then} \quad v_j^i(t+1)=gap_j^i(t),$$

where $gap_j^i(t)$ is the gap between vehicle$_j^i$ and the vehicle ahead.

2. **Determine the location:** We derive the locations based on the speeds:

$$x_j^i(t+1)=x_j^i(t)+v_j^i(t+1), \qquad gap_j^i=x_j^{i+1}-x_j^i-1,$$

where x_j^i is the location of vehicle$_j^i$.

3. **Lane-changing:**

(a) **Changing to the left lane**

i. Based on trigger criterion, the driver tends to change to the left if the vehicle ahead moves so slowly that the driver fails to reach the driver's expected speed:

$$gap_j^i(t)<v_{j,expect}^i.$$

ii. Then, taking the acceleration criterion into account, we have

$$gap_j^i(t)<lfgap_j^i,$$

where $lfgap_j^i$ is the left front gap of vehicle$_j^i$.

iii. Lastly, considering safety, we have

$$lbgap_j^i>lbv_j^i,$$

where $lbgap_j^i$ is the left back gap of vehicle$_j^i$ and lbv_j^i is the speed of the vehicle behind in the left lane.

(b) **Changing to the right lane**

i. If any one of the rules in 3(a) above is not satisfied, the driver cannot change to the left lane.

ii. To consider safety, we have

$$rfgap_j^i>v_j^i,$$

where $rfgap_j^i$ is the right front gap of vehicle$_j^i$.

iii. To consider another safety criterion, we have

$$rbgap_j^i > rbv_j^i,$$

where $rbgap_j^i$ is the right back gap of vehicle$_j^i$ and rbv_j^i is the speed of the vehicle behind in the right lane.

Modeling Using Periodic Boundary Conditions

To run a cellular automaton, we need to specify the boundary conditions and the initial condition. Boundary conditions determine how vehicles move into and out of the system; an initial condition determines the initial distribution of vehicles and their speeds.

Inspired by Nagel and Schreckenberg [1992], we use *periodic boundary conditions*. Periodic boundary conditions assume that the vehicles moving out of the freeway immediately appear again at the front of the system, so the total number of vehicles is constant. Thus, we can accurately define a constant system density and study the performance of the rule with varying density. Moreover, periodic boundary conditions turn our road into a closed system, so it is similar to all vehicles moving on a circle. So we name our extended model the "Ring Road" model.

The Refined Model with Ramps

We refine our model by adding entrance and exit ramps and applying open boundary conditions.

Entrance ramps give vehicles a chance to accelerate to the expected speed. However, most ramps are too short to allow speeding up to, say, 100 km/h. As a result, vehicles in the right-most lane might have to slow down to let vehicles enter, or incoming vehicles might have a hard time entering. Both cases might have deleterious effects on the traffic flow.

Likewise, vehicles have to decelerate to enter an exit ramp, where similar problems can occur.

To introduce ramps in our cellular automaton, we add the following assumptions.

Additional Assumptions in the Refined Model

- **Exit ramp**: The overlap between the freeway and the exit ramp ranges from the 850^{th} cell to the 900^{th} cell, which is 300 m.
- **Entrance ramp**: The overlap between the freeway and the entrance ramp ranges from the 1100^{th} cell to the 1500^{th} cell, which is 300 m.

Additional Laws

- **Off-ramp law**: If the vehicle wants to leave the freeway, it is not allowed to change to left lane after it reaches the 700^{th} cell. At the same time, it will slow down to 3 cells per second. If the vehicle is in the rightmost lane between the 850^{th} cell and the 900^{th} cell, we assume that it can move into the ramp at the next time step if it wants to.

- **On-ramp law**: A vehicle on the entrance lamp, if the laws of changing to the left lane are satisfied, can enter the freeway during the next time step. Otherwise, it will continue to move forward.

Modeling Using Open Boundary Conditions

Our model with ramps is no longer a closed system. Thus, we must use open boundary conditions to determine how vehicles flow in. Considering that the amount of traffic is stochastic and the inputs to the system are discrete, a generally-used approach is to model the entry of vehicles as a Poisson process. Consequently, we assume that the number of vehicles flowing in from the starting point in any interval of length t is Poisson-distributed with mean λt.

Vehicles tend to move off through the exit ramp with a probability p_{exit}. We try to let the number of vehicles from the entrance ramp equal the number of vehicles from the exit ramp. However, as discussed above, some vehicles may fail to exit the freeway and others may fail to enter. We view both kinds of events as bad characteristics of the traffic flow.

Results: Influencing Factors

We give definitions of light and of heavy traffic. Then we explicitly define four factors and change one factor each time in order to analyze how it influences the performance of the KRETP Rule.

From simulations with our cellular automaton, we find that the traffic flow can be classified into two kinds, as shown in the time-space diagrams in **Figure 5**. The diagrams show the trace of every vehicle in the simulation. A gentle trace indicates a low speed; a steep trace indicates a high speed.

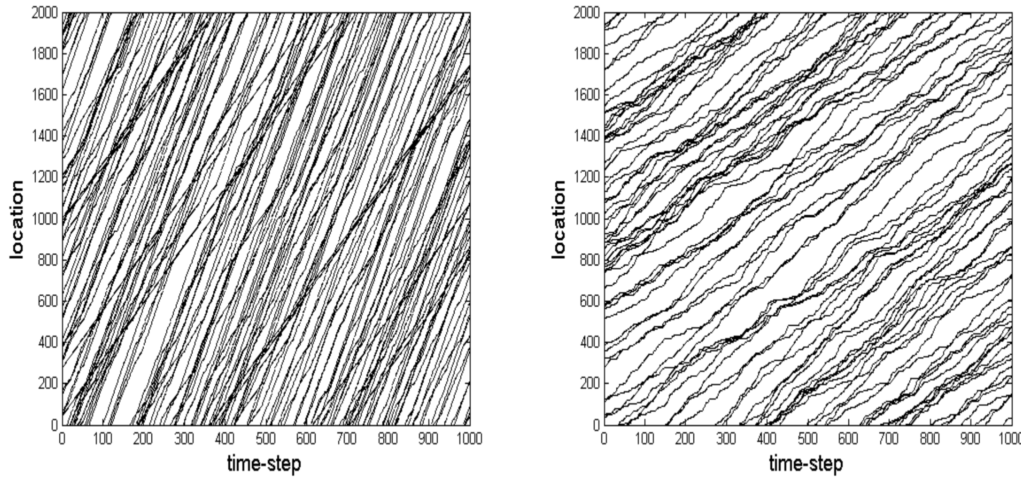
- In **Figure 5a**, we see that vehicles with a high speed can continue at the high speed, which indicates that they are not constrained by slower vehicles. This is light traffic.

- In **Figure 5b**, no vehicle can reach a high speed, which indicates congestion. This is heavy traffic.

Variables and Criteria

We choose as our variables

- the number of lanes,
- the maximum speed limit,
- the minimum speed limit, and
- signaling behavior.



(a) Time-space diagram (occupancy=0.1). (b) Time-space diagram (occupancy=0.4).

Figure 5. Time-space diagrams for different occupancy levels.

To judge the effectiveness of a rule, we propose the following criteria:

- **Flow rate**: The number of vehicles passing a point per unit time:

$$\text{flow rate} = \frac{N}{T}, \quad \text{where } N \text{ vehicles pass a point in time } T.$$

- **Average speed**: The average speed of all vehicles passing a point on a highway or a lane over a specified time period.

$$\text{average speed } \overline{v}(t) = \frac{1}{N(t)} \sum_{j=1}^{3} \sum_{i=1}^{N_j(t)} v_j^i(t),$$

where $N(t)$ is the number of vehicles on the freeway, and $N_j(t)$ is the number of vehicles in the j^{th} lane.

- **Lane utilization ratio**: the ratio between the number of vehicles in the lane to the total number of vehicles on the freeway.

$$\text{lane utilization ratio}_j = \frac{N_j(t)}{N(t)}.$$

- **Sharp braking frequency**: We do not consider accidents in our simulation, since we view accidents as abnormal events and it is difficult to consider abnormal events in microscopic models. Instead, we use sharp braking frequency as an indicator of unsafety. Sharp braking frequency occurs when a vehicle's speed decreases by more than 2 cells per time step. If this happens in our simulation, we assume that it would likely cause an accident in reality.

- **Shift ratio**: The number of lane shifts per unit time.

$$\text{shift ratio} = \frac{N_{\text{shift}}(t)}{N(t)},$$

where $N_{\text{shift}}(t)$ is the number of vehicles changing lanes at the t^{th} time step.

- **Satisfaction**: If a vehicle fails to reach its maximum speed, the driver's satisfaction will decrease. We define the expected time t_{expect} as the time that it takes to drive a given distance at the maximum speed. We define the actual time t_{actual} as the time that it actually takes to drive that distance. Then we derive our criterion by dividing expected time by actual time, so the value of satisfaction ranges from 0 to 1:

$$\text{satisfaction} = \frac{t_{\text{expect}}}{t_{\text{actual}}}.$$

- **Standard deviation of speed**: People might feel uncomfortable if the vehicle keeps accelerating and decelerating. We use the standard deviation of speed to measure this kind of discomfort:

$$\text{Std. deviation of speed} = \frac{1}{N(t)} \sum_{j=1}^{3} \sum_{i=1}^{N_j(t)} \sqrt{\sum_{t=1}^{T} [v_j^i(t) - \overline{v}(t)]^2}.$$

We analyze each of our variables in light of each of our criteria.

[EDITOR'S NOTE: For each of the criteria for each of the variables, the authors offer graphs. We cannot reproduce all of them here; we offer instead in most cases just the authors' conclusions from their figures.]

Variable: Number of Lanes

- **Flow rate:** With low occupancy, the road resources are not fully utilized, so the flow rate is low. With high-occupancy, there may be congestion. We aim for an optimal occupancy.

  From **Figure 6**, we see that the optimal occupancy for a 3-lane freeway and a 4-lane freeway is between 0.2 and 0.3. Thus, in our following analysis, **we let an occupancy of 0.1 represent light traffic and an occupancy of 0.4 represent heavy traffic.**

- **Average speed:** As occupancy increases, the average speed tends to decrease due to congestion. Surprisingly, the curves for 3 lanes and for 4 lanes in **Figure 7** coincide! This result suggests that different numbers of lanes may make little or no difference for average speed.

- **Lane utilization ratio:** As expected, our model demonstrates that utilization of the leftmost lane increases with occupancy.

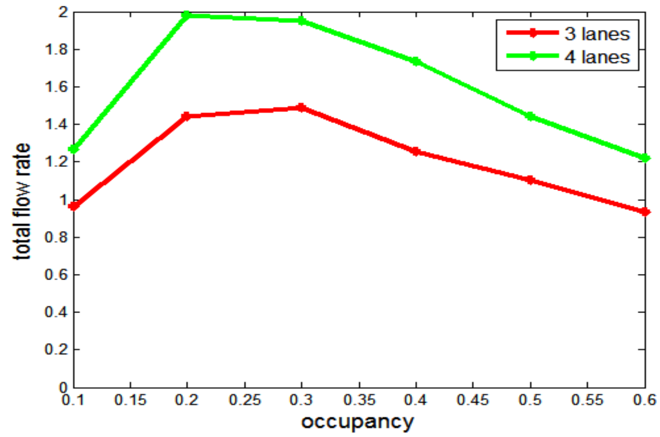


Figure 6. Total flow rate vs. occupancy, for 3 lanes (bottom) and for 4 lanes (top).

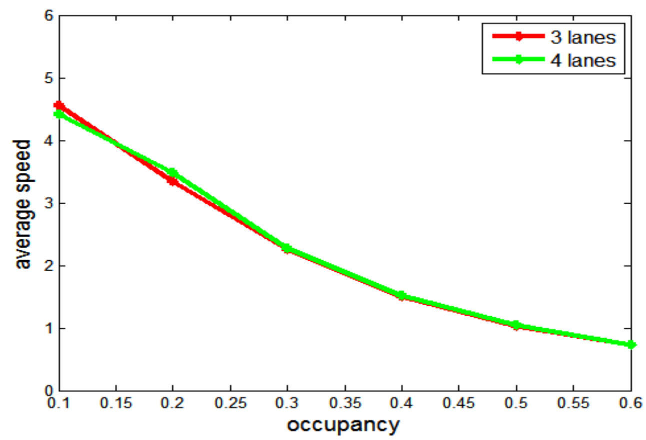


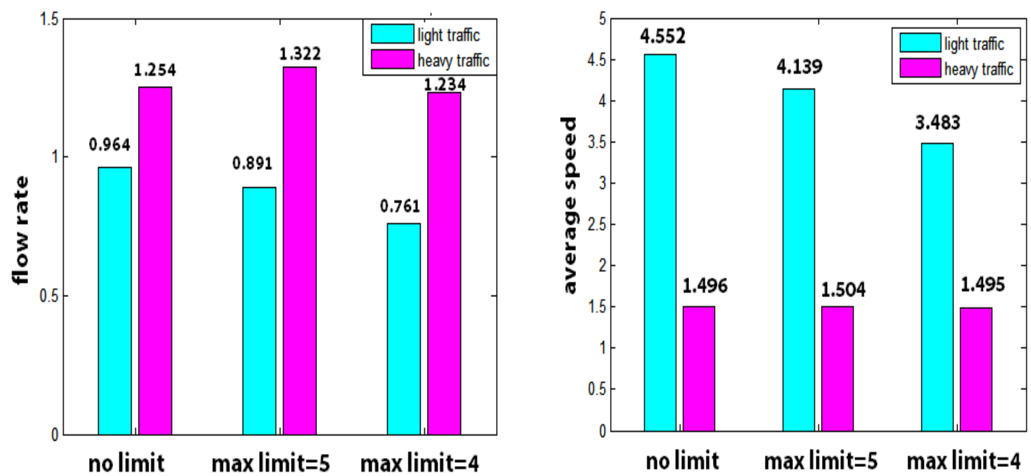
Figure 7. Average speed vs. occupancy, for 3 lanes and for 4 lanes.

- **Sharp braking frequency:** Also as expected, for light traffic, as occupancy increases, sharp braking increases, because the average speed is high. In heavy traffic, the average speed is low, so sharp braking decreases correspondingly.
- **Shift ratio:** For low occupancy, the shift ratio is high, probably because of the low number of cars; but it levels off as occupancy increases.
- **Satisfaction:** As occupancy increases, speeds slow, so satisfaction decreases. But at the same occupancy, there is little difference between a 3-lane and a 4-lane freeway in terms of satisfaction.
- **Standard deviation of speed:** The shapes of the curves for standard deviation are similar to the speed curves and unrelated to the number of lanes.

Variable: Maximum Speed Limit

We study a maximum speed limit of 4 cells per second (86 km/h) and of 5 cells per second (108 km/h) and also no maximum speed limit.

- **Flow rate and average speed:** Vehicles can move at high speed in light traffic, so the speed limit must have a significant influence on the flow rate and on the average speed. In heavy traffic, however, few vehicles can reach the speed limit. However, the speed limit may avoid sharp changes in following distance and hence be beneficial to traffic flow. Therefore, an appropriate speed limit might result in a higher flow rate. We see from **Figure 8** that the results from simulations of our model meet our expectations.



(a) Flow rate. (b) Average speed.

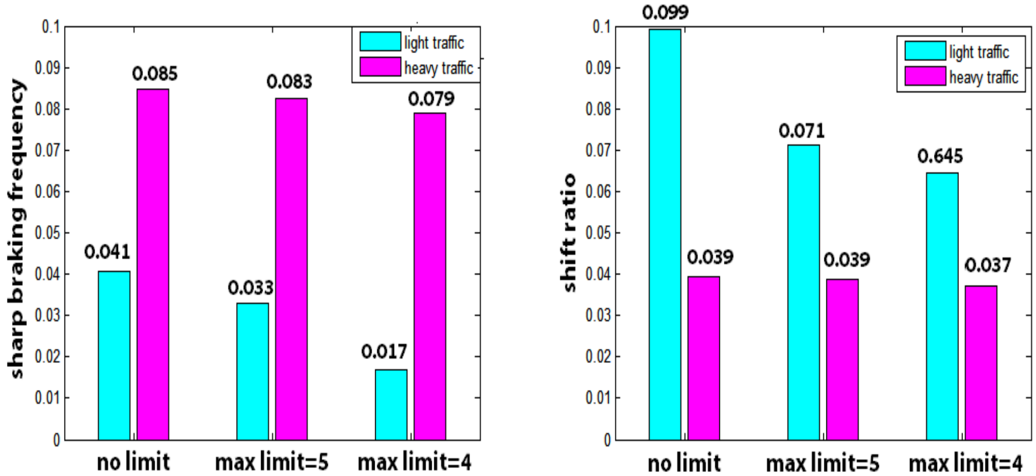
Figure 8. Flow rate vs. speed limit, and average speed vs. speed limit, for light traffic (left bars) and for heavy traffic (right bars).

- **Sharp braking frequency and shift ratio: Figure 9** demonstrates that in light traffic, the speed limit can reduce the sharp braking frequency and the shift ratio, which is beneficial to safety. But there is little effect in heavy traffic.

- **Lane utilization ratio:** As expected, utilization ratio of the leftmost lane in light traffic decreases under a lower speed limit because the limit reduces a driver's willingness to overtake.

Minimum Speed Limit

We study the influence of a *minimum* speed limit by presenting two cases: a minimum speed limit of 3 cells per second (65 km/h), and no limit. By a minimum speed limit, we mean that vehicles will not go more



(a) Sharp braking frequency. (b) Shift ratio.

Figure 9. Sharp braking frequency vs. speed limit, and shift ratio vs. speed limit, for light traffic (left bars) and for heavy traffic (right bars).

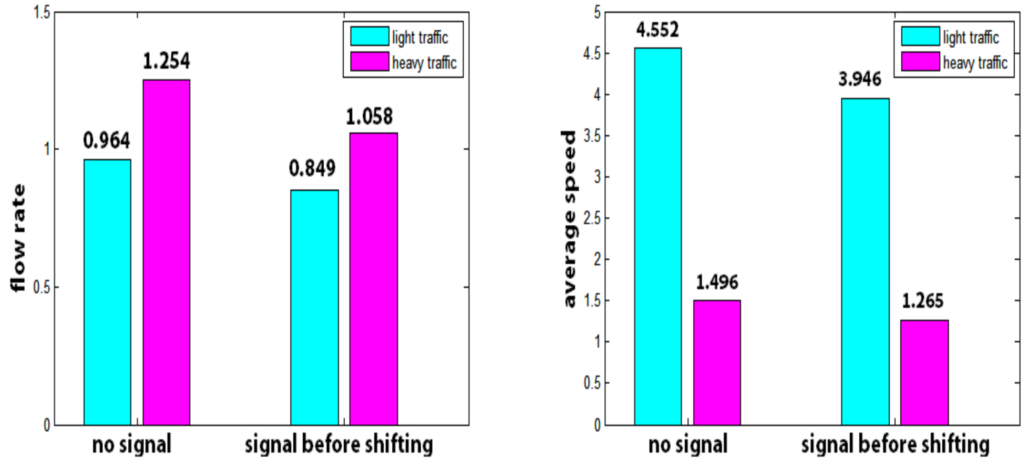
slowly due to random slowing. However, they can decelerate due to safety considerations.

- **Flow rate and average speed:** Opposite to the maximum speed limit, the minimum speed limit plays an important role in heavy traffic but is of little importance in light traffic.
- **Sharp braking frequency and shift ratio:** In light traffic, vehicles cannot slow down in advance due to the speed limit, which in turn increases the sharp braking frequency. In heavy traffic, the frequency decreases. The reason might lie in the low shift ratio and the fluent traffic flow; the conditions for lane-changing are difficult to satisfy.
- **Lane utilization ratio**: In light traffic, the ratio is of little influence. In heavy traffic , the ratio tends to evenly distribute between the three lanes due to the difficulty of changing lanes.

Signal Before Shifting

It is a rule to signal before changing lanes; we add this factor into our model. We assume that a driver must signal first if the lane-changing conditions are satisfied. The related vehicle might decelerate to give way to the driver; then the driver can change lanes in the next time step. We compare the cases with and without this signal mechanism.

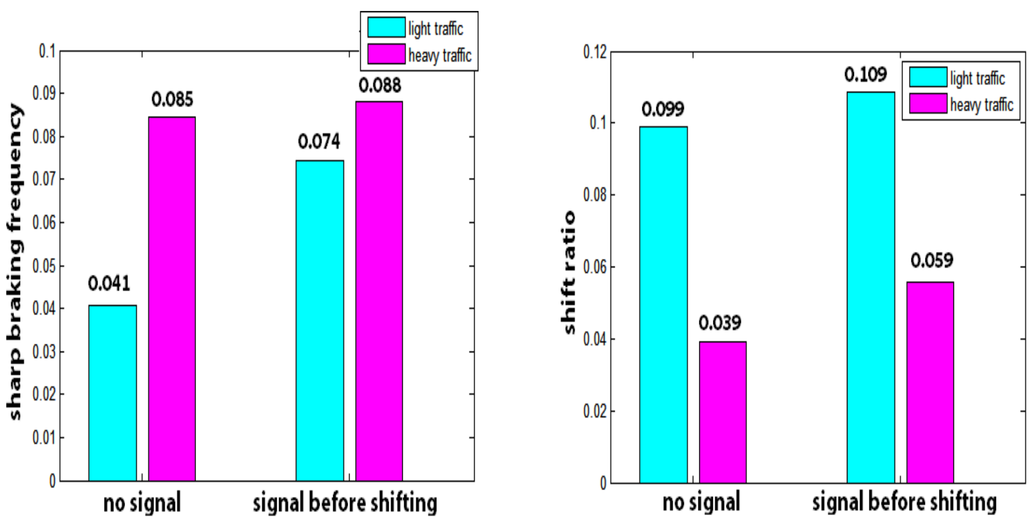
- **Flow rate and average speed:** With signaling, acceleration is constrained, which reduces the flow rate and the average speed. **Figure 10** illustrates this change.



(a) Flow rate vs. signaling. (b) Average speed vs. signaling.

Figure 10. Flow rate and average speed vs. signaling, for light traffic (left bars) and for heavy traffic (right bars).

- **Sharp braking frequency and shift ratio:** Because of vehicles' giving-way behavior, the shift ratio increases and vehicles have to respond to the signal behavior, which will increase sharp braking (**Figure 11**).



(a) Sharp braking frequency vs. signaling. (b) Shift ratio vs. signaling.

Figure 11. Sharp braking frequency and shift ratio vs. signaling, for light traffic (left bars) and for heavy traffic (right bars).

Conclusions

- The number of lanes is not influential under any circumstances.
- The maximum speed limit plays a significant role in light traffic but is of no importance in heavy traffic; for a minimum speed limit, the reverse.
- Signaling behavior reduces the flow rate and average speed while enhancing safety.

Results: The Optimal Rule

We examine five basic rules and design four improved rules. To determine the performance of the rules, we implement a fuzzy synthetic evaluation to consider all the criteria.

Basic Rules of Overtaking

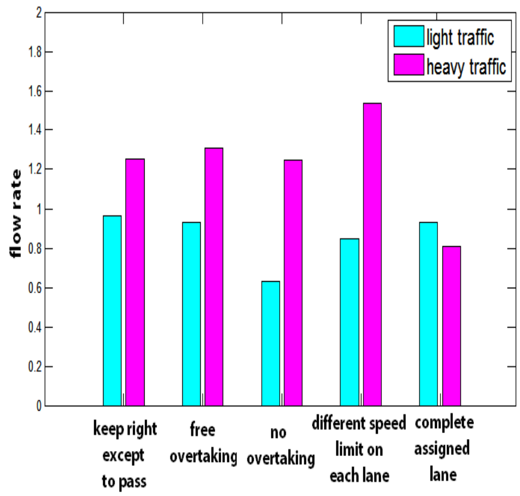
We take a three-lane freeway as an example to state the rules. Apart from the KRETP rule, we present four other basic rules:

- **The free-overtaking rule**: Drivers can overtake or change lanes as they wish.
- **The no-overtaking rule**: Vehicles randomly move into the freeway, but once they are on the freeway, they must stick to their current lane. This rule can be implemented by replacing the dashed lines separating lanes with full lines. This lane-marking bans drivers from changing lanes.
- **The different-speed-limit-on-each-lane rule**: The maximum speed limit ranges from lowest in the rightmost lane to highest in the leftmost lane.
- **The complete-assigned-lane rule**: Cars are assigned to the leftmost lane, buses to the middle lane, trucks to the rightmost lane. No vehicle can change lanes after it moves onto the freeway.

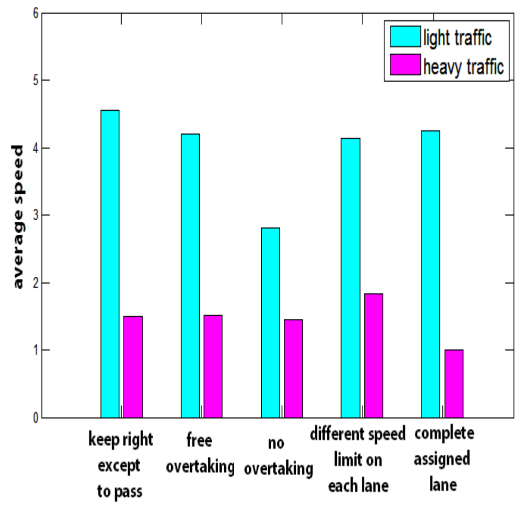
Criteria for the Rules

We adopt flow rate and average speed as criteria for the quality of traffic flow, sharp braking frequency as the criterion for safety, and satisfaction and standard deviation of speed as criteria for people's experience. We analyze the five basic rules with each criterion to see their performance in both light and heavy traffic.

- **Flow rate and average speed:**
  In light traffic, the no-overtaking rule performs the worst because the road resources cannot be fully used. There is no significant difference among other rules (**Figure 12(a)**).



(a) Flow rate vs. rule.



(b) Average speed vs. rule.

Figure 12. Flow rate and average speed vs. rule, for light traffic (left bars) and for heavy traffic (right bars).

In heavy traffic (**Figure 12(b)**), the complete-assigned-lane rule behaves the worst: The leftmost lane, which is assigned to cars, suffers bad congestion while the other two lanes are relatively available. The different-speed-limit-on-each-lane rule performs the best: This rule bans large vehicles from entering the passing lanes and prevents a large number of cars from entering the rightmost lane.

- **Sharp braking frequency:**
  The results are presented in **Figure 13**. In light traffic, the no-overtaking rule performs the worst because vehicles at high speeds encountering slow vehicles can only brake instead of changing lanes. The well-performing rules are KRETP and the complete-assigned-lane rule. The latter too bans overtaking, but the speed is relatively even, so sharp braking seldom occurs.
  In heavy traffic, the free-overtaking rule performs the worst. The different-speed-limit-on-each-lane rule performs the best because it can reasonably allocate the vehicles to three lanes by speed; thus, traffic can flow freely.

- **Satisfaction and standard deviation of speed:**
  Satisfaction rises with speed (no surprise there). In terms of the standard deviation of speed: In light traffic, different-speed-limit-on-each-lane performs the best. Other rules perform almost the same. In heavy traffic, the complete-assigned-lane rule does worst, because of low speed. In this case, the criterion cannot accurately indicate people's experience.

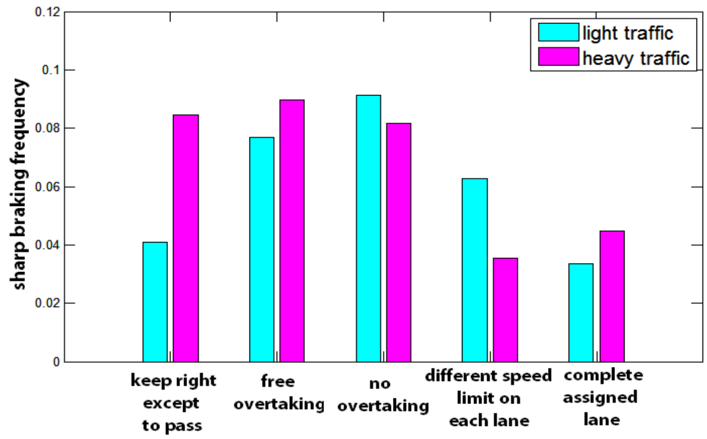


Figure 13. Sharp braking frequency vs. rule, for light traffic (left bars) and for heavy traffic (right bars).

Fuzzy Synthetic Evaluation for Basic Rules

We obtain different results using different criteria. If we want a unique answer, we have to combine criteria into a single criterion. The relative importance of the criteria is hard to determine. Since we have no other information, we implement a fuzzy synthetic evaluation (FSE) [Sadiq et al. 2004], which determines the weight of each criterion based on data.

One way to determine the weights is the coefficient of variation method. If a criterion can differentiate the rules evaluated, the method will assign a large weight to the criterion.

In the following subsections, we use light traffic to introduce the process of FSE. Heavy traffic can be processed in the same way.

Identify alternatives and attributes

The alternatives are the five basic rules and the attributes are the five criteria. The values of each attributes for each alternative are listed in **Table 2**, from which we derive the ideal alternative:

$$u = (u_1^0, u_2^0, u_3^0, u_4^0, u_5^0) = (0.964, 4.552, 0.033, 0.841, 0.813).$$

Determine fuzzy evaluation matrix

The membership function is defined as

$$r_{ij} = \frac{|a_{ij} - u_i^0|}{\max\{a_{ij}\} - \min\{a_{ij}\}}$$

Table 2.
Criteria results for basic rules in light traffic.

	flow rate	average speed	sharp braking frequency	satisfaction	std. deviation of speed
keep-right-except-to-pass	**0.96**	**4.55**	0.04	**0.84**	1.15
free-overtaking	0.93	4.20	0.08	0.79	1.36
no-overtaking	0.63	2.80	0.09	0.53	1.42
different-speed-limit-on-each-lane	0.85	4.13	0.06	0.78	**0.81**
complete-assigned-lane	0.93	4.26	**0.03**	0.81	1.48

Then we have the fuzzy evaluation matrix

$$\mathbf{R} = \begin{bmatrix} 0.000 & 0.000 & 0.127 & 0.000 & 0.508 \\ 0.108 & 0.201 & 0.750 & 0.180 & 0.814 \\ 1.000 & 1.000 & 1.000 & 1.000 & 0.901 \\ 0.357 & 0.242 & 0.504 & 0.205 & 0.000 \\ 0.096 & 0.169 & 0.000 & 0.104 & 1.000 \end{bmatrix}.$$

Using the coefficient of variation method, we define v_j and w_j as:

$$v_j = \frac{s_j}{\bar{x}_j}, \qquad w_j = \frac{v_j}{\sum_{i=j}^{5} v_j}.$$

Then we calculate the weighting vector

$$w = (0.243, 0.226, 0.164, 0.251, 0.117).$$

Aggregate using a fuzzy operator

We use a fuzzy operator to aggregate and obtain the *relative deviation*:

$$F_i = \sum_{j=1}^{5} w_j r_{ij},$$

which measures the distance from a specific alternative to the ideal alternative. The lower the value, the better the alternative.

The Results

The relative deviations in both cases are listed in **Table 3**. Smaller is better.

In light traffic, KRETP has an absolute advantage over other rules; in heavy traffic, the different-speed-limit-on-each-lane rule is the best.

Table 3.
Relative deviations of different rules, in light traffic and in heavy traffic.

Rule	Light traffic	Heavy traffic
keep-right-except-to-pass	**0.08**	0.56
free-overtaking	0.34	0.57
no-overtaking	1.00	0.63
different-speed-limit-on-each-lane	0.28	**0.16**
complete-assigned-lane	0.21	0.67

Improved Rules of Overtaking

We propose four new rules by modifying / combining the five basic rules.

- **The partial-assigned-lane rule**: Cars are assigned to the middle lane and the leftmost lane with permission to change between these two lanes. Buses and trucks are assigned to the rightmost lane and are banned from changing lanes. We present this rule because cars account for 60% of vehicles and have relatively high speed.
- **The truck-on-rightmost-lane-only rule**: Trucks must stick to the rightmost lane, while cars and buses can change among the three lanes. We present this rule because trucks have the lowest speed and they may constrain the speed of others if they are in a passing lane.
- **The minimum-speed-on-leftmost-lane rule**: We set a minimum speed for the leftmost lane.
- **The partial-assigned-lane-and-keep-right rule**: We combine the partial-assigned-lane rule and KRETP: Vehicles are assigned by the partial-assigned-lane rule and cars overtake by KRETP.

Fuzzy Synthetic Evaluation for All Rules

To test the performance of the improved rules, we apply a fuzzy synthetic evaluation to all rules, with the results in **Table 4**.

Conclusions

- **The different-speed-limit-on-each-lane rule is the best in heavy traffic** and hence should be used during rush hour.
- **The partial-assigned-lane-and-keep-right rule is the best in light traffic** and hence should be used at other times.

Sensitivity Analysis

We test our model in both light traffic and heavy traffic for various changes in our assumptions. The analysis proves that our model is not unduly sensitive.

Table 4.
Relative deviations of different rules in light and in heavy traffic.

	Light traffic	Heavy traffic	Overall
keep-right-except-to-pass	0.14	0.52	0.27
free-overtaking	0.37	0.52	0.42
no-overtaking	0.99	0.58	0.85
different-speed-limit-on-each-lane	0.33	**0.12**	0.26
complete-assigned-lane	0.26	0.71	0.41
partial-assigned-lane	0.06	0.56	0.23
trucks-on-rightmost-lane-only	0.17	0.53	0.29
minimum-speed-on-leftmost-lane	0.23	0.32	0.26
partial-assigned-lane-and-keep-right	**0.00**	0.56	0.19

Percentages of Vehicles

Despite the data from Anhui [2013], the percentages of vehicles of different types may vary. Therefore, we change the percentage of large vehicles (40%) by up to 15%. We observe a 17% increase in sharp braking frequency in light traffic, but the other criteria change little. In heavy traffic, all criteria change little. Hence, our model can be used on freeways with varying percentages of vehicles.

Random Slowing

The probability p_{slow} describes random deceleration. We assumed $p_{\text{slow}} = 0.2$, since few data are available. If we increase it by 15%, sharp braking frequency rises 16%, which is acceptable.

Willingness to Change Lane

A driver might choose not to change lanes even if all the other conditions are satisfied. We assumed that the probabilities of willingness to change to the left lane and right lane are 0.5 and 0.7, respectively. We change the probabilities by up to 15% proportionally. The maximum deviation is 7%, which indicates good robustness.

Further Discussions

Countries that Drive on the Left

Although we can imagine that driving on the left mirrors driving on the right, there is one difference, which is human beings: A left-handed person is always a left-handed person. We assumed in our model that the probabilities of being willing to change to the left lane and to the right lane were 0.5 and 0.7. If this tendency does not change under any circumstances, in a left-driving country, drivers will have a higher tendency to move to the right lane—there, the passing lane. This is exactly the same as if we had swapped the two probabilities in our original model.

Our simulation shows a maximum change of 5% in any of our criteria. So we can safely conclude that even if willingness to change lanes differs between left and right, the deviations are small enough to ignore.

Modifications for an Intelligent System

We propose two intelligent systems:

- **The semi-intelligent system**: This system forces vehicles to change to the right lane if the conditions are satisfied, while whether to change to the left lane relies upon human judgment.
- **The complete intelligent system**: This system not only forces vehicles to change to the right lane if the conditions are satisfied, but also forces vehicles to change to the left lane if the corresponding conditions are satisfied.

We run our simulation to examine the performance of the intelligent systems.

- **Flow rate and average speed:**

  In light traffic, the flow rate of the complete intelligent system increases slightly, while that of semi-intelligent system decreases slightly, compared with the situation without an intelligent system. The small impact is due to rich passing-lane resources.

  In heavy traffic, the flow rates of the intelligent systems increase, because forcing vehicles to move back to the rightmost lane releases more passing-lane resources.
- **Sharp braking frequency and shift ratio:** In light traffic, the shift ratio and sharp braking frequency both change slightly. In heavy traffic, the shift ratio increases significantly, as expected. In light traffic, the sharp braking frequency decreases greatly if a complete intelligent system is implemented: If a driver is about to be constrained by the vehicle ahead, the driver can change to the left lane in time to avoid sharp braking.
- **Lane utilization ratio:**

  The overall changes are relatively small.

Additional Research on the Refined Model with Ramps

We present a refined model with ramps. Due to open boundary conditions, we use λ to control the occupancy, which determines whether the traffic is heavy or light. Then we vary the value of p_{exit} to study the model. We have some interesting findings.

- **Flow rate:** The probability p_{exit} to exit has little, if any, impact on the flow rate, partly because we equalize the number of vehicles entering with the number exiting.

- **Average speed:** In the previous analysis, the average speed was always consistent with the flow rate. Adding ramps, however, breaks this consistency, especially in heavy traffic. As p_{exit} increases, a large number of vehicles need to exit through the off-ramp. They decelerate in advance, which causes low average speed.
- **Lane utilization ratio:**
  In heavy traffic, p_{exit} increases utilization of the rightmost lane, because vehicles need to use the rightmost lane to exit.

Failure Ratio

Some vehicles might fail to exit as desired, a common situation in the real world. In light traffic, p_{exit} is irrelevant; vehicles can move to the rightmost lane with ease. In heavy traffic, when p_{exit} is low, the average speed is relatively high, which makes moving to the rightmost lane difficult; when p_{exit} becomes high, the average speed slows down—vehicles have more time to move to the rightmost lane, which in turn reduces the failure ratio.

Strengths and Weaknesses

Strengths

- Our sensitivity analyses show that our models are fairly robust to changes in parameter value.
- We take into account different types of vehicles, based on data. We consider the lengths of vehicles and different maximum speeds, which makes the model closer to reality.
- We come up with various criteria to compare different situations. Hence, an overall comparison can be made based on these criteria.
- The results of our models also agree with common sense and experience.
- We offer a refined model to consider the role of ramps.

Weaknesses

- Factors of human judgments may be over-simplified. To consider that a driver may randomly decelerate and not choose to overtake when possible, we simply define a probability respectively. The actual situation may be more complicated.
- Some of the parameters are based on semi-educated guesses because few data are available. However, based on our sensitivity analysis, they will not make a great difference if slightly changed.
- We did not consider look-ahead by each driver. In our model, drivers change their speed only based on the information of the previous time step. But in fact, they can look ahead further in time, make a prediction, and choose their speed in a more complicated way.

References

Anhui Expressway Company Limited Network. 2013. Traffic flow on Tianchang section of the national road 205. `http://www.anhui-expressway.net/enterprise/quarterselect1.aspx?Year=2013&RoadID=205%u56fd%u9053%u5929%u957f%u6bb5`.

Chowdhury, D., D.E. Wolf, and M. Schreckenberg. 1997. Particle hopping models for two-lane traffic with two kinds of vehicles: Effects of lane-changing rules. *Physica A: Statistical Mechanics and its Applications* 235: 417–439.

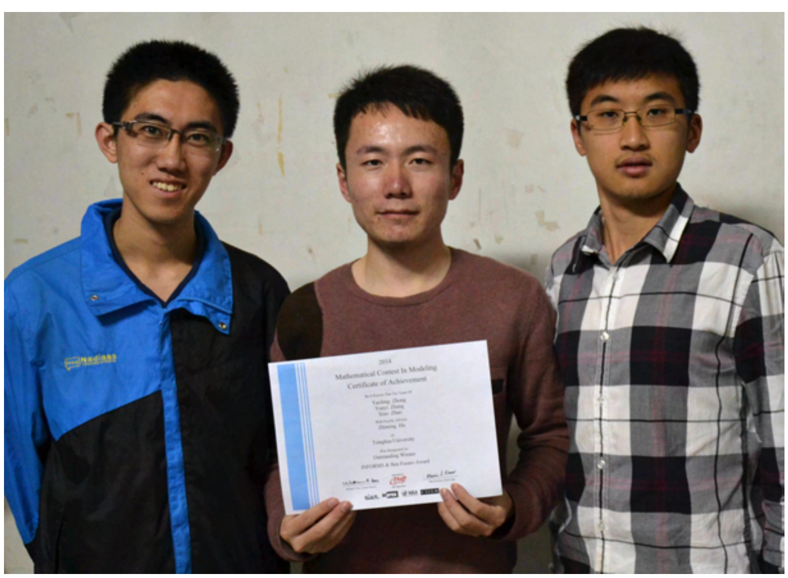
Knospe, W., L. Santen, L., A. Schadschneider, A., and M. Schreckenberg. 1999. Disorder effects in cellular automata for two-lane traffic. *Physica A: Statistical Mechanics and its Applications* 265 (3): 614–633.

Nagel, K., and M. Schreckenberg. 1992. A cellular automaton model for freeway traffic. *Journal de Physique France I*, 2 (12): 2221–2229.

Rickert, M., K. Nagel, M. Schreckenberg, and A. Latour. 1996. Two lane traffic simulations using cellular automata. *Physica A: Statistical Mechanics and its Applications* 231 (4): 534–550.

Roess, R.P., E.S. Prassas, and W.R. Mcshane. 2004. *Traffic Engineering*. 3rd ed. Upper Saddle River, NJ: Pearson Education, Inc.

Sadiq, R., T. Husain, B. Veitch, and N. Bose. 2004. Risk-based decision–making for drilling waste discharges using a fuzzy synthetic evaluation technique. *Ocean Engineering* 31 (16): 1929–1953.



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Judges' Commentary: The Keep Right Papers

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Introduction

The questions for the Keep Right Problem of the 2014 MCM required teams to examine traffic rules and determine a way to assess different rules and practices. The specific task was to examine the "keep right except to pass" rule. Teams were asked to determine how to balance different concerns such as safety and traffic flow.

The majority of teams developed computational models to simulate traffic. The teams generally extended and adapted standard models to use multiple lanes, with the majority of drivers respecting various general rules. Many teams examined a variety of different rules, and one of the primary difficulties was to determine ways to analyze the results of the team's simulations.

This commentary is an overview of the approaches used by the teams on this problem. I first examine the problem itself and discuss the different approaches. Next, I discuss issues associated with presenting the results of a model. Finally, I provide an overview of various topics.

I do not provide an overview of the judging process. This is an important topic discussed in the commentaries from previous years [Black 2009; 2011; 2013]. I highly recommend that both advisors and students read some of the commentaries from previous years that do include information about the judging process, since that information can help to motivate the importance of different aspects of a report.

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The Modeling Problem

I discuss the questions posed as part of the Keep-Right Problem and offer an overview of the modeling approaches employed.

Prior to reading papers, the judges read the problem statement carefully. After reading the statement, each judge reads a random sample of papers. This is done so that the judges get an idea of what the teams are able to address in the short time allotted. We are acutely aware that the time and resource constraints make it difficult to address the problem, and we make every effort to let the teams' efforts—rather than the judges' expectations—drive the event.

The majority of papers made use of either a physics-based model or a cellular automata model. It was not uncommon to read papers that included both approaches, and many teams examined a variety of rules and made comparisons between the differences in their models.

The Questions

There are three parts in the original problem statement.

- The first part of the problem requires teams to "build and analyze a mathematical model to analyze the performance" of a traffic rule, the rule that requires drivers to stay to one side of the road and change lanes only to pass another car. The important aspect, which is open to interpretation, is to "analyze" the model. A wide range of approaches was attempted. At the extremes, some teams focused only on flow rates, and some teams focused only on safety issues. The best papers examined a combination and examined the relationship between these two aspects.
- The second part of the problem required the teams to make an argument as to whether or not the rule made sense in a country where drivers stay on the left side of the road. This was a question that received less attention. The vast majority of teams mentioned it or examined a few simulations, and most teams provided broad insight into the problem.
- The final part was to examine the impact of intelligent systems. The teams were asked to determine what would happen if all of the driving was automated, and the teams were asked to determine how the analysis might change. This part of the question was wide open to interpretation. The wording was vague, and it was interpreted in a wide variety of ways. This was an opportunity for a team to excel and do something that could set their paper apart.

Models

The majority of teams constructed a computational model and conducted simulations that made use of one of two approaches.

- The first common approach was a mechanics-based physics model.
- The second approach was a cellular automata model based on either the Biham–Middleton–Levine model [1992] or the Nagel–Schreckenberg traffic model [1992]. A small number of teams attempted to model traffic flow using a continuous model that results in a partial differential equation [Lighthill and Whitham 1955; Richards 1956].

In both the physics and the cellular automata approaches, the teams constructed computational models and ran multiple simulations under a variety of conditions. Many of the higher-ranked papers first presented a simple model and then looked at a succession of more complicated models. The teams often discussed the relative shortcomings of the models and proposed fixes to address the issues. In doing so, the teams constructed a sequence of models, with analysis, provided a critical review of their models, and posed adaptations to improve their models.

The teams often proposed a set of rules and governing equations for how to react over discrete time steps in their computational model. These rules could be adapted to examine different circumstances with respect to passing other cars and changing lanes. The models had a wide range of features such as how to handle the inflow and the outflow, as well as on- and off-ramps.

Presenting rules in a structured way is a challenge. Many teams simply provided a list of rules and relationships with little or no discussion or motivation. Reading these papers and trying to understand the models was difficult, and it was not clear what the teams actually did and what they simply repeated from what they found in the literature. The papers that were most warmly received by the judges included a narrative that described the relationships, discussed the motivation for the relationships, and offered insights into the individual terms. Additionally, many teams included a flowchart with a brief discussion of the chart in their narrative. Such aids were immensely helpful in trying to understand a team's model.

Finally, a smaller number of teams attempted to construct continuous models based on partial differential equations. Such models were problematic:

- First, it was difficult to incorporate multiple lanes in the resulting models.
- Second, the resulting models often led to nonlinear hyperbolic equations, and they often admit shocks in their solutions. Trying to approximate or find analytic solutions to the resulting equations is problematic.

Presenting Results

One of the biggest challenges in the problem was for the teams to present their results in a coherent, structured way. The majority of teams conducted Monte Carlo simulations. That is, they had to examine the results of multiple simulations, and those simulations were conducted using probabilistic models. Assembling the resulting data, analyzing it, and presenting the results is a difficult task.

The primary way that results were given was to provide sample means from multiple runs. Additionally, it was common to present the results from a single simulation. The results were often given in the forms of tables and graphical representations. Again, the tables often provided sample means, but it is was rare to provide any indication of the variation and distribution in the sample data.

With respect to graphical representation, a common figure was of a "time-space" chart representing the traffic density in both space and time. Common issues associated with such figures is that they were presented with little discussion or were poorly annotated. Teams that were able to provide good descriptions of every figure with proper annotation of the plots had a considerable advantage with respect to how a judge reacted to their paper. The kind of figures necessary to convey the important features of the complex data sets generated require detailed descriptions. A team that simply presented a figure with no discussion or had figures that lacked labels on the axes and titles was at a severe disadvantage.

Finally, most teams generated results that were sample data based on a probabilistic model. Few teams provided adequate statistical measures of their data. Even fewer teams provided any sense of the distribution of their data, with histograms and boxplots an exceedingly rare occurrence. This has been the case for as long as I have been a judge! Such basic practices to communicate and interpret stochastic data were acutely missing this year. It is clear to me that we are failing to provide for our students with respect to developing their understanding about what data is, thinking about sampling, and how to analyze data.

The few teams that recognized that they had data and provided the most basic of statistical analyses had an immediate advantage. Simply reporting a sample standard deviation was enough to make a paper stand out from the rest of the field! The few teams that discussed the distribution of their results in the slightest way immediately demonstrated an understanding of the nature of their work in a way that very few other teams were able to do.

Other Themes

In this section, I discuss a number of topics that are always important—and require discussion every year. First, I give a few notes on the summary and the introduction. Next, I discuss strengths and weaknesses, followed by sensitivity. Finally, I offer a discussion on writing in general.

The Summary and Introduction

Every year, the summary is given a nontrivial weight in the judging. The summary is the first thing that a judge will read, and it is the first impression. In the early rounds of the judging, the relative importance of the summary is magnified. By the later rounds, it is not as important; but that is partly because most of the papers still being read in the later rounds have well-written summaries.

My overall impression is that this year the summaries appeared to be better than in previous events. A good summary should do three things:

- It should provide a context and overview of the problem.
- It should give the reader a good idea of the general approach.
- Finally, it should include an explicit statement of specific results.

The next thing that a judge will read is the introduction. Many teams borrow heavily from the summary for their introduction, and that is a good thing. The introduction should include the same things that are in the summary, and we understand that the teams operate under difficult time constraints. There are some important additions, though, that should be in the introduction:

- First, the introduction should provide more *context and background* information about the problem. The best papers provided background information about the different modeling approaches and gave the reader a sense of the history of the problem. A team can set the tone for the paper by immediately demonstrating a basic understanding of the problem and letting the reader know what they think are the core ideas.
- The introduction should give an explicit statement of the *contents and structure* of the paper. The team should tell the reader what to expect and mitigate any surprises. After reading the introduction, the reader should know what to expect and understand the structure of the whole paper.

The Conclusion

One aspect of the paper that is often overlooked is the conclusion. It is not uncommon, even in the best papers, to have a short conclusion that provides little insight into the problem or the paper itself. The writing of the conclusion is an opportunity for the team to wrap up loose ends and to remind the reader of the full spectrum of activities that have been discussed in the paper.

When a paper ends with a weak statement and no overview of the results or approach, it is a let-down. The conclusion is the last chance impression on the person reading the paper. The teams should take advantage of the conclusion to remind the reader about the tremendous effort required to do the work they are presenting.

Strengths and Weaknesses

A critical part of the modeling process is to stop and engage in a critical view of the model. A team should identify the things that are best and the things that need to be improved, so that the team can demonstrate that they understand and can perform this aspect of the modeling process. No model is perfect, and each model can provide insights due to specific strengths.

Identifying the strengths and weaknesses of a model is something that is stated in the requirements and other materials for the MCM. Every year, the judges assign a relative weight for this part of a paper; and when we pick up a paper we expect to see insights into a model's strengths and weaknesses. Most teams include a separate section in which a bulleted list is given, although few provide an adequate introduction and transition for their lists.

We understand that there are enormous time constraints, so a bulleted list is good. The strengths and weaknesses is not just a bulleted list, though. There should be some commentary within the narrative itself that also discusses these issues. In this year's contest, a large number of teams developed multiple models. The best teams provided transitions that included a discussion of their motivation for their improvements, provided an explicit acknowledgment of the role of model refinement, and identified their model's relative strengths and weaknesses.

Sensitivity

Another aspect of modeling that is given a heavy weight is sensitivity of a model. This is an extremely important idea, yet it is the one part of the modeling process that is given the least attention. A team that performs a structured and detailed sensitivity analysis will stand out. Few teams do it, though.

Some teams have a section on sensitivity, but most of the discussion about sensitivity is either superficial or does not adequately demonstrate an understanding of what sensitivity is. It is important for a team to look at individual terms or parts of a model and ask what happens to the measured responses under some small change. That change can be as simple as looking at a small change in the value of a parameter or a more-nuanced look at what happens to a small change in the model itself. A team can perform a small bit of relatively easy analysis, and that can make a big difference in how a paper is received.

Writing

Writing and grammar are important. Every year, we see teams that appear to have created excellent models and probably performed an excellent set of analyses on their models. Unfortunately, in the eyes of someone reading a paper, the work cannot be better than the writing submitted. A paper with poor grammar or a poor overall structure will not receive a high rating.

Students need to have experience writing mathematics. We want our students to be able to share their ideas. Writing mathematics, though, is a skill, and it is a different skill from mathematics. Small things can make a big difference:

- All equations should be numbered. (This makes it easier to discuss the ideas in the paper with others.)
- A table of contents makes it easier for the reader to understand how a paper is structured.
- Equations and expressions should be part of a sentence and have proper punctuation.
- Incomplete sentences. Really bad.
- Spell-checkers can suggest the wrong word. Spell-checkers are not your friend.
- Transitions are absolutely vital.
- There is a difference between a citation and a reference. Both are necessary.

Students do not get many opportunities to engage in the full process of creating and bringing together complex mathematical ideas, performing an analysis of their ideas, and writing a complete report of all of their work. It is not something that they will do outside of a mathematics course. The inclusion of an expression in a sentence is not intuitive, and students will not know how to do it simply by reading or picking it up on the street. Most students have no issue about including both citations and a list of references in papers for their other courses, but they will not automatically bring that skill into mathematical writing unless explicitly reminded that it is still important.

Students need practice writing in a mathematical context. Writing a full report such as those submitted in the MCM is fundamentally different from almost all of the writing that they normally do. Advisers who take the time to have students practice this skill will be doing an enormous favor for their students, and that practice will aid them well beyond just this event.

Conclusions

To address the Keep-Right Problem, teams were required to develop a model to describe traffic flow. The majority of teams made use of a computational model and examined the results of Monte Carlo simulations. Most teams extended existing models, and the primary differences among entries was in the way that the teams interpreted and presented the results of their simulations.

This was a difficult task in that for most cases the data generated is stochastic in nature. Few teams examined their data using formal statistical techniques, and few were able to present the nature of the variation in their data in a formal manner. Presenting the results of this kind of data in a formal manner is a difficult task, and those who were able to make good use of figures and tables and were able to discuss them in a structured manner had an advantage in the way that their paper was perceived.

In addition to the problems associated with discussing and presenting their model, the teams had to address other important tasks. As is the case every year, the importance of the summary, conclusion, and writing cannot be over-stated. Also, the importance of providing a critical view of the model is vital, and this year the importance of determining the sensitivity of the model had a larger weight than usual.

References

Biham, O., A.A. Middleton, and D. Levine. Self-organization and a dynamical transition in traffic-flow models. *Physical Review A* 46 (10) (1992): R6124–R6127. `http://arxiv.org/pdf/cond-mat/9206001.pdf`.

Black, Kelly. Judge's Commentary: The Outstanding Traffic Circle papers. *The UMAP Journal* 30 (3) (2009): 305–311.

________. Judges' Commentary: The Outstanding Snowboard Course papers. *The UMAP Journal* 32 (2) (2011): 123–129.

________ Judges' Commentary: The Ultimate Brownie Pan papers. *The UMAP Journal* 34 (2–3) (2013): 141–149.

Lighthill, M.J., and G.B. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London* 229 (1178) (1955): 317–345. `https://amath.colorado.edu/sites/default/files/2013/09/1710796241/PRSA_Lighthill_1955.pdf`.

Nagel, Kai, and Michael Schreckenberg, A cellular automaton model for freeway traffic. *Journal de Physique I France* 2 (12) (December 1992): 2221–2229. `http://hal.archives-ouvertes.fr/docs/00/24/66/97/PDF/ajp-jp1v2p2221.pdf`.

Richards, P.I. Shock waves on the highway. *Operations Research* 4 (1) (1956): 42–51.

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Author's Commentary: The Keep Right Papers

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Introduction

My intent in setting this problem was to begin to understand the global properties of the keep-right-except-to-pass (KRETP) rule. One has the intuitive sense that if everyone kept to this rule without exception, then traffic would eventually become chaotic (not necessarily in the technical sense) and throughput would decrease. If we think of the flow of traffic on a multi-lane highway as a stochastic process, the problem was intended to elicit an understanding of how the application of this particular control scheme alters the properties of the process. For example, does the process with KRETP become unstable or chaotic (in the technical sense) as the traffic density increases, and what implications would that have for throughput and safety? If so, might there be an alternative that works better? Or is any control scheme needed at all?

Disappointment

A solution along these lines in full generality is certainly too much to ask of undergraduates, never mind undergraduates under severe time pressure; but I thought it possible that a simple Poisson process model for each lane, with switching between lanes as prescribed by the KRETP rule, might have been tried. I was disappointed to see that such an approach was not considered by any of the teams. The teams construed the problem very

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locally and narrowly, and were satisfied to apply a readily-available cellular automaton model, with minor modifications, to study throughput and safety along a linear stretch of freeway having no on- or off-ramps. The best papers did a creditable job with this pedestrian approach, but I couldn't help feeling that there was still something missing. It was as if the teams were solving a consulting problem and gave the client a solution that met the letter of the requirement but nothing more.

I noticed the same phenomenon in the Snowboard Course (half-pipe) Problem from a few years ago [Giordano 2011; Black 2011]. When I set that problem, my intent was to see if the current half-pipe shape was optimal (in whatever sense would be defined by the teams). In the event, however, teams went online and found a standard definition of half-pipe, including shape, in Wikipedia; and thereafter they confined their solutions to fiddling with dimensions—which was far from interesting and far from what was intended.

Leaving Room for Creativity

In the Keep-Right Problem, teams did well in understanding that it was necessary to create specific measures for throughput and safety so that these properties of the rule could be exposed. This activity was consistent with the MCM's intent of leaving room for team creativity in determining the specific mathematical constructs and expressions that teams will use to solve the problem.

Well-written problems play to this strength, allowing teams a lot of freedom in constructing their approaches and solutions. I would prefer to see teams exercise much more creativity in general. I think this goal should motivate us to write problems in such a way that it becomes clear that finding a model that someone else has already constructed, and modifying it in some minor way, may produce a solution to the problem—but that such an approach is not consistent with the spirit of the MCM. I believe that spirit is to draw out the teams' own ideas, even if they are imperfect or not fully formed, so that they get some practice in going beyond the letter of the requirements to test their own skills in a more challenging way. So I believe that those of us who write problems can learn from this experience and pay more attention to whether a problem statement does enough to support this need without unnecessarily constraining the possible approaches that teams may consider. Indeed, properly written, the problem should encourage the team to range widely over possible solution approaches before settling on something to write up for the competition. (Coaches: Train your teams to spend time at the beginning of the contest period in brainstorming mode, looking for other connections—maybe even train using mind-mapping software to explicitly bring unusual ideas out into the open!)

Criteria for the Keep-Right Problem

For the KRETP problem, certain aspects were essential. Papers that did not consider

- three (or more) lanes of traffic,
- behavior in heavy traffic (a problem requirement),
- throughput and safety (another problem requirement), and
- entrances and exits on a limited-access highway

were downgraded. Similarly, teams that found a model online and did not add any value of their own were not ranked well.

Factors causing papers to be looked at more favorably included

- explicit consideration of a tradeoff between throughput and safety,
- explicit consideration of speed variation, and/or
- the influence of new vehicle technologies (smart roads, inter-car communication, etc.).

Formulating Problems to Evoke Creativity

Overall, as an author, I would have to say that I learned from this experience that the way that a problem is stated is important to getting good results. Problem requirements need to be explicitly called out. Perhaps even more important is the idea that problems should be written so as to encourage creativity over routine solution, and this demand places more responsibility on authors to anticipate the possible approaches that teams might take and subtly encourage approaches that promote more "interesting" solutions.

References

Black, Kelly. Judges' Commentary: The Outstanding Snowboard Course papers. *The UMAP Journal* 32 (2) (2011): 123–129.

Giordano, Frank R. 2011. Results of the 2011 Mathematical Contest in Modeling. *The UMAP Journal* 32 (2) (2011): 99–107.

About the Author

Mike Tortorella is Visiting Professor at RUTCOR, the Rutgers Center for Operations Research at Rutgers, the State University of New Jersey, and Managing Director of Assured Networks, LLC. He retired from Bell Laboratories as a Distinguished Member of the Technical Staff after 26 years of service. He holds the Ph.D. degree in mathematics from Purdue University. His current interests include stochastic flow networks, network resiliency and critical infrastructure protection, and stochastic processes in reliability and performance modeling. Mike has been a judge at the MCM since 1993 and particularly enjoys the MCM problems that have a practical flavor of mathematical analysis of social policy. Mike enjoys amateur radio, playing the piano, and cycling.

Judge's Commentary: The Ben Fusaro Award for 2014

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Introduction

The Ben Fusaro Award honors the Founding Director of the MCM (who continues to serve as a judge for the contest). First awarded in 2004, it recognizes an entry for an "especially creative approach" to the A Problem in the contest, which generally involves continuous mathematics.

The Fusaro Award for 2014 goes to a team from Tsinghua University, Beijing, China, for their paper titled "Keep Right to Keep 'Right.'" This paper, one of the top group of papers designated as Outstanding, stands out for a remarkably flexible model, combined with exceptional research, detailed analysis, and clear writing.

The Problem

The Keep-Right-Except-To-Pass-Rule Problem asked teams to build and analyze a mathematical model to analyze the performance of this rule in light and in heavy traffic. Is this rule effective in promoting greater throughput? If not, teams were to suggest and analyze alternatives that might promote greater throughput, safety, and/or other factors that they deemed important.

My Comments

I offer my comments on the paper, organized by the categories deemed appropriate for this problem by the judges, who gave the most credit for

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model development, analysis / validation, and conclusions.

Summary

As always, a strong summary is essential. The abstract does a thorough job of describing the breadth of their model and methods. However, such a sophisticated model should include more information about the team's conclusions, besides the one sentence about which lane-changing rules work best. That sentence, their main conclusion, deserves to be featured more prominently in the summary.

Format, Clarity, Writing

These are generally excellent in this entry. This paper is a pleasure to read, and it makes many interesting observations. The numerous charts and tables are informative. The organization is very good, particularly for a paper written over just a few days.

One quibble is that the reader must hunt through the paper to find the conclusions. One wishes there were a section at the end to review the conclusions and bring closure to the whole project. An executive summary would be ideal.

Model Development

Like many of the entries on the Keep Right Problem, this paper adapts the cellular automaton model for traffic flow from the literature, called the Nagel-Schreckenberg Model (or N-S model, for short). While many teams chiefly analyze a 2-lane model, similar to the N-S models in the literature, this paper treats 3- and 4-lane traffic throughout, which is something the judges wanted to see.

Their "basic model" includes buses and trucks, not just cars, which is nice. They do a good job justifying their periodic boundary conditions, which is equivalent to saying their road is treated as a large ring. Their "refined model" incorporates entrance and exit ramps, another important feature only the very best entries treated well.

They introduce 9 different rules for lane-changing, which is an exceptional number, and they successfully compare them by the multiple-criteria-decision-making tool called Fuzzy Synthetic Evaluation (FSE). While FSE is not familiar to most judges, the team gives a reasonable explanation of it.

Models are compared under conditions of both light and heavy traffic, which is very important in the analysis.

Analysis/Validation

The basic model is studied from several perspectives, including average speed, utilization of the different lanes, and driver satisfaction. The imposition of a maximum speed limit is studied, as is a minimum speed limit. An exceptional factor the team considers is signaling behavior.

A sophisticated multiple-criteria evaluation method (FSE) is used to compare 9 different lane-changing rules. This is impressive.

The paper includes sections that discuss left-hand side driving as well as intelligent systems. These are concise, and simulations are run. (Some papers went further with these topics.)

However, the team's analysis is sensible. A strength of the paper is that the team tries to explain the conclusions suggested by their simulations.

Conclusions, Extras

This study involves abundant simulations that appear to address all of the issues raised in the problem, which is extraordinary. It includes many extras, particularly the variety of lane-changing rules. As noted before, there could have been better closure.

Sensitivity Analysis

The paper includes extensive analysis of several parameters in the model, more than almost all other entries did.

Strengths/Weaknesses

The paper concludes with a collection of relevant observations. It is clear that with more time and effort the models could be expanded to encompass many more features.

Research

We mention one more factor that was not a separate category for the judges: Research.

This is another strength of this paper. It references several papers on the N-S model. It lists several Wikipedia articles on traffic, a traffic engineering text, and an engineering article using FSE. The team appears to draw useful information from all of these sources, with numerous literature citations in their text. By comparison, even many of the very good papers in the contest do a less-thorough literature search and / or fail to properly cite the literature when they take ideas from it.

About the Author

Dr. Griggs is Carolina Distinguished Professor of Mathematics at the University of South Carolina, where he has supervised 15 doctoral dissertations as well as several master's and undergraduate theses. He is a Fellow of SIAM (the Society for Industrial and Applied Math), which he has served in various roles, including two terms as Editor-in-Chief of the *SIAM Journal on Discrete Mathematics*. He has judged the MCM since 1988, and he wrote or co-wrote six MCM problems.

Evaluation System for College Coaching Legends

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Abstract

To evaluate the performance (evaluation grade) of a coach, we formulate metrics on five aspects: historical record, game gold content, playoff performance, honors and contribution to the sport. Moreover, each metric is subdivided into several secondary metrics, making for a three-tier hierarchical structure. Take playoff performance as an example: We collect post-season results (Sweet Sixteen, Final Four, etc.) each year from the NCAA official Website, from Wikimedia, and so on.

First, we use the Analytic Hierarchy Process (AHP) to determine the weight of each metric on a coach's evaluation. Second, we use Fuzzy Synthetic Evaluation (FSE) to overcome the weakness of excessively subjective factors in AHP. The FSE model is based on data and generates a fuzzy matrix. After that, we apply the entropy method and linear weights to obtain the coaches' evaluation grades.

To evaluate the accuracy of the two models, we define *hit score* to reflect the difference between our results and standard rankings from several authorities, such as ESPN and Sporting News. Take NCAA basketball as a case study: AHP receives a 78.8 hit score while FSE gets 81.8, which indicates that FSE performs better than AHP. Afterwards, we develop an Aggregation Model (AM) combining the two models based on hit score. The top 5 college basketball coaches, in order, are John Wooden, Mike Krzyzewski, Adolph Rupp, Dean Smith and Bob Knight.

The time horizon does make a difference. According to turning points in NCAA history, we divide the previous century into six periods with different time weights, which leads to changes in the rankings.

We apply our model to college women's basketball only to find that gender does not matter.

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The model proves to be effective in other sports. The ranking of college football coaches is: Bear Bryant, Knute Rockne, Tom Osborne, Joe Paterno, Bobby Bowden; and the top 5 coaches in college hockey are Bob Johnson, Red Berenson, Jack Parker, Jerry York, Ron Mason.

We conduct sensitivity analysis on FSE to find the best membership function and calculation rule, and also on the aggregation weight. We find that AM performs better than either AHP or FSE alone. As a creative use, we apply AM to pick the top 3 U.S. presidents: Abraham Lincoln, George Washington, Franklin D. Roosevelt.

We discuss the strengths and weaknesses of our model and present a nontechnical explanation of it.

Introduction

Problem Background

Sports Illustrated is an American sports media franchise owned by media conglomerate Time Warner. The magazine *Sports Illustrated* is looking for the "best all time college coach," male or female, over the previous century, in any sport.

We face mainly four problems:

- Articulate our own metrics and build a mathematical model.
- Set up the evaluation system for the performance of the model.
- Discuss how our model can be applied with a time factor taken into account, or across both genders and all possible sports.
- Analyze the influences of the parameters, then discuss whether our model could be applied more widely.

Previous Research

Finding the best college coaches is an evaluation problem, and there are models to solve such problems. One is the Analytic Hierarchy Process (AHP), developed by Thomas L. Saaty [2008]. The AHP provides a comprehensive and rational framework for structuring a decision problem, representing and quantifying its elements, relating those elements to overall goals, and evaluating alternative solutions [Wikipedia 2014a].

Another model is the Fuzzy Synthetic Evaluation Model (FSE). Fuzzy mathematics is a branch of mathematics related to fuzzy set theory and fuzzy logic [Wikipedia 2014b; Zadeh 1965].

Our Work

We determine the best college coaches, male or female, in different sports. We begin with terminology, definitions, and assumptions. We then define our evaluation standard and the specific evaluation norms that we use in our models, and show some of the data that we collected.

We build two mathematical models to choose the best college coaches, and then consider a combination of the two. We extend our models further to take time, gender, and sport into consideration.

Finally, we provide an overview of our approach and give a nontechnical explanation of our models that sports fans will understand.

Symbols, Definitions and Assumptions

Symbols and Definitions

General Assumptions

- The elements that we take into consideration play a vital role in the evaluation.
- Elements that we ignore do not influence the ranking.
- The data that we have collected is sufficient and accurate.
- There exists an objective and accurate ranking for coaches, and the rankings from selected media reflect the accurate ranking to some extent.

Articulate Metrics

Specify Evaluation Norms

As for the evaluation standard for players, there are mainly five aspects that count: strength, speed, skill, defense, and offense. Similarly, a coach could be evaluated from five aspects: historical record, game gold content, playoff performance, honors, and contribution to the sports.

Historical record:

The team's record undoubtedly accounts for the largest proportion in the coach evaluation. The team's record of total wins a and total losses b could directly reflect coaching ability.

Table 1.
Symbols.

Symbol	Definition
Symbols for evaluation norms	
a_i	wins in year i
b_i	losses in year i
R	average SRS
O	average SOS
n_k	number of times for each class of playoff
k_l	weight of each award
c_i	point for each aspect of contribution
Symbols for Analytic Hierarchy Process	
A	judging matrix
$\lambda_{\max}$	greatest eigenvalue of A
CI	indicator of consistency check
CR	consistency ratio
RI	random consistency index
CW	weight vector for criteria level
AW	weight vector for components level
Y_1	evaluation grade for model I
Symbols for Fuzzy Synthetic Evaluation	
X_i	grades for each aspect
$\mu_j(X_{ij})$	membership function
X_f	the fuzzy matrix
p_{ij}	characteristic weight
e_j	entropy for the evaluation grade
EW	weight vector in entropy method
Y_2	evaluation grade for model II
Symbols for Aggregation Model	
D	average offset distance
W_I	weight for model I
Y	evaluation grade for aggregation model

Game gold content:

If all wins are against weak teams, the wins could not illustrate the real coaching ability. At the same time, the average point difference also makes a difference; it reflects the coaching style, whether a coach is conservative or radical. We choose the following two norms:

- **Simple Rating System (SRS):** The simple rating system works by first finding how many points, on average, a team wins/loses by. For each game, the point differential is then weighted based on how much better or worse than average the point differential is. Let R denote the total SRS:

$$R = \frac{\sum_i SRS_i}{t},$$

where SRS_i is the SRS value for year i and t is the number of years.

- **Strength of Schedule (SOS):** Strength of schedule (SOS) refers to the difficulty of beating the opponent as compared to other teams' difficulty in doing so [Wikipedia 2014d]. This criterion is especially important if teams in a league do not play each other the same number of times. Let O denote the total SOS:

$$O = \frac{\sum_i SOS_i}{t},$$

where SOS_i is the SOS value for year i and t is the number of years.

Playoff performance:

Generally, during the regular season, teams play more games in their conference than outside it, but the country's best teams might not play against each other in the regular season. Therefore, post-season playoff performance, in terms of the rounds reached, is important in evaluating the coach. To quantify the aspect, we count the number of times n_k that the coach's team(s) reached round k of the playoffs. Let $m_{kl} = 1$ if the team reaches round k in year i and 0 otherwise. Then we have

$$n_k = \sum_i m_{ki}.$$

Honors:

There are various honors, such as the Basketball Hall of Fame and the College Basketball Hall of Fame, in addition to such awards as the Naismith College Coach of the Year, Basketball Times National Coach of the Year, and so on. Different awards have different gold content. Let k_i be the weight of an award in year i. The total weights of all the awards for a coach is

$$H = \sum_i k_i.$$

Contribution to sports:

We divide the contribution into five parts:

- **Star players:** Count how many star players the coach has trained.
- **Coaching age:** When the coaching career started and how long it lasted.
- **Tactical innovation:** Did the coach invent any tactical innovations?
- **Performance in international competitions:** Has the coach ever coached in international competitions? If so, how many gold or silver medals?
- **Popularity:** The number of the results when the coach's name is searched in Google.

We assign points for each aspect above: 0 for mediocre, 1 for good, 2 for excellent, then add the points up to form the final grade in this aspect (the full mark is 10):

$$C = \sum_i c_i.$$

Figure 1 illustrates the evaluation norms.

Figure 1. First-level evaluation criteria.

Collect Data

We use men's basketball as an example. We choose the 70 coaches in the National Collegiate Basketball Hall of Fame [Wikipedia 2014c]. We also select five other college coaches who are not in the Hall of Fame but still made significant contributions.

Combining data from Sports Reference [2013], a Website with specific data about coaches, with statistics from searching Wikipedia, we arrive at relative statistics for those 75 college coaches.

Figure 2. Second-level evaluation criteria.

In **Table 2**, FR, SR, SS, EE, FF, RU, and CH refer to rounds achieved in the NCAA Basketball Tournament: First Round, Second Round, Sweet Sixteen, Elite Eight, Final Four, Runner-Up, and Champion.

Table 2.
Sample from data on basketball coaches.

Coach	from	to	years	wins	losses	SRS	SOS	FR	SR	SS	EE	FF	RU	CH
Jim Boeheim	1905	1995	48	719	259	15.81	7.27	5	8	11	2	1	2	1
Jim Calhoun	1972	2001	40	877	382	12.64	4.74	5	5	4	5	1	0	3
Larry Brown	1979	2013	9	210	83	13.08	5.95	0	3	1	0	1	1	1
⋮														
Mike Krzyzewski	1975	2013	39	975	302	20.16	8.78	2	6	6	2	3	4	4

We also collected the college basketball coaching record for each season for every coach. In **Table 3**, we show Larry Brown as an example.

Table 3.
Sample coaching record, for Larry Brown.

Season	wins	losses	SRS	SOS	AP Pre	AP High	AP Final	Result
1979-80	22	10	15.67	6.1	8	7	—	NCAA Runner-up
1980-81	20	7	14.89	5.26	6	3	10	NCAA Second Round
1983-84	22	10	9.76	5.86	17	17	—	NCAA Second Round
1984-85	26	8	11.84	6.27	19	9Z	13	NCAA Second Round
1985-86	35	4	23.18	10.42	5	2	2	NCAA Final Four
1986-87	25	11	13.36	7.73	8	6	20	NCAA Sweet Sixteen
1987-88	27	11	15.71	10.77	7	7		NCAA Champions
2012-13	15	17	−0.59	−1.33	—	—	—	—
2013-14	18	5	13.88	2.45	—	—	—	—

Preprocess the Data

When we collect data from the Internet, we notice that some data are missing—e.g., SRS and SOS. Given that fact, we have to preprocess the data. We fill in the data mainly based on interpolation according to the ranking generated by the other metrics.

Two Models for Coach Ranking

Model I: Analytic Hierarchy Process (AHP)

When we try to obtain the weight of the five aspects of the first-level evaluation and the weight of several second-level evaluation criteria, subjective judgment is ill-considered. So we choose the Analytic Hierarchy Process (AHP) as the way to combine the weighting coefficients of all the indicators in the evaluation system.

The three-level hierarchy structure

The three-level hierarchy structure that contains the criteria level and components level is shown in **Table 4**.

Table 4.
The three-hierarchy structure of our model.

Goal	Criteria	Components
Influence of the coach	Historical Record	Wins Losses
	Game Gold Content	SRS SOS
	Playoff Performance	First Round . . . Champion
	Honors	Different Awards Hall of Fame
	Contribution to sports	Star Player Coaching Age Tactical Innovation International Games Popularity

Obtain the index weights

- Determine the judging matrix. We use the pairwise-comparison method and 1–9 method of AHP to construct the judging matrix $A = (a_{ij})$:

$$a_{ij} = a_{ik}a_{kj},$$

where a_{ij} is set according to the 1–9 method.

- Calculate the eigenvalues and eigenvectors. The greatest eigenvalue $\lambda_{\max}$ of matrix A has corresponding eigenvector $u = (u_1, \ldots, u_n)^T$. Then we normalize u by

$$x_i = \frac{u_i}{\sum_j u_j}.$$

- Do a consistency check. The indicator of consistency is

$$CI = \frac{\lambda_{\max} - n}{n - 1},$$

where n is the dimension of the matrix.

The expression of consistency ratio is

$$CR = \frac{CI}{RI}.$$

Having confirmed the weighting coefficients of all the indicators in the evaluation system, now we quantify the importance of coaches.

CW_i denotes the weight of criteria level factor i, where AW_j is the weight of secondary critical level factor j for the ith critical level, mi denotes the total number of secondary critical factors, and F_j denotes the secondary critical level factor.

The evaluation grade Y_1 should be

$$Y_1 = \sum_{i=1}^{5} CW_i \sum_{j=1}^{mi} AW_j F_j.$$

Results and analysis

We obtain the following results:

- Judging matrix:

$$A = \begin{bmatrix} 1 & 5 & 5/9 & 1 & 1 \\ 1/5 & 1 & 1/9 & 1/5 & 1/5 \\ 7/5 & 7 & 1 & 7/5 & 9/5 \\ 1 & 5 & 5/7 & 1 & 9/5 \\ 1 & 5 & 6/7 & 1/5 & 1 \end{bmatrix}$$

- Weight vector of criteria level:

$$CW = [0.1996\ \ 0.0399\ \ 0.3093\ \ 0.2419\ \ 0.2092].$$

  For this level, $CI = 0.301$, $CR = 0.0269$, satisfying the criterion for consistency of $CI/RI < 0.1$.

- Weight vector of components level:
  - Historical Record: $AW_1 = [1.5\ \ -0.5]$.
  - Game Gold Content: $AW_2 = [0.75\ \ 0.25]$.
  - Playoff Performance:

$$AW_3 = [0.0079\ \ 0.0157\ \ 0.0315\ \ 0.126\ \ 0.252\ 0.5039].$$

All of these weight vectors satisfy $CI/RI < 0.1$.

As for Honors and Contribution to the Sport, we have taken so many awards and factors into consideration, making it hard to determine a weight vector. We arrive at an approximate solution by assigning equal weights to these factors.

Finally, we can obtain the final rankings of the top 10 college basketball coaches using the AHP model.

Table 5.

The top 10 college basketball coaches' grades.

Rank	Name	Grade (Y_1)	Rank	Name	Grade (Y_1)
1	Mike Krzyzewski	0.8426	6	Roy Williams	0.5637
2	John Wooden	0.7334	7	Bob Knight	0.5479
3	Adolph Rupp	0.6048	8	Phog Allen	0.4788
4	Jim Boeheim	0.5985	9	Rick Pitino	0.4683
5	Dean Smith	0.5844	10	Lute Olson	0.4132

Conclusions

- Analyzing the weight vector of criteria level, the highest weight is for Playoff Performance.
- SOS plays a less-important role than SRS in determining the Game Gold Content, and the weight of the Game Gold Content is the lowest.

Model II: Fuzzy Synthetic Evaluation (FSE)

Quantify grades in the five aspects

Fuzzy set theory was designed to supplement the interpretation of linguistic or measured uncertainties for real-world random phenomena.

We have already articulated our metrics for ranking the five aspects: historical record, game gold content, playoff performance, honors, and contribution to sports. Before using fuzzy set theory, we calculate the grades in each of the five aspects using the collected data.

- Calculation rule for historical record:

$$X_1 = \lambda_{\text{win}} a - \lambda_{\text{lose}} b,$$

  where a is the number of wins, b is the number of loses, λ_{win} is the weight for a single win, and λ_{lose} s the weight for single loss:

- Calculation rule for game gold content:

$$X_2 = R\left(1 + \frac{O}{O_{\max}}\right),$$

  where R is the value of SRS, O is the value of SOS, and $O_{\max}$ is the maximum value of SOS in the Strength of Schedule system.

  A coach with higher SRS will also have higher grade in this aspect because the team is always far ahead of its opponents. At the same time, the higher SOS is, the harder games are. So we let the SOS be an addition to SRS.

- Calculation rule for playoff performance:

$$X_3 = \sum_{k=1}^{7} 2^k n_k,$$

  where n_k is the number of appearances for level k of the playoffs.

  In the playoffs, the number of teams decreases by half from one level to the next, hence the weight increases exponentially by a factor of 2.

- Calculation rule for honors:

$$X_4 = H,$$

  where H counts up the awards by weight.

- Calculation rule for contribution to sports:

$$X_5 = C.$$

Determine membership functions

A fuzzy set is defined in terms of a membership function that maps the domain of interest onto the interval $[0, 1]$. The value of the membership function represents the degree, or weighting, that the domain item belongs to the set.

Let X_{ij} denote the X_j value for coach i and $X_{j(\max)}$ be the maximum value for all the coaches. Here we use the normalization function as membership function:

$$\mu_{ij}(X_{ij}) = \frac{X_{ij}}{X_{j(\max)}}.$$

Let N be the total number of coaches. Then we have the $N \times 5$ matrix

$$X_f = \begin{bmatrix} \mu_1(X_{1,1}) & \dots & \mu_1(X_{1,5}) \\ \vdots & \mu_1(X_{i,j}) & \vdots \\ \mu_1(X_{N,1}) & \dots & \mu_N(X_{N,1}) \end{bmatrix}.$$

Determine the weights using entropy method

The entropy method [Dahiya et al. 2007] states that, subject to precisely stated prior data (such as a proposition that expresses testable information), the probability distribution that best represents the current state of knowledge is the one with largest entropy.

To use the entropy method, there are 5 steps:

- Calculate the characteristic weight p_{ij} for the ith coach's jth evaluation grade (X_{ij}) based on the normalized fuzzy matrix

$$p_{ij} = \frac{X_{f(i,j)}}{\sum_{i=1}^{N} X_{f(i,j)}}.$$

- Calculate the entropy for the evaluation grade:

$$e_j = \frac{-1}{\ln N} \sum_{i=1}^{N} p_{ij} \ln p_{ij}.$$

- Calculate the diversity factor for the evaluation grade:

$$g_j = 1 - e_j.$$

- Determine the weight for each evaluation grade:

$$w_j = \frac{g_j}{\sum_{j=1}^{5} g_j}.$$

- Determine the FSE evaluation grade for each coach:

$$Y_2 = WX_f.$$

Table 6.
Results of FSE analysis using entropy.

	X_1	X_2	X_3	X_4	X_5	Y_2
John Wooden	0.04	0.06	0.15	0.13	0.08	0.871
Mike Krzyzewski	0.07	0.06	0.10	0.13	0.08	0.863
Adolph Rupp	0.06	0.06	0.08	0.11	0.02	0.675
Dean Smith	0.06	0.06	0.08	0.04	0.05	0.609
Bob Knight	0.05	0.06	0.06	0.07	0.05	0.605
Roy Williams	0.06	0.05	0.06	0.04	0.08	0.587
Jim Boeheim	0.06	0.03	0.05	0.03	0.01	0.586
Phog Allen	0.04	0.04	0.05	0.05	0.05	0.487
Henry Iba	0.04	0.04	0.04	0.04	0.02	0.466
Lute Olson	0.06	0.04	0.04	0.06	0.077	0.454
g_j	-0.99	-1.00	-1.04	-0.96	-0.93	0.434
w_j	0.18	0.16	0.23	0.20	0.22	0.871

Results and analysis

Characteristic weight, entropy, diversity factor and weight are shown in **Table 6**. We find:

- The weights for each aspect are close to one another.
- Playoff performance (X_3) plays the most important role (with 0.23 weight) in FSE evaluation.
- At the same time, coaches who have amazing game gold content (with only 0.16 weight) might not stand out.

Models Combination

Evaluation of Individual Model

To compare our two models (AHP and FSE), we define the *average offset distance D*.

We collect ranked lists of the top 10 NCAA basketball coaches from several authoritative media such as ESPN, Bleacher Report, Yahoo Sports, and Sporting News (e.g., Merron [2009]). We compare our results to those lists, and average offset distance reflects the difference.

We use the first-order Minkowski distance to denote the average offset distance of the top 10:

$$D = \frac{1}{10n} \sum_{i=1}^{10} \sum_{j=1}^{5} |j - r_j|,$$

where

- n is the number of top-10 ranking lists,
- j is the rank in the ith list, and
- r_j is the ranking of that jth coach in our results.

So $|j - r_j|$ is the difference between the result in the media and ours, and D gives the average difference.

D_α is the average offset distance of the top 5, and D_β is the average offset distance of 6th through 10th.

We define *hit score* as

$$g = \frac{900}{9 + D}, \qquad 0 < g < 100.$$

Table 7.

Results for offset distance.

	AHP	FSE
D_α	1.75	1.15
D_β	3.10	2.85
D	2.4	2.0
g_α	83.7	88.7
g_β	73.4	75.9
g	78.8	81.8

Conclusions

From the results in **Table 7**, we conclude:

- Vertical comparison: For both AHP and FSE, $D_\alpha < D_\beta$, meaning that the results are more reasonable for the top 5 than for the top 10.
- Horizontal comparison: FSE performs better than AHP in both the top 5 and the top 10.

Aggregation Model

AHP is a subjective method, it largely depends on artificial scoring; relatively, Fuzzy Synthetic Evaluation is an objective method, it depends on data. To comprehensively consider the effect of subjective and objective factors, we adopt a linear weighted method:

$$Y = wY_1 + (1 - w)Y_2,$$

where w and $(1 - w)$ are weights that add to 1, Y_1 is the evaluation grade from the AHP model, and Y_2 is the evaluation grade from the FSE model.

To determine the weights, we take D (average offset distance) into consideration. Since smaller average offset distance means the more accurate results, we assign higher weight to the mode with smaller D. Then we get

$$w = \frac{D_2}{D_1 + D_2}.$$

Results and Analysis

Table 8.

Rankings according to the different models.

Rank	AHP	FSE	AM
1	Mike Krzyzewski	John Wooden	John Wooden
2	John Wooden	Mike Krzyzewski	Mike Krzyzewski
3	Adolph Rupp	Adolph Rupp	Adolph Rupp
4	Jim Boeheim	Dean Smith	Dean Smith
5	Dean Smith	Bob Knight	Bob Knight
6	Roy Williams	Roy Williams	Jim Boeheim
7	Bob Knight	Jim Boeheim	Roy Williams
8	Phog Allen	Phog Allen	Phog Allen
9	Rick Pitino	Rick Pitino	Rick Pitino
10	Lute Olson	Henry Iba	Henry Iba

Table 9.

Ranking comparison among the models.

	AHP	FSE	AM
Top 5 hit score	83.7	88.7	88.7
Top 10 hit score	78.8	81.8	82.6

Conclusion

- All our models perform better for the top 5 than the top 10. This fact shows that the top 5 coaches in college basketball history are less controversial than the top 10.
- The results of AM and FSE are very similar. They have the same hit score for the top 5; but for the top 10, AM has a higher hit score. These results shows that using the combination can improve our model.
- Our model AM's final result is: The top 5 coaches in college basketball are John Wooden, Mike Krzyzewski, Adolph Rupp, Dean Smith, and Bob Knight.

Extend Our Models

Gender Does Not Matter

Now we take gender into consideration. We still use basketball as an example, and rank the top 10 college women's basketball coaches for the previous century. Searching the Internet, we collected data on about 50 college women's basketball coaches with 600 wins [Wikipedia 2014e] and 5 other coaches who have established outstanding traditions, earned many awards, and garnered recognition for their colleges. Then we ranked them with our models. In **Table 10**, we compare with the ranking at Yahoo [Michael 2010].

Table 10.
Women's coaches ranked by our AM model and as ranked by Yahoo.

Rank	AM	AM grade	Yahoo
1	Pat Summitt	0.85	Pat Summitt
2	Geno Auriemma	0.84	Geno Auriemma
3	Tara VanDerveer	0.75	Leon Barmore
4	Leon Barmore	0.72	C. Vivian Stringer
5	C. Vivian Stringer	0.61	Tara VanDerveer
6	Sylvia Hatchell	0.59	Jody Conradt
7	Jody Conradt	0.57	Kay Yow
8	Kay Yow	0.55	Gail Goestenkors
9	Sue Gunter	0.48	Sylvia Hatchell
10	Gail Goestenkors	0.44	Sue Gunter

Using the average offset distance, all results of our models are in agreement within reasonable error range (hit score = 87.6), so that we can safely conclude that our models can be applied in general across both genders.

Time Factor Does Make a Difference

Why does time factor matter?

The NCAA Basketball Tournament started in 1939. In the years since, the number of teams participating has increased, the competition has become fiercer, and the tournament has gained in popularity—all of which influence the quality of the evaluation grades.

To quantify the time factor, we attach a weight (1 to 10) to different time periods, mainly based on the turning points that occurred in the period.

Tale 11 shows the critical years in the NCAA history [Wikipedia 2014f].

Table 11.
Weights for different periods in NCAA history.

Years	Turning points	Weight w_i
1913–1939	No national tournament.	5
1939–1951	Two college tournaments: NIT and NCAA, with 8 teams in NCAA.	6
1951–1975	16 teams in NCAA, NIT became second-class competition.	7
1975–1980	32 teams.	8
1980–1985	48 teams.	9
1985–2014	64 teams (plus play-ins).	10

How does the time factor matter?

The weights w_i must be applied in calculating a, b, R, O, and n_k.

For AHP, the top 5 and top 10 hit scores remain nearly unchanged, but several adjoining coaches with close grades change places.

For FSE, both the top 5 and the top 10 hit scores decrease somewhat, and there are larger changes in the rankings. The model appears to be easily influenced by time weights.

The AM model too appears to be easily influenced by time weights, because of the weighting of FSE.

Table 12 shows the final ranking according to AM. Every coach's rank changes except for Bobby Knight at rank 5.

Table 12.
Ranking according to AM without and with time weights.

AM without weight	Grade	AM with weight	Grade
John Wooden	0.857	Mike Krzyzewski	0.920
Mike Krzyzewski	0.830	John Wooden	0.778
Adolph Rupp	0.654	Roy Williams	0.643
Dean Smith	0.602	Jim Boeheim	0.632
Bob Knight	0.590	Bob Knight	0.612
Jim Boeheim	0.588	Dean Smith	0.606
Roy Williams	0.580	Adolph Rupp	0.571
Phog Allen	0.485	Rick Pitino	0.517
Rick Pitino	0.467	Lute Olson	0.460
Henry Iba	0.431	Phog Allen	0.435
top 10 hit score	82.6	top 10 hit score	76.6
top 5 hit score	88.7	top 5 hit score	85.5

What is the effect of considering time?

- The rankings of coaches in earlier eras fall to some extent. Take Phog Allen, for example. He is known as the "Father of Basketball Coaching," but most of his games occurred in 1920–1959, which means that the

NCAA had not started or though started the teams were few. The time weight for that era is relatively low, thus making his ranking fall.

- Coaches in recent years enjoy some superiority. Take Roy Williams and Adolph Rupp, for example. The two coaches' performance are quite close to each other. Rupp was even better in the historical record; but due to the time weight, the historical record for Adolph Rupp does not count that much, and Roy Williams is ahead of him.
- Introducing time weights does not necessarily mean a higher hit score.

The Model Works in Other Sports, Too

There are four steps to apply the model in any sport as you want.

- Step 1: Adjust the metrics according to the sport.
  Different sports may have different playoff rules, so the metric for Playoff Performance should be adapted. Take football, for example: A ranking of playoff bowl games can replace round of the NCAA Basketball tournament.
- Step 2: Adapt the calculation rules according to the feature of the sports. For example, for football, Bowl game should be assigned another weight according to its gold content.
- Step 3: Adjust the time weight according to the history of the sport. For example, for football, before 2006, there was no BSC Bowl.
- Step 4: Solve the aggregation model again and analyze the results.

Following the four steps presented above, we apply the model to determine the top 5 coaches in other two other sports, football and hockey (**Table 13**).

Table 13.

The top 5 coaches in football and in hockey.

Football	Grade	Hockey	Grade
Bear Bryant	0.887	BobJohnson	0.896
Knute Rockne	0.866	RedBerenson	0.873
Tom Osborne	0.854	JackParker	0.853
Joe Paterno	0.787	JerryYork	0.776
Bobby Bowden	0.786	RonMason	0.763

Further Discussion

Sensitivity Analysis for FSE

Vary membership function

For FSE, there are also other available membership functions.

[EDITOR'S NOTE: The authors examine a parametrized version of their original membership function, as well as parametrized versions of two alternative membership functions. They find that the membership function

$$\mu_j(X_{ij}) = \left(\frac{X_{ij}}{X_{j(\max)}}\right)^k$$

with $k = 3$ is the most appropriate in terms of effect on hit score.]

Vary calculation rule

Here we focus on figuring out how the hit score will change with the ratio $\lambda_{\text{win}}/\lambda_{\text{lose}}$, that is, the relative benefit of a win compared to a loss.

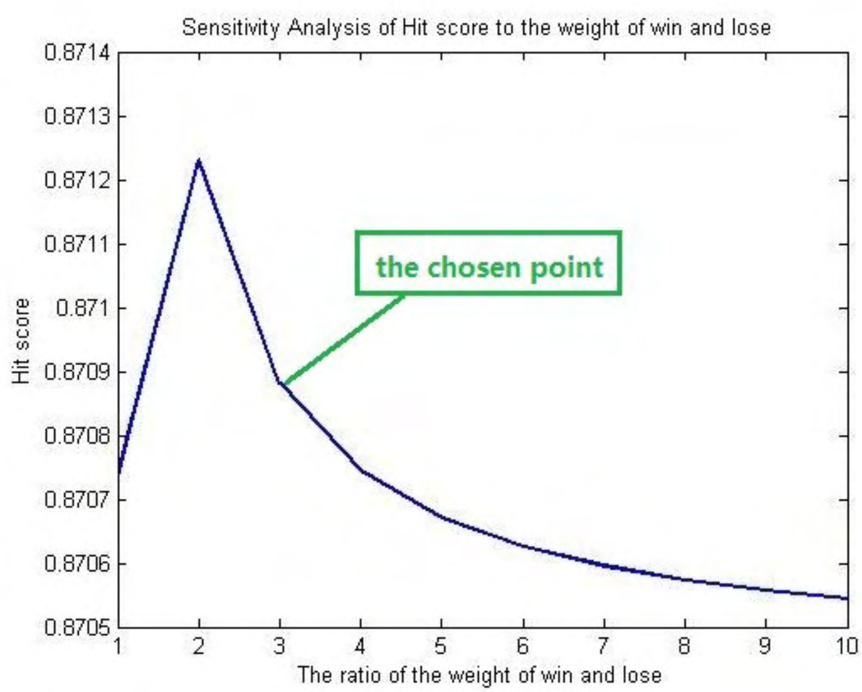


Figure 3. Sensitivity analysis to varying the weights of winning and losing.

If we attach the same weight to winning a game and losing a game, the model will have a poor hit score. If the ratio of the weight of wins and losses is too high, that too will also lead to a bad result. We conclude that the model performs best when the weight of winning a game is *twice* that of losing a game.

Sensitivity Analysis on Aggregation Weight

We analyze how hit score (for AM) and rankings change with varying the weight for AHP (w).

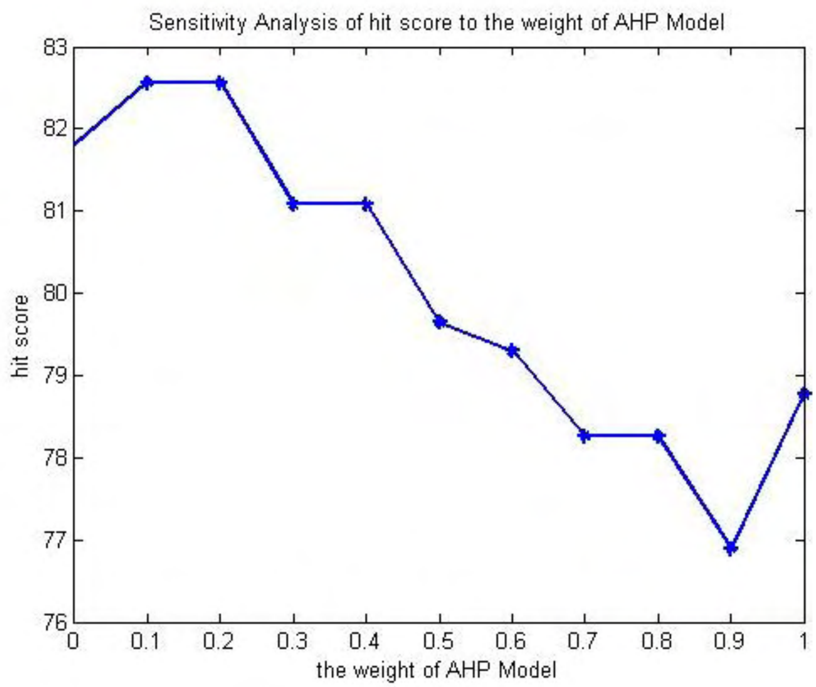


Figure 4. Sensitivity analysis of hit score to weight of AHP.

Since AHP is less accurate than FSE, the hit score of AM would be optimal when the weight of AHP is small. But when the weight of AHP is zero, the hit score doesn't reach the maximum; the maximum hit score is reached when the weight of AHP is 0.1–0.2.

Exploration: Evaluating the Best President

Now we use our models to find the top 10 presidents of the United States. We collect relative data from the Internet [Wikipedia 2014g].

A president can be evaluated on five aspects: personal qualities, presidential achievements, leadership qualities, failures and faults, and popular opinion (**Figure 5**).

The personal qualities include imagination, intelligence and being willing to take risks, while the presidential achievements can be valued in terms of domestic accomplishments, executive appointments, foreign policy accomplishments, and ability to compromise. Leadership qualities can be measured by party leadership ability and relations with Congress. We also take popular opinion into consideration, in terms of polls from C-SPAN, ABC News, Washington College, Gallup, Rasmussen, and 2012 Gallup.

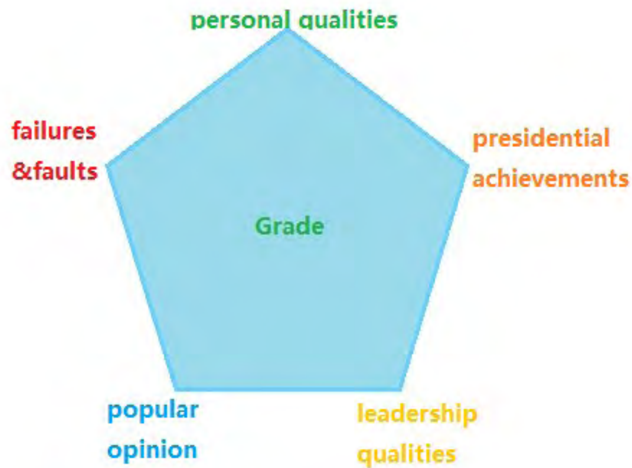


Figure 5. Aspect norms for President of the USA.

The resulting ranking of presidents of the United States is shown in **Table 13**.

Table 13.
Ranking of U.S. presidents.

Rank	Name	Rank	Name
1	Abraham Lincoln	6	Harry S. Truman
2	George Washington	7	Woodrow Wilson
3	Franklin D. Roosevelt	8	Dwight D. Eisenhower
4	Thomas Jefferson	9	James K. Polk
5	Theodore Roosevelt	10	Andrew Jackson

Strengths and Weaknesses

Strengths

- Our metrics for assessment include all the important elements of a coach. Time factor, gender, and category are all discussed in the model.
- We evaluate the performance of a coach from 5 specific perspectives.
- We set up two different models to form an aggregation model (AM). AHP includes more subjective factors, while FSE appears to be more objective. The aggregation model is devoted to make clear the tradeoff between AHP and FSE.

Weaknesses

- We adopt in total 18 indicators to evaluate a coach, but there are still others that we do not take into consideration.
- Weights are everywhere in the model, but some weight assignments might not be the best.

References

Dahiya S., B. Singh, S. Gaur, et al. 2007. Analysis of groundwater quality using fuzzy synthetic evaluation. *Journal of Hazardous Materials* 147 (3): 938–946.

Merron, Jeff. 2009. The Wizard still ranks No. 1. `http://sports.espn.go.com/espn/page2/story?page=list/050304/collegehoopscoaches`.

Michael, Patrick. 2010. Top 10 women's basketball coaches in NCAA history. `http://sports.yahoo.com/ncaa/football/news?slug=ac-7168152`.

Saaty, Thomas L., and Kirti Peniwati. 2008. *Group Decision Making: Drawing out and Reconciling Differences*. Pittsburgh, PA: RWS Publications.

Sports Reference. 2013. `http://www.sports-reference.com/cbb/coaches/`.

Wikipedia. 2014a. Analytic hierarchy process. `http://en.wikipedia.org/wiki/Analytic_hierarchy_process`.

________. 2014b. Fuzzy mathematics. `http://en.wikipedia.org/wiki/Fuzzy_mathematics`.

________. 2014c. National Collegiate Basketball Hall of Fame. `http://en.wikipedia.org/wiki/National_Collegiate_Basketball_Hall_of_Fame`.

________. 2014d. Strength of schedule. `http://en.wikipedia.org/wiki/Strength_of_schedule`.

________. 2014e. List of college women's basketball coaches with 600 wins. `http://en.wikipedia.org/wiki/List_of_college_women's_basketball_coaches_with_600_wins`.

________. 2014f. College basketball. `http://en.wikipedia.org/wiki/College_basketball`.

________. 2014g. Historical rankings of Presidents of the United States. `http://en.wikipedia.org/wiki/Historical_rankings_of_Presidents_of_the_United_States`.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control* 8: 338–353.

Nontechnical Explanation

For better or worse, coaches are often the faces of college sports programs. Different from players, who stay only for a few years, coaches can exert longer influence in the college games. Here is a list of the top 5 coaches in the college basketball, college football, and college hockey:

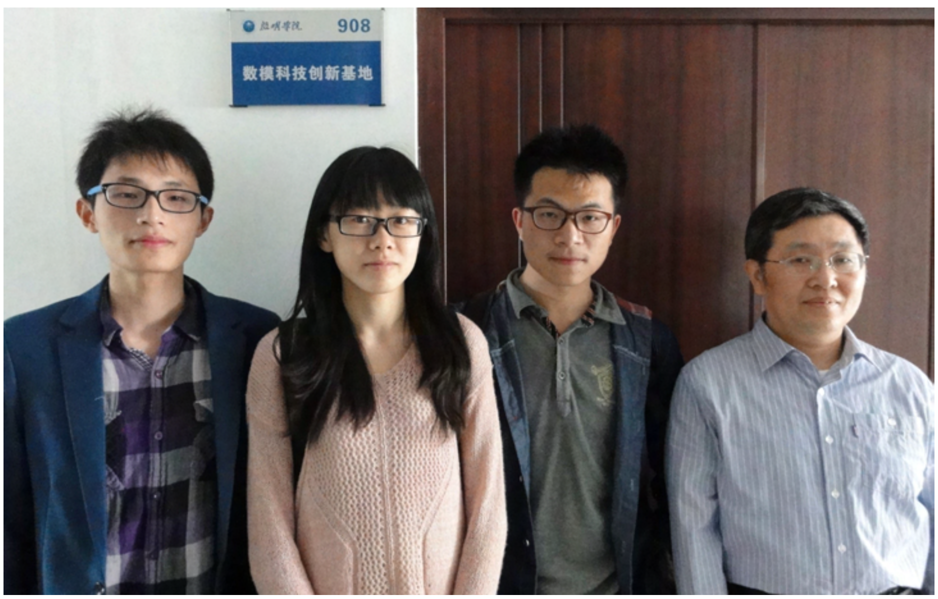
Rank	Basketball	Football	Hockey
1	John Wooden	Bear Bryant	Bob Johnson
2	Mike Krzyzewski	Knute Rockne	Red Berenson
3	Adolph Rupp	Tom Osborne	Jack Parker
4	Dean Smith	Joe Paterno	Jerry York
5	Bob Knight	Bobby Bowden	Ron Mason

The rankings proved to be a difficult task. First, we choose some coaches, who are in the Hall of Fame or who have established outstanding traditions and earned many awards, as our ranking candidates. Then, searching the Internet or other data sources, we try to collect relevant data as detailed as possible. After choosing proper data, we calculate rankings. What's more, we search for existing rankings on the Internet to serve as an evaluation criterion.

We evaluate the coaches in our list of candidates from five aspects. The best college coaches tend to have a good win-loss record. What's more, SRS (Simple Rating System) and SOS (Strength of Schedule) can reflect coaching ability. We also examine each coach's success in the post-season. Taking basketball as an example, the performance could be valued by counting the number of times appearing in the NCAA Tournament as Champion, Runner-up, Final Four, Sweet Sixteen, Second Round, and First Round. In many cases, we take into account coaches' contribution to the sport, as well as honors, such as awards or being in the Hall of Fame.

After collecting and choosing coaches' detailed data, we define the importance of those aspects that can measure coaches' ability, and use the results to give each coach a score. The higher the score, the higher the rank.

We use the data on the best college basketball coach, John Wooden, as an example. In his college coach career, his team won 826 games; and during his sixteen years in the NCAA tournament, he won 10 championships and 12 straight trips to Final Four. John Wooden has been recognized a tremendous number of times for his achievements, including being recognized for his impact on college basketball as a member of the founding class of theNational Collegiate Basketball Hall of Fame. He was also named the Sporting News "Greatest Coach of All Time." With so many honors and awards, Wooden gets the highest score when we rank the coaches and is worthy of the title of best college basketball coach.



Wenchao Ding, Jingling Li, and Feng Xiong, with team advisor Zhibin Han.

Judges' Commentary: The Coach Papers

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The Problem

Sports Illustrated, a magazine for sports enthusiasts, is looking for the "best all-time college coach," male or female, for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer. Does it make a difference which time line horizon that you use in your analysis, that is, does coaching in 1913 differ from coaching in 2013? Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model's top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1–2-page article for *Sports Illustrated* that explains your results and includes a non-technical explanation of your mathematical model that sports fans will understand.

Introduction and Overview

The Coach Problem focused on identifying the factors or metrics for success as a college coach. The problem required students to develop a modeling approach based on these metrics to determine the best coach

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across all sports, gender, and time. In addition, there was the traditional required nontechnical (*Sports Illustrated* article) paper.

I start this commentary with a short review of the mechanics of this year's judging process. I follow the mechanics with a discussion and observations from the judging on various elements of the problem. I then discuss the importance of sensitivity analysis, assumptions and identifying the strengths and weaknesses of a developed model. I finish by addressing some points concerning communication and conclude with a summary.

The Process

Dr. Kelly Black provided an excellent overview of the judging process in his commentary for the Ultimate Brownie Pan Problem of 2013 [2013]. However, I believe it is beneficial to once again review several elements of the process for this year's problem. In general, it is important to understand that the criteria used to identify good papers gradually change as the judging progresses through the triage and final rounds, with the final papers standing out as the best under a wide variety of criteria.

Triage

The primary objective of the triage is to identify the papers that should be given more detailed consideration from the judges.

Every paper is read by at least two judges seeking to determine if the paper contains all of the necessary elements that make it a candidate for more-detailed readings. If a paper addresses all of the issues and appears to have a reasonable model, then judges are likely to identify it as a paper that deserves more attention.

A paper must be clear and concise to do well in the triage, and the paper's summary is critical at this point in the judging. A good summary provides a brief overview of the problem, the paper's structure, and specific results stated in a clear and concise manner. Small things that make a paper stand out include having a table of contents and ensuring that all required questions are addressed in the paper.

Many papers do not do well in the triage because the summary fails to address all of the questions, and the judge decides that a team's efforts will not compare well with the better papers. For example, one critical question overlooked by many papers this year was how their model could be applied across both genders and all possible sports. Fully developing all of the required elements is a critical area often overlooked in papers.

The sensitivity analysis remains one of the weakest elements in many papers, and these papers do not do well during the triage.

In addition, it is vital that the team express their general approach and results as clearly and concisely as possible in the nontechnical position paper. This means providing a broad overview of the problem, the approach, and specific results in clear and concise nontechnical terms. In other words: Can the article be read and understood by someone without an education in mathematics?

These small things make it much easier for a judge to identify the team's effort and for the paper to do well in the triage round. However, the best models and the best effort is not effective if the results are not adequately communicated. It is important to remember that this is a modeling competition and that effective communication is a critical part of the modeling process.

Final

The final consists of multiple rounds of judging over several days. As the rounds progress, the judging criteria shift from identifying papers that warrant further consideration to a process to identify the very best papers.

The first round of the final begins with each judge reading a set of papers and then all judges meeting to discuss the key aspects of the question and what should be included in a "good" paper. This year these aspects included, in addition to all of the required elements:

- a clear discussion of the assessment metrics,
- how and why these metrics were weighted,
- the incorporation of time in the analysis, and
- emphasis placed on the sensitivity analysis portion of the paper.

As the final progresses, each paper is read multiple times, with the final set of papers being read by all judges. In these last rounds, the modeling process and the mathematical integrity of a paper begin to identify the Outstanding papers in the competition.

The Questions

This year's Coach Problem consisted of three major components:

- The first component required teams to determine the mechanism for selecting the best college coach.
- The second component required teams to address the impact of time on their analysis.
- The last component focused on gender as a factor in the best coach selection process and how their models could be applied across all sports.

The Best College Coach

One major aspect a team must address is what is meant by the term "best." Was it number of wins? Was it number of years coaching? Was it popularity? Was it some combination of a set of factors? Many teams did not take the time to clearly develop the purpose of their model or to define "best" but immediately began modeling this aspect of the problem.

In general, teams did not take an approach of developing a generic definition of coaching success. It appears that many teams started their modeling process by first selecting a sport then developing "successful" coaching metrics based on that sport. This approach had a tendency to result in some sport-specific metrics. For example, the number of Bowl Games was a popular metric for college football. This metric becomes a clear problem when attempting to apply the model across all sports, and such a problem should be addressed in the paper.

Better papers first considered carefully the definition of "best" in terms of generic coaching success, then discussed how it could be measured.

The judges were not looking for a specific set of assessment metrics but were looking for those papers that clearly identified and developed their metrics. Most teams developed a set of metrics (anywhere from 5 to 15) that they collectively modeled to develop rankings in each of the three required sports. The better papers tended to develop a set of global metrics that could be applied across all sports and genders and then applied them individually to a set of three different sports.

Many papers treated this requirement as a multi-objective decision-making problem. Popular modeling approaches included the Analytical Hierarchal Process (AHP), Principal Component Analysis (PCA), Fuzzy Comprehensive Evaluation, TOPSIS, Artificial Neural Network, and Dynamic Network Analysis (DNA). However, by far, the most common modeling approach was the AHP. The large volume of papers utilizing the AHP would seem to suggest that this problem was developed for this approach—but that is not the case. There were several very successful papers that did not utilize the AHP approach.

However, many teams, regardless of the modeling approach, failed to recognize the inherit nature of having to assume some sort of weighting mechanism for their metrics in the modeling process. The judges considered the discussion of weighting as a critical criterion for "good" papers. The better papers recognized this, discussed how they developed their weights, and then conducted a sensitivity analysis on their assumed weights.

The Impact of Time

Has the nature of coaching changed over time, or is a coach in 1913 the same as a coach in 2013? An analysis of the impact of time had mixed results and was one of the weaker modeling components in many papers. It appears that many teams ran short of time and provided little analytical effort to this component. The analysis of time was viewed by the judges as a critical criterion for "good" papers. The judges were looking for some recognition that in 100 years of college sports history, the rules of the game, the number of games, the nature and duration of training programs, the social environment, and a host of additional factors have changed—and these changes may influence the metrics used in their model.

Many teams approached this aspect as a time-series analysis problem. These teams evaluated how their model's metrics may have changed over time, to see if there was a correlation between their metric and year. If a team discovered a correlation, typically it was simply noted in the paper that time was important and influenced their model's ranking; but the team usually made no adjustments to their models. The better papers adjusted their metrics, usually the weighting mechanism, and generated a new coach ranking.

Gender

The last question examined how the model could be applied across both genders and all sports. This requirement consisted of two distinct discussion points, addressing both genders and modeling across all sports. Most teams addressed the gender requirement in their paper but were weaker at analyzing how their model could be applied across all sports.

The judges were not looking for any one particular approach to address the gender requirement. Most papers addressed this requirement by modeling a traditional women's college team and ranking the best set of woman coaches. This was presented as evidence that the model worked for both genders. However, a handful of teams used their model to rank a combined list of men- and women-coached teams, developing a top-5 ranking that contained both men and women coaches. The judges viewed this as a superior approach to model the requirement.

In terms of applying the model across all sports, most teams provided their rankings for three different college sports as evidence that the model applied across all sports. Only a handful of papers used the developed model to produce a single ranking that encompassed all sports and genders as evidence. The judges viewed this as a superior approach that went beyond the basic question and requirement of the problem.

Analysis: Assumptions, Sensitivity

The judges realize the limited time available to the teams to complete their models is a considerable constraint, and they do not expect perfect models. However, the judges do expect teams to analyze their models in a structured way and to critically assess their models. A vital part of the mathematical modeling process is this critical analysis of the model. This analysis ranges from examining the impact of the basic assumptions on the modeled conclusions to examining the shortcomings of the techniques employed in the model.

As in previous years, the judging criteria placed a large emphasis on assumptions and sensitivity analysis. Many papers neglected to fully consider these issues and were scored lower by the judges.

Assumptions

The basic assumptions that a team makes is the starting point for their modeling efforts. The judges did not place restrictions on the basic assumptions other than that they need to make sense and be necessary. However, simply listing assumptions is not enough; papers should include a discussion of why they are making the assumption and their potential impact/influence on the model.

It is also important to recognize that stating the assumption is not the end of the process, but that examining the impact on the modeled conclusions when a change in the assumption takes place is a vital part of the modeling process. If changing an assumption results in a change in the coach rankings, then the team should indicate that as a potential weakness.

Sensitivity Analysis

Sensitivity analysis was appropriate and necessary for all modeling approaches. For AHP, sensitivity analysis would have involved varying the weights (or pairwise rankings) to explore what conditions would cause the alternative ranking to change. Many papers included a sensitivity analysis section in their paper but only addressed the theoretical aspects of sensitivity analysis as opposed to actually changing the value of an assumption or parameter to understand the impact.

Communication

Papers were judged on the quality of the writing, with special attention to the summary and to the nontechnical (*Sports Illustrated*) article. In general, the quality of writing is continuing to improve. The strongest summaries this year included a definition of what the team meant by "best," a general overview of the modeling process, and an explicit result of the model analysis.

The judges continue to be surprised by the number of papers where the summary only describes what the team will attempt without telling the results.

Similarly, many of the nontechnical articles focused more on the mathematics and modeling process than on the details of the rankings. A nontechnical article does not mean that numbers are not included. It means that the article can be read meaningfully by someone without an education in advanced mathematics.

Conclusions

The Outstanding teams modeled and presented all the aspects of the problem described in the problem statement, including the fully-developed standard elements (assumptions, sensitivity analysis, strengths and weaknesses, etc.), developed an effective model, explained the modeling choices made, and were clearly and concisely written. The judges continue to be impressed with the quality of the submissions, especially considering the time constraints. The growth in the quality and number of submissions is very encouraging to those who work to promote the practice of good mathematical modeling.

Reference

Black, Kelly. Judges' Commentary: The Ultimate Brownie Pan papers. *The UMAP Journal* 34 (2) (2013): 141–149.

About the Author

Robert Burks is a senior analytic consultant in the Dept. of Defense Analysis at the Naval Postgraduate School. He received his undergraduate degree in Aerospace Engineering from the United States Military Academy, his Master's in Operations Research from the Florida Institute of Technology, and his Ph.D. in Operations Research from the Air Force Institute of Technology. He has wide-ranging research interests, including diffusion of information and epidemiology agent-based modeling. Dr. Burks served as both a triage and final judge on the Coach Problem.

Author's Commentary: The Coach Papers

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Problem B: College Coaching Legends

Sports Illustrated, a magazine for sports enthusiasts, is looking for the "best all-time college coach," male or female, for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer. Does it make a difference which time line horizon that you use in your analysis, that is, does coaching in 1913 differ from coaching in 2013? Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model's top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1–2-page article for *Sports Illustrated* that explains your results and includes a non-technical explanation of your mathematical model that sports fans will understand.

Introduction and Overview

The problem was deliberately written to have a potentially overwhelming amount of data, and to force the student teams to decide what "metrics" they needed to consider to choose the all-time best coach. Good simplifying assumptions could be made so that the problem would be tractable in the

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time allowed yet would still provide useful insights. It was also written so that it could be attempted by students with only lower-division college mathematics.

To recognize the increasing international diversity of the student teams in the MCM, the teams were allowed to choose among all the college sports for either or both male or female athletics. Since the teams participating in the MCM had to analyze a minimum of three college sports, many chose both male and female teams. The most common sports chosen were basketball, football, and baseball. There was some confusion on the part of some international entries, which used professional sports coaches, not college sports coaches. At least one team from the People's Republic of China chose to model college sports in their own country, which was acceptable.

As has been the norm recently, there were required elements in the problem statement. Almost every paper this year included the required nontechnical position paper, although the disparities in the article were remarkable. Any article for a sports magazine that does not contain the *list* of best all-time coaches was considered an extremely poor article!

This commentary will discuss the various elements of the problem, with observations from the judging from the my perspective as the problem's author. I will conclude with a summary.

Readers interested in a discussion of the mechanics of the judging process will find a very good report on the process in [Black 2013].

"Best" College Coach of All Time

Models are constructed to answer questions. Here, we are asked to identify the best all-time college coach. But what did a team mean by "best"? Most teams did not do a very good job of defining what metrics they needed. Most teams dived into the Internet and found lots of data on wins, losses, years coaching, and institutions coached at.

I felt that the number of athletes graduating from the institution with the sports program should be a metric that should be considered—but less than a handful of teams even addressed this point.

The coach's popularity was a variable of interest by many teams. How does one measure popularity over the past 100 years? Again, many teams never stopped to define clearly the purpose of their models, but plunged immediately into chosen models without linking assumptions to the model building process.

The better teams considered carefully what they meant by "best" and included a discussion in their restatement of the problem or elsewhere in the submission.

Models Chosen

Of the 2,871 papers from 606 schools with teams that chose the Coach Problem, well over 90% chose the Analytical Hierarchy Process (AHP) as their model.

AHP is a good tool to rank alternatives based on decision-makers' weights, using pairwise comparison with a consistency ratio less than 0.1. Every one of the papers that used AHP used subjective inputs for obtaining the weights; very few teams did sensitivity analysis on those criterion weights. No teams found the breaking points of the weights that actually altered the ranking of the number-one coach.

Most papers used the AHP or else TOPSIS (Technique of Order Preference by Similarity to Ideal Solution) to incorporate the different elements of the solution into one decision model. Teams used their own judgment to estimate the criterion weights in each case. Since different teams provided different weights and inputs, solutions varied widely even using the same sports and coaches.

The other 10% of the teams used a variety of methods from networks, principal component analysis, genetic algorithms, a grey model [Julong 1989; Giannelli n.d.], linear regression, . . . ; and the list goes on.

Most teams avoided the issue of uncertainty, assuming that the data were accurate but the weights were not and hence needed some analysis. The best teams included an assessment of the sensitivity of their models to changes in their inputs.

Modeling assumptions were very poor across the board by teams. Some teams assumed away the timeline and gender issue, yet these were main parts of the modeling questions:

- **Timeline:** Two issues that were readily apparent over the 100 years were salaries and schedule. Few teams addressed these issues.
- **Gender:** We will say that teams that included female sports automatically considered gender. Those that did not had to do more than just lightly discuss this. One team said that the proportion of differences in coaches were significant but that gender was not an issue.

Sensitivity Analysis and Model Testing

As in previous years, the judging criteria for this problem considered sensitivity analysis as a main component of good analysis for "coaching legends." Many papers neglected to consider these issues and scored lower as a result.

Sensitivity analysis was appropriate for all elements of the models. For AHP or TOPSIS, sensitivity analysis would have involved varying the weights (or pairwise rankings) to explore what conditions would cause the alternative ranking to change.

Model testing took several forms. For prediction models, graphical methods for examining residuals of historical data were often used. Statistical tests of significance were used for regressions. Consistency checks, such as $CR < 0.1$, were used for the AHP. The better papers used these methods and others to convince the reader that the models selected were appropriate.

Communication

Papers were judged on the quality of the writing. Special attention was paid to the abstract and to the nontechnical article.

The quality of writing, in general, is improving from year to year. This is notable in the papers that come from countries where English is not the primary language spoken. About 70% of the Outstanding papers this year were from teams where English was a second language, and that was a record.

The strongest abstracts/articles included a definition of what the team meant by "best," the results of the model, and a simple explanation of how the answer was found. The judges continue to be surprised by the number of papers where the abstract and even the article for *Sports Illustrated* only describes what the team will attempt without describing what they found.

A nontechnical letter or article does not mean that numbers are not included. Rather, it means that it can be read meaningfully by someone without an education in advanced mathematics. Too many of the articles omitted all details of the solution as well as the solution itself!

Papers that labeled figures and tables with informative captions scored higher than those that did not.

The quality of citations was also a discriminator. Papers that cited their sources and provided complete references formatted according to a recognized standard scored higher than those that did not.

Several of the very best papers were a joy to read. The explanations were clear and complete, and the phrasing was almost lyrical. The judges will continue to value outstanding writing.

Summary

The Outstanding teams:

- modeled all the aspects of the problem described in the problem statement,

- included the standard contest discussions (assumptions, sensitivity analysis, strengths and weaknesses, etc.),
- had defensible and useful models,
- explained the modeling choices made, and
- were well written.

Papers that listed model #1 through model #N, without ever reconciling which model was best and which was used in the final analysis, were generally not Outstanding.

The judges were pleased by the teams' submissions. The topic allowed for a wide range of solutions, and the allowed choice of sports provided a diversity of solutions. The growth in the quality and number of submissions is very encouraging to those who work to promote the practice of good mathematical modeling.

References

Black, Kelly. Judges' Commentary: The Ultimate Brownie Pan papers. *The UMAP Journal* 34 (2) (2013): 141–149.

Giannelli, Carlo. n.d. More details about Grey Model. `http://lia.deis.unibo.it/research/SOMA/SmartBuffer/Client/htmlDocs/detailsMobPred.html` .

Julong, Deng. 1989. Introduction to Grey System theory. *Journal of Grey Systems* 1: 1–24. `http://www.researchinformation.co.uk/grey/IntroGreySysTheory.pdf` .

About the Author

Dr. William P. Fox is a professor in the Department of Defense Analysis at the Naval Postgraduate School and teaches a three-course sequence in mathematical modeling for decision making. He received his B.S. degree from the United States Military Academy at West Point, New York, his M.S. at the Naval Postgraduate School, and his Ph.D. at Clemson University. Previously he has taught at the United States Military Academy and at Francis Marion University, where he was the Chair of Mathematics for eight years. He has many publications and scholarly activities including books, chapters of books, journal articles, conference presentations, and workshops. He directs several mathematical modeling contests through COMAP: HiMCM and MCM. His interests include applied mathematics, optimization (linear and nonlinear), mathematical modeling, statistical models for medical research, and computer simulations. He is President-Emeritus of the NPS Faculty Council and President of the Military Application Society of INFORMS.

Judges' Commentary: The Frank Giordano Award for 2014

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Introduction

For the third year, the MCM is designating a paper with the Frank Giordano Award. This designation goes to a paper that demonstrates a very good example of the modeling process. Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. As Frank says,

> It was my pleasure to work with talented and dedicated professionals to provide opportunities for students to realize their mathematical creativity and whet their appetites to learn additional mathematics. The enormous amount of positive feedback I have received from participants and faculty over the years indicates that the contest has made a huge impact on the lives of students and faculty, and also has had an impact on the mathematics curriculum and supporting laboratories worldwide. Thanks to all who have made this a rewarding and pleasant experience!

The Frank Giordano Award for 2014 goes to a team from **Huazhong University of Science and Technology**, School of Mathematics and Statistics, in Wuhan, Hubei, China. This solution paper was in the top group, receiving the designation of Outstanding, and was characterized by

- a high-quality application of the complete modeling process, with clear justifications and examples of how the models could be applied to the coaching data, including an extension of the first two models to a third that gave better results;

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- a careful analysis of the parameters and demonstrated sensitivity analysis;
- originality and creativity in the modeling effort to solve the problem as given and to extend the process to selecting the top U.S. presidents; and
- clear and concise writing, making it a pleasure to read.

The Coach Problem

Sports Illustrated, a magazine for sports enthusiasts, is looking for the "best all-time college coach," male or female, for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer. Does it make a difference which time line horizon that you use in your analysis, that is, does coaching in 1913 differ from coaching in 2013? Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model's top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1–2-page article for *Sports Illustrated* that explains your results and includes a nontechnical explanation of your mathematical model that sports fans will understand.

Solution by the Team

Executive Summary Sheet and *Sports Illustrated* Article

The team's summary was well done and gave the reader a good idea of what to expect. It contained the appropriate specifics with regard to techniques used and comparison of techniques and was both concise and thorough.

Despite a few grammatical errors, the team's article, written in an appropriate nontechnical manner, served as an informative and inviting overview of the issues involved in selecting the top coaches. The decision to highlight John Wooden as a way to clarify the process was excellent.

Assumptions

The assumptions made were very general and somewhat generic. The paper would have been stronger if assumptions were made that applied directly to the models used.

The Models and Methods

The metrics were clearly articulated, with details on how the values associated with each metric were to be determined. The team's explanations included examples, so that the reader could see precisely how coaches' scores were determined. Very few other papers were as thorough in their explanations. The methods used were the Analytical Hierarchy Process (AHP) and Fuzzy Synthetic Evaluation (FSE). After applying each method and comparing the results, they discussed the subjectivity of the AHP method versus the objectivity of the FSE method. To aggregate the results from both methods, they adopted a linear weighted model, Aggregation Model (AM).

Testing Their Models

The primary focus in testing their model was men's college basketball. After determining the top coaches for each method (AHP, FSE, and AM), the team also computed "hit scores" for each by comparing their results to published ratings. They also commented on how, in all their models, the top five positions were less controversial than the top 10.

Extending and Testing Their Models

The team began by applying their model to women's basketball, which has both male and female coaches. After computing the AM scores for these coaches, they determined the top 10 in this field and found their results largely agreed with published rankings. Thus, they concluded that gender was not an issue in their method for determining top coaches.

The team next considered the time factor. Considering that the NCAA basketball tournament did not begin until 1939, then grew from 8 to 16 teams in 1951, and to 32 teams in 1975, 48 teams in 1980, and 64 teams in 1985, the team assigned different weights to each of these periods in their analysis. In applying this factor to each of their three models, they found that the top-10 lists changed their ordering somewhat, but the overall "hit scores" did not change significantly. In their analyses of these results, the team highlighted selected coaches whose positions had changed and explained why that happened. This was a very good example of what distinguished their paper from others.

Finally, the team extended their AM model to football, first listing the metrics that would be different for football versus basketball (for example, bowl games instead of tournaments). Although data for the football coaches were not shown, results for the top five football coaches were given. The team also listed results for the top five hockey coaches, but no change in metrics nor coach data were given. The paper would have been stronger had it been more thorough in application of the AM model to hockey and had it shown the data for football, hockey, and women's basketball coaches.

Sensitivity Analysis

In applying sensitivity analysis, the team demonstrated that their AM model performed better than either their AHP or FSE model alone. Their use of graphs and examples lent clarity and credibility to their analysis. This again distinguished their paper from others.

Extending Their Model Beyond Sports

As an exploration in applying their model more broadly, this team developed metrics to determine the top U.S. presidents. After taking personal qualities, presidential achievements, and leadership qualities into account, they ranked the top 10 presidents. This was what an MCM judge would see as a value-added feature, because it showed that the team was embracing the concept of mathematical modeling, recognizing that the same model can often be applied to very different circumstances.

Recognizing Limitations of the Model

Recognizing the limitations of a model is an important last step in the completion of the modeling process. The team commented on the subjectivity of their weight assignments.

References and Bibliography

The list of references was thorough, and it was very good to see specific documentation of where those references were used in the paper.

Conclusion

The careful exposition in the development and application of the mathematical models, together with the extensions and sensitivity analysis, made this paper one that the judges felt was worthy of the Outstanding designation. The team is to be congratulated on their thoroughness, their clarity, and using the mathematics they knew to create and justify their models. Their presentation made this a very enjoyable and understandable read.

About the Author

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she taught for more than 30 years. She was also a Visiting Professor at the U.S. Military Academy at West Point and taught for five years at California State University, Stanislaus. She chairs the Board of Directors at the Montana Learning Center on Canyon Ferry Lake and serves on the Engineering Advisory Board at Carroll College. She has been a judge for the MCM for 19 years and for the HiMCM for 10 years.

Our Story with the MCM

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Introduction

Just over a year ago, we participated in the 2013 Mathematical Contest in Modeling (MCM) and won the Outstanding winner award for our work on the The Ultimate Brownie Pan Problem. We feel very proud of ourselves because there were only 6 Outstanding winners out of more than 2,000 teams (less than 1%) who worked on this problem. It seems like just a few weeks ago when we were talking about square or circular ovens.

2013
Mathematical Contest In Modeling
Certificate of Achievement
Be It Known That The Team Of
Libin Wen
Jingyuan Wu
Cong Wang
With Faculty Advisor
Yuehui Zhang
Of
Shanghai Jiaotong University
Was Designated As
Outstanding Winner
William P. Fox, Contest Director
Administered by
COMAP
With support from
Marvin Keener, Head Judge
siam. informs MAA CSIAM

Figure 1. The team's certificate.

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All of us were junior s in the Physics Dept. of Shanghai Jiao Tong University at that time. We teamed up for the challenge of the MCM for reasons we discuss later. But none of us had any experience related to mathematical modeling.

What is more embarrassing is that we did not have much time to prepare for the contest after we had decided to enter it. Because the contest was very close to the Chinese Spring Festival, when we had to go home from the university, we then had to communicate with one another in a very inconvenient way—by Web chat application.

However, after our methodical preparation for about 20 days before the competition, we finished our work quite successfully during the competition, although with some further work undone due to the limited time. We can never forget the moment when we heard the exciting news a few months later that we had gotten a top award!

The good news was published on the official Website of our university very quickly. Some of our classmates and teachers congratulated us on our achievement.

Now, we are about to graduate [this essay was written in March 2014]. Jingyuan and Cong are planning to study abroad, and the MCM Outstanding award will help. Libin will start his Ph.D. work at Shanghai Jiao Tong University next year, majoring in physics. We are going to different places, a few months apart; but our MCM story will always unite us.

Libin Wen's Experience

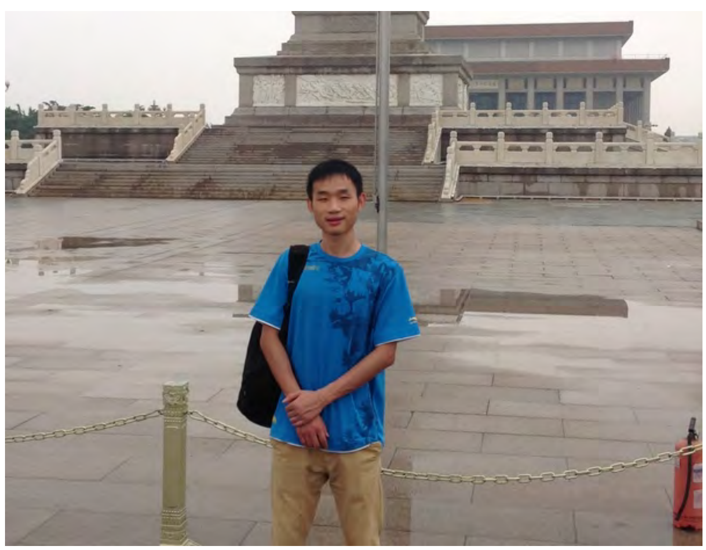

Figure 2. Libin Wen.

I'll begin by telling why I decided to participate in the competition. To be honest, I knew nearly nothing about the MCM until one day when Jingyuan asked me whether I intended to take part in this competition.

I did a survey on the topic of MCM and found that it was something that I was actually familiar with. The general procedure of mathematical modeling is to analyze a practical problem, describe it in a mathematical way, and finally solve it. I suddenly realized that I had always been good at solving problems in this way.

For example, I once analyzed the efficiency of sleep modes of human beings because I felt that my daily life system was not as good as I desired. And I heard that some famous men were used to some special sleep mode, like Leonardo da Vinci, who, it was said, engaged in polyphasic sleep (sleeping multiple times in a 24-hour period). To solve this problem, I defined a quantity to describe how energetic or how tired a person was. Then I thought about the effects of different sleep modes on this quantity. In fact, what I did was to build a model to solve the problem. I wanted to know the differences between the monophasic sleep mode and the polyphasic sleep mode—and then I designed my own sleep mode. So it was quite practical because I was just designing something for myself, to improve the quality of my life.

The point is that I made a model out of my own need for one. And the real needs were just the motivation of mathematical modeling. At that time, I wondered whether my idea about mathematical modeling was right. I knew some guys who had been in these kinds of competition, who thought that mathematical modeling was just games for those competitive men who had spent a lot of time on the preparation. That kind of attitude definitely hurt my confidence. But finally, I decided to give it a try since it was my first time and I need never worry if I got just a Successful Participation award. Besides, I would have a lot of time during the winter vacation; doing something academic would make me feel great. Of course, It was somewhat difficult to refuse the invitation of the two girls.

Some interesting things occurred during our preparation and competition. I got excited when I noticed that all of our members' family names started with letter "W": namely "Wang," "Wen," and "Wu." And so I announced that our team name should be "Group 3W." What a domineering name!

Another amazing story was about my sister and her husband. When I went home a few days before the competition and visited them, they asked me what mathematical modeling is after all. OK, it was indeed difficult to explain clearly to them. So I started searching for a possibly vivid example. The situation was that we were together enjoying the "hot pot," a famous kind of Chinese food, which uses a pan to heat the food. I took it as an example and explained, "For example, mathematical modeling studies something like the pan's heating. If we change the shape of the pan, the food may be heated more evenly. To find the optimal shape of the pan, we can set up a model to solve this problem. And this is mathematical modeling." I assumed that they understood my explanation by their reaction. What made me excited was that the Problem A of the 2013 MCM was exactly about oven pans! Once I saw this question, I could not wait to let the others know this interesting story. What a genius I was! I could predict the questions.

We chose the Ultimate Brownie Pan Problem because we were familiar with the physical principles of the heating process. What impressed me later was that I seemed to realize the benefits of teamwork. Before, I was accustomed to thinking alone. And I never thought about how others could help me out when I was solving a real problem like this. When we discussed together, I found that the others are smarter than I, with very quick minds. Every time when I proposed something, they could think very quickly and give some new ideas. That made perfect sense because I was a slow-mind. I usually thought on a simple issue again and again. And that was the essence of teamwork. We could make up for each other.

Before the competition, I made a schedule to let every member know what we should do and when. It worked very well, though we got behind schedule late in the competition. We had not been planning to stay up late any night. But on the last night, we found that doing an abstract (summary) was not as easy as we expected. Adding the finishing touches, we spent the whole night on the work, not going to bed until 7 A.M. I got more and more excited when it got closer and closer to the end of the contest. When we checked the final work again and again and determined that it had been well done, I felt really satisfied. We had successfully finished four days of work!

Around 6 A.M. that morning, my mother was wakened by the noise from my room and found that I had stayed up through the night. She got very worried about my health and made breakfast for me immediately. I went to bed after breakfast that morning. And later I dreamt about the MCM. I dreamt that our work was not valued by the judges, which was totally different from the result that would come out few months later. Thus, when I heard from Jingyuan that we had gotten the Outstanding Winner designation, I just could not believe it—it was definitely a surprise.

Though I am writing about our MCM story more than a year later, it is just like few days ago. The short four days seem to have given me many many memories. And what I described above is just part of them. I hope that my experience can give you a little feeling about the MCM. Do not believe those rumors about how difficult it is!

Jingyuan Wu's Experience

When Cong Wang asked me during the summer holiday in 2012 whether I had interest in participating in the 2013 Mathematical Contest in Modeling (MCM), I hesitated since I had no experience in mathematical modeling before.

Figure 3. Jingyuan Wu.

In 2013, I took a course called "Computational Physics." The main aim of this course was to enable students to apply some classic models and algorithms to solve physical problems. It was the first time that I learned about mathematical modeling and felt the charm of it.

After one semester's study, I could handle some complex problems, such as the analysis of the behavior of a chaos system and simulation of the growth of clusters. As I gained confidence and passion for mathematical modeling, I recalled the conversation with Cong. As a result, Cong and I decided to take part in 2013 MCM and asked our classmate Libin Wen, who was good at programming, to join us. That was how we, three amateurs, formed a team. With little experience in mathematical modeling, we had never dreamed of winning an Outstanding prize, or even an ordinary prize. What we had done was to try our best to solve the problems and enjoy this exciting journey.

Before the contest, we spent much of our spare time doing preparatory work. We searched almost all the reference books in mathematical modeling in our library and studied classic models. Since this time was during our winter holiday, we were all back in our hometowns and could not contact each other frequently. But we held meetings online every week and chatted about each member's progress. Due to our unremitting efforts and perseverance, we absorbed some classic models, became familiar with programming, and learned how to find information efficiently. A few weeks before the contest, we picked problems in previous contests to train on. We searched for the background knowledge related to the problems, proposed our own models, and finally gave solutions to the problems. After comparing our answers with the answers of the Outstanding Winners, I found that we always paid more attention to figure out sophisticated solutions to the mathematical equations and forgot the importance of combining the theory with the realistic application; we always stayed in a stereotyped thinking pattern and forgot the significance of innovative ideas. Realizing the difference between us and the Outstanding Winners, we gradually learned how to think about a problem.

To take full advantage of our strength in physics, we chose to work on the The Ultimate Brownie Pan Problem. Our task was to determine the optimal shape of a baking pan by considering heat distribution and space utilization. We gave the analytic solution to the heating distribution of the baking pan with particular shapes and implemented Matlab scripts to analyze the heating process of baking pans with various shapes. Based on reasonable assumptions, we created a Rounded-Rectangle Model. During the optimizing process, what made us surprised and excited was a fixed point derived from our mathematical model. After delving into this fantastic point deeply, we drew the conclusion that this fixed point corresponds to a specific shape that would allow manufacturers to meet different customers' preferences equally.

During the contest, we had a pleasant cooperation. We separated our jobs according to our strengths and thought about problems together, which allowed us to work efficiently and effectively. We slept merely 6 hours every day and stayed up all night to finish the paper on the last day of the contest. Although we were too tired to think at the end, we still tried our best to carry out the task and work till the last second. The four days was short, but it meant a lot to us. The four-day journey was full of our efforts, our brainstorming, our perseverance, our excitement, and our sweat.

When we heard the news that we had won the Outstanding designation, we were all excited and could not believe our ears. As a team that was participating in the MCM for the first time, we were so lucky that our paper gained the favors of the referees. The award gave me an extra bonus and helped me to get a National Scholarship. Besides, I am glad to see that our success inspires more students in my department to take part in mathematical modeling contests. Some of them often ask me about the techniques in mathematical modeling and the experience in MCM.

Finally, thanks a lot to the MCM for the award and this wonderful experience!

Cong Wang's Experience

The four days experience of competing in the MCM is significant and memorable to me. During that exhausting but exciting period, our group spent all our time on the contest from 9 A.M. to nearly 1 A.M. the next

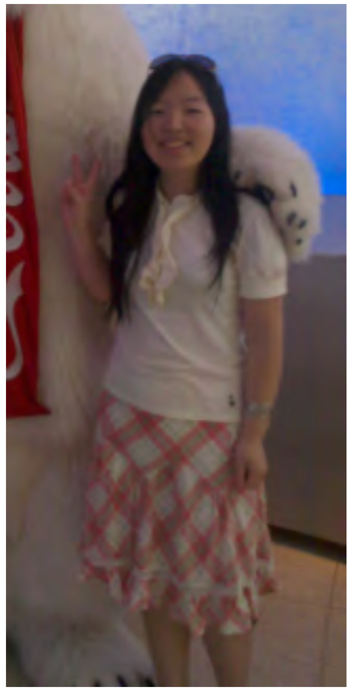

Figure 4. Cong Wang.

morning. Furthermore, the last day, we stayed up the whole night to modify the language of our paper until we were so sleepy that we could hardly open our eyes.

During these days, we did everything as fast as we could to meet the demand of our schedule, which was quite painful. However, when I am reminded of this period, I consider it valuable and happy. What we gained is much more than what we paid in these days.

The four days in the contest, though tiring and exhausting, left a very deep impression on me and helped me to make the decision of changing my major. My undergraduate major is physics, though I am more interested in applied science. In the first two years of my study in the university, I tried different fields, such as chemistry and biology. However, I had not found my favorite field until the MCM contest. When we finally solved the problem, I gained so much joy that I had never felt before, which inspired my interest in modeling and designing. I decided to pursue a master's degree in a field related to modeling and designing. Furthermore, when I apply for such programs, our MCM award is good evidence to support my potential in this field.

I would like to share some tips about what may have helped us in the competition. Thanks to our careful preparation, we did not waste much time. Because we were not in the same cities, we had to try four kinds of communication software before we could enable our discussion through the Internet. We practiced using LaTeX for writing our paper, so it would be more beautiful and concise. We worked together to study algorithms.

These preparations were quite essential for us to cooperate more fluently and save time in the competition.

Apart from the preparation, I consider the procedure that we followed to be very important in the contest. We had to weigh the advantages of taking certain steps and decide the priority of the steps. The problems that are given usually can be solved and extended in several aspects. For example, our problem was to design an appropriate shape of the pan, which could be warmed up most effectively and evenly by the oven. For this problem, we could discuss many aspects of the pan, such as material of the pan, heat distribution of the oven, and so forth. If we had taken all these factors into consideration, the problem would have turned out to be really confusing and complicated. Therefore, what we did first was to simplify the problem. We added several constraints to make the problem easier; it still contained enough factors but we could handle it. After we finished the basic and simplest modeling, we could extend the model to a more practical one one step at a time. As a consequence, we could make sure that we had the ability to address each problem that we confronted and would not spend too much time on the factors that we could not handle. I think that our highly-organized procedures not only saved time for calculating and simulating but also reduced the time of writing the paper. Therefore, we had more time to spend on optimizing our method of modeling.

[EDITOR'S NOTE: The team's entry in the 2013 MCM can be found on the *2013 MCM-ICM CD-ROM*, which contains the press releases for the two contests, the results, the problems, unabridged versions of all the Outstanding papers, and judges' commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772–6627.]

First Experience with Modeling

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Inexperienced

The 2014 COMAP competition was Ramapo College's first team in a modeling competition, as well as our first experience with a mathematical modeling competition. Despite the facts that

- no member of the team had taken a mathematical modeling course,
- we had no formal preparation for this contest, and
- we had finalized the team only three days before the start,

we were able to earn a Meritorious designation.

Through this competition, we learned that mathematical modeling is the ability to take the mathematics learned academically and apply it to real-world situations. There are no prerequisites for this way of thinking, but we found our ability to use data effectively, write well, and be creative were imperative for our success. We had not experienced anything like mathematical modeling before this contest, and it was such a great learning opportunity. It taught us how to process questions and tackle real life situations. Since this competition took place, mathematics and other mathematically-based sciences have become more relatable and exciting, because now we understand how elements of these courses are simply ways of understanding real life.

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Uneasy

At the beginning of the contest, we were uneasy about the nature of the competition and what would be required of us. This feeling of unease was contrasted by our combined passion for mathematics and our confidence in our own abilities. Dobri's progression through advanced mathematics and computer science classes as a dual major added some confidence in our attempt. He was clearly the most technically experienced and established himself as the leader of the group. Princep expressed that his secondary education in Nepal incorporated a high degree of advanced mathematics, but thus far he had taken only general education courses towards his finance degree at Ramapo. Although it comforted Matt to be in the company of such experience, he felt behind, since he was just beginning Calculus I and Computer Science I in his program in chemistry.

Melding

Despite the uncertain nature of our preparation, we found our medley of skills beyond mathematics aided in translating academic mathematics into a practical solution. We began our work the evening that the problems were announced, trying as a group to select one.

The problems seemed impossible. As we discussed potential strategies for each problem, our ideas were all over the place and none seemed particularly effective.

Dobri was very confident in his ability to process data for use in algorithms, so we chose the Coach Problem; sports data are easy to find. From the very beginning, we all agreed that we should measure a coach's ability by the improvement of the team under the coach's mentoring and not just by the raw performance. This required us to define what we meant by "improve a team" and measure "ability."

Our Approach to the Coach

We looked at different ways to judge performance in team-based games, but there seemed very few in line with our idea of improvement. To quantify performance, Dobri found a single-opponent game-scoring algorithm used extensively in chess called the Elo rating system [Wikipedia 2014]. This method provides a way of assigning every competitor a numeric rating at all points in their history.

To implement the Elo rating system, we had to make modifications so that input values were scores from the specific sport, and adjust the scheme for the number of points that are common to that competition. Also, our data were organized by team rather than by coach, so we had to re-work the data to reflect the individual coaches and the ratings of their teams. Each of these steps required independent algorithms outside the rating system itself.

We attempted to use a change in Elo score to rate the coaches. However, we found many inconsistencies with this approach, particularly with coaches who changed teams several times in their careers. Thus, our final model analyzed trends to track consecutive streaks of improvement or degrading performance as given by the Elo score.

Our results showed that top-rated coaches throughout their careers had consistently improved the Elo scores of their teams, which was in line with our vision of a successful coach. Our methodology used a lot of coding to get data into a usable format, so we could create and implement our algorithm; and we had to make modifications to account for inconsistent results.

Liberal Arts Value

Furthermore, we worked very hard on our paper and executive summary to ensure that it was well-written and coherent. *Our liberal arts education helped us coherently express our ideas in this paper.* With two non-native speakers on the team, it was sometimes difficult to express in English the point we wanted to make. However, despite our suffering in literature and humanities courses, we believe that these courses greatly improved our ability to express our ideas correctly.

Our Impressions

We feel that this experience was by far the most productive any of us has had over a weekend, and that made us feel awesome. It was definitely a lot of work, but it also was fun collaborating and doing nonstandard, actual problem-solving mathematics and reasoning.

In the end, we were very proud of our results and incredibly pleased with our unexpected recognition. We had put a lot of effort into our models and our paper and believed that we had done great work. However, we never imagined that we would get to be in the top 10% in the world and top 5% in the U.S. It was such a shock and one of the proudest moments for each of us academically. We couldn't wait to get the certificates and start enjoying the press!

It had been difficult to complete this work and write the paper in only four days. It was a very busy and stressful long weekend, so learning that we had done so well made our sacrifices a bit more tolerable. As trailblazers for the Ramapo College mathematical modeling reputation, we can't wait to see what we (and, we hope, other students) will accomplish in the future.

Reference

Wikipedia. 2014. Elo rating system. `http://en.wikipedia.org/wiki/Elo_rating_system`.

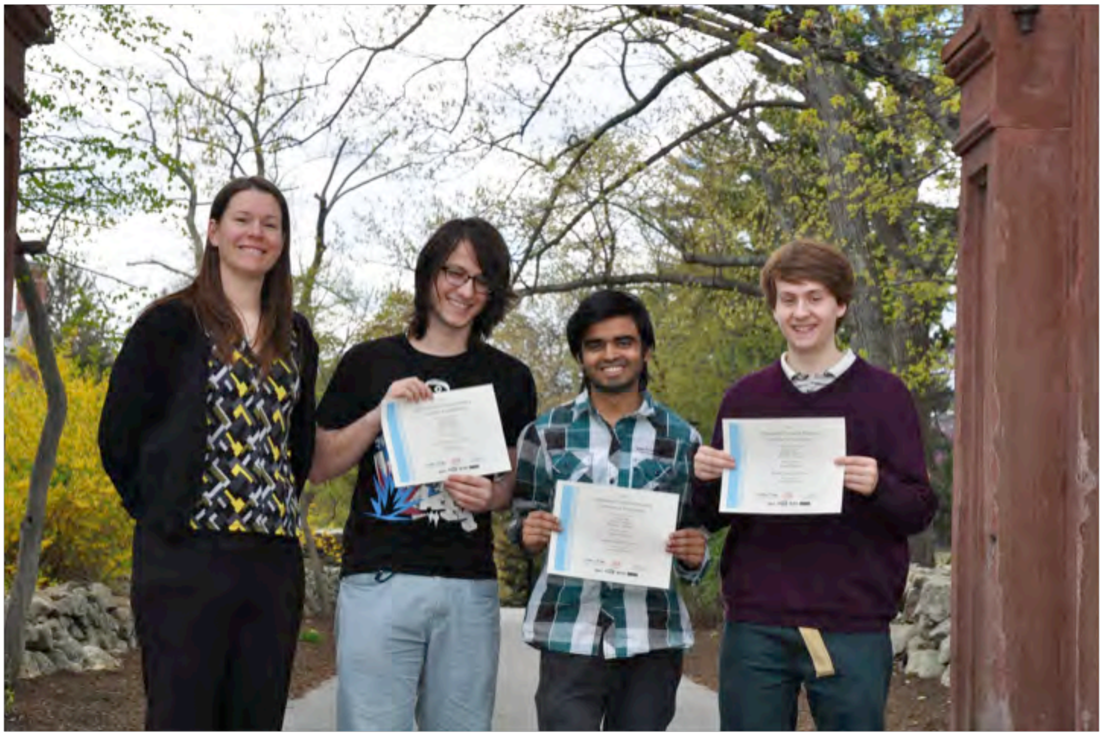
About the Authors

Matthew Marner was a sophomore at Ramapo College. His academic concentration is in chemistry but he has taken a medley of courses, including mathematics and computer science. Spring 2014 was Matt's first experience with calculus and with computer science. Taking both these courses at once was a difficult transition from the less complex math and science courses that came before; however, the difficulty of the courses paid off in increased analytic and computational skills, which contributed greatly to the success of the team.

Princep Shah will be sophomore at Ramapo College in Fall 2014. He is from Nepal and is studying finance with a minor in mathematics. He will be serving as a Residence Assistant and also as a peer facilitator. "I have not set any specific goals yet, but I want to see myself working in the field of investment banking after I graduate."

Dobromir Yordanov, an international student from Bulgaria, will be a senior in Mathematics and Computer Science at Ramapo. "Some of the more exciting classes I've taken are Stochastic Calculus for Finance, Abstract Algebra, Financial Modeling, and Topology. Next year, I'm also planning to take Artificial Intelligence. Going to grad school was never really in question; but as a double major, it's really difficult to decide what program to go after. So far, I am leaning towards mathematics, most likely algebra, topology, or number theory. I have done independent research in cryptography and I intend to do bioinformatics research next year. Currently, I'm a software engineering intern at Google, Inc. working on YouTube; I have years of previous experience in the field, with my first full-time programming job at age 16. Outside of school and work, I've been involved with math club and personal programming projects."

Amanda Beecher is an Assistant Professor of Mathematics at Ramapo College. She earned her Ph.D. from the University at Albany, SUNY in commutative algebra. After finishing her doctoral work, she held a three-year postdoctoral appointment at the United States Military Academy at West Point. Amanda has served as an advisor for the MCM and also as a triage grader, referee, and final judge for the ICM.



Amanda Beecher (team advisor), Dobromir Yordanov, Princep Shah, and Matthew Marner.

Model Students

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Against upper-class mathematics majors, they didn't dream they would make it past the qualifying round, but a team of first-year engineering students thought it would still be fun to try the Mathematical Contest in Modeling in the spring of 2012. Singaporean Dennis Chua ('14 ChemE) first learned of the contest in Math 2930, Differential Equations for Engineers. Chua knew Alvin Wijaya ('15 EE) had already completed the class the previous fall and was taking a more advanced math class, so he asked the Indonesian if he wanted to give it a go. "We needed a third person," says Wijaya. "Jessie Lin seemed like the best choice because she had also taken that class the first semester." Jessie Lin ('15 EE) had competed in math contests in her native Shanghai, but none involved modeling and they were only a few hours long. The modeling competition took four days. "I like using math for applied stuff, that's why I became an engineer," says Lin. "Both of us (Alvin) are pretty good at math, so I thought, 'Why not try it?'" For the initial round of the contest, in which they were competing against other Cornell teams, they were asked to devise a model to determine whether a traffic light would improve pedestrian and vehicular traffic flow at the busy intersection of Tower Road and East Avenue, near Uris Hall.

Wijaya handled the research, making a field trip with Dennis to observe another traffic light on campus, near Carpenter Hall. "I trekked down there at night while it was raining to see how a traffic light works," he says. "The problem statement was super vague, so we went the extra mile and did primary research which is really helpful, because I'm pretty sure not one of the other groups thought of doing that."

"I remember us freezing out there," says Chua. "It was fun." For four days, the three spread out on the floor of Alvin's or Dennis's dorm room to work on their model. "It was like lack of sleep and lots of coffee," says Lin.

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"We had a good time. Because we spent so much time together those four days we got to know each other pretty well and became good friends."

"I basically did like all the math parts. I came up with all the formulas," continues Lin. "We actually had a really good team because we were good at different stuff."

"I was the guy that did most of the coding. I was the kid who knew how to use MATLAB," says Chua. "It was crazy; I was literally writing code for every single minute of the day for four days before the competition. It was a ton of code. My computer couldn't take the programming I was doing. I wanted to run 100,000 iterations but I pressed Enter and it died. I tried just ten and it did the same thing. Finally I tried three and it worked." The team figured a properly timed light would improve flow at the intersection, but did not recommend it. "We found out that by varying the length of the red and green lights, and synchronizing with the lights at the bridge on North Campus, it would improve it," says Wijaya. "But in the end, we concluded that a traffic light there would be bad because students would cross against the light. We determined pedestrian bridges would be better. During class transition periods it would be so dangerous for people to cross there."

The three submitted their paper with little hope of winning. In fact, Chua did not even go with Lin and Wijaya to hear the winners announced. "I was a freshman and I thought there was absolutely no way I could have made it to the next round," he says. "We knew we did well, but because we were competing against junior and seniors, people who had done the competition once or twice, we just felt that experience-wise, we were defeated really bad," says Wijaya. "But we felt we had a chance. We went there not expecting much, maybe top five and we were announced as the second-place winner."

Second place qualified them for a berth in the international competition, in which they would be up against more than 3,600 teams from all over the world. This time they chose Problem A: The Leaves of a Tree, which tasked them with developing mathematical models to estimate the actual weight of the leaves on a tree and to describe and classify them. "They give you one piece of paper with the problem, no information at all," says Chua. "There's no guidelines. There's no data for you to check your models on. It was a really, really a broad thing that required you to spend a whole day just zooming in on the ideas you want to focus on."

Again, Wijaya did the research, finding values to plug into their model, including tree height, branch angles, and bifurcation rates. "We had no clue about trees and leaves and all this bio stuff, so we had to do a ton of research," says Chua. "That really got me looking into bio stuff. I'm now doing all I can to become a biomedical engineer. Hoping to do a biomedical master's when I graduate." He's now assisting biomedical engineering assistant professor Jan Lammerding with muscular dystrophy research. "Jesse was the math whiz who turned out all the formulas," says Chua. "She came up with all these crazy formulas. I don't even know how she did it."

Holed up in a dorm room together for hours on end with little sleep, the students sometimes worried they weren't up to the challenge. "Especially me, because I did the coding, so if I fail, this is not happening," says Chua. "Many times I had to rethink the strategy. Most of the time I was just learning, reading from my textbook. We hit a lot of walls where it was 'Dennis cannot code that.' I'm not a computer science major."

After four days, the students emerged with a simulation-based approach using probabilistic and dynamic models based on established research. "You put all the parameters into this model, like say it's in the alpine region, at what temperature, what time of year, what height of tree you're looking at, and out comes the weight of all the leaves on the tree," says Chua. "For our other model, the result that comes out is what kind of leaves does this tree typically have. Does it have a sword-shaped thin leaf? Does it have a big palm leaf?"

Despite all their hard work, the students had small expectations. "I honestly had no confidence that we were going to win. We were against computer science seniors. They have some really solid coding skills," says Chua. "I knew when the results were going to come out and I didn't bother checking, because there's no way you're going to win. You're one team from Cornell, only freshman, there's no way! We didn't find out until two months later when Alvin checked the Website."

What they found out was that they finished ahead of all other U.S. teams attempting that problem, including MIT, UCLA, and Harvey Mudd College, landing them the Mathematical Association of America Award. Lin couldn't attend, but that summer, Chua and Wijaya presented the team's winning project at the largest annual mathematics conference, MathFest 2012, hosted by the association in Madison, Wis. They were interviewed by renowned mathematician Frank Morgan, the Atwell Professor of Mathematics at Williams College, and subsequently featured in his blog entry about the MathFest 2012 in *The Huffington Post*.

"I would say one of the things that I learned from this is confidence," says Lin. "Even though you are a freshman or sophomore, you can do some pretty amazing stuff."

[Editor's Note:
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Figure 1. From left to right: MCM founder Ben Fusaro (Florida State University), Alvin Wijaya, Dennis Chua, and Mathematical Association of America Prsident Paul Zorn (St. Olaf College).

[EDITOR'S AFTERNOTE:

Bingxuan Dennis Chua also led a team of Cornell engineering students to first place in the IBM Watson Two Worlds Case Competition, with a plan to apply supercomputer Watson's capabilities to technical support for consumer electronics.

In addition to other achievements during his three years at Cornell, Dennis was president of a dance group, was part of a medical brigade in Peru, and nursed lions in South Africa.

He was named a 2014 Merrill Presidential Scholar, the highest recognition given by Cornell University to a graduating senior.

In July 2014, Tau Beta Pi, the engineering honor society, named him among five laureates in its recognition of engineering students who have excelled in areas beyond their technical majors.

After graduation, Dennis began a career in investment banking at Goldman Sachs in New York.]

ICM Modeling Forum

Results of the 2014 Interdisciplinary Contest in Modeling

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Introduction

A total of 1,028 teams from six countries spent a weekend working on an applied modeling problem involving the health of planet Earth in the 16th Interdisciplinary Contest in Modeling (ICM)®. This year's contest began on Thursday, February 6, and ended on Monday, February 10, 2014. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended interdisciplinary modeling problem concerning the state-changes associated with the health of planet Earth. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Six of the papers were judged to be Outstanding by the expert panel of judges.

COMAP's Interdisciplinary Contest in Modeling (ICM) involves students working in teams to model and analyze an open interdisciplinary problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders. The ICM is an example of COMAP's efforts in working towards these goals.

This year's problem was challenging in its demand for teams to utilize aspects of science, mathematics, and analysis in their modeling and prob-

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lem solving. The problem required teams to investigate the relationships involved in network models for determining influence in a large co-author network (Paul Erdős's 511 co-authors) and measuring impact within a set of foundational papers within the discipline of network science. This problem required teams to mine a large data set and understand concepts from the informational sciences to build effective models for these complex phenomena.

The problem contained many multifaceted issues to be analyzed and had several challenging requirements for innovative scientific and mathematical modeling and analysis. In addition to network modeling, informational analysis, and data collection, the teams had to explain the nature of influence and impact in an academic social network and show how their models could be used to help make informed decisions. This year's problem continued the ICM theme of network science for a third year.

The problem also had the ever-present ICM requirements to use thorough data analysis, creative modeling, and scientific methodology, along with effective writing and visualization to communicate their teams' results in a 20-page report. All members of the 1,028 competing teams are to be congratulated for their excellent work and dedication to interdisciplinary modeling and problem solving.

Next year's contest will add a *second interdisciplinary problem* to the available options for MCM/ICM contestants, for a total of four problems instead of three. We will continue the network science theme for one of the problems, and the second problem (Problem D for contestants) will focus on environmental issues. Teams preparing for the 2014 contest should consider reviewing interdisciplinary topics in the areas of network science and social network analysis for the C problem, and human-environment interactions in the areas of environmental science, climatology, food security, and geography for the D problem, and prepare and assemble their teams accordingly.

Finally, we announce the forthcoming publication of the volume *The Interdisciplinary Contest in Modeling: Culturing Interdisciplinary Problem Solving*, edited by Chris Arney and Paul J. Campbell, which will appear later in 2014. Details are given in an announcement following this article.

A Brief History of the ICM

As always, a panel of expert judges read the papers, judged their attributes, debated their merits, and decided on the rankings reported in this article. Looking at the range of topics over the 16 years (**Table 1**), the contest shows its interdisciplinarity with problems involving elements from chemistry, physics, biology, engineering, information science, medicine, business, and network science. The problems also show a balance of public (government) and private (business) issues. Including a second ICM problem for the 2015 contest will give teams more choice and provide variety in the contest problems.

Results and winning papers from the first 15 contests were published in special issues of *The UMAP Journal* (1999–2013). In addition to this special issue of *The UMAP Journal*, COMAP offers a supplementary *2014 MCM-ICM CD-ROM* containing the press releases for this contest and the MCM, the results, the problems, unabridged versions of the Outstanding papers, and judges' commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772–6627.

Table 1.

Participating teams and topics in the first 16 years of the ICM.

Year	Number of teams	Topic
1999	40	Controlling the spread of ground pollution
2000	70	Controlling elephant populations
2001	83	Controlling zebra mussel populations
2002	106	Preserving the habitat of the scrub lizard
2003	146	Designing an airport screening system
2004	143	Designing information technology security for a campus
2005	164	Harvesting and managing exhaustible resources
2006	224	Modeling HIV/AIDS infections and finances
2007	273	Designing a viable kidney exchange network
2008	380	Measuring utility in health care networks
2009	374	Balancing a water-based ecosystem affected by fish farming
2010	356	Controlling ocean debris
2011	735	Measuring the impact of electric vehicles
2012	1,329	Identifying criminals in a conspiracy network
2013	957	Planet Earth's health
2014	1,028	Using networks to measure influence and impact

2014 ICM Problem Statement: Using Networks to Measure Influence and Impact

One of the techniques to determine influence of academic research is to build and measure properties of citation or co-author networks. Co-authoring a manuscript usually connotes a strong influential connection between researchers.

One of the most famous academic co-authors was the 20th-century mathematician Paul Erdős, who had over 500 co-authors and published over 1,400 technical research papers.

It is ironic (or perhaps not!) that Erdős is also one of the influencers in building the foundation for the emerging interdisciplinary science of networks, particularly, through his publication with Alfred Rényi of the paper "On random graphs" [1959].

Erdős's role as a collaborator was so significant in the field of mathematics that mathematicians often measure their closeness to Erdős through analysis of Erdős's amazingly large and robust co-author network (see Grossman [2014]).

The unusual and fascinating story of Paul Erdős as a gifted mathematician, talented problem solver, and master collaborator is provided in many books and at Websites (e.g., O'Connor and Robertson [2000]). Perhaps his itinerant lifestyle, frequently staying with or residing with his collaborators, and giving much of his money to students as prizes for solving problems, enabled his co-authorships to flourish and helped build his astounding network of influence in several areas of mathematics.

To measure such influence as Erdős produced, there are network-based evaluation tools that use co-author and citation data to determine an impact factor of researchers, publications, and journals. Some of these are *Science Citation Index*, H-factor, Impact factor, Eigenfactor, etc. Google Scholar is also a good data tool to use for network influence or impact data collection and analysis. Your team's goal for ICM 2014 is to analyze influence and impact in research networks and other areas of society. Your tasks to do this include:

1. Build the co-author network of the Erdos1 authors (you can use the file from the Website

   `https://files.oakland.edu/users/grossman/enp/Erdos1.html`

   or the one we include at `Erdos1.htm`)[1]. You should build a co-author network of the approximately 510 researchers from the file `Erdos1`, who co-authored a paper with Erdős, but do not include Erdős. This will take some skilled data extraction and modeling efforts to obtain the correct set of nodes (the Erdős co-authors) and their links (connections with one another as co-authors). There are over 18,000 lines of raw data in the `Erdos1` file, but many of them will not be used since they are links to people outside the Erdos1 network. If necessary, you can limit the size of your network to analyze in order to calibrate your influence measurement algorithm. Once built, analyze the properties of this network. (Again, do not include Erdős—he is the most influential and would be connected to all nodes in the network. In this case, it's co-authorship with him that builds the network, but he is not part of the network or the analysis.)

2. Develop influence measure(s) to determine who in this Erdos1 network has significant influence within the network. Consider who has published important works or connects important researchers within Erdos1. Again, assume that Erdős is not there to play these roles.

[1]The file is available at `http://www.comap.com/undergraduate/contests/mcm/contests/2014/problems/ICM_2014.pdf`.

3. Another type of influence measure might be to compare the significance of a research paper by analyzing the important works that follow from its publication. Choose some set of foundational papers in the emerging field of network science either from the attached list (`NetSciFoundation.pdf`)[2] or papers you discover. Use these papers to analyze and develop a model to determine their relative influence. Build the influence (co-author or citation) networks and calculate appropriate measures for your analysis. Which of the papers in your set do you consider the most influential in network science and why? Is there a similar way to determine the role or influence measure of an individual network researcher? Consider how you would measure the role, influence, or impact of a specific university, department, or a journal in network science? Discuss methodology to develop such measures and the data that would need to be collected.

4. Implement your algorithm on a completely different set of network influence data—for instance, influential songwriters, music bands, performers, movie actors, directors, movies, TV shows, columnists, journalists, newspapers, magazines, novelists, novels, bloggers, tweeters, or any data set you care to analyze. You may wish to restrict the network to a specific genre or geographic location or predetermined size.

5. Finally, discuss the science, understanding, and utility of modeling influence and impact within networks. Could individuals, organizations, nations, and society use influence methodology to improve relationships, conduct business, and make wise decisions? For instance, at the individual level, describe how you could use your measures and algorithms to choose who to try to co-author with in order to boost your mathematical influence as rapidly as possible. Or how can you use your models and results to help decide on a graduate school or thesis advisor to select for your future academic work?

6. Write a report explaining your modeling methodology, your network-based influence and impact measures, and your progress and results for the previous five tasks. The report must not exceed 20 pages (not including your cover sheet and summary) and should present solid analysis of your network data; strengths, weaknesses, and sensitivity of your methodology; and the power of modeling these phenomena using network science.

The Results

The 1,028 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each

[2]This list of papers is available at the site noted in the previous footnote.

paper was then read preliminarily by triage judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in late March. The judges classified the 1,028 submitted papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participant	Total
Influence/Impact	6	5	131	367	519	1,028

Outstanding Teams

Institution and Advisor	Team Members
"Who Are the 20%?"	
Southeast University	Chen Wang
Nanjing, China	Mi Gong
Dan He	Zhen Li
"The Research of Influence Based on the Characteristic of a Network"	
National University of Defense Technology	Sheng Zhang
Changsha, China	Ran Cheng
Dan Wang	Danling Zhao
"Influence Measures in Networks"	
Central University of Finance and Economics	Xicheng Miao
Beijing, China	Lingjing Gu
Xiaoming Fan	Yue Xu
"Methods of Measuring Influence Using a Network Model"	
Xidian University	Sijia Jiang
Xi'an, China	Yuke Zhu
Shuisheng Zhou	Ruijie He
"Who Is the Hidden Champion in a Network?"	
Tsinghua University	Yanjun Han
Beijing, China	Yingning Sun
Liping Zhang	Zhonghong Kuang
"A Three-Dimensional Network Impact Analysis Model"	
Tsinghua University	Jiawen Gu
Beijing, China	Lu Chen
Jun Ye	Yuanye Wang

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Director. Additional awards were presented to the team from Southeast University, Nanjing, China, by the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors
Chris Arney, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Joseph Myers, Mathematical Sciences Division, Army Research Office, Research Triangle Park, NC

Associate Directors
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Rodney Sturdivant, Dept. of Statistics, Ohio State University, Columbus, OH

Judges
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Amanda Beecher, Dept of Mathematics, Ramapo College, Mahwah, NJ
Kathryn Coronges, Network Science Center, U.S. Military Academy, West Point, NY
Rachelle DeCoste, Dept. of Mathematics, Wheaton College, Norton, MA
Amy Krakowka, Dept. of Geography and Environmental Sciences, U.S. Military Academy, West Point, NY
Jessica Libertini, Dept. of Mathematics, University of Rhode Island, Kingston, RI
Ziyang Mao, (Lecturer), Dept. of Mathematics and System Science, College of Science, National University of Defense Technology, Changsha, Hunan, P.R. China
Kathleen Snook, COMAP Consultant, Bedford, MA
Robert Ulman, Network Sciences Division, Army Research Office, Research Triangle Park, NC
Jie Wang, Computer Science Dept., University of Massachusetts, Lowell, Lowell, MA

Triage Judges
Eleanor Abernethy, Chris Arney, Kevin Blaine, Peter Charbonneau, Jong Chung, Gabe Costa, Michael Findlay, Hilary Fletcher, Paul Goethals, Tina Hartley, John Jackson, Joseph Lavalle-Rivera, Timothy Povich, Jarrod Shingleton, James Starling, and Shaw Yoshitani
—all of Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Amanda Beecher, Dept. of Mathematics, Ramapo College of New Jersey, Mahwah, NJ
Kathryn Coronges, Dept. of Behavioral Sciences, U.S. Military Academy, West Point, NY
Kevin Cummiskey, Rob Nowicki, and Chris Weld, U.S. Army
Ralucca Gera and Jonathon Roginski, Naval Postgraduate School, Monterey, CA
Michelle Craddock Guinn, Dept. of Mathematics, Belmont University, Nashville, TN
Sheila Miller, Dept. of Mathematics, Bard College, Annandale-on-Hudson, NY
Elizabeth Russell, National Security Agency, MD
Michael Smith, Missile Defense Agency, Huntsville, AL
Csilla Szabo, Dept. of Mathematics, Rensselaer Polytechnic Institute, Troy, NY
Johann Thiel, Dept. of Mathematics, New York City College of Technology, Brooklyn, NY
Robert Wooster, Dept. of Mathematics, College of Wooster, Wooster, OH.

References

Erdős, P., and A. Rényi. 1959. On random graphs, I. *Publicationes Mathematicae* 6: 290–297.

Grossman, Jerry. 2014. The Erdős Number Project. http://www.oakland.edu/enp/.

O'Connor, J.J., and E.F. Robertson. 2000. Paul Erdős. http://www-history.mcs.st-and.ac.uk/Biographies/Erdos.html .

Acknowledgments

We thank:

- the Institute for Operations Research and the Management Sciences (INFORMS) for its support in judging and providing prizes for a winning team, and
- all the ICM judges for their valuable and unflagging efforts.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the team papers here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential ICM advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

The complete roster of participating teams and results is too long to reproduce in the *Journal*. It can be found at the COMAP Website:

`http://www.comap.com/undergraduate/contests/mcm/contests/2014/results/2014_ICM_Results.pdf`

About the Author

Chris Arney graduated from the U.S. Military Academy and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to Research Triangle Park, NC, where he served in various positions in the Army Research Office. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of the ICM. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

Announcement

Later in 2014 COMAP will publish a book devoted to the ICM and to interdisciplinary modeling:

The Interdisciplinary Contest in Modeling: Culturing Interdisciplinary Problem Solving,

edited by Chris Arney and Paul J. Campbell.

This volume contains

- the history of the ICM contest,
- statements of the 16 problems,
- listings and summaries of outstanding teams,
- demographics of contestants and their schools, and
- reflections and helpful advice articles by participants, advisors, judges, and directors.

Chapters describe how to prepare teams and how to develop modeling curricula, along with discussions on the current interdisciplinary academic environment and related literature.

The volume provides an insightful look at trends in educating future interdisciplinary modelers and problem solvers.

The book will be available as a CD-ROM:

`http://216.250.163.249//product/?idx=1441`
ISBN 978–1–933223–53–7

and as a printed book:

`http://216.250.163.249//product/?idx=1440`
ISBN 978–1–933223–52–9

The table of contents is available at the Web pages indicated.

Who Are the 20%?

Chen Wang
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Southeast University
Nanjing, China

Advisor: Dan He

Abstract

The famous 80-20 rule states that for many events, 80% of influence is caused by 20% of those involved. This principle also applies in network science: Only a few nodes have a significant influence and impact on the whole network. We employ a Relation Distance Model and an Authority-Popularity Evaluation Model to measure the 20% and analyze its influence.

For Requirements 1 and 2, we construct the undirected co-author network based on the 511th-order relationship matrix. We propose a Relation Distance Model based on the SNA (social network analysis) technique. It combines three centrality indexes in a vector to calculate the "distance" from the most influential node. Another measure (eigenvector centrality), which takes both degree and the influence of co-authors into consideration, outputs a different rank. Validation of the model is discussed by comparing the two rankings of the top 15 authors in the Erdos1 network: We find ALON, NOGA M., is the most influential person in the network. We show that the degree distribution of the Erdos1 network approximates a power-law distribution, which indicates that it is a scale-free network.

For Requirement 3, we establish an Authority-Popularity Evaluation Model to analyze the depth and the width of the influence of nodes. We calculate an Authority Index by our Modified PageRank Algorithm to measure the depth of impact. We define a Popularity Index as citations per year, to reflect the width of influence. We implement the algorithm on a citation-directed network of 24 papers with weighted nodes. The ranking of the papers is obtained by combining the Authority and Popularity Indexes.

For Requirement 4, we construct for 15 actors a bidirectional network with movie co-stars as links.

For Requirement 5, we discuss two characteristics of a scale-free network: growth and preferential attachment. The philosophy of the dynamics of a scale-free network is revealed to be the "Matthew effect." We propose a

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method to boost influence: Find the shortest links to the most influential author, cooperate step-by-step with the 80% with low closeness-centrality, and finally co-author with the key figure in the field.

We conduct a sensitivity analysis to study the robustness of our algorithm, and the results show a good stability. We further discuss strengths and weaknesses of our models.

Basic Assumptions

Assumption 1. *The strength of co-authorship between two arbitrary Erdos1 authors is the same.*

The accurate strength level of co-authorship between two arbitrary Erdos1 authors is hard to measure. For the sake of simplification, if two authors co-author a paper, the co-authorship index is 1; if not , it is 0.

Assumption 2. *The significance of a research paper is determined by both its citations and its publication date; also, the influence of the journal should be considered.*

The more often a paper is cited or the earlier a paper is published, the more influential it is. Also, a highly influential journal contributes more to the influence of a paper.

Assumption 3. *We assume that the quality of a movie is determined by its IMDb rating [Internet Movie Database 2014]. The popularity of a movie star is measured by the number of Google search results.*

Definitions of symbols employed in this paper are listed in **Table 1**.

Models for Requirement 1 and 2

Data Preprocessing

Before presenting our models, we describe the preprocessing work that we did with the data.

- **Step 1.** We extracted the 511 Erdos1 authors from over 18,000 lines of raw data in the Erdos1 file, by eliminating names without a date but followed by by the date of the first joint paper with Erdős and possibly the number of joint publications (if more than one).

- **Step 2.** To obtain the relationships among the 511 Erdos1 authors, we left out the Erdos2 names from the list of Erdos1, by using the library function `countif()` in Microsoft Excel 2010.

Table 1.
Symbols used.

Variable	Description
	Relation Distance Model
x	Index of a member node
a_{ix}	Relation strength
d_{ix}	Relation distance
$C_d(x)$	Degree centrality of node x
$C_b(x)$	Betweenness centrality of node x
$C_c(x)$	Closeness centrality of node x
$g_{ij}(x)$	Shortest path between i and j that passes through the node x
l_{ix}	Length of the shortest path connecting node i and node x
n	Total number of nodes in a network
$\overrightarrow{A_x}$	Vector containing the three centrality measures
$\overrightarrow{A^C}$	The ideal vector of the node that has the most significant influence within the network
$D_C(x)$	Euclidean distance defined to measure the influence and impact of node x
	Eigenvector Centrality Model
E_x	Eigenvector centrality value of node x
$B=(b_{ij})$	Adjacent matrix of a network
c	Proportional constant
	Revised PageRank Algorithm
$PR_x(0)$	Initial PageRank value of node x
s	Scale constant for the Revised Algorithm
G_{ij}	Relationship matrix
D	Coefficient matrix

- **Step 3.** We constructed the 511th-order co-authorship matrix by executing a Matlab program. For the sake of description, we give each Erdos1 author an ID number by the rule:

  '1' stands for *ABBOTT, HARVEY LESLIE*
  '2' stands for *ACZEL, JANOS D.*
  ...
  '511' stands for *ZIV, ABRAHAM*

Figure 1 shows part of the resulting graph.

Relation Distance Model based on SNA

Overview

To build the Erdos1 network and analyze its properties, we employ the social network analysis (SNA) technique.

Social network analysis refers to methods to analyze social network structures made up of individuals ("nodes") tied (connected) by one or more specific types of interdependency. In our case, the Erdos1 authors are viewed as nodes and co-authorship as links among them.

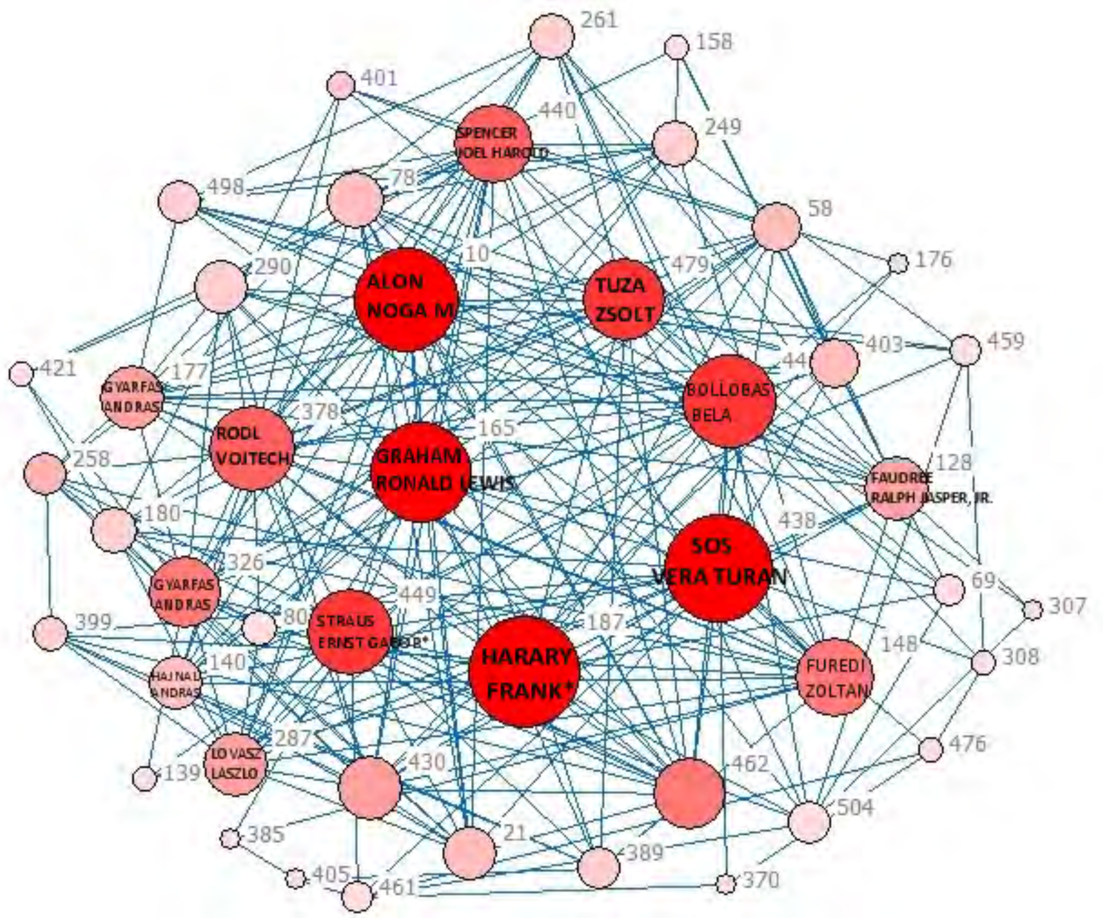

Figure 1. The co-author network of the Erdos1 file. For the sake of visibility, we illustrate just the top 50 Erdos1 authors.

Methodology

- **Step 1.** We calculate centrality measures of the vector $\overrightarrow{A_x}$ of nodes in the network. There are three popular centrality measures [Freeman 1979]:
  - degree centrality $C_d(x)$,
  - betweenness centrality $C_b(x)$, and
  - closeness centrality $C_c(x)$.

  These measures can be used to identify "masters" who have significant influence or impact in a network. These measures are defined as below:

  - **Degree centrality** is defined as $$C_d(x) = \sum_{i=1}^{n} a_{ix},$$ where n is the total number of nodes in a network and a_{ix} is a variable indicating the weighted number of co-authorship between nodes x and i. According to **Assumption 1**, in our Erdos1 network, $a_{ix} = 1$ or 0 for all i.

- **Betweenness centrality** is defined as $C_b(x) = \sum_x^n \sum_j^n g_{ij}(x)$,

  where $g_{ij}(x)$ indicates whether the shortest path between two other nodes i and j passes through the node x.

- **Closeness centrality** is defined as $C_c(x) = \sum_{i=1}^n l_{ix}$,

  where l_{ix} is the length of the shortest path connecting nodes i and x. The shortest paths can be calculated based on the Floyd algorithm.

The centralities above describe different characteristics of nodes in a network:

- **Degree centrality** shows the number of nodes' connections, which also reflects connectivity of nodes in a network. Nodes with high connectivity can be viewed as more influential.
- **Betweenness centrality** shows the number of shortest paths that pass through a node. It also reveals the dependency of a node on other nodes. Large betweenness centrality value of a node is equivalent to high importance in the network.
- **Closeness centrality** measures how far away one node is from other nodes. Small closeness of a node reflects high importance.

We formulate a *measure vector* that contains all three measures:

$$\overrightarrow{A_x} = \left(\frac{C_d(x)}{\max C_d(x)},\ \frac{C_b(x)}{\max C_b(x)},\ \frac{C_c(x)}{\max C_c(x)} \right) = (A_{x1},\ A_{x2},\ A_{x3}).$$

The three elements are all divided by their maximum values so as to be normalized, separately. According to the definition of the three centralities, A_x will achieve its optimal value when degree (A_{x1}) and betweenness (A_{x2}) get to their largest value, 1, and closeness (A_{x3}) gets to its smallest value, 0.

Therefore, an author who has the most significant influence within the network will ideally have measure vector

$$\overrightarrow{A^C} = (A_1^C, A_2^C, A_3^C) = (1, 1, 0).$$

- **Step 2.** Calculate the "distances" among member nodes.

  We denote the Euclidean distance of $\overrightarrow{A_x}$ from the node x to the ideal vector $\overrightarrow{A^C}$ by $D_C(x)$. Based on the idea of the ranking method from the TOPSIS algorithm [Mahmoodzadeh et al. 2007], we call the distance "Influence and Impact Distance," defined as:

$$D_C(x) = \sqrt{(A_{x1} - A_1^C)^2 + (A_{x2} - A_2^C)^2 + (A_{x3} - A_3^C)^2}$$

- **Step 3.** Generate the influence priority list.
  This distance is the key to determine who in the Erdos1 network has significant influence. A greater distance indicates a lower possibility of significant influence. We arrange the Erdos1 authors in a priority list according to Influence and Impact, the value of $D_C(x)$. Nodes with smaller $D_C(x)$ rank higher in the priority list, since $D_C(x)$ is the distance from the node to an ideal "most significant" node.

Results and Analysis

In the three-dimensional graph of **Figure 2**, the 511 points are the 511 Erdos1 authors' measure vectors.

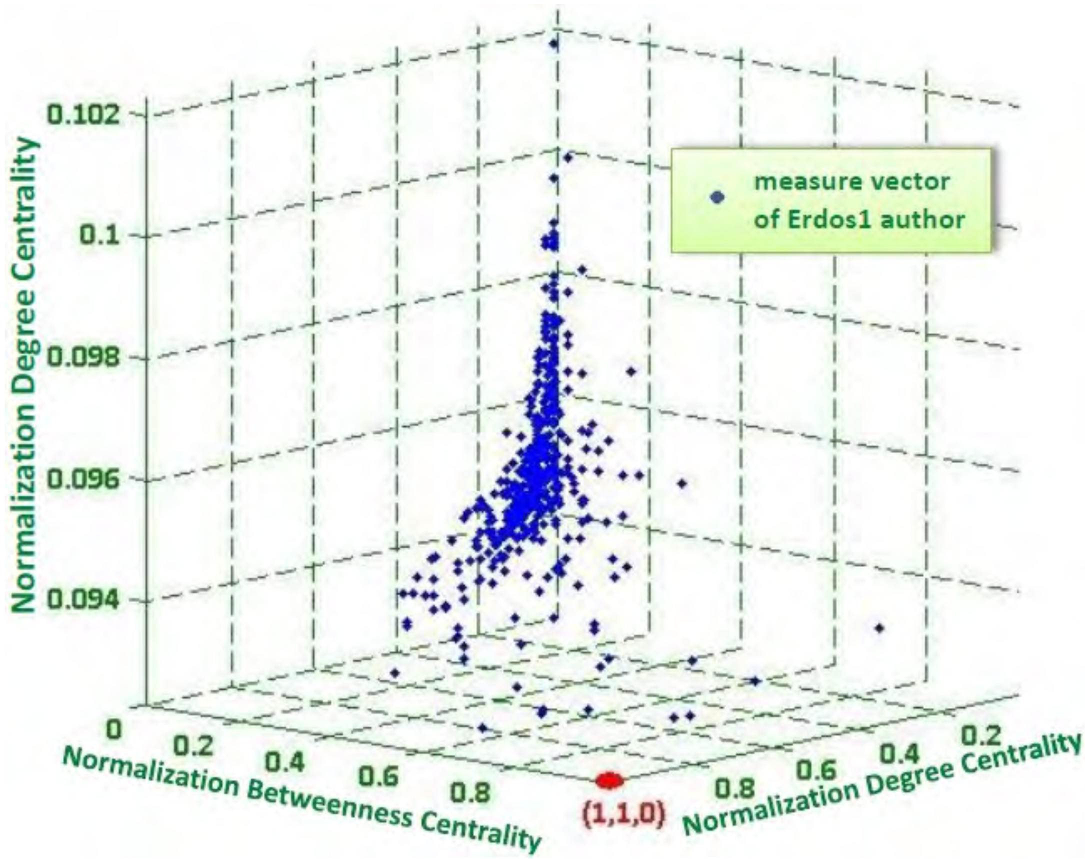

Figure 2. Measure vectors of the 511 Erdos1 authors. The big red point$(1, 1, 0)$ at center bottom stands for the ideal vector of being the most influential.

The top 15 authors of the Influence and Impact priority list are shown in **Table 2**. For the sake of visibility, we choose the top 50 Erdos1 authors in the impact priority list to draw the network in UCINET, as we did in **Figure 1**.

We can see from **Table 2** that $HARARY,\ FRANK*$ has the most significant influence within the co-author network.

Table 2.
Top 15 most influential authors in the Erdos1 network.

Ranking	$D_C(x)$	ID	Name of author
1	0.18	187	*HARARY, FRANK**
2	0.29	438	*SOS, VERA TURAN*
3	0.30	10	*ALON, NOGA M.*
4	0.34	165	*GRAHAM, RONALD LEWIS*
5	0.36	148	*FUREDI, ZOLTAN*
6	0.36	44	*BOLLOBAS, BELA*
7	0.48	479	*TUZA, ZSOLT*
8	0.50	355	*POMERANCE, CARL BERNARD*
9	0.53	449	*STRAUS, ERNST GABOR**
10	0.54	341	*PACH, JANOS*
11	0.58	180	*HAJNAL, ANDRAS*
12	0.65	378	*RODL, VOJTECH*
13	0.70	440	*SPENCER, JOEL HAROLD*
14	0.74	249	*KLEITMAN, DANIEL J.*
15	0.77	399	*SARKOZY, ANDRAS*

Eigenvector Centrality

For the second question in **Requirement 2**, we consider who has published important works or connects important researchers within Erdos1.

The importance of a node is determined by both the number of its neighbor nodes (its degree) and the importance of its neighbor nodes. In graph theory and network analysis, centrality of a vertex measures its relative importance within a graph.

The definition of eigenvector centrality is

$$E_i = c \sum_{j=1}^{n} b_{ij} E_j,$$

where c is the proportional constant and $B = (b_{ij})$ is the adjacency matrix of the network.

Calculating the 511 nodes' eigenvector centrality value in UCINET, we get the top 15 nodes as shown in **Table 3**.

We can see from **Table 3** that *ALON, NOGA M.* is the person who connects more important researchers within Erdos1 than others.

Validation of the Model

Comparing the two ranking methods, the majority of the authors in the first table are also in the second table. However, some authors in the first table cannot be found in the second list. Why? We focus our discussion on *HARARY, FRANK**.

Table 3.
Top 15 connecting important researchers within Erdos1.

Ranking	E-value	ID	Name of author
1	0.26	10	*ALON, NOGA M.*
2	0.23	378	*RODL, VOJTECH*
3	0.21	44	*BOLLOBAS, BELA*
4	0.20	165	*GRAHAM, RONALD LEWIS*
5	0.20	148	*FUREDI, ZOLTAN*
6	0.19	479	*TUZA, ZSOLT*
7	0.18	440	*SPENCER, JOEL HAROLD*
8	0.18	177	*GYARFAS, ANDRAS*
9	0.17	462	*SZEMEREDI, ENDRE*
10	0.16	128	*FAUDREE, RALPH JASPER, JR.*
11	0.16	287	*LOVASZ, LASZLO*
12	0.15	78	*CHUNG, FAN RONG KING (GRAHAM)*
13	0.15	341	*PACH, JANOS*
14	0.15	261	*KOSTOCHKA, ALEXANDR V.*
15	0.15	326	*NESETRIL, JAROSLAV*

The degree of *HARARY, FRANK** is the highest, 44. When we study the relatively most influential authors in the network, we discover that only 30 authors are "masters" within the network. That is to say, most of the co-authors of *HARARY, FRANK** are not highly-influential authors. So, when we consider both the degree and the influence of co-authors, an author whose degree ranks high may not be a "master." So, a weakness of the Relation Distance Model is that it is susceptible to a large degree value.

Properties of the Erdos1 Network

We have already found each node's degree centrality, so we can obtain the degree distribution of the network. Combining maximum-likelihood fitting methods with goodness-of-fit tests proposed by Clauset [2007], we discover that the distribution of degree centrality k in the Erdos1 network is approximately a power-law distribution, with 1.6 (**Figure 3**):

$$P(K > k) \sim k^{-1.6}$$

According to an idea of Barabási and Albert [1999], we can consider the Erdos1 co-authoring network as a scale-free network. In a scale-free network, most nodes have small degree value, only very few nodes have large degree value, and the degree distribution of the scale-free network is approximately a power-law distribution.

Some other properties of the network are:

- The overall graph clustering coefficient is 0.34.
- The average distance (among reachable pairs) is 3.83.

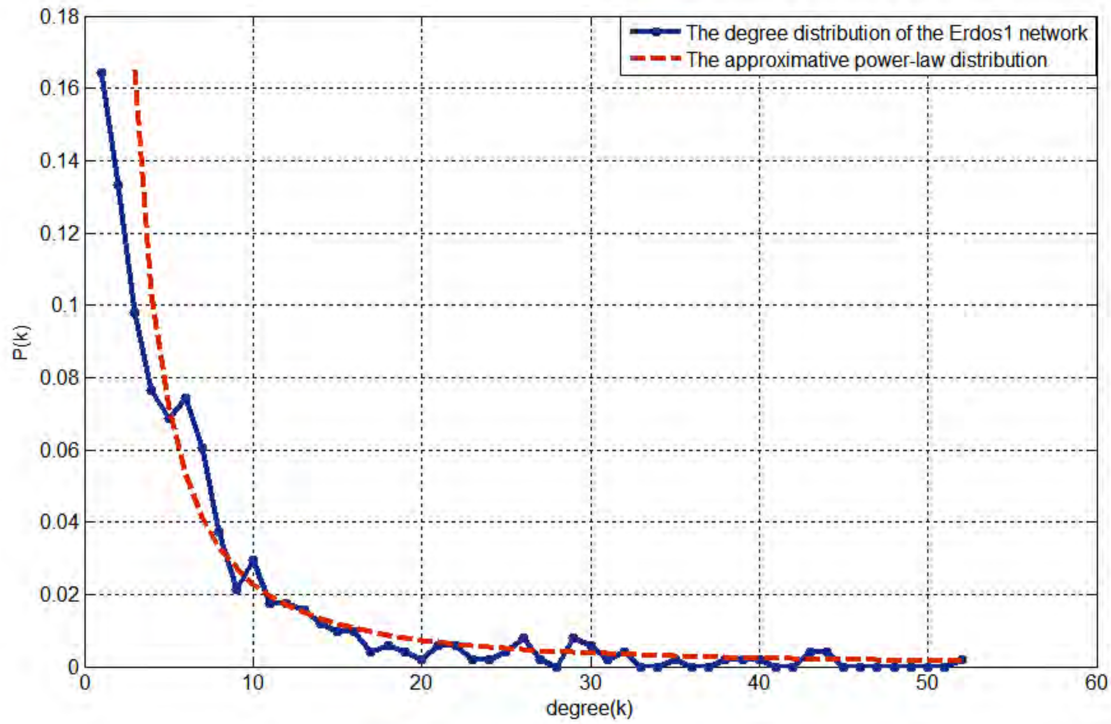

Figure 3. Degree distribution of the Erdos1 network and the approximating power-law distribution.

Authority-Popularity Evaluation Model

The Citation Network for Requirement 3

For **Requirement 3**,we construct a directed network connection graph with weights. The 16 foundational papers listed in the list supplied with the contest question (`NetSciFoundation.pdf`) and 8 additional works that we discovered are considered as the nodes in the Citation network, and the citation relationship as the links. The Citation network is shown in **Figure 4**. The 8 additional works are listed in the **Appendix**.

- **Explanation 1**: *Let paper A be cited by paper B, then the direction of the link between them is from B to A (for the implementing of the PageRank Algorithm).*
- **Explanation 2**: *According to **Assumption 2**, we define the weight of a node as the impact factor of the journal [Impact Factor Search 2014] in which the paper was published.*
- **Explanation 3**: *For the sake of description, we give each paper an ID: 1 for the first paper in* `NetSciFoundation.pdf`*, 2 for the second, 17 for the first paper in the additional list, etc.*

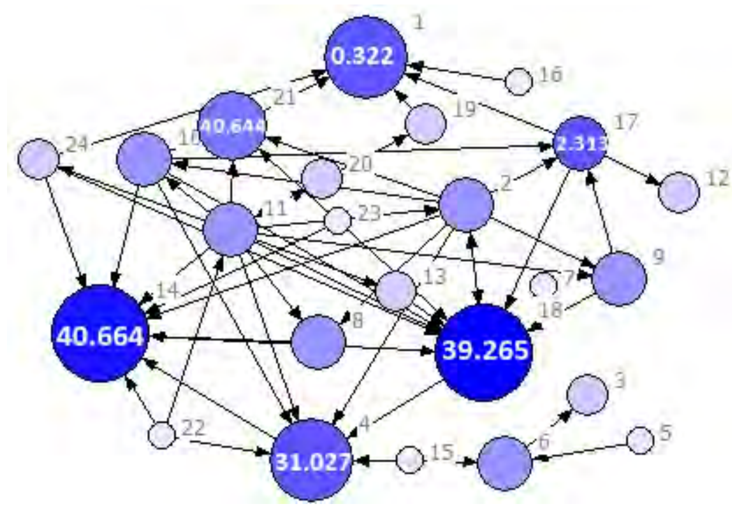

Figure 4. The directed citation network with weights added to nodes.

Authority-Popularity Evaluation Model

The aim of **Requirement 3** is to determine relative influence. For this task, we develop an Authority-Popularity Evaluation Model, which is a combination of the revised PageRank Algorithm and a Normalized Influence Factor. In our model, the PageRank value and the Influence Factor reflect the depth and the width of the impact, respectively.

The Modified PageRank Algorithm

The basic idea of the PageRank Algorithm is that the importance of a node is determined by the quantity and quality of other nodes pointing to it. But when there is a dangling node (a node whose out-degree is 0), random surfing will fail, since the surfer will be "trapped" in the dangling node forever. There are three dangling nodes the Citation Network, so we propose a Modified PageRank Algorithm to measure influence. The Modified PageRank Algorithm includes two steps: Initialization and Iteration.

- **Step 1: Initialization**

  We give an initial PageRank value (PR value) $PR_x(0)$, for $x = 1 \ldots, n$, to all nodes in the network, normalized so that

$$\sum_{x=1}^{n} PR_x(0) = 1$$

  The initial PR value of each paper is defined as the normalized impact factor of the journal in which the paper is published. The absolute impact factor values (collected from the Web [Impact Factor Search 2014]) for each paper are listed in **Table 4**. For instance, the weight of paper 4, "Emergence of scaling in random networks" published in *Science* is equal to the Impact Factor of *Science*, viz., 31.027.

Table 4.
Weights of Works.

1	2	3	4	5	6	7	8
0.32	44.98	0.54	31.03	0.42	3.38	4.38	38.60
9	10	11	12	13	14	15	16
2.31	9.74	5.95	3.54	31.03	38.60	5.02	3.38
17	18	19	20	21	22	23	24
2.31	39.27	1.83	2.54	40.66	9.74	9.74	7.94

A 24-order relationship matrix G is constructed by the rule that if paper j is cited by paper i, then $G_{ij} = 1$; otherwise $G_{ij} = 0$. The coefficient matrix D (also of order 24) is:

$$D = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \cdots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sum(1)} & & & & \\ & \frac{1}{\sum(2)} & & & \\ & & \cdots & & \\ & & & \frac{1}{\sum(23)} & \\ & & & & \frac{1}{\sum(24)} \end{pmatrix},$$

where $\sum(i) = \sum_{j=1}^{24} G_{ji}$.

- **Step 2: Iteration**

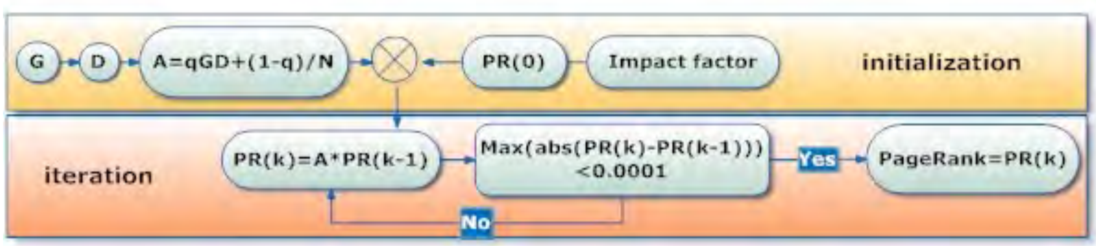

Figure 5. The flowchart of the modified PageRank iteration process.

After k iterations, we get the PageRank value:

$$PR_i(k) = q \sum_j \frac{PR_j(k)}{L(j)} + \frac{1-q}{n}, \qquad i = 1, 2, \ldots, n,$$

where j represents all nodes that point to node i and q is a constant with default value 0.85. We iterate the equation above until the change in PageRank in a single step is small enough; we set this threshold as 0.0001. We get the stable PageRank values $PR(k)$ shown in **Table 5**.

Table 5.
Ranks of works according to the stable PageRank value.

Ranking	$PR_i(k)$	**ID**	**Ranking**	$PR_i(k)$	**ID**
1	0.01575	18	13	0.00094	19
2	0.01217	14	14	0.00060	11
3	0.00937	4	15	0.00060	23
4	0.00784	2	16	0.00051	24
5	0.00493	1	17	0.00051	20
6	0.00278	17	18	0.00051	13
7	0.00183	21	19	0.00047	22
8	0.00150	3	20	0.00047	16
9	0.00142	10	21	0.00047	15
10	0.00142	9	22	0.00047	12
11	0.00142	8	23	0.00047	7
12	0.00111	6	24	0.00047	5

The result shows that the No. 18 work *Random Graphs*, by B. Bollobás, has the highest PageRank. So it is the most authoritative (highest Authority value) work among the 24. Authority reflects the depth of influence.

To find the most influential paper, we have to get the *width* of influence: popularity. We use citations per year to measure popularity, as shown in **Table 6**.

Table 6.
Citations per year of each paper.

ID	citations/year	ID	citations/year
14	1355.5	8	89.0
4	1256.2	1	82.4
2	1104.2	3	72.4
11	965.1	13	69.6
18	458.2	6	54.5
21	281.8	20	34.6
10	211.4	16	27.7
17	178.2	24	19.5
23	170.6	19	16.9
22	142.3	15	11.0
9	117.2	7	0.5
12	97.1	5	0.3

To judge which is the most influential paper, we draw a picture of 24 nodes (**Figure 6**). For the sake of visibility, we use a log-log scale, graphing $\log_{10}(10^6 \times PR_i(k))$ vs. $\log_{10}(10 \times \text{citations/year})$.

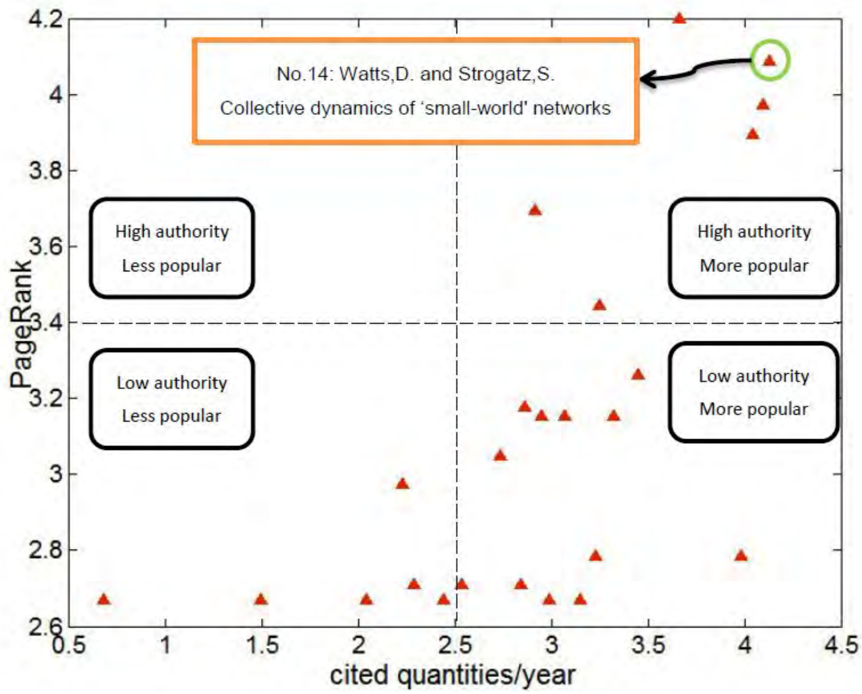

Figure 6. Authority-Popularity Diagram for the citation network

In our Authority-Popularity Evaluation Model, we divide **Figure 6** into four regions:

- high authority, more popular;
- high authority, less popular;
- low authority, more popular; and
- low authority, less popular.

The node in the upper right corner is the most influential paper, since it is both the most authoritative and the most popular one. It is No. 14: "Collective dynamics of 'small-world' networks," by D. Watts and S. Strogatz.

The Co-star Network for Requirement 4

Similar to the Erdős number in mathematics, a Bacon number in the film industry became popular in the late 1990s. The "Game of Kevin Bacon" measures the shortest path to connect an arbitrary actor/actress to Bacon. Inspired by the game, we collect a set of 15 famous Hollywood movie stars as nodes and movies as links to their co-stars to construct a co-star network (**Figure 7**). This is also a directed network connection graph with weights, but the difference is that the weight value is on the link instead of on the node.

- **Explanation 1**: *Let A be the leading actor who is supported by actor B in a movie, then the direction of the link between them is from B to A.*
- **Explanation 2**: *According to **Assumption** 3, we define the weight of the link between two stars as the IMDb rating [Internet Movie Database 2014] of the movie in which they co-starred.*

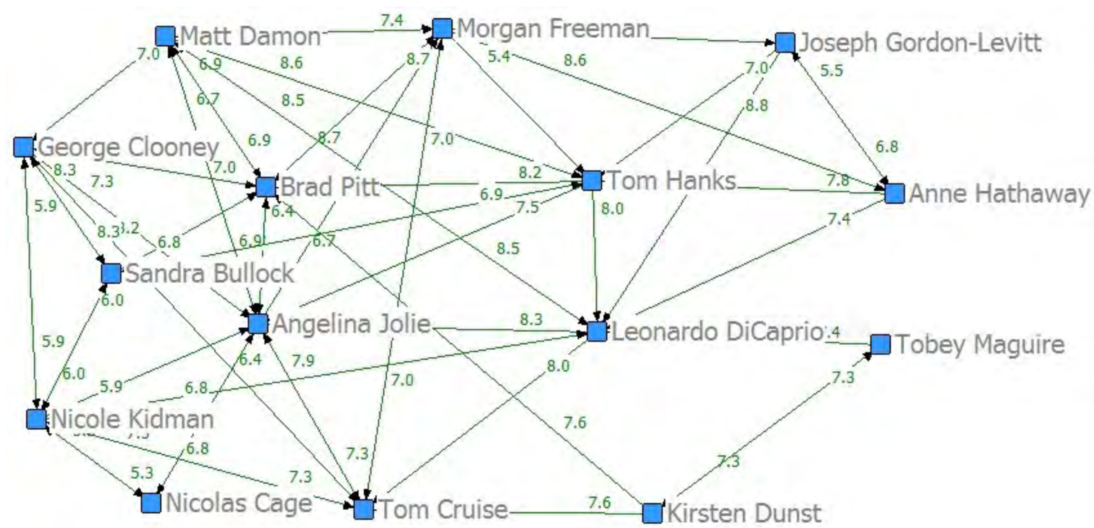

Figure 7. The directed citation network with weight added to links.

Since in the co-star network the weight is on the link instead of on the node, the PageRank algorithm here has to be improved. In the original PageRank algorithm, at every step of the iteration, the PageRank value of node i is distributed uniformly to nodes that i points to. In our improved PageRank algorithm, how much PageRank value that node i distributes to nodes that i points to is determined by the weight (IMDb rating of the movie) of the link. This is reasonable because an excellent movie can make movie stars more influential. For example, suppose that the weight of the link (i to k) is 3 and the weight of the link (i to n) is 4. So the PageRank value that i distribute to k is $\frac{3}{3+4} \times 1$ and the PageRank that i distribute to n is $\frac{4}{3+4} \times 1$, as is shown in **Figure 8**.

So iteration for this algorithm is different. The relationship matrix G in the iteration for the citation network should be revised to G' in the co-star network. We set column j of G as G_j, with G'_j defined similarly. It is clear that if $G_{ij} \neq 0$, then G'_{ij} represents a movie. The elements in G'_j should be proportional to the IMDb rating of the movie in G'_j. For example, assume that movie G_{2j}'s IMDb rating is 3 and G_{4j}'s IMDb rating is 4. Let

$$G_j = [0 \;\; 1 \;\; 0 \;\; 1 \;\; 0]^T$$

So we get

$$G'_{2j} = \tfrac{3}{3+4}(1+1) = \tfrac{6}{7}, \quad G'_{4j} = \tfrac{4}{3+4}(1+1) = \tfrac{8}{7}, \quad \text{and} \quad G'_j = [0 \;\; \tfrac{6}{7} \;\; 0 \;\; \tfrac{8}{7} \;\; 0]^T.$$

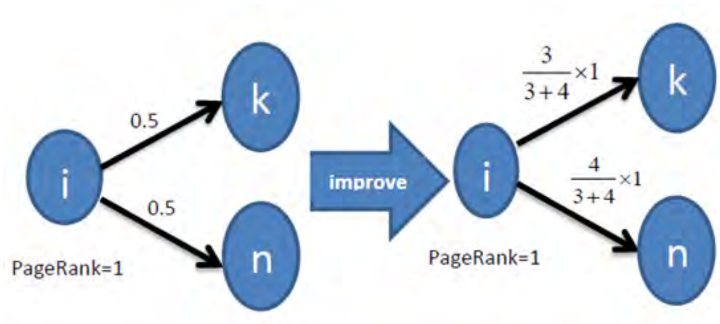

Figure 8. The diagram for the modified PageRank algorithm.

Other steps are the same as previously. We also implement the improved PageRank algorithm in Matlab, with the results shown in **Table 7**.

Table 7.
PR value and number of Google search results for each star.

Name	PR value	Google results (millions)
Angelina Jolie	0.141	242
Tom Cruise	0.117	238
Brad Pitt	0.097	198
Morgan Freeman	0.092	61
Matt Damon	0.087	85
Tom Hanks	0.076	122
Nicole Kidman	0.074	87
Leonardo DiCaprio	0.070	149
George Clooney	0.062	58
Nicolas Cage	0.044	35
Sandra Bullock	0.044	50
Joseph Gordon-Levitt	0.032	20
Anne Hathaway	0.032	86
Kirsten Dunst	0.016	16
Tobey Maguire	0.015	7

As we did earlier for the Erdos1 network, we draw a figure for the 15 nodes in the co-star network. In **Figure 9**, we graph stable PageRank value vs. $\log_{10}$(number of Google search results). It is clear that *Angelina Jolie* is the most influential movie star in this network.

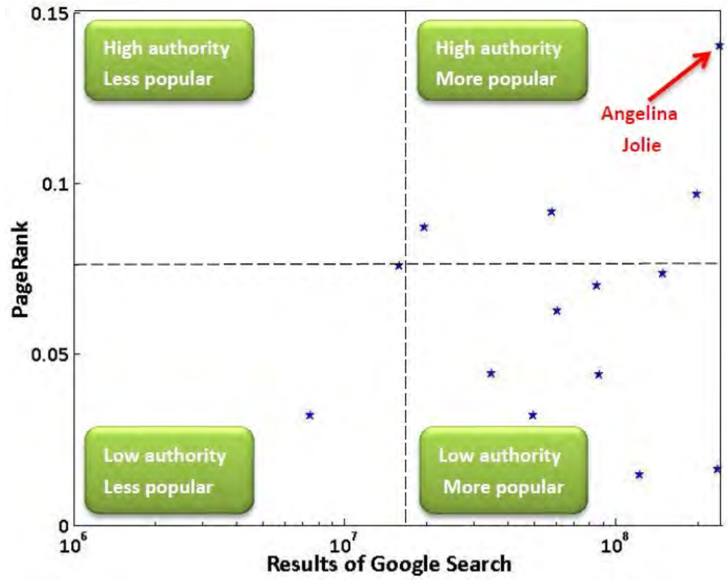

Figure 9. Authority-Popularity Diagram for the co-star network.

Sensitivity Analysis

To analyze the robustness of our model, we perform a sensitivity analysis. In the PageRank algorithm, there is the equation

$$PR_i(k) = q \sum_j \frac{PR_j(k)}{L(j)} + \frac{1-q}{n} \qquad i = 1, 2, \ldots, n$$

where q is a constant, generally called the damping coefficient. The meaning of q is: There is a probability $(1 - q)$ that all of the nodes in the network have the same PageRank value $1/N$. If the coefficient q is not used, the "strong" nodes will be so strong that other nodes find it hard to "survive."

For the co-star network, too, this make sense. The PageRank value of a movie star reflects influence, which can also be seen as the likelihood that a movie wants him to star. But there are always some movies that do not need the most famous actor, and for which all movie stars all have an opportunity to be the star. This situation is reflected by the coefficient q. So the value of q can influence the PageRank of the network. We set the default to be $q = 0.85$, as is standard. With a change in q, we get a corresponding PageRank and can compare this PageRank with the standard one (**Figure 10**).

The results show that in a certain range, the PageRank changes slowly with a change in q. So our model has high stability.

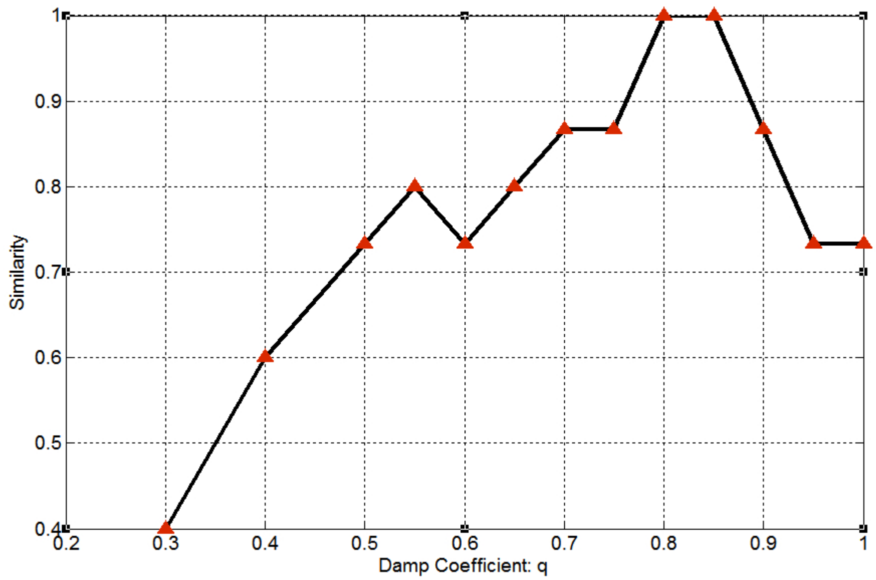

Figure 10. Sensitivity to damping coefficient q.

Strengths and Weaknesses

Strengths

- **Comprehensiveness**: From the perspective of both depth and width, we determine importance/influence based on a variety of indexes.
- **Adaptability and Practicability**: The model we build has good portability; it is suitable for most network analysis.
- **Simplicity and Accuracy**: The programs of the model are easy to understand, and the calculations are precise.
- **Flexibility**: No matter whether weights are added to the nodes or to the links, and whether there are dangling nodes in the network or not, our model can handle the network.

Weaknesses

- **Data Limitations**: If the data on a network are limited, the error may be large.

The Philosophy of a Scale-Free Network

Barabási and Albert [1999] pointed out that an ER (Erdős-Rényi) random graph and a WS (Watts-Strogatz) small-world model neglect two important characteristics of an actual network:

- **Growth**: The scale of an actual network is likely growing; for instance, many papers are published every month. However, in an ER random graph and in a WS small-world model, the number of nodes is fixed.
- **Preferential attachment**: New nodes tend to connect with nodes with high connectivity. This phenomenon is also known as "the rich get richer" or the "Matthew effect" [Wikipedia 2014a]. New papers tend to cite important and influential papers that have been widely quoted. And also, actors always try their best to act with stars.

The dynamics in a scale-free network is the basis for the famous Pareto principle [Wikipedia 2014b] (also known as the 80-20 rule), which states that for many events, roughly 80% of the effects come from 20% of the causes.

Shortcut to Boost Influence

In the real world, "important nodes" are hard for ordinary ones to approach: An ordinary researcher will find it hard to co-author with a leading figure, since the important ones tend to co-author with other influential people. So, how can we boost our influence as quickly as possible? The solution that we propose from our model is illustrated in **Figure 11**:

- Determine the "20%" in the network according to our Relation Distance Model or Authority-Popularity Evaluation Model
- Calculate the closeness centrality of the "80%" to the "20%."
- Cooperate step by step with the "80%" (more approachable, relatively) with low closeness centrality.
- Finally, co-author with the key figure in the field!

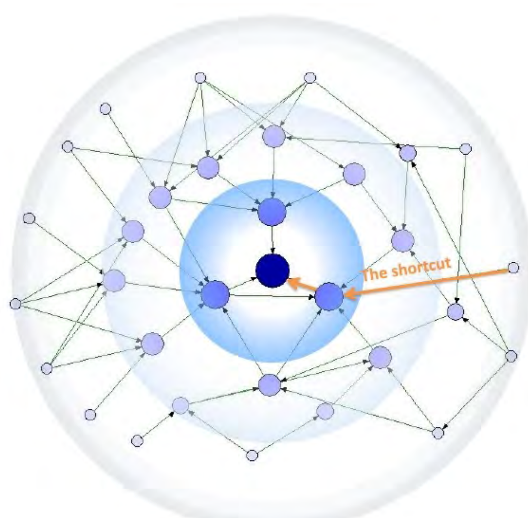

Figure 11. The shortcut to boost influence.

Appendix: The 8 Additional Papers

No. 17: Newman, M.E.J., S.H. Strogatz, S and D.J. Watts. 2001. Random graphs with arbitrary degree distributions and their applications. *Physical Review E* 64 (2): 026118–026134.

No. 18: Bollobás, B. 2001. *Random Graphs*. 2nd ed. New York: Academic Press.

No. 19: Holland, P.W., and S. Leinhardt. 1981. An exponential family of probability distributions for directed graphs. *Journal of the American Statistical Association* 76: 33–65.

No. 20: Snijders, T.A.B. 2002. Markov chain Monte Carlo estimation of exponential random graph models. *Journal of Social Structure* 3 (2).

No. 21: Watts, D.J. 1999. *Small Worlds*. Princeton, NJ: Princeton University Press.

No. 22: Barrat, A., M. Barthélemy, R. Pastor-Sattoras, and A. Vespignani. 2004. The architecture of complex weighted networks. *Proceedings of the National Academy of Sciences* 101 (11): 3747–3752.

No. 23: Amaral, Luís A. Nunes Amaral, Antonio Scala, Marc Barthélemy, and H. Eugene Stanley. 2000. Classes of small-world networks. *Proceedings of the National Academy of Sciences* 97 (21): 11149–11152.

No. 24: Barthélemy, M., and Luís A. Nunes Amaral. 1999. Small-world networks: Evidence for a crossover picture. *Physical Review Letters* 82: 3180.

References

Barabási, Albert.-László., and Réka Albert. 1999. Emergence of scaling in random networks. *Science* 286 (5439): 509–512.

Clauset, Aaron, Cosma Rohilla Shalizi and M.E.J. Newman. 2007. Power-law distributions in empirical data. `arXiv:0706.1062v1`.

Freeman, L.C. 1979. Centrality in social networks: Conceptual clarification. *Social Networks* 1 (3): 215–239.

Impact Factor Search. 2014. `http://www.impactfactorsearch.com`.

Internet Movie Database (IMDb). 2014. `http://www.imdb.com/`.

Mahmoodzadeh, S., J. Shahrabi, M. Pariazar, and M.S. Zaeri. 2007. Project selection by using fuzzy AHP and TOPSIS technique. *World Academy of Science, Engineering and Technology* 30: 333–338.

Reynolds, Patrick. 2013. The Oracle of Bacon. `http://oracleofbacon.org/`.

Wikipedia. 2014a. Matthew effect. `http://en.wikipedia.org/wiki/Matthew_effect`.

Wikipedia. 2014b. Pareto principle. `http://en.wikipedia.org/wiki/Pareto_principle`.

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Judges' Commentary: Measuring Network Influence and Impact

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Introduction

The topic area for this year's Interdisciplinary Contest in Modeling (ICM) was network science. Network science will continue to be the topical area for one of next year's ICM problems. However, there will also be a second ICM problem, involving human-environment interactions in the areas of environmental science, including climatology, food security, and geography. For teams that want to organize early for next year's contest, prepare by studying network modeling or environmental science and assemble a team with one of those subjects in mind.

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The ICM continues to be an opportunity for teams to tackle challenging real-world problems that require a wide breadth of understanding in multiple academic subjects and skill in modeling interdisciplinary phenomena. These kinds of interdisciplinary study and modeling are included in the definition of network science. The complexity of ICM problems, along with the short duration of the contest, requires effective communication and coordination among team members. One of the most challenging issues for ICM teams is to organize and collaborate effectively to use each team member's skills and talents to tackle the diverse nature of ICM problems. Teams that resolve this organizational challenge and co-operate well often submit 20-page solutions that rise to the higher levels of ICM awards.

The Problem Statement

One of the techniques to determine influence of academic research is to build and measure properties of citation or co-author networks. Co-authoring a manuscript usually connotes a strong influential connection between researchers.

One of the most famous academic co-authors was the 20th-century mathematician Paul Erdős, who had over 500 co-authors and published over 1,400 technical research papers.

It is ironic (or perhaps not!) that Erdős is also one of the influencers in building the foundation for the emerging interdisciplinary science of networks, particularly, through his publication with Alfred Rényi of the paper "On random graphs" [1959].

Erdős's role as a collaborator was so significant in the field of mathematics that mathematicians often measure their closeness to Erdős through analysis of Erdős's amazingly large and robust co-author network (see Grossman [2014]).

The unusual and fascinating story of Paul Erdős as a gifted mathematician, talented problem solver, and master collaborator is provided in many books and online Websites (e.g., O'Connor and Robertson [2000]). Perhaps his itinerant lifestyle, frequently staying with or residing with his collaborators, and giving much of his money to students as prizes for solving problems, enabled his co-authorships to flourish and helped build his astounding network of influence in several areas of mathematics.

To measure such influence as Erdős produced, there are network-based evaluation tools that use co-author and citation data to determine impact factor of researchers, publications, and journals. Some of these are *Science Citation Index*, H-factor, Impact factor, Eigenfactor, etc. Google Scholar is also a good data tool to use for network influence or impact data collection and analysis. Your team's goal for ICM 2014 is to analyze influence and impact in research networks and other areas of society.

We summarize the tasks for the teams in this year's ICM problem:

1. Build the co-author network of the 511 Erdos1 co-authors. This will take some skilled data extraction and modeling efforts to obtain nodes (the Erdős co-authors) and their links (connections with one another as co-authors). There are over 18,000 lines of raw data in Erdos1 file, but many of them will not be used since they are links to people outside the Erdos1 network. Analyze the properties of this network.
2. Develop influence measure(s) to determine who in this Erdos1 network has significant influence within the network. Consider who has published important works or connects important researchers within Erdos1.
3. Another type of influence measure might be to compare the significance of a research paper by analyzing the important works that follow from its publication. Choose a set of foundational papers in the emerging field of network science (a possible set was provided). Use these papers to develop a model to determine their relative influence. Which of the papers of the set do you consider is the most influential in network science and why?
4. Implement your algorithm on a completely different set of network influence data.
5. Discuss the science and utility of modeling influence and impact within networks. Could individuals, organizations, nations, and society use influence methodology to improve relationships, conduct business, and make wise decisions?

A Short Historical Reflection on Paul Erdős and the Erdős Co-author Network

Paul Erdős's creative research advanced graph theory, combinatorics, discrete mathematics, and number theory and laid foundations for the applied subjects of computer and network science. He excelled at modeling number systems and graphical structures, and determining their properties. He worked on many of the most important problems in these fields; and, through tireless effort and amazing skill, he became the most prolific and eccentric mathematician of modern times. He published over 1,500 scholarly papers. Paul Hoffman, who wrote Erdős's biography *The Man Who Loved Only Numbers*, wrote, "Erdős's style was one of intense curiosity, a style he brought to everything he confronted" [2012, 21]. The ICM hopes to develop that trait of curiosity in its contestants.

Because of Erdős's extensive collaborations, mathematicians began tracking and counting his collaborators. A special network numbering system was devised such that a person who collaborated by publishing a paper with Erdős was given an Erdős number of 1. Collaboration (publishing) with an Erdős 1 author gave a mathematician an Erdős number of 2, and so on. If there is no chain of co-authorships connecting someone with Erdős, the person's Erdős number is infinite. The result of this effort, using the Erdős Number Project site (`http://www.oakland.edu/enp/`) [Grossman 2014] and data through the MathSciNet service of the American Mathematical Society's *Mathematical Reviews*, is an elaborate collaboration graph of the mathematics research community that captures the connections of over 400,000 authors. Elaborate records and a myriad of statistics are kept on the collaborations and connection record of Erdős. Today the Erdős Number Project Website provides all sorts of trivia, such as the data in **Table 1** on co-author connections to Erdős [Grossman 2014]. The table shows the number of people with Erdős number 1, 2, 3, . . . , according to the electronic data from MathSciNet (slightly different than other data sources).

Table 1.
Numbers of mathematicians with particular Erdős numbers.

Erdős number	Number of mathematicians
0	1
1	504
2	6593
3	33,605
4	83,642
5	87,760
6	40,014
7	11,591
8	3,146
9	819
10	244
11	68
12	23
13	5

Thus, at the moment when this table was tabulated, the median Erdős number was 5, the mean 4.65, and the standard deviation 1.21. In addition to these 268,000 people with finite Erdős number, there are about 50,000 published mathematicians who have collaborated with others but have an infinite Erdős number, and 84,000 who have never published joint work (and therefore also have an infinite Erdős number).

Erdős lived from 1913 to 1996 and spent six decades living out of two tattered suitcases. He would show up on the doorsteps of his colleagues prepared to do work, and they would accommodate him. After working through a problem or two and writing a paper, he would move on to the next research station, hopefully to confront the next problem and find its solution.

Erdős received many awards, which allowed him the freedom to travel and the money to pay student solvers of his legendary challenge problems. His challenge problems were often easy to state but difficult to solve. These numerous cash giveaways to student problem-solvers made Erdős's campus visits special events. Another quotation about Erdős from Hoffman's book reinforces an interdisciplinary perspective: "For Erdős, mathematics was a glorious combination of sciences and art." [1998, 27]

Judges' Criteria

The panel of judges was impressed by the modeling of many teams. Many papers were rich in network modeling methodology and modeling creativity. To ensure that the individual judges assessed submissions on the same criteria, a judging rubric and guide was developed. The general framework used to evaluate submissions is described below. The main thrust of ICM problem-grading is finding and evaluating modeling that includes good science and leads to measurable outcomes and a viable solution.

Executive Summary

It was important in the summary that students succinctly and clearly explained the highlights of their submissions. The executive summary should contain brief descriptions of both the problem and the bottom-line results. One mark of better papers was a summary with a well-connected and concise description of the methodology, results and recommendations.

Modeling

Well-defined measures of influence and impact were needed to build a viable model. Many teams started with standard network centrality measures and modified them to produce influence or impact effects. Other teams calculated other measures from clustering, community building, and dynamic measures. In this problem, teams needed to develop viable influence measure(s) to determine who in the network has significant influence within the network. For some teams, influence was a scalar value; others established a multidimensional vector with several components.

Many teams used network analysis software packages such as Gephi, ORA, Pajek, and UICNET for both calculations and visualizations. In many cases, the resulting mathematical analysis included statistical measures. Some teams used the explicit structures of networks or graphs to determine classic nodal measures and properties. In such cases, critical assumptions such as the directionality of influence and weights of connections within the network led to viable network models.

Better papers discussed the differences in co-author and citation networks by explaining that a co-author network is not directed and a citation network model is directional. Similarly, the Erdős co-author network is now static (nearly 20 years after his death); but the citation network is dynamic, with new citations to papers occurring frequently.

No matter the modeling framework, the assumptions needed for these models and the careful and appropriate development of these models were important in evaluating the quality of the solutions. The better submissions explicitly discussed why key assumptions were made and how these assumptions affected model development.

Stronger submissions presented a balanced mix of mathematics and prose rather than a series of equations and parameter values without explanation. One major discriminator was the use or misuse of arbitrary parameters without any explanation or analysis. Establishing and explaining parameter values in models are at least as significant as making and validating assumptions.

Perhaps the most challenging aspect of this problem was determining the topic and data collection of the test application dataset. Collecting good data where their influence measure was appropriate was a challenge. The judges recognized this challenge and rewarded papers with strong datasets that produced viable network models.

Science

The ICM modelers discussed the science of influence at many levels. Some teams did effective background research and analysis of this aspect of the problem, included elements of their scientific analysis, and described how their model fit into the science of influence. In this case, powerful scientific analysis was performed best by making strong, insightful connections between the precise mathematical measures that teams created and the abstract notions of influence that they produced from social theory.

No matter what level of modeling was performed by the teams, the interdisciplinary nature of this problem was revealed in the science requirements and the background investigation performed by the teams. The ICM students were exposed to the nature of influence in information theory in performing their background research, and the team reports required proper documentation of the team's research sources.

Data/Validity/Sensitivity

Filtering over the 18,000 lines of data to extract the 511 co-author nodes and nearly 1,700 links was a challenge for some teams, as was collecting data for their own test application. Sensitivity analysis to determine the effects of assumptions and data validity were empowering for some of the teams. Sensitivity analysis is especially important for highly-structured and powerful data-rich models such as networks. Some network structures are highly robust and flexible, while others are more fragile and highly sensitive to data errors or changes. While this sensitivity analysis is a challenging element of network modeling, it was important to address this modeling issue in the report. Teams that did this well quickly rose to the top of judges' evaluations.

Strengths/Weaknesses

Discussion of the strengths and weaknesses of the models is where students demonstrate their deeper understanding of what they have created. The utility of a model fades quickly if team members do not understand the limitations or constraints of their assumptions or the implications of their methodology. Networks are non-reductive, complex structures and, therefore, the strengths and weakness are often hidden from direct view or full control of the modeler. Some of the better reports presented these elements despite these challenges.

Communication/Visuals/Charts

To clearly explain solutions, teams used multiple modes of expression including diagrams and graphs, and—for this contest—clearly written English. A report that could not be understood did not progress to the final rounds of judging. Judges were often well informed through the amazing array of powerful charts and graphs that explained both models and results. The graphics shown in **Figures 1–3** provide a glimpse of the richness of this kind of presentation.

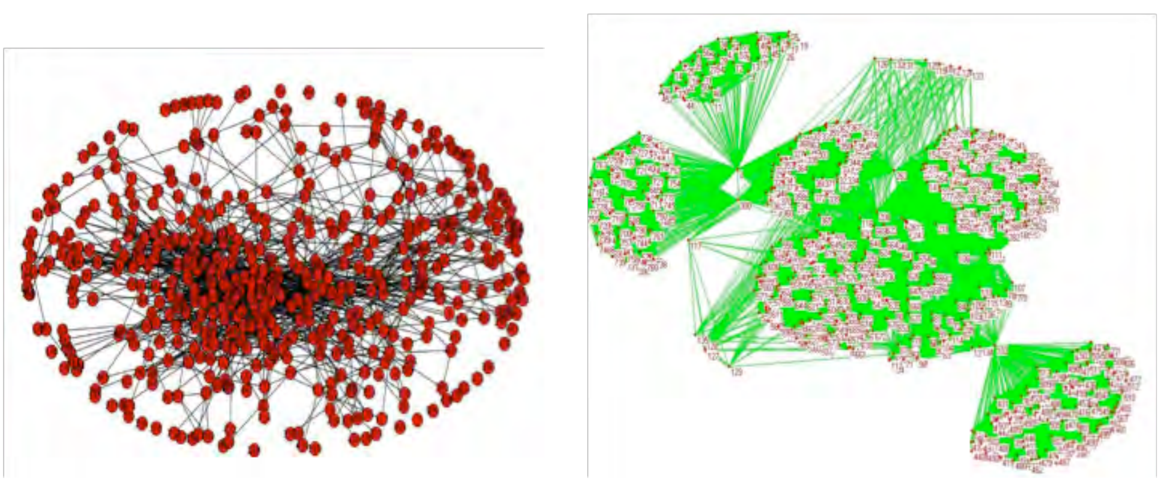


Figure 1. Many teams provided informative network graphs to show the entire co-authors' network model. The graphic on the left is from Team 25425 (Beijing University of Posts and Telecommunications). The graphic on the right is from Team 30407 (Hong Kong Polytechnic University).

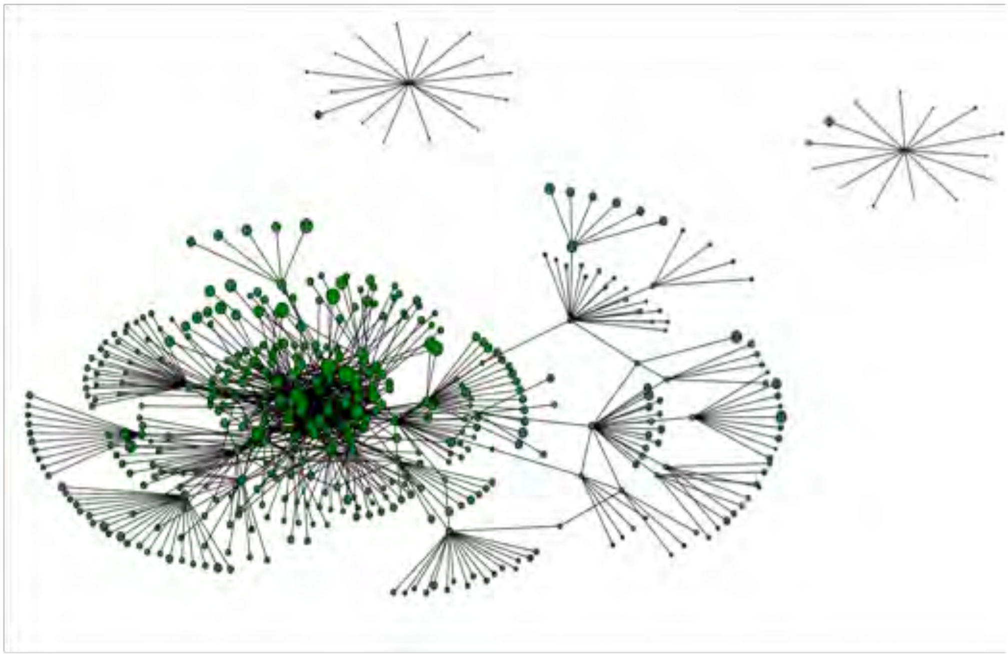


Figure 2. Some reports contained elaborate co-citation network diagrams, like this one from Team 31227 (Humboldt State University).

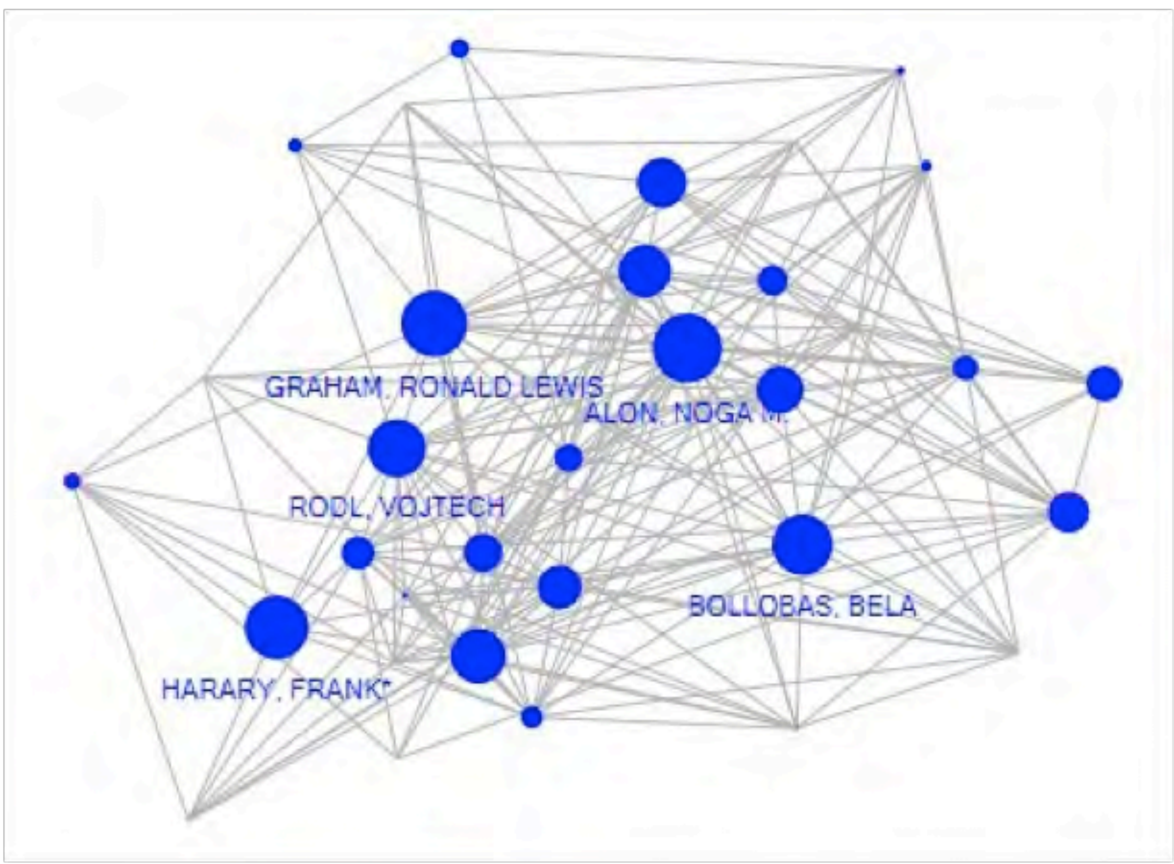


Figure 3. Other graphics zoomed in on significant parts of the co-author network to show details of the most influential co-authors, like this one from the report by Team 26715 (Peking University).

Discussion of the Outstanding Papers

Despite the common dataset and tasks, many different approaches were used by ICM teams to model various aspects of the problem. As a result, the submissions were varied and interesting. Overall, the basic modeling was often sound, creative, and powerful. Those papers that did not reach final judging generally suffered from various shortcomings. Some lacked clear explanation of the structure of their model. They provided some details but not a complete description of their model and its purpose. Others failed to connect their mathematical models to the aspects and basic elements of the science of influence. In general, incomplete or awkward communication was the most significant discriminator in determining which papers reached the final judging stage.

Although the six Outstanding papers used different methodologies, they all addressed the problem in a comprehensive way. These Outstanding papers were generally well written and presented clear explanations of their modeling procedures. In several of the Outstanding papers, a unique or innovative approach distinguished them from the rest of the finalists. Others were noteworthy for either the thoroughness of their modeling or the significance of their results. Summaries of the six Outstanding papers follow.

Central University of Finance and Economics, China: "Influence Measures in Networks"

The team from Central University of Finance and Economics gave their report the title "Influence Measures in Networks" to reflect their focused and quite thorough investigation of network proximity as a proxy of influence among network members. They rightly point to the limitations of some of the traditional network metrics, namely centrality measures, in their inability to efficiently handle link weights and to account for the whole network structure. This group instead combined the Shapley approach, a concept developed in cooperative game theory, with a cohesion measure (KPP-POS, developed by Stephen Borgatti) to capture influence effects. The limitation of this measure is that it can be applied only to undirected networks.

The team's approach to directed networks was somewhat less novel, making modest modification to the frequently used PageRank measure.

The team's approach to combine conventional social network analytic methods with the less obvious, game-theoretic Shapley approach showed a breadth and depth in their handling of this problem. Challenged with developing a more appropriate and meaningful influence measure, they modified Borgatti's cohesion measure to enable inclusion of weighted edges, and more notably, used the Shapley method to account for the rank order of nodes.

As a nice introduction to their analysis, the team provided descriptive information about the networks, such as the degree distribution, path length, and clustering. They also showed that their new metric of influence gives different results than conventional betweenness centrality values. They chose to validate their Shapley-cohesion measure on a weighted network that they built by selecting actors who have collaborated with the popular British actor, Jude Law. Unfortunately, on that dataset, their measure did not perform differently from betweenness centrality. The Jude Law network may have been too small to enable detection of difference among these metrics.

The team's approach to directed networks was one that was used by many ICM teams. However, this team successfully modified the network data to overcome some of the limitations of the PageRank method. Specifically,

- they incorporated additional papers into the citation network to augment the data specified in the problem; and
- they weighted the papers as a function of the number of times they were cited by these endogenous nodes.

The schematic of their model is provided in **Figure 4**.

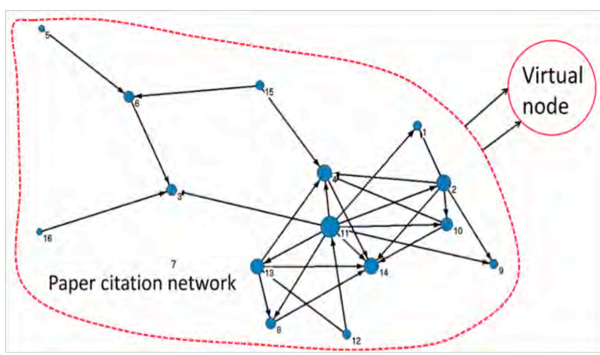


Figure 4. Network model supplemented with additional co-citation papers.

With their modified or optimized PageRank measure, the team found influence rankings among publications that differed from the standard PageRank measure. However, they did not investigate whether those papers that jumped in rank were any more meaningful than the standard rank. In addition, the team did not deal with the dimension of time—ignoring the fact that articles could not be cited by articles that appeared earlier in time, or that articles that were cited more recently or over longer periods of time were probably more influential.

Judges were impressed with the ability to combine the conventional network metrics with game-theoretic approaches. Further, the team showed thoughtfulness about applying network science concepts to a problem of social influence. This paper was well written and contained graphic results such as that given in **Figure 5** to show the nodal degree distribution between the Erdos1 network and a similar-sized random network.

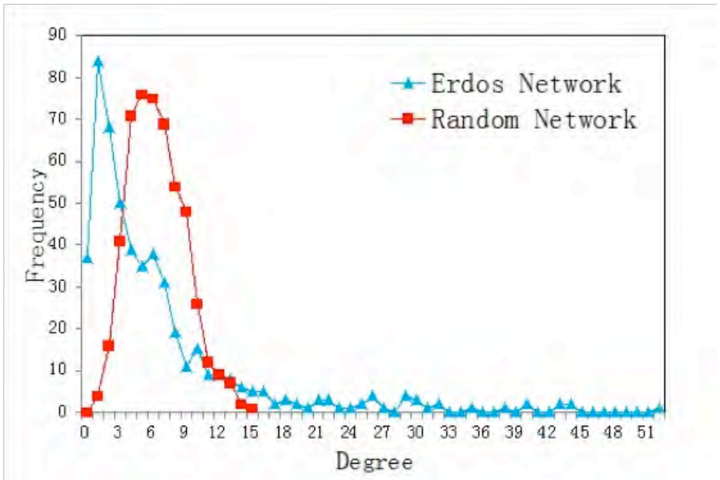


Figure 5. Nodal degree distribution of the Erdős co-author network and a similar-size random graph.

National University of Defense Technology, China: "The Research of Influence Based on the Characteristic of a Network"

The team from National University of Defense Technology took two very different approaches in analyzing the graphs, depending on whether or not the graph was directed or not. For the undirected co-author network, this team explained and computed many of the established network metrics for each node, including degree, betweenness, closeness, and eigenvector centralities. Rather than use all these metrics, the team provided a clear argument for first identifying those with the most authority (those who had published the most number of times with Erdős), and then ranking these authoritative co-authors based only on eigenvector centrality.

To qualitatively validate the results produced by this algorithm, the team did online searches to learn about the careers of each of the top five mathematicians on their results list. For the citation network, this team gave a visual representation of the network, laid out on a temporal axis. The inclusion of time in the network visualization was an excellent example of how something relatively simple can really make a difference in the ease of interpretation of the results.

For the analysis, this team's approach to the citation network relied on the fact that the resulting network is a directed acyclic graph. This team then attempted to identify the most influential paper by examining four different centrality measures, only to discover that their results were inconsistent.

From there, they determined that a topological sorting algorithm would sufficiently decrease runtime compared to a matrix multiplication method such as PageRank.

Leveraging the topological structure of the graph, the team defined and computed a contribution coefficient that took both the number of citations and the timing of those citations into consideration. They discussed how self-citation could influence their results, along with giving a modified model and a sensitivity analysis based on a range of values for their self-contribution coefficient.

Lastly, this team applied their algorithms to construct a directed ownership network of 500 U.S. media corporations and to identify the top media companies. After performing their analysis, they validated their results by looking at published business rankings. Following this, the team provided an insightful list of potential applications and a discussion of the benefits of using network science in business and military decision-making.

The judges were impressed by this paper's strong links between the theory, applications, and mathematics. The visualizations of their networks provided meaningful insights into their analysis as shown in the network graph in **Figure 6**. They calculated many of the standard network measures; but instead of consolidating all of them, they presented a convincing argument for only using certain measures. Additionally, these students demonstrated that they understood the most predictable paths for solving the problems, and they showed how they could use inherent network properties to improve upon the more obvious choices. The judges were very impressed by this team's qualitative approach to validating their results through internet research.

Southeast University, China (INFORMS Winner): "Who are the 20%?"

This team developed a model that they called the Relation Distance Model, which utilized three standard centrality measures (degree centrality, betweenness centrality, and closeness centrality) to construct a normalized centrality measure vector for each node in the network. The Euclidean distance from each node's centrality measure vector to an ideal vector was computed and used to determine the most influential nodes in the network. The team did a nice analysis of the results of their model, and then used eigenvector centrality results as a means to validate the results of their model, which exposed a limitation of the model.

To analyze the citation network, the team developed a model that they called the Authority-Popularity Model. Each paper or node was weighted based on the impact factor of the publishing journal. The model then used the Modified PageRank Algorithm to determine a value representing the importance of each paper in the network, which the team classified as its

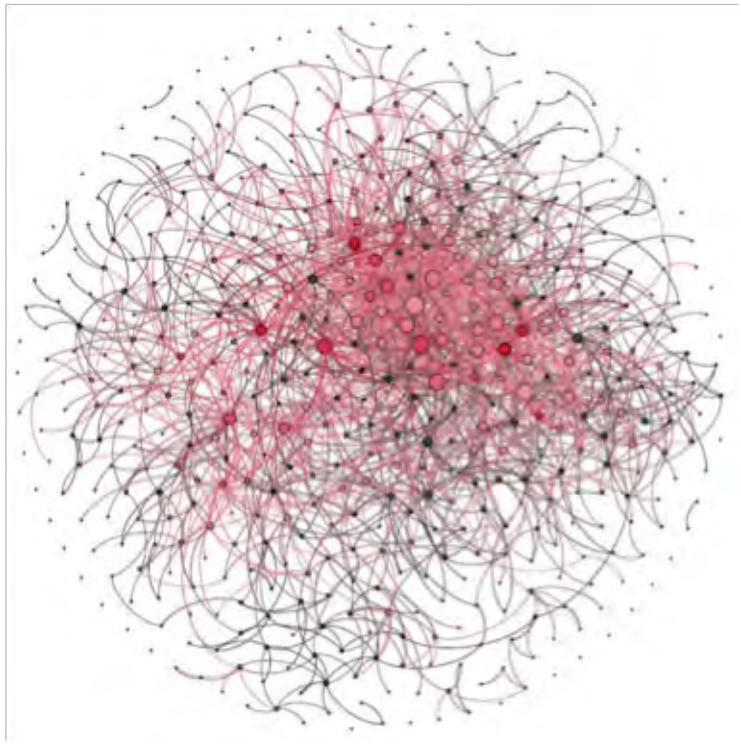


Figure 6. Structure of the Erdos1 network as presented in the report from the Outstanding team from the National University of Defense Technology, China.

level of authority or "depth of influence." Each node was also assigned a value based on the number of times per year that it was cited, which the team classified as its level of popularity or "width of influence." These two scores were then plotted on a log scale to visually segregate those papers that had both high authority and high popularity.

The Authority-Popularity Model was then applied to a co-star network that consisted of 15 movie stars and the links between them. The team recognized that this network would have weighted links (based on the rating of the movie) rather than weighted nodes, and adapted the model accordingly.

A particularly impressive feature of this paper was the strong use of visuals and graphics to clearly present the models and display results in a meaningful way. **Figure 7** is an example. In addition, judges were impressed with the thorough development of each of the models, the analysis of the effects of significant parameters, and the candid discussion of the models' strengths and weaknesses. Finally, the judges appreciated the team's use of their model to propose an innovative method for a researcher to quickly "boost" their influence in order to co-author with a leading figure in network science using the Pareto principle, or 80–20 rule.

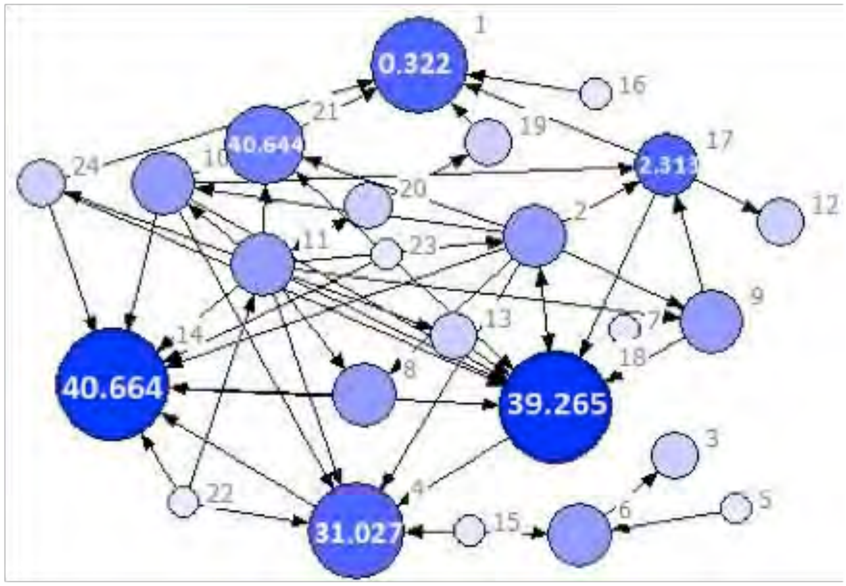


Figure 7. Portrayal of the co-citation network by the Outstanding team from Southeast University, China.

Tsinghua University, China: "Who is the Hidden Champion in a Network?"

This team performed standard network analysis on the Erdős co-author network, considering four standard centrality measures: degree centrality, eigenvector centrality, closeness centrality, and betweenness centrality. They did a very nice job of explaining each centrality measure and interpreting its meaning. The team evaluated the citation network both as a directed and as an undirected graph. While recognizing that the citation network is a directed network, they initially transformed it using symmetric relationships to find two related undirected networks, which they called a co-bibliography network and a co-citation network. The team then evaluated these related networks using the same centrality measures as they used for the co-author network. Next, the team analyzed the citation network as a directed graph, using applicable centrality measures such as the Modified Katz centrality.

The team then applied the methods that they used for the Erdős co-author network analysis to two different data sets: Chinese pop singers and Chinese movie actors. The network for singers had singers as vertices with an edge between two singers if they recorded a song written by the same songwriter. In the network for actors, an edge represents the fact that two actors are in a movie together. Upon completing this analysis, the team then developed a new approach: constructing a bipartite graph of singers-songwriters and actors-films, and using network centrality measures to rank the popularity of each set.

The judges were impressed by the team's clear presentation of the problem and the report's thorough and well-explained analysis. The executive summary and introduction were extremely well written, and the paper concluded with a clear discussion of the strengths and weaknesses of the team's analysis. One innovative aspect of the paper was the team's development and utilization of an algorithm called *stress majorization* to produce a graph of the network, which minimizes distance between vertices that are connected to present an optimal visual depiction of the network.

Tsinghua University, China: "A Three-Dimensional Network Impact Analysis Model Based on Centralizing, Connecting and Spreading Characteristics"

This team from Tsinghua University, as indicated in the title, realized that the task of identifying the most influential node in a network depends on the definition of influence. This team divided the concept of influence into three characteristics:

- centralizing characteristics, which aim to identify the nodes with the most central location in the topology of the network;
- connecting characteristics, which aim to identify the nodes whose positions are crucial to the connectedness of the whole network; and
- spreading characteristics, which aim to identify the nodes that are most capable of promoting flow of information through the network.

For the majority of these characteristics, the team carefully selected established measures from the field of network science and thoughtfully explained how each measure was relevant to its assigned family of characteristics. Specifically, the measures of degree centrality, eigenvector centrality, and page rank were chosen for the centralizing characteristics, while the measures of betweenness centrality, clustering coefficient, and a node removal method for evaluating total loss were chosen for the connecting characteristics. After finding only one established measure, that of closeness centrality, for their spreading characteristics, the team presented the clear development of two new network measures, *spreading breadth index* and *spreading depth index*, both of which factor time into the flow of information through the network.

After calculating all nine of these measures for each node, the team used principal component analysis for each family of characteristics, reducing the measure to a 3-vector, which could be reduced further to a scalar by applying weights based on the goal of the analysis. This approach was then applied to the co-author network, the paper citation network, and the users' comments on a Website for movies that are popular in China. Ultimately, the vectors for the most influential co-authors were presented visually on a three-dimensional graph. The team also presented very meaningful visualizations for their co-author and citation networks.

While many teams used similar sets of established network measures, the judges were impressed by this team's understanding and intuition about how each of these established metrics measured different aspects of influence. When they were not able to find established metrics that measured the elements that they were interested in capturing, this team developed new metrics and wrote detailed explanations of their measurement process. Additionally, this team presented a meaningful way to reduce all of these measures into a scalar, allowing ease of comparison and ranking of nodes. This talented team did a very strong job of connecting the mathematics to the more abstract meaning of influence through clear written prose. Additionally, their paper made excellent use of tables, charts, and graphical representations of their networks to convey results. See **Figure 8** for an example of the team's graphics to display the citation network.

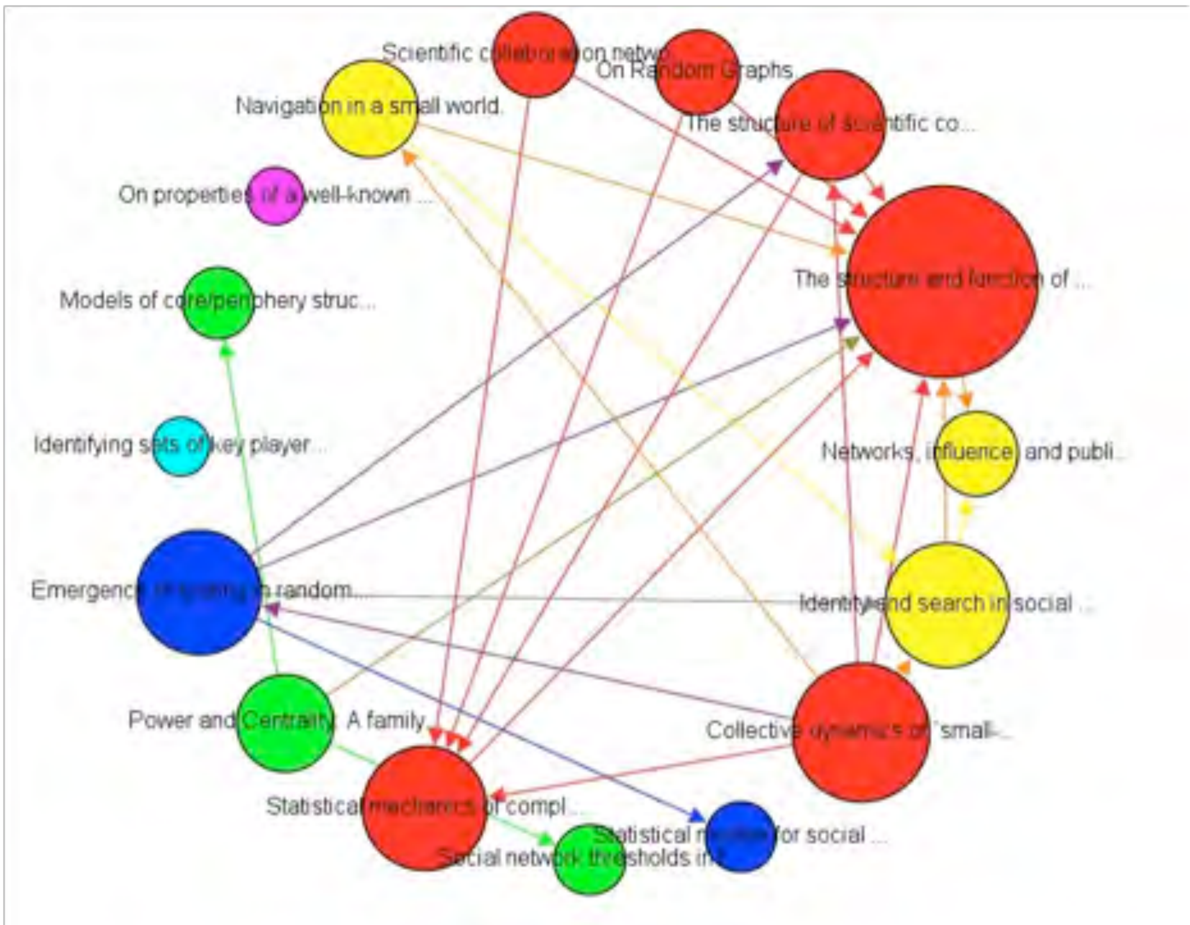


Figure 8. Network of fundamental papers' co-citations, by the Outstanding team from Tsinghua University, China.

The judges realize that given the timeline of the competition, not every team will have an opportunity to tackle masterfully all elements of the problem. This team appears to have done some nice analysis to identify the most influential user based on comments on a Website for popular movies in China; however, this section of the report was not as strongly presented as the other applications. Additionally, although the team did a nice job of explaining some factors that may contribute to the sensitivity of their model, they did not follow through with any computational results in their report.

Xidian University, China: "Methods of Measuring Influence Using Network Model"

The team from Xidian University did excellent work in

- laying out a set of criteria that one should consider in the evaluation of influence,
- how they mapped out these criteria to their specific approach, and finally,
- translating the algorithm components to meaning in a social context.

They developed two new measures of centrality to account for network influence: *importance degree* (combines degree centrality and clustering coefficient) and *influence degree* (weights PageRank with importance degree measure). They focus on these two dimensions of influence, which they attempt to describe: "[importance degree] reflects the researcher's ability to contribute to the ... network by contacting other researchers, while [influence degree]... shows [how] the researcher is affected by ... her/his partners and [how they] can ... [assert] her/his overall influence."

They validate their model with a network of actors who have collaborated with the Chinese movie star Tony Leung Chiu Wai. Analysis of importance degree and influence degree show that these metrics reflect different dimensions of social influence.

Importance degree ranks nodes by combining degree centrality (number of links) and clustering coefficient (links among neighbors), which the team turn into a piecewise function to deal with time of collaboration. In their piecewise function, they account for the year of collaboration between each author and Erdős. They identify the group who collaborated with Erdős in his earlier years as "old researchers," for whom they note "their frequent and early cooperation help them develop and grow in the collaborative network...." One of the most innovative aspects of this team's solution was their thorough handling of the dimension of time on influence.

The team powerfully presents a comparison of the two time periods to examine the changes over time. They found that mathematicians with large differences in their influence before and after their collaboration with Erdős are "young researchers" (those with later collaboration dates). While these researchers were less integrated in the network because they joined the collaboration late, they were successful in creating connections with high-influence researchers, thus drastically improving their influence metrics.

Authors who lose influence over time are those who have many collaborators in the Erdős 2 network (2 steps away from Erdős) but do not integrate into the Erdős 1 network. Judges were uncertain about some elements of the piecewise equation, making this aspect of the paper difficult to judge.

This team found a creative solution to identify social influence. Their algorithms enabled them to analyze these data, providing useful insights about the influence dynamics. For example, they suggest that a researcher can enhance their influence degree by cooperating with highly-influential researchers, even if they are themselves low-influence individuals; and high-influence individuals will lose influence over time if they are unable to collaborate with the core community. Crisp, clear graphical displays such as that in **Figure 9** helped the presentation of this paper.

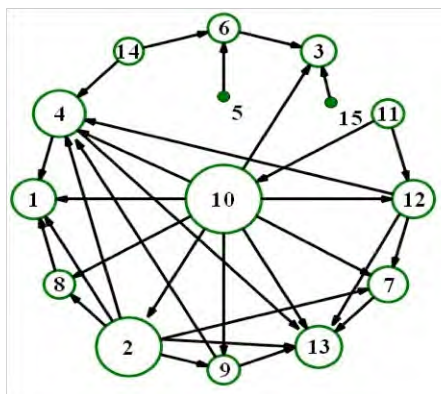


Figure 9. Influence links in the co-citation network, from the paper by the Outstanding team from Xidian University, China.

Conclusion

Among the 1,028 papers, there were many strong and innovative submissions that made judging both exciting and challenging. It was very gratifying to see so many students with the ability to combine modeling, science and effective communication skills in order to understand such large, complex datasets and build viable network models for their analysis.

Recommendations for Future Participants

- **Answer the problem.** Weak papers sometimes do not address a significant part of the problem. Outstanding teams often cover all the bases and then go beyond for some aspects of the problem.
- **Manage your time.** Every year there are submissions that do an outstanding job on one aspect of the problem, then "run out of gas" and are

unable to complete their solution. Outstanding teams have a plan and adjust as needed to submit a complete solution.

- **Coordinate your plan.** It is obvious in weaker papers that the work and writing was spilt between group members, then pieced together into the final report. For example, the output from one model or one step in a process doesn't match the input for the next model or a section appears in the paper that does not fit with the rest of the report. The more your team can coordinate the efforts of its members and integrate the writing, the stronger your final submission will be.
- **Do more than just model.** The model itself is not the solution. Some weak papers present a strong model, then stop. Outstanding teams use their models to produces results, understand the problem and recommend or produce a solution.
- **Explain what you are doing and why.** Weaker submissions tend to use too many equations and too few words. Problem approaches appear out of nowhere. Outstanding teams explain what they are doing and why.

References

Erdős, P., and A. Rényi. 1959. On random graphs, I. *Publicationes Mathematicae* 6: 290–297.

Grossman, Jerry. 2014. The Erdős Number Project. `http://www.oakland.edu/enp/`. Accessed 1 May 2014.

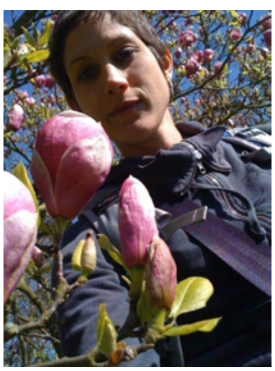
"This page was last updated on April 29, 2014 (but subpages may have been updated more recently). However, the lists of coauthors and the various other statistics on this site are updated about once every five years. The current version was posted on October 20, 2010 and includes all information listed inMathSciNet through mid-2010. The next update will probably occur around 2015."

Hoffman, Paul. 1998. *The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth*. New York: Hyperion.

O'Connor, J.J., and E.F. Robertson. 2000. Paul Erdős. `http://www-history.mcs.st-and.ac.uk/Biographies/Erdos.html`.

About the Authors

Chris Arney graduated from the U.S. Military Academy and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to Research Triangle Park, NC, where he served in various positions in the Army Research Office. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM). In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.



Kate Coronges received a Master's in Public Health and a Ph.D. from the University of Southern California in Human Health Behavior. She was an Assistant Professor in the Dept. of Behavioral Sciences and Leadership and a Research Fellow in the Network Science Center at the U.S. Military Academy for four years. Currently, she works as a Program Manager at the Army Research Office. Her research interests focus on the role of social and organizational network structures, and the dynamics of these networks, in communication patterns and performance of teams, groups, and societies. She is active in shaping and building momentum towards specific domains of social science basic research important in military settings, ranging from small-team dynamics (such as shared decision-making and collective intelligence), to belief and behavior propagation in groups and communities, to public and global policy (to include energy, education, information security and health care systems), and methodological challenges involved with modeling multidimensional and multifunctional systems.

Tina Hartley is an Academy Professor at the U.S. Military Academy and an active-duty officer. Her Ph.D. is in computational mathematics from George Mason University. Tina began her military career as an Air Defense Artillery Officer, and also served as an Operations Research Analyst. She is currently the Director of the Core Mathematics Program at the U.S. Military Academy. She has been an ICM judge for the past six years.

Jessica Libertini started her career as an engineer, earning a B.S. and an M.S. in mechanical engineering from Johns Hopkins University and Rensselaer Polytechnic Institute, respectively. She spent nine years at General Dynamics working on projects ranging from the design of submarines to the development of a multinational layered missile defense system. After earning her Ph.D. in applied mathematics from Brown University in 2008, Jessica left industry and began her academic career at the U.S. Military Academy, where she held the positions of Assistant Professor and National Research Council Fellow. While there, Jessica used her engineering background to motivate students to address large, open-ended, and meaningful questions, both in the classroom and as the coach of the competitive mathematics team. She has since served on the faculty at the University of Rhode Island and is currently an Assistant Professor of Applied Mathematics at Virginia Military Institute. Given her background in engineering, her research spans a wide variety of mathematical approaches to modeling, analyzing, and simulating medical, military, and physical applications; she also participates in the scholarship of teaching and learning, focusing on undergraduate pedagogy and the elements of a successful transition from high school to college.

Developing an Interdisciplinary Mindset with Students Through the ICM

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Overview

This is an account of Simpson College's participation in the Interdisciplinary Contest in Modeling (ICM) from 2010 through 2014. During this time period, there were 4,405 teams that participated in the ICM, 156 which came from the United States. Simpson College fielded 50 of those teams.

While Simpson College did not field any Outstanding teams over this time period, it did field 33% of the Finalist teams from the U.S. and 32% of the Meritorious teams from the U.S. In 2014, Simpson College accounted for 48% of the teams from the U.S. participating in the ICM.

The focus of this article will address what the Simpson College Mathematics Department has done to promote participation in the ICM and what outcomes our students have had.

Part I: Building the Culture

At Simpson College, we pride ourselves on helping students learn oral and written communication skills, collaborative skills, problem-solving skills, critical thinking skills, time management skills, and research skills. As a faculty, we recognize that the ICM is a valuable experience for students in that it addresses the development of all of these skills. Through

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our promotional efforts, this competition has become the hallmark of our program.

We have been deliberate in promoting participation in this competition, starting with the recruiting of students into the program. A significant number of high school seniors who decide to join our program do so because of our success in the ICM. During 2010–14, an average of 16–18 first- and second-year students participated in the ICM. All of the students who participate in the ICM are aware of student outcomes in terms of securing internships, full-time employment, Research Experiences for Undergraduates (REUs), and graduate school acceptance as a direct result of their success with the ICM. This has led to a very high level of persistence with participation in the competition.

To increase their chances of success, the students have learned to form interdisciplinary teams for the ICM. By bringing in students with different academic backgrounds, teams possess a wide variety of problem-solving strategies. Thus, it's not only mathematics majors who are involved in this competition—in 2010-2014, a range of 15–20 different majors participated in the ICM each year.

The fact that so many disciplines are represented in the ICM annually led to the institution providing academic and financial support. Starting in 2013, students were able to receive general education credits for Collaborative Leadership and Information Literacy if they participated in the ICM. Furthermore, Simpson College's Student Government Association recognized the impact that this competition was having across campus and agreed to pay every team's registration fee for the ICM. Finally, in three of the past five years, the College's Public Relations Dept. has produced videos and articles related to the competition that highlight the students' experiences and achievements.

We have now gotten to the point where a critical mass of students consistently participate in the ICM. This has led to a steady increase in the number of teams participating in the ICM. We expect these numbers to persist and increase over the coming years.

Part II: During the Competition

To support our students in this competition, we have been deliberate in our attempts to assure the students have a positive experience during the competition.

A few weeks prior to the ICM, the Mathematics Department builds a wiki for the competition. All logistical information is available to students at this site, including building hours, staff schedules, meal times, and links to resources on formatting their papers. Students can also provide input on what they would like during the competition, such as food, snacks, and beverages.

On Thursday evening before the competition begins, we have a one-hour workshop about logistics and strategies for surviving the ICM. At this meeting, we introduce students to our computer archive of past ICM solutions, which they can use as a resource for formatting their papers. Experienced students provide tips for success—including time management skills, collaborative skills, and writing skills. This meeting leads into a "kick-off party." At this party, light snacks are provided and the problems become available to the students on the COMAP website.

Figure 1. Students at Simpson's kick-off party.

Starting on Friday afternoon of the ICM, every team is assigned its own classroom and provided laptops with appropriate modeling software, for the duration of the competition. Faculty and staff across campus volunteer their time during the competition to provide students access to buildings and to bring snacks for the students. Additionally, we have an IT person on call to assist with technological malfunctions (e.g., printer problems, etc.).

To build community spirit, we stock a break room with appropriate snacks and beverages. Additionally, we organize formal dinners on Friday, Saturday, and Sunday night as a show of support for the students' efforts in the competition.

Part III: Student Outcomes

Over the past decade, we have strived to build an interdisciplinary culture at Simpson College. COMAP's Interdisciplinary Contest in Modeling has played a major role in successfully building this environment, culminating in the results cited from 2010 through 2014.

During this time, our students have played an active role in motivating us to continue supporting their efforts in the ICM. Indeed, students have persisted in participating in the competition in large numbers and maximized their efforts by consistently striving to perform at the highest level.

Two students who exemplify the commitment for participating in the ICM are 2013 graduates Laura Collins and Stephen Henrich. Laura triple-majored with Honors in Mathematics, Physics, and Chemistry and has just finished her first year in the Ph.D. program in Physics at Notre Dame. Stephen triple-majored in Mathematics, Biochemistry, and Philosophy. He has just finished his first year in an M.D./Ph.D. program at Northwestern University. Both students graciously agreed to provide a brief account of their experiences with the ICM and its impact on their career paths.

Laura Collins

I participated in the ICM competition at Simpson College all 4 years that I was there, with a different team each time. My first year, my team received an Honorable Mention, and the last three years my teams received Meritorious rankings for our papers. While I didn't really know what to expect from the competition my first year, every following year I was very excited to have the opportunity to work together with my classmates towards solving a challenging math problem. I even found myself wishing I could participate again this past year when the time came for the competition to start again.

Whenever I try to explain the ICM to my new classmates in graduate school, I mention that it's a 96-hour challenging but very fun math competition, and by the end we completed a 20-page research paper based on mathematical models. Almost immediately my classmates, who have never participated in a competition like the ICM, conclude that I am/was crazy. While they might have a point, I definitely think that it was a worthwhile experience and would participate in the ICM again in a heartbeat.

Participating in the ICM helped me grow not only as a mathematician, but as an overall problem solver. The competition encouraged us to work together and think of new ways to solve real-world problems. Over my four years, I learned a lot about working together as a team, how I work with others under pressure with a deadline, and how it is important to recognize others' strengths and utilize them. The ICM is one of the many experiences I had at Simpson College that helped me realize that I enjoy working on

Figure 2. Laura Collins.

challenging problems collaboratively with others. My experiences in the ICM competition convinced me that research would be an excellent career path for me, and so here I am in graduate school at the University of Notre Dame working towards a Physics Ph.D.

Stephen Henrich

My first time participating in the ICM mathematical modeling competition during my freshman year at Simpson was one of those rare pivotal experiences that will probably stick with me forever. I wasn't quite sure what to expect initially, but I was glad to discover that the competition would be much more than a mere test. We weren't asked to provide definitive answers to close-ended questions—and the judges didn't have an answer sheet. Instead, we were given the task of investigating, in depth, a real-world problem.

In 2010 our problem was the rapidly expanding multi-ton pile of plastic known as the Pacific Ocean Garbage Patch. Over the course of 96 hours, our team of freshman rookies learned a great deal from one another, consumed pounds of nutritionless food, got very little sleep, scrawled on and erased three whiteboards 15 times over, thought up numerous original ideas, and eventually scrapped most of them. Our final product was a new mathematical description of a relationship between atmospheric pressure and the density of plastic in the Patch, which earned a Finalist rank in the competition. I walked away from the weekend feeling that I had experienced for the first time what life might be like as a professional problem solver—as a scientist, mathematician, politician, businessman, or other trained analytic specialist.

I participated in the competition three more times while at Simpson,

Figure 3. Stephen Henrich.

always looking forward to that time of the year when the modeling competition would come around, and Carver Hall would come alive with the excitement of our 20+ teams of amateur problem solvers. My teams were able to tackle an electric vehicle integration proposal, solve a corporate conspiracy, and assess the state of the global energy crisis– earning Honorable Mention, Meritorious, and another Finalist ranking in the process.

Looking back, it is clear that the competitions played a significant role in my decision to undertake a career path in academia as a medical researcher. These short but intense experiences provided a great deal of insight into the stimulating and ever-changing world of research. And each competition confirmed my suspicion that nothing would suit me better than to become a part of it.

Conclusion

We believe that our participation level and student success in the ICM is a result of our intentional efforts in building an interdisciplinary mindset in our students. These efforts start with the recruitment of the students to the program and continue with our mentorship and structure around the competition. The most exciting thing for us as faculty is to see the students follow our lead and run with it. To better understand the interdisciplinary mindset that our students have developed, please watch the video produced by our public relations office [Simpson College 2012].

It is the students who choose the interdisciplinary problem on a regular basis and form teams that are interdisciplinary in nature. This mindset they have developed has allowed them to persist and excel in this competition.

Reference

Simpson College. 2012. Math @ Simpson College. Video, 3:20 min. `https://www.youtube.com/watch?v=EaIYEzzbTIM`.

About the Authors

Rick Spellerberg was raised on an Iowa farm and graduated from Coe College in 1984 with degrees in Mathematics and Physics. He went directly on to the University of Iowa where he completed his Ph.D. in Mathematics in 1990. Since then he has been at Simpson College teaching mathematics. A major focus of his outreach activities in recent years has been promoting COMAP's High School Contest in Modeling (HiMCM®). For the past four summers, he and co-author Heidi Berger have run a workshop called "Molding Mathematical Minds" that prepares a teacher and a team of their students for the HiMCM. This workshop was first funded through a SUMMA-Tensor grant from the Mathematical Association of America and most recently by the National Security Agency.

Heidi Berger earned her degrees in mathematics and physics at Coe College in 2002, a master's in mathematics at University of Nebraska–Lincoln (UNL) in 2004, and a Ph.D. from UNL in 2008. She has just completed her sixth year at Simpson College. Aside from working with the MCM/ICM and the HiMCM workshop at Simpson College, Heidi is passionate about conducting undergraduate research in biomathematics. She has collaborated with Dr. Clint Meyer and about a dozen undergraduates on problems of population ecology, time scales calculus, and restoration processes of ecosystems. She has received numerous minigrants to support this work and is a co-director for CURM, the Center for Undergraduate Research in Mathematics.