The UMAP Journal

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COMAP

Publisher's Editorial

The Face of Things to Come

Solomon A. Garfunkel
Executive Director
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Typically, I use this space to write once a year about the new activities at COMAP. And this has been an amazing year. We have sent three new undergraduate books to be published, all of which followed from work we had done on major NSF projects. Brooks/Cole has published *Mathematics Methods and Modeling for Today's Mathematics Classroom: A Contemporary Approach to Teaching Grades 7-12*, by John Dossey, Frank Giordano, and others (ISBN 0–534–36604–X). This book, designed for use in preservice programs for high school teachers, is a direct result of a Division of Undergraduate Education grant from NSF. The idea behind this grant was to help prepare future high school teachers, both in content and in pedagogy, for the changes in curricula, technology, and assessment that have followed the implementation of the NCTM Standards.

In addition, W.H. Freeman has published two new COMAP texts: *Precalculus: Modeling Our World* (ISBN 0–7167–4359–0) and *College Algebra: Modeling Our World* (ISBN 0–7167–4457–0). Based on our secondary series, *Mathematics: Modeling Our World (M:MOW)*, these texts represent activity-based, modeling-driven approaches to entry-level collegiate mathematics. We hope that they will set a new standard for these courses.

And speaking of new standards, we are also in the process of completing the sixth edition of *For All Practical Purposes*. In this new edition, we greatly expand our coverage of election/voting theory, not surprisingly taking advantage of the data and interest surrounding the 2000 presidential race. We are also adding a section on the human genome, reinforcing the fact that new and important applications of mathematics are being discovered every day.

But perhaps this year's most important accomplishments are the ideas we have generated and the proposals that we have written. I have often joked that I would like to found two new journals: the *Journal of Funded Proposals* and the

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Journal of Unfunded Proposals, if for no better reason than at least I would have a great many more publications. As I write this editorial, I do not know into which category our three new NSF proposals will fall, but I would like to share their contents with you. It is my fondest hope that these will represent the face of COMAP to come.

The first proposal is to revise *M:MOW*. Our four-year comprehensive reform secondary school series was first published in 1998. In the years since publication, we have learned a great deal from early adopters about ways to help them customize the texts to meet local needs, including new standardized tests. It is time to produce a second edition, which we hope will have widespread appeal.

The second proposal is to produce a new liberal arts calculus text with accompanying video and web support. COMAP has not undertaken a major video project in some time and we feel that a series of videos visually demonstrating the importance and applicability of the calculus is a natural extension of our previous efforts. Moreover, we will (if funded) prepare shorter video segments for ease of use on the Web.

The last proposal extends the idea of making materials available on the Web one step further with an ambitious program to produce a series of Web-based courses for present and future teachers, K-12. Again, we plan to make extensive use of new video as well as the interactivity of the Web. Here we hope to use master teachers, with expertise in both content and methods, and use the power of the Internet to reach classrooms and teachers all across the country.

I do not know whether at this time next year we will be working on all of these projects. But I do know that we will continue our efforts to create new materials and support all of you, who make reform of mathematics education possible.

About the Author

Sol Garfunkel received his Ph.D. in mathematical logic from the University of Wisconsin in 1967. He was at Cornell University and at the University of Connecticut at Storrs for eleven years and has dedicated the last 20 years to research and development efforts in mathematics education. He has been the Executive Director of COMAP since its inception in 1980.

He has directed a wide variety of projects, including UMAP (Undergraduate Mathematics and Its Applications Project), which led to the founding of this *Journal*, and HiMAP (High School Mathematics and Its Applications Project), both funded by the NSF. For Annenberg/CPB, he directed three telecourse projects: *For All Practical Purposes* (in which he also appeared as the on-camera host), *Against All Odds: Inside Statistics*, and *In Simplest Terms: College Algebra*. He is currently co-director of the Applications Reform in Secondary Education (ARISE) project, a comprehensive curriculum development project for secondary school mathematics.

Modeling Forum

Results of the 2001 Mathematical Contest in Modeling

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Introduction

A total of 496 teams of undergraduates, from 238 institutions in 11 countries, spent the second weekend in February working on applied mathematics problems in the 17th Mathematical Contest in Modeling (MCM) and in the 3rd Interdisciplinary Contest in Modeling (ICM). This issue of *The UMAP Journal* reports on the MCM contest; results and Outstanding papers from the ICM contest will appear in the next issue, Vol. 22, No. 4.

The 2001 MCM began at 12:01 A.M. on Friday, Feb. 9 and officially ended at 11:59 P.M. on Monday, Feb. 12. During that time, teams of up to three undergraduates were to research and submit an optimal solution for one of two open-ended modeling problems. The 2001 MCM marked the inaugural year for the new online contest, and it was a great success. Students were able to register, obtain contest materials, download the problems at the appropriate time, and enter data through COMAP'S MCM website.

Each team had to choose one of the two contest problems. After a weekend of hard work, solution papers were sent to COMAP on Monday. Nine of the top papers appear in this issue of *The UMAP Journal*.

Results and winning papers from the first sixteen contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2000). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains all of the 20 problems used in the first ten years of the contest and a winning paper for each. Limited quantities of that

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volume and of the special MCM issues of the *Journal* for the last few years are available from COMAP.

This year's Problem A was about bicycle wheels and what edge they may give to a race. Before any contest, professional cyclists make educated guesses about which one of two basic types of wheels to choose for any given competition. The team's Sports Director has asked them to come up with a better system to help determine which kind of wheel—wire spoke or solid disk—should be used for any given race course.

Problem B addressed the evacuation of Charleston, South Carolina during 1999's Hurricane Floyd. Maps, population data, and other specific details were given to the teams. They were tasked with constructing a model to investigate potential strategies. In addition, they were asked to submit a news article that would be used to explain their plan to the public.

Problem A: The Bicycle Wheel Problem

Introduction

Cyclists have different types of wheels they can use on their bicycles. The two basic types of wheels are those constructed using wire spokes and those constructed of a solid disk (see **Figure 1**). The spoked wheels are lighter but the solid wheels are more aerodynamic. A solid wheel is never used on the front for a road race but can be used on the rear of the bike.

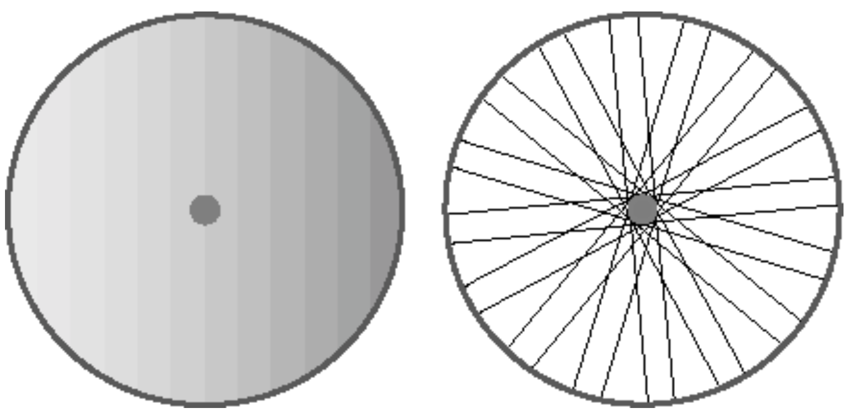

Figure 1. Solid wheel (left) and spoked wheel (right).

Professional cyclists look at a racecourse and make an educated guess as to what kind of wheels should be used. The decision is based on the number and steepness of the hills, the weather, wind speed, the competition, and other considerations.

The *directeur sportif* of your favorite team would like to have a better system in place and has asked your team for information to help determine what kind of wheel should be used for a given course.

The *directeur sportif* needs specific information to help make a decision and has asked your team to accomplish the tasks listed below. For each of the tasks, assume that the same spoked wheel will always be used on the front but that there is a choice of wheels for the rear.

Task 1

Provide a table giving the wind speed at which the power required for a solid rear wheel is less than for a spoked rear wheel. The table should include the wind speeds for different road grades starting from 0% to 10% in 1% increments. (Road grade is defined to be the ratio of the total rise of a hill divided by the length of the road.[1]) A rider starts at the bottom of the hill at a speed of 45 kph and the deceleration of the rider is proportional to the road grade. A rider will lose about 8 kph for a 5% grade over 100 m.

Task 2

Provide an example of how the table could be used for a specific time trial course.

Task 3

Determine if the table is an adequate means for deciding on the wheel configuration and offer other suggestions as to how to make this decision.

Problem B: The Hurricane Evacuation Problem

Evacuating the coast of South Carolina ahead of the predicted landfall of Hurricane Floyd in 1999 led to a monumental traffic jam. Traffic slowed to a standstill on Interstate I-26, which is the principal route going inland from Charleston to the relatively safe haven of Columbia in the center of the state. What is normally an easy two-hour drive took up to 18 hours to complete. Many cars simply ran out of gas along the way. Fortunately, Floyd turned north and spared the state this time, but the public outcry is forcing state officials to find ways to avoid a repeat of this traffic nightmare.

The principal proposal put forth to deal with this problem is the reversal of traffic on I-26, so that both sides, including the coastal-bound lanes, have traffic headed inland from Charleston to Columbia. Plans to carry this out have been prepared (and posted on the Web) by the South Carolina Emergency

[1] If the hill is viewed as a triangle, the grade is the sine of the angle at the bottom of the hill.

Preparedness Division. Traffic reversal on principal roads leading inland from Myrtle Beach and Hilton Head is also planned.

A simplified map of South Carolina is shown in **Figure 2**. Charleston has approximately 500,000 people, Myrtle Beach has about 200,000 people, and another 250,000 people are spread out along the rest of the coastal strip. (More accurate data, if sought, are widely available.)

Figure 2. Highways in South Carolina.

The interstates have two lanes of traffic in each direction except in the metropolitan areas, where they have three. Columbia, another metro area of around 500,000 people, does not have sufficient hotel space to accommodate the evacuees (including some coming from farther north by other routes); so some traffic continues outbound on I-26 towards Spartanburg, on I-77 north to Charlotte, and on I-20 east to Atlanta. In 1999, traffic leaving Columbia going northwest was moving only very slowly.

Construct a model for the problem to investigate what strategies may reduce the congestion observed in 1999. Here are the questions that need to be addressed:

1. Under what conditions does the plan for turning the two coastal-bound lanes of I-26 into two lanes of Columbia-bound traffic, essentially turning the entire I-26 into one-way traffic, significantly improve evacuation traffic flow?

2. In 1999, the simultaneous evacuation of the state's entire coastal region was ordered. Would the evacuation traffic flow improve under an alternative strategy that staggers the evacuation, perhaps county by county over some time period consistent with the pattern of how hurricanes affect the coast?

3. Several smaller highways besides I-26 extend inland from the coast. Under what conditions would it improve evacuation flow to turn around traffic on these?

4. What effect would it have on evacuation flow to establish additional temporary shelters in Columbia, to reduce the traffic leaving Columbia?

5. In 1999, many families leaving the coast brought along their boats, campers, and motor homes. Many drove all of their cars. Under what conditions should there be restrictions on vehicle types or numbers of vehicles brought in order to guarantee timely evacuation?

6. It has been suggested that in 1999 some of the coastal residents of Georgia and Florida, who were fleeing the earlier predicted landfalls of Hurricane Floyd to the south, came up I-95 and compounded the traffic problems. How big an impact can they have on the evacuation traffic flow?

Clearly identify what measures of performance are used to compare strategies.

Required: Prepare a short newspaper article, not to exceed two pages, explaining the results and conclusions of your study to the public.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at Southern Connecticut State University (Problem A) or at the National Security Agency (Problem B). At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree on a score, a third judge evaluated the paper.

Final judging took place at Harvey Mudd College, Claremont, California. The judges classified the papers as follows:

The nine papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries. We list those teams and the Meritorious teams (and advisors) below; the list of all participating schools, advisors, and results is in the **Appendix**.

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Bicycle Wheel	3	27	58	127	215
Hurricane Evacuation	6	43	65	167	281
	9	70	123	294	496

Outstanding Teams

Institution and Advisor	**Team Members**

Bicycle Wheel Papers

"Spokes or Discs?"	
Stellenbosch University	W.D.V. De Wet
Matieland, South Africa	D.F. Maian
Jan H. van Vuuren	C. Mumbeck
"Selection of a Bicycle Wheel Type"	
United States Military Academy	Nicholas J. Howard
West Point, NY	Zachariah R. Miller
Donovan D. Phillips	Matthew R. Adams
"A Systematic Technique for Optimal Bicycle Wheel Selection"	
University College Cork	Michael Flynn
Cork, Ireland	Eamonn Long
James Jo Grannell	William Whelan-Curtin

Hurricane Evacuation Papers

"Project H.E.R.O.: Hurricane Evacuation Route Optimization"	
Bethel College	Nathan M. Gossett
St. Paul, MN	Barbara A. Hess
William M. Kinney	Michael S. Page
"Traffic Flow Models and the Evacuation Problem"	
Duke University	Samuel W. Malone
Durham, NC	Carl A. Miller
David P. Kraines	Daniel B. Neill

"The Crowd Before the Storm"
The Governor's School — Jonathan D. Charlesworth
Richmond, VA — Finale P. Doshi
John A. Barnes — Joseph E. Gonzalez

"Jammin' with Floyd: A Traffic Flow Analysis of South Carolina Hurricane Evacuation"
Harvey Mudd College — Christopher Hanusa
Claremont, CA — Ari Nie
Ran Libeskind-Hadas — Matthew Schnaider

"Blowin' in the Wind"
Lawrence Technological University — Mark Wagner
Southfield, MI — Kenneth Kopp
Ruth G. Favro — William E. Kolasa

"Please Move Quickly and Quietly to the Nearest Freeway"
Wake Forest University — Corey R. Houmand
Winston-Salem, NC — Andrew D. Pruett
Miaohua Jiang — Adam S. Dickey

Meritorious Teams

Bicycle Wheel Papers (27 teams)
Beijing University of Chemical Technology, Beijing, P.R. China (Jiang Guangfeng)
Beijing University of Chemical Technology, Beijing, P.R. China (Wenyan Yuan)
Brandon University, Brandon, Canada (Doug A. Pickering)
California Polytechnic State University, San Luis Obispo, CA (Thomas O'Neil)
Harbin Engineering University, Harbin, P.R. China (Gao Zhenbin)
Harbin Institute of Technology, Harbin, P.R. China (Shang Shouting)
Harvey Mudd College, Claremont, CA (Michael E. Moody)
James Madison University, Harrisonburg, VA (James S. Sochacki)
Jilin University of Technology, Changchun, P.R. China (Fang Peichen)
John Carroll University, University Heights, OH (Angela, S. Spalsbury)
Lafayette College, Easton, PA (Thomas Hill)
Lake Superior State University, Sault Sainte Marie, MI (J. Jaroma and D. Baumann)
Lewis and Clark College, Portland, OR (Robert W. Owens)
Southeast University, Nanjing, P.R. China (Chen En-shui)
Tianjin University, Tianjin, P.R. China (Dong Wenjun)
Trinity University, San Antonio, TX (Fred M. Loxsom)
United States Air Force Academy, USAF Academy, CO (Jim West)
University College Dublin, Dublin, Ireland (Peter Duffy)
University of Western Ontario, London, Canada (Peter H. Poole)
Washington University, St. Louis, MO (Hiro Mukai)

Westminster College, New Wilmington, PA (Barbara T. Faires) (two teams)
Wright State University, Dayton, OH (Thomas P. Svobodny)
Youngstown State University, Youngstown, OH (Thomas Smotzer)
Zhejiang University, Hangzhou, P.R. China (He Yong)
Zhejiang University, Hangzhou, P.R. China (Yang Qifan)
Zhongshan University, Guangzhou, P.R. China (Chen Zepeng)

Hurricane Evacuation Papers (43 teams)
Beijing University of Posts & Telecommunications, Beijing, P.R. China (He Zuguo)
California Polytechnic State University, San Luis Obispo, CA (Thomas O'Neil)
Central South University, Changsha, P.R. China (Zheng Zhou-shun)
Clarion University, Clarion, PA (Jon A. Beal)
Dong Hua University, Shanghai, China (Ding Yongsheng)
East China University of Science & Technology, ShangHai, P.R. China (Liu Zhaohui)
Gettysburg College, Gettysburg, PA (Sharon L. Stephenson)
Hillsdale College, Hillsdale, MI (Robert J. Hesse)
James Madison University, Harrisonburg, VA (Caroline Smith)
Jiading No. 1 High School, Jiading, P.R. China (Wang Yu)
MIT, Cambridge, MA (Dan Rothman)
N.C. School of Science and Mathematics, Durham, NC (Dot Doyle)
National University of Defence Technology, Changsha, P.R. China (Wu Mengda)
National University of Singapore, Singapore, Singapore
(Lim Leong Chye Andrew)
North Carolina State University, Raleigh, NC (Jeffrey S. Scroggs)
Northeastern University, Shenyang, P.R. China (Xiao Wendong)
Pacific Lutheran University, Tacoma, WA (Zhu Mei)
Päivölä College, Tarttila, Finland (Merikki Lappi)
Rose-Hulman Institute of Technology, Terre Haute, IN (David J. Rader)
Rowan University, Glassboro, NJ (Paul J. Laumakis)
Shanghai Foreign Language School, Shanghai, P.R. China (Pan Li Qun)
South China University of Technology, Guangzhou, P.R. China (Lin Jianliang)
Southern Oregon University, Ashland, OR (Lisa M. Ciasullo)
U.S. Military Academy, West Point, NY (David Sanders)
U.S. Military Academy, West Point, NY (Edward Connors)
University of Alaska Fairbanks, Fairbanks, AK (Chris Hartman)
University of Colorado–Boulder, Boulder, CO (Bengt Fornberg)
University of Massachusetts Lowell, Lowell, MA (James Graham-Eagle)
University of North Texas, Denton, TX (John A. Quintanilla)
University of Richmond, Richmond, VA (Kathy W. Hoke)
University of Science and Technology of China, Hefei, P.R. China (Gu Jiajun)
University of South Carolina Aiken, Aiken, SC (Laurene V. Fausett)
University of South Carolina, Columbia, SC (Ralph E. Howard)
University of Southern Queensland, Toowoomba, Queensland, Australia (Tony J. Roberts)
University of Washington, Seattle, WA (James Allen Morrow)
Wake Forest University, Winston-Salem, NC (Miaohua Jiang)

Washington University, St. Louis, MO (Hiro Mukai)
Western Washington University, Bellingham, WA (Saim Ural)
Worcester Polytechnic Institute, Worcester, MA (Bogdan Vernescu)
Wuhan University, Wuhan, P.R. China (Huang Chongchao)
York University, Toronto, Ontario, Canada (Juris Steprans)
Zhejiang University, Hangzhou, P.R. China (He Yong)
Zhejiang University, Hangzhou, P.R. China (Yang Qifan)

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, gave a cash prize and a three-year membership to each member of the teams from Stellenbosch University (Bicycle Wheel Problem) and Lawrence Technological University (Hurricane Evacuation Problem). Also, INFORMS gave free one-year memberships to all members of Meritorious and Honorable Mention teams. The Lawrence Tech team presented its results at the annual INFORMS meeting in Washington DC in April.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The teams were from U.S. Military Academy (Bicycle Wheel Problem) and Wake Forest University (Hurricane Evacuation Problem). Each of the team members was awarded a $300 cash prize and the teams received partial expenses to present their results at a special Minisymposium of the SIAM Annual Meeting in San Diego CA in July. Their schools were given a framed, hand-lettered certificate in gold leaf.

The Mathematical Association of America (MAA) designated one Outstanding team from each problem as an MAA Winner. The teams were from University College Cork (Bicycle Wheel Problem) and Wake Forest University (Hurricane Evacuation Problem). With partial travel support from the MAA, both teams presented their solutions at a special session of the MAA Mathfest in Madison WI in August. Each team member was presented a certificate by MAA President Ann Watkins.

Judging

Director
Frank R. Giordano, COMAP, Lexington, MA

Associate Directors
Robert L. Borrelli, Mathematics Dept., Harvey Mudd College,
Claremont, CA

Patrick Driscoll, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY
William Fox, Mathematics Dept., Francis Marion University, Florence, SC
Michael Moody, Mathematics Dept., Harvey Mudd College,
Claremont, CA

Bicycle Wheel Problem

Head Judge
Marvin S. Keener, Executive Vice-President, Oklahoma State University,
Stillwater, OK

Associate Judges
Ronald Barnes, University of Houston Downtown, Houston TX (MAA)
Kelly Black, Mathematics Dept., University of New Hampshire,
Durham, NH (SIAM)
David Elliott, Institute for System Research, University of Maryland,
College Park, MD (SIAM)
Ben Fusaro, Mathematics Dept., Florida State University,
Tallahassee, FL
Mario Juncosa, RAND Corporation, Santa Monica, CA
John Kobza, Texas Tech University, Lubbock, TX (INFORMS)
Dan Solow, Mathematics Dept., Case Western Reserve University,
Cleveland, OH (INFORMS)

Hurricane Evacuation Problem

Head Judge
Maynard Thompson, Mathematics Dept., University of Indiana,
Bloomington, IN

Associate Judges
Paul Boisen, National Security Agency, Ft. Meade, MD (Triage)
James Case, Baltimore, Maryland
Courtney Coleman, Mathematics Dept., Harvey Mudd College,
Claremont, CA
Lisette De Pillis, Harvey Mudd College, Claremont, CA
William Fox, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY
Jerry Griggs, University of South Carolina, Columbia, SC
Jeff Hartzler, Mathematics Dept., Pennsylvania State University Middletown,
Middletown, PA (MAA)
Deborah Levinson, Compaq Computer Corp., Colorado Springs, CO
Veena Mendiratta, Lucent Technologies, Naperville, IL
Don Miller, Dept. of Mathematics, St. Mary's College, Notre Dame, IN (SIAM)
Mark R. Parker, Mathematics Dept., Carroll College, Helena, MT (SIAM)

John L. Scharf, Carroll College, Helena, MT
Lee Seitelman, Glastonbury, CT (SIAM)
Kathleen M. Shannon, Salisbury State University, Salisbury, MD
Michael Tortorella, Lucent Technologies, Holmdel, NJ
Marie Vanisko, Carroll College, Helena, MT
Cynthia J. Wyels, Dept. of Mathematics, Physics, and Computer Science, California Lutheran University, Thousand Oaks, CA

Triage Sessions:

Bicycle Wheel Problem

Head Triage Judge
Theresa M. Sandifer, Southern Connecticut State University, New Haven, CT

Associate Judges
Therese L. Bennett, Southern Connecticut State University, New Haven, CT
Ross B. Gingrich, Southern Connecticut State University, New Haven, CT
Cynthia B. Gubitose, Western Connecticut State University, Danbury, CT
Ron Kutz, Western Connecticut State University, Danbury, CT
C. Edward Sandifer, Western Connecticut State University, Danbury, CT

Hurricane Evacuation Problem

Head Triage Judge
Paul Boisen, National Security Agency, Ft. Meade, MD

Associate Judges
James Case, Baltimore, Maryland
Peter Anspach, Jennifer Mcgreevy, Erin Schram, Larry Wargo, and 7 others from the National Security Agency

Sources of the Problems

The Bicycle Wheel Problem was contributed by Kelly Black, Mathematics Dept., University of New Hampshire, Durham, NH. The Hurricane Evacuation Problem was contributed by Jerry Griggs, Mathematics Dept., University of South Carolina, Columbia, SC.

Acknowledgments

The MCM was funded this year by the National Security Agency, whose support we deeply appreciate. We thank Dr. Gene Berg of NSA for his coordinating efforts. The MCM is also indebted to INFORMS, SIAM, and the MAA, which provided judges and prizes.

I thank the MCM judges and MCM Board members for their valuable and unflagging efforts. Harvey Mudd College, its Mathematics Dept. staff, and Prof. Borrelli were gracious hosts to the judges.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. Please peruse these student efforts in that context.

To the potential MCM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Appendix: Successful Participants

KEY:

P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

A = Bicycle Wheel Control Problem
B = Hurricane Evacuation Problem

INSTITUTION	CITY	ADVISOR	A	B
ALABAMA				
Huntingdon College	Montgomery	Robert L. Robertson		P,P
ALASKA				
University of Alaska Fairbanks	Fairbanks	Chris Hartman		M
ARIZONA				
McClintock High School	Tempe	James S. Gibson	P	
CALIFORNIA				
California Lutheran University	Thousand Oaks	Sandy Lofstock		H,P
California Poly. State University	San Luis Obispo	Matthew J. Moelter	P	
		Thomas O'Neil	M	M
California State University	Bakersfield	Maureen E. Rush		P
Christian Heritage College	El Cajon	Tibor F. Szarvas	P	
Harvey Mudd College	Claremont	Ran Libeskind-Hadas		O,H
		Michael E. Moody	M	H
Occidental College	Los Angeles	Ramin Naimi		P
University of California	Berkeley	Brian W. Curtin		P,P
COLORADO				
Colorado College	Colorado Springs	Peter L. Staab	H	H
Mesa State College	Grand Junction	Edward K. Bonan-Hamada		H,P
Regis University	Denver	Linda L. Duchrow		P
United States Air Force Academy	USAF Academy	James S. Rolf	P	
		Jim West	M	
University of Colorado	Colorado Springs	Gregory J. Morrow	P	
	Boulder	Bengt Fornberg	H	M
University of Southern Colorado	Pueblo	James N. Louisell	H	
CONNECTICUT				
Sacred Heart University	Fairfield	Peter Loth		P
Southern Conn. State University	New Haven	Therese L. Bennett	H	
DISTRICT OF COLUMBIA				
Georgetown University	Washington	Andrew J. Vogt	P	P

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For team advisors from China and Singapore, we have endeavored to list family name first, with the help of Susanna Chang '03.

Spokes or Discs?

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Introduction

It is well known that disc wheels and standard spoked wheels exhibit different performance characteristics on the race track, but as yet no reliable means exist to determine which is superior for a given set of conditions.

We create a model that, taking the properties of wheel, cyclist, and course into account, may provide a definitive answer to the question, "Which wheel should I use today?" The model provides detailed output on wheel performance and can produce a chart indicating which wheel will provide optimal performance for a given environmental and physical conditions.

We use laws of physics, plus data from various Web sites and published sources, then numerical methods to obtain solutions from the model.

We demonstrate the use of the model's output on a sample course. Roughly speaking, *standard spoked wheels perform better on steep climbs and trailing winds, while disc wheels are better in most other cases*.

We did some validation of the model for stability, sensitivity, and realism. We also generalised it to allow for a third type of wheel, to provide a more realistic representation of the choice facing the professional cyclist today.

A major difficulty was obtaining reliable data; sources differed or even contradicted one another. The range of the data that we could find was insufficient, jeopardizing the accuracy of our results.

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Analysis of the Problem

Consider the system of a cyclist and the racing cycle. The cyclist provides the energy to drive the bicycle against the forces of drag (from contact with air), friction (from contact between wheels and ground), and gravity (which opposes progress up a slope). Furthermore, when accelerating, the cyclist must provide the energy to set the wheels rotating, due to their moment of inertia.

The primary problem is to determine, for a given set of conditions, which type of rear wheel is the most effective. "Effectiveness" is what a specific rider desires from the equipment. We assume that the rider desires to complete the course in as short a time as possible, or with the least possible energy expenditure. These definitions are closely linked: A rider who expends more energy to maintain a certain speed will soon tire and will therefore have a lower maintainable speed.

The differences between standard 32-spoke wheels and disc wheels lie in weight and in their aerodynamic properties. Given the right wind conditions (which we investigate), the disc wheels should allow air to pass the cyclist/cycle combination with less turbulence, that is, less drag. However, disc wheels weigh more, which affects the amount of power required to move the wheels up a slope and to begin rotating the wheels from rest (such as when accelerating).

To examine which type of wheel performs the best under which conditions, we need to determine which wheel allows the greatest speed given specific conditions or, equivalently, which wheel requires the least power to drive.

The greatest difficulty is that air resistance is a function of speed while speed is a function of air resistance. The model needs to utilise numerical methods to calculate the speed that a rider can maintain with the given parameters.

There are other factors to consider, too. Disc wheels are not very stable in gusty wind conditions, since they provide a far greater surface area to cross-winds; with a greater moment of inertia, they accelerate more slowly; and their greater weight may provide more grip on wet roads.

Assumptions and Hypothesis

We investigated the performance of the wheel types noted in **Table 1**. The wheel data do not conform to any specific make or model but are typical.

Table 1.

Types of wheels.

Type	Standard 32-spoke	Aero wheel (trispoke)	Solid disc wheel
Diameter (m)	0.7	0.7	0.7
Mass (kg)	0.8	1.0	1.3

We assume that the cyclist uses a standard spoke wheel in the front and either a disc wheel or standard 32-spoked wheel at the back. We also briefly examine aero wheels, which are not solid but more aerodynamic than standard spoked wheels. We refer to the three types as standard, aero, and disc.

We assume that the rider and cycle frame (excluding wheels) exhibit the same drag for all wind directions.

Table 2.

Symbols used.

Symbol	Unit	Definition
A	m^2	area of rider/bicycle exposed to wind
c_a	dimensionless	variable coefficient of axial air resistance for specific wheel
c_{rr}	dimensionless	constant of rolling resistance
c_w	dimensionless	constant of air resistance
D	m^2	reference area of wheel
F_{ad}	Newton (N)	axial air resistance (against the cyclist's direction of motion)
F^*_{ad}	Newton (N)	axial air resistance on a bicycle with box-rimmed spoked wheels in a headwind
F_g	Newton (N)	effect of gravity on the cyclist
F_{rr}	Newton (N)	rolling resistance
g	m/s^2	gravitational constant
M	kg	mass of cyclist and cycle
P	Watt (W)	rider's effective power output
v_{bg}	m/s	speed of the bike relative to the ground
v_{wb}	m/s	speed of the wind relative to the bike
v_{wg}	m/s	speed of the wind relative to the ground
α	degree	angle of the rise
β	degree	yaw angle, the angle between the direction opposing bicycle motion and perceived wind (A relative headwind has a yaw of 0°.)
γ	degree	angle between wind direction (v_{wg}) and direction of motion (A straight-on headwind has $\gamma = 180°$.)
ψ	percent	grade of hill, the sine of α, the angle of the rise
ρ	kg/m^3	air density

Forces at Work

For a bicycle moving at a constant velocity, there are three significant retarding forces (**Figure 1**):

rolling resistance, due to contact between the tires and road;

gravitational resistance, if the road is sloped; and

air resistance, usually the largest of the three.

When accelerating, the rider also uses energy to overcome translational and rotational inertia, although the model does not take these into account.

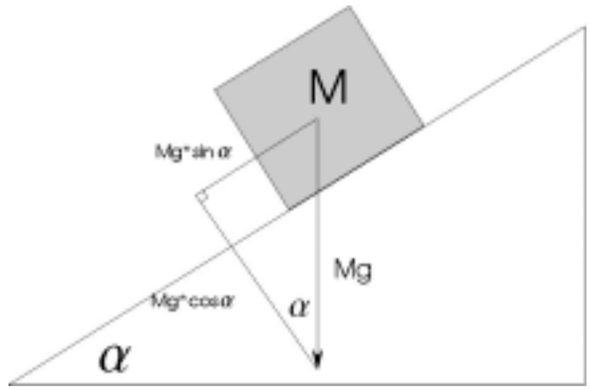

Figure 1. Diagram of forces.

The forces of rolling resistance and gravity are as follows:

$$\begin{aligned} F_{rr} &= c_{rr} \cdot (\text{normal force}) \\ &= c_{rr} M g \cos\alpha \\ &= c_{rr} M g \cos \arcsin \psi \\ &= c_{rr} M g \sqrt{1-\psi^2}, \\ F_g &= M g \sin\alpha = M g \psi. \end{aligned}$$

Calculating the axial drag force is more complicated. The air resistance is $f = \frac{1}{2}\rho \cdot c_w A v^2$, where v is the speed of the air relative to the object. Since we assume that the area of the rider/frame exposed to the wind is constant, we have (neglecting the additional drag on the wheels caused by yaw and type of wheel)

$$F^*_{ad} = \tfrac{1}{2}\rho \cdot A v_{wb}^2 \cos\beta \qquad \text{(axial component)}.$$

Observe the sketches in **Figure 2**. From them we derive that

$$\begin{aligned} v_{wb}^2 &= v_{wg(\text{axial})}^2 + v_{wg(\text{side})}^2 \\ &= \big(v_{bg} + v_{wg}\cos(180^\circ - \gamma)\big)^2 + \big(v_{wg}\sin(180^\circ - \gamma)\big)^2 \\ &= (v_{bg} - v_{wg}\cos\gamma)^2 + (v_{wg}\sin\gamma)^2 \\ &= {v_{bg}}^2 - 2v_{bg}v_{wg}\cos\gamma + v_{wg}^2. \end{aligned}$$

Also,

$$\beta = \arctan\left(\frac{v_{wg(\text{side})}}{v_{wg(\text{axial})}}\right) = \arctan\left(\frac{v_{wg}\sin\gamma}{v_{bg} - v_w\cos\gamma}\right).$$

The axial air drag on the rear wheel [Tew and Sayers 1999] is

$$F_{ad(\text{wheel})} = 0.75 \cdot \tfrac{1}{2}\rho \cdot c_a D \cdot v_{wb}^2;$$

the 0.75 is because a rear wheel experiences 75% of the drag of a wheel in free air, due to interference of the gear cluster, frame, cyclist's legs, and so forth.

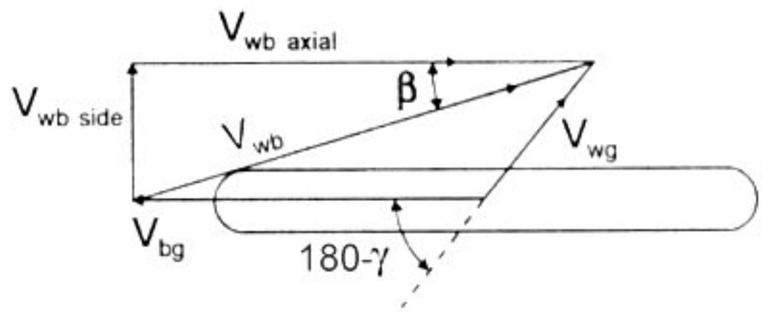

Figure 2a. Wind speed relative to wheel.

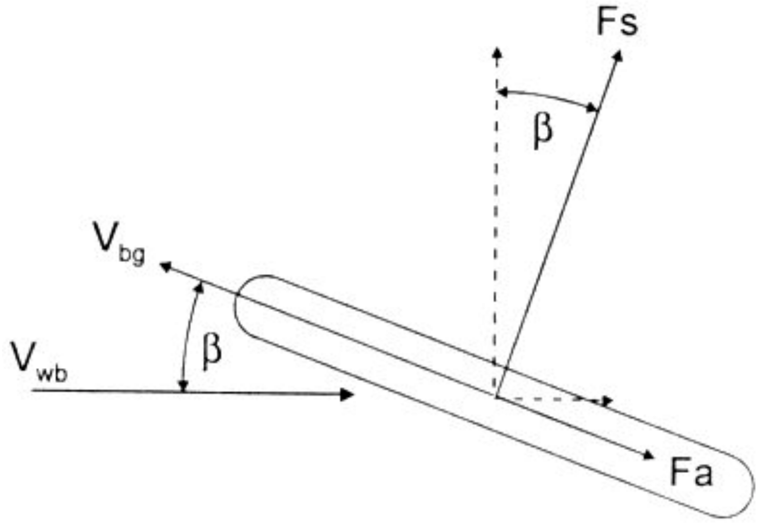

Figure 2b. Forces on wheel.

For the three basic types of wheel, Tew and Sayers [1999] give typical curves of the axial drag coefficient c_a vs. yaw angle ($0° \leq \beta \leq 30°$); this interval accounts for the majority of conditions experienced by a rider. We approximate these curves by straight lines (a close match).

The curves for different relative wind speeds are very much alike for the standard wheel and the aero wheel. The disc wheel, however, shows major variation for different relative wind speeds.

Since the axial drag coefficient must be zero at $\beta = 90°$, and by observing the shape of the curves, we extrapolated to larger yaw angles using a sine-shaped curve through $c_a = 0$ at $\beta = 90°$, with an appropriate scaling to ensure continuity. Without wind-tunnel testing, the accuracy cannot be guaranteed.

Comparing the percentage of power dissipated by drag on one wheel (according to the model) with the data of Tew and Sayers [1999], we found a high degree of agreement. Typically, 1% to 10% (depending on wheel type) of the power is dissipated by drag on the wheels.

From $F_{ad(\text{wheel})}$ we subtracted the drag experienced by a normal (box-rimmed) wheel under headwind, since it was already taken account in F^*_{ad}.

The axial air drag on the bicycle is thus given by

$$F_{ad} = F_{ad}^{*} + F_{ad(\text{wheel})} = \tfrac{1}{2}\rho v_{wb}^{2}[c_w A\cos\beta + 0.75(c_a - 0.06)D],$$

where 0.06 is the coefficient of axial drag for a normal wheel in a headwind. The $\cos\beta$ gives the component in the direction of motion of the cyclist.

Calculating Results for a Typical Rider

For data, we used standard values [Analytic Cycling 2001] for a road racer near sea level in normal atmospheric conditions:

$$\begin{aligned}
M &= 80\text{ kg}, & g &= 9.81\text{ m/s}^2,\\
c_{rr} &= 0.004, & c_w &= 0.5,\\
A &= 0.5\text{ m}^2, & D &= 0.38\text{ m}^2 \quad \text{(for a 700 mm wheel)},\\
\rho &= 1.226\text{ kg/m}^3 & & \text{(could be changed to incorporate altitude).}
\end{aligned}$$

We calculated that, to maintain a speed of 45 km/h on a level road (as in the problem description), the rider must deliver 340 W of effective pedaling power.

The Computer Program

We wrote a computer program in Pascal that calculates the speed that the rider can sustain for a given v_{wg}, γ, and ψ. It does this by trying a speed and determining the wattage necessary to sustain the speed. If the wattage is too high, the speed is lowered; otherwise, the speed is increased. Every time the solution point is crossed, the step size is reduced. The process is carried out until the wattage used is within a tolerance 0.01 W to P.

To take into account the effect of drag on different types of wheels, our program does the following:

1. The wind direction, wind speed relative to the ground, and slope of the road (γ, v_{wg}, and ψ) are provided as inputs.
2. The program tries a value for v_{bg}.
3. F_{rr} and F_g are calculated.
4. From γ, v_{wg}, ψ, and v_{bg}, we calculate v_{wb} and β.
5. From v_{wb} and β, we calculate F_{ad}.
6. We calculate the wattage by using the formula $P = (F_{rr} + F_g + F_{ad})v_{bg}$.
7. We compare the calculated value of P to the known value of 340 W.

8. We try a new value for v_{bg}, depending on whether the wattage required for the previous value of v_{bg} was higher or lower than the available 340 W.

9. We repeat this process from Step 3 until the maximum maintainable speed is determined.

Since the wheel that requires the least power in a set of circumstances also enables the highest speed, we used our program to vary the speed of the wind and show which wheel is best for the circumstances. **Figure 3** shows a screen shot. The dark colour represents blue and the light colour red. Each of the 11 horizontal strips represents a road gradient, ranging from 0 at the top to 0.1 at the bottom in 0.01 increments.

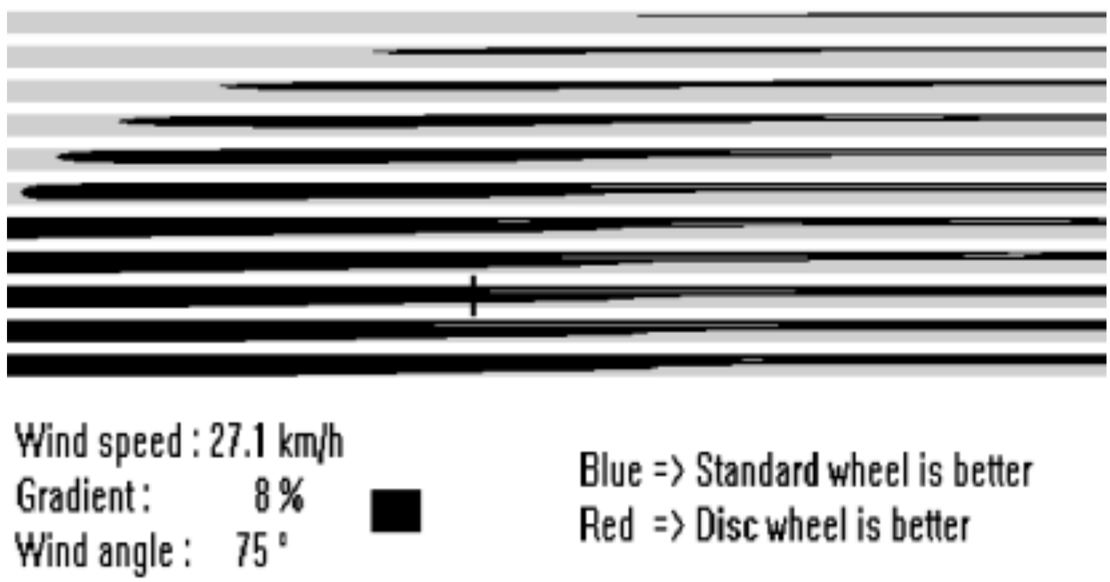

Figure 3. Screen shot from program.

The horizontal axis is wind speed, from 0 km/h at the left to 63.9 km/h at the right in 0.1 km/h increments. The vertical axis of each bar is the wind angle (relative to track), from 0° to 180° in 15° increments.

We have a very compact representation showing the transition wind speeds for a range of wind angles and road gradients. The user is provided a crosshair to move over any point on the graph. The colour of a pixel indicates the better wheel to use for the corresponding gradient, wind speed, and wind angle.

To generate a table of transition speeds (**Table 3**), we read off the points at which transitions occur. This might seem cumbersome, but developing an algorithm to find the transition points is very difficult, since the number of transitions is not known beforehand and the functions exhibit irregular behaviour.

To use the table, look up the particular entry corresponding to the road grade and the angle that the wind makes with the forward direction of the bike. An entry of S means that a standard wheel performs better for all wind speeds, a D indicates that a disc wheel is better at all wind speeds.

Table 3.

Which wheel to use, as a function of road grade and wind angle.

Road grade	Angle of wind (in degrees) relative to bike's direction 0	15	30	45
<0	D	D	D	D
0	D	D46.5S	D36.6S	D
0.01	D	D27.8S	D22.6S	D21.3S47.3D
0.02	D44.5S	D16.5S	D13.3S	D12.3S53.8D
0.03	D24.2S	D9.4S47.6D54.5S	D7.3S56.6D	D6.5S
0.04	D12.3S	D4.8S	D3.5S42.0D	D3.0S
0.05	D4.2S	S	S33.9D	S
0.06	S	S63.4D	S54.8D61.8S	S38.6D46.2S
0.07	S	S	S55.6D59.1S	S32.3D46.5S
0.08	S	S	S	S28.0D45.8S
0.09	S	S	S	S24.9D44.8S
0.10	S	S	S	S

Road grade	Angle of wind (in degrees) relative to bike's direction 60	75	90	105
<0	D	D	D	D
0	D	D	D	D
0.01	D22.1S33.6D	D	D	D
0.02	D12.7S39.1D	D14.5S32.0D	D	D
0.03	D6.4S43.2D	D6.9S35.1D	D8.5S30.6D	D14.2S23.8D
0.04	D2.9S47.9D	D2.8S37.2D	D3.2S33.4D	D3.8S29.6D
0.05	S	S40.0D	S34.7D	S32.4D
0.06	S	S42.8D	S35.7D	S34.2D
0.07	S	S46.6D	S37.5D	S35.5D
0.08	S	S	S39.2D	S36.4D
0.09	S	S	S40.6D	S37.2D
0.10	S	S	S42.1D	S37.8D

Road grade	Angle of wind (in degrees) relative to bike's direction 120	135	150	165	180
<0	D	D	D	D	D
0	D	D	D	D	D
0.01	D	D	D	D	D
0.02	D	D	D	D	D
0.03	D	D	D	D	D
0.04	D6.1S22.1D	D	D	D	D
0.05	S28.0D	S17.4D	D	D	D
0.06	S31.1D	S23.7D	S15.4D	S8.3D	S1.7D
0.07	S33.3D	S27.9D	S19.8D	S13.7D	S6.5D
0.08	S34.9D	S30.6D	S23.0D	S17.5D	S10.5D
0.09	S36.3D	S32.5D	S25.5D	S20.2D	S13.8D
0.10	S37.4D	S34.1D	S27.6D	S22.6D	S16.8D

The other entries can be decoded as follows: A number between two letter entries indicates at which wind speed a transition occurs; the first letter indicates which wheel is most efficient at lower speeds, and the second number which wheel is best at higher speeds. For example, S28.0D indicates that standard wheels are better at speeds below 28.0 km/h. An entry of D6.1S22.1D indicates that the standard wheel performs better at speeds between 6.1 and 22.1 km/h, while the disc wheel performs better at all other wind speeds.

The table applies for wind speeds up to 64 km/h. Strong winds are very rare and a disc wheel will cause major stability problems in these conditions.

As an aside, we created graphs comparing standard, aero, and disc wheels simultaneously and allowed for negative gradients as well. The aero wheel dominated in most conditions.

Applying the Table to a Sample Course

We designed a simple time-trial course. The map of the course and a view of the elevation are given in **Figure 4**. The course consists of four different segments, with each turning point labelled with a letter. The data for each point are in **Table 4**.

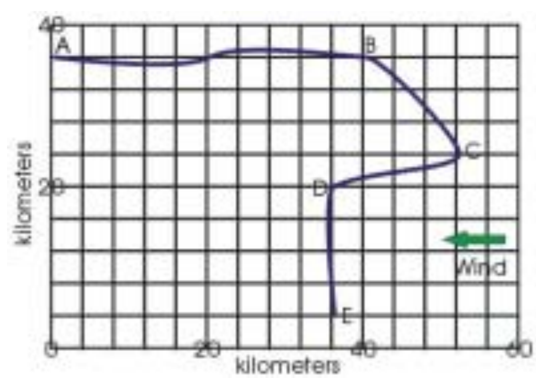

Figure 4a.

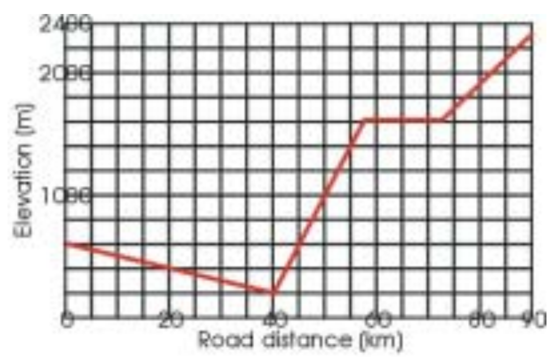

Figure 4b.

Table 4.
Details of the sample course.

Point	Map coordinates (km, km)	Elevation (m)
A (Start)	(0,36)	600
B	(40,36)	200
C	(52,24)	1600
D	(36,20)	1580
E (Finish)	(36,4)	2350

Assume that the wind is blowing at 25 km/h in the direction shown on the map. For each segment, we compute the gradient and the angle of the segment with the wind by trigonometry. The length of each segment is slightly longer than the straight-line distance, because the road is not perfectly straight.

We look at **Table 3** to determine the best wheel for each section of the course. For instance, in the second section, the gradient is 0.08 and the angle 135°; according to the table, the standard wheel is better at a wind speed below 30.6 km/h, so at 25 km/h, a standard wheel is better for this section. We fill in the other entries in a similar manner (**Table 5**).

Table 5.

Best wheel for each section of the sample course.

Section	Distance (km)	Wind angle (°)	Gradient	Best wheel
AB	40.8	180	−0.01	Disc
BC	17.5	135	0.08	Standard
CD	16.7	14	0.00	Disc
DE	16.2	90	0.05	Standard

The disc wheel and the standard wheel both win in two segments. However, the disc wheel wins over 58 km of the course, while the standard wheel wins over only 33 km. Thus, the table advises that the cyclist use the disc wheel.

Getting More Refined Results

For each section, we calculated the expected speed for the rider with each wheel. We add the times for the individual sections to obtain an estimate of the total time, obtaining **Table 6**. The table shows two interesting results:

- The disc wheel beats the standard wheel by about 50 s. This is consistent with the result obtained earlier in this section.
- The aero wheel is almost 2 min faster than the disc wheel!

Table 6.

Total time for sample course for each wheel.

		Standard wheel		Aero wheel		Disc wheel	
Section	Length (km)	Speed (km/h)	Time taken (h:min:sec)	Speed (km/h)	Time taken (h:min:sec)	Speed (km/h)	Time taken (h:min:sec)
AB	40.8	32.73	1:14:47.6	33.27	1:13:34.8	33.46	1:13:09.7
BC	17.5	12.56	1:23:35.9	12.68	1:22:48.5	12.47	1:24:12.1
CD	16.7	59.52	0:16:50.1	60.13	0:16:39.8	59.97	0:16:42.5
DE	16.2	19.72	0:49:17.4	19.94	0:48:44.8	19.58	0:49:38.6
Total	91.2		3:44:31		3:41:48		3:43:43

Validating the Model

Sensitivity Analysis

Does the table generated for one rider with a specific set of physical attributes apply to another rider, and if not, can the model easily be adjusted?

To determine whether the same table could be used for different riders, we varied one of the rider's parameters, either power output, mass, or cross-sectional surface area, while keeping the others constant. In these analyses we found that

- Changing any one or any combination of the parameters P, A, or M does not affect the basic pattern but slightly distorts (shifts, scales, or skews) it.
- Every rider-cycle combination would need its own chart for determining which wheel to use at which speed.

Other Validation

We compared our model's output to data available at Analytic Cycling [2001], which provides interactive forms. Our model's output matched their output almost exactly for all the different combinations of input parameters that we used. Unfortunately, this site does not make provision for wind speed or angle, so this part of our model could not be compared.

We tested the model with a completely different set of parameters approximating a very powerful sports car ($P = 300$ kW, $C_d = 0.3$, $A = 2.3$ m^2, $M = 1100$ kg). We kept the other parameters the same. Our model predicted a top speed of 320 km/h on a level road, which seemed very realistic.

Error Analysis

We were concerned about the disc wheel "islands" that showed up in our graphical output at wind speeds of 40–50 km/h, wind angles of 30°–60°, and higher gradients (see **Figure 3**). They probably are due to the peculiar behaviour of disc wheels in crosswinds. Since we extrapolated the drag coefficient function, we have no way of knowing whether this strange behaviour is realistic or not.

Model Strengths

If a rider can obtain reasonably accurate course data (something that is not difficult at all), then the rider can determine exactly what type of rear wheel to use for a race by referencing this information to a chart or computer.

The model has many parameters (air density, coefficients, rider mass, etc.) that can be adjusted to account for various situations.

It is easy to extend the model to include the front wheel of the bicycle.

Model Weaknesses

When the angle of the wind with the cyclist changes, there are a number of factors that influence the amount of drag that the cyclist experiences. The most obvious is the changing surface area; a cyclist from the side presents a far larger surface area to the wind than a cyclist heading into the wind.

Less obvious, but still a large contributing factor, is that the drag coefficient is a function of the shape of the object. A cyclist from the side is far less streamlined and has a higher drag coefficient.

Both these factors are extremely difficult to model. The cross-sectional surface area of a complex three-dimensional shape could be the subject of a paper on its own, and determining the drag coefficient of the same complex shape would require empirical tests.

As a result, we ignored these effects and assume that the drag coefficient and cross-sectional surface area of the rider are the same for all directions. Strong cross-winds would cause too much rider instability to even consider using disc wheels, so it would not really be necessary to investigate drag in such cases.

We also ignored the effects of the energy needed to overcome the rotational and translational inertia for each wheel. We cannot comment on the effect of these forces, since we did not have time to implement these effects.

The model is only as accurate as the data used, and much of the available data on wheel drag is, at best, dubious. Many wheel manufacturers exaggerate the performance of their brands, while rival manufacturers quickly denounce their findings. Further, few data were available for yaw angles greater than 30°, and we were forced to extrapolate, leading to a high degree of uncertainty.

Using the table requires specific information about rider characteristics, some of which may be difficult to obtain, such as the surface area of rider and bicycle.

Conclusion

Our model provides a good means for making an informed decision as to which wheel to use in a particular situation, but it needs more accurate data and refinement.

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Selection of a Bicycle Wheel Type

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Introduction

We present a model that compares the performance of various wheels over a user-determined course. We approach the modeling problem by beginning with Newton's Second Law of Motion: The sum of the forces acting on an object equals the mass of that object multiplied by its acceleration.

We identify the four principal forces that contribute to a cyclist's motion: applied force, drag force, gravity, and rolling resistance. We further classify drag force into three components: the cyclist and bicycle frame, the front wheel, and the rear wheel.

Drag force is dependent on a cyclist's velocity, and the force of gravity is dependent on a cyclist's position. Thus, our force equation is a function of cyclist position and cyclist velocity.

We can then arrange Newton's Second Law to yield a second-order differential equation. Given position S, velocity dS/dt, acceleration d^2S/dt^2, mass m, and a force function F, the differential equation is

$$\frac{d^2S}{dt^2} = \frac{F\left(S, \frac{dS}{dt}\right)}{m}.$$

To implement our model, we created a computer software program that allows a user to input numerous pieces of data, including course layout, elevation profile, wind, weather conditions, and cyclist characteristics. The software iterates the differential equation using the fourth-order Runge-Kutta method. The software reports the preferred wheel choice based on the data.

As a real-world application of our model, we analyze the 2000 Olympic Cycling time-trial race. Over that course, a disk wheel provided a considerable advantage over a spoked wheel.

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Problem Analysis

We must determine whether a spoked wheel (lighter but less aerodynamic) or a disk wheel (more aerodynamic but heavier) is more power-efficient. The choice depends on many factors, including the number and steepness of hills, the weather, wind speed and direction, and type of competition.

We are not striving to discover which wheel is best for all situations. To the contrary, we are interested in which wheel outperforms other wheels given specific conditions. Given accurate data concerning factors such as hills, weather, wind, and competition type, a good model will be able to determine and recommend which wheel is preferred.

Our Approach

We determine equations for each force that acts on the cyclist and the bicycle. These forces are not constant throughout a race. For example, air drag increases with the square of velocity.

We apply Newton's Second Law of Motion to the cyclist-bicycle system: The sum of the forces acting on an object equals mass of the object times its acceleration. The sum of the forces is a function of both position on the course and velocity of the cyclist. Because the forces are different at different positions, the power required with a given type of wheel is also different. Our approach is to apply the same power to both types of wheels and determine how long it takes to traverse the course. The wheel that allows the cyclist to complete the course in the shortest time requires less power, that is, is more power-efficient.

A more comprehensive model would incorporate wind based upon a probabilistic function. However, to do so would be at odds with our goal: We wish to determine performance times for two types of wheels given a constant set of conditions. If we included probabilistic functions, differing performance times would be due to both wheel differences and random fluctuations instead of just to wheel differences.

Let the position of the bicycle on the course be S with components S_x, S_y, and S_z. The second-order differential equation for acceleration is

$$\frac{d^2S}{dt^2} = \frac{F\left(S, \frac{dS}{dt}\right)}{m},$$

where d^2S/dt^2 is the acceleration, $F(S, dS/dt)$ is the total force, and m is the total mass of the system.

We determine the time that it takes to complete a course by solving this equation numerically.

Assumptions

- Weather conditions (temperature, humidity, wind direction, and wind speed) are uniform over the course and constant throughout the race. Because wind varies over time, and terrain significantly changes wind speed and direction, larger courses with greater variability in elevation are most affected by this assumption. However, due to the unpredictability of the weather, a general wind direction and speed is probably the most detailed information to which the rider will have access.
- Both wheel types use the same tire.
- Turning does not significantly affect power efficiency of the wheel, speed of the rider, or acceleration of the rider.
- Based on the previous assumption, we assume that the bicycle moves in a linear path in 3-D space.
- The cyclist applies power according to the function that we develop in the **Model Design** section below, where we introduce other assumptions associated with developing this function.
- The drag coefficient for the rider plus bike frame is the same for all riders and does not change as a function of yaw angle. The drag coefficient is 0.5 and the cross sectional area is 0.5 m^2 [Analytic Cycling 2001a].
- The wheels do not slip in any direction as they roll over the course.
- Other riders have no effect on the aerodynamic characteristics of the bike-rider system. This means that we ignore the effects of drafting (which can reduce drag by up to 25%).
- The rider uses a conventional 36-spoke wheel on the front of the bike.
- The rotational moments of inertia for disc wheels and spoke wheels are approximately 0.1000 kg-m^2 and 0.0528 kg-m^2, respectively. In reality, these must be determined experimentally.

Model Design

We identify the forces that act on a bicycle and rider:

- The forward force that the rider applies with pedaling.
- The drag force that opposes the motion of the bicycle. Since we are concerned with analyzing wheel performance, we divide the total drag force into three components:

  - The drag force F_f on the front wheel.
  - The drag force F_r on the rear wheel.
  - The drag force F_B on the bicycle frame and rider.

- The force of gravity F_g that either opposes or aids motion due to the road grade.

- The force of rolling resistance F_{rr} due to the compression and deformation of air in the tires.

Because we assume that the bicycle travels in a linear path, we need consider only components of these forces that act co-axially (i.e., parallel to the bicycle's direction of movement). This will remain realistic so long as the assumption that the wheels do not slip remains true, because the static frictional force between the wheels and the ground prevents movement normal to the velocity.

Consequently, we do not need to treat the forces as vectors; we must note only whether they aid or oppose the bike's movement.

The Force that the Rider Applies

The rider applies a force to the pedals, which the gears translate to the wheels of the bicycle. Competitive bicycle racers generally shift gears to maintain a constant force on the pedals as well as a constant pedaling rate (cadence) whenever the sum of the other forces oppose the motion (i.e., going up a hill or into the wind) [Harris Cyclery 2001]. In other words, the cyclist attempts to exert constant power.

However, as the rider moves downhill, gravity aids the effort. As the bicycle gains speed, the rider's pedaling results in a diminished effect on speed because drag forces increase with the square of speed. Eventually, at a speed v_{cutoff}, the rider ceases pedaling.

We model the power P_a that the rider inputs as a function of speed v_g:

$$P_a = \begin{cases} 0, & \text{if } v_g \geq v_{\text{cutoff}}; \\ P_{\text{avg}}, & \text{if } v_g < v_{\text{cutoff}}, \end{cases}$$

where P_{avg} is the average power that the rider can sustain; its value varies from rider to rider and with the type of race. Typical values range from 200 W for casual riders to between 420–460 W for Olympic athletes on long-distance road races, to as much as 1500 W in sprint races [Seiler 2001].

If we assume that there is no energy lost in the translation of the power between the pedal and the wheel (i.e., in the gears), then by conservation of energy the power goes into either rotating the wheels or moving the bicycle forward:

$$P_a = P_w + P_f, \tag{1}$$

where P_w is the power to rotate the wheels and P_f is the power to drive the bicycle forward.

From elementary physics, the rotational kinetic energy of an object is $\frac{1}{2}I\omega^2$, where I is the rotational moment of inertia and ω is the angular velocity of the object. For the front and rear wheels, we have

$$K_f = \tfrac{1}{2}I_f\omega^2, \qquad K_r = \tfrac{1}{2}I_r\omega^2,$$

where I_f and I_r are the rotational moments of inertia of the front and rear wheels. The total rotational energy K_T of the wheels is then

$$K_T = K_f + K_r.$$

The power due to the rotation of the object is the time derivative of the rotational energy:

$$P_w = \frac{dK_T}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}I_f\omega^2 + \tfrac{1}{2}I_r\omega^2\right) = (I_f + I_r)\omega\omega'.$$

The angular velocity ω of an object equals its ground speed v_g divided by its radius R, while its angular acceleration ω' is a/R.

Substituting these into the above equation yields

$$P_w = (I_f + I_r)\frac{v_g a}{R^2}.$$

If we solve for the power P_a that pushes the bike forward, then divide both sides of **(1)** by v_g, we obtain the applied force F_A that pushes the bicycle forward:

$$P_f = P_a - P_w, \qquad F_A = \frac{P_a}{v_g} - (I_f + I_r)\frac{a}{R^2}.$$

The Drag Forces on the Bicycle

The drag force acting on an object moving through a fluid is

$$F = \tfrac{1}{2}C\rho AV^2,$$

where ρ is the density of the fluid (air), A is the cross-sectional area of the object, V is its velocity relative to the fluid, and C is the coefficient of drag that must be determined experimentally.

Air density, which can have a significant effect on drag forces, depends on temperature, pressure, and humidity. Pressure depends on altitude and weather. Most baseball fans can attest to the significant affects of air density on drag forces: Baseballs carry much farther in Coors Field in Denver, Colorado, because the high altitude leads to a low pressure, which means that the air density is less as well.

We consider all of these factors that affect air density in our model, via calculations in the **Appendix** [EDITOR'S NOTE: We omit the appendix.]. The most important aspect is that the air density is a function of the bike's elevation, S_z. The bike's acceleration d^2S/dt^2 depends on the drag forces, which depend on air density, which depends on the bike's position. This means that the differential equation that we develop will be second-order, because the second derivative of position depends on the position.

The Movement of the Air and Bicycle

The air through which the bicycle moves is not stagnant—wind blowing over the course has a significant effect. We represent the wind as a vector field $\vec{V}_A$ with a magnitude and direction that are uniform over the racecourse and constant for the duration of the race.

The cyclist's speed over the ground is v_g; thus, the velocity is $v_g\vec{u}$, where $\vec{u}$ is a unit vector in the bicycle's direction of movement. We now consider the air's velocity relative to the bicycle instead of the bicycle's velocity relative to the air, because this is an easier way of thinking about the problem and the magnitudes of these two velocities are equal. Since the bike's velocity over the ground is $v_g\vec{u}$, the air's velocity relative to the bike due to the bike's motion is $-v_g\vec{u}$.

The total velocity of the air moving relative to the bike is a function of two motions: the air's movement relative to the ground in the form of wind, $\vec{V}_A$, and the air's movement relative to the bike due to the bike's movement over the ground, $-v_g\hat{u}$, where we use the hat over u to denote that u is a unit vector. The total speed is then the magnitude of the sum of these two velocities (**Figure 1**),

$$v_T = \|\vec{V}_A - v_g\hat{u}\|.$$

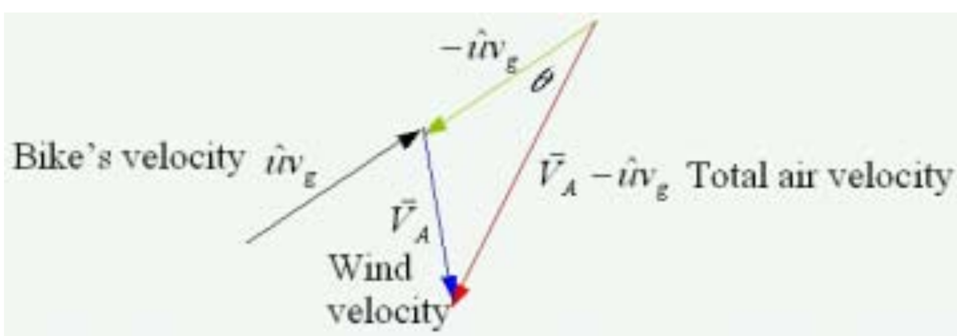

Figure 1. Components of total air velocity.

The yaw angle θ is the angle between the bicycle's axis of movement and the air direction, which we find from the dot product, $\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$, where

θ is the angle between the vectors. We have

$$-v_g\hat{u}\cdot(\vec{V}_A - v_g\hat{u}) = v_g\|\vec{V}_A - v_g\hat{u}\|\cos\theta,$$

$$\theta = \arccos\left[\frac{-v_g\hat{u}\cdot(\vec{V}_A - v_g\hat{u}}{v_g\|\vec{V}_A - v_g\hat{u}\|}\right].$$

The bicycle's aerodynamic characteristics, and thus the drag forces, change with θ. Furthermore, because the bicycle does not always head into the wind, the overall drag force has components both normal and axial to the rider's path. We assume that the normal component is negligible and consider only the axial component (the component parallel to the cycle's axis of travel).

The Wheels

The axial drag force on the wheels largely depends on the yaw angle of the air moving past them and must be calculated experimentally for different types of wheels. For experimental results, we rely on Greenwell et al. [1995], who used a wind tunnel to determine the axial drag coefficient at different yaw angles for various commercially available wheels.

For each type of wheel, we plotted axial drag coefficient vs. yaw angle. From the plots, we constructed a polynomial regression of axial drag coefficients as a function of the yaw angle. Greenwell et al. considered the reference cross-sectional area of each wheel S_{ref} to be the total side cross-sectional area of the wheel,

$$S_{\text{ref}} = \pi R^2.$$

Thus, the effects of the cross-sectional area changing with yaw angle are included in the axial drag coefficient. Consequently, to use their results, we must use the same reference area. The axial drag force on the front wheel is then

$$F_f = K_W C_F v_T^2,$$

where $K_W = \frac{1}{2}\rho S_{\text{ref}}$ and C_F is the axial drag coefficient of the front wheel at yaw angle θ.

An interesting result of Greenwell et al. is that drag forces on the rear wheel are generally reduced by about 25% due to aerodynamic effects of the seat tube. This means that the axial drag force on the rear wheel is

$$F_R = (0.75)K_W C_R v_T^2,$$

where C_F is the axial drag coefficient of the rear wheel at yaw angle θ.

The Bicycle Frame and Rider

Unfortunately, we were not able to obtain data relating the drag coefficient of the rider and frame to the yaw angle, so we could consider only the effects of the component of the wind that is parallel to the bicycle's velocity.

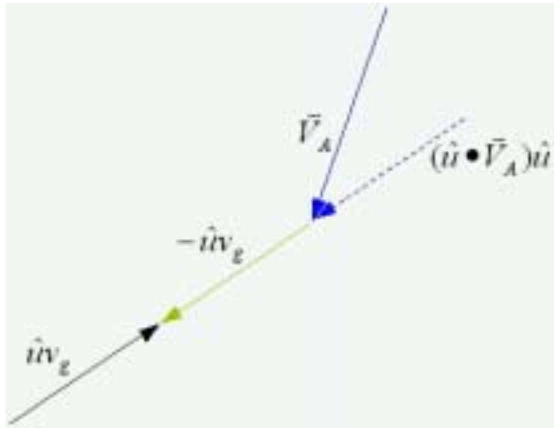

Figure 2. Drag force on the frame and rider.

In other words, for the drag force on the frame and rider, we consider only the vector projection of the wind onto the rider's velocity (**Figure 2**). The total air velocity is then the sum of this projection and the negative of the rider's velocity. We find the total drag force on the frame and rider to be

$$F_B = K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2,$$

where $K_B = \frac{1}{2} C_B \rho A$, A is the cross-sectional area of bicycle frame and rider, and C_B is the drag coefficient of bicycle frame and cyclist.

Force of Gravity

If the bicycle is on a hill, the component of gravitational force that is in the direction of the motion is

$$F_g = m_T g \sin\phi,$$

where m_T is the total mass of the bicycle and rider, g is the acceleration of gravity, and ϕ is the angle at the bottom of the hill. But since the road grade is $G = \sin\phi$, we have$F_g = m_T g G$.

Force of Rolling Resistance

Because the wheels have inflatable tires, the compression of the air within the tires causes a resistance to their rolling. This rolling resistance is a reaction to the rolling of the tires, which means that it will be 0 so long as the tires are not rotating but proportional to the total weight of the bicycle and rider when they are. Thus,

$$F_{rr} = \begin{cases} C_{rr} m_T g, & \text{if } v_g \neq 0; \\ 0, & \text{if } v_g = 0, \end{cases}$$

where C_{rr}, the coefficient of rolling resistance, is about 0.004 for most tires.

Summing the Forces

We sum the forces that act in the model:

$$F_A - F_B - F_f - F_r - F_g - F_{RR} = m_T a, \tag{2}$$

where

$F_A = \dfrac{P_a}{v_g} - (I_f + I_r)\dfrac{a}{R^2}$ (forward force that rider exerts),

$F_B = K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u})\|^2$ (drag force on bike frame and rider),

$F_F = K_W C_F v_T^2$ (drag force on front wheel),

$F_R = 0.75 K_W C_R v_T^2$ (drag force on rear wheel),

$F_g = m_T g G$ (force of gravity),

$F_{rr} = C_{rr} m_T g$ if $v_g \neq 0$, 0 if $v_g = 0$ (force of rolling resistance).

The forward force of the rider depends on acceleration. Since we want to have all the acceleration terms together, we first group them:

$$\left(\frac{P_a}{v_g} - (I_f + I_r)\frac{a}{R^2}\right) - F_B - F_f - F_r - F_g - F_{RR} = m_T a,$$

$$\frac{P_a}{v_g} - F_B - F_f - F_r - F_g - F_{RR} = a\left(m_T + \frac{I_f + I_r}{R^2}\right).$$

Substituting the other forces into **(2)** , we obtain

$$\frac{P_a}{v_g} - K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2 - K_W C_F v_T^2 - 0.75 K_W C_R v_T^2 - m_T g G = \left(m_T + \frac{I_f + I_r}{R^2}\right) a.$$

Solving for a, we find the second-order differential equation

$$a = \frac{d^2 S}{dt^2} = \frac{\dfrac{P_a}{v_g} - K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2 - K_W C_F v_T^2 - 0.75 K_W C_R v_T^2 - m_T g G}{m_T + \dfrac{I_f + I_r}{R^2}}.$$

Completing the Model

This differential equation is too complicated to solve analytically, so we solve it numerically using a fourth-order Runge-Kutta (RK4) approximation method [Burden and Faires 1997]. This method is generally more accurate than other numerical approximation methods such as Euler's method, especially at points farther away from the start point.

To make the approximation, we first define acceleration based on time t, position (elevation S_z), and speed v_g:

$$a = a(t, S_z, v_g)$$

The RK4 method uses a weighted approximation of the acceleration at a given time t_k with speed v_{g_k} to determine the speed at some time t_{k+1} in the future, where $t_{k+1} = t_k + h$, with h the time-step size.

$$(v_g)_{k+1} = (v_g) + \frac{1}{6}\, h \left(W_{K1} + 2W_{K2} + 2W_{K3} + W_{K4}\right),$$

where

$$\begin{aligned} W_{K1} &= a(t_k, v_{g_k}), \\ W_{K2} &= a\left(t_k + \tfrac{h}{2}, v_k + \tfrac{hW_{K1}}{2}\right), \\ W_{K3} &= a\left(t_k + \tfrac{h}{2}, v_k + \tfrac{hW_{K2}}{2}\right), \\ W_{K4} &= a(t_k + h, v_k + hW_{K3}). \end{aligned}$$

As with any numerical approximation method, we must know the initial speed v_{g_0}. Then, with the velocity computed at a given time, we calculate the position of the bike at time t_{k+1} as

$$\vec{S}_{k+1} = \vec{S}_k + h(v_g)_k \hat{u}$$

from the initial position of the bike $\vec{S}_0$. Again, because we consider only axial forces acting on the bike and ignore turning, we can model only a bike moving in a straight line; this means that $\hat{u}$, the unit vector in the direction of the velocity, is constant.

Model Validation

To validate our model, we developed a computer program to simulate traversal of a course (see screen display in **Figure 3**). We inputted a map of the 2000 Olympic Games time-trial course [NBC Olympics 2000a]. The prevailing wind speed and direction for Sydney on 27 September 2000 (the day of the Olympic finals for the road race) was 10 m/s at 315° [Analytic Cycling 2001]. We assume that the winner would maintain an average power of 450 W.

Using our model, we calculated that cyclists would complete 15 laps of the 15.3 km course in a time between 5 h 18 min and 5 h 47 min (depending on which wheel is used). In Sydney that day, Jan Ullrich of Germany won the race in a time of 5 h 29 min. As this result falls within the range of predicted values, we believe that our model is relatively accurate.

However, to test truly the validity of the model, we would need to experiment with various riders and wheel types for which we know the drag

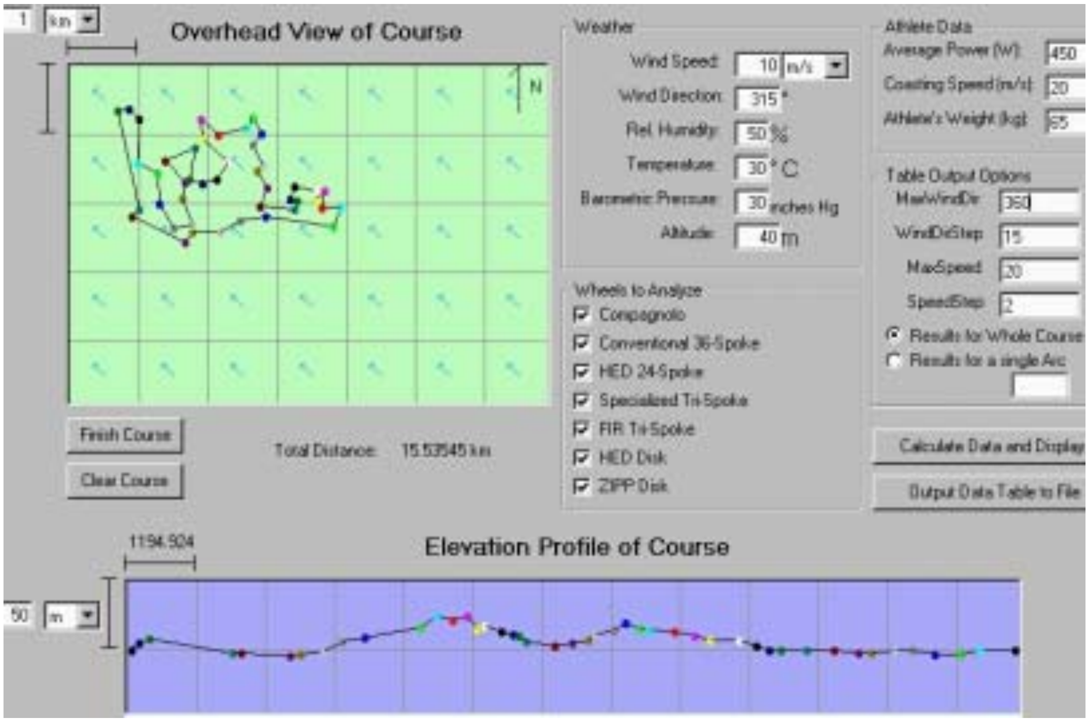

Figure 3. 2000 Olympic Games time-trial course.

coefficient as a function of yaw angle. The riders could use an exercise bike to determine their average power outputs. We would then have them traverse a course with different wheel types. After inputting the course in our model, we could compare the times that the model predicted with the experimental results.

Model Application

Table Creation

We construct tables for varying road grades and wind speeds. Since the direction of the wind has an appreciable effect on wheel performance, we create three tables: one for a headwind, one for a crosswind, and one for a tailwind. We applied our model to a hill 1.0 km long, beginning with a velocity of 10 m/s. The results in tabular form are in **Table 1**.

Table Analysis

Our model accounts for many additional factors other than wind speed and road grade; these contributions are lost if the results are pressed into table

Table 1.

Preferred wheel type for a hill 1.0 km long and starting speed 10 m/s.

Headwind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Either	Either	Disc	Disc	Disc
1	Spoke	Spoke	Disc	Disc	Disc	Disc
2	Spoke	Either	Disc	Disc	Disc	Disc
3	Spoke	Spoke	Either	Spoke	Either	Either
4	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
5	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
6	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
7	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
8	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
9	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
10	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke

Tailwind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
1	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
2	Spoke	Spoke	Spoke	Spoke	Spoke	Either
3	Spoke	Spoke	Spoke	Spoke	Spoke	Disc
4	Spoke	Spoke	Spoke	Spoke	Spoke	Either
5	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
6	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
7	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
8	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
9	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
10	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke

Crosswind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Disc	Disc	Disc	Disc	Disc
1	Spoke	Disc	Disc	Disc	Disc	Spoke
2	Spoke	Disc	Disc	Disc	Disc	Spoke
3	Spoke	Disc	Disc	Disc	Disc	Spoke
4	Spoke	Disc	Disc	Disc	Disc	Spoke
5	Spoke	Disc	Disc	Disc	Disc	Spoke
6	Spoke	Disc	Disc	Disc	Disc	Spoke
7	Spoke	Disc	Disc	Disc	Disc	Spoke
8	Spoke	Disc	Disc	Disc	Disc	Spoke
9	Spoke	Disc	Disc	Disc	Disc	Spoke
10	Spoke	Disc	Disc	Disc	Either	Spoke

form. Additionally, cycling races generally do not consist of one large hill of a uniform grade; there are typically many turns, hills, and valleys.

As a result, we recommend against using the tables that we provide! Our software implementation of the model allows the entry of *all* factors relating to the course that affect wheel choice.

Typically, course layout and elevation profile are available well in advance of a cycling race. We recommend that users of our software input the course and run multiple scenarios based on varying wind speeds and directions.

Results and Conclusions

We analyzed 7 different wheels: 5 spoked (Compagnolo, Conventional 36-Spoke, HED 24-Spoke, Specialized Tri-Spoke, and FIR Tri-Spoke) and 2 disc (HED Disc and ZIPP Disc). We chose these wheels because we could find data relating the drag coefficients to the yaw angle of the air. After running our model over varying courses and conditions, we came to some interesting conclusions, which mesh well with what intuition would suggest.

Crosswinds

In crosswinds, disc wheels dramatically outperform spoked wheels, since the drag coefficients for disc wheels decreases sharply as the yaw angle increases. As yaw angle increases to around 20°, drag coefficients for disc wheels drop dramatically. For the HED Disc, the drag coefficients actually become negative at larger yaw angles, indicating that the wheel acts like a sail and helps propel the cycle forward instead of slowing it down! The ZIPP Disc drag coefficients do not become negative but drop very close to zero at larger yaw angles. Consequently, the difference in speed between discs and spokes in a crosswind is significant. If a course has a strong crosswind (greater than 20 mph), then a disc wheel can make time differences on the order of 20% or more. However, *even in a light crosswind, the two disc wheels that we analyzed outperformed every spoked wheel*.

Direct Head and Tail Winds

In direct head and tail winds, spoked wheels slightly outperform disc wheels, because both types have similar drag coefficients at small yaw angles and spoked wheels are lighter.

Spoked wheels generally outperform disc wheels when going uphill in the absence of wind, because they are lighter, whereas disk wheels outperform spoked wheels when going downhill, because they are heavier. When there is a head wind or a tail wind when going up a hill, spokes generally still outperform discs; however, the introduction of any cross wind with a yaw angle much greater than 15° or 20° causes the smaller disc drag coefficients to outweigh the mass differences. This means that disc wheels are more efficient than spoked wheels.

Long Races

In long races, disc wheels generally outperform spoked wheels, because the smaller drag forces on the disc have a more significant effect on overall performance. Generally, in a race of any length much over 5 km, discs are more efficient, because the wind acts on them for longer amounts of time than in short races, which means that aerodynamic characteristics are more important than mass differences.

Short Races

In short races, where acceleration is more important, the spoked wheels, with their smaller masses and moments of inertia, outperform the disc wheels.

Strengths of the Model

Robustness. We derive our model from basic physical relationships and limit our use of assumptions. In every case where an assumption is required, we substantiate it with evidence and reasoning that illustrates why the assumption is valid.

Grounded in theory and research. We constructed our model based on both theory (Newton's second law) and research (Greenwell et al. [1995]).

Ease of use. Although the physical concepts behind our model are relatively simple to understand, the mathematical derivations and calculations are difficult to perform. We created a user-friendly computer program with a graphical interface that allows anyone with a basic knowledge of computers to input the required data. The program performs the mathematical calculations and reports the preferred wheel choice, along with calculated stage times for the race.

Weaknesses of the Model

Representing the course. We represent the course as a sequence of points in three-dimensional space, with the path from point to point represented by a straight line. This is not an accurate representation of any course; the traversal of hills, valleys, and curves all create nonlinear movement. Accuracy could be increased through a detailed entering of the course with a larger number of location nodes.

Approximation of wheel drag data. Greenwell et al. [1995] reported data for yaw angle 0° through 60° only in 7.5° increments. We developed interpolating polynomials through this range. We would conduct further tests at various yaw angles.

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A Systematic Technique for Optimal Bicycle Wheel Selection

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Introduction

We present a theoretical investigation of the dynamics and aerodynamics of a racing bicycle. We identify the dominant physical mechanisms and apply Newtonian mechanics principles to obtain a differential equation that gives the power output required of the rider. We then approximate the time-averaged power difference between the two types of rear wheel. We develop an easy-to-read, unambiguous, and comprehensive table that allows a person familiar with track and wind conditions to select the correct wheel type. We apply this table in the analysis of a time-trial stage of the Tour de France. We compare and contrast our choice of wheel with that of the leading competitors, with enlightening results.

Our criterion on the wind speed and direction gives predictions that agree with the exhaustive experimental data considered. We complement our criterion with suggestions based on the experience of an extensive range of experienced cyclists. We provide an informative critique of our model and suggest innovative ways to enhance the wheel-choice criterion.

Assumptions

- The spoked wheel is the Campagnolo Vento 16 HPW clincher wheel, used by the ONCE cycling team [ONCE Cycling Team Website 2000]. Reported mass = 1.193 kg [Hi-Tech Bikes 2001].

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Table 1.
Model Inputs and Symbols.

s	distance over which the average power is calculated
k	road grade
u	initial speed of the rider before any incline
c_r	coefficient of air drag for the bike and the rider frame
A_r	cross-sectional area of bike and rider frame
ρ	air density
$m(spk)$	mass of spoked rear wheel
$m(sld)$	mass of disc rear wheel
c_r	coefficient of rolling resistance
g	9.81m/s^2 acceleration due to gravity
c_{fw}	drag coefficient of front wheel
$c_{rw}(sld)$	drag coefficient of solid wheel
$c_{rw}(spk)$	drag coefficient of spoked wheel
r_{rw}	radius of rear wheel
μ	constant of deceleration
ϕ	angle of wind with respect to direction of bike (degrees)
v_f	speed of the wind with respect to the cyclist
I_{rw}	moment of inertia of the rear wheel
I_{fw}	moment of inertia of the front wheel
v_w	wind speed
ν	kinematic viscosity of air

- The density and pressure of the air across the bike are essentially constant. (See **Appendix**. [EDITOR'S NOTE: We omit the **Appendix**.])
- For most of the journey, the bike travels in a straight line ($\pm 10\%$). This is reasonable, since cyclists avoid turning as much as possible and they have a wide enough road to achieve a straight line.
- The solid wheel is taken to be the HED disc tubular freewheel, the solid wheel of choice in the Tour de France. Reported mass = 1.229 kg [Hi-Tech Bikes 2001].
- The drag coefficients of both wheels are independent of the wind speed in the range of raceable weather conditions and vary significantly with the relative direction of the wind [Flanagan 1996].
- The drag area of a typical crouched racer is 0.3 m^2 [Compton 1998].
- The radius of the wheel is 35 cm [Compton 1998].
- The efficiency of the drive train is essentially 100%, reasonable for élite racing bikes [Pivit 2001].
- The coefficient of rolling resistance between road and wheel rubber is 0.007 [Privit 2001].
- The moment of inertia of a bicycle wheel about an axis through its centre and perpendicular to the plane of the wheel is $I = \frac{1}{9}mr^2$, where m is the mass

of the wheel and r is the radius. This agrees with empirical data [Compton 1998].

- The deceleration to terminal speed on a uniform incline is constant to within $\pm 2\%$.
- The terminal speed on a uniform incline is reached at 100 m up the incline.
- The deceleration of a rider on a slope is proportional to the gradient of the slope (given in the problem statement).
- The power is averaged over 100 m of acceleration, where the acceleration is that calculated in the **Appendix**. This power is used in determining the wind speed criterion for solid wheels.
- All of the drag due to the rider and bike frame is due to the rider cross-sectional area, since the area of the rider is much greater.
- The rolling resistance $c_{rr}mg$ is the same for the solid wheel as for the spoked wheel. The tires surrounding the wheels are identical and the mass difference between the wheels is only about 0.1% of the overall mass of the bike plus rider.

The Wind Speed Criterion

We have from Newton's Second Law that $F = ma$, where F is the force acting the object, m is its mass, and a is its net acceleration. For the bicycle, the force equation can be written as follows:

$$\text{Force applied by rider} = \text{Retarding Forces} + ma + \text{rotational acceleration},$$

where m is the mass of the rider-bike system. The retarding forces consist of

- the drag due to the bike frame and the rider,
- the drag due to the individual wheels,
- the friction due to the motion of the wheels in the air, and
- the rolling resistance of the wheels on the surface.

The formulas for these torques and forces are

$$
\begin{aligned}
\text{drag force of bike frame and rider} &= \frac{c_w A \rho v_f^2}{2}, \\
\text{drag force due to front wheel} &= \frac{c_{fw} A \rho v_f^2}{2}, \\
\text{drag force due to rear wheel} &= \frac{c_{rw} A \rho v_f^2}{2}, \\
\text{frictional torque} &= 0.616 \pi \rho \omega^{3/2} \nu^{1/2} r^4, \\
\left(m_r + m_b + m_{fw} + \frac{I_{fw}}{r^2} + m_{rw} + \frac{I_{rw}}{r^2} \right) a &= \text{Resultant Force.}
\end{aligned}
$$

With this in mind, the power is averaged. The criterion on the wind speed v_w becomes:

$$v_w > \sqrt{\frac{m_{rw}(sld) - m_{rw}(spk)}{(\sigma_{spk} - \sigma_{sld})} + \left(\frac{\gamma}{s}\right)^2 - \frac{\lambda}{s}} + \frac{\gamma}{s},$$

where σ, λ, and γ are as given in the **Appendix**. If the quantity on the left side is not positive, then the solid wheel is always best. The quantity σ is effectively an aerodynamic term, whereas γ and λ can be thought of as representing the acceleration, and time of acceleration respectively. Note that γ is dependent on the wind direction ϕ.

The Minimum Wind Speed Table

Table 2 gives the wind speed below which the power for the spoked wheel is less, for various rider speeds. A 0 means that for *any nonzero* wind speed, the solid wheel requires less power; ∞ means that the rider will not be able to continue up the slope for 100 m.

Course Example

Time-trial course: Stage 1, Tour de France 2000 (**Figure 1**): A 16.5 km circuit starting at the Futuroscope building in Chassenuil-du-Poitou, France [Tour de France 2000].

Modeling the course: The course is modeled by a quadrilateral. This is a faithful representation of the course [CNN/Sports Illustrated Website 2000]. This course is predominantly level (**Figure 2**) save for a 1 km climb at a gradient of 3.7%. This climb is treated as the crucial feature of the time trial course, insofar as wheel choice is concerned.

Table 2.

Wind speed (in m/s) below which the power for the spoked wheel is less.

a.

Rider speed of 10 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	16	0	0	0	0
0.01	23	0	0	0	0
0.02	29	1	0	0	3
0.03	34	3	0	0	5
0.04	39	4	0	0	7
0.05	43	6	1	0	8
0.06	48	7	3	0	10
0.07	51	8	4	0	11
0.08	55	10	5	1	13
0.09	59	11	6	2	14
0.10	∞	∞	∞	3	∞

b.

Rider speed of 12.5 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	13	0	0	0	0
0.01	21	0	0	0	0
0.02	25	0	0	0	0
0.03	26	0	0	0	2
0.04	32	2	0	0	3
0.05	36	3	0	0	5
0.06	41	4	0	1	7
0.07	45	5	0	2	8
0.08	48	7	1	3	10
0.09	52	8	2	4	11
0.10	59	9	3	5	12

c.

Rider speed of 14 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	12	0	0	0	0
0.01	19	0	0	0	0
0.02	25	0	0	0	0
0.03	30	0	0	0	0
0.04	35	0	0	0	1
0.05	39	1	0	0	3
0.06	43	3	0	0	5
0.07	47	4	0	0	6
0.08	50	5	0	1	8
0.09	54	6	0	2	9
0.10	57	7	1	3	10

Weather: The air temperature was about 20°C. The wind was variable west to southwest, between 20 and 30 kph (5.6–8.3 m/s) [Official Tour de France Website 2000].

Figure 1. The course.

North points towards the top of the page. The climb is westward, meaning that the riders faced the wind at an angle varying from 0° to 45° to the wind: exactly the range of applicability of the table. The average speed of the top ten finishers was about 14 m/s. We analyse the climb from the perspective of a rider who begins the climb at that speed and whose speed levels off after 100 m of the climb. Knowing the speed of the rider to be 14 m/s and the gradient of the slope to be approximately 4%, we can apply **Table 2c**. The minimum wind speed in the table is 1.44 m/s (assuming that the wind is at an angle to the direction of motion). On the basis of the climb alone, the solid wheel is the better choice. In fact, assuming that the wind is at an angle to the rider for most of the journey (wind that "follows" a cyclist around is unlikely), the solid wheel is better at *all* nonzero wind speeds (assuming level terrain and rider speed of 14 m/s).

The recommended wheel for this race is therefore the solid wheel. In this time trial, U.S. Postal Team and the Spanish ONCE Team used a solid rear wheel and a tri-spoke front wheel (a more aerodynamically efficient version

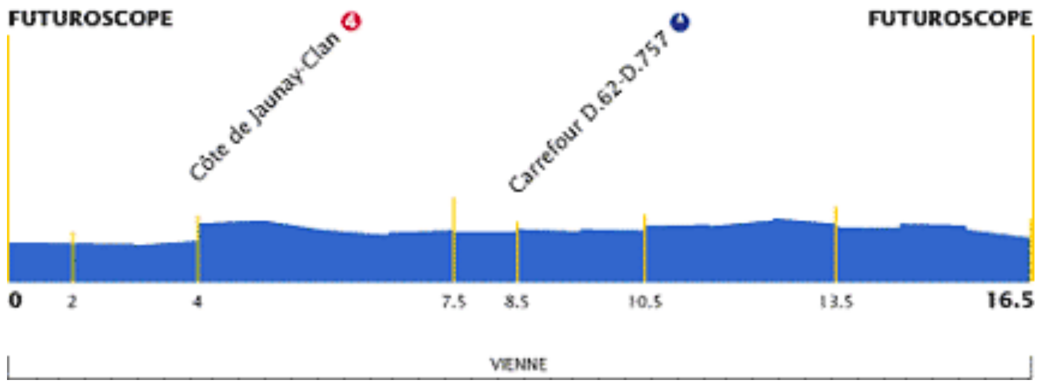

Figure 2. Profile of the course.

of the standard spoked wheel). The two teams had five riders in the top ten [Official Tour de France Website 2000; Pediana 2000].

The Adequacy of the Table

Comparing Its Predictions with Experimental Data

On an indoor circuit, the gradient and the wind can be considered to be zero. The corresponding entries in **Table 2**, for a direct headwind (180°) at zero gradient, are nonzero, indicating that spoked wheels are better. This prediction is borne out by the empirical results on an indoor circuit [Beer 1999] (**Table 3**).

Table 3.
Measurements made on a 1-km indoor circuit.

Watts		Spoked Rear Wheel	Solid Rear Wheel
100	time (min:sec)	2:22.58	2:26.78
	speed (ms^{-1})	6.97	6.77
200	time (min:sec)	1:51.75	1:52.55
	speed (ms^{-1})	8.90	8.83
300	time (min:sec)	1:36.98	1:39.70
	speed (ms^{-1})	10.25	9.97

The times correspond to seven laps of 147 m. The test cyclist rode at power outputs of 100 W, 200 W, and 300 W. The tests were repeated with no more than a 2% time variation; the spoked rear wheel has the better performance by a margin greater than the experimental error.

However, for a road circuit (where one would expect variable and nonzero wind velocities), **Table 2** predicts that for nonzero cross winds and slopes of a gradient less than 5%, the solid wheel is best, based on minimising aerodynamic losses. Thus, if the solid wheel is the right choice for a particular cyclist speed, then the solid wheel choice is valid for all greater cyclist speeds, given the same atmospheric conditions. Experimental data in **Table 4** back up this claim.

Table 4.

Measurements made on a 7.2-km road circuit.

Watts		Spoked Rear Wheel	Solid Rear Wheel
200	time (min:sec)	15:22	13:53
	speed (ms^{-1})	7.81	8.63

The circuit consisted of a 7.2-km looped course of rolling hills (gradient $< 5\%$) with varying wind conditions (nonzero wind speeds, unlike the indoor track). The power output of the cyclist was approximately 200 W. The tests were repeated with no more than a 2.3% time variation. For the road circuit, the solid rear wheel is the better choice (again by more than the experimental error). According to the appropriate table, the solid rear wheel is the more efficient for most of the conditions that were encountered. Thus, the predictions of the model are once more confirmed.

Additional Factors Not Considered by the Table

Stability of the bike: Stability is a major factor in cycling. A cyclist wants to concentrate on putting the maximum possible power through the pedals. If the bicycle is unstable, the jerking of the handlebars in response to sudden gusts of wind will be very disruptive. In general, bicycles with standard spoked wheels front and rear are the least affected by changing crosswinds, but bikes with solid wheels are fastest. A solid front wheel can increase the rider's pedaling wattage by 20% to 30%, but it removes all the bicycle's self-centering effects, making for a difficult ride. A solid rear wheel has similar but milder effects [Cobb 2000].

Turning radius: This factor is important if the course contains many turns or if maneuvering in response to other riders is necessary. A greater turning radius loses time at each turn, where more maneuverable riders may overtake.

Rider comfort: The solid wheel also creates the problem of rider comfort. As it has no spokes, it cannot flex to absorb any shocks due to irregularities of the road surface. Towards the end of a long race, a rider's concentration may be impaired due to the resulting discomfort, causing a drop in performance [Bicycle Encyclopedia 2000].

Very few cyclists ride with two solid wheels (1 out of 177 in the time trial that we studied), despite the aerodynamic superiority—steering problems negate the gain. Similarly, we have shown the superiority of the solid rear wheel in race conditions, yet in the long-distance ($150+$ km) stages of the Tour de France, all riders use spoked wheels: Solid wheels have insufficient maneuverability and steering problems cause mental fatigue near the end of the stage.

Thus, the *directeur sportif* should consider the length of the race, the wind conditions, and the quality of the road surface in making the decision.

Error Analysis

The error was determined by calculating the differential.

The wind direction may vary considerably over a given course due to local effects (e.g., buildings, trees). We chose an error of 10% for $c_{fw}(\phi)$ and $c_{rw}(\phi)$.

Air density can usually be determined to a high accuracy but varies by location. Therefore, we chose a 1% error, since the necessary equipment is unlikely to be available to the *directeur sportif* at the time of the race.

Using a wind tunnel, drag coefficients can be determined to a very high precision (to within 0.02kg); but variables such as pedaling speed, wheel rotation, and rider posture introduce inaccuracies that cannot be easily determined. The main difficulty is that the flowfield about a rider in a wind tunnel is not ergodic, primarily as a result of airflow irregularities caused by pedaling. Therefore, we chose an error of 3% for c_r [Flanagan 1996].

For each rider/bicycle combination, the *directeur sportif* should determine the appropriate masses and dimensions. We assume that they can be determined to a high degree of accuracy ($\sim 0.1\%$) with the exception of the rider's cross section, in which we take an error of 2%. From error analysis in the **Appendix**, we find

$$\frac{\triangle v_w}{v_w} = 16\%.$$

Strengths and Weaknesses

Our model clearly and concisely states which wheel should be used under the various conditions. The model is based on equations that are sufficiently versatile so that further factors pertaining to a particular situation may be included as required, without any need to construct new equations.

We could not verify the model experimentally under conditions of the high power output of élite cyclists. However, in data available for various lesser power outputs, we could not discern any noticeable trends in time differential between the different wheels with respect to increasing power.

Our model does not contain a quantitative analysis of the wind speeds at which the solid wheel is unacceptably unstable. However, this question depends largely on the abilities and preferences of each individual rider and requires detailed local information about road conditions and wind variability.

No data were available regarding the coefficients of drag in a tail wind. However, since the cyclist (~ 15 m/s) in general travels faster than the wind (race conditions $< 10\,\mathrm{m/s}$), only the crosswind effects are important, and these are adequately handled in the tables.

The major weakness of the model is assuming that the rider's speed does not differ much from the rider's average speed over the duration of the race. Moreover, we could find no evidence to support the problem's assumption that a rider reaches terminal velocity after 100 m of a slope.

Conclusion

- When there is a head wind, the spoked wheel is better. If the course is flat, the solid wheel is better for strong winds; however, the wind (particularly if gusting) may cause instability problems.
- When the wind is not weak (> 5 m/s) and strikes the wheel at an angle, the solid wheel is nearly always better. Even a small component of wind perpendicular to the rider direction makes the solid wheel the better choice.
- If the circuit has many tight turns or involves riding in close company with other cyclists, the solid wheel's lack of maneuverability dictates the spoked wheel; otherwise, the risk of an accident and injury is unacceptable.
- The region of superiority of the solid wheel increases with rider speed. Since the power required to overcome air resistance goes as the cube of the velocity, the aerodynamic savings of the solid wheel become more important with higher speed.

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Dr. Ann Watkins, President of the Mathematical Association of America, congratulating MAA Winners Eamonn Long, Michael Flynn, and William Whelan-Curtin, after they presented their model at MathFest in Madison, WI, in August. [Photo courtesy of Ruth Favro.]

Author-Judge's Commentary: The Outstanding Bicycle Wheel Papers

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Introduction: The Problem

Professional bicycle racers have a wide variety of wheel types available to them. The types of wheels range from the familiar spoked wheels, to wheels with three or four blades, to solid wheels. The spoked wheels have the lowest mass but have the highest friction forces due to interactions with air. The solid wheels have the most mass but have the lowest friction forces. The question posed was to demonstrate a method to determine what kind of wheel to use for a given race course.

The problem focused on the two most basic types of wheels, the spoked wheel and the solid wheel. Three tasks were given:

- Find the wind speeds for which one wheel has an advantage over the other for particular inclines.
- Demonstrate how to use the information in the first task to determine which wheel to use for a specific course.
- Evaluate whether the information provided in the first task achieved the overall goal.

The UMAP Journal 22 (3) (2000) 253–256.

General Remarks on the Solution Papers

As is the case each year, many fine papers were submitted. The papers were judged on both their technical merit and on their presentation. The submissions in which both aspects were superior received the most attention. The problem required that many assumptions had to be made; because of the severe time restrictions, it was extremely important to be able to choose assumptions that did not make the problem too simple but still relevant.

For example, a number of submissions concentrated on the *yaw angle*, the angle that the wind makes with respect to the direction of movement of the bicycle. While some of these submissions were quite good, it appeared that others spent so much time trying to figure out how to deal with this complicated aspect that sufficient progress was not made with respect to the other parts of the problem. Moreover, it was often difficult to read and interpret the resulting descriptions of the teams' efforts.

While the assumptions were important, it was also important in developing a mathematical model for this problem to stay consistent with the basic definitions of mechanics. There were a number of entries in which Newton's Second Law, the torque equations, or the power was not correctly identified. There was also some confusion about units. Such difficulties represented a key division between the lower and higher rankings.

Approaches

Overall, there were two different approaches:

- The first approach focused on the mechanics of a bicycle moving on an incline. The forces acting on the bicycle and rider were used to find the equations of motion from Newton's Second Law and the torque equations. The equations could then be used to isolate the force acting to move the bicycle forward.

  > The main difficulty with this approach was in isolating and identifying the relevant force based on the equations from Newton's Second Law and the corresponding torque equations. In many cases, it was difficult to identify exactly how the system of equations was manipulated and how the equations were found. The submissions in this category that were highly rated did an excellent job of displaying and referring to the free-body diagrams, as well as discussing how the relevant force was isolated by manipulating the system of equations.

- The second approach focussed on the aerodynamic forces acting on the wheels, then calculated the power to move the wheels forward. For the spoked wheel, the total force acting on the wheel was found by adding the effects on each spoke (along its entire length) with its respective orientations. For the solid wheel, the forces acting on the whole wheel were found with respect to the wind yaw angle.

> This second approach turned out to be a difficult one. In some cases, it was hard for the judges to identify the approach and what assumptions were being made. The submissions in this category were also more likely to concentrate on the yaw angle and its associated complications. The teams that carefully structured their approach and clearly identified each step stood out.

For either approach, there were different assumptions that could be made about the motion of the bicycle and rider. The most common approach was to make some assumption about either the acceleration or the steady-state velocity as the bike and rider moved along the hill. The second most common assumption was to assume that the rider provided a constant power output and then work backwards to isolate the forces acting on the wheels. For the most part, the judges did not question the technical merits of these kinds of assumptions. The judges concentrated instead on whether or not the submissions presented a clear and consistent case based on the given assumptions.

Fulfilment of the Tasks

There were many fine entries in which the first task (provide a table) was addressed. The first task was the most specific and straightforward part of the problem. The factor that set the entries apart was in how the two remaining tasks (use the table in a time trial, determine if the table is adequate) were addressed and presented. The majority of submissions discussed the second task by dividing the race course into discrete pieces; the total power could then be found by adding up the power requirements over each piece. This part of the submissions often seemed to have received the least amount of attention by the different teams and was often the hardest part to read and interpret.

The analysis and qualitative comparisons within each submission were crucial in determining how a team's efforts were ranked. Many teams provided an adequate formulation for the first task in the problem but addressed the other tasks in a superficial manner. The real opportunity to express a deeper understanding of the problem and show some creativity lay in how the remaining aspects of the problem were approached.

The entries that most impressed the judges went further in their analysis. In particular, a small number of entries approached the third task by noting that the real goal was to minimize the time spent on a particular race course. By assuming that the rider would expend a constant power output, the equations of motions from Newton's Second Law could then be found. The position of the rider on the course at any given time could then be approximated through a numerical integration of the resulting system of equations. For a given racecourse, the total time on the course could be found for different wheel configurations. A simple comparison of total times determined which wheel to use for the course.

The submissions that went beyond the stated problem and stayed true to the original goal received the most attention from the judges. Such papers showed creative and original thought, and they truly stood apart from the rest. Moreover, they showed the deepest understanding of the task at hand.

About the Author

Kelly Black is visiting Utah State University and is on sabbatical leave from the University of New Hampshire. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his Master's and Ph.D. from the Applied Mathematics program at Brown University. His research is in scientific computing and has interests in computational fluid dynamics, laser simulations, and mathematical biology.

Project H.E.R.O.: Hurricane Evacuation Route Optimization

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Introduction

Through modeling and computer simulation, we established an evacuation plan for the coastal region of South Carolina in the event of an evacuation order.

We derive nine evacuation routes running from the coastal region inland. Based on geography, counties are given access to appropriate routes. Combining flow theory with geographic, demographic, and time constraints, we formulate a maximum flow problem. Using linear optimization, we find a feasible solution. This solution serves as a basis for our evacuation model. The validity of the model is confirmed through computer simulation.

A total evacuation (1.1 million people) in 24 to 26 hours is possible only if all traffic is reversed on the nine evacuation routes.

Terms and Definitions

Flow F**:** the number of cars that pass a given point per unit time (cars per hour per lane, unless otherwise specified).

Speed s**:** the rate of movement of a single car (mph, unless otherwise specified).

Density k**:** the number of cars per unit length of roadway (cars per mile per lane, unless otherwise specified).

The UMAP Journal 22 (3) (2001) 257–269.

Headway distance h_d: the space between the back of the leading car and the front of the immediately trailing car (ft). (Note: This is *not* a standard definition of headway distance.)

Headway time h_t: the time required to travel headway distance.

Car Length C: length from front bumper to rear bumper of a single car (feet).

Goals

Our first priority is to maximize the number of people who reach safety; in terms of our model, we must maximize the flow of the entire system. A secondary goal is to minimize the total travel time for evacuees; this means that we must maximize speed. As we establish, these goals are one and the same.

Assumptions

- **Vehicles hold 2 people on average.** This seems reasonable, based on the percentage of the population who would be unable to drive themselves and those who would carpool.
- **Vehicles average 17 ft in length.** This is based on a generous average following a quick survey of car manufacturers' Web sites.
- **Vehicles have an average headway time of 3 s.** This is based on numbers for driver reaction time, found in various driving manuals.
- **50 mph is a safe driving speed.**
- **Merging of traffic does not significantly affect our model.** See the **Appendix** for justification.
- **Highways 26, 76/328, and 501 are 4-lane.** [Rand McNally 1998]
- **Safety is defined as 50 mi from the nearest coastal point.** Counties that lie beyond this point will not be evacuated [SCAN21 2001].
- **Only the following counties need to be evacuated:** Allendale, Beaufort, Berkeley, Charleston, Colleton, Dorchester, Georgetown, Hampton, Horry, Jasper, Marion, Williamsburg, and a minimal part of Florence County (based on the previous assumption).
- **Myrtle Beach will not be at its full tourist population during a hurricane warning.** This seems reasonable because tourists do not like imminent bad weather.

- **The evacuation order will be given at least 24 to 26 h prior to the arrival of a hurricane.** This is based on the timeline of the 1999 evacuation [Intergraph 2001].
- **Boats, trailers, and other large vehicles will be limited from entering the main evacuation routes.** Being able to evacuate people should have a higher priority than evacuating property.
- **If we can get everyone on a road within 24 h and keep traffic moving at a reasonable speed, everyone should be at a safe zone within 25 to 26 h.** This is based on our assumption of what a safe zone is and our assumption of average speed.

Developing the Model

Abstracted Flow Modeling

Upon inspecting the evacuation route map, we decided that there are only nine evacuation routes. There appear to be more, but many are interconnected and in fact merge at some point. By identifying all bottlenecks, we separated out the discrete paths.

Using this nine-path map in combination with the county map, we constructed an abstracted flow model with nodes for each county, merge point, and destination, so as to translate our model into a form for computer use.

A Brief Discussion of Flow

The flow F is equal the product of density and speed: $F = ks$ [Winston 1994]. We can find the density k of cars per mile by dividing 1 mi = 5,280 ft by the sum $C + h_d$, the length of a car plus headway distance (in ft), so

$$F = ks = \frac{5280s}{C + h_d}.$$

Using the fact that headway distance h_d is speed s (ft/s) times headway time h_t (sec), we have

$$F = \frac{5280s}{C + sh_t} = \frac{5280}{\frac{C}{s} + h_t}.$$

Increasing s increases F. This result is exciting, because it shows that maximizing flow is the same as maximizing speed. The graph of F versus s gives even more insight (**Figure 1**). Increases in speed past a certain point benefit F less and less. So we might sacrifice parts of our model to increase low speeds but not necessarily to increase high speeds.

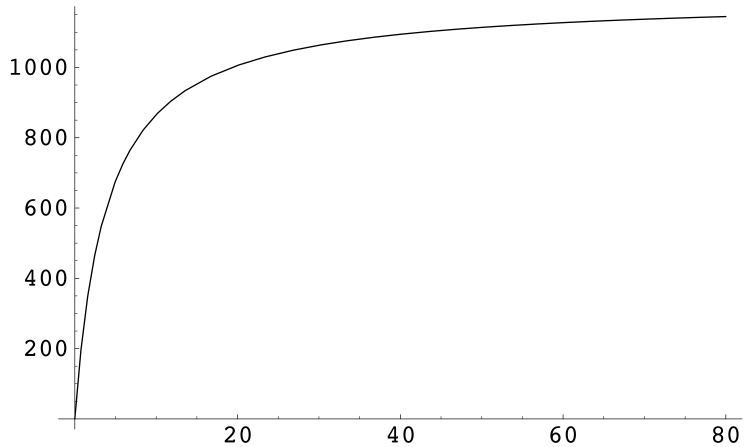

Figure 1. Flow vs. speed.

According to our assumptions, we have $C = 17$ ft and $h_t = 3$ sec, and converting to units of miles and hours, we get

$$F = \frac{1}{\frac{17}{5280s} + \frac{1}{1200}},$$

or

$$s = \frac{17}{5280} \cdot \frac{1200F}{1200 - F}. \quad \textbf{(1)}$$

At our assumed maximum safe speed of $s = 50$ mph, we have $F = 1114$ cars/h.

Determining Bounds

We combined our knowledge of county populations with the 24-h deadline and generated a minimum output flow for each county. We also determined the maximum flow for each node-to-node segment, based on the number of lanes. It would be unrealistic to assume that each segment would reach optimal flow, so we set maximum flow at 90% of optimal flow. This reduction in flow is meant to cover problems that arise from accidents, slow drivers, less than ideal merging conditions, or other unexpected road conditions. Putting $F = 0.9F_{\text{opt}} = (0.9)(1113.92)$ into **(1)**, we find $s \approx 19.6$ mph. We decided that this is an acceptable minimal speed.

Finding a Feasible Solution

We used the linear optimizing program LINDO to find a feasible solution; the solution takes 26 h. Since this scenario does not take into account geographical convenience, we did some minor hand-tweaking. The final product is in the **Appendix**.

The Simulation

To confirm the feasibility of our model, we conducted a computer simulation using Arena simulation software. The simulation encompassed 24 h of traffic flow on the nine evacuation routes assuming 90% flow efficiency. The model assumed that there was an unlimited number of vehicles ready to enter the simulation in all counties. The time headway between entering vehicles was considered to be normally distributed with a mean of 3 s and a standard deviation of 1 s. The simulation verified our model.

Implementation Requirements

For optimal performance of our model:

- Evacuees must follow the evacuation routes. The State of South Carolina should notify specific communities or households which route to take.
- Flow must be monitored on all evacuation routes; this requires metering entry of evacuees onto the evacuation routes. Allowing vehicles to enter an evacuation route too quickly may result in congestion at bottlenecks.
- Advance notification that there will be ticketing by photograph could enforce the restriction on towing boats and trailers, which might otherwise be ignored.

Applying the Model

Requirement 1

If an evacuation order included both Charleston and Dorchester counties 24 h prior to the predicted arrival of a hurricane, it would be necessary to reverse all four lanes of I-26 to ensure the evacuation of the entire population of the two counties. In our simulation runs, all of the exit routes from Charleston and Dorchester ran at full capacity (all lanes reversed, 90% of maximum flow) for 24 h to evacuate the counties completely. If the lanes are not reversed, it is doubtful that the two counties could evacuate in a timely fashion.

Requirement 2

To optimize use of the available bandwidth while ensuring that the entire population is displaced inland within 24 h, we opted for a simultaneous evacuation strategy: All counties begin evacuation at the same time.

Since hurricanes typically arrive in South Carolina moving northward, a staggering strategy would evacuate southernmost counties first. Our model has discrete evacuation routes servicing each part of the coastline, so it is not necessary to stagger evacuation. For example, since Beaufort County, which would be among the first counties to be hit in the case of a hurricane, and Horry County, which would be hit significantly later, do not depend on the same evacuation route, nothing is gained by delaying the evacuation of Horry County until Beaufort County has cleared out.

Requirement 3

To evacuate the entire coastal region within 24 h, we found it necessary to turn around traffic on *all* designated evacuation routes. With greater time allowance, not all routes would need to be turned around.

Requirement 4

Our model directs approximately 480,000 evacuees to Columbia. This surge entering a city of 500,000 would undoubtedly disrupt traffic flow. While three major interstates head farther inland from Columbia and could easily accommodate the traffic from the coast, the extreme congestion within the city would disrupt the flow coming into Columbia from the coast. It would be best to set up temporary shelters around the outskirts of Columbia (and at other destination sites) to avoid having too many people vying for space within Columbia itself.

Requirement 5

Because heavy vehicles take up more road space and generally require a greater headway time, they adversely affect our model. Heavy vehicles are allowable if they are the only available means of transportation, as may be the case for tourists in recreational vehicles. Boats and trailers are strictly forbidden on the evacuation routes. A rule of one car per household can be announced, but the model can probably handle up to two cars per household. Our assumption of two people per car can still hold up, given the number of people who are unable to drive themselves to safety.

Requirement 6

With the flow and time constraints defined within our model, the entrance of large numbers of additional evacuees onto the designated evacuation routes from I-95 would cause serious disruptions. The traffic on I-95 coming from Georgia and Florida may turn west onto any nonevacuation roadway. Ideally, better evacuation routes could be established within Georgia and Florida to minimize their evacuees entering South Carolina.

Limitations

Time forced us to simplify our model. Here are extensions that we would have liked to include:

Factor in the "first-hour" effect. The western counties could potentially have full use of the evacuation routes for a limited time at the very beginning of the evacuation. The population of the eastern counties would take time to reach the western counties; but once they did, both groups would have to share the route.

Do more work with the impact of large vehicles.

Explore headway time in greater detail. We know that headway time is not dependent on speed in an ideal world, but does human psychology make headway time dependent on speed? Also, although we assume that all vehicles exhibit the same stopping pattern, we know that car size and brake condition have an effect. We would like to find a more accurate concept of headway time.

Inspect all nine routes on-site. We assume that any road listed by the SCDOT as a hurricane evacuation route is well maintained and appropriate for that use, and that these are the only appropriate routes.

Expand the complexity of our model. We kept the number of paths to a manageable level, but it would be nice to factor in the smaller routes.

Develop mechanisms to implement our model. This would involve planning out a block-by-block time-table, metering techniques, merging techniques, traffic reversal techniques, and large-vehicle restriction techniques.

Add accidents, breakdowns, and other problems to the model specifically, rather than just lumping them into "efficiency."

Study the potential costs of complete traffic reversal.

Study the population fluctuations of Myrtle Beach, so that we would know how many tourists to expect in the event of a hurricane.

Authorities Fear Floyd Repeat, Enlist Help of Undergraduates

Arden Hills, MN, Feb. 12—Responding to complaints over the disorder of South Carolina's 1999 coastal evacuation in preparation for Hurricane Floyd, authorities enlisted the aid of three undergraduates from Bethel College. The task set before the three young mathematicians was to plan an orderly and timely evacuation of South Carolina's coastal region.

A denial of funding for travel expenses prevented the students from making an on-site inspection, but they managed to get a feel for the territory based on maps and census reports. Using a technique they dubbed "Abstracted Flow Modeling," the team constructed several computer models of what a full coastal evacuation would involve. Using all the tools available to them and a little human intuition, the trio created a 26-hour scenario for the full evacuation of more than 12 counties.

Such an evacuation would involve all people in the area being divided among 9 separate evacuation routes and released in a timed fashion. Using the timings and routings suggested by the Bethel team, the entire coast could be evacuated within a reasonable time and the travel time for individual vehicles could be kept to a minimum.

Compared to the 1999 evacuation, when fewer than 800,000 people were evacuated, the Bethel model can evacuate more than 1.1 million people. Much of this increase can be attributed to grid-lock prevention, lane-doubling, and access restriction to the main highways.

Concerned citizens should be on the lookout for announcements concerning route assignments and departure timings for their neighborhoods.

— Nathan Gossett, Barbara Hess, and Michael Page in Arden Hills, MN

Appendix

Abstracted Flow Model

We created the flow model in **Figure A1** to represent what we perceived as 9 routes from the coast of South Carolina to the interior of the state. Rectangles represent locations and ovals represent junctions. The arrows represent flow direction. The flow assigned to various segments can be found in **Table A1**.

Table A1.
Flow rates by route and county.

County	Population (thousands)	Junction	Flow (cars/min)
Jasper	17	1a	5.9
Hampton	19	1b	6.6
Allendale	11	1c	3.8
Beaufort	113	1a	17.1
		2a	22
Colleton	38	2a	4.4
		2b	4.4
		3	4.4
Charleston	320	2b	2.6
		3	29
		4	17.6
		26	51
		5	5.5
		6a	5.5
Dorchester	91	4	15.8
		26	15.8
Berkeley	142	5	27.4
		6a	21.4
Georgetown	55	6a	3
		6b	16.1
Williamsburg	37	6a	3.5
		6b	9.3
Florence	125	6b	4
		7b	4
Horry	179	6b	4
		7	51
		8	33.4
Marion	34	7b	5.9
		7a	5.9

Merge Considerations

We do not want congestion at merges, so we maintain a constant traffic density in a "merge zone." Let A and B be the pre-merge flows and C be the post-merge outflow. Then we must have $C = A + B$ to avoid congestion.

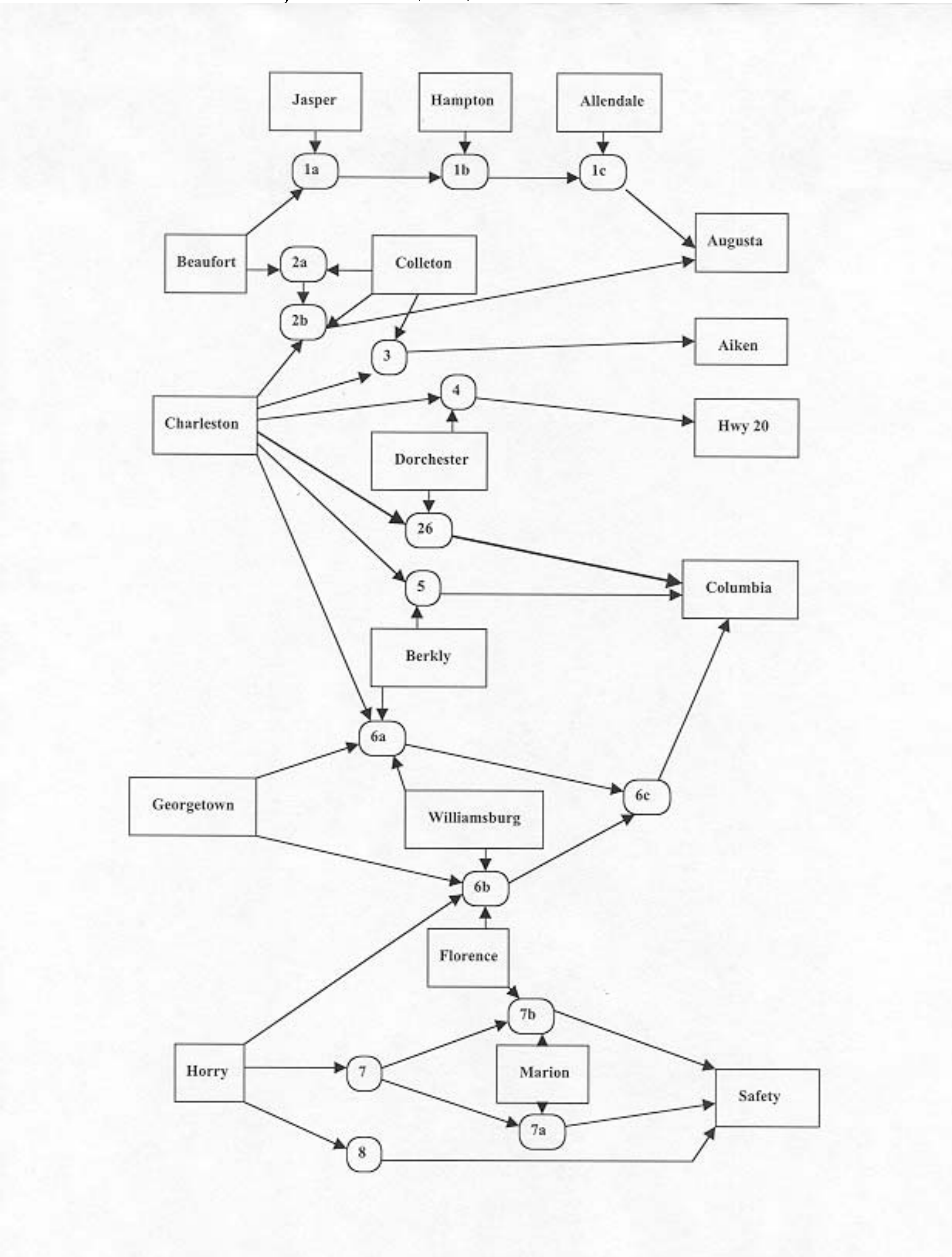

Figure A1. Abstracted flow model.

We must regulate the pre-merge flows to prevent congestion and allow for maximum post-merge flow.

If the density of merging traffic is large enough, we set aside one or two lanes for merging for about a mile prior to the merge point. We investigate whether this is possible without lowering the pre-merge flows A and B, and when this strategy is physically possible and beneficial.

Does shifting main-road traffic left to open a merge lane(s) increase the main road's flow (before merge traffic is added) and thus increase its contribution to post-merge flow? If so, then the post-merge flow would be

$$(A + \text{increase}) + B = (C + \text{increase}).$$

But we fixed $C = A + B$ as the maximum flow, so A or B or both must decrease. Is this reduction really a concern?

Recall that $F = ks$, where F is flow, k is density, and s is speed. Let N be the number of cars on a one-mile stretch and L be the number of lanes in the direction of concern. The total flow of the road is the product of the lane flow and the number of lanes:

$$Lks = L(N/L)s = Ns.$$

To shift one lane left, we must move N/L vehicles to $L-1$ lanes, adding $\frac{N/L}{L-1}$ vehicles to each non-merge lane, thus giving a new lane population of

$$\frac{N}{L} + \frac{N/L}{L-1} = \frac{N}{L} \cdot \frac{L}{L-1}.$$

So the new flow is

$$(L-1)\left(\frac{N}{L} \cdot \frac{L}{L-1}\right) s = Ns.$$

No change! We do not have to reduce pre-merge flows to add a merge lane (assuming that the merge lane modification is physically possible). The same argument confirms that pre-merging works for shifting two lanes.

When are these shifts physically possible? Let $m = N/L$, and let q be a lane's carrying capacity per mile, which is a function of s for fixed h_t. Then for clearing one merge lane to be physically possible, we need

$$\frac{m}{L-1} \leq q - m.$$

If two lanes need to be shifted left, the same process yields the requirement

$$\frac{2m}{L-2} \leq (q - m).$$

Moving everything to the right side of the inequalities gives two functions that are greater than or equal to 0, each a function of L and m. We fix $L = 2$, 3, and 4 lanes and graph the functions to see when they are within the constraints.

Figure A2 shows valid and invalid ms given four lanes (two lines for two possible shifts, single or double). We have $q = 309.83$, the carrying capacity for a mile-long lane with $s = 50$, and $h_t = 3$ seconds. The function must lie on or above the horizontal axis for creation of a merge lane.

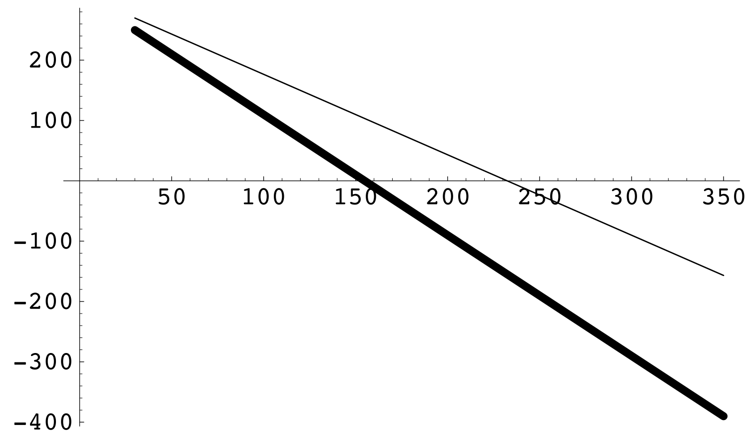

Figure A2. Feasibility functions for one lane (thin line) and two lanes (thick line), as a function of the number of cars per mile ($m = N/L$).

The merge patrol officer can determine q by multiplying the number of cars counted in one minute by $6/5$. If the ratio of merging traffic to main-road traffic is higher than $1 : 20$, there may be enough disruption that merge efficiency could benefit from a merge lane(s). With this ratio, and 90% flow, there would be one car merging every minute.

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Traffic Flow Models and the Evacuation Problem

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Introduction

We consider several models for traffic flow. A steady-state model employs a model for car-following distance to derive the traffic-flow rate in terms of empirically estimated driving parameters. We go on to derive a formula for total evacuation time as a function of the number of cars to be evacuated.

The steady-state model does not take into account variance in speeds of vehicles. To address this problem, we develop a cellular automata model for traffic flow in one and two lanes and augment its results through simulation.

After presenting the steady-state model and the cellular automata models, we derive a space-speed curve that synthesizes results from both.

We address restricting vehicle types by analyzing vehicle speed variance. To assess traffic merging, we investigate how congestion occurs.

We bring the collective theory of our assorted models to bear on five evacuation strategies.

Assumptions

- Driver reaction time is approximately 1 sec.
- Drivers tend to maintain a safe distance; tailgating is unusual.
- All cars are approximately 10 ft long and 5 ft wide.
- Almost all cars on the road are headed to the same destination.

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Terms

Density d: the number of cars per unit distance.

Occupancy n: the proportion of the road covered by cars.

Flow q: the number of cars per time unit that pass a given point.

Separation distance s: the average distance between midpoints of successive cars.

Speed v: the average steady-state speed of cars.

Travel Time: how long a given car spends on the road during evacuation.

Total Travel Time: the time until the last car reaches safety.

The Steady-State Model

Development

Car-following is described successfully by mathematical models; following Rothery [1992, 4-1], we model the average separation distance s as a function of common speed v:

$$s = \alpha + \beta v + \gamma v^2, \tag{1}$$

where α, β, and γ have the physical interpretations:

$$\begin{aligned} \alpha &= \text{the effective vehicle length } L, \\ \beta &= \text{the reaction time, and} \\ \gamma &= \text{the reciprocal of twice the maximum average deceleration} \\ &\quad\ \text{of a following vehicle.} \end{aligned}$$

This relationship allows us to obtain the optimal value of traffic density (and speed) that maximizes flow.

Theorem. *For $q = kV$ (the fundamental equation for traffic flow) and* **(1)**, *traffic flow q is maximized at*

$$q^* = (\beta + 2\gamma^{1/2}L^{1/2})^{-1}, \qquad v^* = (L/\gamma)^{1/2}, \qquad k^* = \frac{\beta(\gamma/L)^{1/2} - 2\gamma}{\beta^2 - 4\gamma L}.$$

Proof: Consider N identical vehicles, each of length L, traveling at a steady-state speed v with separation distance given by **(1)**. If we take a freeze-frame picture of these vehicles spaced over a distance D, the relation $D = NL + Ns'$

must hold, where s' is the bumper-to-bumper separation. Since $s' = s - L$, we obtain $k = N/D = N/(NL + Ns') = 1/(L + s') = 1/s$. We invoke **(1)** to get

$$k = \frac{1}{\alpha + \beta v + \gamma v^2}.$$

This is a quadratic equation in v; taking the positive root yields

$$v(k) = \frac{1}{2\gamma}\sqrt{4\,\frac{\gamma}{k} + (\beta^2 - 4\gamma L)} - \frac{\beta}{2\gamma}.$$

Applying $q = kv$, we have

$$q(k) = \frac{k}{2\gamma}\sqrt{4\,\frac{\gamma}{k} + (\beta^2 - 4\gamma L)} - \frac{k\beta}{2\gamma}.$$

Differentiating with respect to k, setting the result equal to zero, and wading through algebra yields the optimal values given. □

Interpretation and Uses

We can estimate q^*, k^*, and v^* from assumptions regarding car length (L), reaction time (β), and the deceleration parameter (γ). If we let $L = 10$ ft, $\beta = 1$ s, and $\gamma \approx .023$ s^2/ft (a typical value [Rothery 1992]), we obtain

$$q^* = 0.510 \text{ cars/s}, \qquad v^* = 20.85 \text{ ft/s}, \qquad k^* = 0.024 \text{ cars/ft}.$$

A less conservative estimate for γ is $\gamma = \frac{1}{2}(a_f^{-1} - a_l^{-1})$, where a_f and a_l are the average maximum decelerations of the following and lead vehicles Rothery [1992]. We assume that instead of being able to stop instantaneously (infinite deceleration capacity), the leading car has deceleration capacity twice that of the following car. Thus, instead of $\gamma = 1/2a = .023$ s^2/ft, we use the implied value for a to compute $\gamma' = \frac{1}{2}\left(a^{-1} - 2a^{-1}\right) = \frac{1}{2}\gamma = 0.0115$ s^2/ft and get

$$q^* = 0.596 \text{ cars/s}, \qquad v^* = 29.5 \text{ ft/s} \approx 20 \text{ mph}, \qquad k^* = .020 \text{ cars/ft}.$$

Going 20 mph in high-density traffic with a bumper-to-bumper separation of 40 ft is not bad.

The 1999 evacuation was far from optimal. Taking 18 h for the 120-mi trip from Charleston to Columbia implies an average speed of 7 mph and a bumper-to-bumper separation of 7 ft.

Limitations of the Steady-State Model

The steady-state model does not take into account the variance of cars' speeds. Dense traffic is especially susceptible to overcompensating or undercompensating for the movements of other drivers.

A second weakness is that the value for maximum flow gives only a first-order approximation of the minimum evacuation time. Determining maximum flow is distinct from determining minimum evacuation time.

Minimizing Evacuation Time with the Steady-State Model

Initial Considerations

The goal is to keep evacuation time to a minimum, but the evacuation route must be as safe as possible under the circumstances. How long on average it takes a driver to get to safety (Columbia) is related to minimizing total evacuation time but is not equivalent.

A General Performance Measure

A metric M that takes into account both maximizing traffic flow and minimizing individual transit time T is

$$M = W\,\frac{N}{lq} + (1 - W)\,\frac{D}{v},$$

where $0 \leq W \leq 1$ is a weight factor, D is the distance that to traverse, l is the number of lanes, and N is the number of cars to evacuate. This metric assumes that the interaction between lanes of traffic (passing) is negligible, so that total flow is that of an individual lane times the number of lanes. Given W, minimizing M amounts to solving a one-variable optimization problem in either v or k. Setting $W = 1$ corresponds to maximizing flow, as in the preceding section. Setting $W = 0$ corresponds to maximizing speed, subject to the constraint $v \leq v_{\text{cruise}}$, the preferred cruising speed; this problem has solution $M = D/v_{\text{cruise}}$. The model does not apply when cars can travel at v_{cruise}.

Setting $W = 1/2$ corresponds to minimizing the total evacuation time

$$\frac{N}{lq} + \frac{D}{v}.$$

The evacuation time is the time D/v for the first car to travel distance D plus the time N/lq for the N cars to flow by the endpoint.

To illustrate that maximizing traffic flow and maximizing speed are out of sync, we calculate the highest value of W for which minimizing M would result in an equilibrium speed of v_{cruise}. This requires a formula for the equilibrium value v^* that solves the problem

$$\text{minimize } M(v) = W\,\frac{N(L + \beta v + \gamma v^2)}{lv} + (1 - W)\,\frac{D}{v}$$

$$\text{subject to } 0 < v \leq v_{\text{cruise}}.$$

The formula for $M(v)$ comes from **(1)**, $q = kv$, and $k = 1/s$. Differentiating with respect to v, setting the result equal to zero, and solving for speed yields

$$v^* = \min\left\{v_{\text{cruise}}, \sqrt{\frac{1}{\gamma}\left[L + \frac{(1-W)}{W}\cdot\frac{Dl}{N}\right]}\right\}.$$

For v_{cruise} to equal the square root, we need

$$W = \left(1 + \frac{N}{Dl}\left(v_{\text{cruise}}^2\gamma - L\right)\right)^{-1}.$$

Using $N = 160{,}000$ cars, $D = 633{,}600$ ft (120 mi), $l = 2$ lanes, $v_{\text{cruise}} = 60$ mph $=$ 88 ft/s, $\gamma = .0115$ s^2/ft, and $L = 10$ ft, we obtain $W \approx 1/11$. Thus, minimizing evacuation time in situations involving heavy traffic flow is incompatible with allowing drivers to travel at cruise speed with a safe stopping distance.

Computing Minimum Evacuation Time

From the fact that $T = 2M$ when $W = 1/2$, we obtain

$$q^* = k^*v^*, \qquad v^* = \sqrt{\frac{1}{\gamma}\,[L + Dl/N]},$$

$$k^* = \frac{\beta\gamma^{1/2}[L + Dl/N]^{-1/2} - 2\gamma\frac{[L+\frac{1}{2}Dl/N]}{[L+Dl/N]}}{[\beta^2 - 4\gamma L] - \gamma\frac{[Dl/N]^2}{[L+Dl/N]}}.$$

The minimum evacuation time is

$$T^* = \frac{N}{lq^*} + \frac{D}{v^*}.$$

Predictions of the Steady-State Model

For N large, evacuation time minimization is essentially equivalent to the flow maximization (**Figure 1**), and it can be shown analytically that

$$\lim_{N\to\infty}\frac{T_{\text{flow}}(N)}{T_{\min}(N)} = 1.$$

The predicted evacuation time of 40 h for $N = 160{,}000$ seems reasonable. We can evaluate the impact of converting I-26 to four lanes by setting $l = 4$ in the equation for minimum evacuation time, yielding $T \approx 23$ h. For the steady-state model, this prediction makes sense, since the model does not deal with the effect of the bottleneck that will occur when Columbia is swamped by evacuees. The bottleneck would be compounded by using four lanes instead of two. On balance, however, doubling the number of lanes would lead to a net decrease in evacuation time.

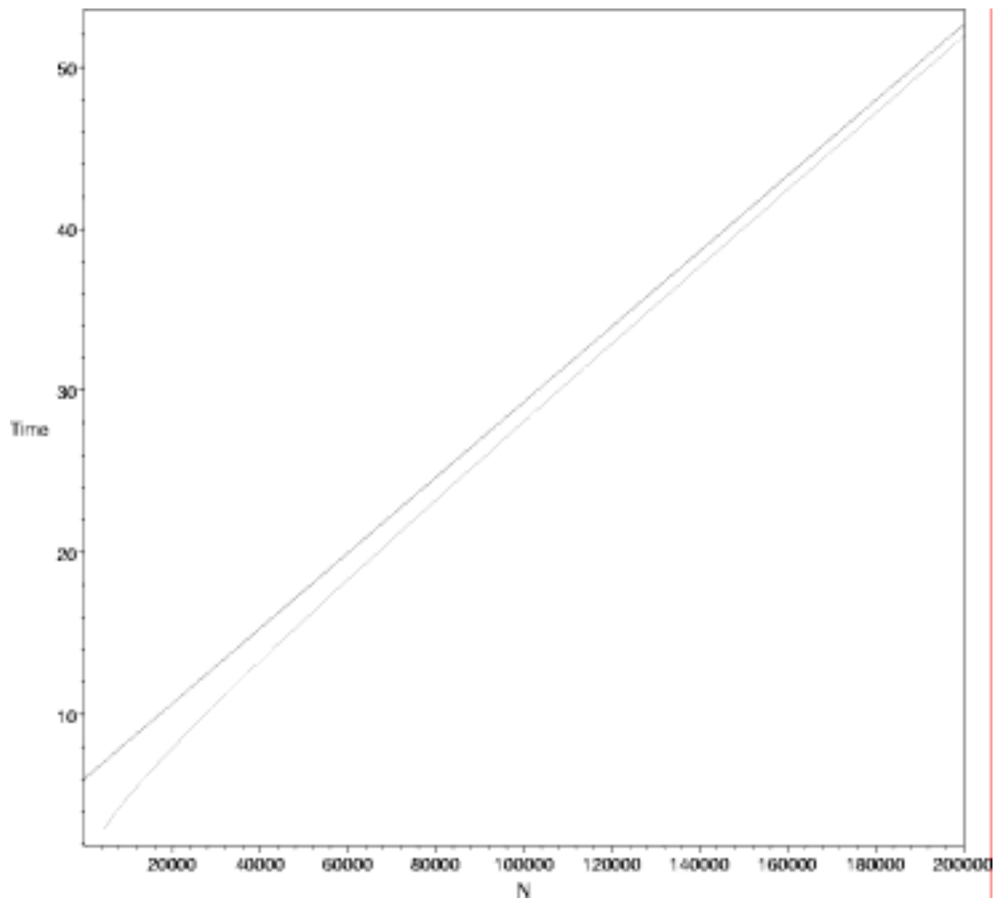

Figure 1. Comparison of minimum evacuation time (lower line) and maximum flow evacuation time (upper line).

One-Dimensional Cellular Automata Model

Development

In heavy traffic, cars make repeated stops and starts, with somewhat arbitrary timing; a good model of heavy traffic should take this randomness into account but also be simple enough to give an explicit formula for speed.

We divide a single-lane road into cells of equal length. A cell contains one car or no car. A car is *blocked* if the cell directly in front of it is occupied. At each time state, cars move according to the following rules:

- A blocked car does not move.
- If a car is not blocked, it advances to the next cell with probability p.

The decisions of drivers to move forward are made independently.

A traffic configuration can be represented by a function $f : \mathbb{Z} \to \{0, 1\}$, where $f(k) = 1$ if cell k contains a car and $f(k) = 0$ if not. Probability distributions on the set of all such functions are called *binary processes*.

Given a process X, define a process $I_p(X)$ according to the following rule:

$$\begin{array}{lr} \text{If} & \big(X(i), X(i+1)\big) = (1,0) \\ \text{then} & \big(I_p(X)(i), I_p(X)(i+1)\big) = \begin{cases} (0,1), & \text{with probability } p; \\ (1,0), & \text{with probability } 1-p. \end{cases} \end{array}$$

This rule is identical to the traffic flow rule given above: If X represents the traffic configuration at time t, $I_p(X)$ gives the traffic configuration at time $t+1$.

We are interested in what the traffic configuration looks like after several iterations of I. Let $I_p^n(X)$ mean I_p applied n times to X. The formula for traffic speed in terms of density comes from the following theorem[1]:

Theorem. *Suppose that X is a binary process of density d. Let*

$$r = \frac{1 - \sqrt{1 - 4d(1-d)p}}{2pd}$$

and let $\mathcal{M}_{p,d}$ denote the Markov chain with transition probabilities

$$0 \longrightarrow \begin{cases} 0, & \text{w/ prob. } 1-r; \\ 1, & \text{w/ prob. } 1-r; \end{cases} \qquad 1 \longrightarrow \begin{cases} 0, & \text{w/ prob. } r; \\ 1, & \text{w/ prob. } r. \end{cases}$$

The sequence of processes $X, I_p(X), I_p^2(X), I_p^3(X), \ldots$ converges to $\mathcal{M}_{p,d}$.

Here "density" means the frequency with which 1s appear, analogous to the average number of cars per cell. This theorem tells what the traffic configuration looks like after a long period of time.

Knowing the transition probabilities allows us to compute easily the average speed of the cars in $\mathcal{M}_{p,d}$: the average speed is the likelihood that a randomly chosen car is not blocked and advances to the next cell at the next time state:

$$\begin{aligned} v &= Pr\left[I(\mathcal{M}_{p,d}(i)) = 0 \,|\mathcal{M}_{p,d}(i) = 1\right] \\ &= Pr\left[\mathcal{M}_{p,d}(i+1) = 0 \,|\mathcal{M}_{p,d}(i) = 1\right] \\ &\quad \cdot Pr\left[I(\mathcal{M}_{p,d}(i)) = 0 \,|\mathcal{M}_{p,d}(i) = 1 \text{ and } \mathcal{M}_{p,d}(i+1) = 0\right] \\ &= rp = \left(\frac{1 - \sqrt{1 - 4d(1-d)p}}{2pd}\right) p = \frac{1 - \sqrt{1 - 4d(1-d)p}}{2d}. \end{aligned} \qquad \textbf{(2)}$$

The model does not accurately simulate high-speed traffic and does not take into account following distance, and the stop-and-start model of car movement is not accurate when traffic is sparse. The model is best for slow traffic (under 15 mph) with frequent stops.

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[1]This theorem is the result of previous research [Miller 2001] by an author of this paper. The **Appendix** on binary processes (written during the contest period) gives a relatively complete proof of the result. [EDITOR'S NOTE: We unfortunately must omit the **Appendix**.]

Low Speeds

We must set three parameters:

Δx = the size of one cell, the space taken up by a car in a tight traffic jam; we set it to 15 ft, slightly longer than most cars.

Δt = the length of one time interval, the shortest time by a driver to move into the space in front; we take this to be 0.5 s.

p = the movement probability, representing the proportion of drivers who move at close to the overall traffic speed; we let $p = 0.85$.

In **(2)** we insert the factor $(\Delta x/\Delta t)$ to convert from cells/time-state to ft/s:

$$v = \left(\frac{\Delta x}{\Delta t}\right) \frac{1 - \sqrt{1 - 4d(1-d)p}}{2d}.$$

Density d is in cars per cell, related to *occupancy* n by $d = (15\,\text{ft}/10\,\text{ft}) \times n = 3n/2$. **Table 1** gives v for various values of n and k.

Table 1.
v for various values of n and k.

n	K ft^{-1}	d	v (mph)
0.60	0.060	0.90	2
0.55	0.055	0.83	4
0.50	0.050	0.75	5
0.45	0.045	0.68	8
0.40	0.040	0.60	10
0.35	0.035	0.53	12
0.30	0.030	0.45	14
0.25	0.025	0.38	15
0.20	0.020	0.30	16

One Lane

To explore the one-dimensional cellular automata model, we wrote a simple simulation in C++. The simulation consists of a 5,000-element long (circular) array of bits, with a 1 representing a car and a 0 representing a (car-sized) empty space. The array is initialized randomly based on a value for the occupancy n: An element is initialized to 1 with probability n, or to 0 with probability $1 - n$. The array is iterated over 5,000 time cycles: On each cycle, a car moves forward with probability p if the square in front of it is empty. The flow q is calculated as the number of cars N passing the end of the array divided by the number of time cycles (i.e., $q = N/5000$), and thus the average speed of an individual car in cells per time cycle is $v = q/n = N/5000n$.

Table 2.
Comparison of simulation and equation values for speed.

n	Speed $p = 1/2$ Simulation	Speed $p = 1/2$ Equation	Speed $p = 3/4$ Simulation	Speed $p = 3/4$ Equation
0.2	0.433	0.438	0.694	0.697
0.4	0.356	0.349	0.592	0.589
0.6	0.234	0.232	0.392	0.392
0.8	0.108	0.110	0.171	0.174

Table 2 shows results for various values of the occupancy n and probability p, verifying the accuracy of the one-dimensional cellular automata equation **(2)**.

The value for p should be related to the mean and standard deviation of v_{cruise}. The mean and standard deviation of a binary random variable are p and $\sqrt{p(1-p)}$. We have

$$\frac{p}{\sqrt{p(1-p)}} = \frac{\mu}{\sigma} \qquad \longrightarrow \qquad p = \frac{1}{1 + \left(\frac{\sigma}{\mu}\right)^2}.$$

For $\mu(v_{\text{cruise}}) = 60$ mph and $\sigma(v_{\text{cruise}}) = 5$ mph, we get $p = 144/145$. Now, $L = \mu t p$; so assuming $L = 10$ ft, $p = 144/145$, and $\mu = 60$ mph $= 88$ ft/s, we obtain a time step of 0.113 s.

We now use the model to predict how fast (on average) a car moves in a single lane, as a function of the occupancy. We consider the "relative speed" v_{rel}, the average speed divided by the (mean) cruise speed. The average speed is given by the one-dimensional cellular automata equation, and the cruise speed is p cells per time cycle, so this gives us

$$v_{\text{rel}} = \frac{v_{\text{avg}}}{v_{\text{cruise}}} = \frac{1 - \sqrt{1 - 4n(1-n)p}}{2pn}.$$

Using $p = 144/145$, we calculate v_{rel} and v_{avg} as a function of n (**Table 3**).

The model predicts that for low occupancy the average speed will be near the cruise speed, but for occupancies greater than 0.5 the average speed will be significantly lower. The cellular automata model does not take following distance into account; thus, it tends to overestimate v_{avg} for high speeds and is most accurate when occupancy is high and speed is low.

Table 3 also shows flow rate $q = nv_{\text{avg}}/L$, in cars/s, as a function of occupancy. The flow rate is symmetric about $n = 0.5$. Each car movement can be thought of as switching a car with an empty space, so the movement of cars to the right is equivalent to the movement of holes to the left.

The model fails, however, to give a reasonable value for the maximum flow rate: 4.1 cars/s $\approx$ 14,600 cars/h, about seven times a reasonable maximum rate [Rothery 1992]. The reason is that the cell size equals the car length, a correct

Table 3.
Relative speed, average speed, and flow rate as a function of occupancy.

n	v_{rel} (ft/s)	v_{avg} (ft/s)	flow rate (cars/s)
0.1	.999	88	0.9
0.2	.998	88	1.8
0.3	.995	88	2.6
0.4	.987	87	3.5
0.5	.923	81	4.1
0.6	.658	58	3.5
0.7	.426	38	2.6
0.8	.249	22	1.8
0.9	.111	10	0.9

approximation only as car speeds approach zero and occupancy approaches 1. For $n \geq 0.5$, we should have a larger cell size; so we assume that cell size equals car length plus following distance and that following distance is proportional to speed. Assuming a 1 s following distance, we obtain cell size as

$$C = L + v_{\text{avg}} \times (1 \text{ sec}).$$

But we do not know the value of v_{avg} until we use the cell size to obtain it! For n large, we can assume that $v_{\text{avg}} \approx v_{\text{cruise}}$ and find an upper bound on cell size:

$$C = L + v_{\text{cruise}} \times (1 \text{ sec}) = 98 \text{ ft}.$$

We divide the original flow rate by the increased cell size to obtain a more reasonable flow rate:

$$q = \frac{4.063 \text{ cars/s}}{98 \text{ ft}/10 \text{ ft}} = 0.415 \text{ cars/s} \approx 1{,}500 \text{ cars/h}.$$

This is likely to be an underestimate; for greater accuracy, we must find a method to compute the correct cell size before finding the speed. We address this problem later.

Two Lanes

We expand the one-dimensional model. The simulation consists of a two-dimensional (1000×2) array of bits. The array is initialized randomly and then iterated over 1,000 time cycles: On each cycle, a car moves forward with probability p if the cell in front of it is empty. If not, provided the cells beside it and diagonally forward from it are unoccupied, with probability p the car changes lanes and moves one cell forward.

Like the one-lane simulation, the two-lane one is correct only for high densities and low speeds, since it uses cell size equal to car length. Hence, we do not use the two-lane simulation to compute the maximum flow rate. However, since cell size affects flow rate by a constant factor, we can compare flow rates by varying parameters of the simulation. In particular, we use this model to examine how the flow rate changes with the variance of speeds.

"But I Want to Bring My Boat!"

There are two main types of variance in speeds:

- σ_t^2 of traveling speed (random fluctuations in the speed of a single vehicle over time), and
- σ_m^2 of mean speed (variation in the mean speeds of all vehicles).

In the one-lane simulation, we assumed that $\sigma_m = 0$ and $\sigma_t = 5$ mph; this choice was reflected in the calculation of p, since $p = 1/[1 + (\sigma_t/\mu)^2]$ for every car. When we take σ_m into account, each car gets a different value of p:

- Choose the car's mean speed μ randomly from the normal distribution with mean v_{cruise} and standard deviation σ_m.
- The car's traveling speed will be normally distributed with mean μ and standard deviation σ_t.
- The car's transition probability p is

$$p = \frac{\mu}{v_{\text{cruise}} + \lambda\sigma_m} \left(\frac{1}{1 + \left(\frac{\sigma_t}{\mu}\right)^2} \right),$$

  where λ is a constant best determined empirically. We use $\lambda = 0$, a conservative estimate of the change in flow rate as a function of σ_m.

We consider what effect σ_t and σ_m have on the speed at a given occupancy. Considering the cars' movement as a directed random walk, increasing σ_t increases randomness in the system, causing cars to interact (and hence block one other's movement) more often, decreasing average speed.

The effects of σ_m are even more dramatic: Cars with low mean speeds impede faster cars behind them.

We ran simulations in which we fixed $n = 0.5$ and varied both σ_m and σ_t from 0 to 15 mph. For each pair of values, we calculated average flow rate for the one- and two-lane simulations.

Table 4 shows the effects of lane-changing by comparing maximum flows for the two-lane model with twice that for the one-lane model. For small σ_t and σ_m, allowing cars to switch lanes does not increase the flow rate much; for high values, the two-lane model has a much higher flow rate. Each 5 mph increase in σ_m results in an 11–16% decrease in flow rate (two-lane model, $\sigma_t = 0$), while each 5 mph increase in σ_t results in a 5–7% decrease in flow rate (two-lane model, $\sigma_m = 0$). Both variances dramatically affect flow rate, and σ_m is more significant than σ_t.

Table 4.
Two-lane average flow / Twice the one-lane average flow.

σ_m	σ_t 0	5	10	15
0	979/976	923/908	856/830	805/776
5	822/757	817/746	790/729	753/683
10	699/537	691/518	659/485	642/469
15	588/393	569/366	540/319	518/292

"So, Can I Bring My Boat?"

We consider how variations in vehicle type affect σ_m, σ_t, and flow rate. Most large vehicles (boats, campers, semis, and motor homes) travel more slowly than most cars. A significant proportion of large vehicles results in increased σ_m and hence a lower flow rate.

As a simplified approximation, we assume that there are two types of vehicles: fast cars ($\mu = \mu_1$) and slow trucks ($\mu = \mu_2$), with proportion α of slow trucks α. We calculate

$$\begin{aligned}\sigma_m^2 &= \alpha(\mu_2 - \bar{\mu})^2 + (1-\alpha)(\mu_1 - \bar{\mu})^2 \\ &= \alpha\big[\mu_2 - \big(\mu_1 - (\mu_1 - \mu_2)\alpha\big)\big]^2 + (1-\alpha)\big(\mu_1 - (\mu_1 - \mu_2)\alpha\big)^2 \\ &= (\mu_1 - \mu_2)^2\big(\alpha^2(1-\alpha) + \alpha(1-\alpha)^2\big) = (\mu_1 - \mu_2)^2\alpha(1-\alpha).\end{aligned}$$

Thus, $\sigma_m = (\mu_1 - \mu_2)\sqrt{\alpha(1-\alpha)}$. We now assume that fast cars travel at $\mu_1 = 70$ mph and slow trucks at $\mu_2 = 50$ mph and find σ_m as a function of α. Random fluctuations in vehicle speed are likely to depend more on driver psychology than on vehicle type, so we assume $\sigma_t = 5$ mph. We interpolate linearly in **Table 4** to find the flow rate (cars per 1,000 time cycles) as a function of the proportion of slow vehicles α (**Table 5**).

Table 5.
Flow rate as a function of proportion of slow vehicles.

α	σ_t	flow rate	% reduction in flow
0	0	923/908	0/0
.01	1.99	881/844	4.6/7.0
.02	2.80	864/817	6.4/10.0
.05	4.36	831/767	10.0/15.5
.1	6.00	792/700	14.1/22.9
.2	8.00	741/609	19.7/32.9
.5	10.0	691/518	25.1/43.0

The flow rate is decreased significantly by slow vehicles: If 1% of vehicles are slow, the flow rate decreases by 5%; if 10% of vehicles are slow, the flow rate decreases by 15%. The effects of σ_m are magnified if vehicles are unable to pass slower vehicles; so if the highway went down to one lane at any point (due to construction or accidents, for example), the flow rate would be reduced even further. Hence, we recommend no large vehicles (vehicles that may potentially block multiple lanes) and no slow vehicles (vehicles with a significantly lower mean cruising speed). Exceptions could be made if a family has no other vehicle and for vehicles with a large number of people (e.g., buses). Slow-moving vehicles should be required to stay in the right lane and families should be encouraged to take as few vehicles as possible.

The Space-Speed Curve

To determine optimal traffic flow rates, we can combine the one-dimensional cellular automata and the steady-state models to get a good estimate of the relationship between speed v and the separation distance s.

$s \leq 15$: There is essentially no traffic flow: $v(s) = 0$.

$15 \leq s \leq 30$: Traffic travels at between 0 and 12 mph and the one-dimensional cellular automata model applies; v is approximately a linear function of s.

$30 \leq s \leq 140$: Traffic travels between 12 and 55 mph and the steady-state model is appropriate; v is again approximately a linear function of s, with less steep slope.

$140 \leq s$: Traffic travels at the speed limit of 60 mph.

Incoming Traffic Rates

The optimal flow is determined by the optimal flow through the smallest bottleneck. However, the time of travel (which is a more important measure for our purposes) is affected by other factors, including the rate of incoming traffic. If incoming traffic is heavy, congestion occurs at the beginning of the route, decreasing speed and increasing travel time for each car.

How does congestion occur and how much does it influence travel time? Consider the one-dimensional cellular automata model with $p = 1/2$. Represent the road by the real line and let $F(x,t)$ denote the density of cars at point x on the road at time t. (For our purposes now, the cells and cars are infinitesimal in length.) Suppose that the initial configuration $F(x,0)$ is given by

$$F(x,0) = \begin{cases} 1, & \text{if } x < 0; \\ 0, & \text{if } x \geq 0. \end{cases}$$

This represents a dense line of cars about to move onto an uncongested road.

We omit units for the time being. By formulas derived earlier, the speed $v(x_0,t_0)$ at position x_0 and time t_0 is given by

$$v(x_0,t_0) = \frac{1 - \sqrt{1 - 2F(x_0,t_0)[1 - F(x_0,t_0)]}}{2F(x_0,t_0)},$$

while speed must also equal the rate at which the number of cars past point x is increasing; that is,

$$v(x_0, t_0) = \frac{d}{dt}\left(\int_x^\infty F(x,t)dx\right)(t_0).$$

Thus,

$$\frac{dF}{dt} = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{1-\sqrt{1-2F(1-F)}}{2F}\right).$$

This is a partial differential equation whose unique solution is

$$F(x,t) = \begin{cases} 1, & \text{if } x/t < -\frac{1}{2}; \\ \frac{1}{2} - \frac{(x/t)}{\sqrt{2-4(x/t)^2}}, & \text{if } -\frac{1}{2} \le x/t \le \frac{1}{2}; \\ 0, & \text{if } \frac{1}{2} < x/t. \end{cases}$$

Thus, after a steady influx of cars for a period of Δt, the resulting congestion is

$$\frac{1}{2} - \frac{\frac{x}{\Delta t}}{\sqrt{2-4\left(\frac{x}{\Delta t}\right)^2}}$$

and the congestion ends at $x = \Delta t/2$.

Thus, the extent of the congested traffic is *linear* in Δt. So if there is a steady influx of cars onto a highway, the extent of the resulting congestion is directly proportional to how long it takes them to enter, thus to the number N of them. Likewise, the time for the congestion to dissipate is proportional to N.

This allows us to evaluate staggering evacuation times for different counties. Suppose that n counties have populations $P_1, \ldots, P_n$. If all evacuate at the same time, the effect of the resulting traffic jam on total travel time is proportional to the product of the extent of the jam and the time before it dissipates:

$$\begin{aligned} \Delta T_{\text{travel time}} &= c_1 \cdot c_2 (P_1 + \cdots + P_n) \cdot c_3 (P_1 + \cdots + P_n) \\ &= c_1 c_2 c_3 (P_1 + \cdots + P_n)^2 \end{aligned}$$

for some constants c_1, c_2, c_3. If the evacuations are staggered, the effect is

$$\Delta T_{\text{travel time}} = c_1 c_2 c_3 P_1^2 + \cdots + c_1 c_2 c_3 P_n^2 = c_1 c_2 c_3 \left(P_1^2 + \cdots + P_n^2\right).$$

Now, $P_1^2 + \cdots + P_n^2 < (P_1 + \cdots + P_n)^2$; so unless one of the counties has a much larger population than the rest, the difference between these two values is relatively large. We therefore recommend staggering counties.

The Effects of Merges and Diverges

While the steady-state model is a reasonably accurate predictor of traffic behavior on long homogeneous stretches of highway, we must also consider how to deal with the effects of road inhomogeneities: merges of two lanes into a single lane and "diverges" of one lane into two lanes. To do so, we apply the principle of conservation of traffic [Kuhne and Michalopolous 1992]. Assuming that there are no sources or sinks in a region, we have

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0,$$

where q is flow rate (cars/s), k is density (cars/ft), x is location (ft), and t is time (s). Let the merge or diverge occurs at a specific point x. In the steady state (i.e., for $\partial D/\partial t = 0$), we have $\partial q/\partial x = 0$, so flow conserved at a junction.

For a flow q_s merging or diverging flows q_1 and q_2, we have $q_s = q_1 + q_2$. If proportion P ($0 < P < 1$) of the flow q_s going to (or coming from) q_1, and we know either density (k_s or k_1), we can solve for the other density:

$$q_1 = Pq_s, \qquad k_1 v_1 = P k_s v_s, \qquad k_1 v(k_1) = P k_s v(k_s).$$

From the steady-state model, for the given values of the constants, we know

$$v(k) = \begin{cases} 88 \text{ ft/s}, & 0 < k < 0.0056; \\ 21.7\left(\sqrt{.08 + \frac{.092}{k}} - 1\right), & 0.0056 < k < 0.1. \end{cases}$$

Assuming that both densities are greater than the free-travel density $k = .0056$, we can set

$$k_1\left(\sqrt{.08 + \frac{.092}{k_1}} - 1\right) = P k_s \left(\sqrt{.08 + \frac{.092}{k_s}} - 1\right).$$

Given either k_s or k_1, we can solve numerically for the other. Then we can find the speeds associated with each density using the above expression for $v(k)$.

Solving the equation gives two values; we assume that the density is greater on the single-lane side of the junction (i.e., density increases at a merge and decreases at a diverge). Also, if solving produces a speed v_1 that is larger than v_{cruise}, we set $v_1 = v_{\text{cruise}}$ and calculate $n_1 = q_1/v_1$.

How is the steady-state flow rate determined on a path with merges and diverges? Following Daganzo [1997], we consider a bottleneck to be a location (such as a merge or diverge) where queues can form and persist with free flow downstream. The *bottleneck capacity* is the maximum flow rate through the bottleneck, which (with Daganzo) we assume to be constant. If a steady-state flow greater than the bottleneck capacity attempts to enter the bottleneck, the queue size will increase until it stretches all the way back to its origin. At that point, the steady-state flow is blocked by the queue of cars and decreases to the bottleneck capacity. Hence, the maximum steady-state flow rate along a path is the minimum capacity of all bottlenecks along the path.

Parallel Paths

From A to B, let there be multiple parallel paths $p_1, \ldots, p_m$ with bottleneck capacities $c_1, \ldots, c_m$. The maximum steady-state flow rate from A to B is the minimum of:

- the bottleneck capacity of the diverge at point A,
- the bottleneck capacity of the merge at point B, and
- the sum of the bottleneck capacities of all paths p_i.

If a path has no bottlenecks, its capacity is the maximum flow rate predicted by the steady-state model.

We focus on maximizing flow rate rather than minimizing evacuation time, since for a large number of cars, maximizing flow gives near-minimal evacuation time. The maximum flow rate $q_{\max}$ from Charleston to Columbia is

$$q_{\max} = \min\left(\sum_i q_i, c_0, c_f\right),$$

where q_i is the maximum flow rate of path i, c_0 is the bottleneck capacity of Charleston, and c_f is the bottleneck capacity of Columbia. The flow rates q_i are

$$q_i = \min(b_1 \ldots b_n, q_{i,ss}),$$

where $b_1, \ldots, b_n$ are the capacities of bottlenecks along the given route and $q_{i,ss}$ is the maximum flow along that route as predicted by the steady-state model.

We first consider evacuation with no bottlenecks along I-26. Denoting the steady-state value $q_{\text{I-26},ss}$ by q_I, this gives us $q_{\max} = \min(q_I, c_0, c_f)$. Which factor limits $q_{\max}$? We cannot achieve $q_I \approx 2{,}000$ cars/h if if either $c_0 < 2{,}000$ (traffic jam in Charleston) or $c_f < 2{,}000$ (traffic jam in Columbia). With the traffic in Columbia splitting into three different roads, there should be less congestion there than with everyone merging onto I-26 in Charleston. Hence, we assume that $c_0 < c_f$, so the limiting factor is c_0 if $c_0 < 2{,}000$ and q_I if $c_0 > 2{,}000$. The value of c_0 is best determined empirically, perhaps by extrapolation from Charleston rush-hour traffic data or from data from the 1999 evacuation.

Effects of Proposed Strategies

Reversing I-26

Reversing I-26 doubles q_I to 4,000 cars/h. It is likely to increase c_0 as well, since cars can be directed to two different paths onto I-26 and are thus less likely to interfere with the merging of cars going on the other set of lanes. On the other hand, twice as many cars will enter the Columbia area simultaneously, and the unchanged capacity c_f may become the limiting factor. It may be possible to increase c_f by rerouting some of the extra traffic to avoid Columbia, or even turning around traffic on some highways leading out of Columbia.

Thus, this strategy is likely to improve evacuation traffic flow.

Reversing Other Highways

A similar argument applies to turning around the traffic on the smaller highways. Each highway adds some capacity to the total $\sum_i q_i$, increasing this term, but each highway's capacity is significantly less than q_I, and increasing the number of usable highways has unclear effects on c_0. It may increase capacity by spreading out Charleston residents to different roads, or crossing evacuation routes may lead to traffic jams. As with the reversal of I-26, reversal of smaller highways does not affect c_f (unless crossing evacuation routes becomes a problem in Columbia). More important, the interactions between highways (merges and diverges) may lead to bottlenecks, reducing capacity. In fact, interactions between these highways and I-26 could cause bottlenecks that slow the flow on I-26, perhaps offsetting the extra capacity of the smaller highways. Thus, it is safer not to turn around traffic on the secondary highways or to encourage using these as evacuation routes.

Temporary Shelters

Establishing temporary shelters in Columbia, to reduce the traffic leaving that city, could be useful if only some of the cars are directed into Columbia; thus the flow of traffic in the Columbia area would be split into four streams rather than three, possibly increasing c_f. Nevertheless, we hesitate to recommend this strategy, since the actual effects are likely to be the opposite. Evacuees entering Columbia are likely to create congestion there, making it difficult for traffic to enter, resulting in a major bottleneck. Without careful regulation, more people will try to stay in Columbia than the available housing, and frantic attempts of individuals driving around looking for housing will exacerbate the bottleneck. Hence, it is most likely that c_f will decrease significantly, probably becoming the limiting factor on maximum flow rate.

Staggering Traffic Flows

Staggering is likely to reduce the time for an average car to travel from Charleston to Columbia while leaving the value of the steady-state flow rate q_I unchanged. Hence, staggering decreases total evacuation time; it may also increase maximum flow rate, since it decreases the number of cars traveling toward I-26 at any one time, reducing the size of the c_o bottleneck. Increasing the capacity c_o, however, increases the flow rate only when c_o is the limiting factor.

Evacuees from Florida and Georgia

Since evacuation time is proportional to number of cars over flow rate, out-of-state evacuees add to total evacuation time unless they take a route that does not intersect the paths of the South Carolina evacuees. However, it is very hard to constrain the routes of out-of-state evacuees, since they come from a variety of paths and are unlikely to be informed of the state's evacuation procedures. In particular, major bottlenecks are likely at the intersections of I-26 with I-95 and of I-95 with I-20 and U.S. 501. If many cars from I-95 attempt to go northwest on I-26 toward Columbia, q_I will no longer equal 2,000 cars/h but instead the capacity of the I-26/I-95 bottleneck. This is likely to reduce q_I significantly and likely make q_{I-26} the limiting factor.

A similar argument suggests that I-95 traffic will impede the flow of traffic west from Myrtle Beach by causing a bottleneck at the I-95/I-20 junction. Traffic flow from Myrtle Beach is less than from Charleston, and many of the cars from I-95 may have already exited at I-26; so the bottleneck at the I-20 junction is likely to be less severe. Nevertheless, the flow of evacuees from Florida and Georgia has the potential to reduce dramatically the success of the evacuation.

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Modeling Hurricane Evacuation Strategies

DURHAM, NC, FEB. 12— Hurricanes pose a serious threat to citizens on the South Carolina coastline, as well as other beach dwellers in Florida, Georgia, and other neighboring states. In 1999, the evacuation effort preceding the expected landfall of Hurricane Floyd led to a monumental traffic jam that posed other, also serious, problems to the more than 500,000 commuters who fled the coastline and headed for the safe haven of Columbia. Several strategies have been proposed to avoid a future repeat of this traffic disaster.

First, it has been suggested that the two coastal bound lanes of I-26 be turned into two lanes of Columbia bound traffic. A second strategy would involve staggering the evacuation of the coastal counties over some time period consistent with how hurricanes affect the coast, instead of all at once. Third, the state might turn around traffic flow on several of the smaller highways besides I-26 that extend inland from the coast. The fourth strategy under consideration is a plan to establish more temporary shelters in Columbia. Finally, the state is considering placing restrictions on the type and number of vehicles that can be brought to the coast.

In the interest of the public, we have developed and tested several mathematical models of traffic flow to determine the efficacy of each proposal. On balance, they suggest that the first strategy is sound and should be implemented. Although doubling the number of lanes will not necessarily cut the evacuation time in half, or even double the flow rate on I-26 away from the coast, it will significantly improve the evacuation time under almost any weather conditions.

Our models suggest that staggering the evacuation of different counties is also a good idea. Taking such action on the one hand will reduce the severity of the bottleneck that occurs when the masse of evacuees reaches Columbia, and on the other hand could potentially increase average traffic speed without significantly increasing traffic density. The net effect of implementing this strategy will likely be an overall decrease in coastal evacuation time.

The next strategy, which suggests turning traffic around on several smaller highways, is not so easy to recommend. The main reason for this is that the unorganized evacuation attempts of many people on frequently intersecting secondary roads is a recipe for inefficiency. In places where these roads intersect I-26, the merging of a heightened volume of secondary road traffic is sure to cause bottlenecks on the interstate that could significantly impede flow. To make a strategy of turning around traffic on secondary roads workable, the state would have to use only roads that have a high capacity, at least two lanes, and a low potential for traffic conflicts with other highways. This would require competent traffic management directed at avoiding bottlenecks and moving Charleston traffic to Columbia with as few evacuation route conflicts as possible.

The fourth proposal, of establishing more temporary shelters in Columbia, is a poor idea. Because it is assumed that travelers are relatively safe once they reach Columbia, the main objective of the evacuation effort should be minimizing the transit time to Columbia and the surrounding area. It is fairly clear that increasing the number of temporary shelters in Columbia would lead to an increased volume of traffic to the city (by raising expectations that there will be free beds there) and exacerbate the traffic problem in the city itself (due to an increased demand for parking). Together, these two factors are sure to worsen the bottleneck caused by I-26 traffic entering Columbia and would probably increase the total evacuation time by decreasing the traffic flow on the interstate.

The final proposal of placing limitations on the number and types of vehicles that can be brought to the beach is reasonable. Families with several cars should be discouraged from bringing all of their vehicles and perhaps required to register with the state if the latter is their intention. Large, cumbersome vehicles such as motor homes should be discouraged unless they are a family's only option. Although buses slow down traffic, they are beneficial because they appreciably decrease the overall number of drivers. In all cases, slow-moving vehicles should be required to travel in the right lane during the evacuation.

In addition to the strategies mentioned above, commuters in the 1999 evacuation were acutely aware of the effect on traffic flow produced by coastal residents of Georgia and Florida traveling up I-95. We have concluded that, when high-volume traffic flows such as these compete for the same traffic pipeline, the nearly inevitable result is a bottleneck. A reasonable solution to this problem would be to bar I-95 traffic from merging onto I-26 and instead encourage and assist drivers on I-95 to use the more prominent, inland bound secondary roads connected to that interstate.

To conclude, we think that combining the more successful strategies suggested could lead to a substantial reduction in evacuation time, the primary measure of evacuation success. Minimizing the number of accidents that occur en route is also important, but our models directed at the former goal do not make compromises with the latter objective. In fact, the problem of minimizing accidents is chiefly taken care of by ensuring that traffic flow is as orderly and efficient as possible.

— Samuel W. Malone, Carl A. Miller, and Daniel B. Neill in Durham, NC

The Crowd Before the Storm

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Introduction

Applying safety regulations and flow-density equations, we find the maximum rate of flow through a lane of road is 1,500 cars/h, occurring when cars travel at 27.6 mph.

We construct a computer simulation that tracks the exit of cars through South Carolina's evacuation network. We attempt to optimize the network by reversing opposing lanes on various roads and altering the time that each city should begin evacuating, using a modified genetic algorithm.

The best solution—the one that evacuates the most people in 24 h—involves reversing all the opposing lanes on evacuation routes. Increasing the holding capacity of Columbia is only marginally helpful. Georgia and Florida traffic on I-95 is only mildly detrimental, but allowing people to take their boats and campers greatly decreases the number of people that can be evacuated.

Background on Evacuation Plans

After the 1999 evacuation, the South Carolina Department of Transportation (SCDOT) designated evacuation routes for all major coastal areas, including 14 different ways to leave the coast from 32 regions. The routes take evacuees past I-95 and I-20. Although officers direct traffic at intersections, traffic on roads not in in the plan may have long waits to get onto roads in the plan. Moreover, the South Carolina Emergency Preparedness Division (SCEPD) does not call

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for any traffic type limitations (i.e., all campers, RVs, and cars with boats are allowed) [South Carolina Department of Public Safety 1999].

Assumptions

Assumptions About Hurricanes

- There is exactly one hurricane on the East Coast of the United States at the time of the evacuation.
- The hurricane, like Floyd, moves along the South Carolina coast. Most Atlantic hurricanes that reach the United States follow a northeasterly path along the coast [Vaccaro 2000].

Assumptions About Traffic Flow

- All cities act as points. The smaller streets within a city do not affect flow in and out of a city.
- The capacity of a city is the sum of its hotel rooms and the number of cars that can fit on the city's roads.
- The flow between intersections is constant.
- Density of traffic between intersections is constant.
- Charlotte and Gastonia in North Carolina; Spartanburg, Greenville, and Anderson in South Carolina; and Augusta, Georgia are infinite drains, meaning that we do not route people beyond them. Flow out of these cities should not create traffic jams. The cities are also large and therefore should be able to accommodate most if not all incoming evacuees.
- After the order to evacuate is issued, vehicles immediately fill the roads.
- Traffic regulators attempt to maintain the ideal density, using South Carolina's GIS system.
- All motorized vehicles are 16 ft long. This takes into account the percentage of motorcycles, compact cars, sedans, trucks, boats, and RVs and their lengths.
- On average, three people travel in one vehicle.
- The traffic that enters I-95 from Georgia or Florida stays on I-95 and travels through South Carolina.

Assumptions About People

- All people on the coast follow evacuation regulations immediately.
- Drivers obey the speed limit and keep a safe following distance.

Flow-Density Relationship

Flow is the number of vehicles passing a point on the road per unit time. The flow q on a road depends on the velocity v and density k of vehicles on the road:

$$q = kv. \tag{1}$$

Empirical studies suggest that velocity and density are related by [Jayakrishnan et al. 1996]:

$$v = u_f \left(1 - \frac{k}{k_j}\right)^a, \tag{2}$$

where k_j is the density of a road in a traffic jam, a is a parameter dependent on the road and vehicle conditions, and u_f, free velocity, is speed at which a vehicle would travel if their were no other vehicles on the road. Generally, the free velocity is the speed limit of the road.

We substitute **(2)** into **(1)** to obtain flow as a function of density:

$$q = ku_f \left(1 - \frac{k}{k_j}\right)^a. \tag{3}$$

This equation is linear in the free velocity. To find the ideal density that produces the fastest flow, we take the first derivative of the flow with respect to density and set it equal to zero:

$$u_f \left(1 - \frac{k}{k_j}\right)^a - au_f \frac{k}{k_j}\left(1 - \frac{k}{k_j}\right)^{a-1} = 0.$$

Solving for k, we find the ideal density k_i:

$$k_i = \frac{k_j}{a+1}.$$

Assuming that all roads behave similarly, we find a numerical value for the ideal density. Jam density is generally between 185 and 250 vehicles/mile [Haynie 2000]; we use the average value of 218 vehicles/mile. By fitting **(2)** to Kockelman's flow-density data for various cars, road conditions, and driver types in Kockelman [1998], we find that a has an average value of 3 (**Figure 1**). Therefore, the ideal density is 54 vehicles per mile.

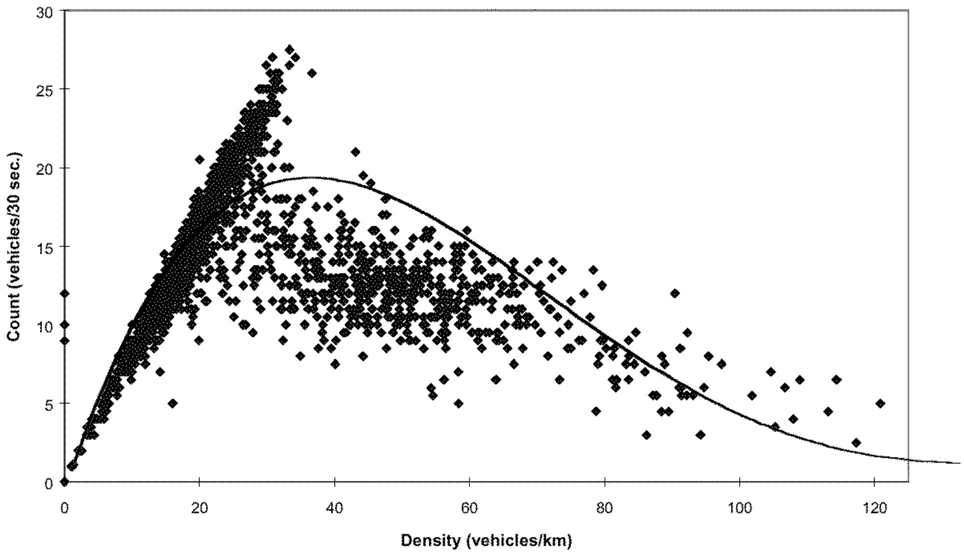

Figure 1. Plot of observed counts vs. density. Data from Kockelman [1998] with our curve of fit of the form **(2)**.

To account for reaction time, vehicles must be spaced at least 2 s apart [NJDOT 1999]. For vehicles spaced exactly 2 s apart, we can find the density of a road where all vehicles are traveling at speed v. The distance d_c required by a vehicle traveling at speed v is the sum of the vehicle's length and its following distance:

$$d_c = l + \frac{2v}{3600},$$

where the units are miles and hours. The maximum safe density of a road is the maximum number of vehicles on the road (the length of the road divided by the space required for each vehicle) divided by the length of the road:

$$k = \frac{1}{l + \frac{2v}{3600}}.$$

If each car is 16 ft (3.03×10^{-3} mi) long and the density is ideal, then the maximum safe velocity of the vehicles is 27.6 mph.

Knowing the ideal density and the maximum safe velocity at that density, we use **(1)** to find the maximum flow:

$$q = kv = 1500. \qquad \textbf{(4)}$$

The free velocity parameter is needed to find the flow at situations other than ideal. Using **(2)**, we find that the free velocity is 65.2 mph—close to the highway speed limit, thus validating the approach for finding the free velocity. Substituting the known and derived values of free velocity, jam density, and the exponential parameter into **(3)**, we quantify the flow-density relationship:

$$q = 62.5k\left(1 - \frac{k}{218}\right)^3.$$

Traffic Flow Model

Mapping the Region

We programmed in Java a simplified map of South Carolina that consists of 107 junctions (cities) and 154 roads. A junction is an intersection point between two or more roads. A road connects exactly two junctions. Our map includes most of the roads in by SCEPD's model and many more. Data for the number of cars, boats, and campers in each city used in the computer model can be found in the **Appendix**. [EDITOR'S NOTE: We omit the **Appendix**.]

Behavior of Cities

Each point in the program stores a city's population and regulates traffic flow into and out of the roads connected to it. First, it flows cars out of the city into each road. The desired flow out—the maximum number of vehicles that the road can take—is defined by the flow-density equation **(4)** for the road that the cars are entering. If the total number of vehicles that can be exited exceeds the evacuee population of the city, then the evacuees are distributed proportionally among the roads with respect to the size of the road.

Next, the city lets vehicles in. The roads always try to flow into the city at the ideal flow rate. The city counts the total number of cars being sent to it and compares this to its current evacuee capacity. If the evacuee capacity is less than the number of vehicles trying to enter, the city accepts a proportion of the vehicles wanting to enter, depending on the road size. A check in the program ensures that the number of vehicles taken from the road does not exceed the number of cars on the road at that time.

After repeating the entering and exiting steps for each road, the city recalculates its current evacuee population, removing all the vehicles that left and adding all the vehicles that entered.

Behavior of Roads

We define each road by its origin junction, destination junction, length, and number of lanes. The number of lanes is the number of lanes in a certain direction under nonemergency circumstances. For example, a road that normally has one lane north and one lane south is considered a one-lane highway. If the number of lanes on a road changes between cities, we use the smaller number of lanes. To analyze the possibility of turning both lanes to go only north or only south, our program would double the number of lanes.

During an evacuation, traffic never needs to flow in both directions, because the net flow of a road that flowed equally in two directions would be zero. Therefore, each road has a direction defined by its origin junction and destination junction. While the origin junction is normally the point closer to the coast, the program analyzes the possibility of having the road flow from its "destination" to "origin." In some cases, this could provide the optimal flow out of the coastal areas by finding alternative routes.

We model traffic congestion as a funnel. As long as vehicles are on the road, they attempt to exit at the ideal flow rate. However, if the road begins to fill, then the number of vehicles entering the road varies depending on the flow-density equation **(4)**. We determine the new density of the road via

$$D_i = \frac{n_i + \Delta n_i}{d},$$

where n_i is the initial number of vehicles on the road, Δn_i is the difference between the cars entered and cars exited, and d is the length of the road (mi).

Optimization Algorithm

We use a modified genetic optimization algorithm, beginning from South Carolina's current solution into the evacuation. The simulation stores a possible solution as two chromosomes: a city chromosome, storing the time to start evacuating coastal cities, and a road chromosome, storing directions and reversals of the roads. Stored with the solution is the number of people left in the evacuation zone after 24 h, the usual advance notice for evacuation.

The simulation randomly chooses a chromosome and a gene to mutate. We use a uniform distribution to choose first the chromosome, then the gene, and finally a value for that gene. City genes can take the value of any time step between 0 and 12 h before starting to evacuate. Road direction can be in either the specified direction, the reversed direction, or closed. Opposing lanes can be either reversed or not reversed. If the changed chromosome leaves fewer people in the evacuation region in 24 h, it replaces the old chromosomes.

Results

Knowing that the maximum flow of a one-lane road is 1,500 cars per hour, we first tested South Carolina's current evacuation route with our modified flow equations, which allow for more people to leave the evacuation region. After 24 h, 556,000 people (58% of the people needing to leave) were still left in the cities needing evacuation. If only I-26 was reversed, the people left dropped to 476,000 (50%). However, if people were allowed to take their boats and campers with them, the number left was 619,000. Therefore, boats, campers, and extra cars generally should not be allowed to evacuate.

After 10,000 iterations, the solutions were still improving. Therefore, we restarted the program with all roads beginning with lanes reversed and cities evacuating immediately. After 10,000 iterations, the program could not find a better solution than one in which 233,000 people (25%) are left.

If I-95 is too congested due to traffic from Georgia and South Carolina (i.e., I-95 is not used in the simulation), the number of people left is 254,000, 9.4% more than if the highway had been clear.

Increasing Columbia's evacuee capacity helps the evacuation only marginally, removing only 948 more people from danger.

Regardless of the situation, the solutions always have cities that start evacuating immediately: Staggered solutions are not optimal.

Stability Analysis

We tested the flow equations in the evacuation simulation by varying the exponential parameter, the free velocity, and the jam density. When the exponential parameter was increased or decreased by 1, the number of people evacuated changed by only 0.2% and 1.0%. Doubling or halving the free velocity caused variations of 3.4%. Doubling and halving the jam density caused variations of 2.2%. The length of the car did not affect the number of people emptied from the city because the following distance was so large compared to the length of the car. Finally, we doubled and halved the iterative time step of the evacuation simulation, which did not change number of people evacuated. Thus, this model was robust in every variable tested.

Strengths and Weaknesses

Strengths

- The model can be used for any evacuation. For, example, if a meteor were predicted to hit the Atlantic Ocean and flood a strip of land 50 mi wide along the Atlantic Coast, this model could be used to evacuate residents of South Carolina to areas not affected by the flood.

- Moreover, the program is flexible enough to work for any possible map; it is not specific to the individual roads and cities of South Carolina.

- The model is stable with regards to all variables tested, and the optimization algorithm runs very quickly.

Weaknesses

- The greatest weakness of this model is that it assumes that people will follow directions: use the two-second following distance rule, travel at the speed limit, and travel on assigned roads.

- The model assumes density homogeneity along each road after each iteration, while in reality the density varies.

- The model can handle only situations where the roads are empty at the beginning of the simulation.
- We underestimated the holding capacity of the cities, leading to a slower exit from the unsafe regions. Thus, although the relative changes in the results are probably correct, the actual number of people in danger after 24 hours is probably fewer.
- The optimization algorithm can get stuck in local minima.

Conclusion

Reversing traffic on all evacuation routes evacuates the most people. Traffic from Georgia and Florida is not a problem, but many boats and campers would significantly decrease flow.

Hence, we suggest that roads be reversed and that to maintain maximum flow, traffic regulators not allow the number of cars in a stretch of road to exceed 54.4 per lane per mile, by regulating on- and off-ramps at cities.

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The Crowd Before the Storm: Improved Hurricane Evacuation Routes Planned

COLUMBIA, SC, FEB. 12— A new mathematical model prepared for the South Carolina Emergency Preparedness Division should end aggravating and dangerous backups when a hurricane threatens South Carolina. In response to the monstrous traffic jams that turned I-26 northbound out of Charleston into a parking lot during the Hurricane Floyd evacuation, the new model distributes traffic over several smaller routes.

One of the new program's controversial traffic-control methods is to make *all* lanes of traffic on evacuation routes to run in the same direction, away from the beach, to improve traffic flow.

People with campers, RVs, boats, and more than one car should leave as early as possible if a hurricane is predicted; once the new plan is implemented, families may be permitted only one car, to reduce traffic.

The model counteracts these minor inconveniences by evacuating 75% of the population at risk within 24 hours, compared to 42% under South Carolina's current plan.

Although more people may be evacuated, don't expect to get out of the region too quickly. The model predicts that the fastest evacuation will occur if all cars travel at 28 mph. At that rate, it will take you 4 hours to get out of the evacuation region.

The model used a "genetic algorithm" approach, which involves testing possible solutions against each other and "breeding" new ones from the best ones so far. Further analysis showed that reversing lanes of all roads along the evacuation routes is the best method for quick evacuation. The computer tested 10,000 minor changes without finding a more effective solution.

Increasing the housing and parking capacity of Columbia by constructing a shelter there would be only mildly helpful. On a more positive note, residents do not need to worry about traffic from Georgia and Florida slowing the evacuation.

— Jonathan David Charlesworth, Finale Pankaj Doshi, and Joseph Edgar Gonzalez, in Richmond, Virginia.

Jammin' with Floyd: A Traffic Flow Analysis of South Carolina Hurricane Evacuation

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Introduction

We analyze the 1999 Hurricane Floyd evacuation with a traffic-flow model, explaining the extreme congestion on I-26. Then we look at the new South Carolina Hurricane Evacuation Plan, which includes lane reversals. We analyze their effect; they would significantly benefit traffic leaving Charleston. With lane reversals, the maximum number of vehicles passing any point on I-26 is 6,000 cars/h.

We develop two plans to evacuate the South Carolina coast: the first by geographic location, the second by license-plate parity.

We explore the use of temporary shelters; we find that I-26 has sufficient capacity for oversized vehicles; and we determine the effects of evacuees from Georgia and Florida.

Traffic Flow Model

The following definitions and model are taken directly from Mannering and Kilareski [1990, 168–182].

The primary dependent variable is level of service (LOS), or amount of congestion, of a roadway. There are six different LOS conditions, A through F, with A being the least congested and F being the most congested. We focus on the distinction between levels E and F.

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- *Level of Service E* represents operating conditions at or near capacity level. All speeds are reduced to a low but relatively uniform value, normally between 30 and 46 mph.

- *Level of Service F* is used to define forced or breakdown flow, with speeds of less than 30 mph. This condition exists wherever the amount of traffic approaching a point exceeds the amount that can traverse that point. Queues form behind such locations.

If we enter LOS F, the roadway has exceeded its capacity and the usefulness of the evacuation has broken down. An evacuation strategy that results in a highway reaching LOS F is unacceptable.

For a given highway, we can determine the maximum number of vehicles that can flow through a particular section while maintaining a desired level of service. To make this more concrete, we define the characteristic quantity *maximum service flow*.

Definition. *Maximum Service Flow* (MSF_i) for a given level of service i, assuming ideal roadway conditions, is the maximum possible rate of flow for a peak 15-min period, expanded to an hourly volume and expressed in passenger cars per hour per lane (pcphpl). To calculate the MSF of a highway for a given LOS, we multiply the road's capacity under ideal conditions by the volume-to-capacity ratio for the desired LOS. More formally,

$$\mathrm{MSF}_i = c_j \, \frac{v}{c}_i, \tag{1}$$

where c_j is the capacity under ideal conditions for a freeway with Design Speed j, and $(v/c)_i$ is the maximum volume-to-capacity ratio associated with LOS i. For highways with 60- and 70-mph design speeds, c_j is 2,000 pcphpl [Transportation Research Board 1985]. Since LOS E is considered to be "at capacity," $(v/c)_{\mathrm{E}} = 1.0$. The design speed of a road is based mostly on the importance and grade of the road; roads that are major and have shallower grades have higher design speeds. The elevation profile along I-26 shows that South Carolina is flat enough to warrant the highest design speed.

An immediate consequence of **(1)** is that to maintain $\mathrm{MSF}_{\mathrm{E}}$ or better (which we consider necessary for a successful evacuation), the number of passenger cars per hour per lane must not exceed 2,000 for any highway.

For it to be useful in model calculations, we need to convert the maximum service flow to a quantity that conveys information about a particular roadway. This quantity is known as the service flow rate of a roadway.

Definition. The *service flow rate* for level of service i, denoted SF_i, is the actual maximal flow that can be achieved given a roadway and its unique set of prevailing conditions. The service flow rate is calculated as

$$\mathrm{SF}_i = \mathrm{MSF}_i N f_w f_{\mathrm{HV}} f_p, \tag{2}$$

in terms of the adjustment factors:

N: the number of lanes,

f_w: the adjustment for nonideal lane widths and lateral clearances,

f_{HV}: effect of nonpassenger vehicles, and

f_p: the adjustment for nonideal driver populations.

We assume that the lanes on I-26 and other highways are ideal (i.e., $f_w = 1$): at least 12 ft wide with obstructions at least 6 ft from traveled pavement [Mannering and Kilareski 1990]. To account for driver unfamiliarity with reversed lanes and stress of evacuation, we set $f_p = 0.7$ for reversed lanes and $f_p = 0.8$ for normal lanes, in accordance with Mannering and Kilareski [1990]. The model also employs an adjustment factor, denoted f_{HV}, for reduction of flow due to heavy vehicles such as trucks, buses, RVs, and trailers. Later we discuss the effects of heavy vehicles on traffic flow.

Strengths and Weaknesses

This model is easy to implement, the mathematics behind it is quite simple, and it is backed by the National Transportation Board. We establish its reliability by using it to predict traffic flow patterns in the 1999 evacuation.

We assume that the number of lanes does not change, which requires that there are no lane restrictions throughout the length of the freeway and no lanes are added or taken away by construction.

The major weakness of our model is that it fails to take into account the erratic behavior of people under the strain of a natural disaster.

The simplicity of our model also limits its usefulness. It can be applied only to normal highway situations, not to a network of roads.

Improving Evacuation Flow

Gathering data from a various sources, we estimate the number of vehicles used in the 1999 evacuation. According to Dow and Cutter [2000], 65% of households that were surveyed chose to evacuate. About 70% of households used one vehicle or fewer, leaving 30% of households taking two vehicles. Of the evacuees, 25% used I-26 during the evacuation. Based on population estimates [County Population Estimates ... 1999] and average number of people per household [Estimates of Housing Units ... 1998], and assuming a relatively uniform distribution of people per household, we calculate the number of vehicles used during the evacuation (**Table 1**).

Table 1.
Evacuation participation estimates for Hurricane Floyd, in thousands.

	Population	Evacuees	Evacuating Households	Vehicles	Vehicles on I-26
Southern	187	122	47	61	—
Central	553	359	139	181	—
Northern	233	152	59	76	—
Total	973	632	245	319	61

Reversing Lanes

According to our model, the capacity of a highway is directly proportional to the number of lanes. This implies that lane reversal would nearly double the capacity of I-26.

Approximately 319,000 vehicles were used to evacuate the coastal counties of South Carolina. Of evacuees surveyed by Dow and Cutter [2000], 16.3% evacuated between noon and 3 P.M. on Sept. 14. Assuming independence between the above factors, in the hours between 9 A.M. and noon, I-26 must have been clogged by an attempted influx of about 3,300 vehicles/h. Even if evenly distributed, this was more than the 3,200 vehicles/h that the two Columbia-bound lanes of I-26 could take under evacuation conditions. The result was LOS F—a large traffic jam. Our model predicts that this jam would have lingered for hours, even after the influx of vehicles had died down.

What if the coastal-bound lanes of I-26 were reversed? With corrections for nonideal conditions, our model predicts an $\mathrm{SF_E}$ of 6,000 pcphpl. Therefore, reversing the lanes of I-26 has the potential to increase service flow rate by a factor of 1.6.

Simultaneous Evacuation Strategies

By Hurricane Path

Hurricanes sweep from south to north. Because a hurricane commonly travels at a speed of less than 30 mph, the southernmost counties of South Carolina would be affected at least two hours before the northernmost ones.

However, analysis indicates that a staggered evacuation strategy would not improve the speed of the evacuation. The evacuation routes are largely parallel to one another and rarely intersect. Thus, the evacuation of each county should affect only the traffic on evacuation routes of nearby counties. Therefore, postponing evacuation of counties farther from the hurricane would be counterproductive.

By County

What about avoiding simultaneous evacuation of adjacent counties? We recommend evacuating Jasper, Beaufort, Charleston, Georgetown, and Horry counties in the first wave, and leaving Hampton, Colleton, Dorchester, and Berkeley until 3–6 h later, depending on the time of day. This solution would decrease the probability of traffic reaching LOS F on any highway without significantly delaying the evacuation. The nearby state of Virginia has a similar plan for evacuating county by county [Virginia Hurricane . . . 1991].

By License Plate Number

By dividing cars into two categories, depending on the parity of the last digit on their license plate, we could separate traffic into two waves without giving preference to residents of any county. Our solution would request that the even group evacuate 3–6 h after the odd group was given the evacuation order. This would spread out the hours of peak evacuation traffic, resulting in improved traffic conditions and decreased risk of LOS F being reached. A comparison of **Figures 1** and **2** demonstrates the change in time distribution of evacuation when half of the drivers evacuate six hours later. Clearly, the distribution is much smoother, reducing the likelihood of reaching LOS F.

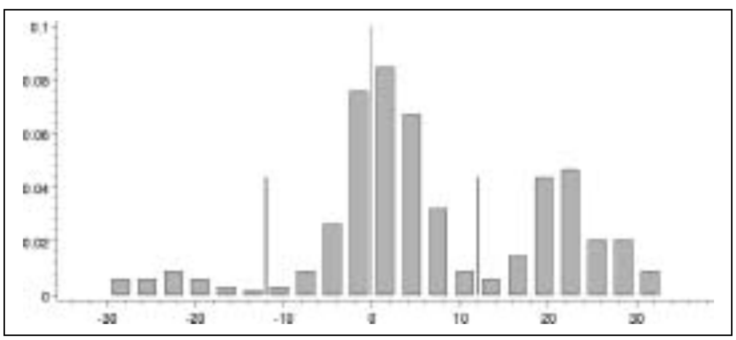

Figure 1. Hurricane Floyd: Fraction of evacuating population vs. hours after the 1999 mandatory evacuation order. (Data from Dow and Cutter [2000].)

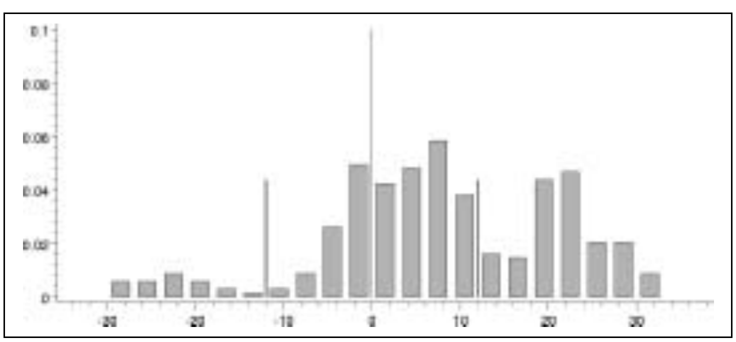

Figure 2. Even/odd license plate plan: Our projected fraction of the evacuating population vs. hours after the mandatory evacuation order.

Lane Reversal on Smaller Highways

As only 29% of evacuees took I-26 or I-95, the majority took smaller roads inland. Because our model results in linear growth of flow with number of available lanes, lane reversals should improve evacuation rates on all roads. Because the evacuation routes are nearly perpendicular to the coastline, there is little risk of opposing traffic being disrupted by these reversals. The logistics of such an action, however, might be prohibitive.

The number of personnel needed to facilitate the I-26 lane reversal is 206 [South Carolina Emergency Preparedness Division 2000]. Smaller highways have less distance between exits, which suggests that more personnel per mile would be needed to blockade highway entrances. The total length of highway on all evacuation routes is approximately ten times as great as the length of I-26. Therefore, a truly prodigious amount of human effort would be necessary to implement lane reversals on all evacuation routes.

It would be imprudent to spend resources for what we estimate to be only a marginal gain in actual highway use. According to Dow and Cutter [2000], these alternative routes did not even approach capacity during the last evacuation. Since our license-plate evacuation strategy increases the overall throughput of major evacuation routes through lane reversal and smoother time distribution, there is no reason to expect a heavier load on state and country roads.

So, there is little evidence to support the utility of lane reversal on all smaller roads. Still, reversing lanes on a small number of evacuation routes might prove useful. The three major population centers of the coast (Beaufort, Charleston, and Horry counties) have different evacuee distributions. Therefore, those highways which most merit reversal are I-26, I-501 from Myrtle Beach to Marion and 301 from Marion to Florence, and the southern corridor from Beaufort County to the Augusta area.

Effect of Additional Temporary Shelters

In 1999, South Carolina housed about 325,000 people in shelters [Dow and Cutter 2000]. In a hurricane, one-third of evacuees go to each of shelters, family and friends, and commercial establishments [Zelinski and Kosinski 1991, 39–44]. According to South Carolina Hurricane Information [2001], the number of predesignated shelters in Columbia is insignificant. However, there must be an efficient way to funnel the evacuees to the evacuation sites, such as a central coordination center with an up-to-date list of where the next group of cars should go.

Vehicle Type Restrictions

Although our model generally calculates flow using only normal passenger cars, it is not difficult to take other types of vehicles into account. The equation used to calculate the heavy-vehicle adjustment factor is

$$f_{\mathrm{HV}} = \frac{1}{1 + 0.6P}, \tag{3}$$

where P is the proportion of nonpassenger vehicles (RVs, trailers, and boats).

Using this equation, our model predicts an upper bound on the proportion of nonpassenger vehicles that occur without causing LOS F. We demonstrate this with a sample calculation using I-26. Earlier, we estimated the SF_{E} of I-26, including reversed lanes, as 6,000 pcphpl. We also estimated that a maximum of 3,300 vehicles/h would enter I-26, ignoring the possibility of spikes in activity. Therefore, the minimum safe value of f_{HV} is approximately 0.55, which means that I-26 has enough leeway to support any mix of passenger cars and heavy vehicles. There is no need to restrict large vehicles on I-26.

Georgians, Floridians, and the I-95 Corridor

According to Georgia's hurricane evacuation plan [Hurricane Evacuation Routes 2001], I-95 is not a valid evacuation route. However, thousands of Floridians and Georgians flocked north on I-95 during Hurricane Floyd. In Savannah, the most popular evacuation route was I-16, which goes directly away from South Carolina [Officials deserve high marks ... 1999]. In South Carolina, as shown in **Figures 1** and **2**, the farther away the destination, the smaller the percentage of the evacuee population that plans to go there. Taking all this into account, a realistic upper bound for the percentage of Georgians or Floridians using I-95 is 20%.

Any population entering South Carolina on I-95 from Georgia or Florida is mostly bound for major cities; an upper bound on the traffic headed through Columbia would be 75%. Since Floyd's landfall was extraordinarily unpredictable, we propose that it was one of the largest evacuations that will affect South Carolina.

Our reasoning is as follows: Hurricanes of lesser strength have fewer evacuees. If the landfall of the hurricane is more southerly, there is less need to evacuate South Carolina and North Carolina and so there will be less traffic on the I-26. Lastly, if the hurricane tends more towards the north, the number of evacuee drivers from Georgia and Florida taing I-95 will be decreased greatly. So, we can take Floyd as a relative upper bound on evacuees.

From CNN's coverage of the lead up to Hurricane Floyd's landfall, in Georgia, we know that "the evacuation orders affected 500,000 people." We bound this rough estimate by 600,000. So the upper bound of people using I-95 can be estimated as $(0.20)(0.75)(600{,}000) \approx 90{,}000$, or about 45,000 vehicles, spread over a two-day period. From Dow and Cutter [2000], we know that about 10% of South Carolina evacuees used I-95, so the Georgians and Floridians effectively doubled the traffic on I-95, which is a huge impact on the model that we have proposed.

Improvements in the Model

Our model needs additional evacuation data. With precise statistics regarding number of evacuees, routes taken, time distributions, and traffic conditions, we could apply it to a greater variety of situations.

Additional refinements might be made to the parameters of the model with information on the highways themselves. The lane widths and distances to roadside obstacles affect the service flow rate, and knowing the exact layout of the highways would enable us to take them into account.

We could also use information regarding the resources available to the state: how many personnel and vehicles would be available to run lane reversals.

With sufficient information, we could use this model to create a simulation of a hurricane evacuation. We would treat the highways of South Carolina as edges in a network flow problem and run a discrete computer simulation to test our premises and conclusions regarding evacuation policies.

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Maurice Knocks on Door, No One Home

COLUMBIA, S.C. (AP)— Hurricane season has come with a fury here to South Carolina, where Hurricane Maurice, the 13th named storm of the season, bears down on Charleston this evening. A record 80% of the population has been evacuated through the new evacuation plan.

When Hurricane Floyd narrowly missed South Carolina in 1999, the lack of preparedness for an evacuation of such a magnitude was highly evident. The state government asked the Research All Day Corporation (RAD Corp.) to come up with a new evacuation plan that would help the coastal residents escape the ferocity of a similar storm.

The RAD Corporation's team of world-class hurricane experts and top-notch traffic engineers analyzed the situation and developed a new evacuation plan. "The basic idea of the plan stems from simple math," explained Dr. K. Esner, Director of RAD Modeling. "Two sets of two lanes almost doubles the evacuation rate."

When asked to explain further, Dr. Esner continued, "On I-26, where there was a colossal traffic jam in 1999, we decided to reverse the flow of the coastal-bound lanes at the first decision of a mandatory evacuation." In this way, people leaving Charleston, the most populous city in South Carolina, could take either the normal two lanes of I-26 or the two "contra-flow" (reversed) lanes of I-26 all the way to Columbia.

The evacuees in the many Columbia shelters seemed in good spirits. There was much less traffic-related annoyance than was felt in 1999. John C. Lately, a British resident of Myrtle Beach, joked, "You realize that in England, driving on the left is commonplace; I felt right at home."

There was nothing but praise for the RAD engineers. "A remarkable difference was seen between the chaos of evacuating for hurricane Floyd in 1999 and the evacuation today," said Joseph P. Riley, Jr., mayor of Charleston. "This time, the drive between Charleston and Columbia took 4 hours instead of 18. And it's a good thing, too; this time the storm didn't miss."

Another feature of the new evacuation plan was the breakup of the evacuating public into two groups. "One of our concerns about the 1999 Floyd evacuation was the volume of cars all trying to access the emergency roads at the same time," explained Dr. Esner. He continued, "To alleviate the traffic volume pressure, we wanted to divide the population into two groups. We had two different proposals; we could break up the population geographically or basically divide the population right down the middle, using even/odd license plate numbers."

With the proposed RAD plan, people with even license plates left in the first group, right when the evacuation order was given, and people with odd license plates or vanity plates left starting 6 hours later. "I thought the plan was crazy," remarked Charles Orange, a 24-year-old Charleston resident. "They told us to evacuate by license plate number; you'd never think high school math would help you one day, but this is one time it did!" he exclaimed.

Using the 1999 data, the RAD researchers calculated that breaking the evacuating population into two equal groups and delaying one group by 6 hours led to a condition where the volume of cars at no time exceeded the maximum volume that the road could handle. In this way, there was no problem with traffic jams, and Charleston became a ghost town, safe for Maurice to make its appearance.

The majority of the people left on the beaches are surfers and media, but even they are sparse in number; all that remains is a hurricane without an audience.

Hurricane Maurice could not be reached for comment.

— Christopher Hanusa, Ari Nieh, Matthew Schnaider, in Claremont, Calif.

Blowin' in the Wind

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Introduction

We present a model to determine the optimal evacuation plan for coastal South Carolina in the event of a large hurricane. The model simulates the flow of traffic on major roads. We explored several possible evacuation plans, comparing the time each requires.

Traffic flow can be significantly improved by reversing the eastbound lanes of I-26 from Charleston to Columbia. By closing the interchange between I-26 and I-95 and restricting access to I-26 at Charleston, we can reduce the overall evacuation time from an original 31 h to 13 h.

However, a staggered evacuation plan, which evacuates the coastline county by county, does not improve the evacuation time, since traffic from each coastal population center interferes little with traffic flowing from other areas being evacuated. Although reversing traffic on other highways could slightly improve traffic flow, it would be impractical. Restrictions on the number and types of vehicles could speed up the evacuation but would likely cause more problems than improvements.

Theory of Traffic Flow

We require a model that simulates traffic flow on a large scale rather than individual car movement. We take formulas to model traffic flow from Beltrami [1998]. Although traffic is not evenly distributed along a segment of road, it can be modeled as if it were when large segments of road are being considered. We can measure the traffic density of a section of road in cars/mi. The traffic

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speed u at a point on the road can be calculated from the density according to the formula

$$u(r) = u_m \left(1 - \frac{\rho}{\rho_m}\right),$$

where ρ is the traffic density, u_m is the maximum speed of any car on the road, and ρ_m is the maximum traffic density (with no space between cars). We define the *flow* of traffic at a point on the road as the number of cars passing that point in a unit of time. The flow q can be easily calculated as

$$q(\rho) = \rho u.$$

It is the flow of traffic that we desire to optimize, since greater flow results in a greater volume of traffic moving along a road.

Assumptions

- During an evacuation, there is an average of 3 people per car. This is reasonable, since people evacuate with their entire families, and the average household in South Carolina has 2.7 people, according to the 1990 census.
- The average length of a car on the road is about 16 ft.
- In a traffic jam, there is an average of 1 ft of space between cars.
- The two above assumptions lead to a maximum traffic density of

$$\frac{5280 \text{ ft/mile}}{17 \text{ ft/car}} = 310 \text{ cars/mile/lane}.$$

- The maximum speed is 60 mph on a 4-lane divided highway, 50 mph on a 2-lane undivided country road.
- Vehicles follow natural human tendencies in choosing directions at intersections, such as preferring larger highways and direct routes.
- The traffic flow of evacuees from Florida and Georgia on I-95 is a continuous stream inward to South Carolina.
- When vehicles leave the area of the model, they are considered safely evacuated and no longer need to be tracked.
- There will not be traffic backups on the interstates at the points at which they leave the area of the model.
- A maximum of 30 cars/min can enter or exit a 1-mi stretch of road in a populated area, by means of ramps or other access roads. Up to the maximum exit rate, all cars desiring to exit a highway successfully exit.

- The weather does not affect traffic speeds. The justifications are:
  - During the early part of the evacuation, when the hurricane is far from the coast, there is no weather to interfere with traffic flowing at the maximum speed possible.
  - During the later part of the evacuation, when the hurricane is approaching the coast, traffic flows sufficiently slowly that storm weather would not further reduce the speed of traffic.
- There is sufficient personnel available for any reasonable tasks.

Objective Statement

We measure the success of an evacuation plan by its ability to evacuate all lives from the endangered areas to safe areas between announcement of mandatory evacuation and landfall of the hurricane; the best evacuation plan takes the shortest time.

Model Design

The Traffic Simulator

Our traffic simulator is based on the formulas above. Both space and time are discretized, so that the roads are divided into 1-mi segments and time is divided into 1-min intervals. Vehicles enter roads at on-ramps in populated areas, leave them by off-ramps, and travel through intersections to other roads.

Each 1-mi road segment has a density (the number of cars on that segment), a speed (mph), and a flow (the maximum number of cars that move to the next 1-mile segment in 1 min). Each complete road section has a theoretical maximum density ρ_m and a practical maximum density ρ'_m (accounting for 1 ft of space between cars), which can never be exceeded.

Moving Traffic Along a Single Road

The flow for each road segment is calculated as

$$q(\rho) = \frac{\rho u}{u_m}.$$

If the following road segment is unable to accommodate this many cars, the flow is the maximum number of cars that can move to the next segment.

Moving Traffic Through Intersections

When traffic reaches the end of a section of road and arrives at an intersection, it must be divided among the exits of the intersection. For each intersection, we make assumptions about percentages of cars taking each direction, based on the known road network, the capacities of the roads, and natural human tendencies. If a road ends at an intersection with no roads leading out (i.e., the state border), there is assumed to be no traffic backup; traffic flow simply continues at the highest rate possible, and the simulation keeps track of the number of cars that have left the model.

Conflicts occur when more cars attempt to enter a road section at an intersection than that road section can accommodate. Consider a section of road that begins at an intersection. Let:

$q_{\max} = \rho'_m - \rho$ = the maximum influx of cars the road can accommodate at the intersection,

$q_1, \ldots, q_n$ = the flows of cars entering the road at an intersection, and

$q_{\text{in}} = \sum q_i$ = the total flow of cars attempting to enter the road at the intersection.

If $q_{\text{in}} > q_{\max}$, then we adjust the flow of cars entering the road from its entrance roads as follows:

$$q'_i = \frac{q_i}{q_{in}}\, q_{\max}.$$

Therefore, q'_i is the number of cars entering the road from road i. The flow of traffic allowed in from each road is distributed according to the flow trying to enter from each road. Clearly, $\sum q'_i = q_{\max}$.

Simulating Populated Areas

A section of road that passes through a populated area has cars enter and leave by ramps or other access roads. We assume that the maximum flow of traffic for an access ramp is 30 cars/min. We estimate the actual number of cars entering and leaving each road segment based on the population of the area.

Cars cannot enter a road if its maximum density has been reached. For simplicity, however, we assume that cars desiring to exit always can, up to the maximum flow of 30 cars/min per exit ramp.

We desire to know how the population of each populated area changes during the evacuation, so that we can determine the time required. Therefore, we keep track of the population in the areas being evacuated, Columbia, and certain other cities in South Carolina. If all people have been evacuated from an area, no more enter the road system from that area.

Areas do not have to be evacuated immediately when the simulation starts. Each area may be assigned an evacuation delay, during which normal traffic is simulated. Once the delay has passed, traffic in the area assumes its evacuation behavior.

Completing an Evacuation

The six coastal counties of South Carolina (where Charleston includes the entire Charleston area) and the roads leading inland from these areas must be evacuated. When the population of these areas reaches zero, and the average traffic density along the roads is less than 5 cars/mi, the evacuation is complete and the simulation terminates.

Implementing the Model

We implemented the model described above in a computer program written in C++. The logic for the main function is as follows: For each road, we let traffic exit, resolve traffic at intersections, move traffic along the rest of the road, and finally let cars enter the road. We loop until the evacuation is complete.

Traffic flow is considered simultaneous; the traffic flow along every road is determined before traffic densities are updated. However, exits occur first and entrances last, to accurately simulate traffic at access ramps.

Model Results

Simulating the 1999 Evacuation

To simulate the evacuation of 1999, we prepared a simplified map that includes the interstates, other the 4-lane divided highways, and some 2-lane undivided roads. We simulated the evacuation of the coastal counties—Beaufort, Jasper, Colleton, Georgetown, and Horry (including Myrtle Beach)—and the Charleston metro area. The inland areas we considered are Columbia, Spartanburg, Greenville, Augusta, Florence, and Sumter. In addition, we simulated large amounts of traffic from farther south entering I-95 N from the Savannah area. A map of the entire simulation is shown in **Figure 1**.

The results of running this simulation with conditions similar to those of the actual evacuation produced an evacuation time of 31 h to get everyone farther inland than I-95. This is significantly greater than the actual evacuation time and completely unacceptable. The increase in time can be explained by two features of the actual evacuation that are missing in the simulation:

- Only 64% of the population of Charleston left when the mandatory evacuation was announced [Cutter and Dow 2000; Cutter et al. 2000]; our model assumes that everyone leaves.

- Late in the day, the eastbound lanes of I-26 were reversed, eliminating the congestion.

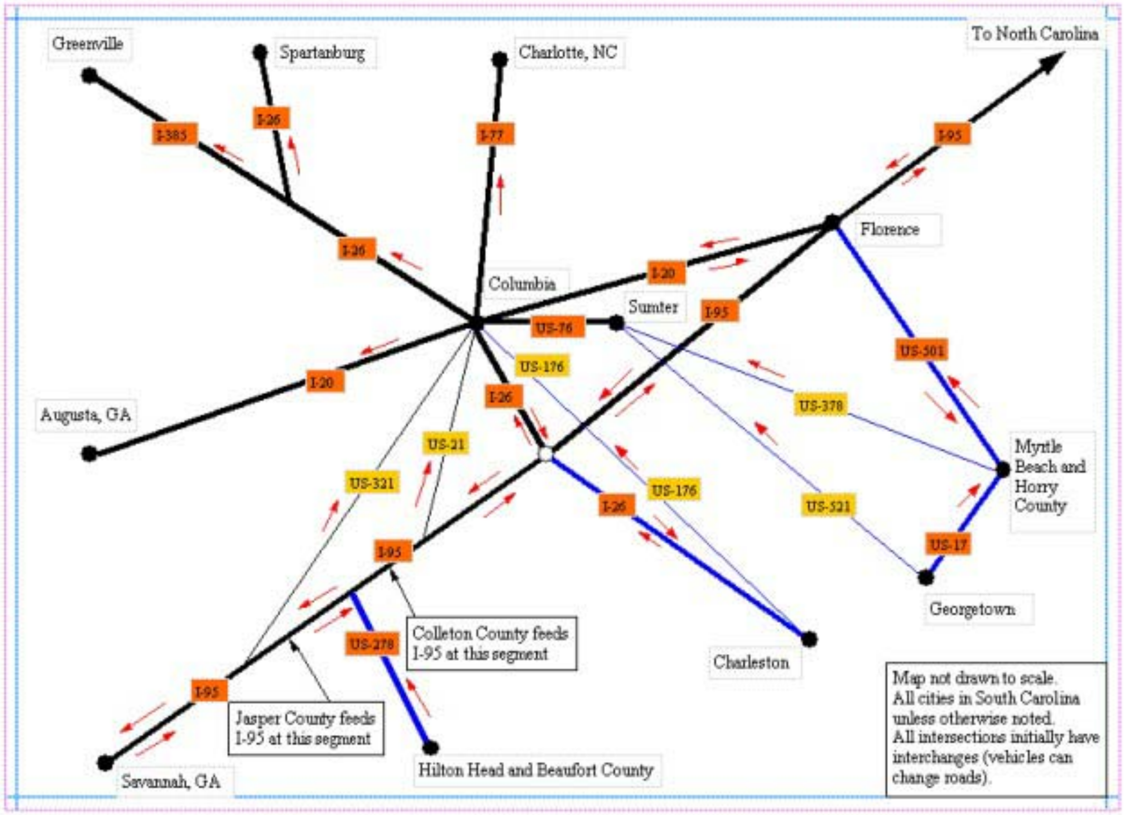

Figure 1. Map of the simulation.

Simulating Reversal of I-26

In this simulation, I-26 E was turned into a second 2-lane highway leading from Charleston to Columbia. The evacuation time was reduced to 19 h. Under all conditions tested, reversing traffic on the eastbound lanes of I-26 significantly reduces evacuation time.

Simulating a Staggered Evacuation

A staggered evacuation of the coastal counties of South Carolina, going from south to north with 1 h delays, decreases the time for evacuation to 15.5 h—2.5 longer than the best time (described below). This is because the second-slowest county to evacuate, Horry County, is the northernmost and the last to evacuate. An analysis of the evacuation routes used reveals why there is no improvement: The roads for the large counties do not intersect until they reach Columbia. Given that the evacuation of Charleston County takes 13 h, the evacuations of the other large counties (Horry and Beaufort) would need to be advanced or delayed at least this much to have any effect.

Reversing Other Highways

Reversing traffic on smaller highways might improve traffic flow, but this is not a practical option. None of the roads besides I-26 is a controlled-access road; therefore, it is impossible to ensure that the traffic entering the reversed lanes would all move in the desired direction. A single vehicle entering and attempting to travel in the undesired direction would cause a massive jam.

The possible minor highways to Columbia that could be reversed are U.S. highways 321, 176, 521, 378, 501, and 21. All are non-controlled-access roads, meaning that there are no restrictions on where vehicles may exit or enter. Together, they have 450 mi of roadway. A quick examination of U.S. 501, the highest-capacity of these, reveals two intersections per mile with other roads. Considering this as typical, there are 900 intersections outside of towns that would need to be blocked. Factoring in the no fewer than 60 towns along the way, the blocking becomes prohibitive.

Therefore, reversal of minor highways leading inland is not feasible. The only road that can be feasibly reversed is I-26.

Adding Temporary Shelters to Columbia

According to our simulation, the population of the Columbia area after the evacuation (in the best-case scenario) was 1,147,000, a massive number above the 516,000 permanent residents. If more temporary shelters were established in Columbia, there would be less traffic leaving the city and therefore more congestion within the city. This would reduce the rate at which traffic could enter Columbia and lead to extra traffic problems on the highways leading into it. The effect of this congestion is beyond our computer simulation.

We investigated buildings for sheltering evacuees. Using `smartpages.com` to search for schools, hotels, and churches in the Columbia area, we found the numbers of buildings given in **Table 1**. We assumed an average capacity for each type of building. According to the table, Columbia can shelter 1,058,251; this leaves a deficit of 89,000.

Table 1.
Post-evacuation sheltering in the two counties (Richland and Lexington) that Columbia occupies.

Type	Buildings in Richland	in Lexington	Total	People sheltered Per building	Number
Permanent residents					516,251
Schools—general *	83	113	196	900	176,400
Hotels/motels	80	32	112	500	56,000
Churches	568	386	954	250	238,500
Schools—other**	63	16	79	900	71,100
				Total	1,058,251

*We assume that schools average 600 students and can shelter 900.
**Includes academies but excludes beauty schools, trade schools, driving schools, etc.

However, Charlotte NC had only a very small increase in population due to evacuation (from 396,000 to 411,000). The people that Columbia cannot shelter can easily find shelter in Charlotte.

Restricting Vehicle Types and Vehicle Numbers

Restrictions on numbers and types of vehicles would indeed increase the speed of the evacuation. However, there are no reliable ways to enforce such restrictions. Consider the following arguments:

- Forbidding camper vehicles may be unsuccessful, since for a sizable fraction of tourists the camper is their only vehicle.
- Restricting the number of vehicles to one per family:
  - The record-keeping involved would be prohibitive.
  - For some families, more than one vehicle is needed to carry all of the family members.

The I-95 Traffic Problem

We assume that if the interchange is not closed, at least 75% of the people coming up from Florida and Georgia on I-95 will take I-26 to Columbia. This is because the next major city reachable from I-95 is Raleigh, 150 mi further on. In our simulation, not closing this intersection (but keeping the eastbound lanes of I-26 reversed) increases the evacuation time to 19 h.

The Best Simulated Evacuation Plan

By altering various model parameters, we reduced the overall evacuation time to 13 h:

- Reverse the eastbound lanes of I-26.
- Close the exit on I-95 N leading to I-26 W.
- Limit the flow of traffic from Charleston to I-26 W.

The third item is necessary to reduce congestion along I-26 in the Charleston area. If too many cars are allowed on, the speed of traffic in Charleston drops significantly. Although this unlimited access results in a greater average speed on the section of I-26 between Charleston and the I-95 interchange, the slowdown in the Charleston area is exactly the type of backup that caused complaints in 1999 and resulted in a greater total time to evacuate the city.

Conclusions

It is possible to evacuate coastal South Carolina in 13 h. Assuming that a hurricane watch is issued 36 h prior to landfall, the state can allow an ample delay between voluntary evacuation announcement and a subsequent mandatory order. However, state agencies must take considerable action to ensure that the evacuation will go as planned:

- Close the interchange between I-26 and I-95. Traffic on I-26 must remain on I-26; traffic on I-95 must remain on I-95.
- The two eastbound lanes of I-26 must be reversed immediately upon the mandatory evacuation order.
- In Charleston, restrict entrance to I-26 to 15 cars / min at each entrance ramp.

Everyone in the areas to evacuate must be notified. Within South Carolina, the existing Emergency Alert System includes many radio stations that can inform the public of the incoming hurricane, the steps to take during evacuation, and which roads to use.

Residents must be more convinced to evacuate than they were during Hurricane Floyd. Appropriate measures must be taken to ensure that residents evacuate and evacuate far enough inland.

Model Strengths and Weaknesses

Strengths

The model's predictions have a number of features found in a real evacuation or other high-density traffic flow:

- An initial congested area around the entrance ramps gives way to a high-flow area when there is no entering traffic.
- Overall traffic speed in high-flow areas is around 35 mph.
- Merging traffic causes a major decrease in flow.

Weaknesses

The model does not take into account

- **city streets**, which are important in moving people from the highways to shelter in Columbia.
- **accidents**. A single accident or breakdown could result in several hours of delay. Tow trucks should be stationed at regular intervals along major roads.

- **local traffic on the non-controlled-access highways**, which would slow traffic on those roads.

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Students Develop Optimal Coastal Evacuation Plan

SOUTHFIELD, MICH., FEB. 12—During September 13–15, 1999, Hurricane Floyd threatened landfall along the coast of South Carolina. In response to weather advisories and a mandatory evacuation order from the governor, hundreds of thousands of people simultaneously attempted to evacuate the coastal regions including Charleston and Myrtle Beach, causing unprecedented traffic jams along major highways. Although the evacuation was successful in that no lives were lost (largely since Floyd did not have as great an impact in the expected area), the evacuation was a failure in that it was not executed quickly nor completely enough to ensure the safety and well-being of all evacuating citizens had Hurricane Floyd made landfall in the Charleston area.

Since that problematic evacuation in 1999, state officials have been working on plans for a safe, efficient evacuation of the South Carolina coast, preparing for the event that a hurricane like Floyd threatens the coast again. They posed the problem to teams of mathematicians all over the country.

After working for four days, a group of talented students evolved a specific plan to safely and quickly evacuate every coastal county in South Carolina (nearly 1 million people) within 13 hours, using a computer simulation of their own design. The plan involves the reversal of the two coastal-bound lanes on Interstate 26 (the main east-west highway), as well as traffic control and detours throughout the major roads heading inland.

The students' plan guides the mass traffic flow to areas the students felt were capable of sheltering large numbers of evacuees. The main destination was Columbia, the capital and largest inland city in South Carolina. Other destinations were Spartanburg, Florence, Sumter, and Greenville in South Carolina; Augusta in Georgia; and Charlotte in North Carolina. The plan also accounted for the possibility of very heavy traffic coming northward from Georgia and Florida on I-95, fleeing from the same hurricane, which could adversely affect the evacuation in South Carolina.

Additionally, the students set forth plans to shelter the more than 1 million people who would be in Columbia after the evacuation is complete. By making use of all the city's schools, hotels, motels, and churches as shelters, nearly all the evacuees could be sheltered. The few remaining evacuees could easily find shelter north in Charlotte, which in 1999 received few evacuees.

— Mark Wagner, Kenneth Kopp, and William E. Kolasa in Southfield, Mich.

Please Move Quickly and Quietly to the Nearest Freeway

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Introduction

We construct a model that expresses total evacuation time for a given route as a function of car density, route length, and number of cars on the route. When supplied values for the last two variables, this function can be minimized to give an optimal car density at which to evacuate.

We use this model to compare strategies and find that any evacuation plan must first and foremost develop a method of staggering traffic flow to create a constant and moderate car density. This greatly decreases the evacuation time as well as congestion problems.

If an even speedier evacuation is necessary, we found that making I-26 one-way would be effective. Making other routes one-way or placing limits on the type or number of cars prove to be unnecessary or ineffective.

We also conclude that other traffic on I-95 would have a negligible impact on the evacuation time, and that shelters built in Columbia would improve evacuation time only if backups were forming on the highways leading away from the city.

Prologue

As Locke asserted [1690], power is bestowed upon the government by the will of the people, namely to protect their property. A government that cannot provide this, such as South Carolina during an act of God as threatening as Hurricane Floyd and his super-friends, is in serious danger of revolutionary

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overthrow by the stranded masses marching from the highways to the capitol. Therefore, South Carolina must find the most effective evacuation program—one that not only provides for the safety of its citizens but also allows for households to rescue as many of their vehicles (read: property) as possible. Pitted against the wrath of God and Nature, one can only hope the power of mathematical modeling can protect the stability of South Carolinian bureaucracy.

Since the goal is to create a useful model that even an elected official can use, our model operates most effectively on the idea that government agencies are poor at higher-level math but good at number-crunching. Our model provides a clear, concise formula for weighing relative total evacuation times and likely individual trip times. This is crucial in deciding how to order a wide-area evacuation while maintaining public approval of the operation and preventing a coup d'état.

Our model shows that of the four strategies for evacuation suggested by the problem statement, staggering evacuation orders is always the most effective choice rather than simply reversing I-26. After that, applying any one of the other options, like lane reversal on I-26 and / or on secondary evacuation routes, can improve the evacuation plan. However, using more than one of these techniques results in a predicted average driver speed in excess of the state speed limit of 70 mph.

Of the three additional methods, we find that the most effective is to make I-26 one-way during peak evacuation times. Implementing the same plan on secondary highways would require excessive manpower from law enforcement officials, and regulating the passenger capacity is too difficult a venture in a critical situation.

We also find that, given the simplifications of the model, I-95 should have a negligible effect.

Furthermore, the construction of shelters in Columbia would facilitate the evacuation only if the highways leading away from Columbia were causing backups in the city.

Analysis of the Problem

To explain the massive slowdowns on I-26 during the 1999 evacuation, our team theorized that substantially high vehicle density causes the average speed of traffic to decrease drastically. Our principal goal is to minimize evacuation times for the entire area by maximizing highway throughput, that is, the highest speed at which the highest density of traffic can travel. Given this fact, we seek to find the relationships between speed, car density, and total evacuation time.

Assumptions

To restrict our model, we assume that all evacuation travel uses designated evacuation highways.

We assume that traffic patterns are smoothly and evenly distributed and that drivers drive as safely as possible. There are no accidents or erratically driving "weavers" in our scenario. This is perhaps our weakest assumption, since this is clearly not the case in reality, but it is one that we felt was necessary to keep our model simple.

Our model also requires that when unhindered by obstacles, drivers travel at the maximum legal speed. Many drivers exceed the speed limit; however, we do not have the information to model accurately the effects of unsafe driving speeds, and a plan designed for the government should avoid encouraging speeding.

As suggested by the problem, we simplify the actual distribution of population across the region placing 500,000 in Charleston, 200,000 in Myrtle Beach, and an even distribution of the remaining approximately 250,000 people.

Multiple-lane highways and highway interchanges are likely to be more complicated than our approximation, but we simplify these aspects so that our model will be clear and simple enough to be implemented by the government.

Distribution of traffic among the interstate and secondary highways in our model behaves according to the results of a survey, which indicates that 20% of evacuees chose to use I-26 for some part of their trip.

The Model

We begin by modeling the traffic of I-26 from Charleston to Columbia, as we believe that understanding I-26 is the key to solving the traffic problems.

We derived two key formulas, the first $s(\rho)$ describing speed as a function of car density and the second $e(\rho)$ describing the total evacuation time as a function of car density:

$$s(\rho) = \sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}, \qquad e(\rho) = \frac{L\rho + N}{\rho s(\rho)}.$$

The constants are:

k = braking constant,

l = average length of cars in feet, and

b = buffer zone in feet.

The variables are:

ρ = car density in cars/mi,

L = length of highway in feet, and

N = number of cars to be evacuated.

The method is to maximize $e(\rho)$ for a given N and L, which gives us an optimal car density.

Derivation of the Model

The massive number of variables associated with modeling traffic on a micro basis leads to a very complex and difficult problem. Ideally, one could consider such factors as a driver's experience, his or her psychological profile and current mood, the condition of the mode of transportation, whether his or her favorite Beach Boys song was currently playing on the radio, etc. Then one could use a supercomputer to model the behavior of several hundred thousand individuals interacting on one of our nation's vast interstate highways. Instead, our model analyzes traffic on a macro basis.

The greater the concentration of cars, the slower the speed at which the individuals can safely drive. What dictates the concentration of cars? Well, the concentration is clearly related to the distance between cars, since the greater the distance, the smaller the concentration, and vice versa. On any interstate highway, drivers allot a certain safe traveling distance between their car and the car directly in front of them, to allow time to react. Higher speeds require the same reaction time but consequently a greater safe traveling distance. How do we determine what the correct distance between cars at a given speed? The braking distance d of a car is proportional to the square of that car's speed v. That is, $d = kv^2$ for some constant k. The value for k is 0.0136049 ft-hr^{2}mi^2; we derive this value by fitting $\ln d = \ln k + b \ln v$ with data from Dean et al. [2001]; the fit has $r^2 = .99999996$.

However, the distance between cars is an awkward measurement to use. Our goal is to model traffic flow. With our model, we manipulate the traffic flow until we find its optimal value. The distance between cars is hard to control, but other values, such as the concentration or density of the cars in a given space, are much easier to control.

How do we find the value of the car density? To start, any distance can be subdivided into the space occupied by cars and the space between cars. The space occupied by cars can be assumed to be a multiple of the average car length l. The space between cars is clearly related to the braking distance, but the two are not necessarily the same. The braking distance at low speeds (< 10 mph) is less than a foot. However, ordinary experience reveals that even at standstill traffic, the distance between cars is still much greater than a foot; drivers still leave a buffer zone in addition to the safe breaking distance. Then each car has a space associated with it, given by $d + l + b$, where b is the average buffer zone in feet. Since this expression is of the form of 1 car per unit distance, this is in itself a density. We can also convert this to more useful units, such as cars per mile:

$$\rho = \frac{\text{cars}}{\text{mi}} = \frac{\text{cars}}{\text{ft}} \times \frac{5280 \text{ ft}}{mi} = \frac{5280}{d+b+l} \times \frac{\text{cars}}{\text{mi}} = \frac{5280}{kv^2+b+l} \frac{\text{cars}}{\text{mi}}.$$

Solving for v gives

$$s(\rho) = v = \sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}.$$

At this point we can substitute $k = 0.0136049$ ft $\cdot$ h^2/mi^2, $l = 17$ ft (from researching sizes of cars), and $b = 10$ ft (from our personal experience) and graph speed as a function of density (**Figure 1**).

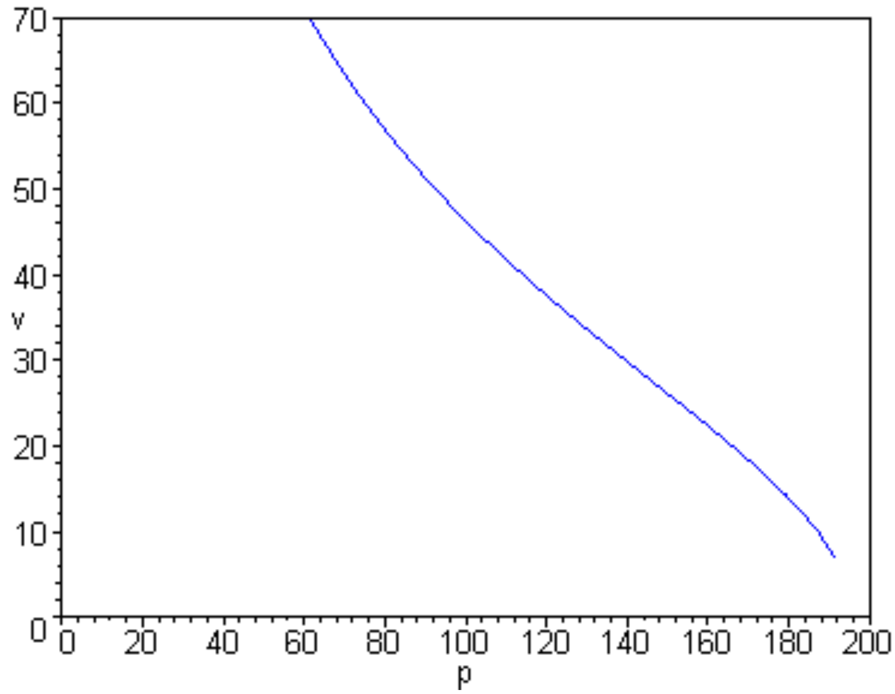

Figure 1. Speed as a function of density.

Note the maximum density at 195.6 cars/mi. To understand why a maximum density exists, consider the case $b = 0$; as $v \to 0$, the distance between the cars approaches zero.

We now determine how long it takes this group of cars to reach their destination. For now, we say that the group reaches its goal whenever the first car arrives. This is a simple calculation: We divide the length of the road L by the average speed of the group

$$t(\rho) = \frac{L}{s(\rho)} = \frac{L}{\sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}}.$$

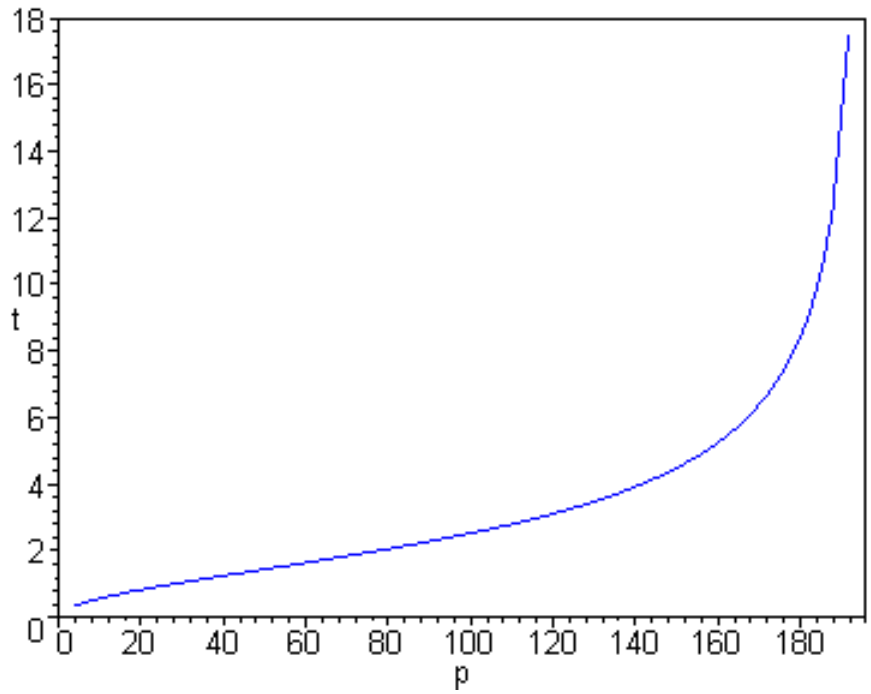

Figure 2. Evacuation time as a function of density.

We refer to this function as t because it gives the time of the trip. **Figure 2** gives t for I-26 between Charleston and Columbia, which has a length of 117 mi.

However, the real problem is not simply evacuating one group of cars, it is quickly evacuating a large number of cars. Our goal poses an interesting dilemma: If we evacuate in a stream of low density, they will travel very fast but evacuation takes an extremely long time. If we evacuate in a stream of very high car density (which is what happened during Hurricane Floyd), many people will move at once but they will move very slowly. We must seek the middle ground.

We express the total evacuation time as a function of car density; then taking the minimum gives the optimum car density.

Unfortunately, the concept of average car density doesn't work as well over large distances. The problem with traffic flow is that it tends to clump, creating high car densities and thus low speeds. However, the instantaneous car density won't vary much for a small distance, such as a mile, compared to the average car density over the entire highway. To look ahead to the problem: Making I-26 one-way will certainly help facilitate evacuation, but it won't help nearly as much as staggering the evacuation flow. Staggering is the only way to realistically create a constant traffic flow and thus an average car density that is more or less constant over the entire length of the highway.

Suppose that we take the N cars that need to be evacuated and subdivide them into groups, each consisting of the number of cars that there are in 1 mi. Call these groups *packets*. Each packet is 1 mi long.

We look at two cases, one where we send only one group and another where we send more than one group. For the first case, we assume that all N cars fit in one packet, so $0 < N < 196$, where 196 is the maximum car density for our values of buffer zone and average car length. Everyone is not technically evacuated until the end of the packet arrives safely, so we need to add to $t(\rho)$ the additional time for the end of the packet to arrive:

$$e(\rho) = t(\rho) + \frac{1}{s(\rho)}.$$

We call this expression e because it express total evacuation time as a function of car density.

For the second case, we can say that the packets travel like a train, as you can't release another packet until the first packet is a mile away. Evacuation time is the sum of the time for the first packet to arrive plus the time until the last packet arrives. Since the packets arrive in order, and they are all one mile long, the time it takes the last packet to arrive is equal to the number of packets times the time it takes the packets to move 1 mi. The equation is

$$e(\rho) = t(\rho) + \frac{1}{s(\rho)}\frac{N}{\rho}.$$

For one packet, this second equation simplifies to the first.

After some algebra, we arrive at

$$e(\rho) = \frac{L\rho + N}{\rho s(\rho)}.$$

The total evacuation time e is simply a function of three variables: the length of the highway L, the number of cars N, and the car density ρ. The speed $s(\rho)$ is itself a function of ρ and three other constants (the braking constant k, the average car length l, and the buffer zone b). Setting $L = 117$ and $N = 65{,}000$ produces the graph in **Figure 3**, which has a clear minimum.

Application of the Model

The model should first deal with I-26. We assume that evacuation along this road takes the longest. We limit our consideration to Charleston, Columbia, and I-26 between them.

Modeling I-26 Traffic Flow

The problem states that the evacuation consists of 500,000 residents from Charleston, 200,000 from Myrtle Beach, and 250,000 others. However, the model need not evacuate all 950,000. The evacuation rate of 64% for Hurricane Floyd was one of the highest evacuation rates ever seen for a hurricane.

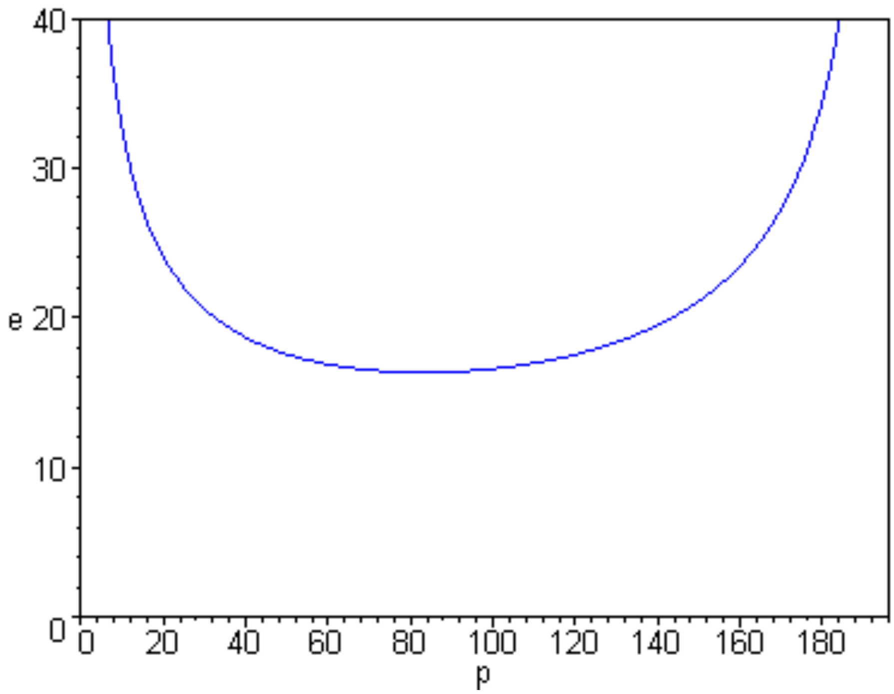

Figure 3. Total evacuation time as a function of density.

Many individuals, whether elderly, financially unable to make the trip, or just darn stubborn remain close to home rather than join the throng of frantic drivers fleeing for their lives. Further, only about 20% of those who left did so along Interstate 26. Taking 20% of 64% of 950,000 gives 122,000 as the number of people that we can expect to evacuate on I-26.

We must convert number of people to cars, the unit in our model. In South Carolina there are 3,716,645 people and 1,604,000 households, or 2.3 people per household. Further research allowed us to find the average number of cars taken for each household. Dow and Cutter [2001] included **Table 2** on the number cars taken by each household.

Table 2.

Cars taken by each household in 1999 Hurricane Floyd evacuation [Dow and Cutter 2001].

Cars	% of households
0	3%
1	72%
2	21%
3+	4%

Since the number of households taking more than three cars is merely a fraction of the 4% of the population evacuating, we assume that a household takes 0, 1, 2, or 3 cars. Thus, we find a weighted average of 1.26 cars/household.

We now calculate the average number of people per car:

$$\frac{\text{people}}{\text{car}} = \frac{\text{people/household}}{\text{cars/household}} = 1.83.$$

We divide the evacuation population by the average number of people per car to find that 66,655 cars need to be evacuated.

Both $s(\rho)$ and $t(\rho)$ are independent of the number of cars to evacuate: $s(\rho)$ is based on highway information that stays constant throughout the model, and $t(\rho)$ is based on the length of the highway. The only function dependent on the numbers of cars is total evacuation time, $e(\rho)$. Substituting values for b, l, N, and L, we arrive at

$$e_l(\rho) = \frac{117\rho + 66655}{\rho\dfrac{1}{0.0136049}\left(\dfrac{5280}{\rho} - 27\right)}.$$

After finding the minimum value, we arrive at

$$\begin{aligned} \min \rho_l &= 83 \text{ cars/mi}, \qquad & s &= 52 \text{ mph}, \\ t &= 2 \text{ h } 15 \text{ min}, \qquad & e_l &= 18 \text{ h}. \end{aligned}$$

Our model does not evacuate people very quickly, but there is a significant decrease in average trip time $t(\rho)$ for an individual car.

Our model applies to only one lane of traffic. If cars on I-26 were allowed to travel in only one lane but at optimal density, the total evacuation time would be approximately 18 h, with each car making the journey in just over 2 h at an average speed of 52 mph.

We assume that adding another highway lane halves the number of cars per lane and find

$$\begin{aligned} \min \rho_l &= 73 \text{ cars/mi}, \qquad & s &= 58 \text{ mph}, \\ t &= 2 \text{ h } 0 \text{ min}, \qquad & e_l &= 10 \text{ h}. \end{aligned}$$

Although the average trip time slightly decreased and speed slightly increased, the most striking result of opening another lane is the halving of total evacuation time.

Turning the entire I-26 into one-way traffic turns I-26 into a pair of two-lane highways rather than a four-lane highway; adding another pair of lanes to our already existing pair of lanes again halves the number of cars to be evacuated per lane.

$$\begin{aligned} \min \rho_l &= 58 \text{ cars/mi}, \qquad & s &= 68 \text{ mph}, \\ t &= 1 \text{ h } 40 \text{ min}, \qquad & e_l &= 6 \text{ h}. \end{aligned}$$

Conclusions from the I-26 Model

The problem explicitly mentions four means by which traffic flow may be improved:

- turning I-26 one-way,
- staggering evacuation,
- turning smaller highways one-way, or
- limiting the number or type of cars.

Staggering

Contrary to the emphasis of the South Carolina Emergency Preparedness Division (SCEPD), the primary proposal should not be the reversal of southbound traffic on I-26 but rather the establishment of a staggering plan. One glance at the total evacuation time vs. car density graph reveals the great benefits gained from maintaining a constant and moderate car density on the highway. The majority of the problems encountered during Hurricane Floyd, such as the 18-hour trips to Columbia or incidents of cars running out of gas on the highway, would be solved if a constant car density existed on the highway.

However, while our model assumes a certain constant car density, it does not provide a method for producing such density. Therefore, the SCEPD should produce a plan that staggers the evacuation to maintain a more constant car density. One proposal was a county-by-county stagger. SCEPD should take the optimal car density and multiply that by the optimal speed to arrive at an optimal value of cars/hour. The SCEPD should then arrange the stagger so that dispersal of cars per hour is as close as possible close to the optimal value.

Making I-26 One-Way

In addition to the staggering plan, making I-26 one-way reduces the total evacuation time from 10 h to 6 h. Thus, while staggering should always be implemented, reversal of traffic on I-26 should supplement the staggering plan when the SCEPD desires a shorter evacuation time.

The Other Options

This leaves two more strategies for managing traffic flow: turning smaller highways one-way and limiting the number or types of car taken per household. Both of these can be implemented easily in our model. Turning smaller highways one-way would encourage evacuees to take back roads instead of I-26, making percentage of evacuees taking I-26 less than 20%, reducing our value for N. Restricting the number of cars per household would also reduce N. Likewise, disallowing large vehicles would reduce l, the average length of vehicles.

Considering that the optimal speed calculated after making I-26 one-way is 68 mph, and that adding other evacuation strategies would raise the optimum average speed above the lawful limit of 70 mph, additional strategies would be unnecessary.

Adding Myrtle Beach

The next route needing consideration is 501/I-20 leaving Myrtle Beach, with 200,000 people. Unfortunately, we lack statistics on how many people took the 501/I-20 evacuation route. We can, however, make a guess using a ratio. We assume that an equal proportion of residents of Myrtle Beach evacuate using the main route as in Charleston, so that the number taking the highway leaving a city is directly proportional to the city's size. This suggests that 49,000 people leave Myrtle Beach in 49,000/1.84 = 26,600 cars, or 13,300 cars/lane. Combining this with an $L = 150$ mi, the distance between Myrtle Beach and Columbia, we can apply our model to arrive at

$$\begin{aligned} \min \rho_l &= 46 \text{ cars/mi}, & s &= 80 \text{ mph}, \\ t &= 1 \text{ h } 50 \text{ min}, & e_l &= 5 \text{ h } 28 \text{ min}. \end{aligned}$$

We cap the speed at 70 mph and determine a preferred car density of 56 cars/mi. This figure produces a new set of optimal values:

$$\begin{aligned} \min \rho_l &= 56 \text{ cars/mi}, & s &= 80 \text{ mph}, \\ t &= 2 \text{ h}, & e_l &= 5 \text{ h } 30 \text{ min}. \end{aligned}$$

This calculation also confirms one of our basic assumptions, that I-26 between Charleston and Columbia is the limiting factor: Evacuation from Myrtle Beach using two lanes still takes less time (5 h) than evacuation using 4 lanes on I-26 from Charleston (6 h). From this we can conclude that making traffic roads leading from Myrtle Beach one-way would be unnecessary. We assume that applying our model to other smaller highways will lead to similar results.

Adding Intersections and I-95

To simplify the model, we assume that intersecting routes of equal density contribute traffic to the adjoining route so that the difference in densities always seeks a balance. Intersections between roads of similar relative density do not cause unexpected spikes in density on either road. Their effect is therefore negligible.

On the case of two interstates of unequal traffic densities, the volume of traffic on the busier route may cause a substantial change in density on the adjoining route. We assume a normalizing tendency at interchanges, so drivers are not likely to change routes without immediate benefit of higher speed. We also assume that on interstates over long distances with only intermittent junctions, there is a normalizing tendency of traffic to distribute itself. Therefore, though I-95 may cause congestion problems, in our model the traffic on I-95 has a negligible effect on the overall evacuation traffic.

Adding Columbia and the Rest of the World

We view Columbia as a distribution center. Columbia accepts a certain number of cars per unit of time, dispenses another number of cars per unit of time to the rest of the world, and retains a certain amount of the traffic that it receives. In the case of an evacuation, this retention reflects evacuees who stay with families or find hotel rooms.

For example, staggered one-way traffic on I-26 yields optimal car density to be 58 cars/mi and the optimal speed of 68 mph. Multiplying these yields a flow of 3,944 cars/h into Columbia.

If the highways leading to the rest of the world can handle a large traffic flow in cars per unit time, evacuation time will not be affected. If backups outside of Columbia are a problem, building more shelters would help because Columbia would retain more of the incoming traffic. However, if the highways leading elsewhere can handle the flow, shelters are unnecessary.

Strengths and Weaknesses

One strength of the model is its formulaic practicality. With reliable measurements of traffic density and speed, the evacuation volume predictions predictions should be useful.

A second and more important strength of the model is its use for comparison. Its prediction can be considered a reference point for experimentation. After all, we are looking for improvements but not necessarily exact results. The model offers a range of possible values, and this is an advantage.

The simplifications and approximations taken in this model introduce obvious weaknesses, particularly in our concept of car density. In simplifying traffic flow, we assumed a homogeneity over distance, but this isn't likely to happen. Much of our model and its derivation hinge on the assumption that traffic density can be controlled.

With unconvincing generality, we calculate ranges of average speeds and assume that all drivers drive as fast and as safely as possible. However, it is likely that traffic will clump and cluster behind slower drivers.

A potential source of congestion, intersections, was simplified by assuming that relative traffic densities seek a balanced level. In reality, different routes are preferred over others and imbalances in traffic density are likely to occur.

The only way to test the model is to collect data during very heavy traffic. However, traffic as great as during Hurricane Floyd is rare.

In short, the weaknesses of the model are primarily related to its simplicity. However, it is that same simplicity that is its greatest strength.

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Dr. Ann Watkins, President of the Mathematical Association of America, congratulating MAA Winner Adam Dickey after he presented the team's model at MathFest in Madison, WI, in August. [Photo courtesy of Ruth Favro.]

Mathematical Commission Streamlines Hurricane Evacuation Plan

COLUMBIA, SC, FEB. 12— As long as the western coast of Africa stays where it is, global climate patterns will continue to have it in for South Carolina. That proverbial "it" is the hurricane season. Some may point their fingers at God and curse His ways, but cooler heads will eventually surrender—shutter their windows, pack their cars, and drive to Columbia for the next few days, mood unsettled.

However, a report issued from the governor's office earlier this week announced that scientists have designed a new plan for coastal emergency evacuation.

Prompted by public disapproval of evacuation tactics used during the mandatory evacuation ordered during Hurricane Floyd, the new study sought to find the source of the congestion problems that left motorists stranded on I-26 for up to 18 hours wondering why they even attempted evacuation at all.

"The government should have known that we didn't have the roads to get everybody out," one Charleston resident commented, "and we just had the speed limits raised, so I would've expected a quicker escape rout."

Governor Jim Hodges commissioned the evacuation study last Friday, expecting a quick response. So far, the results seem plausible and practical.

The private commission developed a mathematical model to describe the traffic flow that caused the backups responsible for the evacuation problems. Then, by manipulating the model and combining it with statistical survey data collected after the evacuation, the commission developed a report evaluating the various current suggestions to alleviate issues.

The findings of the commission suggest that the best way to avoid evacuation backups is to stagger and sequence the county and metropolitan evacuation orders so that main routes do not exceed the critical traffic density that caused the slowdown.

As explained by one member of the commission, "the model does not get into the gritty details of complicated traffic modeling, but it does present a useful framework for evaluating crisis plans and traffic routing."

The solution garnered some skeptical criticism among mathematics researchers state-wide. "The commission is missing the point here," commented one researcher at the Hazards Research Lab in the Department of Geology at the University of South Carolina. "The problem with Hurricane Floyd was an anomalously high rate of evacuation among coastal residents—this is a sociopsychological problem and not a mathematical one."

Despite the cool reception, the commission is confident in its findings. According to the report, a general staggering of traffic will far exceed the benefits of other methods of congestion control that have been suggested, such as reversing the eastbound lanes of I-26 and some secondary highways.

"We find that advance planning with a mind to reduce the traffic surge associated with quickly ordered mandatory evacuations is the most useful way to improve evacuations," said a spokesman for the commission.

Despite the current concerns, the proving grounds for the findings of the commission will not surface until this fall, when the tropical swells off the coast of Africa begin heading our way again.

—Corey R. Houmand, Andrew D. Pruett, and Adam S. Dickey in Winston–Salem, NC.

Judge's Commentary: The Outstanding Hurricane Evacuation Papers

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Introduction

Once again, Problem B proved to be quite challenging—both for the student teams and for the judges! The students were challenged by a multifaceted problem with several difficult questions posed, and the judges were challenged to sort through the wide range of approaches to find a small collection of the best papers. It is worth reminding participants and advisors that Outstanding papers are not without weaknesses and even mathematical or modeling errors. It is the nature of judging such a competition that we must trade off the strengths, both technical and expository, of a given paper with its weaknesses, and make comparisons between papers the same way.

The approaches taken by this year's teams can be divided into two general categories:

Macroscopic: Traffic on a particular highway or segment of highway was considered to be a stream, and a flow rate for the stream was characterized. Among the successful approaches in this category were fluid dynamics and network flow algorithms.

Microscopic: These can be considered car-following models, where the spacing between and the speeds of individual vehicles are used to determine the flow. Among the successful approaches were discrete event simulations (including cellular automata) and queuing systems.

The UMAP Journal 22 (3) (2000) 337–343.

By far, the most common approach was to determine that the flow q, or flux, is a function of the density ρ of cars on a highway and the average speed v of those cars: $q = \rho v$. Successful approaches identified the following characteristics of the basic traffic flow problem:

- When the vehicle density on the highway is 0, the flow is also 0.
- As density increases, the flow also increases (up to a point).
- When the density reaches its maximum, or *jam density* ρ_0, the flow must be 0.
- Therefore, the flow initially increases, as density does, until it reaches some maximum value. Further increase in the density, up to the jam density, results in a reduction of the flow.

At this point, many teams either derived from first principles or used one of the many resources available on traffic modeling (such as Garber and Hoel [1999]) to find a relationship between the density and the average speed. Three of the common macroscopic models were:

- a linear model developed by Greenshield:

$$v = v_0 \left(1 - \frac{\rho}{\rho_0}\right), \qquad \text{so} \qquad q = \rho v_0 \left(1 - \frac{\rho}{\rho_0}\right);$$

- a fluid-flow model developed by Greenberg:

$$v = v_0 \log \frac{\rho}{\rho_0}, \qquad \text{so} \qquad q = \rho v_0 \log \frac{\rho}{\rho_0};$$

  or

- a higher-order model developed by Jayakrishnan:

$$v = v_0 \left(1 - \frac{\rho}{\rho_0}\right)^a, \qquad \text{so} \qquad q = \rho v_0 \left(1 - \frac{\rho}{\rho_0}\right)^a,$$

where v_0 represents the speed that a vehicle would travel in the absence of other traffic (the speed limit). By taking the derivative of the flow equation with respect to speed (or density), teams then found the optimal speed (or density) to maximize flow.

Many teams took the optimal flow from one of the macroscopic approaches and used it as the basis for a larger model. One of the more common models was simulation, to determine evacuation times under a variety of scenarios.

In order to make it beyond the Successful Participant category, teams had to find a way *realistically* to regulate traffic density to meet these optimality conditions. Many teams did this by stipulating that ramp metering systems (long term) or staggered evacuations (short term) could be used to control traffic density.

There were a number of mathematically rigorous papers that started with a partial differential equation, derived one of the macroscopic formulas, determined appropriate values for the constants, calculated the density giving the optimal flow, and incorporated this flow value into an algorithm for determining evacuation time. In spite of the impressive mathematics, if no plan was given to regulate traffic density, the team missed an important concept of the MCM: the realistic application of a mathematical solution to a real-world problem.

One key to successful model building is to adapt existing theory or models properly to the problem at hand, so judges see little difference between deriving these equations from first principles and researching them from a book. Whether derived or researched, it is imperative to demonstrate an understanding of the model you are using.

The Judging

No paper completely analyzed all 6 questions, so the judges were intrigued by what aspects of the problem that a team found most important and/or interesting. We were similarly interested in determining what aspects of the problem a team found least relevant and how they divided their effort among the remaining questions. To be considered Outstanding, a paper had to meet several minimum requirements:

- the paper must address all 6 questions,
- all required elements (e.g., the newspaper article) must be included, and
- some sort of validation of the model must be included.

We were also particularly interested in how teams modeled the I-26/I-95 interchange and the congestion problem in Columbia. Many teams chose to treat Columbia as the terminal point of their model and assumed that all cars arriving there would be absorbed without creating backups.

To survive the cut between Honorable Mention and Meritorious, a paper had to have a unique aspect on some portion of the problem. Two examples that come to mind are a unique modeling approach or some aspect of the problem analyzed particularly well. Thus, papers that failed to address all questions or had a fatal weakness that prevented their model from being extended could still be considered Meritorious. The Meritorious papers typically had very good insight into the problem, but deficiencies as minor as missing parameter descriptions or model implementation details prevented them from being considered Outstanding.

The Outstanding Papers

The six papers selected as Outstanding were recognized as the best of the submissions because they:

- developed a solid model which allowed them to address all six questions, and analyze at least one very thoroughly;
- made a set of clear recommendations;
- analyzed their recommendations within the context of the problem; and
- wrote a clear and coherent paper describing the problem, their model, and their recommendations.

Here is a brief summary of the highlights of the Outstanding papers.

The Bethel College team used a basic car-following model to determine an optimal density, which maximized flow, for individual road segments. They then formulated a maximum flow problem, with intersections and cities as vertices and road segments as arcs. The optimal densities were used as arc capacities, the numbers of vehicles to be evacuated from each city were used as the sources, and cities at least 50 miles inland were defined to be sinks. Each city was then assigned an optimal evacuation route, and total evacuation times under the different scenarios were examined.

The Duke team also used a basic car-following model from the traffic-modeling literature. This model provided the foundation of a one-dimensional cellular automata simulation. They did a particularly good job of defining evacuation performance measures—maximum traffic flow and minimum transit time, and analyzing traffic mergers and bottlenecks—aspects of the problem ignored by many other teams.

What discussion of Outstanding papers would be complete without a Harvey Mudd team? Of the teams that utilized literature-based models, this team did the best job of considering advanced parameters—including road grade, non-ideal drivers, and heavy-vehicle modification. They also did a very good job of comparing their model with the new South Carolina evacuation plan, recognizing the bottleneck problem in Columbia, and analyzing the impact of extra drivers from Florida and Georgia on I-95. Their entry was a nice example of a simple model that was well analyzed and thoroughly explained.

The Virginia Governor's School team began their analysis by reviewing the current South Carolina evacuation plan, a baseline to compare their model against. They researched the literature to find traffic-flow equations and then used a genetic algorithm to assign road orientation and evacuation start times for cities. They did an exceptionally good job of analyzing the sensitivity of their model to changes in parameter values.

The INFORMS prizewinner, from Lawrence Technical University, combined Greenshield's model with a discrete event simulation. The judges saw this entry as a solid paper with logical explanations and a good analysis. The team's model handled bottlenecks, and the team used a simulation of the actual 1999 evacuation to validate their model.

The MAA and SIAM winner, from Wake Forest University, derived a car-following model from first principles, which was then incorporated in a cellular automata type model. Like many of the best approaches, the parameters for their model came from the 1999 evacuation. They provided a thoughtful, not necessarily mathematical, analysis of intersections and I-95.

Advice

At the conclusion of our judging weekend, the judges as a whole offered the following comments:

Follow the instructions

- Answer all required parts.
- Make a precise recommendation.
- Don't just copy the original problem statement, but provide us with your interpretation.

Readability

- Make it clear in the paper where the answers are.
- Many judges find it helpful to include a table of contents.
- Pictures and graphs can help demonstrate ideas, results, and conclusions.
- Use discretion: If your paper is excessively long (we had a paper this year that was over 80 pp, not including computer program listing!), you should probably reconsider the relevance of all factors that you are discussing. Depending on what round of judging your paper is being read, judges typically have between 5 and 30 minutes to read it.

Computer Programs

- Make sure that all parameters are clearly defined and explained.
- When using simulation, you must run enough times to have statistically significant output. A single run isn't enough!
- Always include pseudocode and/or a clear verbal description.

Reality Check

- Why do you think your model is good? Against what baseline can you compare/validate it?
- How sensitive is your model to slight changes in the parameters you have chosen? (sensitivity analysis)

- Complete the analysis circle: Are your recommendations practical in the problem context?

Before the final judging of the MCM papers, a first (or triage) round of judging is held. During triage judging, each paper is skimmed by two or three judges, who spend between 5 and 10 minutes each reading the paper. Typically, when you send your paper off to COMAP, you have about a 43% chance of being ranked higher than Successful Participant. If, however, you survive the triage round, you have about an 80% chance of being ranked higher than Successful Participant. Head triage judge Paul Boisen offers the following advice to help you survive triage.

Triage Judge Tips

- Your summary is a key component of the paper; it needs to be clear and contain results. A long list of techniques can obscure your results; it is better to provide only a quick overview of your approach. The Lawrence Technical University paper is a good example of a clear and concise summary.
- Your paper needs to be well organized—can a triage judge understand the significance of your paper in 6 to 10 minutes?

Triage Judge Pet Peeves

- Tables with columns headed by Greek letters or acronyms that cannot be immediately understood.
- Definitions and notation buried in the middle of paragraphs of text. A bullet form is easier for the frantic triage judge!
- Equations without variables defined.
- Elaborate derivations of formulas taken directly from a text. It is better to cite the book and perhaps briefly explain how the formula is derived. It is most important to demonstrate that you know how to use the formulas properly.

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About the Author

After receiving his B.A. in Mathematics and Computer Science in 1984, Mark Parker spent eight years working as a systems simulation and analysis engineer in the defense industry. After completing his Ph.D. in Applied Mathematics at the University of Colorado–Denver in 1995, he taught mathematics and computer science at Eastern Oregon University for two years. He then spent three years on the faculty at the U.S. Air Force Academy teaching mathematics and operations research courses. He now shares a teaching position with his wife, Holly Zullo, at Carroll College in Helena, MT, and spends as much time as he can with his three-year-old daughter Kira.

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