1 Problem

$\xi \sim Pois(\lambda)$

η - number of successful outcomes of Bernoulli trials, if $\xi = k$ and we perform k trials with the probability of success p

$\eta \overset{?}{\sim} Pois(p\lambda)$

$$\mathbb{P}(\eta = j) = \sum_{k \geqslant j} \mathbb{P}(\eta = j | \xi = k)\mathbb{P}(\xi = k) = \sum_{k \geqslant j} (C_k^j p^j (1-p)^{k-j})(e^{-\lambda}\frac{\lambda^k}{k!}) =$$

$$= \sum_{k \geqslant j} \frac{k!}{j!(k-j)!} p^j (1-p)^{k-j} e^{-\lambda} \frac{\lambda^k}{k!} = \frac{p^j}{j!}\lambda^j e^{-\lambda} \sum_{k \geqslant j} \frac{((1-p)\lambda)^{k-j}}{(k-j)!} =$$

$$= \frac{(p\lambda)^j}{j!} e^{-\lambda} e^{(1-p)\lambda} = \frac{(p\lambda)^j}{j!} e^{-p\lambda}$$

2 Problem

$t_1 \sim N(30, 100)$, f_{t_1} - probability density function

$t_2 \sim N(20, 25)$, f_{t_2} - probability density function

$\mathbb{P}(check1) = \mathbb{P}(check2) = 0.5$

$$\mathbb{P}(check2 | t = 10) = \frac{\mathbb{P}(check2, t = 10)}{\mathbb{P}(t = 10)} =$$

$$= \frac{\mathbb{P}(t = 10 | check2)\mathbb{P}(check2)}{\mathbb{P}(t = 10 | check1)\mathbb{P}(check1) + \mathbb{P}(t = 10 | check2)\mathbb{P}(check2)} \overset{\epsilon \to 0}{=}$$

$$\overset{\epsilon \to 0}{=} \frac{\mathbb{P}(t_2 < 10 + \epsilon) - \mathbb{P}(t_2 < 10 - \epsilon)}{\mathbb{P}(t_1 < 10 + \epsilon) - \mathbb{P}(t_1 < 10 - \epsilon) + \mathbb{P}(t_2 < 10 + \epsilon) - \mathbb{P}(t_2 < 10 - \epsilon)} =$$

$$= \frac{\int_{10-\epsilon}^{10+\epsilon} f_{t_2}(z)dz}{\int_{10-\epsilon}^{10+\epsilon} f_{t_1}(z)dz + \int_{10-\epsilon}^{10+\epsilon} f_{t_2}(z)dz} = \frac{f_{t_2}(10)2\epsilon}{f_{t_1}(10)2\epsilon + f_{t_2}(10)2\epsilon} =$$

$$= \frac{\frac{1}{\sqrt{2\pi}5} e^{-\frac{(10-20)^2}{2 \cdot 25}}}{\frac{1}{\sqrt{2\pi}10} e^{-\frac{(10-30)^2}{2 \cdot 100}} + \frac{1}{\sqrt{2\pi}5} e^{-\frac{(10-20)^2}{2 \cdot 25}}} = \frac{\frac{1}{5}e^{-2}}{\frac{1}{10}e^{-2} + \frac{1}{5}e^{-2}} = \frac{2}{3}$$