例 6.4 A107270010 企管二甲 楊宜洨

$$V(x_i) = \sigma^2 = E(x_i^2) - \mu^2 = E(x^2) - E^2(x)$$

$$V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right) = \frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

$$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i-\bar{x})^2 / (n-1) \quad 為母体變異數 \sigma^2 \text{ 不偏估計量}$$

$$\hat{\theta}_1 = \sum_{i=1}^{n}(x_i-\bar{x})^2 / n \qquad = \qquad 偏誤估計量$$