

例 6.4

1107270010 企管 = 甲 楊直波

$$V(X_i) = \sigma^2 = E(X_i^2) - \mu^2 = E(X^2) - E^2(X)$$

$$V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right) \\ = \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n} \sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right) \\ = \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

$$\hat{\theta}_2 = \sum_{i=1}^n (X_i - \bar{X})^2 / (n-1) \text{ 為母體變異數 } \sigma^2 \text{ 不偏估計量}$$

$$\hat{\theta}_1 = \sum_{i=1}^n (X_i - \bar{X})^2 / n \text{ 偏誤估計量}$$