$$\vec{\nabla} \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{-x}{\sqrt{x^2+y^2}} & \dfrac{-y}{\sqrt{x^2+y^2}} & 0 \end{bmatrix}$$

$$= \left[\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}\left(\frac{-y}{\sqrt{x^2+y^2}}\right)\right]\mathbf{i} - \left[\frac{\partial}{\partial x}0 - \frac{\partial}{\partial z}\left(\frac{-x}{\sqrt{x^2+y^2}}\right)\right]\mathbf{j} + \left[\frac{\partial}{\partial x}\left(\frac{-y}{\sqrt{x^2+y^2}}\right) - \frac{\partial}{\partial y}\left(\frac{-x}{\sqrt{x^2+y^2}}\right)\right]\mathbf{k}$$

$$= 0\,\mathbf{i} - 0\,\mathbf{j} + \left(\frac{-2xy}{x^2+y^2} - \frac{-2xy}{x^2+y^2}\right)\mathbf{k}$$

$$= 0\,\mathbf{i} - 0\,\mathbf{j} + 0\,\mathbf{k}$$