Lista 01 de cálculo 3

1. sabendo que $\mathbf{r}(t) \in \mathbb{R}^3$, determine o domínio da função

   (a) $\mathbf{r}(t) = \dfrac{1}{t}\,\mathbf{i} + \sqrt{4-t}\,\mathbf{j}$

   (b) $\mathbf{r}(t) = \arcsin t\,\mathbf{i} + \ln(t+1)\,\mathbf{j}$

   (c) $\mathbf{r}(t) = \sqrt{t^2+2}\,\mathbf{i} + \sqrt{4-t}\,\mathbf{j} + \cot t\,\mathbf{k}$

   (d) $\mathbf{r}(t) = \sqrt{t^2-9}\,\mathbf{i} + \ln|t-3|\,\mathbf{j} + (t^2+2t-8)\,\mathbf{k}$

   (e) $\mathbf{r}(t) = \tan t\,\mathbf{i} + \sqrt{4-t^2}\,\mathbf{j} + \dfrac{1}{2+t}\,\mathbf{k}$

2. Dadas as funções vetoriais $\mathbf{F}$ e $\mathbf{G}$ em $\mathbb{R}^3$, e sabendo que f e g são funções em $\mathbb{R}$, calcule $(\mathbf{F}+\mathbf{G})(t)$, $(\mathbf{F}-\mathbf{G})(t)$, $(f\mathbf{F})(t)$, $(f\mathbf{G})(t)$, $(\mathbf{F}\circ g)(t)$ e $(\mathbf{G}\circ g)(t)$

   (a) $\mathbf{F}(t) = (t+1)\,\mathbf{i} + (t^2-1)\,\mathbf{j} + (t-1)\,\mathbf{k}$
   $\mathbf{G}(t) = (t-1)\,\mathbf{i} + \mathbf{j} + (t+1)\,\mathbf{k}$
   $f(t) = t-1$
   $g(t) = t+1$

   (b) $\mathbf{F}(t) = (4-t^2)\,\mathbf{i} + 4\,\mathbf{j} - (4-t^2)\,\mathbf{k}$
   $\mathbf{G}(t) = t^2\,\mathbf{i} + (t^2-4)\,\mathbf{j} - 4\,\mathbf{k}$
   $f(t) = \dfrac{1}{2-t}$
   $g(t) = t+1$

   (c) $\mathbf{F}(t) = \cos t\,\mathbf{i} - \sin t\,\mathbf{j} + t\,\mathbf{k}$
   $\mathbf{G}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} - t\,\mathbf{k}$
   $f(t) = \sin t$
   $g(t) = \arcsin t$

   (d) $\mathbf{F}(t) = \sec t\,\mathbf{i} + \tan t\,\mathbf{j} - 2\,\mathbf{k}$
   $\mathbf{G}(t) = \sec t\,\mathbf{i} - \tan t\,\mathbf{j} + t\,\mathbf{k}$
   $f(t) = \cos t$
   $g(t) = \arccos t$

3. sabendo que $\mathbf{r}(t) \in \mathbb{R}^3$, determine o limite, se existe

   (a) $\lim_{t\to -1} \mathbf{r}(t)$; $\mathbf{r}(t) = \dfrac{t^2-1}{t+1}\,\mathbf{i} + \dfrac{t+1}{t-1}\,\mathbf{j} + |t+1|\,\mathbf{k}$

   (b) $\lim_{t\to 0} \mathbf{r}(t)$; $\mathbf{r}(t) = \dfrac{1-\cos t}{t}\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$

   (c) $\lim_{t\to 0} \mathbf{r}(t)$; $\mathbf{r}(t) = \dfrac{\ln(t+1)}{t}\,\mathbf{i} + \sinh t\,\mathbf{j} + \cosh t\,\mathbf{k}$

4. sabendo que $\mathbf{r}(t) \in \mathbb{R}^3$, determine o intervalo de continuidade

   (a) $\mathbf{r}(t) = t^2\,\mathbf{i} + \ln(t-1)\,\mathbf{j} + \dfrac{1}{t-2}\,\mathbf{k}$

   (b) $\mathbf{r}(t) = (t-1)\,\mathbf{i} + \dfrac{1}{e^t-1}\,\mathbf{j} + \dfrac{|t-1|}{t-1}\,\mathbf{k}$

   (c) $\mathbf{r}(t) = \sin \pi t\,\mathbf{i} - \tan \pi t\,\mathbf{j} + \cot \pi t\,\mathbf{k}$

   (d) $\mathbf{r}(t) = \begin{cases} \dfrac{\sin t}{t}\,\mathbf{i} + \dfrac{1-\cos t}{t}\,\mathbf{j} + \dfrac{1-e^t}{t}\,\mathbf{k} & \text{se } t \neq 0 \\ \mathbf{i} - \mathbf{j} & \text{se } t = 0 \end{cases}$

5. Obter a equação cartesiana das seguintes curvas

   (a) $\mathbf{r}(t) = \dfrac{1}{2}t\,\mathbf{i} + 3t+5\,\mathbf{j}$

   (b) $\mathbf{r}(t) = (t-1)\,\mathbf{i} + (t^2-2t+2)\,\mathbf{j}$

   (c) $\mathbf{r}(t) = (s^2-1)\,\mathbf{i} + (s^2+1)\,\mathbf{j} + 2\,\mathbf{k}$

6. Determine a equação paramétrica:

(a) $2x^2 - 8y + 4 = 0$

(b) $y - \dfrac{1}{x-1} = 0; \quad x > 1$

(c) $x^2 + y^2 - 6x + 8y = 0$

(d) $y = 2x^2, \quad z = x^3$

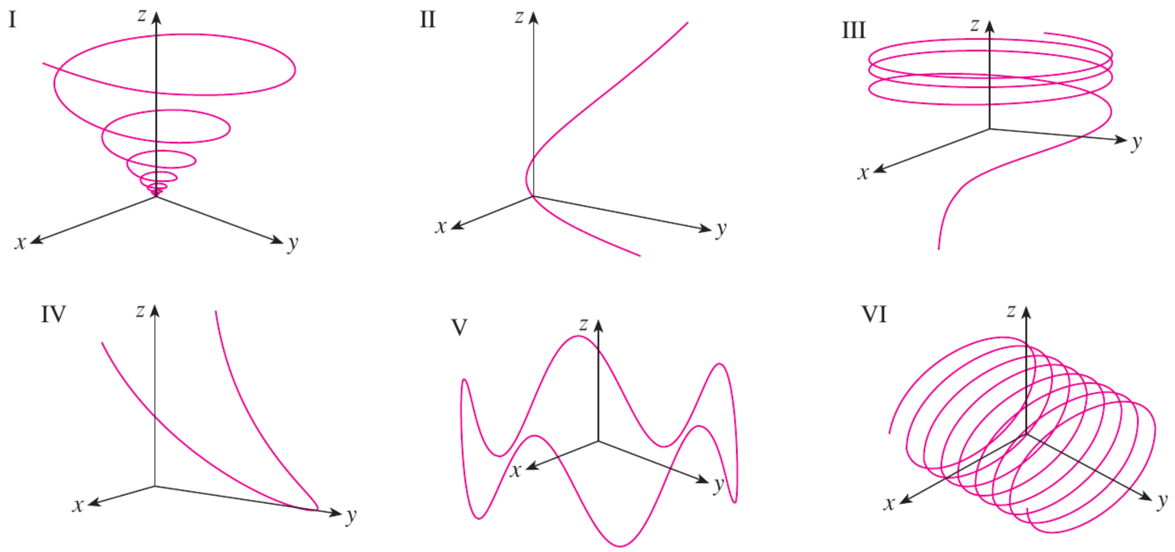
(e) $y = 2(x+1)^2 + y^2, \quad z = 2$

(f) O segmento de reta que passa por $A = (2,1,2)$ e $B = (-1,1,3)$

(g) $x + y + z = 1, \quad z = x - 2y$

(h) $x^2 + y^2 + z^2 = 2y, \quad z = y$

7. Dada as figuras, selecione a qual equação corresponde, justifique matematicamente sua resposta



(a) $\mathbf{r}(t) = \cos 4t\,\mathbf{i} + t\,\mathbf{j} + \sin 4t\,\mathbf{k}$

(b) $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + e^t\,\mathbf{k}$

(c) $\mathbf{r}(t) = e^{-t}\cos 10t\,\mathbf{i} + e^{-t}\sin 10t\,\mathbf{j} + e^{-t}\,\mathbf{k}$

(d) $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \sin 5t\,\mathbf{k}$

(e) $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \ln t\,\mathbf{k}$

8. Encontre a equação cartesiana para as curvas com as seguintes equações paramétricas

(a) $\mathbf{r}(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j} + t\,\mathbf{k}$

(b) $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \sin^2 t\,\mathbf{k}$

9. Encontre o ponto ou a curva de interseção de:

(a) $\mathbf{r}(t) = t\,\mathbf{i} + (2t - t^2)\,\mathbf{k}$ com $z = x^2 + y^2$

(b) $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j}$ com $x^2 + y^2 = 5$

(c) $x^2 + y^2 = 4$ e $z = xy$

(d) $z = \sqrt{x^2 + y^2}$ e $z = 1 + y$

(e) $z = 4x^2 + y^2$ e $y = x^2$

10. Encontre $\mathbf{r}'(t)$ e $\mathbf{r}''(t)$

(a) $\mathbf{r}(t) = \arctan t\,\mathbf{i} + \arcsin t\,\mathbf{j} + \arccos t\,\mathbf{k}$

(b) $\mathbf{r}(t) = (e^{3t} + 2)\,\mathbf{i} + 2e^{3t}\,\mathbf{j} + 3 \cdot 2^t\,\mathbf{k}$

(c) $\mathbf{r}(t) = tan 3t\,\mathbf{i} + \ln \sin t\,\mathbf{j} + \dfrac{1}{t}\,\mathbf{k}$

(d) $\mathbf{r}(t) = at\cos 3t\,\mathbf{i} + b\sin^3 t\,\mathbf{j} + c\cos^3 t\,\mathbf{k}$

(e) $\mathbf{r}(t) = \dfrac{t}{t+1}\,\mathbf{i} + \dfrac{1}{t}\,\mathbf{j}$; suponha $t = -1/2$

(f) $\mathbf{r}(t) = \dfrac{5t-2}{2t+1}\,\mathbf{i} + \ln(1 - t^2)\,\mathbf{j} + 5\,\mathbf{k}$

11. Determine o vetor tangente à curva definida pela função vetorial no ponto indicado $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + \mathbf{k}$

(a) $\mathbf{r}(t) = \sin t\,\mathbf{i} + (t^2 - \cos t)\,\mathbf{j} + e^t\,\mathbf{k}$, $t_0 = 0$

(b) $\mathbf{r}(t) = t^2\,\mathbf{i} + (2t - 1)\,\mathbf{j} + t^3\,\mathbf{k}$, $t_0 = 2$

(c) $\mathbf{r}(t) = \ln t\,\mathbf{i} + \dfrac{t-1}{t+2}\mathbf{j} + t\ln t\,\mathbf{k}$, $t_0 = 1$

(d) $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \sin 2t\,\mathbf{k}$, $t_0 = \pi/2$

12. Encontre a equação paramétrica da linha tangente às curvas que tem as seguintes equações paramétricas no ponto específico

(a) $\mathbf{r}(t) = \left(1 + 2\sqrt{t}\right)\,\mathbf{i} + (t^3 - t)\,\mathbf{j} + (t^3 + t)\,\mathbf{k}$; $P_0 = (3, 0, 2)$

(b) $\mathbf{r}(t) = e^t\,\mathbf{i} + te^t\,\mathbf{j} + te^{t^2}\,\mathbf{k}$; $P_0 = (1, 0, 0)$

(c) $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^- t \sin t\,\mathbf{j} + e^- t\,\mathbf{k}$; $P_0 = (1, 0, 1)$

(d) $\mathbf{r}(t) = \ln t\,\mathbf{i} + 2\sqrt{t}\,\mathbf{j} + t^2\,\mathbf{k}$; $P_0 = (0, 2, 1)$

(e) Encontre o ponto de interseção das linhas tangente da curva $\mathbf{r}(t) = \sin \pi t\,\mathbf{i} + 2\sin \pi t\,\mathbf{j} + \cos \pi t\,\mathbf{k}$ que passam pelos pontos definidos por $t = 0$ e $t = 0.5$.

13. Esboçar as curvas seguentes, representando o sentido positivo de percurso. Obter uma parametrização da curva dada, orientada no sentido contrario

(a) $\mathbf{r}(t) = (2 + 3\cos t)\,\mathbf{i} + (1 = \sin t)\,\mathbf{j}$, $t \in [0, 2\pi]$

(b) $\mathbf{r}(t) = (t)\,\mathbf{i} + (t + 2)\,\mathbf{j} + (2t + 1)\,\mathbf{k}$, $t \in [0, 1]$

(c) $\mathbf{r}(t) = (1 + \cos t)\,\mathbf{i} + (1 + \sin t)\,\mathbf{j} + (2t)\,\mathbf{k}$, $t \in [0, 4\pi]$

(d) $\mathbf{r}(t) = (2\cos^3 t)\,\mathbf{i} + \left(2\sin^3 t\right)\,\mathbf{j}$, $t \in \left[0, \dfrac{\pi}{2}\right]$

14. Encontre as funções mais gerais possíveis, sabendo que as funções abaixo são as suas derivadas $(\mathbf{r}'(t))$

(a) $\tan t\,\mathbf{i} + \dfrac{1}{t}\mathbf{j}$

(b) $\ln t\,\mathbf{i} + t^2\,\mathbf{j}$

(c) Se $\mathbf{r}'(t) = e^t \sin t\,\mathbf{i} + \cos t\,\mathbf{j} - e^t\,\mathbf{k}$e $\mathbf{r}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$

(d) $\dfrac{1}{4+t^2}\mathbf{i} + \dfrac{4}{1-t^2}\mathbf{j}$

(e) $e^{3t}\,\mathbf{i} + e^{-3t}\,\mathbf{j} - te^{3t}\,\mathbf{k}$

(f) Se $\mathbf{r}'(t) = \dfrac{1}{t+1}\mathbf{i} - \tan t\,\mathbf{j} + \dfrac{t}{t^2-1}\mathbf{k}$ e $\mathbf{r}(0) = 4\,\mathbf{i} - 3\,\mathbf{j} + 5\,\mathbf{k}$

15. Verificar quais das seguentes curvas são suaves

(a) $\mathbf{r}(t) = 2\,(t - \sin t)\,\mathbf{i} + 2\,(1 - \cos t)\,\mathbf{j}$, $t \in [-1, 1]$

(b) $\mathbf{r}(t) = 3\cos^3 t\,\mathbf{i} + 3\sin^3 t\,\mathbf{j}$, $\quad t \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

(c) $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$, $\quad t \in [0, 2\pi]$

16. Determine o comprimento de arco

(a) $\mathbf{r}(t) = (t + 1)\,\mathbf{i} - t^2\,\mathbf{j} + (1 - 2t)\,\mathbf{k}$; $-1 \leq t \leq 2$

(b) $\mathbf{r}(t) = \sin 2t\,\mathbf{i} + \cos 2t\,\mathbf{j} + 2t^{3/2}\,\mathbf{k}$; $0 \leq t \leq 1$

(c) $\mathbf{r}(t) = 4t^{3/2}\,\mathbf{i} + 2\sin t\,\mathbf{j} + 3\cos t\,\mathbf{k}$; $0 \leq t \leq 2$

(d) $\mathbf{r}(t) = t^2\,\mathbf{i} + \left(t + \dfrac{1}{3}t^3\right)\,\mathbf{j} + \left(t - \dfrac{1}{3}t^3\right)\,\mathbf{k}$; $0 \leq t \leq 1$

17. Escreva a função de comprimento de arco, l, desde $t = 0$

(a) $\mathbf{r}(t) = (\sin t - t\cos t)\ \mathbf{i} + (\cos t + t\sin t)\ \mathbf{j}$

(b) $\mathbf{r}(t) = 2\,(t - \sin t)\ \mathbf{i} + 2\,(1 - \cos t)\ \mathbf{j}$

(c) $\mathbf{r}(t) = \cosh t\,\mathbf{i} + \sinh t\,\mathbf{j} + t\,\mathbf{k}$

(d) $\mathbf{r}(t) = e^t\cos t\,\mathbf{i} + e^t\sin t\,\mathbf{j} + e^t\,\mathbf{k}$

18. Reparametrize pelo comprimento de arco as seguentes curvas

(a) $\mathbf{r}(t) = 2t\,\mathbf{i} + \dfrac{2}{3}\sqrt{8t^3}\,\mathbf{j} + t^2\,\mathbf{k}$
$t \in [0, 3]$

(b) $\mathbf{r}(t) = a\cos^3 t\,\mathbf{i} + a\sin^3 t\,\mathbf{j}$
$t \in \left[0, \dfrac{\pi}{2}\right]$

(c) $\mathbf{r}(t) = (1 - t)\ \mathbf{i} + (2 + 2t)\ \mathbf{j} + 3t\,\mathbf{k}$
$t \in [0, 1]$

(d) $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 4t\,\mathbf{j} + 2\sin t\,\mathbf{k}$
$t \in \left[0, \dfrac{\pi}{2}\right]$

19. Verificar se as curvas dadas estão parametrizadas pelo comprimento de arco.

(a) $\mathbf{r}(t) = \ln(t + 1)\ \mathbf{i} + \left(\dfrac{s^3}{3} + s^2\right)\ \mathbf{j}$
$t > 0$

(b) $\mathbf{r}(t) = 4\cos\dfrac{t}{4}\,\mathbf{i} + 4\sin\dfrac{t}{4}\,\mathbf{j}$
$t \in [0, 8\pi]$

(c) $\mathbf{r}(t) = a\cos\dfrac{t}{c}\,\mathbf{i} + 4\sin\dfrac{t}{c}\,\mathbf{j} + b\dfrac{t}{c}\,\mathbf{k}$
$c^2 = a^2 + b^2$

(d) $\mathbf{r}(t) = (2t - 1)\ \mathbf{i} + (t + 2)\ \mathbf{j} + t\,\mathbf{k}$
$t > 0$

20. Dar o dominio das seguentes funções vetoriais

(a) $\mathbf{q}(x, y) = \dfrac{1}{xy}\,\mathbf{i} + \sqrt{xy}\,\mathbf{j}$

(b) $\mathbf{u}(x, y, z) = x^2y\,\mathbf{i} + y\,\mathbf{j} + \sqrt{z}\,\mathbf{k}$

(c) $\mathbf{v}(x, y, z) = y\,\mathbf{j} + \sqrt{x + z}\,\mathbf{k}$

(d) $\mathbf{r}(x, y, z) = \sqrt{2 - x^2 - y^2}\,\mathbf{i} +$
$\sqrt{1 - x^2 = y^2}\,\mathbf{j} + z\,\mathbf{k}$