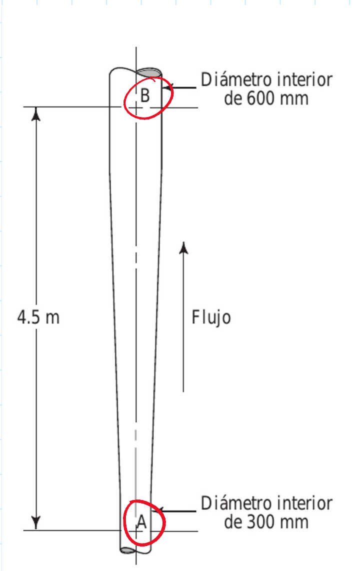
Problema 1

Thursday, October 1, 2020 10:31 AM



Ecuación de continuidad

Para fluidos incompresibles en E.E.:

$$Q_A = Q_B = Q$$

$$V_A A_A = V_B \cdot A_B = Q$$

$$V_A = \frac{Q}{A_A} = \frac{Q}{\frac{\pi D_A^2}{4}} \quad ; \quad V_B = \frac{Q}{A_B} = \frac{Q}{\frac{\pi D_B^2}{4}}$$

Ecuación de Bernoulli:

$$\underbrace{\frac{p_A}{\gamma}}_{\text{Carga de presión}} + \underbrace{\frac{1}{2}\frac{V_A^2}{2g}}_{\text{carga de velocidad}} + \underbrace{z_A}_{\text{carga de elevación}} = \frac{p_B}{\gamma} + \frac{1}{2}\frac{V_B^2}{2g} + z_B$$

Carga total

$$p_B = \gamma\left[\frac{p_A}{\gamma_w} + \frac{1}{2g}\left(V_A^2 - V_B^2\right) + (z_A - z_B)\right]$$

$$p_B = \gamma\left(\frac{p_A}{\gamma} + \frac{1}{2g}\left[\left(\frac{Q}{A_A}\right)^2 - \left(\frac{Q}{A_B}\right)^2\right] + (z_A - z_B)\right)$$

$p_A = 66.2$ kPa ← Si es manométrica, p_B también es manométrica
← Si es absoluta, p_B también es absoluta

No cambia la resolución del problema

$\gamma = 9.81$ kN

$Q = 0.37\ m^3/s$

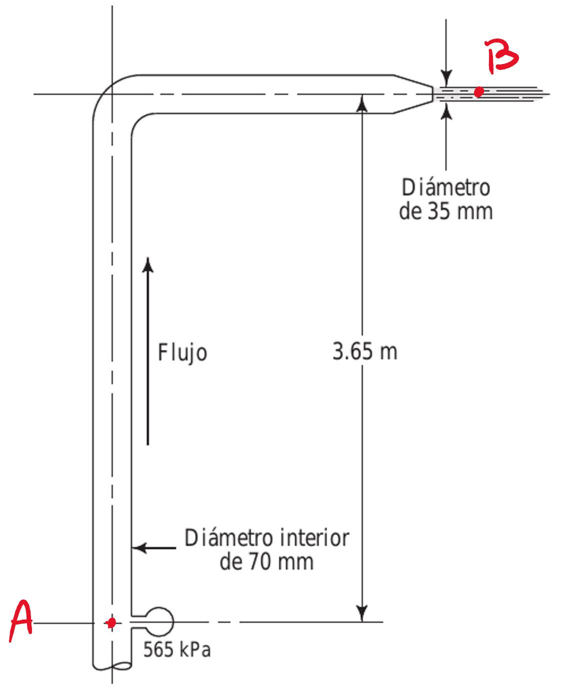
$A_A = \frac{\pi D_A^2}{4} = 0.071\ m^2$

$A_B = \frac{\pi D_B^2}{4} = 0.283\ m^2$

$$\boxed{\Rightarrow p_B = 34.9\ \text{kPa}}$$

Problema 2

Thursday, October 1, 2020 10:53 AM



Eq. Continuidad:

$$Q_A = Q_B = Q$$

$$V_A = \frac{Q}{A_A} = \frac{Q}{\frac{\pi D_A^2}{4}}$$

$$V_B = \frac{Q}{A_B} = \frac{Q}{\frac{\pi D_B^2}{4}}$$

Ecuación de Bernoulli:

$$\frac{P_A}{\gamma} + \frac{1}{2g}\left(\frac{Q}{A_A}\right)^2 + Z_A = \cancel{\frac{P_B}{\gamma}}^{\,0\ (\text{man})} + \frac{1}{2g}\left(\frac{Q}{A_B}\right)^2 + Z_B$$

$$\Rightarrow Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1}\left[(Z_B - Z_A) - \frac{P_A}{\gamma}\right]}$$

$$A_A = \frac{\pi D_A^2}{4} = 0.0039 \text{ m}^2$$

$$A_B = \frac{\pi D_B^2}{4} = 0.0010 \text{ m}^2$$

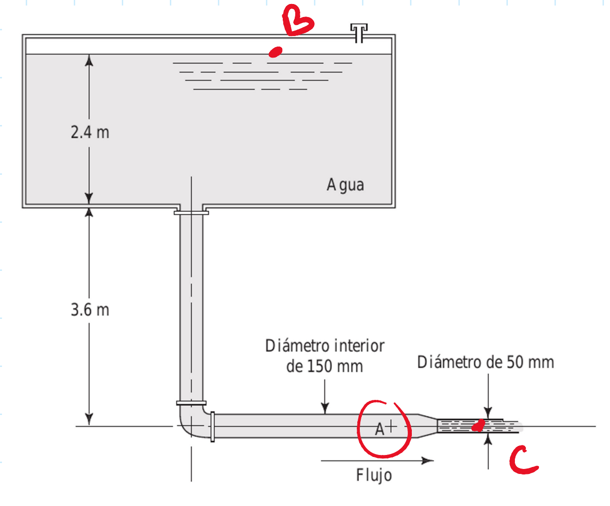
$$Z_B - Z_A = 3.65 \text{ m}$$

$$P_A = 5.65 \text{ kPa (man)}$$

$\therefore Q = 0.032 \ m^3/s$

Problema 3

Thursday, October 1, 2020 11:03 AM



Eq. de Bernoulli:

$$\overset{=0\ (\text{man})}{\cancel{\frac{p_c}{\gamma}}} + \frac{1}{2g} V_c^2 + z_c = \overset{=0\ (\text{man})}{\cancel{\frac{p_B}{\gamma}}} + \frac{1}{2g} \overset{\approx 0}{\cancel{V_B^2}} + z_B$$

$$Q_B = Q_C = Q$$

$$V_B \cdot A_B = V_c \cdot A_c$$

$$\Rightarrow V_B = V_c \cdot \frac{A_c}{A_B}$$

Con $A_c <<< A_B$

$$\rightarrow V_B <<< V_c$$

Además (con $V_B <<< V_c$):

$$\left(\frac{V_B}{V_c}\right)^2 <<< \left(\frac{V_B}{V_c}\right)$$

$$\Rightarrow V_B \approx 0$$

$$\Rightarrow \quad \frac{1}{2g} V_c^2 + z_c = z_{os}$$

$$\boxed{V_c = \sqrt{2g\,(z_{os} - z_c)}}$$ TEOREMA DE TORRICELLI

$$V_c = 10.85 \text{ m/s}$$

$$Q = V_c \cdot A_c = V_c \cdot \frac{\pi D_c^2}{4} = 0.021 \frac{m^3}{s}$$

B) Eq. de Bernoulli:

Puntos A y C:

$$\frac{P_A}{\gamma} + \frac{1}{2g} V_A^2 + \cancel{z_A} = \cancel{\frac{P_c}{\gamma}} + \frac{1}{2g} V_c^2 + \cancel{z_c}$$

$z_A = z_c$ — 0 (man) — $z_A = z_c$

$$P_A = \frac{\gamma}{2g}\left(V_c^2 - V_A^2\right)$$

Eq. de Continuidad:

$$Q_A = Q_c = Q$$

$$V_A \cdot A_A = V_c \cdot A_c$$

$$\Rightarrow V_A = V_c \cdot \frac{A_c}{A_A} = V_c \left(\frac{D_c}{D_A}\right)^2$$

$$P_A = \frac{\gamma}{2g}\left(V_c^2 - \left[V_c \left(\frac{D_c}{D_A}\right)^2\right]^2\right)$$

$$2g \quad (\qquad [\quad (D_A) \quad] \quad)$$

$V_C = 10.85 \ m/s$

$D_C = 50 \times 10^{-3} \ m$

$D_A = 150 \times 10^{-3} \ m$

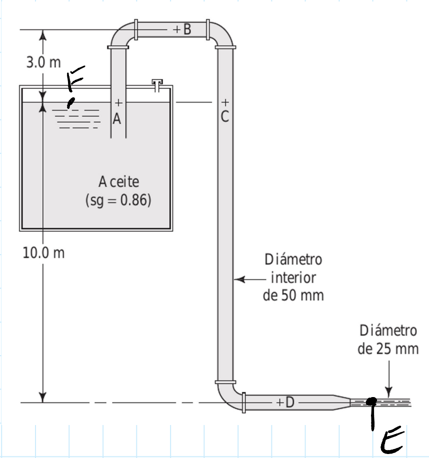
$\gamma = 9.81 \ kPa/m$

$g = 9.81 \ m/s^2$

$\therefore \ p_A = 58.1 \ kPa$

Problema 4

Wednesday, September 30, 2020 10:56 AM



Ecuación de continuidad

$$Q_A = Q_B = Q_C = Q_D = Q_E = Q$$

En el estanque:

$$A_F >>> A_A \rightarrow V_F \approx 0$$

$$p_F = 0 \quad (\text{manométrica})$$

En el chorro:

$$p_E = 0$$

A) Flujo volumétrico

Ecuación de Bernoulli: puntos E y F:

$$\cancel{\frac{p_E}{\gamma}} + \frac{1}{2}\frac{V_E^2}{g} + Z_E = \cancel{\frac{p_F}{\gamma}} + \frac{1}{2}\cancel{\frac{V_F^2}{g}} + Z_F$$

$$V_E = \sqrt{2g(Z_F - Z_E)} = 3.10 \text{ m/s}$$

$$Q = V_E \cdot A_E = 0.0015 \text{ m}^3/\text{s}$$

$$\boxed{A_E = \frac{\pi D_E^2}{4}}$$

$$V_A = V_B = V_C = V_D = \frac{Q}{A_A} = 0.78 \text{ m/s}$$

B) Presión en A:

Comparando A y F:

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{Z_A} = \cancel{\frac{p_F}{\gamma}} + \cancel{\frac{V_F^2}{2g}} + \cancel{Z_F} \quad \leftarrow Z_A = Z_F$$

$$p_A = -\frac{V_A^2}{2g}\cdot\gamma = -0.03 \frac{\left(\frac{m}{s}\right)^2}{m/s^2} \cdot 9.81 \frac{kPa}{m} = -0.3 \text{ kPa (man)}$$

$$p_A = 0.3 \text{ kPa (vac)}$$

Presión en B:

Comparando B y F:

$$p_B \quad V_B^2 \quad + Z_B = \cancel{p_F} \quad \cancel{V_F^2} \quad Z_F$$

Comparamos B y F:

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \cancel{\frac{p_F}{\gamma}} + \cancel{\frac{V_F^2}{2g}} + z_F$$

$$p_B = \gamma\left(-\frac{V_B^2}{2g} + (z_F - z_B)\right) = -29.7 \text{ kPa (man)}$$

$$\Rightarrow p_B = 29.7 \text{ kPa (vac)}$$

Presión en C:

$$\frac{p_C}{\gamma} + \frac{V_C^2}{2g} + \cancel{z_C} = \cancel{z_F}$$

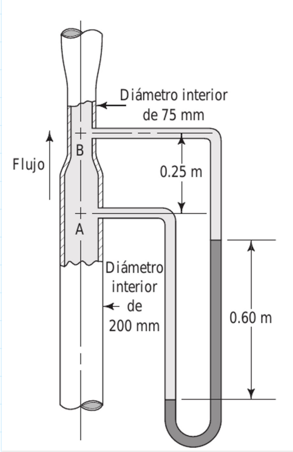
$$p_C = \gamma\left(-\frac{V_C^2}{2g}\right) = \gamma\left(-\frac{V_A^2}{2g}\right) = 0.3 \text{ kPa (vac)}$$

Presión en D:

$$\frac{p_D}{\gamma} + \frac{V_D^2}{2g} + z_D = z_F$$

$$p_D = \gamma\left(-\frac{V_D^2}{2g} + (z_F - z_D)\right) = 97.8 \text{ kPa (man)}$$

problema 6



Ecuacion de continuidad

$$Q_A = Q_B = Q$$

$$\Rightarrow V_A A_A = V_B A_B = Q.$$

$$V_A = \frac{Q}{A_A} \quad ; \quad V_B = \frac{Q}{A_B}$$

Ecuación de Bernoulli

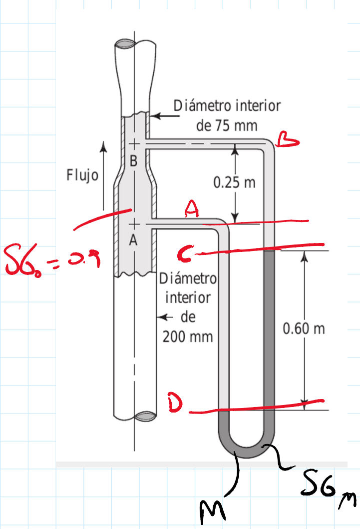
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{P_A}{\gamma} + \frac{\left(\frac{Q}{A_A}\right)^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{\left(\frac{Q}{A_B}\right)^2}{2g} + z_B$$

$$Q^2 \cdot \frac{1}{2g}\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right) = \frac{P_B - P_A}{\gamma} + (z_B - z_A)$$

$$\Rightarrow Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1}\left[\frac{P_B - P_A}{\gamma} + (z_B - z_A)\right]}$$

Del manómetro:



$$P_A + \cancel{\gamma_o(z_A - z_C)} + \gamma_o(z_C - z_D) - \gamma_m(z_C - z_D)$$
$$- \cancel{\gamma_o(z_A - z_C)} - \gamma_o(z_B - z_A) - P_B = 0$$

$$\left(\frac{P_B - P_A}{\gamma_o}\right) = (z_C - z_D) - \frac{\gamma_m}{\gamma_o}(z_C - z_D) - (z_B - z_A)$$

$$\frac{SG_m}{SG_o}$$

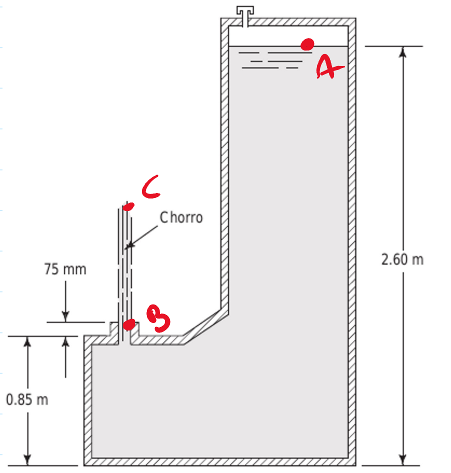
$$\left(\frac{P_B - P_A}{\gamma_o}\right) = -0.583 \text{ m}$$

$$\therefore Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1}\left[\frac{P_B - P_A}{\gamma} + (z_B - z_A)\right]} = 0.0114 \text{ m}^3/\text{s}$$

Problema B

Thursday, October 1, 2020 11:23 AM

$P_A = P_B = P_C = 0$ (man)



Ecuación de Bernoulli:

$$\underbrace{\cancel{\frac{P_A}{\gamma}}}_{0\,(\text{man})} + \cancel{\frac{1}{2g} V_A^2} + Z_A = \underbrace{\cancel{\frac{P_B}{\gamma}}}_{0\,(\text{man})} + \frac{1}{2g} V_B^2 + Z_B$$

$A_A >>> A_B$

$$\frac{1}{2g} V_B^2 + Z_B = Z_A$$

Puntos B y C:

$$\cancel{\frac{P_B}{\gamma}} + \frac{1}{2g} V_B^2 + Z_B = \cancel{\frac{P_C}{\gamma}} + \cancel{\frac{1}{2g} V_C^2} + Z_C$$

Punto máximo

$$Z_C = \frac{1}{2g} V_B^2 + Z_B = Z_A$$

$$\boxed{Z_C = 2.6\ m \text{ (desde la base del estanque)}}$$

De igual forma:

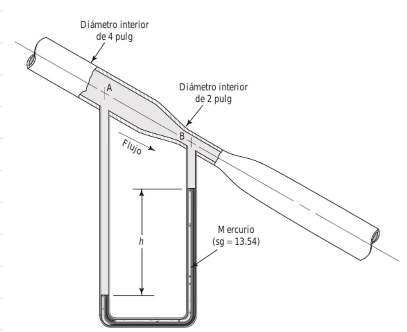
Ec. Bernoulli puntos A y C:

$$\underbrace{\cancel{\frac{P_A}{\gamma}}}_{0\,(\text{man})} + \underbrace{\cancel{\frac{V_A^2}{2g}}}_{V_A \approx 0} + Z_A = \underbrace{\cancel{\frac{P_C}{\gamma}}}_{0\,(\text{man})} + \underbrace{\cancel{\frac{V_C^2}{2g}}}_{0\,(\text{punto máximo})} + Z_C$$

$$\Rightarrow Z_A = Z_C = 2.6\ m$$

Problema 7

Wednesday, September 30, 2020 11:44 AM



Ecuación de continuidad

$$Q_A = Q_B = Q$$

$$V_A A_A = V_B A_B = Q$$

$$V_A = \frac{Q}{A_A} \quad ; \quad V_B = \frac{Q}{A_B}$$

A) Ecuación de Bernoulli:

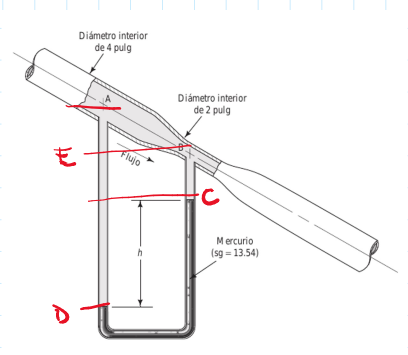
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{P_A}{\gamma} + \frac{\left(\frac{Q}{A_A}\right)^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{\left(\frac{Q}{A_B}\right)^2}{2g} + z_B$$

$$Q^2 \cdot \frac{1}{2g}\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right) = \frac{P_B - P_A}{\gamma} + (z_B - z_A)$$

$$\Rightarrow \quad Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1}\left[\frac{P_B - P_A}{\gamma} + (z_B - z_A)\right]}$$

En el manómetro:



$$P_A + \gamma_w(z_A - z_B) + \gamma_w\cancel{(z_E - z_C)} + \gamma_w(z_C - z_D) - \gamma_{Hg}(z_C - z_D) - \gamma_w\cancel{(z_E - z_C)} - P_B = 0$$

$$\Rightarrow \quad \frac{(P_B - P_A)}{\gamma_w} + (z_B - z_A) = (z_C - z_D) - \frac{\gamma_{Hg}}{\gamma_w}(z_C - z_D)$$

$$\Rightarrow \quad Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1}\left[(z_C - z_D) - \frac{\gamma_{Hg}}{\gamma_w}(z_C - z_D)\right]}$$

$$\Rightarrow Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right) \Big[\underbrace{\quad\quad}_{h} \quad \frac{\gamma}{\gamma_w} \quad \underbrace{\quad\quad}_{h} \Big]}$$

$$\gamma / \gamma_w \downarrow SG_{Hg}$$

$$Q = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1} \left[h - SG_{Hg} \cdot h \right]}$$

$$Q = 0.0277 \ m^3/s$$

B) Si $V_B = 10 \ ft/s \rightarrow h = ?$

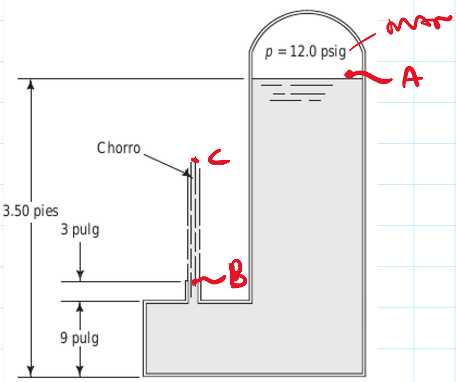
$$Q = V_B \cdot A_B = \sqrt{2g\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)^{-1} \left[h - SG_{Hg} \cdot h \right]}$$

Resolviendo iterativamente:

$$h = 0.0354 \ m$$

Problema 9

Wednesday, September 30, 2020 12:35 PM



Ecuación de Bernoulli punto A y B:

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}}^{\approx 0} + z_A = \cancel{\frac{P_B}{\gamma}}^{0\ (man)} + \frac{V_B^2}{2g} + z_B$$

$$\frac{V_B^2}{2g} = \frac{P_A}{\gamma} + (z_A - z_B)$$

$$V_B = \sqrt{2g\left[\frac{P_A}{\gamma} + (z_A - z_B)\right]} = 13.09 \text{ m/s}$$

Ecuación de Bernoulli puntos B y C:

$$\cancel{\frac{P_B}{\gamma}}^{0\ (man)} + \frac{V_B^2}{2g} + z_B = \cancel{\frac{P_C}{\gamma}}^{0\ (man)} + \cancel{\frac{V_C^2}{2g}}^{= 0\ (\text{máximo})} + z_C$$

$$z_C = \frac{V_B^2}{2g} + z_B = 9.5 \text{ m}$$

Ecuación de Bernoulli puntos A y C:

$$\cancel{\frac{P_C}{\gamma}} + \cancel{\frac{V_C^2}{2g}} + z_C = \frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + z_A$$

$$z_C = \frac{P_A}{\gamma} + z_A$$

$$\Rightarrow z_C - z_A = \frac{P_A}{\gamma} > 0$$