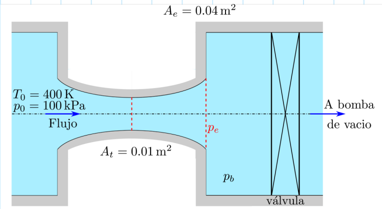
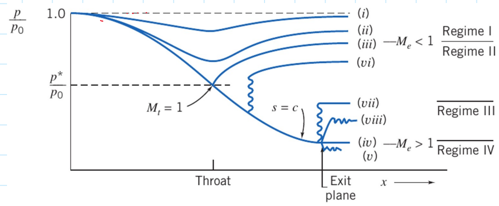
Problema 4

Monday, January 4, 2021 6:06 PM





a) condición de diseño

Flujo isoentrópico en todo el sistema

$$P_0 = P_{0e}$$

$$T_0 = T_{0e}$$

$$A^* = A_e^* = A_t = 0.01\ m^2$$

$$\frac{A_e}{A^*} = \frac{0.04\ m^2}{0.01\ m^2} = 4 = \frac{(1 + 0.2 Ma_e^2)^3}{1.728\ Ma_e}$$

$$\Rightarrow Ma_e = 2.94 \leftarrow \text{supersónico}$$

$$\frac{P_{0e}}{P_e} = 33.56 \rightarrow P_e = 2.97\ kPa$$

condición de diseño : $P_b = P_e = 2.97\ kPa$

b) • El flujo está estrangulado

$\hookrightarrow Ma_T = 1$

• El flujo en la zona divergente es subsónico

$\hookrightarrow Ma_e < 1$

• Todo el flujo es isoentrópico

$$P_{0e} = P_0$$

$$T_{0e} = T_0$$

$$A_e^* = A^* = 0.01\ m^2$$

$$\frac{A_e}{A^*} = \frac{0.04\ m^2}{0.01\ m^2} = 4 = \frac{(1 + 0.2 Ma_e^2)^3}{1.728\ Ma_e}$$

$\hookrightarrow Ma_e < 1$

$\hookrightarrow Ma_e = 0.1465$

$\therefore Ma_e < 1$

$\hookrightarrow Ma_e = 0.1465$

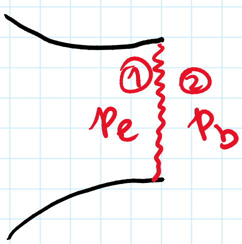
$\hookrightarrow \dfrac{P_{0e}}{P_e} = 1.015$

$$\Rightarrow \quad P_e = \frac{100\,kPa}{1.015} = 98.5\ kPa$$

$$P_e = P_b = 98.5\ kPa$$

c) La presión mínima para que existan ondas de choque normales va a estar dada por la formación de una onda de choque normal en el plano de salida

↳ Todo el flujo es isentrópico **HASTA** el plano de salida



De a) : $P_e = 2.97\ kPa$

$Ma_e = 2.94$

$$P_e = P_1$$
$$Ma_e = Ma_1$$

$$Ma_2 = Ma_b$$
$$P_2 = P_b$$

P_b estará dado por las relaciones de onda de choque normales:

$$\frac{P_2}{P_1} = \frac{P_b}{P_e} = \frac{1}{(k+1)}\left(2k\,\underset{Ma_e}{Ma_1^2} - (k-1)\right)$$

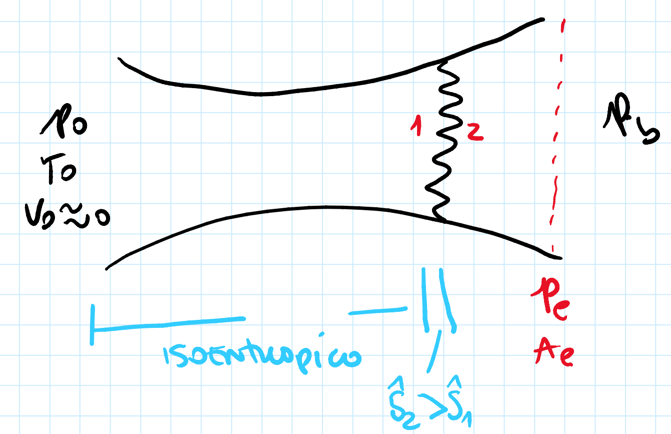
$\underline{P_b} = 9.9175$

$\overline{P_e}$

$$P_b = 9.9175 \cdot P_e = 9.9175 \cdot 2.94\ kPa$$

$$P_b = 29.2\ kPa$$

d) $P_b = 50\ kPa$



$\xrightarrow{\qquad}$ isoentrópico $\Rightarrow P_b = P_e$

$$\frac{\overset{50\,kPa}{P_e}}{\underset{P_0 = 100\,kPa}{P_{0_1}}} \cdot \frac{\overset{0.04\,m^2}{A_e}}{\underset{0.01\,m^2}{A_t}} = \underbrace{\left(\frac{P_e}{P_{0_e}}\right) \cdot \left(\frac{A_e}{A_e^*}\right)}_{\text{relaciones gases ideales}}$$

$$\frac{50\ kPa}{100\ kPa} \cdot \frac{0.04\ m^2}{0.01\ m^2} = 2 = \left(\frac{P_e}{P_{0_e}}\right) \cdot \left(\frac{A_e}{A_e^*}\right)$$

$$2 = \left(1 + \frac{k-1}{2} Ma_e^2\right)^{\frac{-k}{k-1}} \cdot \frac{1}{Ma_e} \left(\frac{1 + 1/2(k-1)\, Ma_e^2}{1/2\,(k+1)}\right)^{\frac{1}{2}\frac{k+1}{k-1}}$$

$$\Big\downarrow\ k = 1.4$$

$$Ma_e = 0.297$$

$$\hookrightarrow \frac{P_e}{P_{0_e}} = 0.944$$

$$\hookrightarrow \frac{A_e}{A_e^*} = 2.118$$

Además :

$$\frac{P_{0_2}}{P_{0_1}} = \frac{A_t}{A_e} \cdot \frac{A_e}{A_e^*} = \frac{0.01\,m^2}{0.04\,m^2} \cdot 2.118 = 0.5295$$

$$\Rightarrow \quad P_{0_2} = 0.5295 \cdot P_{0_1} = 0.5295 \cdot 100\,kPa$$

$$P_{0_2} = 52.95\ kPa$$

$$\frac{P_{0_e}}{P_e} = \frac{52.95\,kPa}{50\ kPa} = 1.059 = \left(\frac{P_e}{P_{0_e}}\right)^{-1} = (0.944)^{-1}$$

$$\frac{T_{0_e}}{T_e} = \left(\frac{P_{0_e}}{P_e}\right)^{\frac{k-1}{k}} = (1.059)^{\frac{0.4}{1.4}} = 1.016$$

$$T_{0_e} = T_{0_1} \Rightarrow T_e = 392.96$$

Para calcular Ma_1 :

$$\frac{P_{0_2}}{P_{0_1}} = 0.5295 = \left[\frac{(k+1)\,Ma_1^2}{2+(k-1)\,Ma_1^2}\right]^{\frac{k}{k-1}} \left[\frac{k+1}{2k\,Ma_1^2-(k-1)}\right]^{1/(k-1)}$$

$\downarrow$ Al resolverlo

$$Ma_1 = 2.425 \leftarrow \text{supersónico}$$

$$\frac{P_0}{P_1} = 15.2 \longrightarrow P_1 = \frac{100\,kPa}{15.2} = 6.6\ kPa$$

flujo isoentrópico

$$\frac{T_0}{T_1} = 2.18 \longrightarrow T_1 = \frac{400\,K}{2.18} = 183.5\,K$$

$$Ma_2 = \sqrt{\frac{(k-1)Ma_1^2 + 2}{2k\,Ma_1 - (k-1)}} \Rightarrow Ma_2 = 0.5205$$

$Ma_1 \longrightarrow$ $$\frac{P_2}{P_1} = 3.2428 \Rightarrow P_2 = P_1 \cdot 3.2428 = 21.4\,kPa$$

6.6 kPa

$$\frac{T_2}{T_1} = 2.0643 \Rightarrow T_2 = T_1 \cdot 2.0643 = 378.8\,K$$

183.5 K

Para el área transversal :

$$\left(\frac{A_1}{A^*}\right) = 2.425 \Rightarrow A_1 = A_T \cdot 2.425 = 0.1 \cdot 2.425$$

$$A_1 = 0.2425\,m^2$$

flujo isoentrópico

considerando Ma_1