ECE 242 Homework 4 (Due Nov 18, 5 PM PT)

NOTE: You will notice that some selected problems are given answers through "Show that" statements. Make sure you show all your work clearly for every problem. Throughout the assignment, $u(t)$ is the unit step function, $r(t)=tu(t)$ and $p(t)=u(t)-u(t-1)$

Complete the following exercises and submit answers as PDF (File>Save and Export As... > PDF) (You may also use : https://www.vertopal.com/ suggested by a student)

1. The impulse response function h(t) : y(t) = (x∗h)(t) or h[n] : y[n] = (x∗h)[n] is given for several continuous and discrete time systems. For each impulse response determine whether or not the associated system is (i) causal and (ii) stable. Provide reasoning for your conclusions.

a. $h(t) = e^{0.5(t-1)}u(t)$

b. $h(t) = (u(t+1) - u(t-1))(1-t)^2$

c. $h[n] = a^{-|n|}cos(\frac{\pi}{3}n), |a| < 1$

d. $h[n] = u[n+2] - 2u[n-1]$

Ans.

a. i) Causal because it only depends on past and present t values

ii) The system is not stable because $e^{0.5(t-1)}$ is unbounded

b. i) Not causal because depends on (t+1)

ii) Stable because even though as t approaches $-\infty (1-t)^2 is unbounded, the function is only defined from positive t = 1 onwards$

c. i) Causal, doesn't depend on future values of t

ii) Stable because all of the functions are bounded

d. i) Not causal, depends on $u[n+2]$ which is in the future

ii) Not stable, as n approaches infinity, the function will stay constant at -1

2. Consider the series composition of three LTI systems S_1, S_2 and S_3:

$$x(t) \to S_1 \to S_2 \to S_3 \to y(t)$$

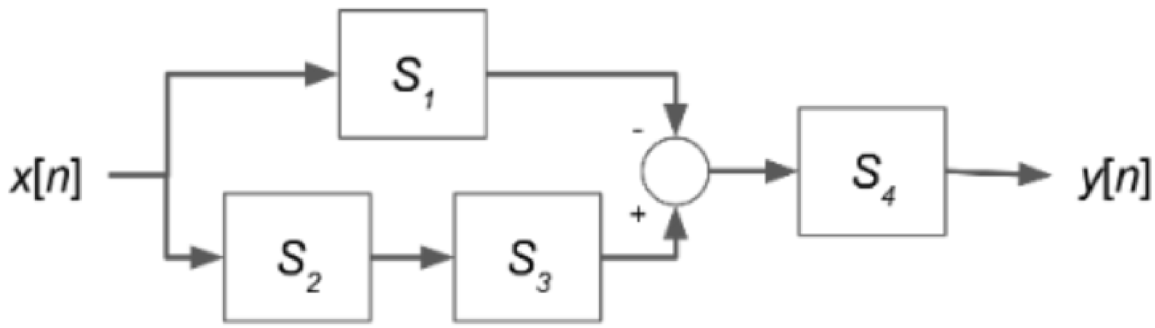
When x(t) = u(t), y(t) is observed to be equal to a bounded, differentiable signal g(t). Given that the impulse response functions for system S_1 is $h_1(t) = \delta(t-1)$ and $S_3[x(t)] = 3\frac{d}{dt}x(t)$, find the impulse response function $h_2(t)$ for system S_2.

Ans.

$h_2(t) = \int_{-\infty}^{\infty} tdt$

3. A set of LTI systems {S_i} are composed in series and parallel as indicated below: Compute the overall impulse response h[n]:y[n]=(x∗h)[n] given each individual subsystem impulse response:

- $h_1[n] = \delta[n-1]$
- $h_2[n] = e^{-n}u[n]$
- $h_3[n] = e^{-\frac{n}{2}}u[n-1]$
- $h_4[n] = \delta[n] - \delta[n-1]$



Ans.

$h[n] = e^{-\frac{3n}{2}}u[n-1](-\delta[n-1])$ $y[n] = x[n] * h[n]$

4. For each system shown, state whether or not the given observed input/output pair x(t),y(t) indicates the system is not time-invariant. Explain your reasoning.

a. $cos(\pi t) \overset{T_a}{\rightarrow} 3sin(\pi(t+1))$

b. $e^{-2t}cos(\pi t) \overset{T_b}{\rightarrow} e^{-2t}sin(\pi t)u(t)$

c. $a^{-n} \overset{T_c}{\rightarrow} ba^{-n-2}, |a| > 1, b \in R$

b. $cos(\frac{\pi}{4}n) + e^{-\frac{\pi}{2}|n|} \overset{T_d}{\rightarrow} sin(\frac{\pi}{4}n)$

Ans.

5. Compute the output signal y for the following paired inputs and LTI system impulse responses x and h, respectively:

a. $x(t) = e^{-\frac{t}{3}}u(t), h(t) = u(t) + \delta(t-1)$

b. $x(t) = e^{-t}cos(\frac{\pi}{2}t)u(t+1), h(t) = e^{-2t}u(t)$

c. $x[n] = e^{-(j\frac{\pi}{4}+1)n}u[n], h[n] = u[n]$

Ans.

a. $y(t) = -3e^{-\frac{t}{3}} - 3e^{\frac{1-t}{3}} + 6$

b. $y(t) = e^{-t}\cos\frac{\pi}{2}t$

c. $y(t) = \sum_{k=0}^{n} e^{-(j\frac{\pi}{4}+1)k}$

6. For a given fundamental frequency ω_0 and fourier coefficient sequence c_k, compute the original time-domain signal x(t) = $\sum_n c_k e^{jk\omega_0 t}$ in reduced form (in this case, no complex exponentials). If a fourier coefficient is not stated for a given index, assume that it is zero.

a. $\omega_0 = \frac{\pi}{4}, c_{-1} = \frac{1}{2j}e^{\frac{j\pi}{3}}, c_1 = -\frac{1}{2j}e^{\frac{-j\pi}{3}}$

b. $\omega_0 = \frac{\pi}{2}, c_{-3} = 2, c_0 = 1, c_3 = 2(1+j)$

Ans.

a. $x(t) = \sin(\frac{\pi}{4}t + \frac{\pi}{3})$

b. $x(t) = 1 + 2(1+j)\cos(\frac{3\pi}{2}t) - 2\sin(\frac{3\pi}{2}t)$

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