ECE 242 Homework 2 (Due Oct 26, 5 PM PT)

For t∈R,n∈Z: **u(t)** and **u[n]** represent the unit step functions and **r(t)** and **r[n]** represent the ramp functions in continuous and discrete time domains. **p(t)** = u(t)−u(t−1) is the unit pulse signal defined in class.

Complete the following exercises and submit answers as PDF (File>Save and Export As... > PDF) (You may also use : https://www.vertopal.com/ suggested by a student)

1. For each of the following systems T{·}, determine if the system is (i) memoryless, (ii) causal, (iii) stable, (iv) linear, (v) time-invariant. Prove your statements using the definitions for these conditions covered in lecture.

a. $T\{x(t)\} = -x(t+4)x(t)$

b. $T\{x[n]\} = x[4n+1]$

c. $T\{x[n]\} = \begin{cases} x[n+1] & n \geq 0 \\ x[n] & n < 0 \end{cases}$

d. $T\{x(t)\} = \int_{-\infty}^{-t} x(\tau)d\tau$

e. $T\{x(t)\} = \int_{-\infty}^{0} \tau^{-2}x(t-\tau)d\tau$

Ans.

a. (i) Not memoryless, ex) if t = 0, y(0) depends on x(4) which is in the future

(ii) Not causal, ex) if t = 0, y(0) depends on x(4) which is in the future

(iii) For $x(t) \leq M$ for all t

$|y(t)| = |-x(t+4)x(t)| = |x(t+4)||x(t)| \leq M^2$

Therefore stable

(iv) Not linear, ex) If x(t) = t, then y(t) =

$t^2 + 4t$

y(t+1) =

$t^2 + 2t + 1 + 4t + 4 = t^2 + 6t + 5$

x(t+1) = $t+1$ which is

$\neq t^2 + 6t + 5$

(v) Time-Invariant,

$y(t-t_0) = -x(t+4-t_0)x(t-t_0)$

$$Tx[t - t_0] = -x(t + 4 - t_0)x(t - t_0)$$

$$y(t) = Tx[t]$$

b. (i) Not memoryless, ex) if n = 0 y[n] = x[1] which is in the future

(ii) Not causal, ex) if n = 0 y[n] = x[1] which is in the future

(iii) For x[n] $\leq$ M for all n

$$|y[n]| = |x[4n + 1]| \leq M$$

Therefore stable

(iv) Not linear, if x[n] = n then at n = 0, y[n] = 1 which is nonlinear (v) If we assume x[n] to equal n, then $T\{x[n - n_0]\} = 4n - 4n_0 + 1$ whereas, $y[n] = 4n + 1$ and $y[n - n_0] = 4n - n_0 + 1$ Therefore it is Time Invariant

c. (i) Not memoryless, ex) if n = 0, y[n] = x[n+1] which depends on a future value

(ii) Not causal, ex) if n = 0, y[n] = x[n+1] which depends on a future value

(iii) For x[n] $\leq$ M for all n

$|y[n]| = |x[n + 1]|$ for n ≥ 0 and $|y[n]| = |x[n]|$ for n $\leq$ 0

$|y[n]| \leq M$ and
$|y[n]| \leq M$

Therefore stable

(iv) Not linear, if x[n] = n and n = 0 then y[0] = 1, which is nonlinear (v)
$$y[n - n_0] = x[n - n_0 + 1] or x[n - n_0]$$

$$Tx[n - n_0] = x[n - n_0 + 1] or x[n - n_0]$$

$$y[n - n_0] = Tx[n - n_0]$$

Therefore Time-Invariant

d. (i) Memoryful, the value of y(t) depends only on future values of t, for instance if t < 0, the integral of x(t) will contain some values of x(t > 0) as a result of the negative t upper bound.

(ii) Noncausal, for the same reason as above, y(t) may depend on a future value of t when the whole integral is computed. (iii) Not stable, ex) x(τ) = u(τ)then y(t) = r(t) which is an unbounded function (iv) Linear, T{ax(t)} = $a\int_{-\infty}^{-t} x(\tau) = a\int_{-\infty}^{-t} x(\tau)$ = aT{x(t)}

T{x_1(t) + x*2(t)} = $ |int*{-\infty}^{-t}{x_1(\tau) + x*2(|tau)} = | |int*{-\infty}^{-t}{x*1(|tau)} + |int*{-\infty}^{-t}{x_2(\tau)} = T{x_1(t)} + T{x_2(t)} \
$Therefore both additivity and scaling proved$ \ $(v) Not time invariant, if x($ \tau

$)= 1 then$ x_1(t) = t and x_2(t) = x_1(t - t_0) = t-t_0 $then$ \ y_1(t) = \frac{1}{2}t^2 and y_2(t) = \frac{1}{2}t^2 - 3t_0 \ne y_1(t - t_0) = \frac{1}{2}(t-t_0)^2 \ \ $

e. (i) Memoryless, the value of y(t) depends only on x(τ)

(ii) Causal, a memoryless system is always causal, y(t) depends only on x(τ)

(iii) Not stable, ex) if x(t) = 1 then y(t) = $-\tau^{-1}$ from $-\infty$ to 0 which is unbounded

(iv) Not linear, if x(t) = 1 then y(t) = $-\tau^{-1}$ from $-\infty$ to 0 which has a nonzero value and is therefore unbounded

(v) $y_1(t-t_0) = \int_{-\infty}^{0} \tau^{-2} x(t-t_0-\tau) = Tx(t-t_0)$ therfore time invariant

2. For T {·} in question 1(a), 1(b), and 1(c), determine if the system is invertible. If yes, compute the inverse. If no, demonstrate this fact mathematically.

Ans.

1a) is not invertible because t = 0 and t = 4 both give the same output of 0 for x(t) = t

1b) Invertible, inverse is y[$\frac{(n-0.25)}{4}$]

1c)Invertible inverse is $y[n-1] n \geq 0, y[n] n < 0$

3. For an LTI sytem T {·}, an input x1(t) = p(t) is paired with an output signal y1(t) = T{x1(t)} = p(t) − p(t − 1). Knowing this, compute T{x2(t)} when x2(t) = p(t) − 2p(t − 1). Show all the steps and assumptions.

Ans.

$T\{x(t)\} = x(t) - x(t-1)$

$T\{x_2(t)\} = p(t) - 2p(t-1) - (p(t-1) - 2p(t-2))$

$= p(t) - 3p(t-1) + 2p(t-2)$

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