EE 399 SPRING QUATER 2023
Instructor: **J. Nathan Kutz**

HOMEWORK #2:
DUE: Midnight on 4/17
Download **yalefaces.mat**.

`https://drive.google.com/file/d/1pqs5WAO7FKVL9GBZkvws6cB3ztLxwdQS/view?usp=sharing`

This file has a total of 39 different faces with about 65 lighting scenes for each face (2414 faces in all). The individual images are columns of the matrix $\mathbf{X}$, where each image has been downsampled to 32×32 pixels and converted into gray scale with values between 0 and 1. So the matrix is size 1024×2414. To important the file, use the following

```
import numpy as np
from scipy.io import loadmat
results=loadmat('yalefaces.mat')
X=results['X']
```

(a) Compute a 100×100 correlation matrix $\mathbf{C}$ where you will compute the dot product (correlation) between the first 100 images in the matrix $\mathbf{X}$. Thus each element is given by $c_{jk} = \mathbf{x}_j^T \mathbf{x}_k$ where $\mathbf{x}_j$ is the jth column of the matrix. Plot the correlation matrix using pcolor.
(b) From the correlation matrix for part (a), which two images are most highly correlated? Which are most uncorrelated? Plot these faces.
(c) Repeat part (a) but now compute the 10×10 correlation matrix between images and plot the correlation matrix between them.
$[1, 313, 512, 5, 2400, 113, 1024, 87, 314, 2005]$.
(Just for clarification, the first image is labeled as one, not zero like python might do)
(d) Create the matrix $\mathbf{Y} = \mathbf{X}\mathbf{X}^T$ and find the first six eigenvectors with the largest magnitude eigenvalue.
(e) SVD the matrix $\mathbf{X}$ and find the first six principal component directions.
(f) Compare the first eigenvector $\mathbf{v}_1$ from (d) with the first SVD mode $\mathbf{u}_1$ from (e) and compute the norm of difference of their absolute values.
(g) Compute the percentage of variance captured by each of the first 6 SVD modes. Plot the first 6 SVD modes