Math 20D Notes

Introduction to Differential Equations

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Taught by Professor Tu

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1   Introduction to Differential Equations

To motivate our definition of a differential equation, we first begin by talking about what an algebraic equation is. When we are given an algebraic equation like:

$$x^2 + 5x - 6 = 0$$

Our goal is to find the value of x. Here, x is the unknown. We need to find a solution (i.e. a number) for x such that the equation is satisfied.

In a differential equation, we are essentially doing the same thing. Given a differential equation, we need to find the unknown function that satisfies it. For instance, if we are given:

$$f'(x) - x = 0$$

It follows that $f'(x)$ is the unknown; that is, we need to find the function that satisfies this.

1.1   Why Study Differential Equations?

To answer this question, suppose we have a radioactive substance. Now, suppose the decay rate is 0.2 (in other words, the probability of decay at every instance is 0.2, or 20%). It follows that every atom could decay together or by themselves (depends on the chance).

Let A be the number of original atoms left. Let $A(t)$ be a function of t and let $A(0) = 10^{10}$ atoms. Then, we let:

$$\frac{dA}{dt} = -20\% \cdot A = \boxed{-0.2A}$$

Which is the same as saying:

$$A'(t) = -0.2A(t)$$

Which our goal is to find A.