

1 Machine (Floating-Point) Numbers (Section 2.1, Continued)

(Example.) Let $x = \frac{2}{3}$.

(1) What is the binary form of x ?

The algorithm for finding the binary form of the decimal is as follows:

- Given x , multiply it by 2. If the integer part of the result is 1, set the i th bit to 1. Otherwise, set it to 0.
- If the i th bit is 1, subtract $x \times 2$ by 1.
- Repeat the above until one of the following occurs:
 - You hit exactly 1, or
 - You hit 23 bits after the binary point (most likely, you'll see that the bits repeat in some way).

$$\frac{2}{3} \cdot 2 = \frac{4}{3} \geq 1 \implies 1$$

$$\frac{1}{3} \cdot 2 = \frac{2}{3} < 1 \implies 0$$

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\vdots

This gives us the binary representation $0.1010101010\dots$. Normalizing this gives us

$$(1.010101010101010101010101010101\dots)_2 \times 2^{-1}.$$

(2) Find x_- and x_+ .

Note that x_- is just what we found in the previous step, but with 23 bits to the right of the binary point,

$$(1.01010101010101010101010)_2 \times 2^{-1}.$$

Then, x_+ is

$$(1.01010101010101010101011)_2 \times 2^{-1}.$$

(3) What is $\text{fl}(x)$?

