1 Similar Matrices (5.4)

Two matrices, $A, B \in \mathbb{R}^{n\times n}$, are **similar** if there exists an invertible matrix $S \in \mathbb{R}^{n\times n}$ such that $AS = SB$. Equivalently,

$$A = SBS^{-1} \qquad B = S^{-1}AS.$$

A and B are called **orthogonally similar** if S is orthogonal and $A = SBS^{-1}$. IN this case, we actually have $A = SBS^T$.

Theorem 1.1

Similar matrices have the same eigenvalues.

That is, if $B = S^{-1}AS$ and v is an eigenvector of A to the eigenvalue λ, then $S^{-1}v$ is an eigenvector of B with respect to λ.