1 Secant Method (Section 3.3)

The secant method was motivated by Newton's method.

1.1 One-Variable Version

Recall from Newton's that

$$x_{m+1} = x_m - \frac{f(x_m)}{f'(x_m)}, \qquad m \geq 0.$$

Instead, we'll approximate $f'(x_m)$ by a difference quotient:

$$f'(x_m) \approx \frac{f(x_m) - f(x_{m-1})}{x_m - x_{m-1}}.$$

The secant method makes use of this; that is,

$$x_{m+1} = x_m - \frac{f(x_m)}{\frac{f(x_m)-f(x_{m-1})}{x_m - x_{m-1}}} = x_m - f(x_m)\frac{x_m - x_{m-1}}{f(x_m) - f(x_{m-1})}.$$

for $m \geq 1$.

Remarks:

- Method starts with x_0, x_1
- Only one function call used, $f(x_{m-1})$ stored from before.

1.2 Algorithm

This algorithm takes in the following

- M: the maximum number of iterations.
- δ: the tolerance interval, $|a - b| < \delta$.
- ϵ: the difference.

Algorithm 1 Secant Method

function SECANT$(a, b, M, \delta, \epsilon)$
    $f_a \leftarrow f(a)$
    $f_b \leftarrow f(b)$
    for $k \leftarrow 2$ to M **do**
        if $|f_a| > |f_b|$ **then**
            $f_a \leftrightarrow f_b$ ▷ Swap f_a and f_b
            $a \leftrightarrow b$ ▷ Swap a and b
        end if
        $s \leftarrow \frac{a-b}{f_a - f_b}$
        $b \leftarrow a$
        $f_b \leftarrow f_a$
        $a \leftarrow a - s \cdot f_a$
        $f_a \leftarrow f(a)$
        if $|f_a| < \epsilon$ or $|a - b| < \delta$ **then**
            break
        end if
    end for
    return a
end function