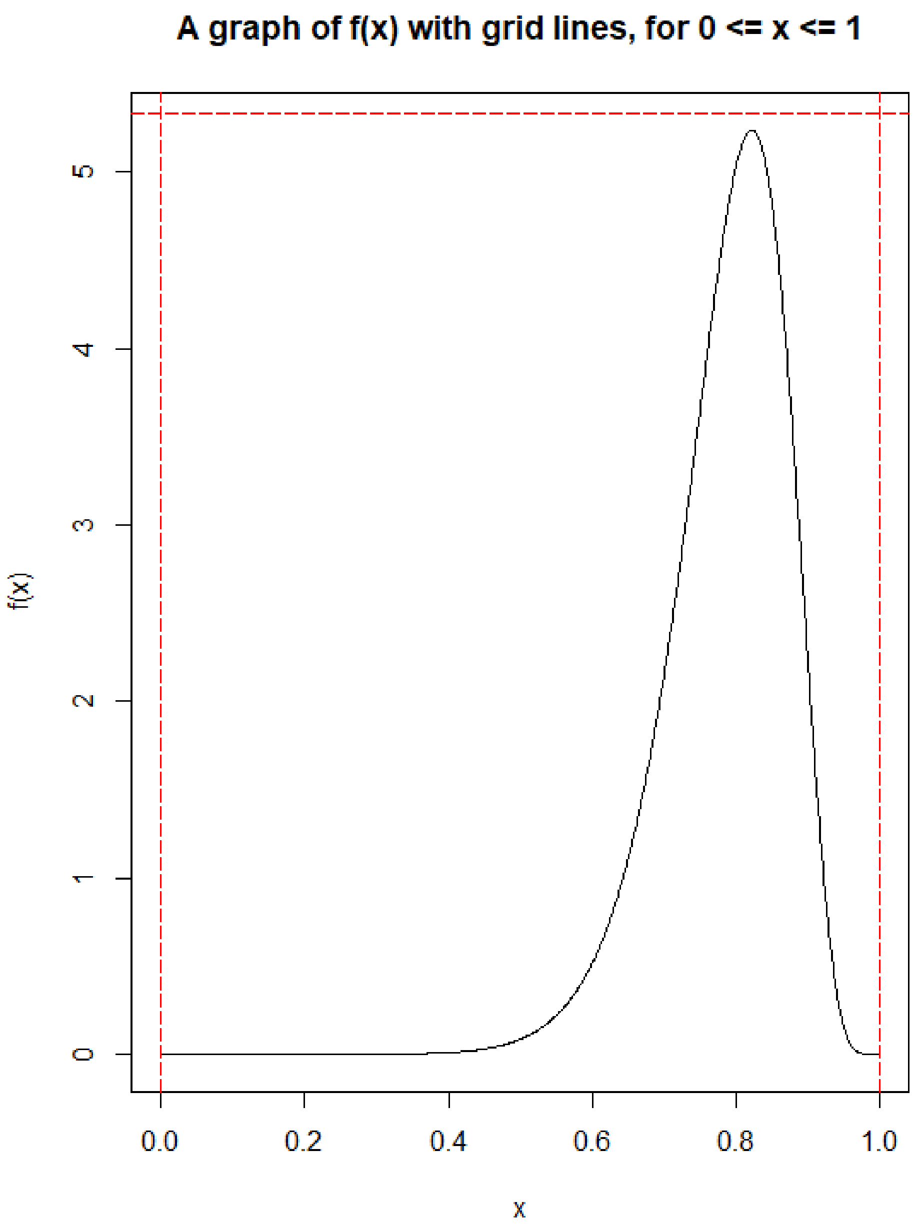
Ewan Walker 200818067

MAS2906 – Mini Project

1) a)

a = 11 b = 8 f(x) = $(11)(8)\mathrm{x}^{(11-1)}(1-\mathrm{x}^{11})^{(8-1)}$
$= 88\mathrm{x}^{(10)}(1-\mathrm{x}^{11})^{(7)}$

b)



c) 0.9967817

```r
set.seed(200818067)

a = sample (3:14 , 1) ; b = sample (3:14 , 1)
x = seq(0,1,0.001)
y = a*b*(x^(a-1))*((1-(x^a))^(b-1))

plot(x,y, xlab = "x" , ylab="f(x)" , type = "l", main="A graph of f(x) with g

abline(v = 0, lty = 5, col = "red")
abline(v = 1, lty = 5, col = "red")
abline(h = max(y)+0.1, lty = 5, col = "red")

MCI = function(N) {

  #set a tally variable for hits under the graph, above y=0
  no_of_hits = 0

  for(i in 1:N){

  #Generate x and y coords
  x0 = runif(1, 0, 1)
  y0 = runif(1, 0, max(y)+0.1)

  #f(x) at x
  f = a*b*(x0^(a-1))*((1-(x0^a))^(b-1))

  if(y0 < f){  #if the point is under the curve then
      no_of_hits = no_of_hits + 1
    }
  }

  P = no_of_hits / N            #The proportion under the curve
  area_under_curve = P*(1*(max(y)+0.1))       #The total area under
  return(area_under_curve)
}

MCI(1e6)
```

37:9 (Top Level) R Script

Console Terminal Background Jobs

R 4.2.1 · ~/

```
>
> abline(v = 0, lty = 5, col = "red")
> abline(v = 1, lty = 5, col = "red")
> abline(h = max(y)+0.1, lty = 5, col = "red")
>
> MCI = function(N) {
+
+   #set a tally variable for hits under the graph, above y=0
+   no_of_hits = 0
+
+   for(i in 1:N){
+
+   #Generate x and y coords
+   x0 = runif(1, 0, 1)
+   y0 = runif(1, 0, max(y)+0.1)
+
+   #f(x) at x
+   f = a*b*(x0^(a-1))*((1-(x0^a))^(b-1))
+
+   if(y0 < f){  #if the point is under the curve then
+       no_of_hits = no_of_hits + 1
+     }
+   }
+
+   P = no_of_hits / N            #The proportion under the curve
+   area_under_curve = P*(1*(max(y)+0.1))       #The total area under
+   return(area_under_curve)
+ }
>
> MCI(1e6)
[1] 0.9967817
>
```

2) a,b,c)

2 a) $f_X(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}$ for $x \geq 0$, σ const

$F_X(x)$?

pdf $\xrightarrow{\text{integrate}}$ cdf

$F_X(x) = \int f_X(x)\,dx = \int \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}\,dx$

From 0 to x so use dummy $x = t$,

$\implies \frac{1}{\sigma^2}\int_0^x t\, e^{\frac{-t^2}{2\sigma^2}}\,dt$

Substitution / by parts?

if $u = \frac{-t^2}{2\sigma^2}$, $\frac{du}{dt} = \frac{-t}{\sigma^2}$, $dt = \frac{-\sigma^2}{t}\,du$

$\implies \frac{1}{\sigma^2}\int_0^x \frac{-t\sigma^2}{t} e^u\,du$

$= -\int_0^x e^u\,du = -\left[e^u\right]_0^x$

$= -e^{\frac{-x^2}{2\sigma^2}} - -e^0 = 1 - e^{\frac{-x^2}{2\sigma^2}}$

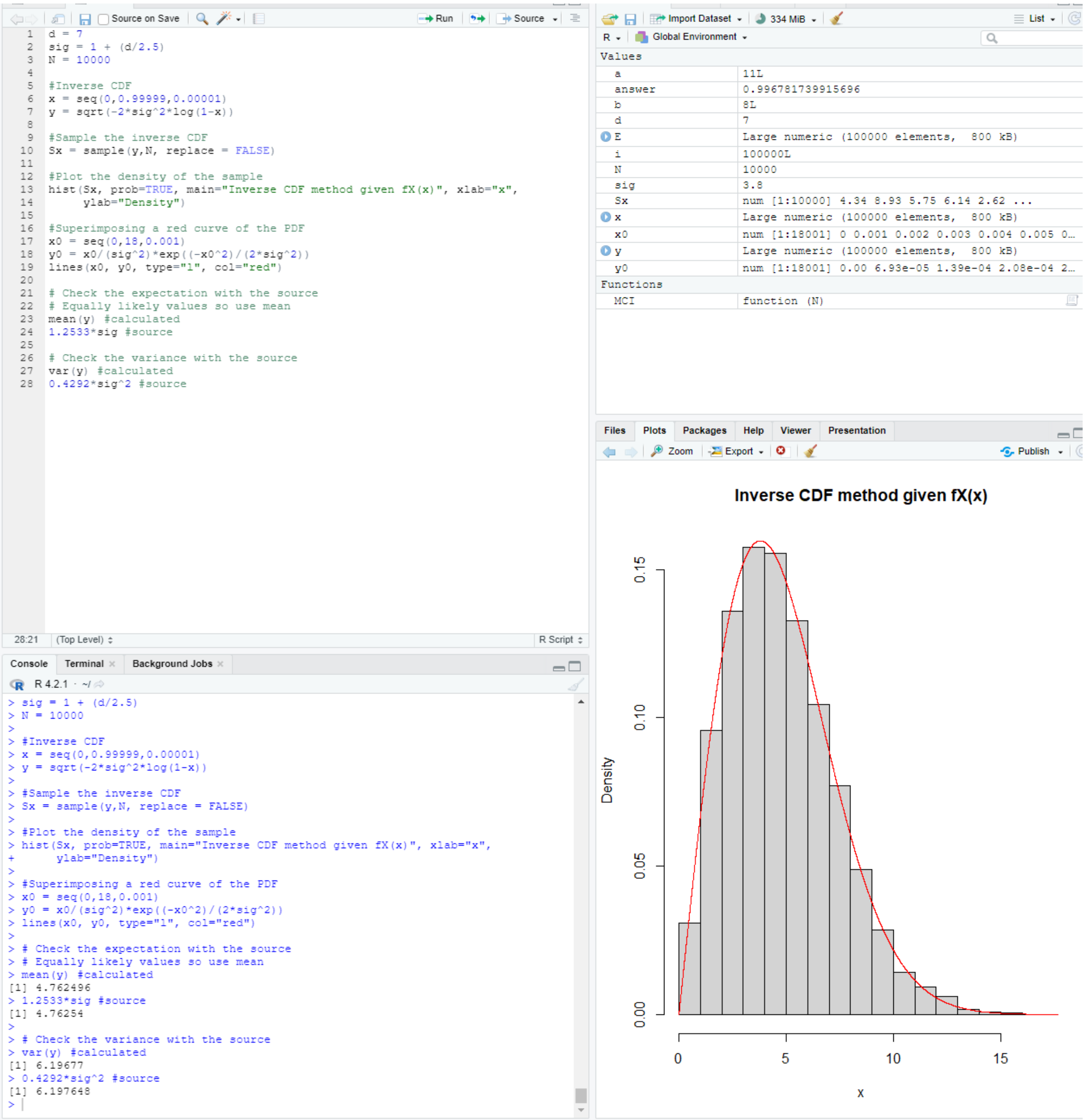
Now $F_X^{-1}(x)$

Let $y = 1 - e^{\frac{-x^2}{2\sigma^2}}$

$\ln(1-y) = \frac{-x^2}{2\sigma^2}$

$x = \sqrt{-2\sigma^2 \ln(1-y)}$

$F_X^{-1}(x) = \sqrt{-2\sigma^2 \ln(1-x)}$ for $0 \leq x < 1$



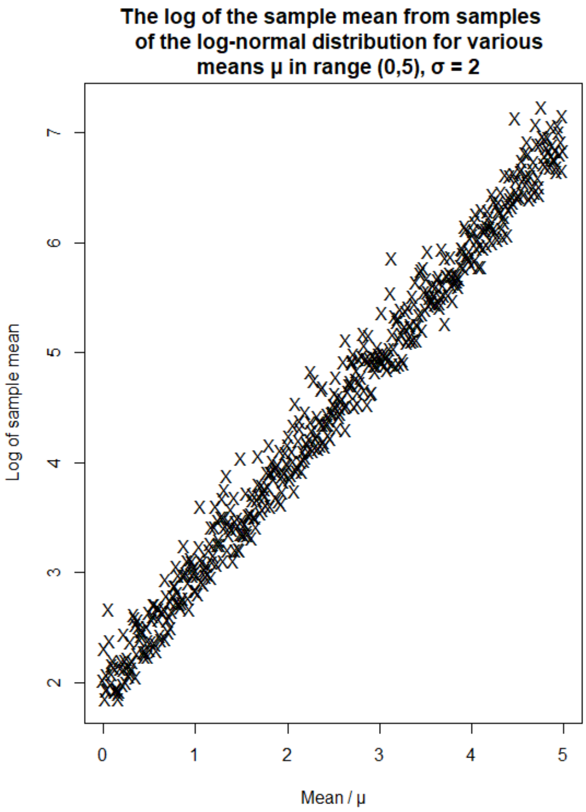
d) The name of the distribution is the Rician Distribution.

e) From the source [Rayleigh and Rician Distributions (ques10.com)](), the variance is defined as 0.4292σ^2. This is coherent with my var(y). Similarly, equal means. I couldn't specifically find the expectation online.

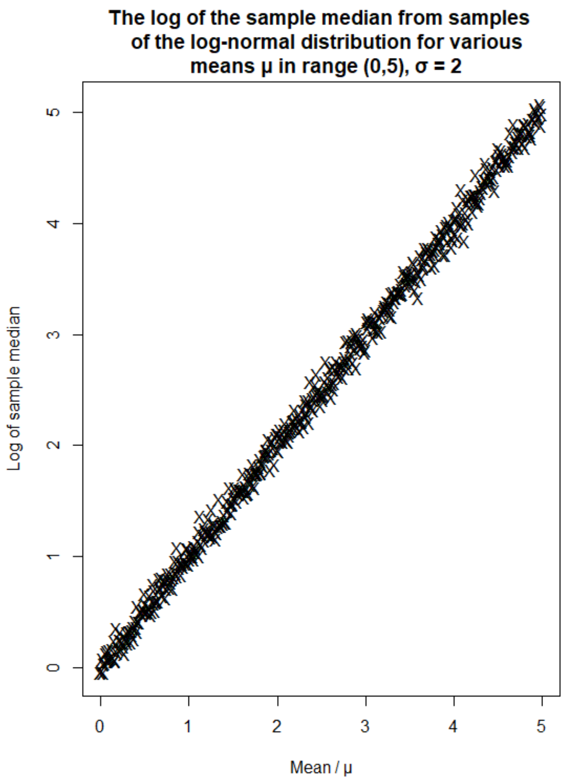
3) a)

```
LN = function(N, mu, sigma){
  return(exp(rnorm(N , mean = mu, sd = sigma ) ))
}
```

b)



c)



d) In conclusion, both the sample mean and medians of the LN distribution are directly proportional to the mean provided (constant of proportionality k=1, i.e. y = x + c). However, the log of the sample mean has a y intercept of 2 (i.e. y = x + 2); whereas, the log of the sample median is identical to the mean provided (i.e. y = x).

4) a) fX(x) = $\frac{8^{11}x^{11-1}e^{-8x}}{(11-1)!} = \frac{8^{11}x^{10}e^{-8x}}{(10)!}$

b) sigma = 3.8

(4b) $F_Y(y) = Pr(Y \leq y) = Pr\left(\frac{X}{3.8} \leq y\right) = Pr\left(X \leq \frac{19}{5}y\right)$

$= F_X\left(\frac{19}{5}y\right)$

$f_Y(y) = \frac{d}{dy}\left[F_Y(y)\right] = \frac{d}{dy}\left[F_X\left(\frac{19}{5}y\right)\right]$

$= f_X\left(\frac{19}{5}y\right)\left(\frac{19}{5}\right)$

$= \frac{19}{5}\left(\frac{8^{11}\left(\frac{19}{5}y\right)^{10}e^{-8\left(\frac{19}{5}y\right)}}{10!}\right)$

c) Pr(Y < E[Y]) = 0.3743

I also did a little plot when making sense of what the question was asking of me / the process. *(A hard question for 3 marks, unless I completely missed something. Integration, probably, but it's just before the deadline so..).*

$Pr(Y < E[Y]) = Pr(Y < E[\frac{X}{\sigma}])$

$= Pr(Y < \frac{1}{\sigma} E[X])$

$= Pr(Y < 3.8(\sum x_i \cancel{x}(Pr(x_i))))$

$3.8(\sum x_i Pr(x_i)) = 3.8(154.2927)$

$= 586.3124$

$Pr(Y < 586.3124)$

Find pdfs of Y in R and ~~sum~~ accumulate, then sum ones less than 586.3124. Use this sum in the x sequence for the relevant prob.

```
x=seq(0,1,0.0001)

fx = (8^11 * x^10 * exp(-8*x)) / factorial(10)

fy = (19/5) * (8^11*(((19/5)*x)^10)*exp(-8*(19/5)*x)) / factorial(10)

Fy = rep(0,length(fy))
for(i in 2:length(fy)){
  Fy[1] = fy[1]
  Fy[i] = fy[i]+Fy[i-1]
}

Ex = sum(x*fx)
Ey = Ex*3.8
x[ sum(Fy < Ey) ]

plot(x,Fy, type="l", ylab="Accumulative y")
```

35:1 (Top Level) R Script

Console Terminal Background Jobs

R 4.2.1 · C:/Users/Super Warrior/OneDrive - Newcastle University/Y2/Statistics with R/

```
>
> fy = (19/5) * (8^11*(((19/5)*x)^10)*exp(-8*(19/5)*x)) / factorial(10)
>
> Fy = rep(0,length(fy))
> for(i in 2:length(fy)){
+   Fy[1] = fy[1]
+   Fy[i] = fy[i]+Fy[i-1]
+ }
>
> Ex = sum(x*fx)
> Ey = Ex*3.8
> x[ sum(Fy < Ey) ]
[1] 0.3743
> x=seq(0,1,0.0001)
>
> fx = (8^11 * x^10 * exp(-8*x)) / factorial(10)
>
> fy = (19/5) * (8^11*(((19/5)*x)^10)*exp(-8*(19/5)*x)) / factorial(10)
>
> Fy = rep(0,length(fy))
> for(i in 2:length(fy)){
+   Fy[1] = fy[1]
+   Fy[i] = fy[i]+Fy[i-1]
+ }
>
> Ex = sum(x*fx)
> Ey = Ex*3.8
> x[ sum(Fy < Ey) ]
[1] 0.3743
>
> plot(x,Fy, type="l", ylab="Accumulative y")
>
```

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