Eliminacja Gaussa i Faktoryzacja LU

Rachunek Macierzowy i Statystyka Wielowymiarowa

Ewa Pelc
Ewa Żukowska

Faktoryzacja LU

Rozmiar macierzy: 20 + 5 = 25

```
n <- 25
A <- matrix(runif(n^2), nrow = n)
```

Pseudokod algorytmu LU

1. Zainicjuj $\mathbf{L}$ jako macierz identycznościową, $\mathbf{I}$ o wymiarach $n \times n$ oraz $\mathbf{U} = \mathbf{A}$.
2. Dla $i = 1, \ldots, n$ wykonaj krok 3.
3. Dla $j = i + 1, \ldots, n$ wykonaj kroki 4-5.
4. Ustaw $lji = uji / uii$.
5. Wykonaj $Uj = (Uj - lji * Ui)$ (gdzie Ui, Uj reprezentują odpowiednio wiersze i oraz j macierzy $\mathbf{U}$).

Algorytm faktoryzacji LU:

```
LU_factorization <- function(A) {
  n <- nrow(A)
  L <- matrix(0, nrow = n, ncol = n)
  U <- matrix(0, nrow = n, ncol = n)
  print(A)

  for (i in 1:n) {
    for (j in i:n) {
      U[i, j] <- A[i,j] - L[i,1:(i-1)] %*% U[1:(i-1),j]
    }
    for (j in i:n) {
      L[j,i] <- (A[j,i] - L[j,1:(i-1)] %*% U[1:(i-1),i]) / U[i,i]
    }
    L[i, i] <- 1
  }

  return(list(L = L, U = U))
}
```

Sprawdzenie poprawności LU faktoryzacji

W celu sprawdzenia poprawności faktoryzacji zdefiniowałyśmy funkcję, która porównuje dwa obiekty dopuszczając ustalony błąd.

```
is_allclose <- function(a, b, tol = 1e-10) {
  max_diff <- max(abs(a - b))
  return(max_diff < tol)
```

```
}
```

Sprawdzenie, czy A = LU

```
if (is_allclose(A, L %*% U)) {
  print("LU faktoryzacja jest poprawna.")
} else {
  print("LU faktoryzacja jest niepoprawna.")
}
```

Pseudokod algorytmu LU z pivotingiem

1. Zainicjuj macierze $\mathbf{L} = \mathbf{P} = \mathbf{I}$ o wymiarach $n \times n$ oraz $\mathbf{U} = \mathbf{A}$.
2. Dla każdego $i = 1, \ldots, n$ wykonaj kroki 3 - 4, 8.
3. Niech $k = i$.
4. Dopóki $uii = 0$, wykonuj kroki 5-7.
5. Zamień miejscami wiersz Ui z wierszem $Uk+1$.
6. Zamień miejscami wiersz Pi z wierszem $Pk+1$.
7. Zwiększ wartość k o 1.
8. Dla każdego $j = i+1, \ldots, n$ wykonaj kroki 9 - 10.
9. Ustaw wartość $lji = uji/uii$. Wykonaj operację $Uj = Uj - lji * Ui$ (gdzie Ui, Uj reprezentują odpowiednio i-ty i j-ty wiersz macierzy $\mathbf{U}$).

```
            [,1]         [,2] [,3]         [,4]         [,5] [,6]         [,7]         [,8]         [,9]         [,10]        [,11]        [,12]
 [1,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [2,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [3,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [4,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [5,] -1.110223e-16  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [6,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00
 [7,] -2.775558e-17  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [8,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [9,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00
[10,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00
[11,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00
[12,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00 -2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[13,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[14,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[15,]  0.000000e+00  0.000000e+00    0  0.000000e+00 5.551115e-17    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[16,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0 -2.775558e-17  0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00
[17,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00  0.000000e+00 -2.220446e-16
[18,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[19,]  1.110223e-16 -8.881784e-16    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[20,] -1.110223e-16  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00
[21,]  0.000000e+00  0.000000e+00    0 -2.775558e-17 0.000000e+00    0  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00 -1.110223e-16  0.000000e+00
[22,]  1.110223e-16  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[23,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[24,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00 -8.881784e-16  0.000000e+00  0.000000e+00  0.000000e+00
[25,]  0.000000e+00  0.000000e+00    0  0.000000e+00 0.000000e+00    0  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
            [,13]        [,14]        [,15]        [,16]        [,17]        [,18]        [,19]        [,20]        [,21]        [,22] [,23]        [,24]
 [1,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [2,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [3,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [4,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [5,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [6,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [7,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00     0  0.000000e+00
 [8,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
 [9,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[10,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[11,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[12,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0 -1.110223e-16
[13,]  0.000000e+00  0.000000e+00 0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16  0.000000e+00     0  0.000000e+00
[14,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[15,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00     0  0.000000e+00
[16,]  0.000000e+00  0.000000e+00 1.110223e-16 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[17,] -2.220446e-16  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[18,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[19,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[20,] -5.551115e-17 -1.110223e-16 0.000000e+00  0.000000e+00 -8.881784e-16  0.000000e+00  0.000000e+00  0.000000e+00 -2.220446e-16     0  0.000000e+00
[21,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[22,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
[23,]  2.220446e-16  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  1.110223e-16 1.110223e-16  0.000000e+00     0  0.000000e+00
[24,]  0.000000e+00  5.551115e-17 0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16  0.000000e+00 -2.220446e-16  0.000000e+00  0.000000e+00     0  0.000000e+00
[25,]  0.000000e+00  0.000000e+00 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00     0  0.000000e+00
          [,25]
```

W trakcie obliczeń numerycznych może wystąpić utrata precyzji, co prowadzi do drobnych różnic w wynikach.

Należy jednak zwrócić

Algorytm faktoryzacji LU z pivotingiem:

```
LU_factorization_with_pivoting <- function(A) {
  n <- nrow(A)
  L <- diag(n)  # Macierz L inicjujemy jako macierz jednostkową
  P <- diag(n)  # Macierz permutacji P inicjujemy jako macierz jednostkową
  U <- A        # Macierz U inicjujemy jako macierz A

  for (i in (1:(n-1)) ){
    k <- i

    # Szukanie elementu maksymalnego w kolumnie i
    while (U[i, i] == 0) {
      if (k > n) {
        return("Macierz jest zdegenerowana.")
      }
      U[c(k, k+1), ] <- U[c(k+1, k), ]
```

```
      P[c(k, k+1), ] <- P[c(k+1, k), ]
      k <- k + 1
    }

    # Eliminate entries below i with row operations on U
    # and reverse the row operations to manipulate L
    factor <- U[(i+1):n, i] / U[i, i]
    L[(i+1):n, i] <- factor
    U[(i+1):n, ] <- U[(i+1):n, ] - outer(factor, U[i, ])
  }

  return(list(P = P, L = L, U = U))
}
```

```
LU_decomposition_with_pivoting <- LU_factorization_with_pivoting(A)
L <- LU_decomposition_with_pivoting$L
U <- LU_decomposition_with_pivoting$U
```

Sprawdzenie, czy A = PLU:

```
if (is_allclose(A, P %*% L %*% U)) {
  print("LU faktoryzacja z pivotingiem jest poprawna.")
} else {
  print("LU faktoryzacja z pivotingiem jest niepoprawna.")
}
```

Poniższa macierz przedstawia różnicę, między macierzą A, a zaproponowanym przez nas rozkładem LU. Jak widać, na niektórych pozycjach wartości są różne, jednak różnica ta jest rzędu 10^16.

Funkcje wbudowane mogą stosować bardziej zaawansowane strategie wyboru elementu głównego w kroku faktoryzacji, co może prowadzić do dokładniejszych wyników.

```{r}
A - L %*% U
```

```
             [,1]         [,2]         [,3]         [,4]         [,5]         [,6]         [,7]         [,8]         [,9]        [,10]        [,11]        [,12]        [,13]
 [1,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [2,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [3,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [4,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [5,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  5.551115e-17  0.000000e+00
 [6,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00
 [7,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00 -5.551115e-17 -1.110223e-16  0.000000e+00  0.000000e+00
 [8,] -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17  0.000000e+00  5.551115e-17
 [9,]  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  2.220446e-16 -2.220446e-16  0.000000e+00
[10,]  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00  2.220446e-16
[11,] -2.775558e-17  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16 -1.110223e-16  0.000000e+00 -1.110223e-16
[12,] -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[13,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  4.440892e-16  2.220446e-16  0.000000e+00
[14,]  5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  3.330669e-16  4.440892e-16  0.000000e+00  0.000000e+00  0.000000e+00  2.220446e-16
[15,] -2.775558e-17  0.000000e+00  0.000000e+00  2.775558e-17  0.000000e+00 -1.110223e-16  1.110223e-16  0.000000e+00  1.110223e-16  0.000000e+00  2.220446e-16 -2.220446e-16  0.000000e+00
[16,] -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -2.775558e-17 -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[17,] -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  8.881784e-16 -4.440892e-16 -1.665335e-16  0.000000e+00
[18,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16 -5.551115e-17  0.000000e+00 -8.673617e-19  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17
[19,]  0.000000e+00  0.000000e+00  5.551115e-17  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  1.665335e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
[20,]  0.000000e+00  0.000000e+00 -5.551115e-17 -5.551115e-17  0.000000e+00  0.000000e+00 -1.110223e-16  5.551115e-17  2.220446e-16 -2.220446e-16  0.000000e+00  0.000000e+00  2.775558e-17
[21,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  1.110223e-16 -4.163336e-16  1.110223e-16 -2.220446e-16  0.000000e+00 -2.775558e-17 -2.220446e-16
[22,] -1.110223e-16  0.000000e+00  3.469447e-18  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00
[23,]  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  2.220446e-16  5.551115e-17 -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  2.775558e-16
[24,]  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-16  0.000000e+00  0.000000e+00  0.000000e+00 -4.996004e-16 -5.551115e-17
[25,] -2.220446e-16 -5.551115e-17  0.000000e+00 -1.110223e-16  1.110223e-16  0.000000e+00 -1.110223e-16 -1.110223e-16  0.000000e+00  0.000000e+00  4.440892e-16  2.220446e-16  0.000000e+00
             [,14]        [,15]        [,16]        [,17]        [,18]        [,19]        [,20]        [,21]        [,22]        [,23]        [,24]        [,25]
 [1,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [2,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [3,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [4,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [5,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [6,]  0.000000e+00  1.110223e-16 -5.551115e-17  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 [7,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  2.775558e-17  0.000000e+00 -1.110223e-16  0.000000e+00 -5.551115e-17  0.000000e+00  0.000000e+00 -1.110223e-16
 [8,] -5.551115e-17  0.000000e+00  0.000000e+00  0.000000e+00 -2.220446e-16  1.110223e-16 -1.110223e-16  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00
 [9,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  5.551115e-17 -1.110223e-16  2.220446e-16  2.220446e-16 -1.110223e-16  0.000000e+00  5.551115e-17
[10,]  0.000000e+00  0.000000e+00 -2.220446e-16 -2.220446e-16  5.551115e-17  1.110223e-16  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00 -5.551115e-17  1.110223e-16
[11,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00  0.000000e+00 -1.110223e-16 -2.220446e-16  0.000000e+00
[12,]  0.000000e+00  0.000000e+00  0.000000e+00 -8.881784e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00 -2.220446e-16  0.000000e+00
[13,] -5.551115e-17  0.000000e+00  0.000000e+00  4.440892e-16  2.220446e-16 -2.220446e-16  3.330669e-16  0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00 -2.220446e-16
[14,]  0.000000e+00  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00 -4.440892e-16  0.000000e+00  4.440892e-16  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00
[15,]  0.000000e+00 -2.220446e-16  0.000000e+00  1.110223e-16 -4.440892e-16  1.110223e-16  4.440892e-16  0.000000e+00  0.000000e+00  0.000000e+00  2.220446e-16 -2.220446e-16
[16,]  0.000000e+00  2.220446e-16 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  0.000000e+00  2.220446e-16  4.440892e-16  0.000000e+00  2.220446e-16
[17,]  1.110223e-16  0.000000e+00 -4.440892e-16 -6.661338e-16 -2.220446e-16  0.000000e+00  0.000000e+00 -2.220446e-16  2.220446e-16  0.000000e+00  0.000000e+00 -2.220446e-16
[18,]  1.110223e-16  0.000000e+00 -2.220446e-16  0.000000e+00  1.110223e-16  0.000000e+00  0.000000e+00  3.330669e-16 -4.440892e-16  4.440892e-16  2.220446e-16  0.000000e+00
[19,] -1.110223e-16 -1.110223e-16  0.000000e+00 -4.440892e-16 -1.110223e-16  4.440892e-16  1.110223e-16  0.000000e+00  1.110223e-16  0.000000e+00 -2.220446e-16 -1.110223e-16
[20,]  2.220446e-16 -2.220446e-16  0.000000e+00  0.000000e+00 -2.220446e-16  0.000000e+00 -6.661338e-16  0.000000e+00 -2.220446e-16  0.000000e+00  0.000000e+00 -2.775558e-17
[21,]  0.000000e+00  3.330669e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00 -1.110223e-16  5.551115e-16 -2.220446e-16  0.000000e+00
[22,]  0.000000e+00 -4.440892e-16  0.000000e+00  0.000000e+00  2.220446e-16  4.996004e-16 -1.110223e-16  1.110223e-16  0.000000e+00  0.000000e+00 -4.440892e-16  4.440892e-16
[23,]  0.000000e+00  4.440892e-16  0.000000e+00  0.000000e+00  0.000000e+00 -5.551115e-17  0.000000e+00 -1.110223e-16  0.000000e+00  0.000000e+00  0.000000e+00  2.220446e-16
[24,]  0.000000e+00  0.000000e+00 -4.440892e-16  4.440892e-16 -1.110223e-16  0.000000e+00  4.440892e-16 -4.440892e-16  0.000000e+00  0.000000e+00  4.440892e-16  0.000000e+00
[25,]  0.000000e+00  0.000000e+00  0.000000e+00  4.440892e-16  0.000000e+00  0.000000e+00  2.220446e-16  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
```

Eliminacja Gaussa

Macierz rozmiaru dzień urodzenia + miesiąc urodzenia

```
date <- 28 + 6
birthday_matrix <- matrix(sample(1:9, date*date, replace=TRUE), nrow=date)
```

Wektor wyrazów wolnych do układu równań

```
v <- t(matrix(runif(date, min=-10, max=20), nrow=1))
```

Pseudokod algorytmu eliminacji Gaussa generującego jedynki

1. Dla każdego wiersza 'row' od 1 do (n - 1):

- a. coef_div = m_copy[row, row]
- b. Dla każdego wiersza 'j' od (row + 1) do n:

- i. coef_sub = m_copy[j, row]
- ii. m_copy[j,] = m_copy[j,] - m_copy[row,] * coef_sub

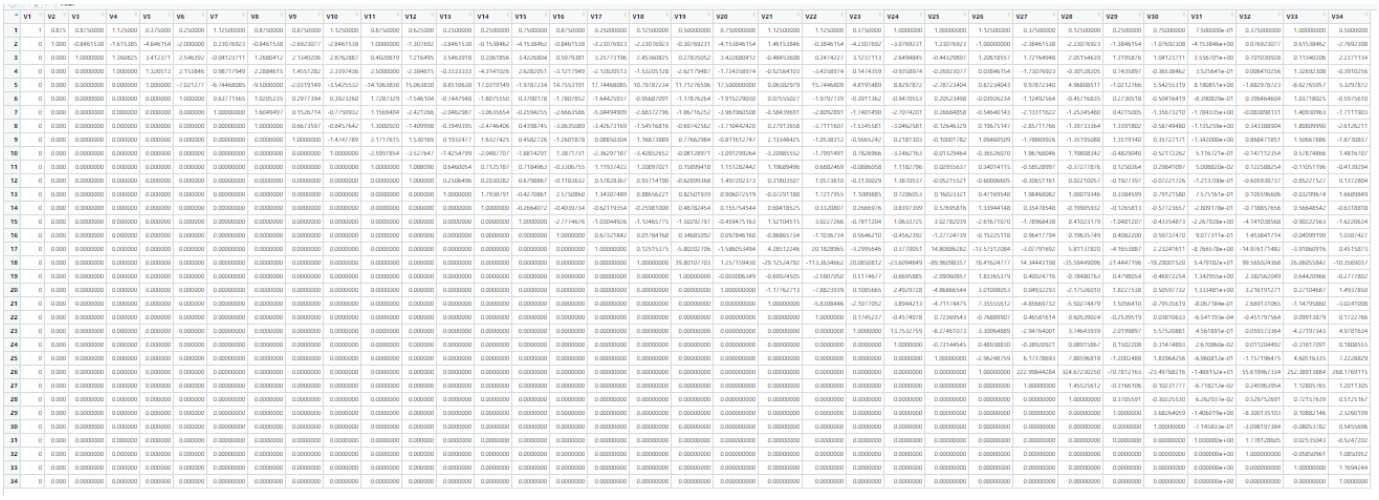
2. Jeśli ostatni wiersz nie jest znormalizowany (m_copy[n, n] != 1):

- a. m_copy[n,] = m_copy[n,] / m_copy[n, n]

3. Zwróć macierz (bez ostatniej kolumny) i ostatnią kolumnę (wektor b) z m_copy.

Algorytm eliminacji Gaussa generujący jedynki na przekątnej

```
gauss <- function(m, b) {
  m_copy <- m
  n <- nrow(m)

  m_copy <- cbind(m_copy, b)

  for (row in 1:(n - 1)) {
    coef_div <- m_copy[row, row]
    m_copy[row, ] <- m_copy[row, ] / coef_div

    for (j in (row + 1):n) {
      coef_sub <- m_copy[j, row]
      m_copy[j, ] <- m_copy[j, ] - m_copy[row, ] * coef_sub
    }
  }

  # Normalizacja ostatniego wiersza, jeśli to konieczne
  if (m_copy[n, n] != 1) {
    m_copy[n, ] <- m_copy[n, ] / m_copy[n, n]
  }

  # Zwrócenie macierzy i wektora b
  return(list(matrix = m_copy[, -ncol(m_copy)], vector = m_copy[, ncol(m_copy)]))
}
```



Pseudokod algorytmu eliminacji Gaussa z pivotingiem

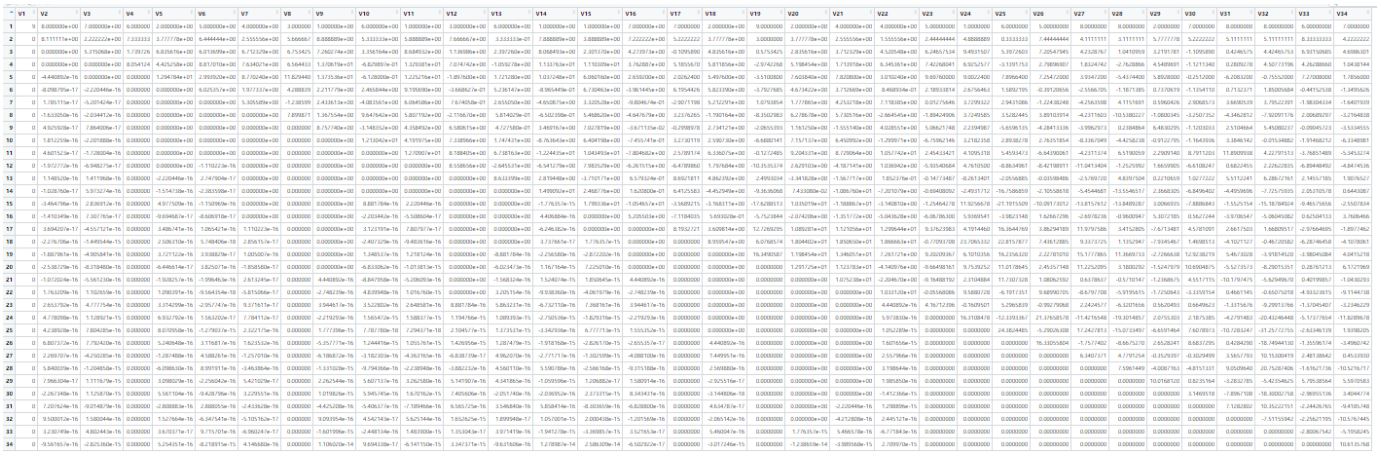
1. Dla każdej kolumny 'i' od 1 do (n - 1):

- a. Znajdź pivot jako maksymalną wartość w kolumnie 'i' dla wierszy od (i + 1) do n.
- b. Znajdź indeks wiersza z maksymalnym pivotem: max_row_index = indeks wiersza z maksymalnym pivotem + i.
  - Jeśli pivot > m_copy[i, i]: Zamień miejscami wiersze 'i' i 'max_row_index' w macierzy 'm_copy'.
  - Dla każdego wiersza 'j' od (i + 1) do n:
    - i. Oblicz współczynnik coef = m_copy[j, i] / m_copy[i, i].
    - ii. Odjęcie wiersza 'i' pomnożonego przez coef od wiersza 'j'.

2. Zwróć macierz (bez ostatniej kolumny) i ostatnią kolumnę (wektor b) z m_copy.

Algorytm eliminacji Gaussa z pivotingiem

```
gauss_with_pivot <- function(m, b) {
  m_copy <- m
  n <- nrow(m)

  m_copy <- cbind(m_copy, b)

  for (i in 1:(n - 1)) {
    pivot <- max(m_copy[(i + 1):n, i])
    max_row_index <- which.max(m_copy[(i + 1):n, i] == pivot) + i

    if (pivot > m_copy[i, i]) {
      temp_row <- m_copy[i, ]
      m_copy[i, ] <- m_copy[max_row_index, ]
      m_copy[max_row_index, ] <- temp_row
    }

    for (j in (i + 1):n) {
      coef <- m_copy[j, i] / m_copy[i, i]
      m_copy[j, ] <- m_copy[j, ] - m_copy[i, ] * coef
    }
  }

  # Return the matrix and b vector separately
  return(list(matrix = m_copy[, -ncol(m_copy)], vector = m_copy[, ncol(m_copy)]))
}
```



Pseudokod funkcji solve()

1. Wykonaj eliminację Gaussa na macierzy 'm' i wektorze 'b' za pomocą funkcji 'func', która zwraca macierz współczynników i wektor 'b' po eliminacji.
2. Przypisz macierz współczynników do 'coef_matrix' i wektor 'b' do 'b_vector'.
3. Określ liczbę równań 'n' na podstawie liczby wierszy macierzy 'coef_matrix'.
4. Zainicjuj wektor wynikowy 'x' jako pusty wektor numeryczny o długości 'n'.
5. Wsteczne podstawianie:

- a. Dla każdego i od n do 1, malejąco:
  - i. Oblicz x[i] jako iloraz b_vector[i] i coef_matrix[i, i].
  - ii. Jeśli i < n, oblicz x[i] jako różnicę b_vector[i] i sumy iloczynów elementów coef_matrix[i, (i + 1):n] i x[(i + 1):n], podzieloną przez coef_matrix[i, i].

6. Zwróć wektor wynikowy 'x'.

Funkcja solve() rozwiązująca układ równań

```
solve <- function(m, b, func) {
  # Gauss elimination
  result <- func(m, b)
  coef_matrix <- result$matrix
  b_vector <- result$vector
  print(coef_matrix)

  n <- nrow(coef_matrix)
  x <- numeric(n)

  # Wsteczne podstawianie
  for (i in seq(n, 1, -1)) {

    x[i] <- b_vector[i]/coef_matrix[i, i]
    if (i < n) {
      x[i] <- (b_vector[i] - sum(coef_matrix[i, (i + 1):n] * x[(i + 1):n])) / coef_matrix[i, i
    }
  }
```

```
  return(x)
}
```

Obliczenie wyników korzystając z obu algorytmów eliminacji Gaussa

```
# calculate results for Gauss and Gauss with Pivot
x6 <- solve(birthday_matrix, v, gauss)
x7 <- solve(birthday_matrix, v, gauss_with_pivot)
```

Sprawdzenie czy rozwiązania są identyczne dla obu metod eliminacji

```
identical(x6, x7)
```

W celu sprawdzenia poprawności (czy oryginalna macierz przemnożona przez wektor X rozwiązań jest równa wektorowi b) ponownie wykorzystujemy funkcję *is_allclose()*, która dopuszcza ustalony błąd

```
# check if correct - Gauss
r1 <- birthday_matrix %*% x6
is_allclose(v,r1)
```

```
# check if correct - Gauss with pivot
r2 <- birthday_matrix %*% x7
is_allclose(v,r2)
```

```
> # check if correct - Gauss
>
> r1 <- birthday_matrix %*% x6
> identical(r1, v)
[1] FALSE
> v - r1
                [,1]
 [1,]  7.549517e-15
 [2,]  3.552714e-15
 [3,] -8.881784e-16
 [4,] -3.552714e-15
 [5,] -2.131628e-14
 [6,]  7.105427e-15
 [7,] -3.375078e-14
 [8,]  1.065814e-14
 [9,]  1.731948e-14
[10,]  5.062617e-14
[11,]  2.398082e-14
[12,]  3.552714e-15
[13,] -3.375078e-14
[14,]  2.486900e-14
[15,] -5.151435e-14
[16,]  1.754152e-14
[17,]  0.000000e+00
[18,] -1.687539e-14
[19,] -1.314504e-13
[20,] -1.243450e-14
[21,]  4.884981e-14
[22,] -2.309264e-14
[23,] -5.382361e-13
[24,]  2.944311e-13
[25,] -3.224421e-12
[26,] -1.776357e-14
[27,]  1.026734e-12
[28,] -4.085621e-14
[29,]  1.822542e-12
[30,]  1.225686e-13
[31,]  2.122302e-12
[32,] -2.257750e-12
[33,] -8.881784e-15
[34,] -2.238210e-13
> is_allclose(v,r1)
[1] TRUE
```

```
> r2 <- birthday_matrix %*% x7
> identical(r2, v)
[1] FALSE
> v - r2
                [,1]
 [1,]  6.217249e-15
 [2,]  4.973799e-14
 [3,]  2.220446e-15
 [4,] -1.776357e-14
 [5,]  2.131628e-14
 [6,] -3.552714e-15
 [7,] -6.217249e-15
 [8,] -3.552714e-15
 [9,]  9.769963e-15
[10,]  2.398082e-14
[11,]  2.220446e-14
[12,] -5.329071e-15
[13,]  1.776357e-14
[14,]  4.973799e-14
[15,] -1.332268e-14
[16,] -3.574918e-14
[17,]  2.309264e-14
[18,] -3.019807e-14
[19,]  1.687539e-14
[20,] -2.664535e-14
[21,] -1.509903e-14
[22,]  8.881784e-16
[23,]  1.953993e-14
[24,]  1.554312e-15
[25,]  7.327472e-15
[26,]  1.065814e-14
[27,] -7.105427e-15
[28,]  1.154632e-14
[29,]  1.598721e-14
[30,] -5.329071e-15
[31,]  1.110223e-15
[32,]  5.506706e-14
[33,]  0.000000e+00
[34,] -3.552714e-15
> is_allclose(v,r2)
[1] TRUE
```