S1336 - Project 1

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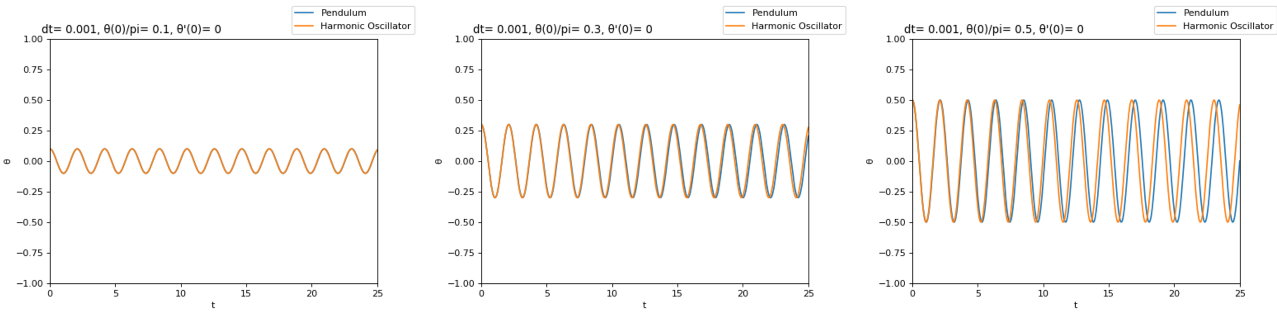
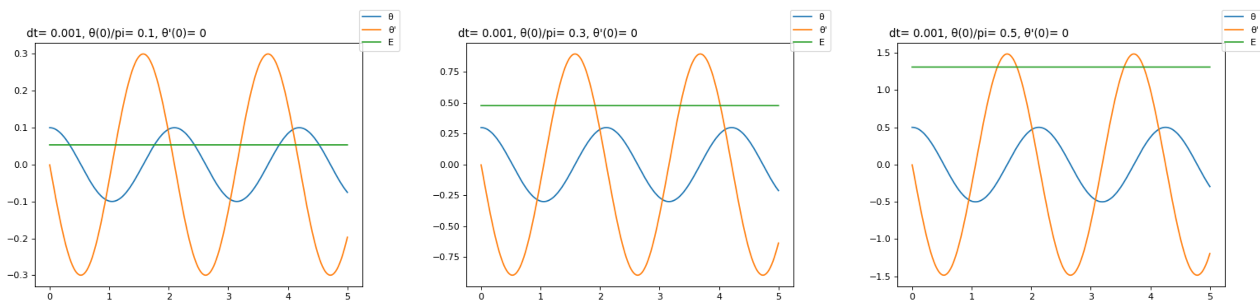
1.1

Pendulums vs harmonic oscillators



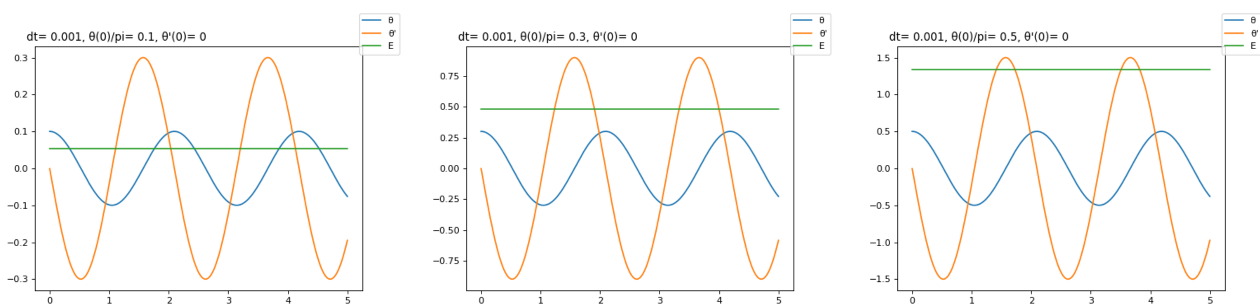
In this first part, it can be seen that a numerical solution for a pendulum has longer period than for that of a harmonic oscillator. Increasing the initial angle increases the amplitude of oscillations.

Pendulums: general solution



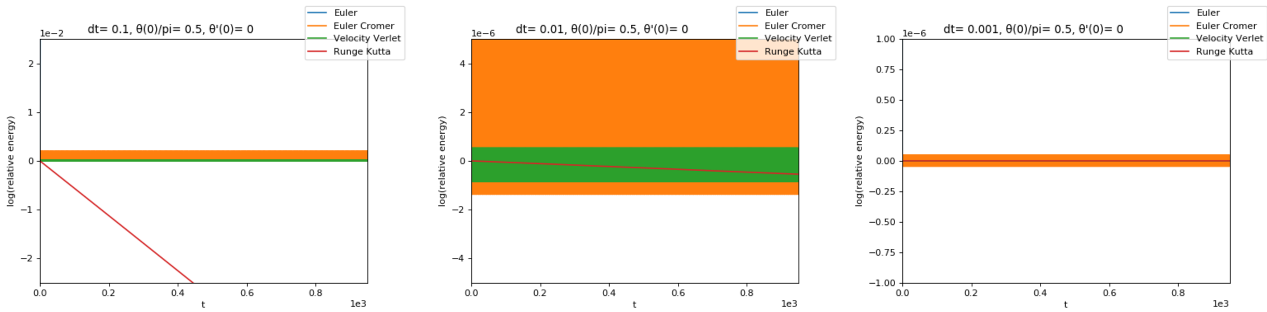
It can be seen that the solution doesn't scale linearly with increased starting angle.

Harmonic oscillators: general solution



More or less the same result as for a pendulum, though with a slightly shorter period as previously shown.

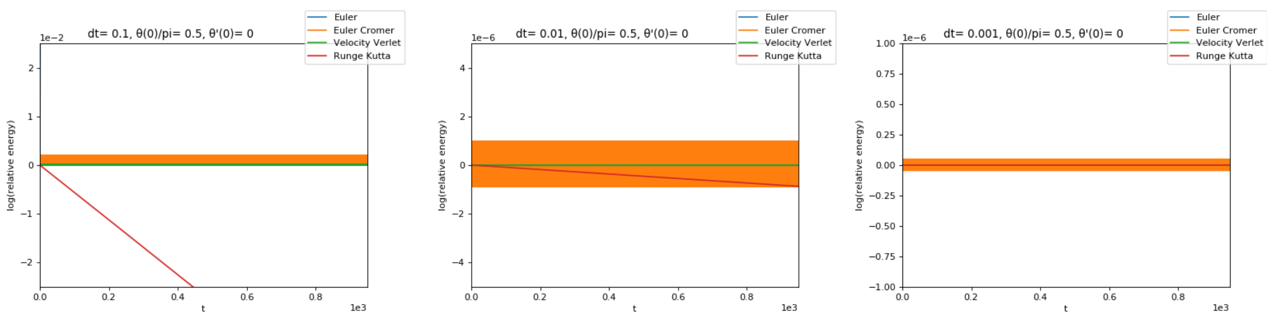
Rolling mean of energy for pendulum



We can't even see the simulation using Euler in this plot - it shot right up within mere seconds. Runge Kutta becomes "good" on timescales on the order of 1000 seconds for $\Delta t \approx 0.01$. For longer timescales, we need a smaller Δt.

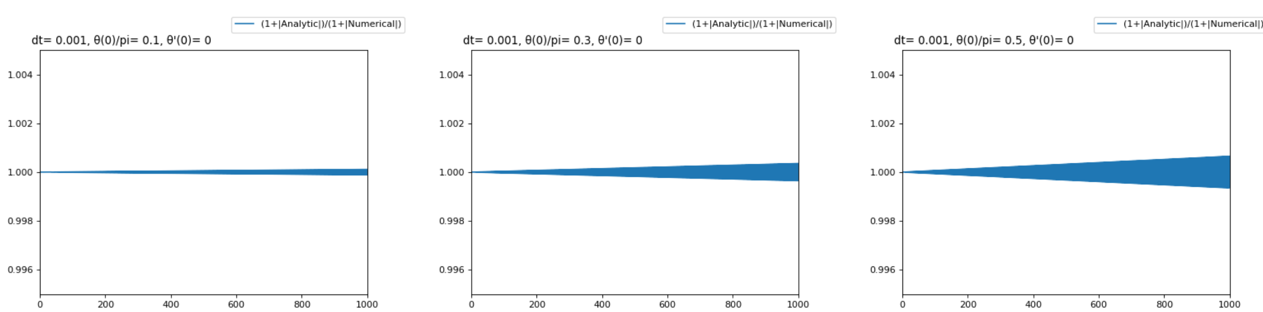
Both Euler-Cromer and Velocity Verlet have energies that oscillate fast enough to cause aliasing effects (hence why the rolling average of those are a solid block of color).

Rolling mean of energy for harmonic oscillator



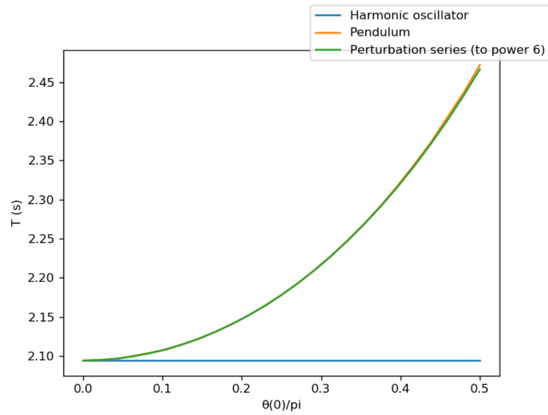
For the harmonic oscillator, we get basically the same results as for the pendulum but the energies oscillate less for Euler-Cromer and Velocity Verlet.

Harmonic oscillators: Numerical vs analytic



Using Verlet integration: sees a drift in period, but the numerical integration is stable.

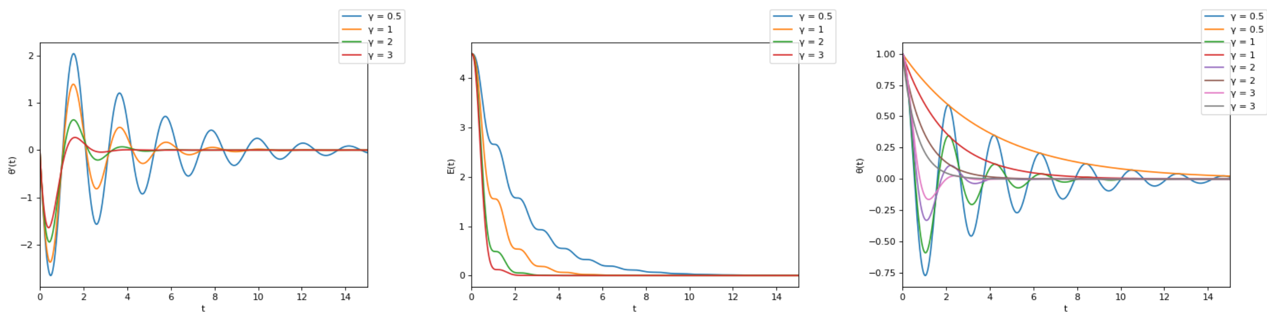
1.2



Harmonic oscillator has constant period, smaller than pendulum. The perturbation series does not approximate the harmonic oscillator well.

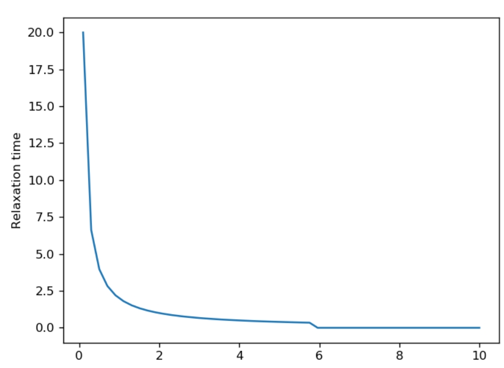
1.3

Dampened harmonic oscillator



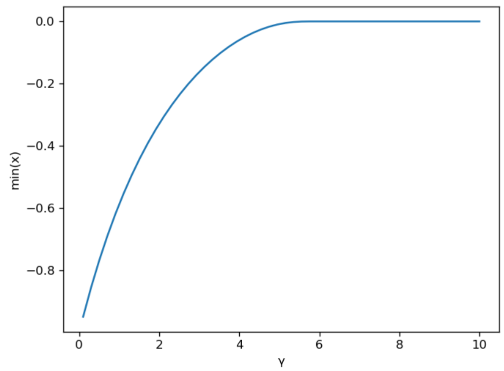
Energy decreases over time with dampening.

Relaxation time as function of γ



Fitting $f(t) = Ae^{-Bt}$ to local peaks of simulated oscillations.

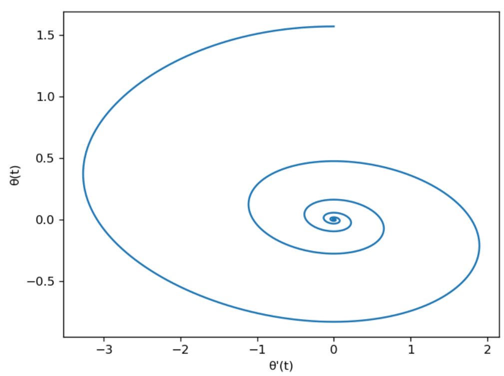
Minimum x as function of γ



Energy decreases over time with dampening.

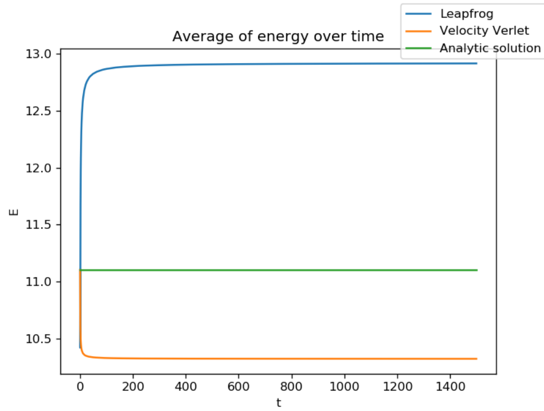
1.4

Phase space portrait of dampened pendulum



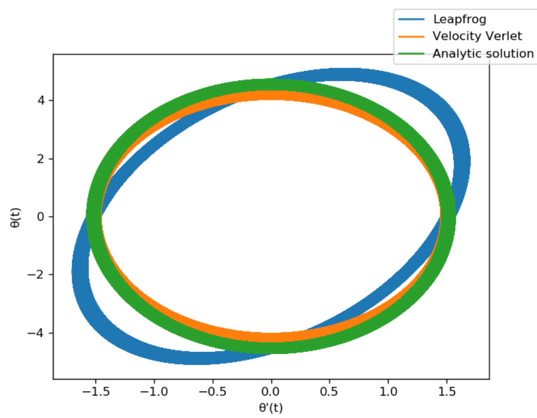
1.5

Energy average



Both are energy preserving over long time, even for silly large time steps.

Energy average



Leapfrog produces different trajectory.