S1336 - Project 2

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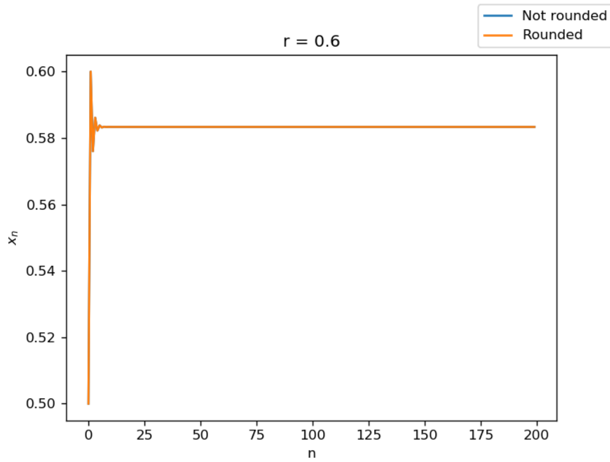
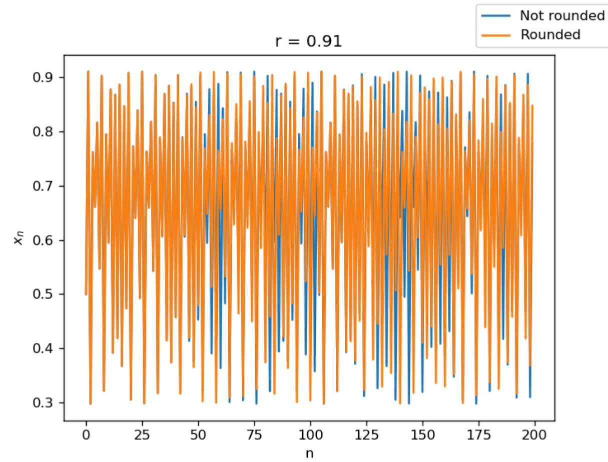
2.1

Effects of rounding errors





It can be concluded that when $r < r_\infty$, rounding doesn't have much of an effect on the system.

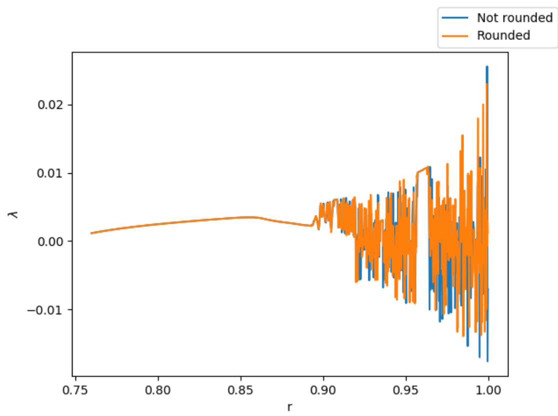
When $r > r_\infty$, rounding does however have an effect on the system.

2.2

If settling for $n = 200$, we can use the following formula to calculate λ:

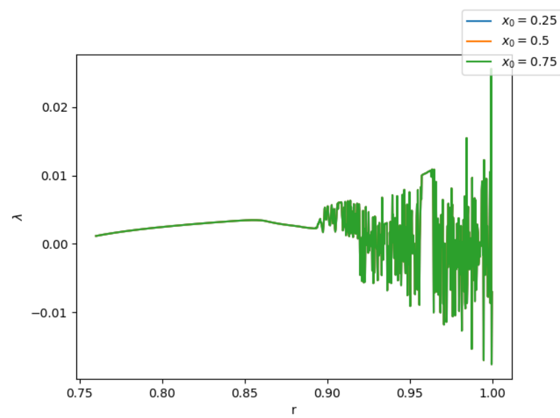
$$\lambda = \frac{1}{n} \sum_{i=0}^{i=n-1} \log \left| \frac{\Delta x_{i+1}}{\Delta x_i} \right|$$

The Lyapunov exponent for a population model



We see that for $0.76 < r < r_\infty$, when the system isn't chaotic, the sign of λ is positive. Effects of rounding doesn't affect λ unless $r > r_\infty$.

The dependence on initial value



In this plot, we see that selecting different x_0 has little to no effect on the calculation of λ.

2.3

Lorenz attractor

To figure out which parts of space get attracted in to the basin of the Lorenz attractor, trajectories for different starting conditions randomly sampled on the interval

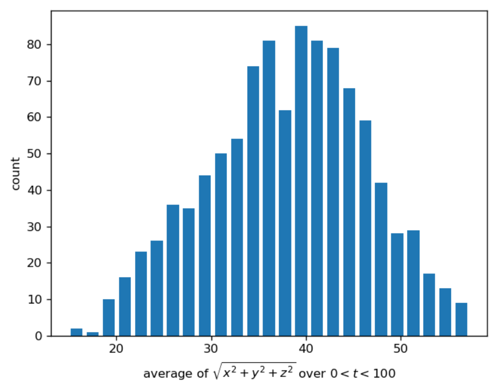
$$x(0) \in (-1000, 1000), \quad y(0) \in (-1000, 1000), \quad z(0) \in (-1000, 1000)$$

are computed.

The time averaged "radius" is calculated for each trajectory:

$$r = \frac{1}{n} \sum_{i=0}^{n} \sqrt{x(i\Delta t)^2 + y(i\Delta t)^2 + z(i\Delta t)^2}$$

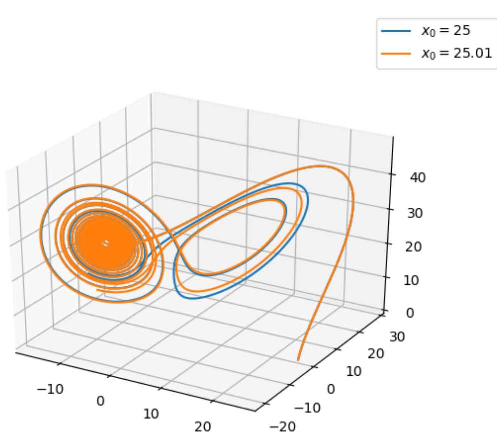
25 CPU hours later, for a time step of $\Delta t = 0.00008$ and a time boundary of $0 < t < 150$, the following histogram can be plotted:

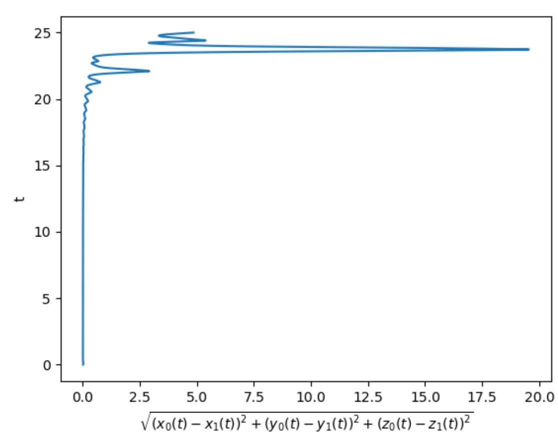


It shows that for all (roughly 1000) computed trajectories, they all average to within a distance 60 from $x = 0, y = 0, z = 0$. We can thus conclude that it's very unlikely that a solution doesn't get attracted into the basin.

2.4

Sensitivity to initial values

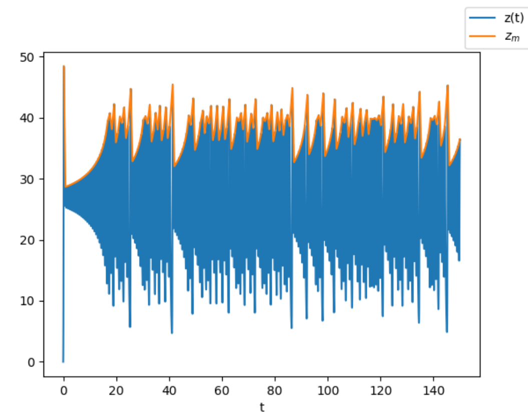




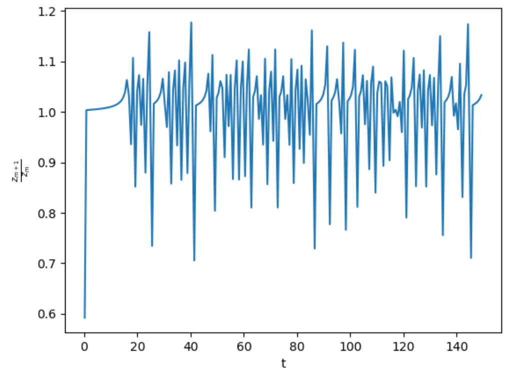
It can be seen that even for a small difference in initial value (0.01), the solutions differ much after some time.

2.5

Chaos or not?



We see that a solution starting in $x(0) = 10, y(0) = 0, z(0) = 0$ has an average of about $z = 28$, coincidentally the same number as the value for r given in the task.



It's clear that the ratio $\frac{z_{m+1}}{z_m}$ isn't strictly greater than unity.