

S1336 - Project 6

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10.10 Numerical solution of the potential within a rectangular region

Subtask a)

Determine the potential $V(x, y)$ in a square region with linear dimension $L = 10$. The boundary of the square is at a potential $V = 10$. Choose the grid size $\Delta x = \Delta y = 1$

Guess the exact form of $V(x, y)$

How many iterations are necessary to achieve 1% accuracy?

Decrease the grid size by a factor of two, and determine the number of iterations that are now necessary to achieve 1% accuracy.

Subtask b)

Consider the same geometry as in part (a), but set the initial potential at the interior sites equal to zero except for the center site whose potential is set equal to four.

Does the potential distribution evolve to the same values as in part (a)?

What is the effect of a poor initial guess?

Are the final results independent of your initial guess?

Subtask c)

Now modify the boundary conditions so that the value of the potential at the four sides is 5, 10, 5, and 10, respectively. (Start with a reasonable guess for the initial values of the potential at the interior sites and iterate until 1% accuracy is obtained.)

Sketch the equipotential surfaces.

What happens if the potential is 10 on three sides and 0 on the fourth?

10.11 Gauss-Seidel relaxation

Subtask a)

Use a modification of the program used in 10.10 so that the potential at each site is updated sequentially. In this way the new potential of the next site is computed using the most recently computed values of its nearest neighbor potentials.

Are your results better, worse, or about the same as for the simple relaxation method?

Subtask b)

Imagine coloring the alternate sites of a grid red and black, so that the grid resembles a checkerboard. Modify the program so that all the red sites are updated first, and then all the black sites are updated.

Do your results converge any more quickly than in part (a)?

10.26 The multigrid method

Subtask a)

Write a program that implements the multigrid method using Gauss-Seidel relaxation on a checker-board lattice. Test your program on a 44 grid whose boundary sites are all equal to unity, and whose initial internal sites are set to zero.

Subtask b)

The exact solution for part (a) gives a potential of unity at each point.

How many relaxation steps does it take to reach unity within 0.1% at every site by simply using the 44 grid?

How many steps does it take if you use one coarse grid and continue until the coarse grid values are within 0.1% of unity?

Is it necessary to carry out any fine grid relaxation steps to reach the desired accuracy on the fine grid?

Next start with the coarsest scale, which is just one site. How many relaxation steps does it take now?

Subtask c)

Repeat part (b), but change the boundary so that one side of the boundary is held at a potential of 0.5.

Experiment with different sequences of prolongation, restriction, and relaxation.