Satellite ADCS Demonstration

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1 Introduction

This project assignment is to verify that the given ADCS requirements can be met with a baseline design for a satellite. The following report will develop an attitude simulator and combine analysis results step by step to prove that the requirements can be met.

2 Requirements

1. Maneuver 1: Slew 30° cross-track.
   (a) Perform the maneuver in less than 30 minutes.
   (b) The steady state error must be $< 0.1°$.
2. Hold attitude constant, i.e. keep the body frame aligned with the inertial frame.
   (a) Pointing accuracy must be $> 0.15°$
3. From an initial state $\vec{\omega} = (0.05, 0.05, 0.05)$ rad/sec, bring the spacecraft to $\vec{\omega} = (0, 0, 0), \psi = \phi = \theta = 0$.
   (a) The maneuver must be performed in less than 12 hours.
   (b) The steady state error must be $< 0.1°$

3 Parameters

4 No Disturbance Torques

To begin the project, we are to plot the roll, yaw, and pitch time histories for 3 orbits, starting with the following initial conditions:

$$(\omega_1, \omega_2, \omega_3) = (-0.0008, 0.0006, 0.00085) \text{ rad/s} \tag{1}$$
$$(\psi, \phi, \theta) = (0.09, -0.07, 0.15) \text{ rad} \tag{2}$$

r	7000 km
i	83°
date	March 21, 2023
Ω	$-70°$
D	3 Am^2, for all axes
I	$\begin{bmatrix} 4500 & 0 & 0 \\ 0 & 6000 & 0 \\ 0 & 0 & 7000 \end{bmatrix} kg - m^2$
A_{plate}	$[5, 7, 7]$ m^2
G	6.674×10^{-11} N m^2/kg^2
M	5.9722×10^{24} kg

Table 1: Given Satellite Parameters

To achieve this, we take the standard Euler equations for the rotational motion and integrate. We first assume no disturbance torques. The integrable equations are:

$$I_{11}\dot{\omega}_1 + (I_{33} - I_{22})\omega_2\omega_3 = 0 \tag{3}$$

$$I_{22}\dot{\omega}_2 + (I_{11} - I_{33})\omega_1\omega_3 = 0 \tag{4}$$

$$I_{33}\dot{\omega}_3 + (I_{22} - I_{11})\omega_1\omega_2 = 0 \tag{5}$$

$$\dot{\phi} = (\omega_1 \sin\theta + \omega_2 \cos\theta - n \sin\psi \cos\phi) / \cos\psi \tag{6}$$

$$\dot{\psi} = (\omega_1 \cos\theta - \omega_2 \sin\theta + n \sin\phi) \tag{7}$$

$$\dot{\theta} = \omega_3 (\omega_1 \sin\theta + \omega_2 \cos\theta) \tan\psi - n \cos\phi / \cos\psi \tag{8}$$

To proceed, we calculate n, which is given by the mean motion of the orbit, calculated as

$$n = \frac{2\pi}{T} \tag{9}$$

where T is the orbital period, given by

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \tag{10}$$

Finally, we have all we need to plot the yaw, roll, and pitch time histories, given that the disturbance torques are zero, meaning $\tau_1 = \tau_2 = \tau_3 = 0$. I used the SciPy method `solve_ivp` to integrate each equation at the same time. This produces the following plots:

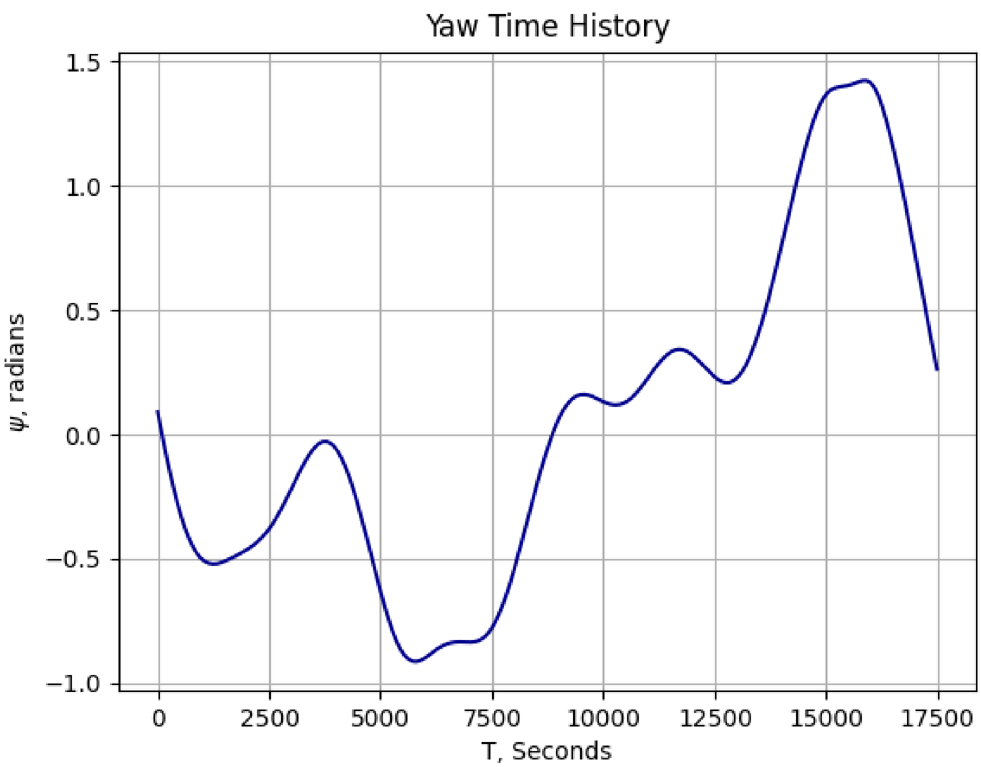

Figure 1: Yaw Time History

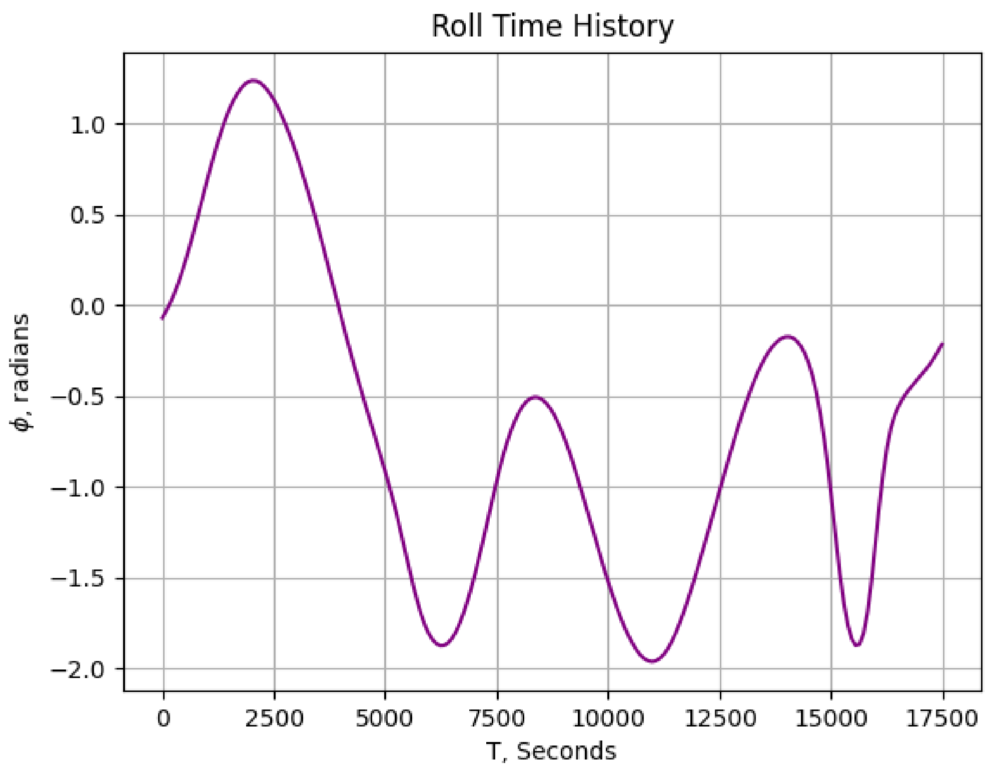

Figure 2: Roll Time History

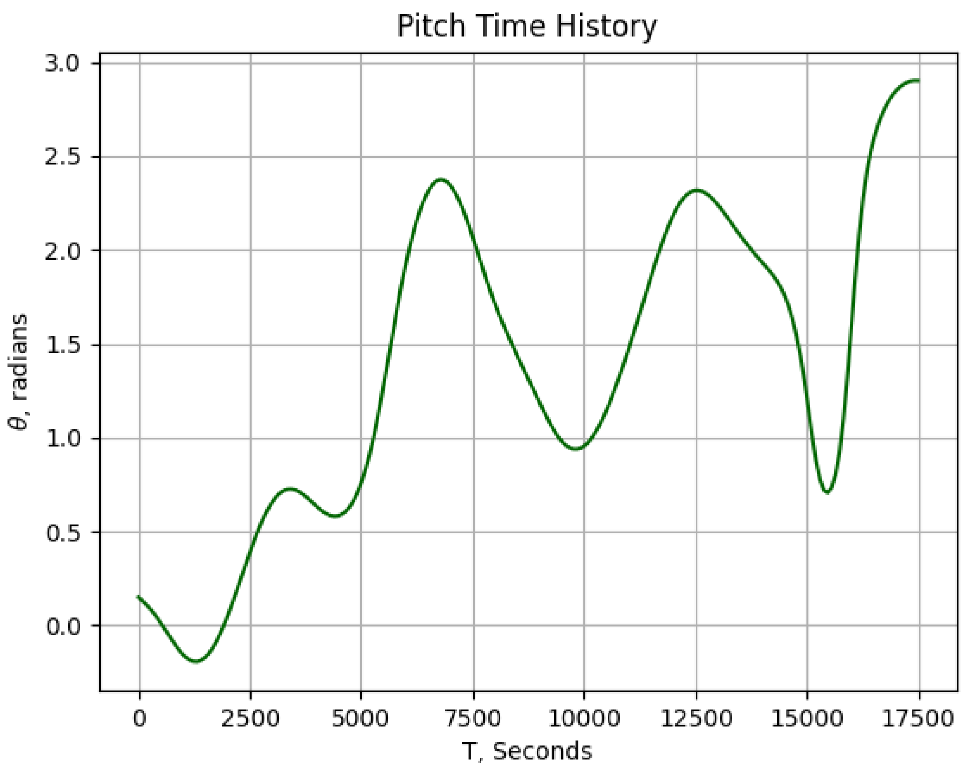

Figure 3: Pitch Time History

5 Adding the Disturbance Torques

There are three main sources of disturbance torques: the gravity gradient, Earth's magnetic field, and solar pressure. These disturbances can be added to the right hand side of the equations of motion (Equations 3 - 8).

B_0	3.11×10^{-5} Tesla
$\mathbf{m}$	7.95×10^{15} Am^2
P	4.644×10^{-6} N/m^2
ρ_s	$(0.8, 0.2, 0.2)$
ρ_d	$(0.1, 0.1, 0.1)$
CM_{plates}	$\begin{bmatrix} 0.1 & 0.1 & 0 \\ 2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

Table 2: Additional Constants

5.1 Gravity Gradient

The gravity gradient can be described with

$$\mathbf{M}_g = 3n^2 \begin{bmatrix} R_{21}R_{31}(I_{33} - I_{22}) \\ R_{11}R_{31}(I_{11} - I_{33}) \\ R_{11}R_{21}(I_{22} - I_{11}) \end{bmatrix} \tag{11}$$

where

$$R = R_{BO}(\phi, \theta, \psi) \tag{12}$$

where n is the mean motion of the satellite about the Earth, i is the inclination of the orbit, and Ω is the longitude of the ascending node.

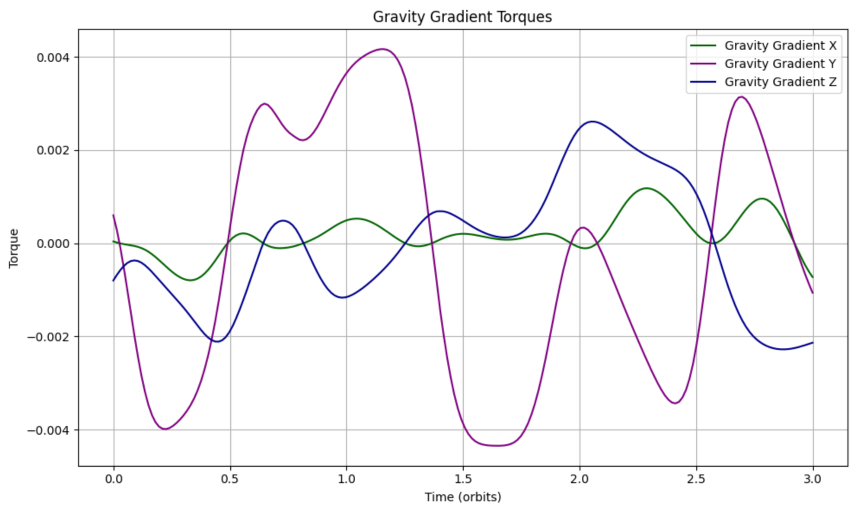

Figure 4: Gravity Gradient Torque

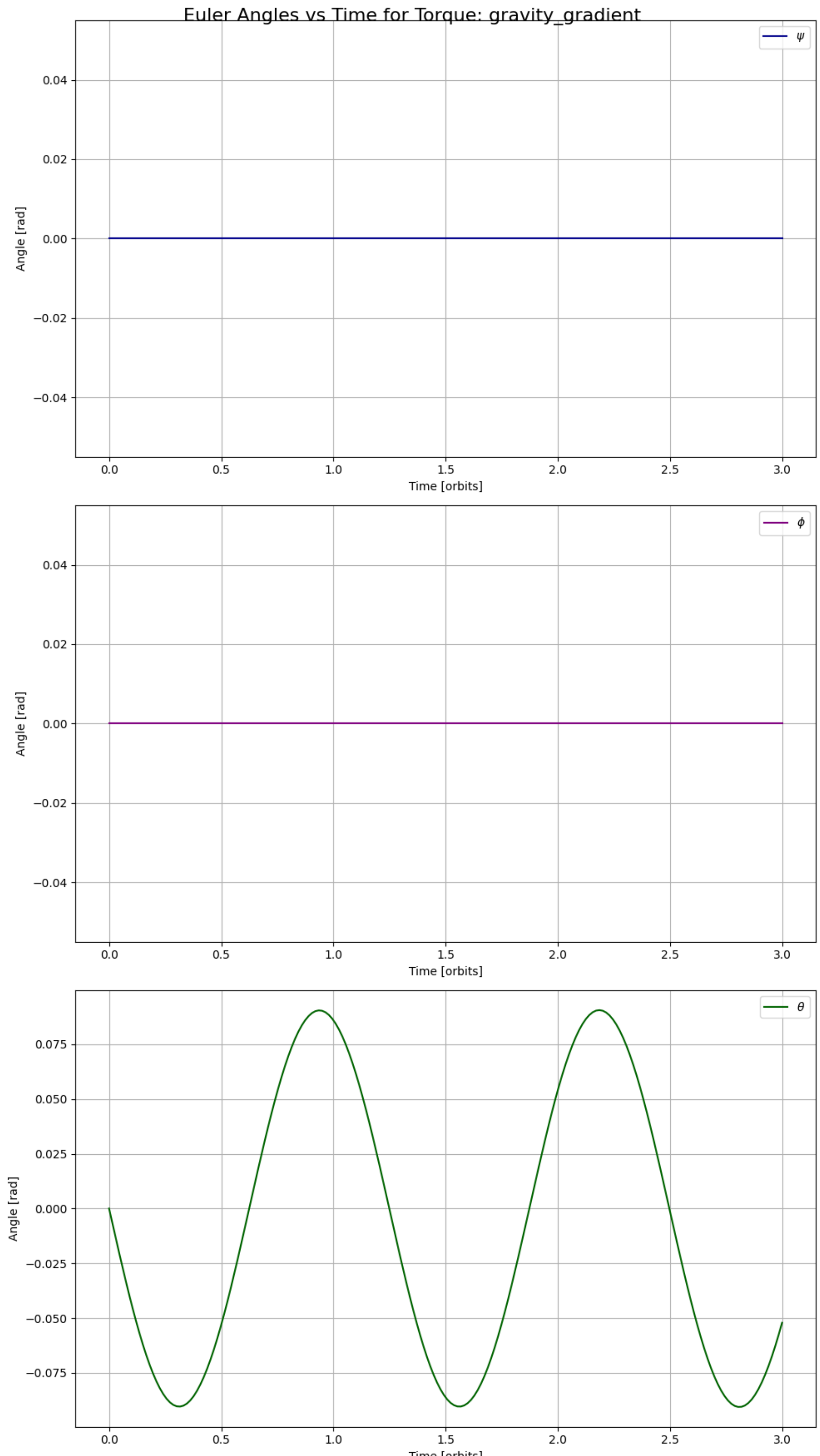

Figure 5: Case 1: Euler Angles Considering Only Gravity Gradient Torque

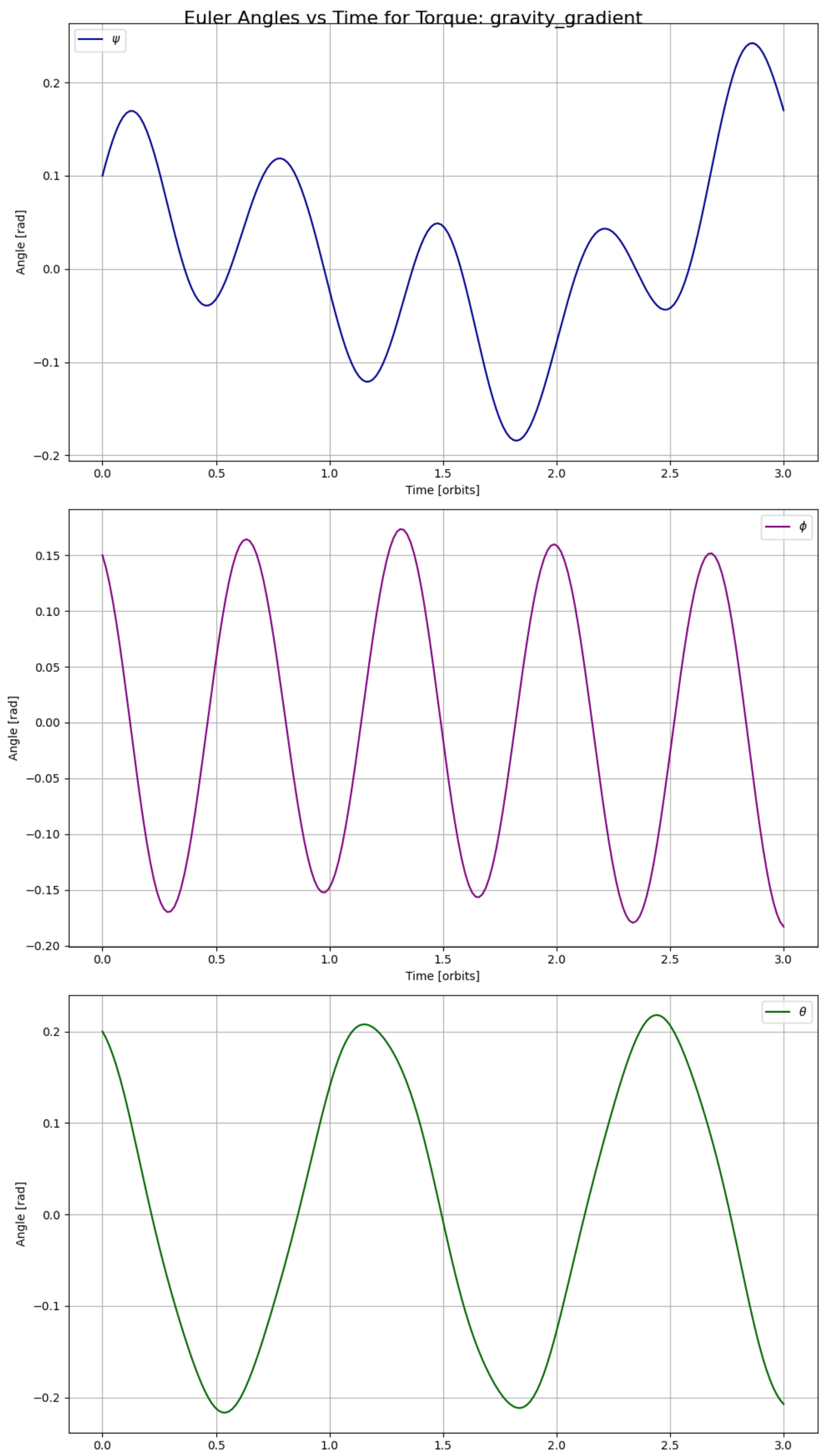

Figure 6: Case 2: Euler Angles Considering Only Gravity Gradient Torque

5.2 Earth's Magnetic Field

The torque Earth's magnetic field exerts on the satellite can be described with

$$\tau_{mag} = \mathbf{M} \times \mathbf{B} \tag{13}$$

where $\mathbf{M} = (3, 3, 3)\ Am^2$. For this,

$$B(\mathbf{r}) = B_0 \left(\frac{R_e}{r} \right)^3 [3(\hat{\mathbf{m}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{m}}] \tag{14}$$

where $B_0 = 3.11 \times 10^{-5}$ Tesla. $\mathbf{m}$ is the magnetic dipole in the ECEF frame, meaning it was taken as a constant and converted to the orbital frame.

$$\mathbf{B} = \frac{\mathbf{m}}{R_e^3} \begin{bmatrix} \cos nt \sin i \\ 2 \sin nt \sin i \\ \cos i \end{bmatrix} \tag{15}$$

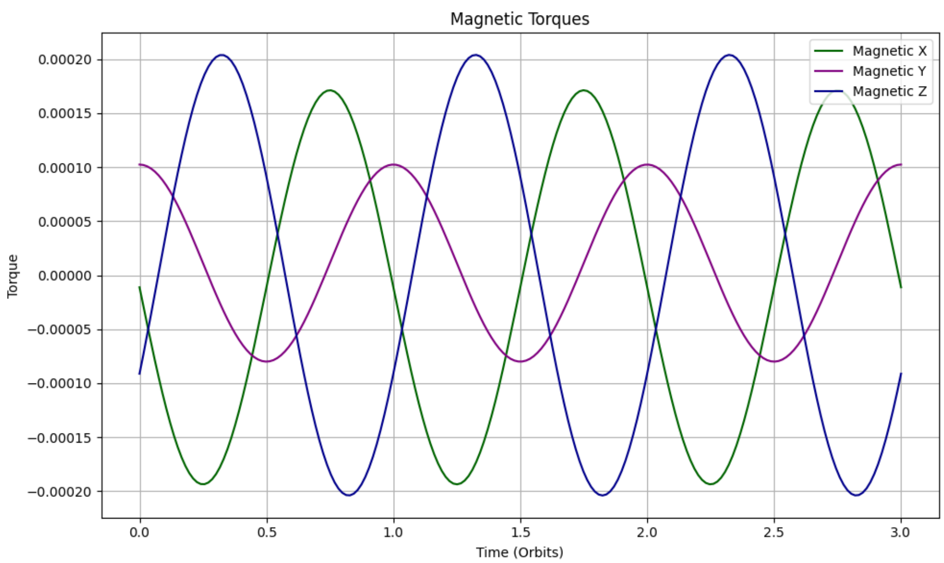

Figure 7: Earth' Magnetic Field Torque

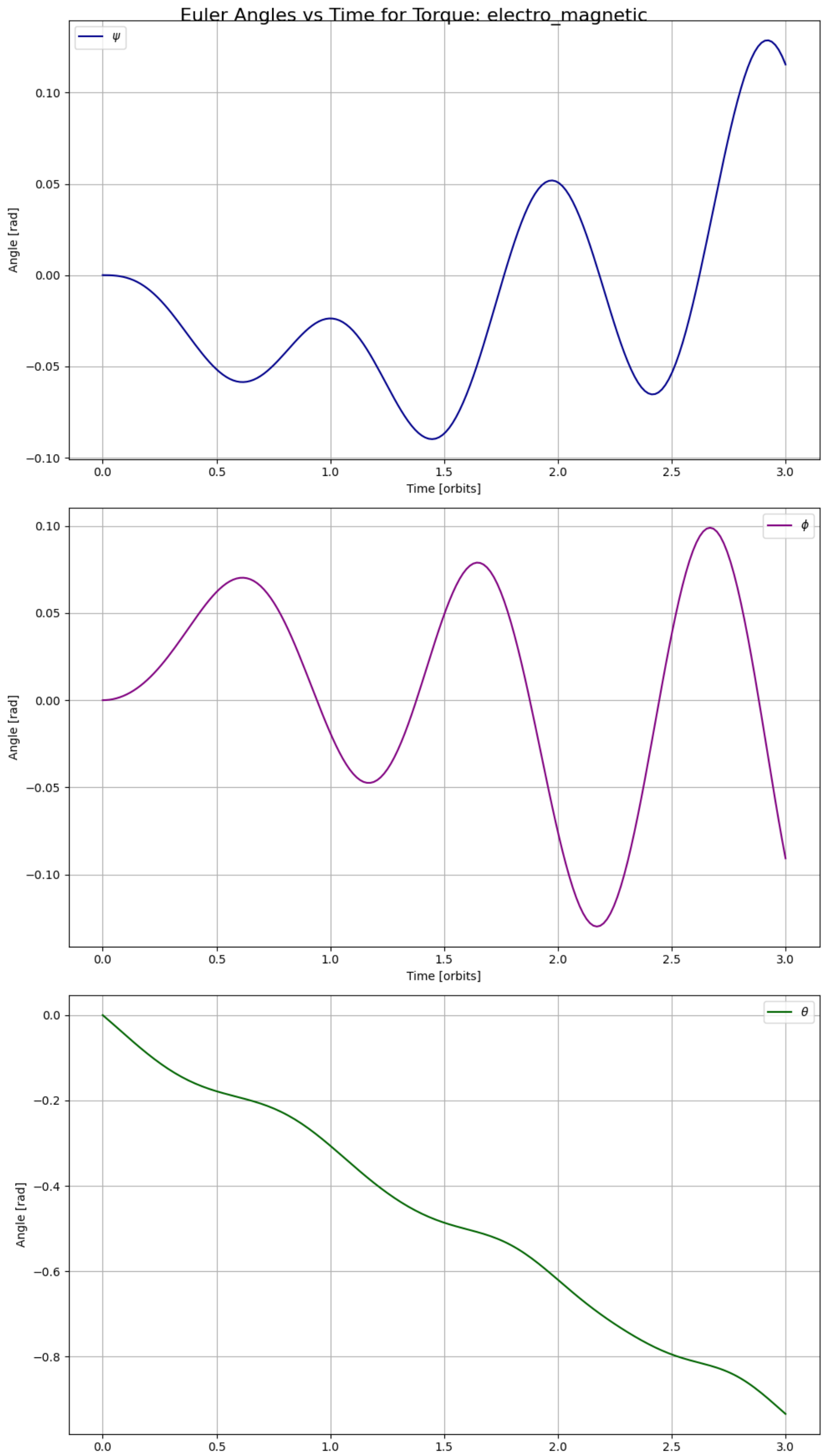

Figure 8: Case 1: Euler Angles Considering Only Electromagnetic Torque

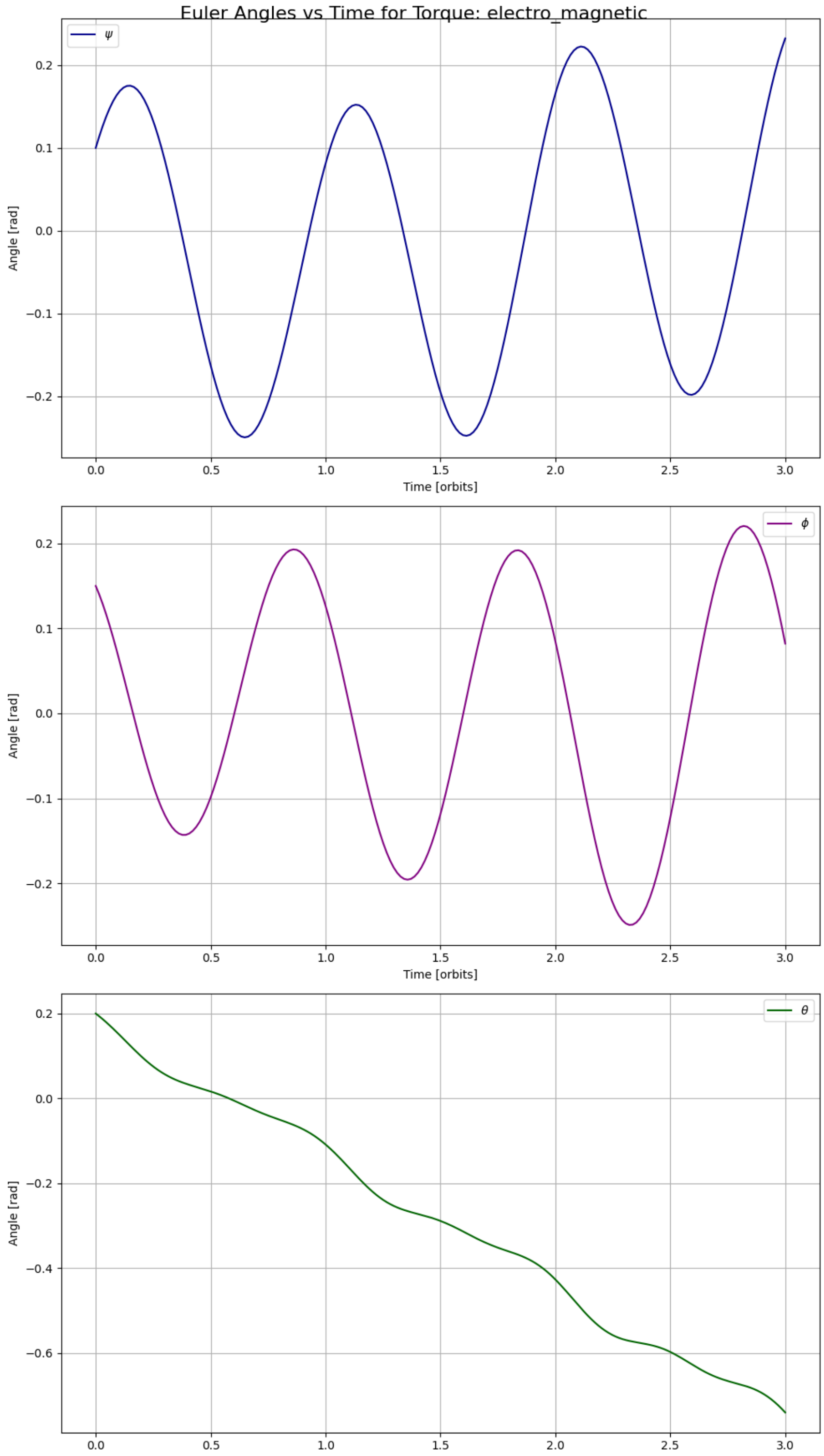

Figure 9: Case 2: Euler Angles Considering Only Electromagnetic Torque

5.3 Solar Pressure

The solar torque is given by

$$\tau_{sp} = CM_{plates}\mathbf{F}_{sr} \tag{16}$$

where A is the areas of the solar panels and $\mathbf{F}_{sr}$ is the force of the solar pressure. This is found using

$$\mathbf{F}_{sr} = PA(\hat{n} \cdot \hat{s}_{body})(1-\rho_s)\hat{s}_{body} + \left(\rho_s + \frac{2}{3}\rho_d\right)\hat{n}. \tag{17}$$

where P is a solar constant given in Table 2. Subsequently, we must find $\hat{s}_{body}$ as we only have it in the orbit frame. This involves a coordinate transform.

$$\hat{s}_{ecliptic} = \begin{bmatrix} \cos\left(\frac{2\pi}{365\ days}t\right) \\ \sin\left(\frac{2\pi}{365\ days}t\right) \\ 0 \end{bmatrix} \tag{18}$$

$$\hat{s}_{body} = \mathbf{R}_{BO}R_3(nt)R_1(i)R_3(\Omega)R_1(-23.5°)\hat{s}_{ecliptic} \tag{19}$$

The solar pressure can also be modeled as

$$A + B\cos\left(\omega_0 t + \phi_{offset}\right) \tag{20}$$

TO model it this way, we could take our data and perform a Fourier transform.

A_x	$1.3 \cdot 10^{-6}$
B_x	$0.22 \cdot 10^{-6}$
A_y	$1.3 \cdot 10^{-6}$
B_y	$0.22 \cdot 10^{-6}$
A_z	$0.16 \cdot 10^{-6}$
B_z	$0.15 \cdot 10^{-6}$

Table 3: Approximate Values for Solar Torque

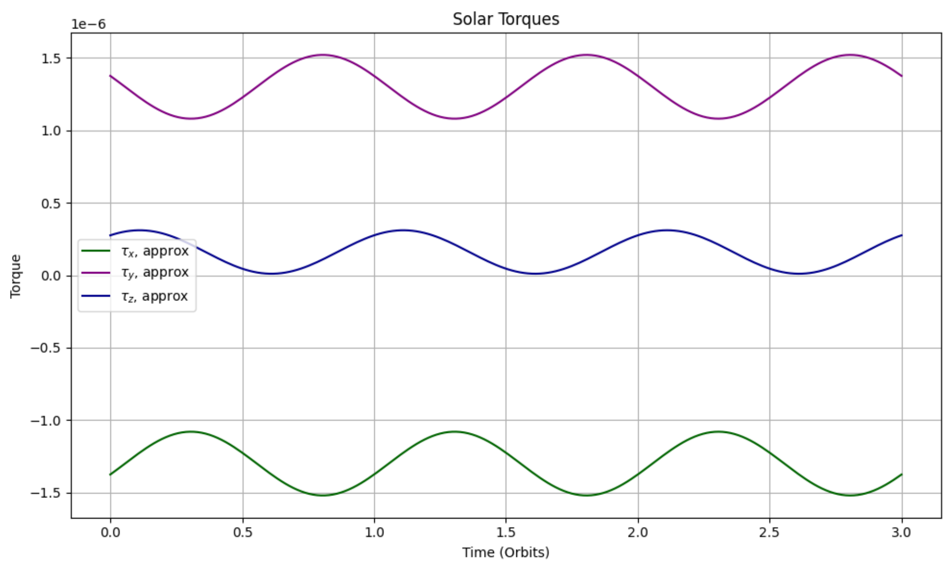

Figure 10: Approximated Solar Torques

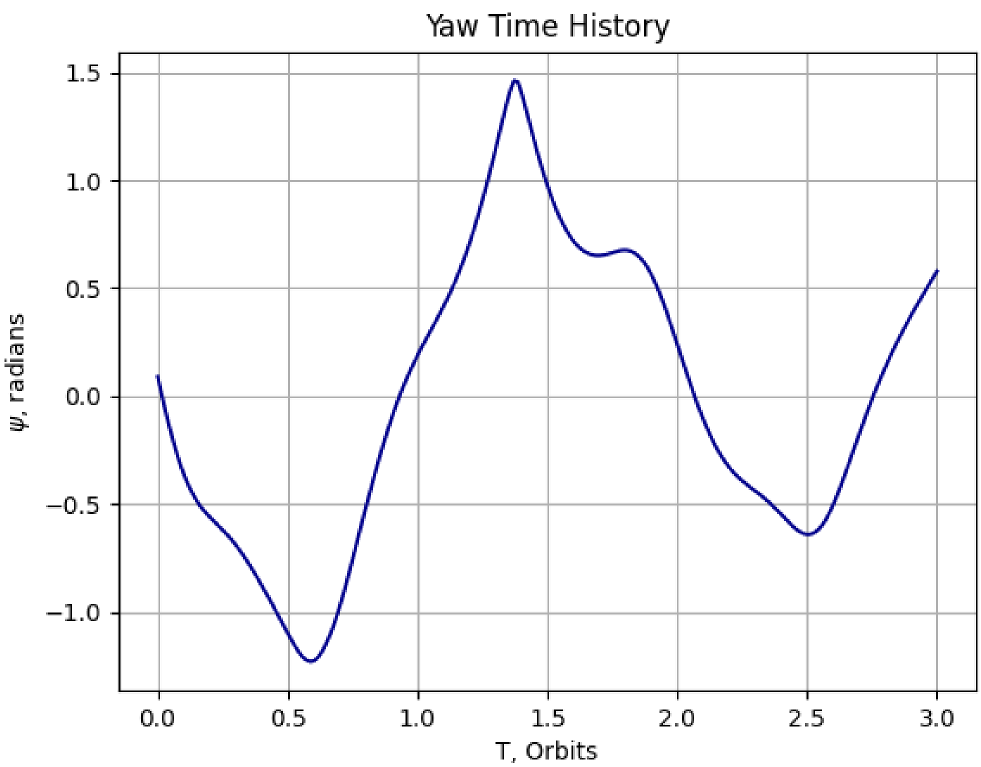

Figure 11: Yaw Time History, with Disturbances

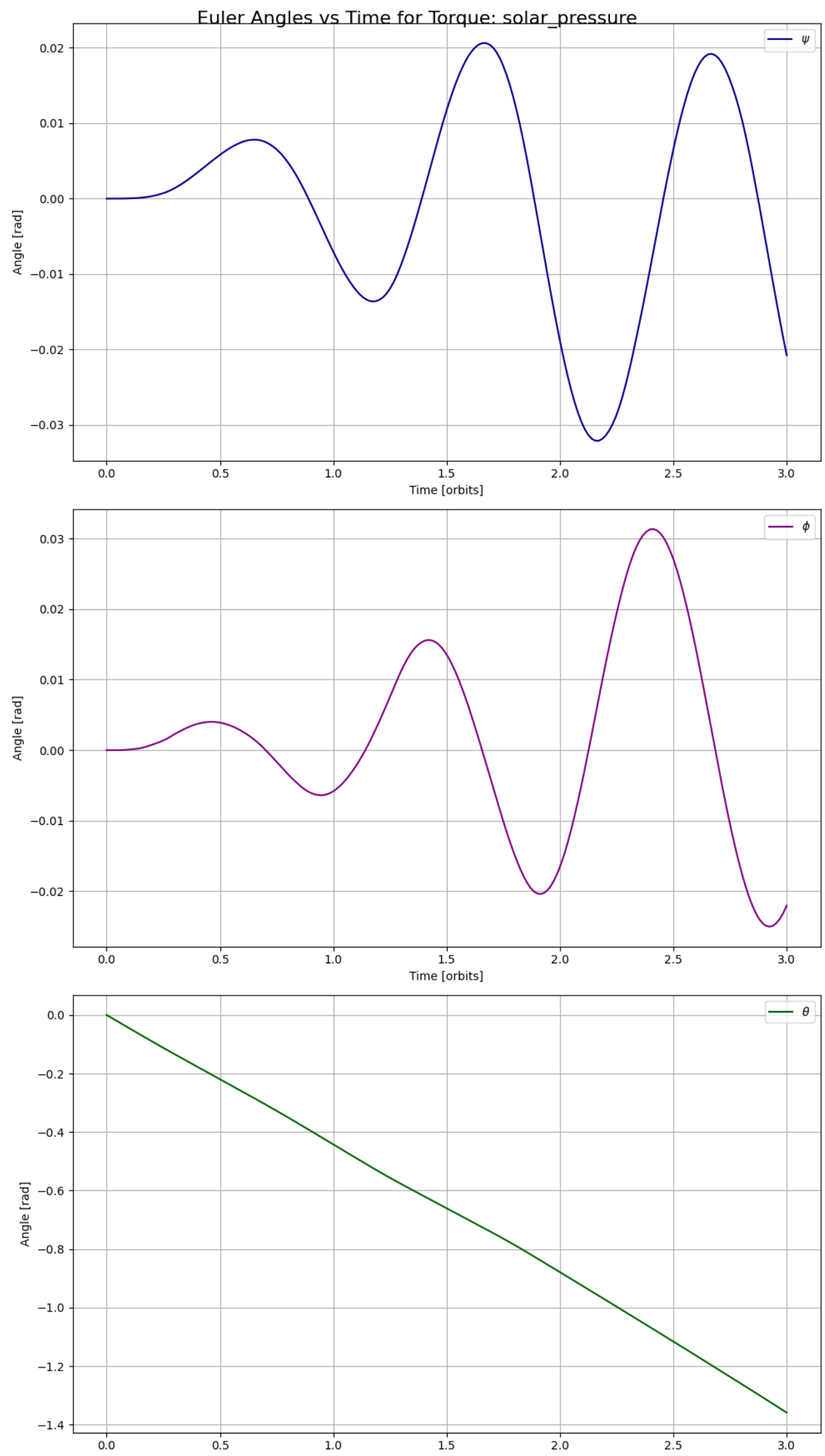

Figure 12: Case 1: Euler Angles Considering Only Solar Pressure Torque

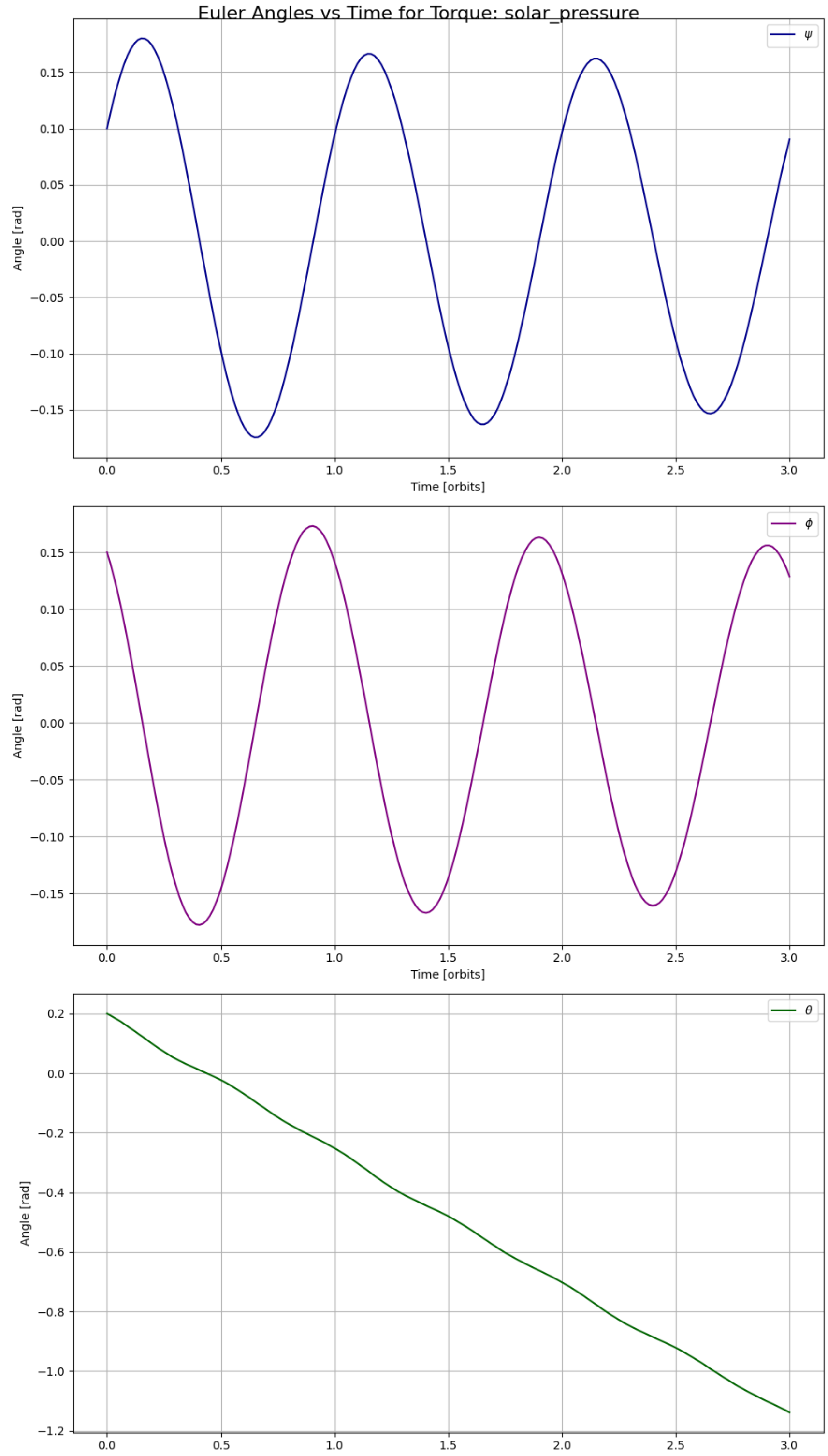

Figure 13: Case 2: Euler Angles Considering Only Solar Pressure Torque

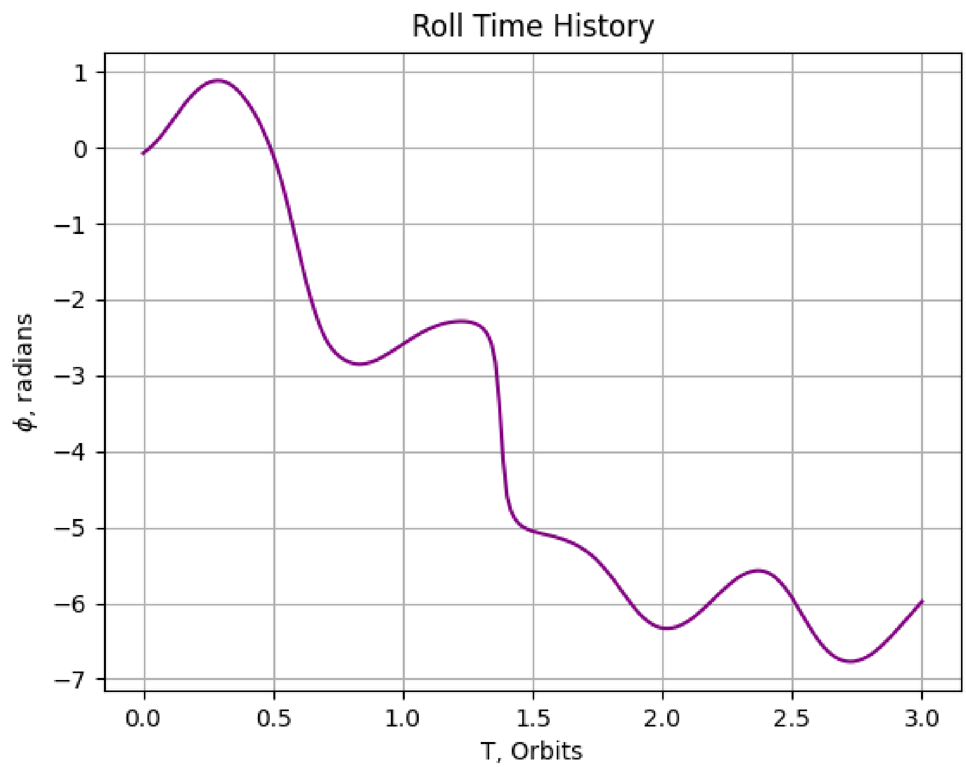

Figure 14: Roll Time History, with Disturbances

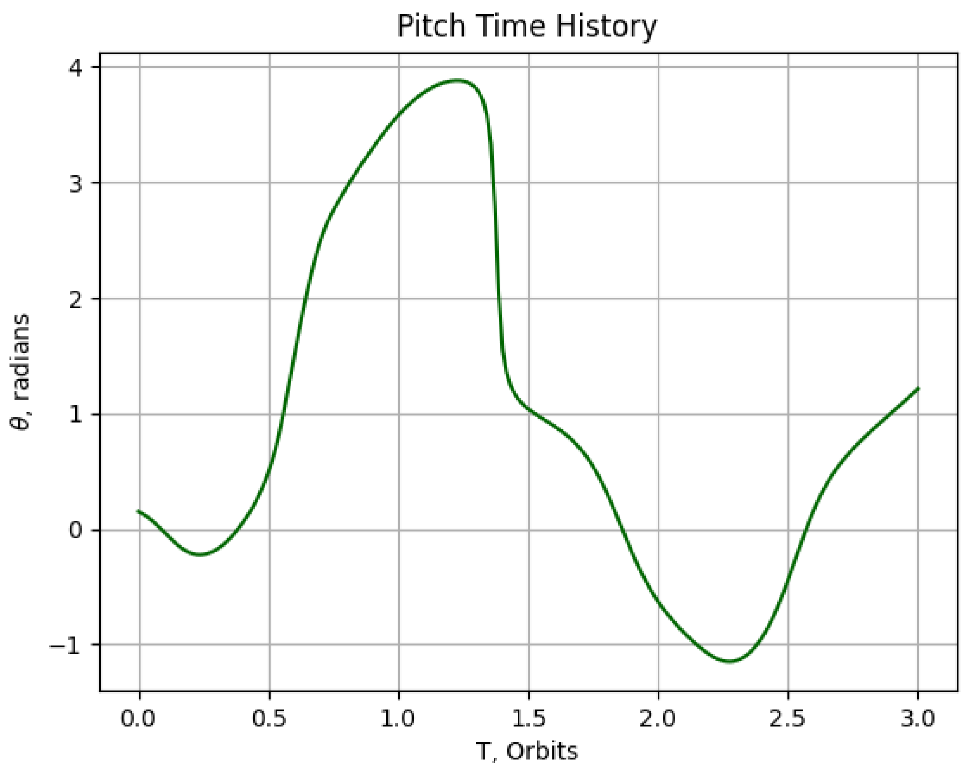

Figure 15: Pitch Time History, with Disturbances

6 Adding Control Torque

For the final part of the project we want to implement a PD or a PID controller for the satellite. This changes the equations of motions a bit:

$$I_{11}\dot{\omega}_1 + \dot{h}_1 + (I_{33} - I_{22})\omega_2\omega_3 + (h_3\omega_2 - h_2\omega_3) = \tau_1 \tag{21}$$

$$I_{22}\dot{\omega}_2 + \dot{h}_2 + (I_{11} - I_{33})\omega_1\omega_3 + (h_1\omega_3 - h_3\omega_1) = \tau_2 \tag{22}$$

$$I_{33}\dot{\omega}_3 + \dot{h}_3 + (I_{22} - I_{11})\omega_1\omega_2 + (h_2\omega_1 - h_1\omega_2) = \tau_3 \tag{23}$$

$$\dot{\phi} = (\omega_1 \sin\theta + \omega_2 \cos\theta - n \sin\psi \cos\phi) / \cos\psi \tag{24}$$

$$\dot{\psi} = (\omega_1 \cos\theta - \omega_2 \sin\theta + n \sin\phi) \tag{25}$$

$$\dot{\theta} = \omega_3 (\omega_1 \sin\theta + \omega_2 \cos\theta) \tan\psi - n \cos\phi / \cos\psi \tag{26}$$

The control law we are using is for a PD controller:

$$\dot{h} = k_\theta(\tau_\theta\dot{\theta} + \theta) \tag{27}$$

This control law can be applied to each axis so that

$$\dot{h}_1 = k_\psi(\tau_\psi\dot{\psi} + \psi) \tag{28}$$

$$\dot{h}_2 = k_\phi(\tau_\phi\dot{\phi} + \phi) \tag{29}$$

$$\dot{h}_3 = k_\theta(\tau_\theta\dot{\theta} + \theta) \tag{30}$$

To slew 30°cross-track, we take the following initial conditions:

ω_1	0
ω_2	0
ω_3	ω_0
ψ	0
ϕ	0
θ	0
ψ_{goal}	30°
ϕ_{goal}	0
θ_{goal}	0

Table 4: Initial Values for Slewing Mode

To detumble, we take these initial conditions: For the system integration, we can simplify the disturbance torques and only consider the solar pressure. Applying this simplification and adjusting the gains, k_θ and τ_θ for each axis until the desired result is achieved. Without carefully performing the tuning of the gains, I was unable to get nice results.

ω_1	0
ω_2	0
ω_3	ω_0
ψ	0.1
ϕ	0.15
θ	0.2
ψ_{goal}	0
ϕ_{goal}	0
θ_{goal}	0

Table 5: Initial Values for Detumble Mode

6.1 Slewing 30° Cross-Track

Here are the results for slewing cross-track. I set the gains to In general, these were the only gains

k_θ	0
τ_θ	0
k_ψ	1
τ_ψ	1
k_ϕ	1
τ_ϕ	1

Table 6: Slewing Gains

to get me near 30 ° for the yawing angle. It did not quite reach all the way to 30 however. In doing this slew, I was unable to subdue the pitch, no matter what the gains were. The roll was also unstable.

6.2 Detumbling

The system did not necessarily stabilize for the detumbling mode either, no matter the gains I tried. I believe we would have better results implementing a PID controller. I had a lot of trouble controlling the satellite in this way.

k_θ	0.1
τ_θ	10
k_ψ	0.2
τ_ψ	0.9
k_ϕ	0.5
τ_ϕ	4

Table 7: Detumbling Gains

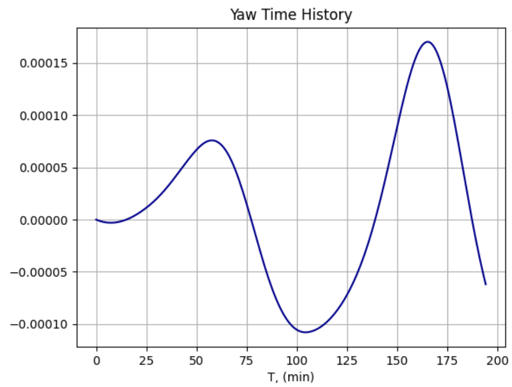

Figure 16: Yaw Control Slew Mode

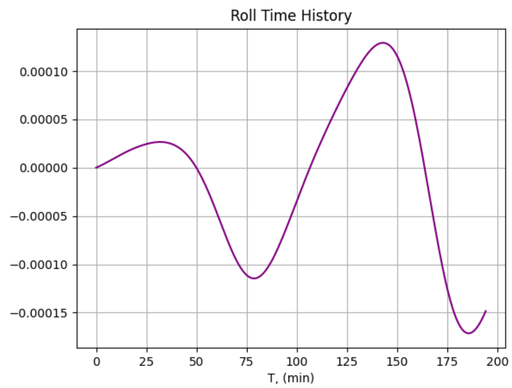

Figure 17: Roll Control Slew Mode

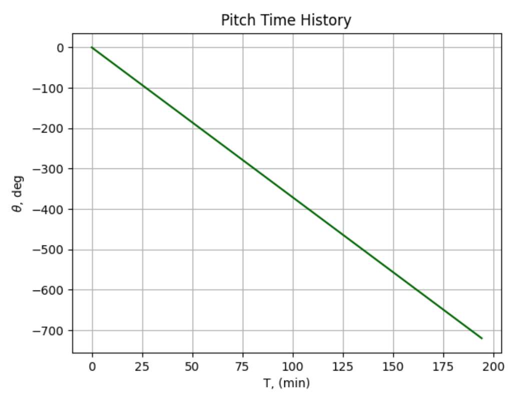

Figure 18: Pitch Control Slew Mode

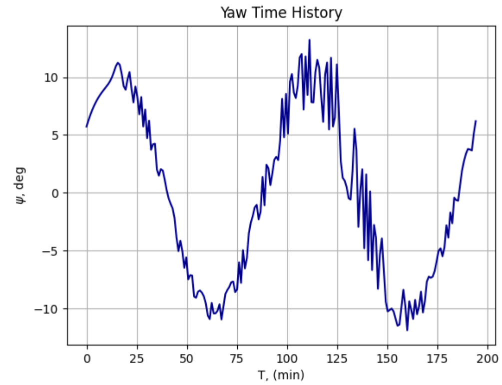

Figure 19: Yaw Control Detumble Mode

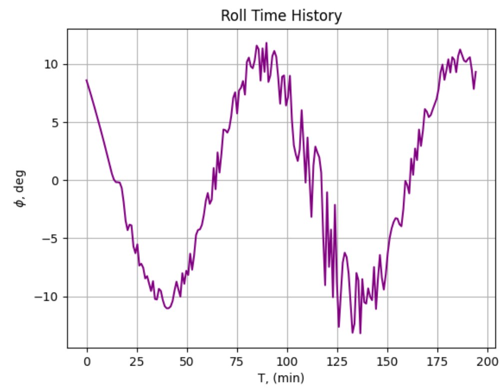

Figure 20: Roll Control Detumble Mode

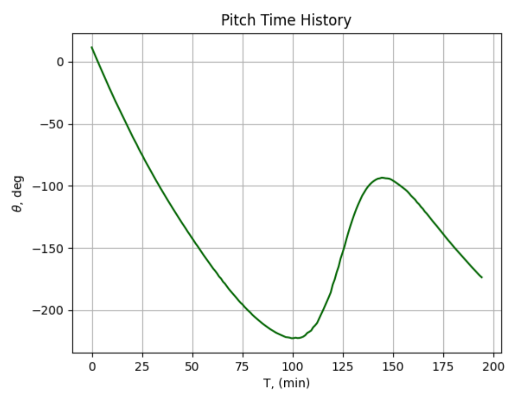

Figure 21: Pitch Control Detumble Mode