Lab01

Question1:

Which of the following functions are increasing? eventually nondecreasing? If you remember techniques from calculus, you can make use of those.

(1) $f(x) = -x^2$

f ‘ (x) is -2x

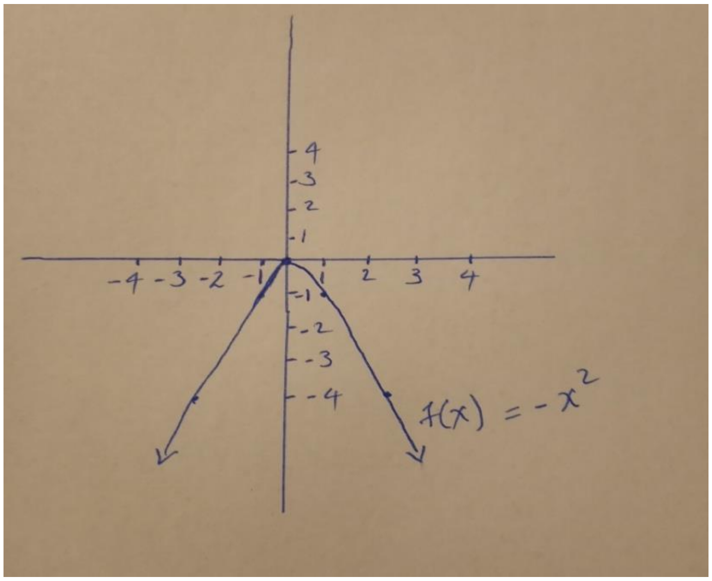
-From the derivative we can check by using positive and negative numbers

Let x= -6

f(x) = -2x = -2(-6) → **+12.**

Let x=6

f(x) = -2x = -2(6) → **-12**



So the function is increasing from -∞ towards ‘0’ and it is decreasing from 0 towards ∞

But the graph is eventually decreasing.

(2) $f(x) = x^2 + 2x + 1$

the derivative of f(x) is

f ‘ (x) = 2x + 2

-From the derivative we can check by using positive and negative numbers

Let $x = 4$

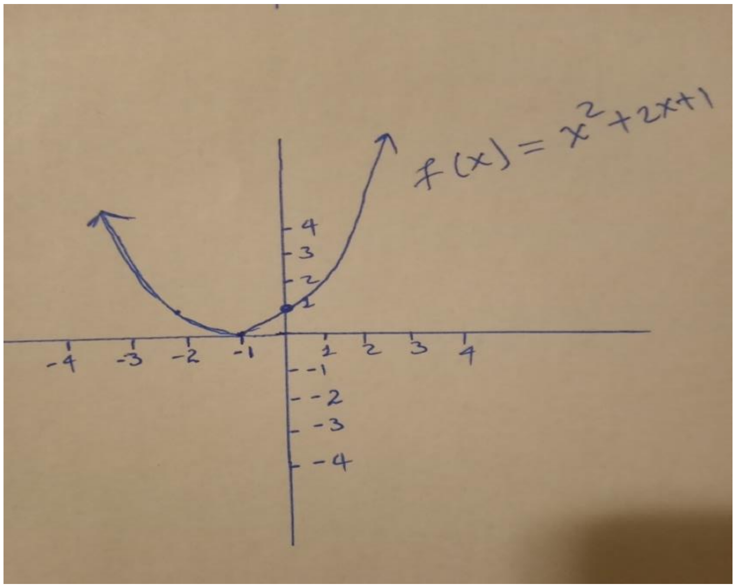
$f'(x) = 2x + 2$

$= 2(4) + 2 \rightarrow \underline{\mathbf{10}}$

Let $x = -4$

$f'(x) = 2x + 2$

$= 2(-4) + 2 \rightarrow \underline{\mathbf{-6}}$



** the function is decreasing from $-\infty$ to wards -1 and is increasing from -1 towards ∞.

Eventually the graph is increasing.

(3) $f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

-From the derivative we can check by using positive and negative numbers

Let $x = 3$

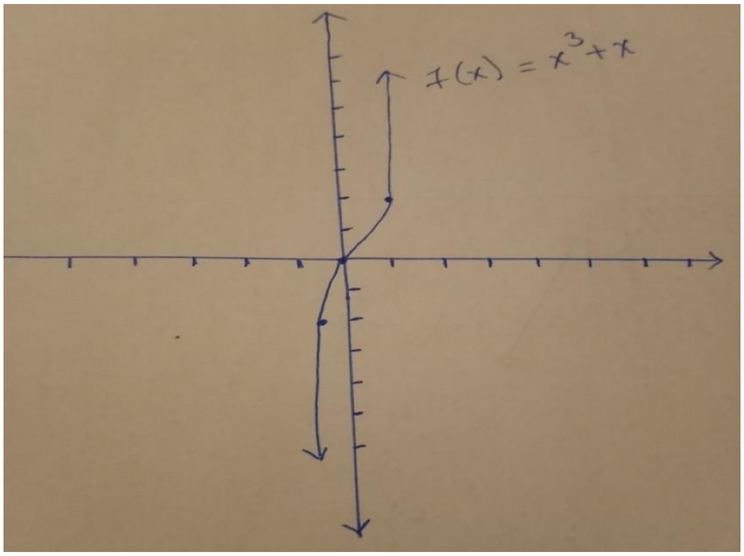
$F'(x) = 3x^2 + 1$

$= 3(3)^2 + 1 \rightarrow \underline{\mathbf{28}}$

Let $x = -3$

$F'(x) = 3x^2 + 1$

$= 3(-3)^2 + 1 \rightarrow \underline{\mathbf{28}}$



** The function is increasing.

It is eventually increasing.

Answer:

1) It is not eventually non decreasing
2) It is eventually non decreasing
3) It is increasing

Question2:

Consider the following pairs and functions f, g. Decide if it is correct to say that, asymptotically, f grows no faster than g, g grows no faster than f, or both.

(1) $f(x) = 2x^2$, $g(x) = x^2 + 1$

$\lim_{n \to \infty} f(x)/g(x) \to 2x^2/ x^2 + 1$

Since both functions have the same degree so it will grow both.
Or we can take derivatives of both f(x) and g(x)

$\lim_{n \to \infty} 2x^2/ x^2 + 1 = \lim_{n \to \infty} 2x^2/ x^2 (1 + 1/ x^2)$ by canceling x^2

$= \lim_{n \to \infty} 2/ (1 + 1/ x^2)$, if we replace n by ∞ denominator will be 1

$= \lim_{n \to \infty} 2$, since we got constant, each function grows no faster than the other.

(2) $f(x) = x^2$, $g(x) = x^3$

$\lim_{x \to \infty} f(x) / g(x)$

$= \lim_{x \to \infty} x^2 /x^3$, by canceling x^2 on both numerator and denominator

$\to \lim_{x \to \infty} 1/ x$

So f(x) grows no faster than g(x)

(3) $f(x) = 4x + 1$, $g(x) = x^2 – 1$

$= \lim_{x \to \infty} f(x) / g(x)$

$= \lim 4x + 1/x^2 - 1$

$x \to \infty$

$= \lim x(4 + 1/x) / x(x - 1/x)$,

$x \to \infty$

$= \lim (4 + 1/x) / (x - 1/x).$,

$x \to \infty$

this gives us undefined so we will try by derivatives

$\lim_{x \to \infty} f(x) / g(x)$

$= \lim_{x \to \infty} x^2 / x^3 = \lim_{x \to \infty} 2x / 3x^2$, if we cancel 'x' on both we will get

$= \lim_{x \to \infty} 2/3x$ if we replace x by ∞ we will get 0.
so we can conclude
f(x) grows no faster than g(x).

Answer:

1) Both functions grow no faster than each other.
2) f(x) grows no faster than g(x).
3) f(x) grows no faster than g(x).

Question3: GCD program

```java
package algorithm.lab01;

public class GCDProb1 {

    public static void main(String[] args) {

        System.out.println(GCDProb1.getGCD(8, 4));
    }

    public static int getGCD(int m, int n) {

        int smaller = m > n ? n: m;
        int gcd = 1;
        for (int i =2; i <= smaller; i ++){
            if ( m % i == 0 && n % i == 0){
                gcd = i;
            }
        }
        return gcd;
    }

}
```

<u>Question4:</u> Greedy Strategies

<u>Answer:</u>

The algorithm does not work. Consider S = {1, 2, 4, 5}, k = 10. Using the greedy strategy, the algorithms populate the set T with 1, 2, 4 and then cannot continue, so the final value is T = {1, 2, 4}. Since the sum of elements in T is not 7, the return is (incorrectly) null.

<u>Question5:</u> SubsetSum problem

This is correct. We must show that the sum of the elements of T' = T − {$s_{n−1}$} is k − $s_{n−1}$. But the sum of the elements of T (which is the set T' U {$s_{n−1}$}) is k. Since $s_{n−1} \notin$ T', the sum of the elements of T' must be $s_{n−1}$ less than the sum of the elements of T; that is the sum of the elements of T0 is k−$s_{n−1}$. (Note that it is possible that the only element of T is $s_{n−1}$.

In that case the sum of elements of T is $s_{n−1}$ (so it must be that k = $s_{n−1}$). Then the sum of elements of T' = T −{$s_{n−1}$}, which is now empty, must be k −k = 0; since the sum of an empty set of integers is 0, this result is still correct.)

If T = {S0, S1, S2, ….. S_{n-2} , S_{n-1}}

K = So +S_1 + S, …… + S_{n-2} + S_{n-1}

Then if we subtract S_{n-1} from T

T' = T - S_{n-1} = {So, S1, S2, …… S_{n-2} }. Then we should subtract the same from our K

K_0 = {So + S1+ S2, …… + S_{n-2} }.

So to get the SubsetSum of T' equal to K_0 we have to subtract equal value that we subtract from T'

Which is

K_0 = k - S_{n-1}