Lab 2 Solutions, Problems 1, 2, 3, 4

(1) Determine the asymptotic running time of the following procedure (an exact computation of number of basic operations is not necessary):

```
int[] arrays(int n) {
    int[] arr = new int[n];
    for(int i = 0; i < n; ++i){
       arr[i] = 1;
    }
    for(int i = 0; i < n; ++i) {
        for(int j = i; j < n; ++j){
            arr[i] += arr[j] + i + j;
        }
    }
    return arr;

}
```

Solution. The first for loop takes $\mathrm{O}(n)$. The second (nested) for loop requires $\mathrm{O}(n^2)$. Asymptotic running time: $\mathrm{O}(n) + \mathrm{O}(n^2) = \mathrm{O}(n^2)$.

(2) See the Java file Merge.java. It is easy to see that there is essentially just one loop depending on n (the sum of the lengths of the two input arrays), so running time is $\mathrm{O}(n)$.

(3) (a) $1 + 4n^2$ is $\mathrm{O}(n^2)$

(b) $n^2 - 2n$ not in $\mathrm{O}(n)$

(c) $\log n$ is $\mathrm{o}(n)$

(d) n not $\mathrm{o}(n)$

Solution to (a). Let $c = 5$ and $n_0 = 1$. Then, whenever $n \geq n_0$,

$$1 + 4n^2 \leq n^2 + 4n^2 = 5n^2 = cn^2.$$

Solution to (b). Given postive c and natural number n_0, we find $n \geq n_0$ so that $n^2 - 2n \not\leq cn$: Let n be such that $n > \max\{c + 2, n_0\}$. Then

$$n^2 - 2n > cn \text{ if and only if } n - 2 > c \text{ if and only if } n > c + 2.$$

That last inequality is true, so the first one is also true.

Solution to (c). This is difficult to solve using the definition directly. We can say that since the limits at ∞ of $\log(n)$ and n "exist" (limits are ∞ in each case), we may use the limit version of the defintion of o. Doing so, we get

$$\lim_{n\to\infty} \frac{\log n}{n} = \lim_{n\to\infty} \frac{\frac{c}{n}}{1} = \lim_{n\to\infty} \frac{c}{n} = 0.$$

Therefore, $\log n$ is $\mathrm{o}(n)$

Solution to (d). We need to find $c > 0$ such that for every positive integer n_0 there is a positive integer $n \geq n_0$ such that $n > cn$. We choose $c = \frac{1}{2}$. Then for any n (in particular, for any $n \geq n_0$ for any choice of n_0), we have $n > cn$, as required.

(4) Power Set: See the Java file PowerSet.java.