Lab08

Question1:

a. Start with an empty stack of integers. You will attempt to do a sequence of pushes and pops so that the sequence of pops will be a specified permutation of 1, 2, 3 ,4 , 5, 6. You will be able to do exactly 6 push operations and 6 pop operations. The first push pushes 1 onto the stack; the next pushes 2; and so forth. The sixth push pushes 6 onto the stack.
   For this exercise, we will let S denote a push operation and X a pop operation. Example: The sequence SSSSSSXXXXXX outputs 654321.

   a. Describe a sequence of pushes and pops that would produce output 325641 (or explain why it is not possible)

      Answer:

      Push(1)
      Push(2)
      Push(3)
      Pop() → 3
      Pop() → 2
      Push(4)
      Push(5)
      Pop() → 5
      Push(6)
      Pop() → 6
      Pop() → 4
      Pop() → 1

   b. Describe a sequence of pushes and pops that would produce output 154623 (or explain why it is not possible)

      Answer:

      Push(1)
      Pop() → 1
      Push(2)
      Push(3)
      Push(4)
      Push(5)
      Pop() → 5
      Pop() → 4
      Push(6)
      Pop() → 6
      Pop() → 3

      This won't work because pop will generate 3 and then 2 (not 2->3)

b. Suppose we store n keys in a hash table of size m = n^2 using a hash function h randomly chosen from a Universal class H of hash functions. Assume that X is a random variable that counts the number of collisions. Show that the Expected number of Collisions is < 1/2.

Answer:

There are (n_{c2}) pairs of keys that may collide; each pair collides with probability 1/m if h is chosen At rando2m from a Universal hash function, H. Let x be a random variable that counts the number of collisions. So the expected number of collision will be:

$e(x) = (n_{c2}) . 1/n^2 \Rightarrow [n!/2!(n-2)!].1/n^2$

$= [n(n-1)\cancel{(n-2)!} / 2! \cancel{(n-2)!}].1/n^2$
$= n(n-1).1/2n^2$
$= n^2-n/2\ n^2$
$= \cancel{n^2} (1-1/n) / 2\ \cancel{n^2}$
$= (1 - 1/n)/2$, which is less than ½

½[(1-1/n)]< 1/2

Question2:

For each integer n = 1,2,3,..., 7, determine whether there exists a red-black tree having exactly n nodes, with *all of them black.* Fill out the chart below to tabulate the results:

Answer:

Num nodes *n*	**Does there exist a red-black tree with *n* nodes, all of which are black?**
1	Yes
2	No
3	Yes
4	No
5	No
6	No
7	Yes

Question3:

For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes and exactly one red node. Fill out the chart below to tabulate the results:

Answer:

Num nodes *n*	**Does there exist a red-black tree with *n* nodes that has exactly one red node?**
1	No
2	Yes
3	No
4	Yes
5	Yes
6	No
7	No