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3. Sampel acak $X_1, X_2, \ldots, X_n$ dari distribusi dengan fkp :

$f(x) = \frac{1}{\theta} \cdot e^{-\frac{x}{\theta}}, \; x > 0 \quad \Rightarrow X_i \sim \text{Gamma}(1, \theta)$

$\bar{X}$ adalah rata-rata sampel acak tersebut.

$Y = \frac{2n\bar{X}}{\theta}$, buktikan Y berdistribusi chi-kuadrat.

Jawab :

$$M_Y(t) = E(e^{tY}) = E(e^{\frac{2nt}{\theta}\bar{X}}) = E(e^{\frac{2t}{\theta}\sum X_i})$$

$$= E(e^{\frac{2t}{\theta}(X_1 + X_2 + X_3 + \ldots + X_n)})$$

$$= E(e^{\frac{2t}{\theta}X_1} \cdot e^{\frac{2t}{\theta}X_2} \cdot e^{\frac{2t}{\theta}X_3} \ldots e^{\frac{2t}{\theta}X_n})$$

$$= E(e^{\frac{2t}{\theta}X_1}) \cdot E(e^{\frac{2t}{\theta}X_2}) \cdot E(e^{\frac{2t}{\theta}X_3}) \ldots E(e^{\frac{2t}{\theta}X_n}) \qquad \text{saling bebas}$$

$$= [E(e^{\frac{2t}{\theta}X})]^n = [M_X(\tfrac{2t}{\theta})]^n$$

Perhatikan $X \sim \text{Gamma}(1, \theta)$ maka diperoleh

$M_X(\frac{2t}{\theta}) = (1 - \theta \cdot \frac{2t}{\theta})^{-1} = (1-2t)^{-1}$

$M_Y(t) = [M_X(\frac{2t}{\theta})]^n = [(1-2t)^{-1}]^n = (1-2t)^{-n}$

FPM Distribusi chi kuadrat : $X \sim \chi^2_{(r)} \Rightarrow M_X(t) = (1-2t)^{-\frac{r}{2}}$

$-\frac{r}{2} = -n$

$r = 2n$

Diperoleh Y berdistribusi chi kuadrat dengan derajat bebas $2n$. $Y \sim \chi^2_{(2n)}$