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Kuis 1 Fungsi Kompleks

1. a) · $u(x,y) + iv(x,y)$

misal : $z = x+iy$.

$f(z) = z^2 + 3z^3$

$= (x+iy)^2 + 3(x+iy)^3$

$= x^2 + 2xyi - y^2 + 3(x^3 + 3x^2yi - 3xy^2 - iy^3)$

$= x^2 + 2xyi - y^2 + 3x^3 + 9x^2yi - 9xy^2 - 3iy^3$

$= (x^2 - y^2 + 3x^3 - 9xy^2) + i(2xy + 9x^2y - 3y^3)$

· $u(r,\theta) + iv(r,\theta)$

misal : $z = r(\cos\theta + i\sin\theta)$.

$f(z) = r^2(\cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta) + 3r^3(\cos^3\theta + 2i\cos^2\theta\sin\theta - \sin^2\theta\cos\theta + i\sin\theta\cos^2\theta - 2\sin^2\theta\cos\theta - i\sin^3\theta)$

$= r^2(\cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta) + 3r^3(\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\sin^2\theta\cos\theta - i\sin^3\theta)$

$= r^2(\cos^2\theta - \sin^2\theta + 3r\cos^3\theta - 9r\sin^2\theta\cos\theta) + ir^3(\cos\theta + 9r\cos^2\theta\sin\theta - 3r\sin^3\theta)$

b) · $u(x,y) + iv(x,y)$

misal : $z = x+iy$.

$\frac{i}{x+iy} = \frac{i(x-iy)}{x^2+y^2} = \frac{ix+y^2}{x^2+y^2}$

$f(z) = i\bar{z} + Im(i/z)$

$= i(x-iy) + Im\left(\frac{i}{(x+iy)}\right)$

$= ix + y + \frac{x}{x^2+y^2} = \frac{x}{x^2+y^2} + y + ix$

· $u(r,\theta) + iv(r,\theta)$

misal : $z = r(\cos\theta + i\sin\theta)$.

$f(z) = i\bar{z} + Im(i/z)$

$= ir(\cos\theta - i\sin\theta) + Im\left(\frac{i}{r(\cos\theta+\sin\theta)}\right)$

$\frac{i}{r(\cos\theta+i\sin\theta)} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta} = \frac{i\cos\theta + \sin\theta}{r(\cos^2\theta + \sin^2\theta)} = \frac{i\cos\theta + \sin\theta}{r}$

$f(z) = ir\cos\theta + r\sin\theta + \frac{\cos\theta}{r} = r\sin\theta + \frac{\cos\theta}{r} + ir\cos\theta$

2. $u(x,y) = x^2 - y^2 + x$ dan $v(x,y) = 2xy - y$

$W = u(x,y) + iv(x,y)$

$= x^2 - y^2 + x + i(2xy - y)$

$= (x+iy)^2 + x - iy$

$= z^2 + \bar{z}$

3. $f(z) = \frac{z(z^2 + (2-i)z - 2i)}{z-i}$

$\lim_{z\to i} \frac{z(z-i)(z+2)}{z-i} = \lim_{z\to i} z(z+2)$

$= i^2 + 2i = 2i - 1$

4. $W = z - 2i$; $z = 0, 2i, 1, i, -2i, 1+i$

$z_1 = 0 \rightarrow W_1 = -2i$

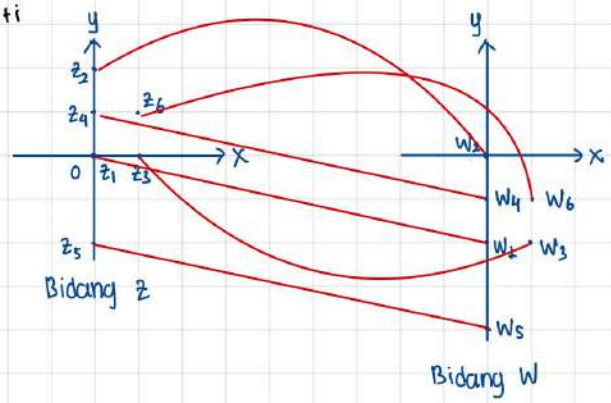
$z_2 = 2i \rightarrow W_2 = 0$

$z_3 = 1 \rightarrow W_3 = 1 - 2i$

$z_4 = i \rightarrow W_4 = -i$

$z_5 = -2i \rightarrow W_5 = -4i$

$z_6 = 1+i \rightarrow W_6 = 1 - i$



b. $W = e^x \cos y + i e^x \cos y$; $z = 0, 2\pi, -2\pi, 1-\pi i, -2+\pi i/2$

$z_1 = 0 + 0i \rightarrow w_1 = 1$

$z_2 = 2\pi + 0i \rightarrow w_2 = e^{2\pi}$

$z_3 = 0 - 2\pi i \rightarrow w_3 = 1$

$z_4 = 1 - \pi i \rightarrow w_4 = -e$

$z_5 = -2 + \frac{\pi}{2} i \rightarrow w_5 = e^{-2} \cdot i$