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2.  $X_1, X_2, X_3, \dots, X_8$  sampel acak dari distribusi dengan fkp :

$$p(x) = \begin{cases} \frac{1}{3} & , x = 1, 2, 3 \\ 0 & , x \text{ lainnya} \end{cases}$$

$$Y = \frac{1}{n} \sum_{i=1}^n X_i$$

Jawab

• Rata-rata

$$\mu_Y = E(Y) = E\left(\frac{1}{8} \sum_{i=1}^8 X_i\right) = \frac{1}{8} E(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8)$$

$$= \frac{1}{8} [E(X_1) + E(X_2) + \dots + E(X_8)] = \frac{1}{8} \cdot 8 E(X) = E(X) \quad \text{karena } X_1, X_2, \dots, X_8 \text{ sampel acak}$$

$$E(X) = \sum_x x p(x) = \sum_{x=1}^3 x \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} = \frac{6}{3} = 2$$

$$\mu_Y = E(X) = 2$$

• Varians

$$\text{Var}(Y) = \text{Var}\left(\frac{1}{8} \sum_{i=1}^8 X_i\right) = \frac{1}{64} \text{Var}(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8)$$

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$$

↳ 0 karena saling bebas

$$\text{Var}(Y) = \frac{1}{64} [\text{Var}(X_1) + \text{Var}(X_2) + \dots + \text{Var}(X_8)]$$

$$= \frac{1}{64} \cdot 8 \text{Var}(X) = \frac{1}{8} \text{Var}(X)$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_x x^2 p(x) = \sum_{x=1}^3 x^2 \frac{1}{3} = 1 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} + 9 \cdot \frac{1}{3} = \frac{14}{3}$$

$$\text{Var}(X) = \frac{14}{3} - (2)^2 = \frac{14}{3} - \frac{12}{3} = \frac{2}{3}$$

$$\text{Var}(Y) = \frac{1}{8} \text{Var}(X) = \frac{1}{8} \cdot \frac{2}{3} = \frac{1}{12}$$