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1. X_1, X_2, X_3 adalah sampel acak dari suatu populasi dengan fkp :

$f(x) = 3(1-x)^2, 0 < x < 1$

a. Buktikan bahwa fkp dari statistik order ke -3 adalah

$$g(y_3) = \frac{3!}{(3-1)!(3-3)!}[F(y_3)]^{3-1}[1-F(y_3)]^{3-3} f(y_3)$$

$$= 3\,[F(y_3)]^2 [1-F(y_3)]^0 f(y_3)$$

$$= 3\,[F(y_3)]^2 f(y_3)$$

Jawab :

Diperoleh fkp gabungan y_1, y_2, y_3 sebagai berikut :

$g(y_1, y_2, y_3) = 3!\ f(y_1) f(y_2) f(y_3) \quad , 0 < y_1 < y_2 < y_3 < 1$

Akan dicari fkp marginal $g(y_3)$

$$g(y_3) = \int_0^{y_3}\int_{y_1}^{y_3} 6\ f(y_1) f(y_2) f(y_3)\ dy_2\, dy_1$$

$$= 6\ f(y_3) \int_0^{y_3} f(y_1)\, dy_1 \int_{y_1}^{y_3} f(y_2)\, dy_2 \quad \rightarrow \text{ Sifat Integral}$$

$$= 6 \cdot f(y_3) \int_0^{y_3} dF(y_1) \int_{y_1}^{y_3} dF(y_2)$$

$$= 6\ f(y_3) \int_0^{y_3} dF(y_1)\,[F(y_3) - F(y_1)]$$

$$= 6 \cdot f(y_3) \int_0^{y_3} F(y_3)\, dF(y_1) - F(y_1)\, dF(y_1)$$

$$= 6\ f(y_3) \left[F(y_3) F(y_1) \Big]_0^{y_3} - \tfrac{1}{2}(F(y_1))^2 \Big]_0^{y_3} \right]$$

$$= 6\ f(y_3) \left[(F(y_3))^2 - 0 - \tfrac{1}{2}(F(y_3))^2 + \tfrac{1}{2}(F(0))^2 \right]$$

$$= 6\ f(y_3)\ \tfrac{1}{2}\ (F(y_3))^2 \qquad = 3\ f(y_3)\ [f(y_3)]^2$$

b. Misal Y_1 adalah nilai minimum dari ketiga sampel acak tersebut, tentukan fkp Y_1

$$g_k(y_k) = \begin{cases} \frac{n!}{(k-1)!(n-k)!}[F(y_k)]^{k-1}[1-F(y_k)]^{n-k} f(y_k) & , a < y_k < b \\ 0 & , y_k \text{ lainnya} \end{cases}$$

$$g(y_1) = \frac{3!}{(1-1)!(3-1)!}[F(y_1)]^{1-1}[1-F(y_1)]^{3-1} f(y_1) \ , 0 < y_1 < 1$$

$$= 3\ [1-F(y_1)]^2 f(y_1)$$

$$= 3\ [1 + (1-y_1)^3 - 1]^2\ 3\ (1-y_1)^2$$

$$= 3\ [(1-y_1)^3]^2\ 3\ (1-y_1)^2$$

$$= 9\ (1-y_1)^8$$

$$F(y_1) = \int_0^{y_1} 3(1-x)^2 dx = -3\int_0^{y_1}(1-x)^2\, d(1-x)$$

$$= -[(1-x)^3]_0^{y_1} = -(1-y_1)^3 + 1$$