

Introducing an AR Model

TIME SERIES ANALYSIS IN PYTHON



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Mathematical Description of AR(1) Model

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

- Since only one lagged value on right hand side, this is called:
 - AR model of order 1, or
 - AR(1) model
- AR parameter is ϕ
- For stationarity, $-1 < \phi < 1$

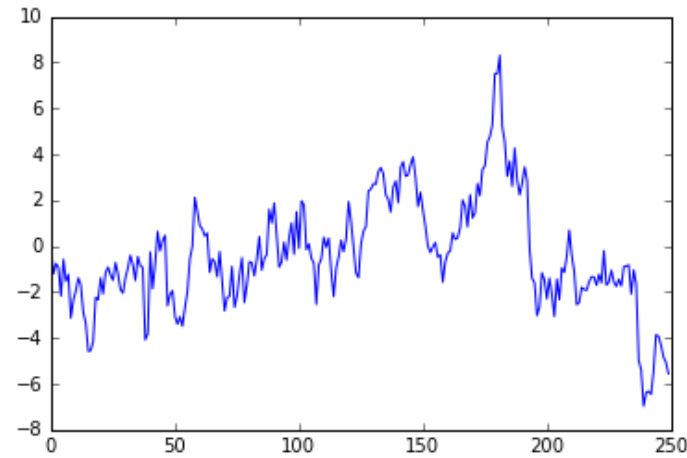
Interpretation of AR(1) Parameter

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

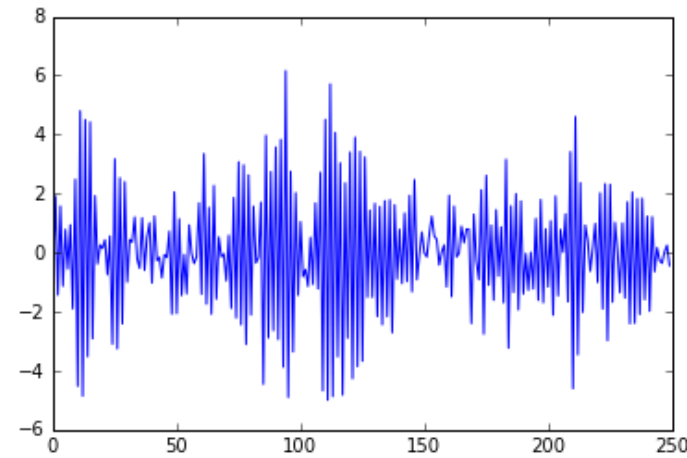
- Negative ϕ : Mean Reversion
- Positive ϕ : Momentum

Comparison of AR(1) Time Series

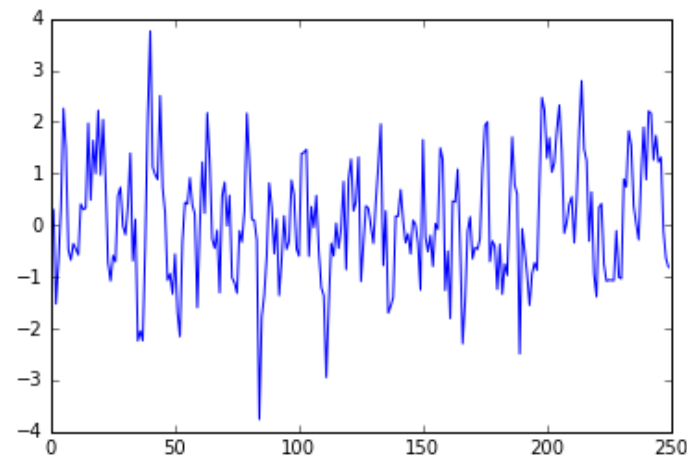
- $\phi = 0.9$



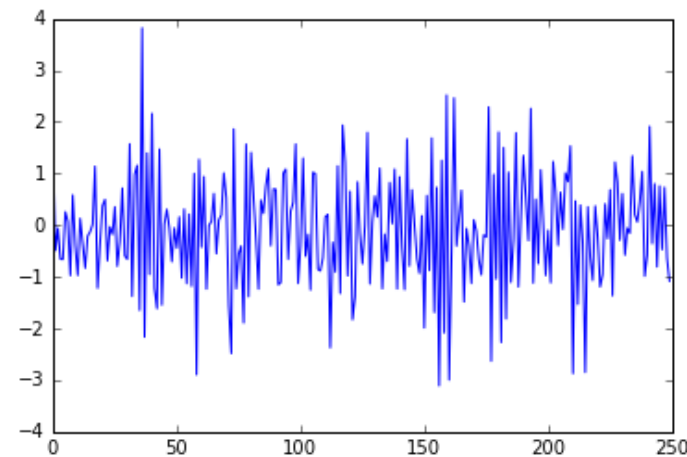
- $\phi = -0.9$



- $\phi = 0.5$

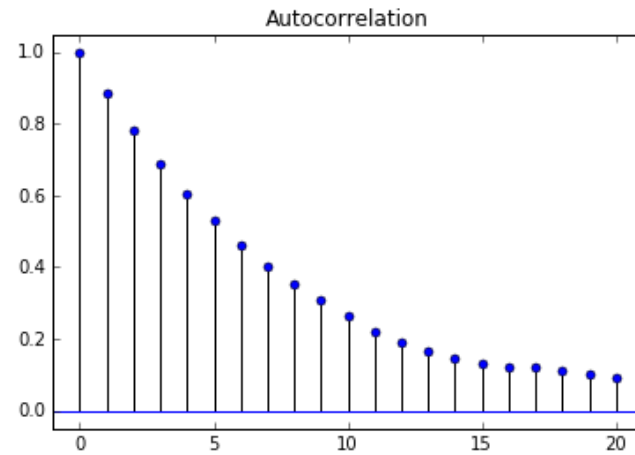


- $\phi = -0.5$

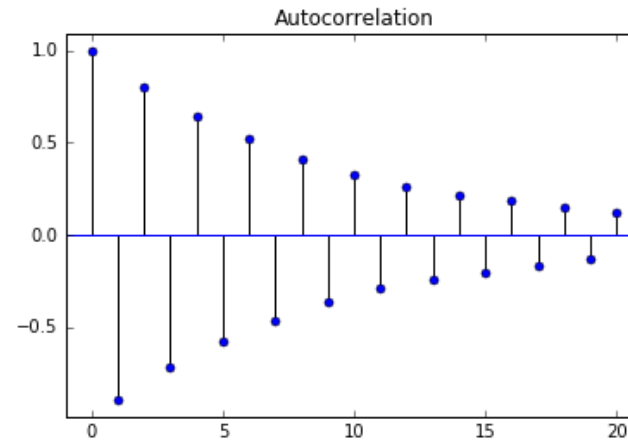


Comparison of AR(1) Autocorrelation Functions

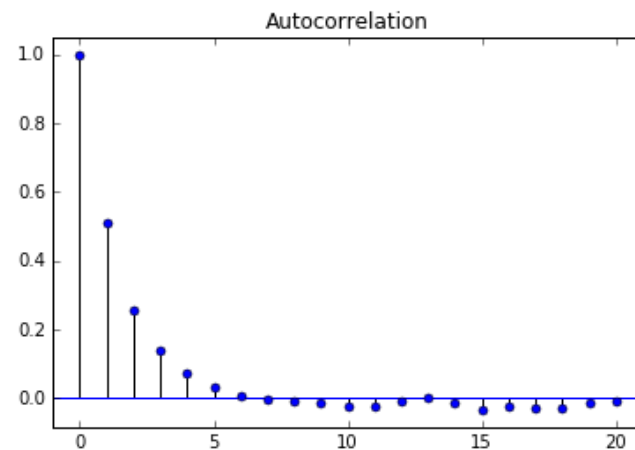
- $\phi = 0.9$



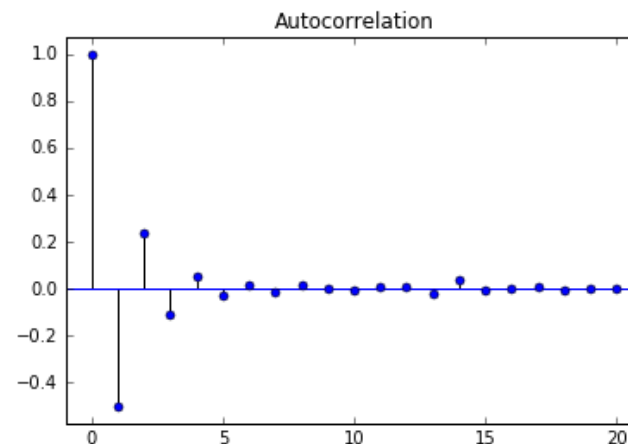
- $\phi = -0.9$



- $\phi = 0.5$



- $\phi = -0.5$



Higher Order AR Models

- AR(1)

$$R_t = \mu + \phi_1 R_{t-1} + \epsilon_t$$

- AR(2)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \epsilon_t$$

- AR(3)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \phi_3 R_{t-3} + \epsilon_t$$

- ...

Simulating an AR Process

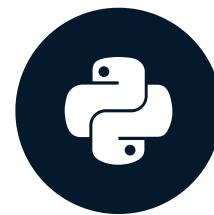
```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1, -0.9])
ma = np.array([1])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

Let's practice!

TIME SERIES ANALYSIS IN PYTHON

Estimating and Forecasting an AR Model

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Estimating an AR Model

- To estimate parameters from data (simulated)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

Estimating an AR Model

- Full output (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.summary())
```

```

                    ARMA Model Results
=====
Dep. Variable:          y      No. Observations:      5000
Model:                ARMA(1, 0)  Log Likelihood      -7178.386
Method:              css-mle    S.D. of innovations      1.017
Date:                Fri, 01 Dec 2017  AIC              14362.772
Time:                15:34:50    BIC              14382.324
Sample:              0      HQIC              14369.625

=====

```

	coef	std err	z	P> z	[95.0% Conf. Int.]	
const	-0.0361	0.152	-0.238	0.812	-0.333	0.261
ar.L1.y	0.9054	0.006	151.020	0.000	0.894	0.917

```

                    Roots
=====

```

	Real	Imaginary	Modulus	Frequency
AR.1	1.1045	+0.0000j	1.1045	0.0000

```

=====
```

Estimating an AR Model

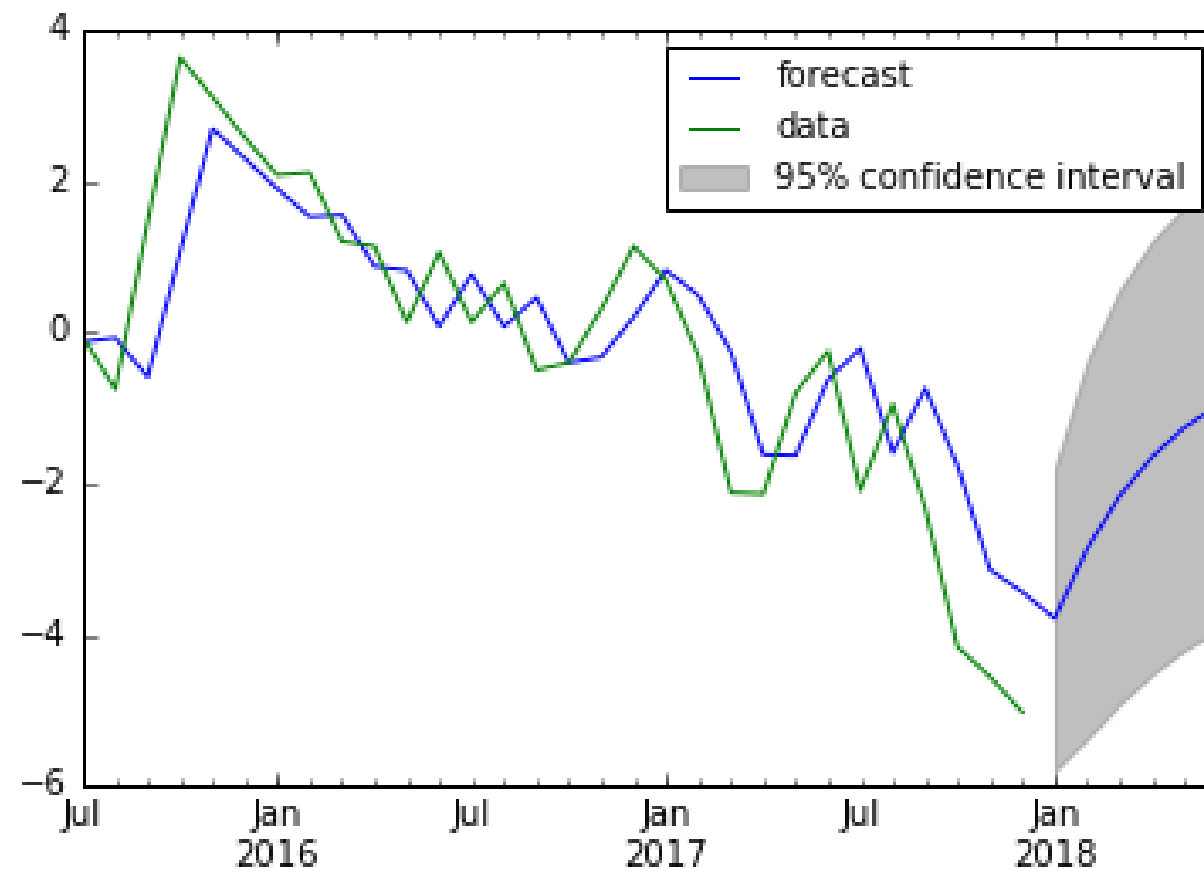
- Only the estimates of μ and ϕ (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.params)
```

```
array([-0.03605989,  0.90535667])
```

Forecasting an AR Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```



Let's practice!

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Choosing the Right Model

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Identifying the Order of an AR Model

- The order of an AR(p) model will usually be unknown
- Two techniques to determine order
 - Partial Autocorrelation Function
 - Information criteria

Partial Autocorrelation Function (PACF)

$$R_t = \phi_{0,1} + \boxed{\phi_{1,1}} R_{t-1} + \epsilon_{1t}$$

$$R_t = \phi_{0,2} + \phi_{1,2} R_{t-1} + \boxed{\phi_{2,2}} R_{t-2} + \epsilon_{2t}$$

$$R_t = \phi_{0,3} + \phi_{1,3} R_{t-1} + \phi_{2,3} R_{t-2} + \boxed{\phi_{3,3}} R_{t-3} + \epsilon_{3t}$$

$$R_t = \phi_{0,4} + \phi_{1,4} R_{t-1} + \phi_{2,4} R_{t-2} + \phi_{3,4} R_{t-3} + \boxed{\phi_{4,4}} R_{t-4} + \epsilon_{4t}$$

⋮

Plot PACF in Python

- Same as ACF, but use `plot_pacf` instead of `plt_acf`
- Import module

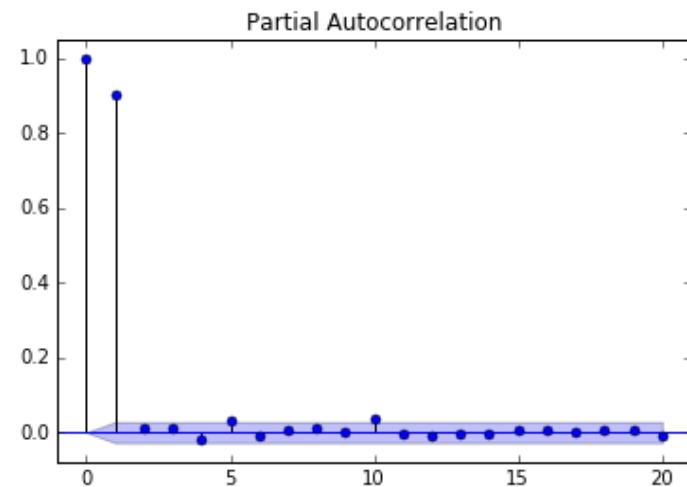
```
from statsmodels.graphics.tsaplots import plot_pacf
```

- Plot the PACF

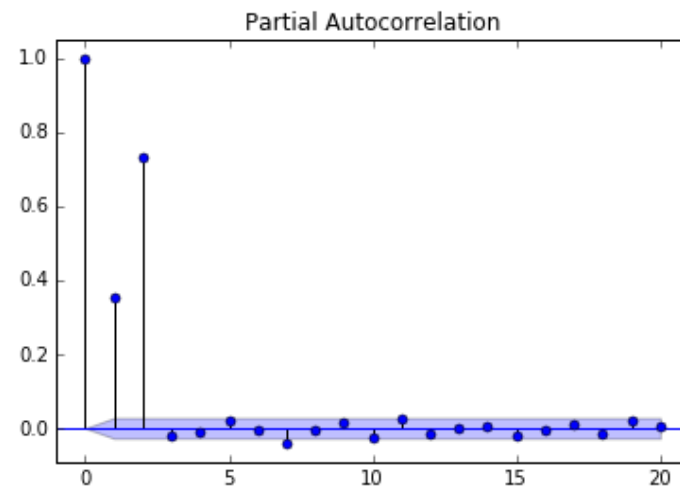
```
plot_pacf(x, lags= 20, alpha=0.05)
```

Comparison of PACF for Different AR Models

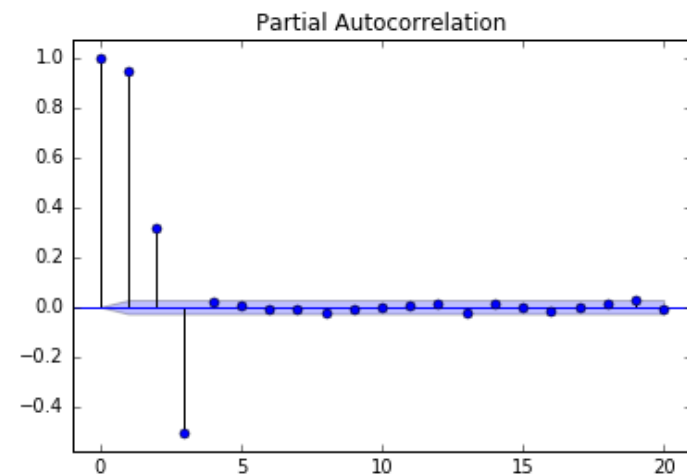
- AR(1)



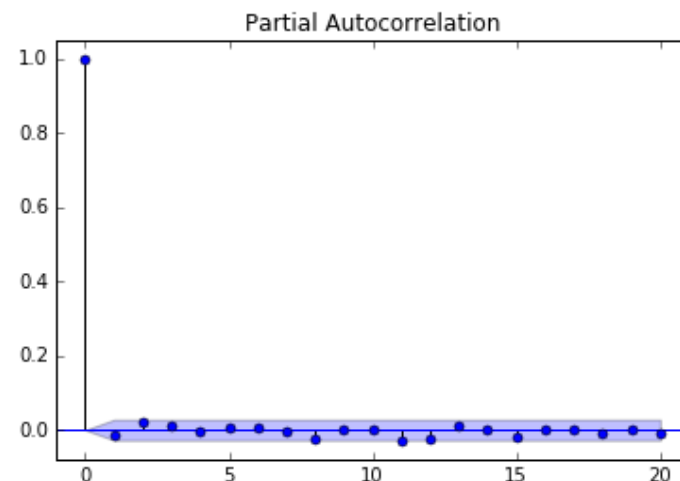
- AR(2)



- AR(3)



- White Noise



Information Criteria

- Information criteria: adjusts goodness-of-fit for number of parameters
- Two popular adjusted goodness-of-fit measures
 - AIC (Akaike Information Criterion)
 - BIC (Bayesian Information Criterion)

Information Criteria

- Estimation output

```
=====
                        ARMA Model Results
=====
Dep. Variable:          y      No. Observations:          2500
Model:                  ARMA(2, 0)  Log Likelihood          -3536.481
Method:                 css-mle   S.D. of innovations          0.996
Date:                  Fri, 29 Dec 2017  AIC              7080.963
Time:                  22:53:24      BIC              7104.259
Sample:                0      HQIC              7089.420
=====

=====
                        coef      std err          z      P>|z|      [95.0% Conf. Int.]
-----
const          0.0054      0.010      0.517      0.605      -0.015      0.026
ar.L1.y        -0.6130      0.019     -32.243      0.000      -0.650     -0.576
ar.L2.y        -0.3109      0.019     -16.351      0.000      -0.348     -0.274
=====
                        Roots
=====
                        Real      Imaginary      Modulus      Frequency
-----
AR.1          -0.9859      -1.4982j      1.7935      -0.3426
AR.2          -0.9859      +1.4982j      1.7935      0.3426
=====
```

Getting Information Criteria From `statsmodels`

- You learned earlier how to fit an AR model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

- And to get full output

```
result.summary()
```

- Or just the parameters

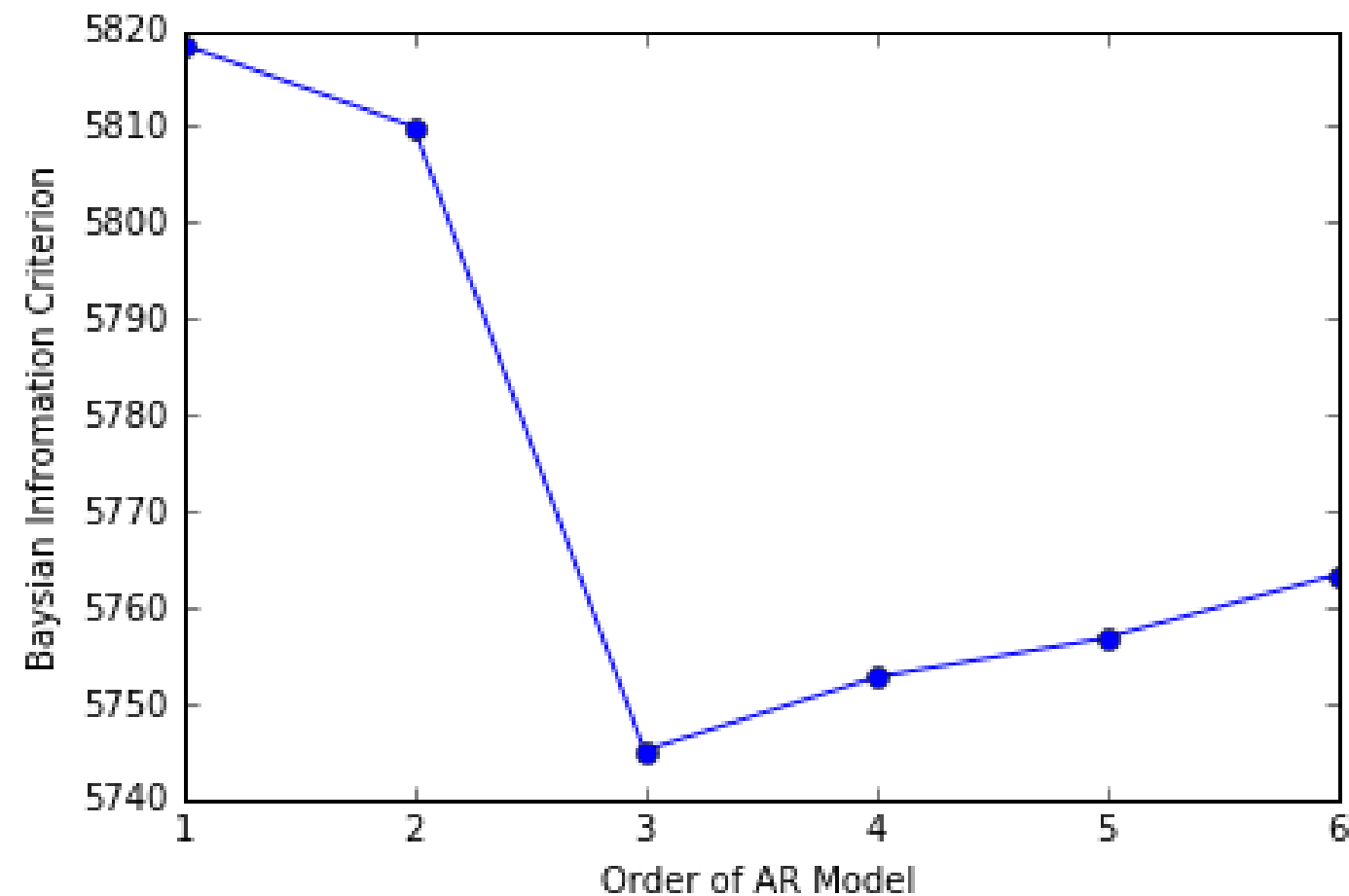
```
result.params
```

- To get the AIC and BIC

```
result.aic
result.bic
```

Information Criteria

- Fit a simulated AR(3) to different AR(p) models
- Choose p with the lowest BIC



Let's practice!

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