Why do we need GARCH models

GARCH MODELS IN PYTHON

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Course overview

GARCH: Generalized AutoRegressive Conditional Heteroskedasticity

- Chapter 1: GARCH Model Fundamentals
- Chapter 2: GARCH Model Configuration
- Chapter 3: Model Performance Evaluation
- Chapter 4: GARCH in Action

What is volatility

- Describe the dispersion of financial asset returns over time
- Often computed as the standard deviation or variance of price returns
- The higher the volatility, the riskier a financial asset

How to compute volatility

- Step 1: Calculate returns as percentage of price changes

$$return = \frac{P_1 - P_0}{P_0}$$

- Step 2: Calculate the sample mean return

$$mean = \frac{\sum_{i=1}^{n} return_i}{n}$$

- Step 3: Calculate the sample standard deviation

$$volatility = \sqrt{\frac{\sum_{i=1}^{n} (return_i - mean)^2}{n-1}} = \sqrt{variance}$$

Compute volatility in Python

Use pandas `pct_change()` method:

```
return_data = price_data.pct_change()
```

Use pandas `std()` method:

```
volatility = return_data.std()
```

Volatility conversion

- Convert to monthly volatility from daily:

(assume 21 trading days in a month)

$$\sigma_{monthly} = \sqrt{21} * \sigma_d$$

- Convert to annual volatility from daily:

(assume 252 trading days in a year)

$$\sigma_{annual} = \sqrt{252} * \sigma_d$$

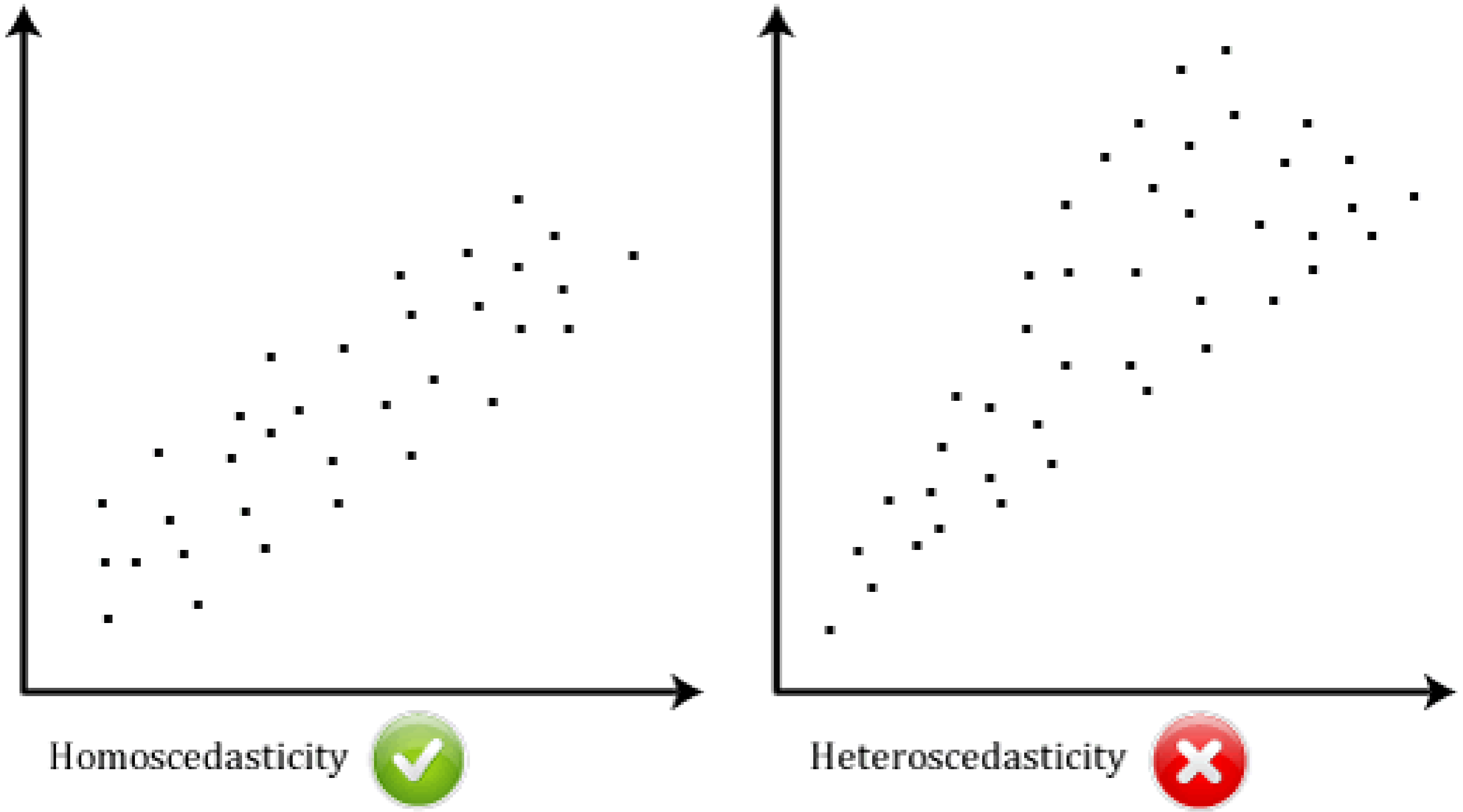
The challenge of volatility modeling

Heteroskedasticity:

- In ancient Greek: "different" (hetero) + "dispersion" (skedasis)
- A time series demonstrates varying volatility systematically over time

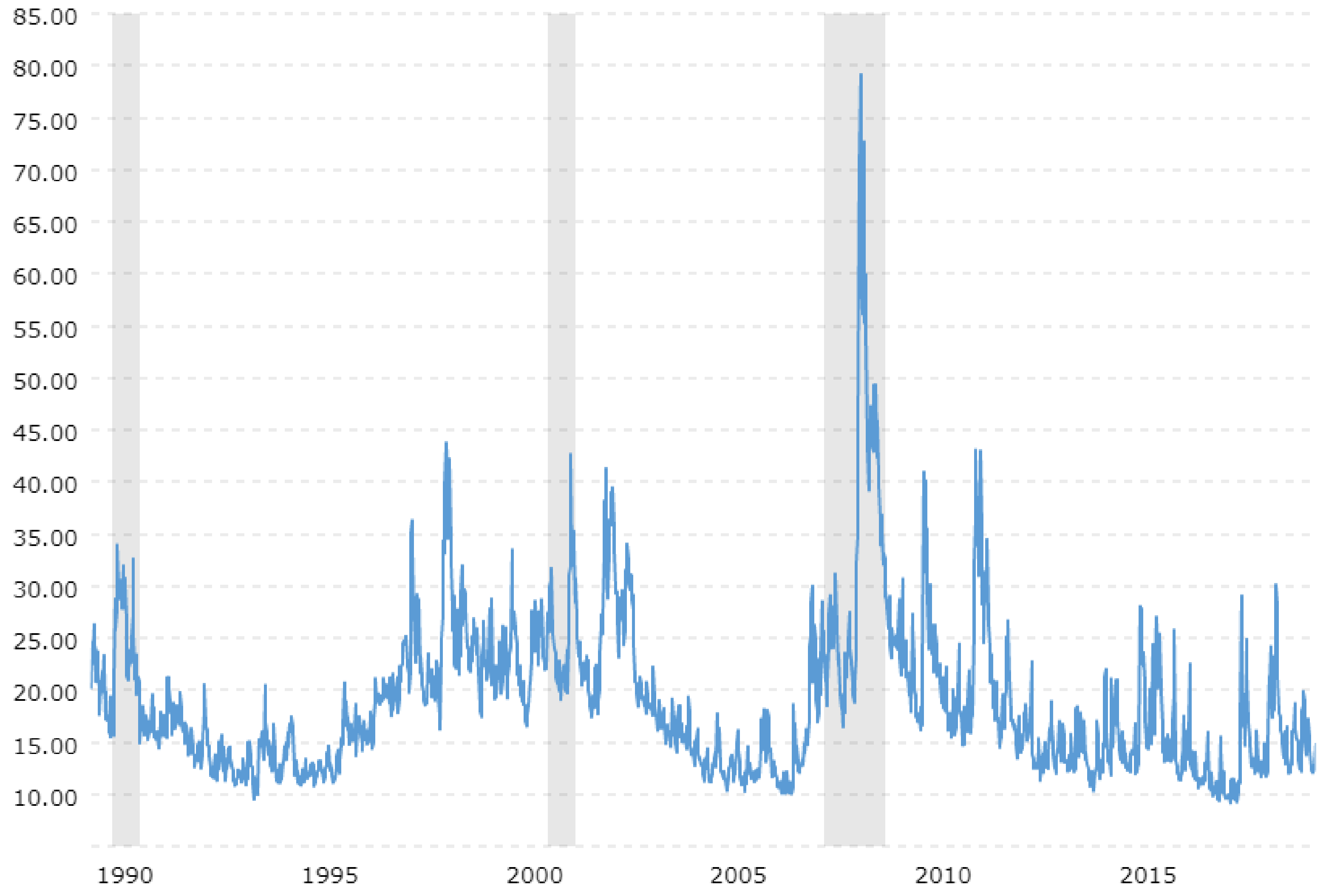
Detect heteroskedasticity

Homoskedasticity vs Heteroskedasticity



Volatility clustering

VIX historical prices:



Let's practice!

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What are ARCH and GARCH

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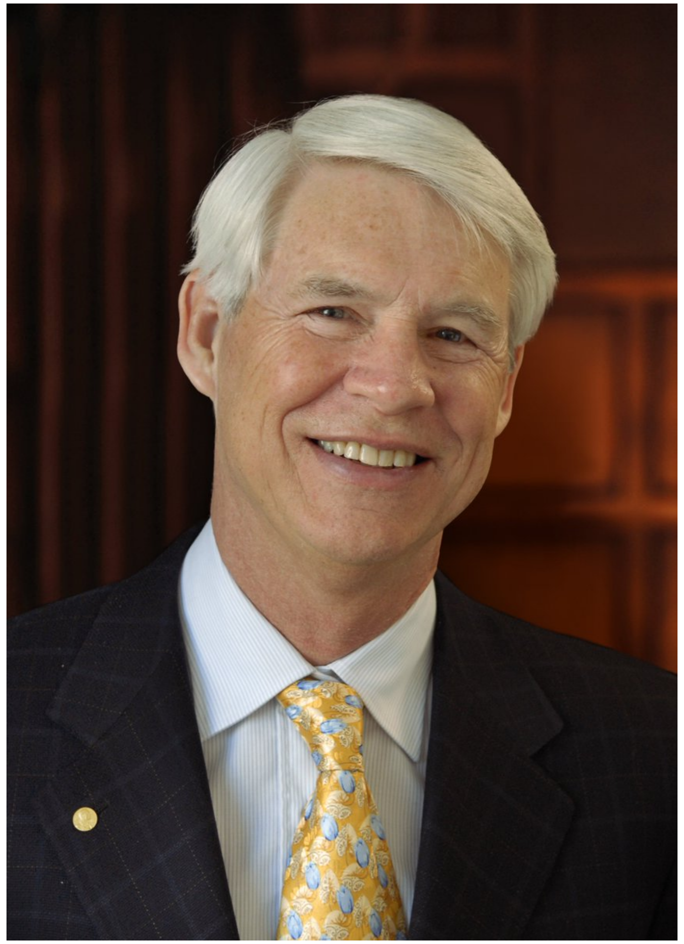
First came the ARCH

- Auto Regressive Conditional Heteroskedasticity
- Developed by Robert F. Engle (Nobel prize laureate 2003)

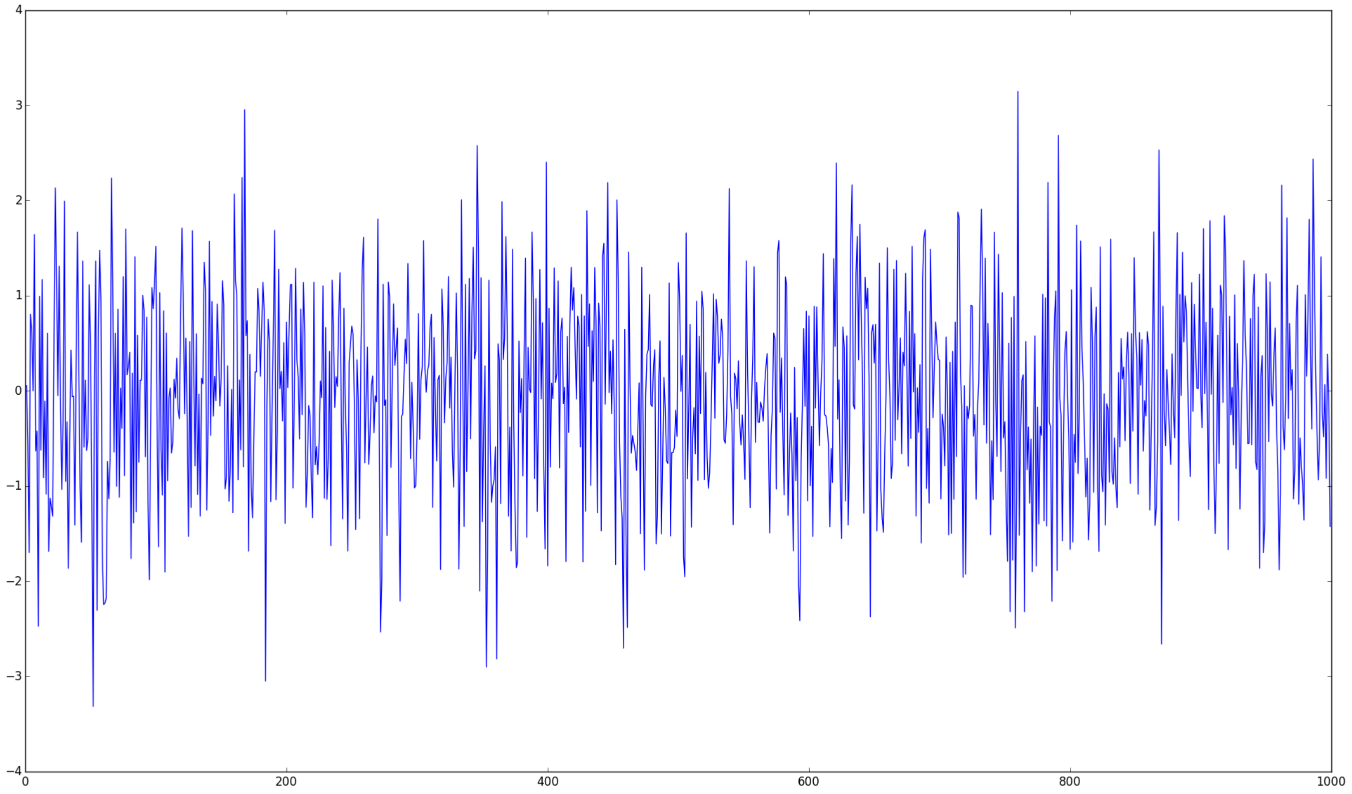


Then came the GARCH

- "Generalized" ARCH
- Developed by Tim Bollerslev (Robert F. Engle's student)

Related statistical terms

White noise (z): Uncorrelated random variables with a zero mean and a finite variance



Residual = predicted value - observed value

Model notations

Expected return:

$$\mu_t = Expected[r_t | I(t-1)]$$

Expected volatility:

$$\sigma^2 = Expected[(r_t - \mu_t)^2 | I(t-1)]$$

Residual (prediction error):

$$r_t = \mu_t + \epsilon_t$$

Volatility is related to the residuals:

$$\epsilon_t = \sigma_t * \zeta(WhiteNoise)$$

Model equations: ARCH

$$ARCH(p) : \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$$

$$ARCH(1) : \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2$$

Model equations: GARCH

$$GARCH(p,q): \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$GARCH(1,1): \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Model intuition

- Autoregressive: predict future behavior based on past behavior
- Volatility as a weighted average of past information

GARCH(1,1) parameter constraints

To make the GARCH(1,1) process realistic, it requires:

- All parameters are non-negative, so the variance cannot be negative.

$$\omega, \alpha, \beta >= 0$$

- Model estimations are "mean-reverting" to the long-run variance.

$$\alpha + \beta < 1$$

long-run variance:

$$\omega/(1 - \alpha - \beta)$$

GARCH(1,1) parameter dynamics

- The larger the α, the bigger the immediate impact of the shock
- The larger the β, the longer the duration of the impact

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How to implement GARCH models in Python

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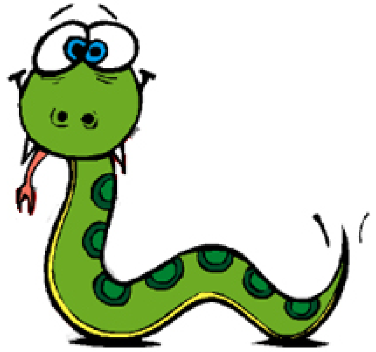
Python "arch" package

```
from arch import arch_model
```



[1] Kevin Sheppard. (2019, March 28). bashtage/arch: Release 4.8.1 (Version 4.8.1). Zenodo. http://doi.org/10.5281/zenodo.2613877

Workflow

Develop a GARCH model in three steps:

1. Specify the model
2. Fit the model
3. Make a forecast

Model specification

Model assumptions:

- Distribution: `"normal"` (default), `"t"` , `"skewt"`
- Mean model: `"constant"` (default), `"zero"` , `"AR"`
- Volatility model: `"GARCH"` (default), `"ARCH"` , `"EGARCH"`

```
basic_gm = arch_model(sp_data['Return'], p = 1, q = 1,
                      mean = 'constant', vol = 'GARCH', dist = 'normal')
```

Model fitting

Display model fitting output after every n iterations:

```
gm_result = gm_model.fit(update_freq = 4)
```

```
Iteration:      4,   Func. Count:     34,   Neg. LLF: 2783.005885607893
Iteration:      8,   Func. Count:     61,   Neg. LLF: 2771.9886612376513
Iteration:     12,   Func. Count:     85,   Neg. LLF: 2771.963828246998
Optimization terminated successfully.    (Exit mode 0)
            Current function value: 2771.9638282462456
            Iterations: 12
            Function evaluations: 85
            Gradient evaluations: 12
```

Turn off the display:

```
gm_result = gm_model.fit(disp = 'off')
```

Fitted results: parameters

Estimated by "maximum likelihood method"

```
print(gm_result.params)
```

```
mu           0.077239
omega        0.039587
alpha[1]     0.167963
beta[1]      0.786467
Name: params, dtype: float64
```

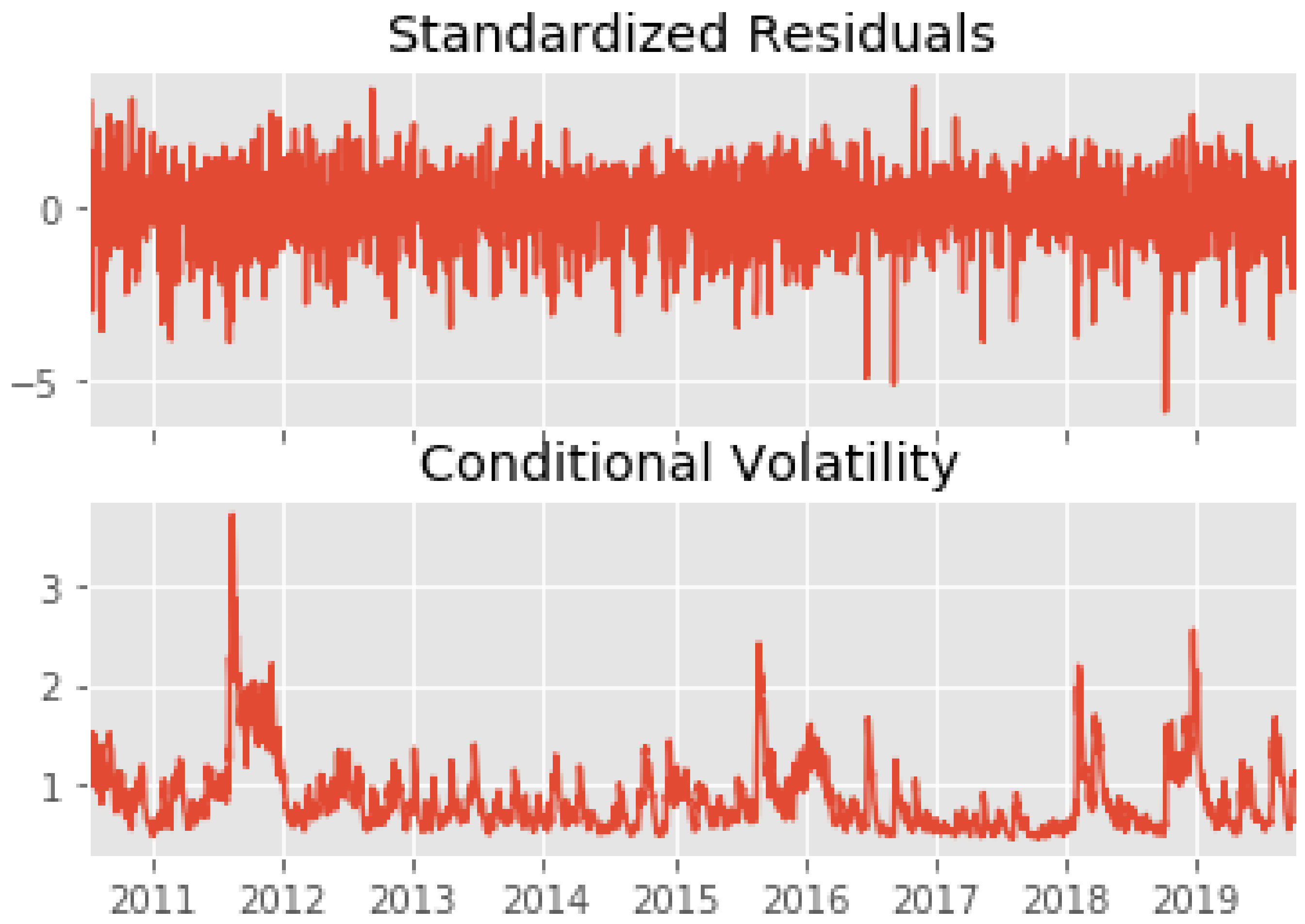
Fitted results: summary

```
print(gm_result.summary())
```

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  Return   R-squared:                      -0.001
Mean Model:              Constant Mean   Adj. R-squared:                 -0.001
Vol Model:                       GARCH   Log-Likelihood:               -2771.96
Distribution:                   Normal   AIC:                           5551.93
Method:             Maximum Likelihood   BIC:                           5574.95
                                         No. Observations:                 2336
Date:                 Mon, Dec 02 2019   Df Residuals:                     2332
Time:                         12:54:53   Df Model:                            4
                               Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0772  1.445e-02      5.345  9.031e-08 [4.892e-02,  0.106]
                             Volatility Model
==============================================================================
                 coef    std err          t      P>|t|        95.0% Conf. Int.
------------------------------------------------------------------------------
omega          0.0396  9.181e-03      4.312  1.619e-05 [2.159e-02,5.758e-02]
alpha[1]       0.1680  2.690e-02      6.243  4.284e-10     [  0.115,  0.221]
beta[1]        0.7865  2.722e-02     28.897 1.303e-183     [  0.733,  0.840]
==============================================================================
```

Fitted results: plots

```
gm_result.plot()
```



Model forecasting

```
# Make 5-period ahead forecast
gm_forecast = gm_result.forecast(horizon = 5)
```

```
# Print out the last row of variance forecast
print(gm_forecast.variance[-1:])
```

```
                 h.1       h.2       h.3       h.4       h.5
Date
2019-10-10  0.994079  0.988366  0.982913  0.977708  0.972741
```

h.1 in row "2019-10-10": 1-step ahead forecast made using data up to and including that date

Let's practice!

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