Significance testing of model parameters

GARCH MODELS IN PYTHON

Chelsea Yang
Data Science Instructor

Do I need this parameter?

- Is it relevant
- KISS: keep it simple stupid

- Always prefer a parsimonious model

Hypothesis test

- Null hypothesis (H0): a claim to be verified
- H0: parameter value = 0
- If H0 cannot be rejected, leave out the parameter

Statistical significance

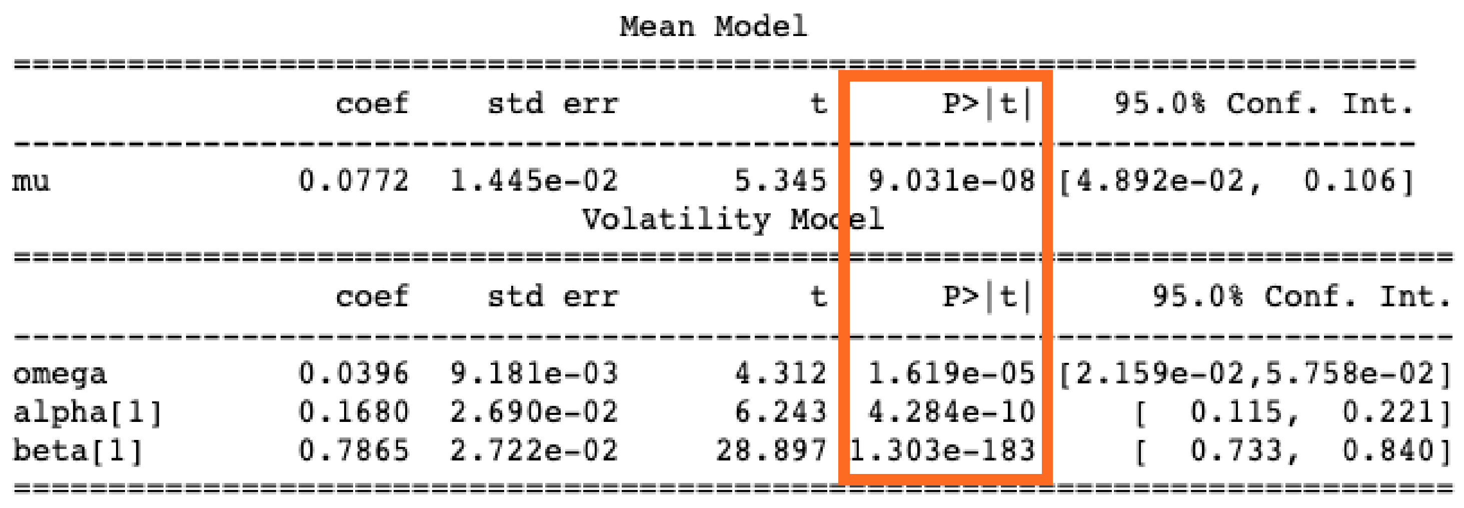
- Quantify having the observed results by chance
- Common threshold: 5%

P-value

- The odds of the observed results could have happened by chance
- The lower the p-value, the more ridiculous the null hypothesis looks
- Reject the null hypothesis if p-value < significance level

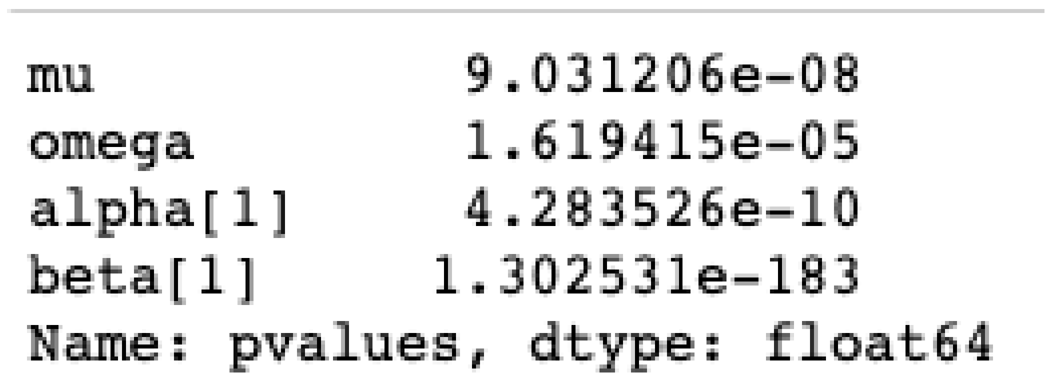
P-value example

```
print(gm_result.summary())
```



```
                               Mean Model
=============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
-----------------------------------------------------------------------------
mu             0.0772  1.445e-02      5.345  9.031e-08 [4.892e-02,  0.106]
                            Volatility Model
===============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-------------------------------------------------------------------------------
omega          0.0396  9.181e-03      4.312  1.619e-05 [2.159e-02,5.758e-02]
alpha[1]       0.1680  2.690e-02      6.243  4.284e-10     [  0.115,  0.221]
beta[1]        0.7865  2.722e-02     28.897 1.303e-183     [  0.733,  0.840]
===============================================================================
```

```
print(gm_result.pvalues)
```



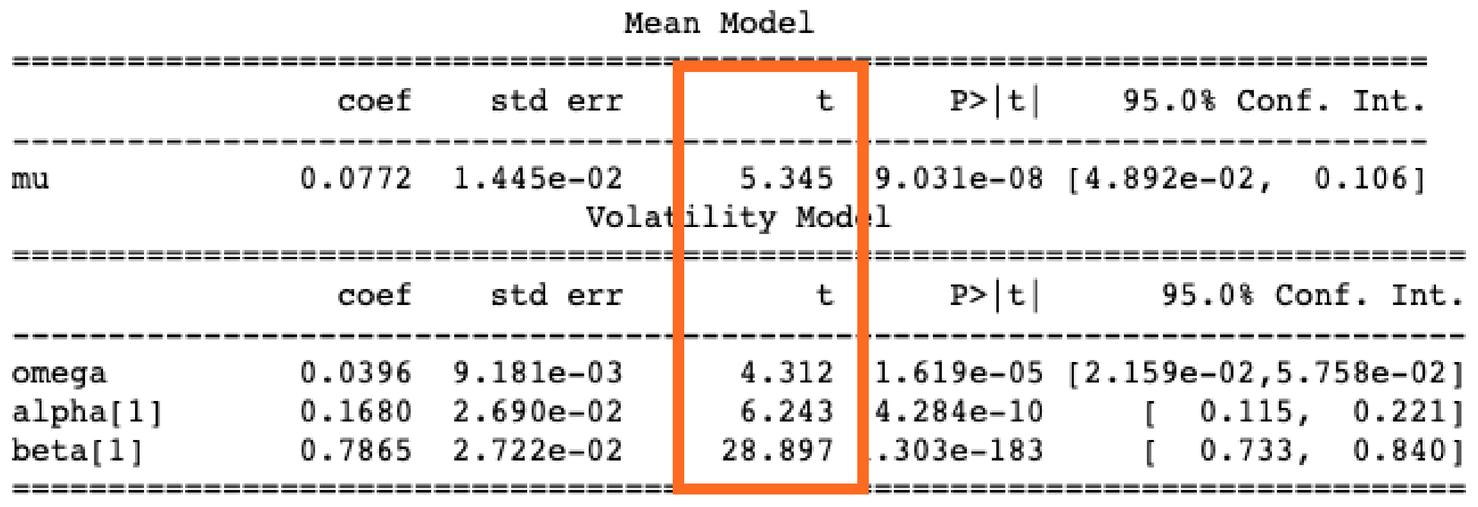
```
mu              9.031206e-08
omega           1.619415e-05
alpha[1]        4.283526e-10
beta[1]        1.302531e-183
Name: pvalues, dtype: float64
```

T-statistic

- T-statistic = estimated parameter / standard error
- The absolute value of the t-statistic is a distance measure
- If |t-statistic| > 2: keep the parameter in the GARCH model

T-statistic example

```
print(gm_result.summary())
```



```
                                Mean Model
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beta[1]        0.7865  2.722e-02     28.897  .303e-183     [  0.733,  0.840]
==============================================================================
```
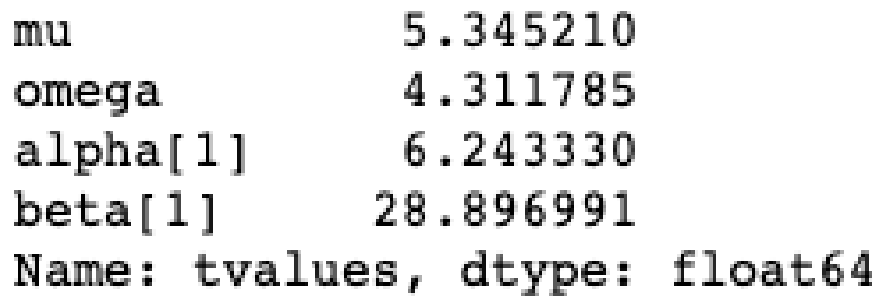

```
print(gm_result.tvalues)
```



```
mu           5.345210
omega        4.311785
alpha[1]     6.243330
beta[1]     28.896991
Name: tvalues, dtype: float64
```

```
# Manual calculation
t = gm_result.params/gm_result.std_err
```

Let's practice!

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Validation of GARCH model assumptions

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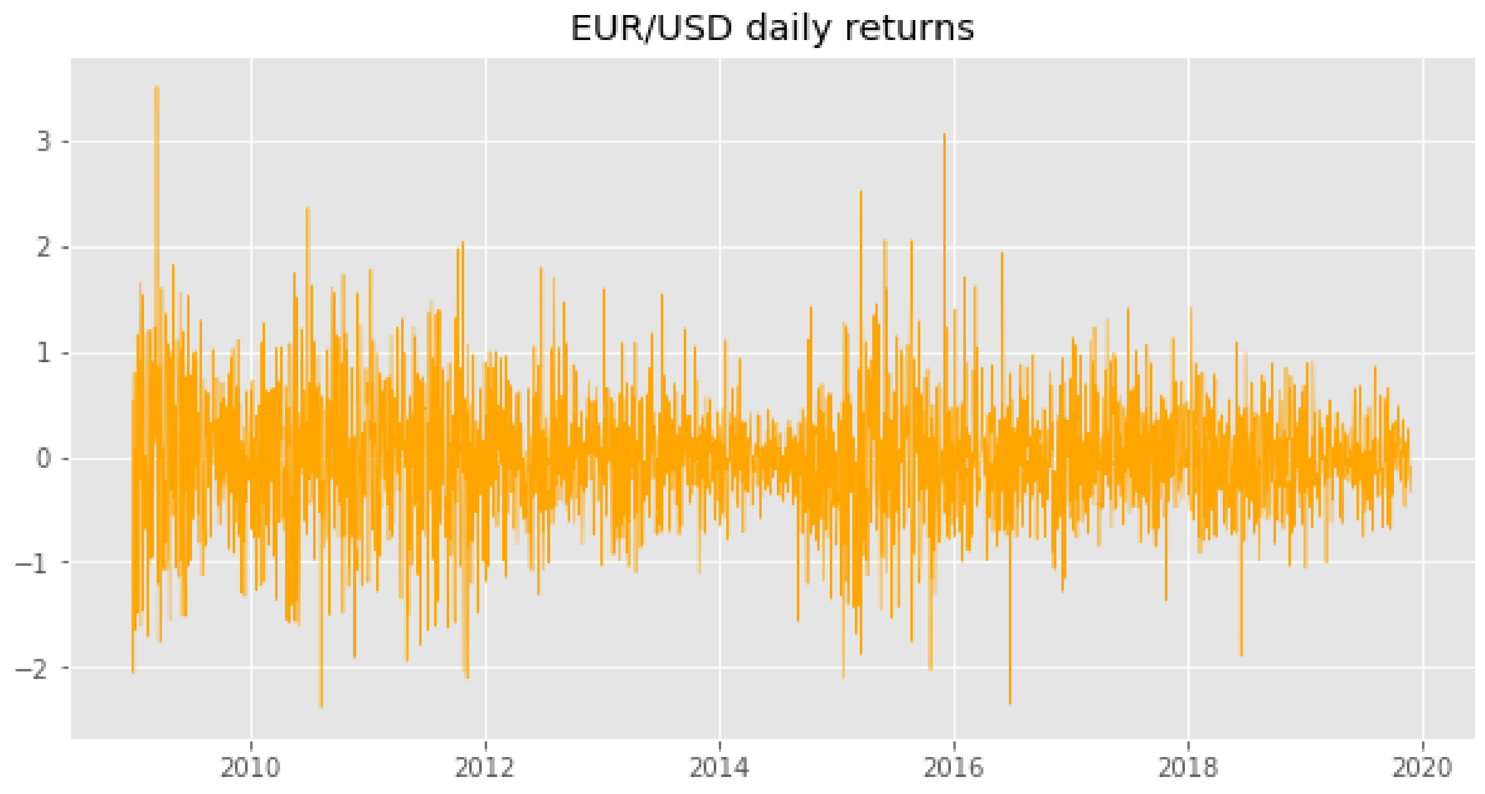
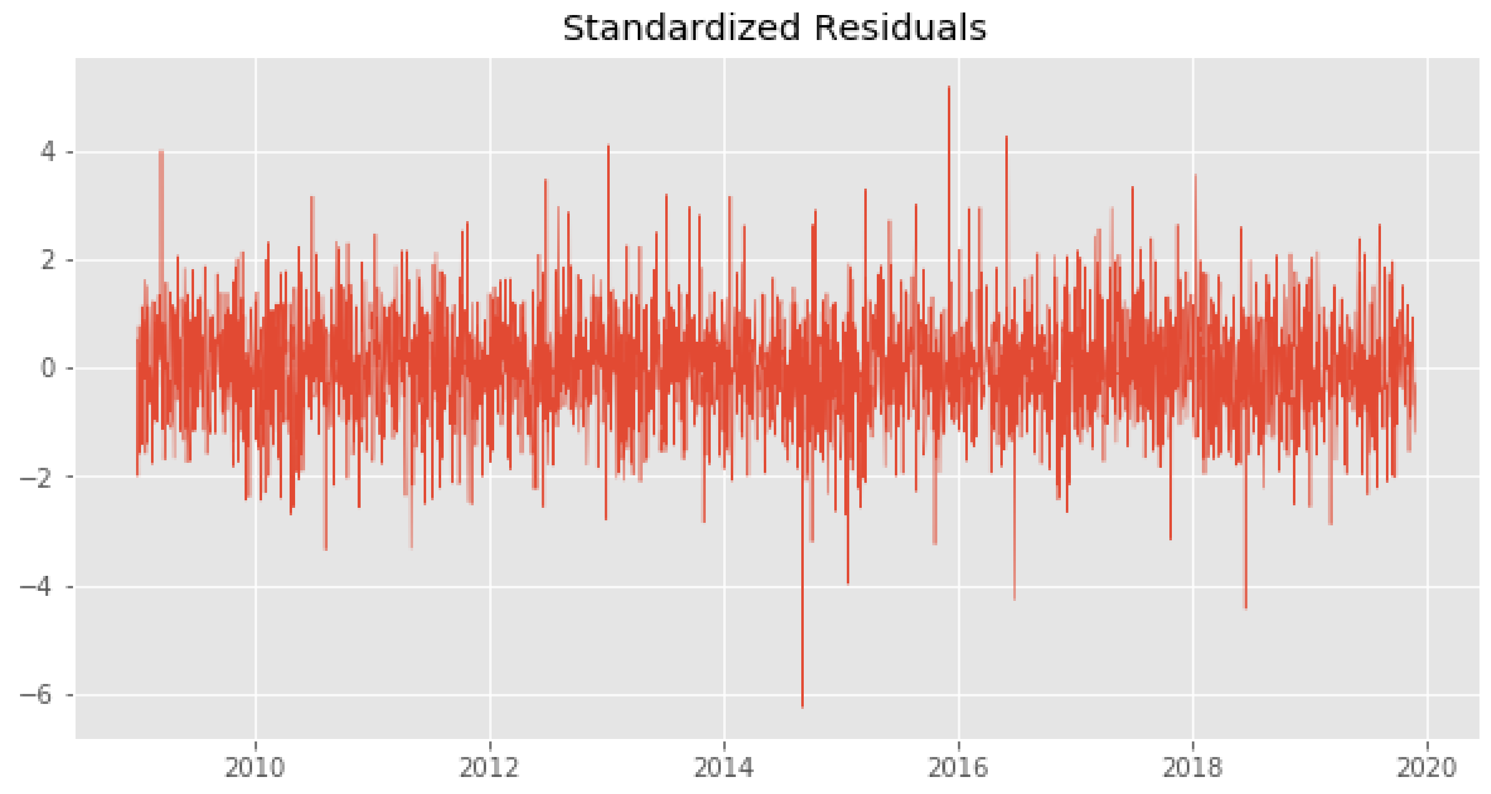
Visual check





Autocorrelation

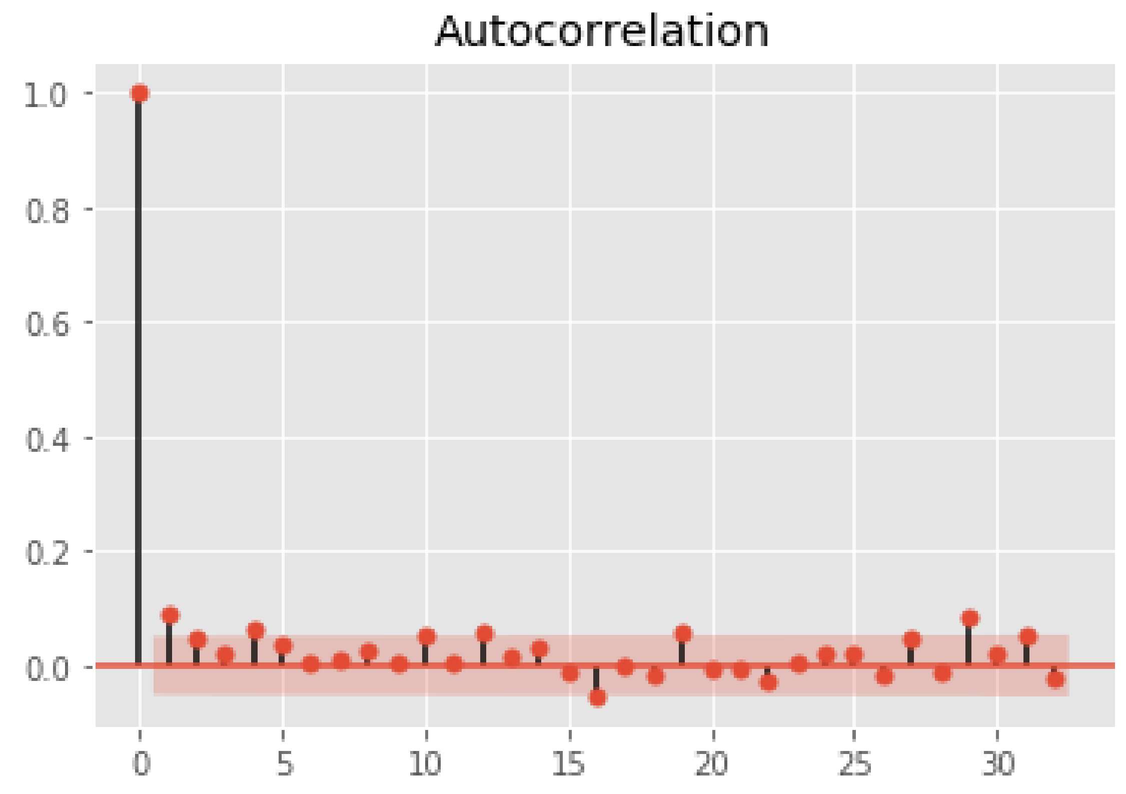
- Describe the correlation of a variable with itself given a time lag
- Existence of autocorrelation in the standardized residuals indicates the model may not be sound

To detect autocorrelation:

- ACF plot
- Ljung-Box

ACF plot

- ACF: AutoCorrelation Function
- ACF Plot: visual representation of the autocorrelation by lags



Red area in the plot indicates the confidence level (alpha = 5%)

ACF plot in Python

```python
from statsmodels.graphics.tsaplots import plot_acf


plot_acf(my_data, alpha = 0.05)
```

Ljung-Box test

- Test whether any of a group of autocorrelations of a time series are different from zero
- H0: the data is independently distributed
- P-value < 5%: the model is not sound

Ljung-Box test Python

```
# Import the Python module
from statsmodels.stats.diagnostic import acorr_ljungbox
```

```
# Perform the Ljung-Box test
lb_test = acorr_ljungbox(std_resid , lags = 10)
```

```
# Check p-values
print('P-values are: ', lb_test[1])
```

Let's practice!

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Goodness of fit measures

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Goodness of fit

Can model do a good job explaining the data?

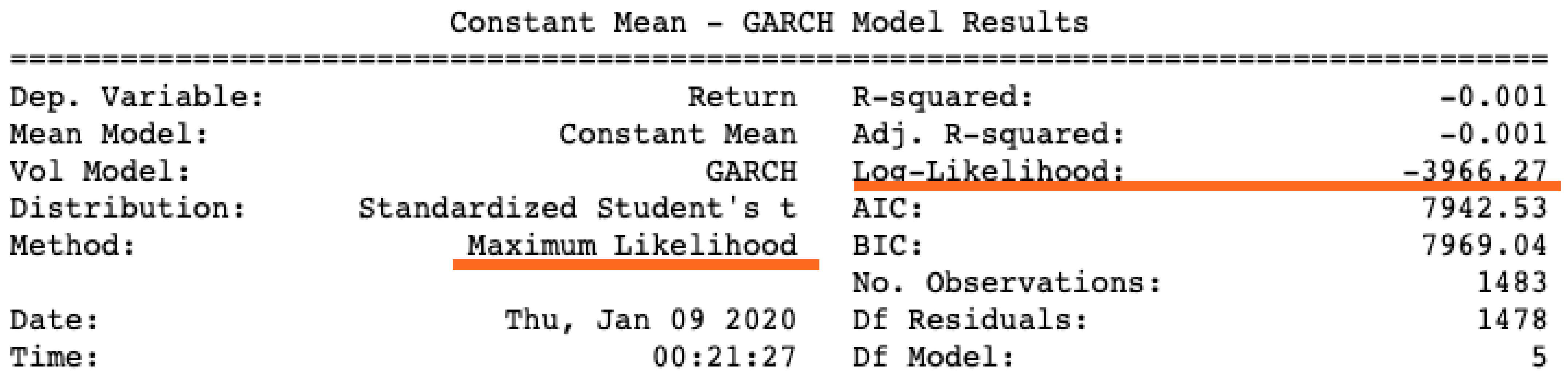
1. Maximum likelihood
2. Information criteria

Maximum likelihood

- Maximize the probability of getting the data observed under the assumed model
- Prefer models with larger likelihood values

Log-likelihood in Python

- Typically used in log form: log-likelihood



```
                     Constant Mean - GARCH Model Results
==========================================================================================
Dep. Variable:                        Return   R-squared:                          -0.001
Mean Model:                    Constant Mean   Adj. R-squared:                     -0.001
Vol Model:                             GARCH   Log-Likelihood:                   -3966.27
Distribution:     Standardized Student's t   AIC:                               7942.53
Method:                   Maximum Likelihood   BIC:                               7969.04
                                               No. Observations:                     1483
Date:                       Thu, Jan 09 2020   Df Residuals:                         1478
Time:                               00:21:27   Df Model:                                5
```

```
print(gm_result.loglikelihood)
```

Overfitting

- Fit in-sample data well, but perform poorly on out-out-sample predictions
- Usually due to the model is overly complex

Information criteria

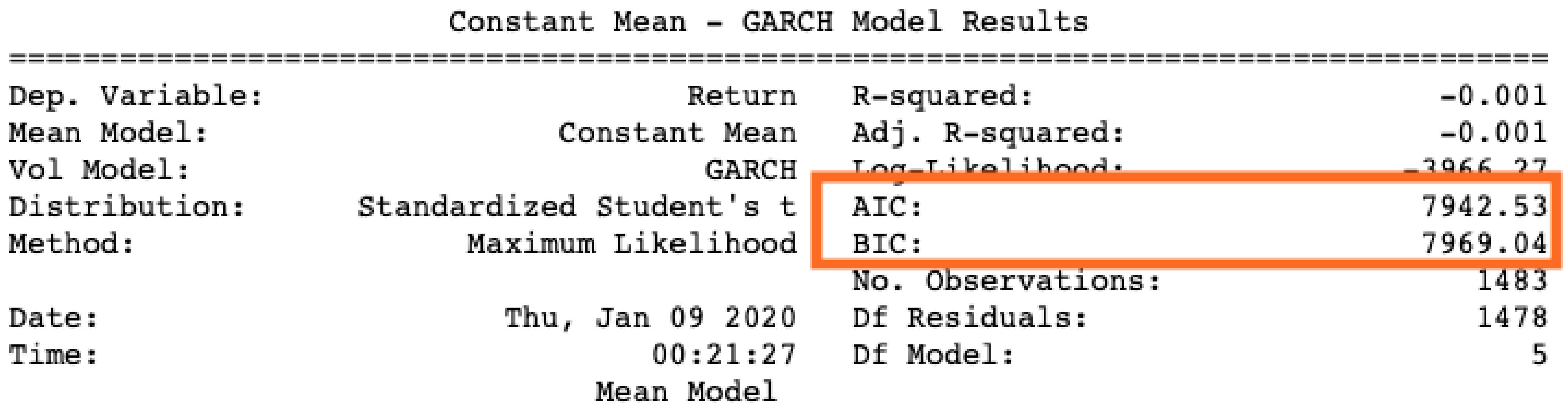
- Measure the trade-off between goodness of fit and model complexity
- Likelihood + penalty for model complexity
- AIC: Akaike's Information Criterion
- BIC: Bayesian Information Criterion

_Prefer models with the lower information criterion score _

AIC vs. BIC

- Generally they agree with each other
- BIC penalizes model complexity more severely

AIC/BIC in Python



```
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Dep. Variable:                     Return   R-squared:                  -0.001
Mean Model:                 Constant Mean   Adj. R-squared:             -0.001
Vol Model:                          GARCH   Log-Likelihood:           -3966.27
Distribution:      Standardized Student's t   AIC:                     7942.53
Method:                Maximum Likelihood   BIC:                       7969.04
                                            No. Observations:             1483
Date:                    Thu, Jan 09 2020   Df Residuals:                 1478
Time:                            00:21:27   Df Model:                        5
                              Mean Model
```

```python
print(gm_result.aic)
print(gm_result.bic)
```

Let's practice!

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GARCH model backtesting

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Backtesting

- An approach to evaluate model forecasting capability
- Compare the model predictions with the actual historical data

In-sample vs. out-of-sample

- In-sample: model fitting
- Out-of-sample: backtesting

MAE

Mean Absolute Error

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}_i|$$

MSE

Mean Squared Error

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

Calculate MAE, MSE in Python

```python
from sklearn.metrics import mean_absolute_error, mean_squared_error
```

```python
# Call function to calculate MAE
mae = mean_absolute_error(observation, forecast)
```

```python
# Call function to calculate MSE
mse = mean_squared_error(observation, forecast)
```

Let's practice!

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