Ec 172, PS 4 Rework

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1 Problem 1

Part (d): For those values of n for which no pure Nash equilibria exist, find a correlated equilibrium in which the expected utility to every player is $1 - 1/n$.

Suppose we randomly select one of the n players to be the "loser," and all other players to be winners. We then send a signal to the designated loser that they are to choose X. Then, ignoring the designated loser, there are an even number of players remaining, so the player to the immediate right of the designated loser is told to choose X and all players thereafter are told to alternate as in part (a), suggesting utilities of 1 for all but the loser.

In this variant of the game, a correlated equilibrium will be that in which all players obey the signal given to them with probability 1. Then the expected utility of any player i will be

$$E(u_i) = P(\text{winning}|\text{see } X)P(\text{seeing X}) + P(\text{winning}|\text{see } Y)P(\text{seeing } Y)$$

$$= (0)\left(\frac{1}{n}\right) + (1)\left(\frac{n-1}{n}\right) = 1 - \frac{1}{n}.$$

This result follows since every player obeys their signal with probability 1, so the probability that a player is a winner given that they are told to choose X will be 0, and the probability that a player is a winner given that they are told to choose Y is 1.

2 Problem 2

Consider the following situation.

There are two players, 1 and 2, and a coin is flipped for each of them. The result of the coin flip is displayed on a screen behind each player, so player 1 can see the result of player 2's flip and vice versa, but a given player cannot see the result of her own flip. If both players flip heads, then the screens will display the results in green; if player 1 gets heads and player 2 gets tails, the screens will display in red. Otherwise, they will be gray. Unfortunately for player 1, she happens to be red-green colorblind. Let A be the event that one or more of the flips between the players is tails.

Now suppose player 1 gets heads and player 2 gets tails, so A has occurred. Then player 1 can see that player 2 got tails, so player 1 knows A occurred. Also, player 1 knows that player 2 must know A occurred, since either both flips were tails or player 1 got heads and player 2 got tails; in the latter case, player 2 would see that the screen behind player 1 is red, so player 2 would know that she must have flipped tails.

Furthermore, player 2 knows that A occurred since player 2 will look behind player 1, see the heads, note that the screen is red, and deduce that she must have flipped tails. Since she now knows that she flipped tails, she also knows that player 1 knows that A has occurred.

However, it is not true that player 1 knows that player 2 knows that player 1 knows that A has occurred. To player 1, it is possible that she flipped either heads or tails. Then, if the result is tails for both, player 2 would not know whether she got heads or tails, even though she would know A occurred. Then to her, player 1 could be looking at either heads or tails; if player 1 is looking at heads, she will not know A occurred but otherwise she will. Thus player 1 does not know whether player 2 knows that player 1 knows A has occurred.

We will now formalize this situation by defining its governing knowledge space $(N, \Omega, \{T_i\}_{i \in \mathbb{N}}, \{t_i\}_{i \in \mathbb{N}})$:

1. $N = \{1, 2\}$ are players 1 and 2.
2. $\Omega = \{HH, HT, TH, TT\}$ are all combinations of coin flips for players 1 and 2, respectively.
3. $T_1 = \{H, T\}, T_2 = \{T, \text{green } H, \text{red } H\}$ are the "types" (the signals) players 1 and 2, respectively, can see.
4. $t_1(HH) = t_1(TH) = H, t_1(HT) = t_1(TT) = T; t_2(HH) = \text{green } H, t_2(TH) = t_2(TT) = T, t_2(HT) = \text{red } H$ are the signals players 1 and 2, respectively, receive in each state of the world.

Then, within this knowledge space, we also define $A = \{TH, TT, HT\}$ to be the states in which there is at least one tail flipped, and in our particular situation $\omega = \{HT\} \in \Omega$. Then $K_1 A = \{TT, HT\}, K_2 A = \{HT\}$, so $K_2 K_1 A = \{HT\}$, and thus $K_1 K_2 K_1 A = \{\emptyset\}$, as desired.