

# Ec 172, PS 1 Resubmission

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April 25, 2023

## 1 Problem 3

*Problem (a): Find a subgame perfect equilibrium of the described game.*

Denote by  $P(i > j)$  the probability that die  $i$  yields a number greater than die  $j$  when rolled. Thus, from elementary probability theory,

$$P(a > b) = \frac{5}{9}, P(b > a) = \frac{4}{9},$$

$$P(a > c) = \frac{4}{9}, P(c > a) = \frac{5}{9},$$

$$P(b > c) = \frac{5}{9}, P(c > b) = \frac{4}{9}.$$

Note, then, that for every die  $i$ , there exists a die  $j \in \{a, b, c\}$  such that  $P(j > i) = \frac{5}{9}$ . In particular, if the first player chooses  $a$ , the second may choose  $c$ ; if the first chooses  $b$ , the second may choose  $a$ ; and if the first chooses  $c$ , the second may choose  $b$ .

Regardless of the die the first player picks, the second may select one such that the utility of the second player is  $\frac{5}{9}$ , and that of the first player becomes  $\frac{4}{9}$ .

Thus the SPEs of this game may be described as follows: Player 1 selects any one of  $a, b$ , or  $c$ . If player 1 picks  $a$ , player 2 picks  $c$ ; if player 1 picks  $b$ , player 2 picks  $a$ ; and if player 1 picks  $c$ , player 2 picks  $b$ .